Multiplicity distributions in QCD jets and jet topics

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Based on: arXiv:2503.24200, arXiv:2509.06158

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第十六届 QCD 相变与相对论重离子物理研讨会

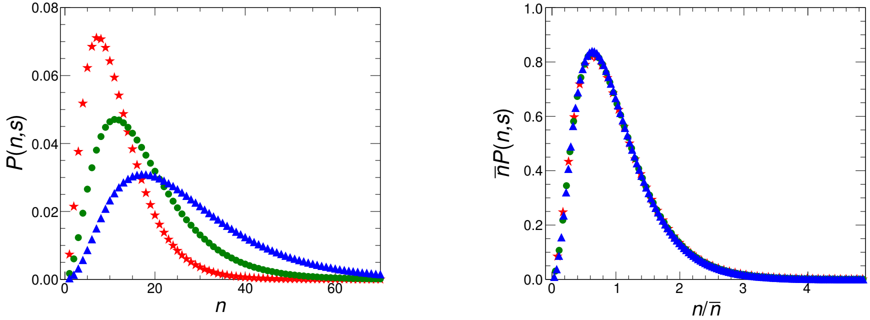
Koba-Nielsen-Olesen (KNO) scaling

- **KNO scaling** describes the universal behavior of multiplicity distributions:

$$\bar{n}P(n,s) = \Psi(n/\bar{n})$$

Koba, Nielsen, Olesen, NPB **40**, 317 (1972)

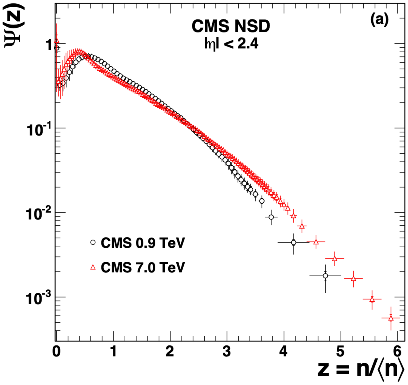
- $\boldsymbol{\bar{n}}$: mean multiplicity
- $\boldsymbol{P(n,s)}$: probability to produce n final-state particles in a collision at $\sqrt{s}$
- $\boldsymbol{\Psi(x)}$: universal function with $\int \mathrm{d}x\, \Psi(x) = 1$ and $x = n/\bar{n}$
- **KNO scaling was proposed before the discovery of QCD**



Violation of KNO scaling in *pp* collisions

- **KNO scaling support:**
  - e^+e^- collisions, Deep Inelastic Scattering (DIS)
- **KNO scaling violation:**
  - *pp* collisions

CMS, JHEP **01**, 079 (2011)



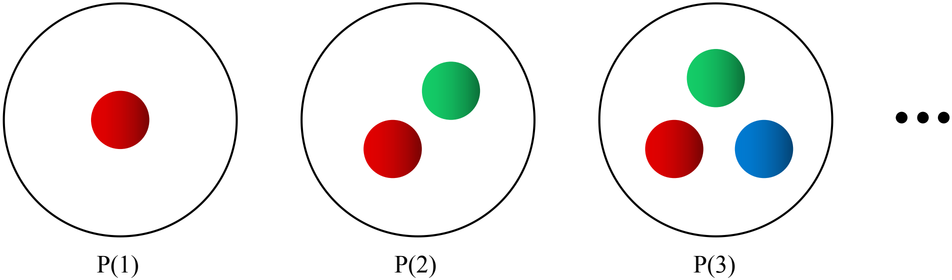
Multiplicity distributions in QCD jets

- Early studies of KNO scaling in jets within pQCD

  Konishi, Ukawa, and Veneziano, NPB **157**, 45 (1979)

  Bassetto, Ciafaloni, and Marchesini NPB **163**, 477 (1980)

- **Multiplicity distributions in QCD jets with p_T and R**
- Multiplicity distributions can be calculated by the generating functions (GFs)



Generating functions in QCD jets

Ellis, Stirling, and Webber, 2 (2011)

Dremin and Gary, Phys. Rept. **349**, 301 (2001)

- **GF** for a jet initiated by a parton a at scale Q is defined as

$$Z_a(u,Q) \equiv \sum_{n=0}^{\infty} u^n P_a(n,Q), \quad \text{with } Z_a(1,Q)=1$$

- The evolution of the GF with the initial jet scale Q satisfies

$$\frac{\partial}{\partial \ln Q} Z_a(u,Q) = \int dz \frac{\alpha_s}{\pi} \boldsymbol{\hat{P}_{a\to bc}(z)} \left[Z_b(u,zQ)Z_c(u,(1-z)Q) - Z_a(u,Q)\right]$$

- Here α_s is evaluated at the scale $z(1-z)\bar{Q}$, and **the splitting functions**

$$\hat{P}_{g\to gg}(z) = 2C_A\left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right]$$

$$\hat{P}_{g\to q\bar{q}}(z) = T_F[z^2 + (1-z)^2]$$

$$\hat{P}_{q\to gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right]$$

Asymptotic KNO scaling functions in QCD jets

Dokshitzer, Fadin, and Khoze, Z. Phys. C **18**, 37 (1983)

Dokshitzer, Khoze, Mueller, and Troian, (1991)

- **Asymptotic limit: $Q \to \infty$**
- **Asymptotic KNO scaling function** is defined as

$$\Psi_a(n/\bar{n}_a) \equiv \lim_{Q\to\infty} [\bar{n}_a P_a(n, Q)]$$

- Its **Laplace transform** can be expressed as

$$\boldsymbol{\Phi_a(\beta)} \equiv \lim_{Q\to\infty} Z_a(e^{-\frac{\beta}{\bar{n}_a}}, Q) = \lim_{Q\to\infty} \sum_{n=0}^{\infty} \frac{e^{-\beta\frac{n}{\bar{n}_a}}}{\bar{n}_a} [\bar{n}_a P_a(n, Q)]$$

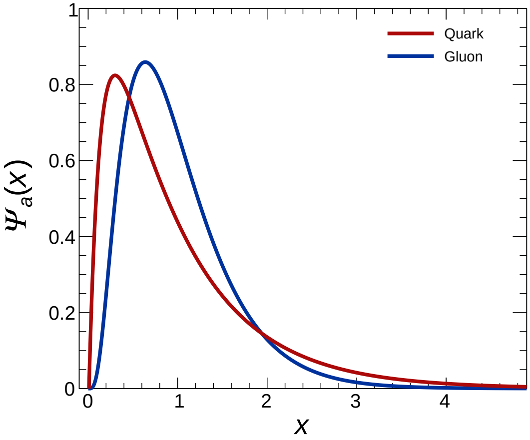
$$\equiv \int_{\mathbf{0}}^{\infty} \mathbf{dx}\ \boldsymbol{\Psi_a(x)}\mathbf{e}^{-\boldsymbol{\beta}\mathbf{x}} = \sum_{k=0}^{\infty} \frac{(-\beta)^k}{k!} f_a^{(k)}$$

- The asymptotic KNO scaling functions $\Psi_a(x)$ are given by the inverse Laplace transform of $\Phi_a(\beta)$

$$\Psi_a(n/\bar{n}_a) \equiv \lim_{Q\to\infty} [\bar{n}_a P_a(n, Q)] = \int \frac{d\beta}{2\pi i} \Phi_a(\beta) e^{\beta\frac{n}{\bar{n}_a}}$$

Asymptotic KNO scaling functions in QCD jets within DLA

- Asymptotic KNO scaling functions in **Double Log Approximation (DLA)**
- $a = q$ for quark jets and $a = g$ for gluon jets — Dokshitzer, Fadin and Khoze, Z. Phys. C 18, 37 (1983)
- There is a clear difference between quark and gluon jets — Bassetto, Nucl. Phys. B **303**, 703 (1988)



Evolution equation in QCD jets within DLA

- In DLA, the splitting function suffices:

$$\hat{P}_{a\to ga}(z) \approx 2C_a/z$$

with $C_a = C_g = C_A$ for gluon jets and $C_a = C_q = C_F$ for quark jets

- Evolution equation in DLA:

$$\frac{\partial}{\partial \ln Q} Z_a(u,Q) = Z_a(u,Q) c_a \int \frac{dz}{z} \boldsymbol{\gamma_0^2} \left[Z_g(u,zQ) - 1 \right]$$

with the anomalous dimension $\boldsymbol{\gamma_0} = \sqrt{\frac{2N_c \boldsymbol{\alpha_s}}{\pi}}$

- Here, $c_q \equiv C_F/N_c = 4/9$ for quark jets and $c_g \equiv C_A/N_c = 1$ for gluon jets
- In our calculations, the initial jet scale $\boldsymbol{Q = p_T R}$

Multiplicity probability distributions in QCD jets within DLA

- Restricting to DLA phase space in $zQ = k^0\theta > k_\perp > Q_0$, GF is given by:

$$Z_a(u,y) = u\exp\left\{c_a\int_0^y \mathrm{d}\bar{y}\,(y-\bar{y})\gamma_0^2[Z_g(u,\bar{y})-1]\right\}$$

where $y \equiv \ln(Q/Q_0)$ and $\bar{y} \equiv \ln(k_\perp/Q_0)$

Dokshitzer, Khoze, Mueller, and Troian, (1991)

- Multiplicity probability distributions:

$$P_a(n,Q) = \frac{1}{n!}\frac{\partial^n}{\partial u^n} Z_a(u,Q)\Big|_{u=0}$$

- Recursive form in DLA:

$$P_a(1,Q) = \exp\left\{-c_a\int_0^y \mathrm{d}\bar{y}\,(y-\bar{y})\gamma_0^2\right\},$$

$$P_a(n+1,Q) = \sum_{k=1}^{n}\frac{k}{n}P_a(n+1-k,Q)\times c_a\int_0^y \mathrm{d}\bar{y}\,(y-\bar{y})\gamma_0^2 P_g(k,\bar{y}),$$

which satisfies the normalization condition $\sum_{n=0}^{\infty} P_a(n,Q) = 1$

Mean multiplicity in QCD jets within DLA

- Mean multiplicity from the first derivative of the GF at $u = 1$:

$$\bar{n}_a(Q) = 1 + c_a \int_0^y \mathrm{d}\bar{y}\ (y - \bar{y})\gamma_0^2 \bar{n}_g(\bar{y})$$

- Relation between quark and gluon jets in DLA:

$$\bar{n}_q(Q) - 1 = c_q[\bar{n}_g(Q) - 1]$$

- Mean multiplicity for gluon jets (from the second-order ODE):

$$\bar{n}_g = \begin{cases} \cosh(\gamma_0 y) & \text{for fixed coupling} \\ z_1 \left\{ I_1[z_1]K_0[z_2] + K_1[z_1]I_0[z_2] \right\} & \text{for running coupling} \end{cases} \tag{1}$$

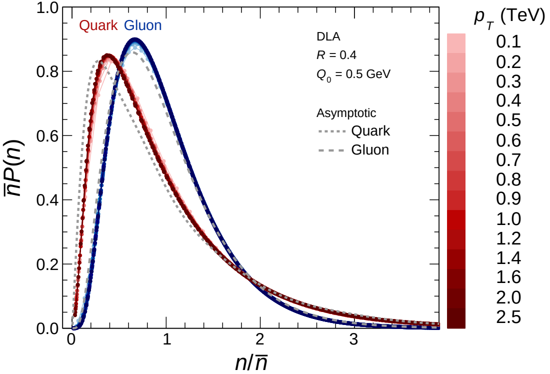
- From multiplicity probability distributions:

$$\bar{n}_a(Q) = \sum_{n=1}^{\infty} n P_a(n, Q) \tag{2}$$

Here, the agreement of these two approaches has been verified in our calculations

KNO scaling in QCD jets within DLA

- $n = 1000$, p_T range: $0.1 - 2.5$ TeV, $R = 0.4$, $Q_0 = 0.5$ GeV
- Calculations performed with running α_s
- **$\bar{n}P(n)$ for quark and gluon jets approach universal scaling functions**
- Convergence depends on Q_0: **$Q/Q_0 \gg 1$ required**
- Slight deviations from the asymptotic DLA



Multiplicity distributions in inclusive jets

- Multiplicity distributions in inclusive jets expressed as:

$$P(n) = r_q P_q(n) + r_g P_g(n)$$
$$\bar{n} = r_q \bar{n}_q + r_g \bar{n}_g$$
$$r_a \equiv \frac{\mathrm{d}\sigma_a/\mathrm{d}p_T}{\mathrm{d}\sigma_q/\mathrm{d}p_T + \mathrm{d}\sigma_g/\mathrm{d}p_T}$$

- Leading-order (LO) differential cross section in pQCD:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} = 2p_T \sum_{a,b,c,d} \int \mathrm{d}y_c \mathrm{d}y_d x_a f_{a/p}(x_a, \mu^2) x_b f_{b/p}(x_b, \mu^2) \frac{\mathrm{d}\hat{\sigma}_{ab\to cd}}{\mathrm{d}\hat{t}}$$

- KNO scaling for inclusive jets:

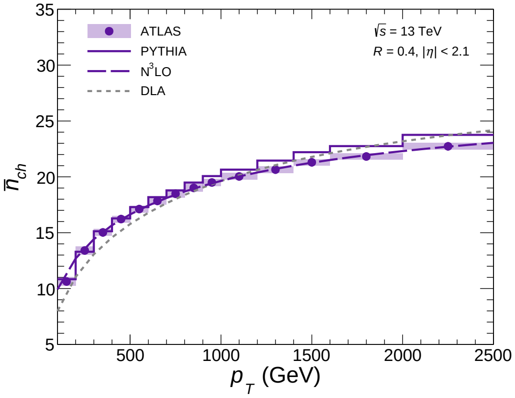
$$\bar{n}P(n) = \bar{n}[r_q P_q(n) + r_g P_g(n)] = r_q \frac{\bar{n}}{\bar{n}_q} \Psi_q(x\bar{n}/\bar{n}_q) + r_g \frac{\bar{n}}{\bar{n}_g} \Psi_g(x\bar{n}/\bar{n}_g)$$

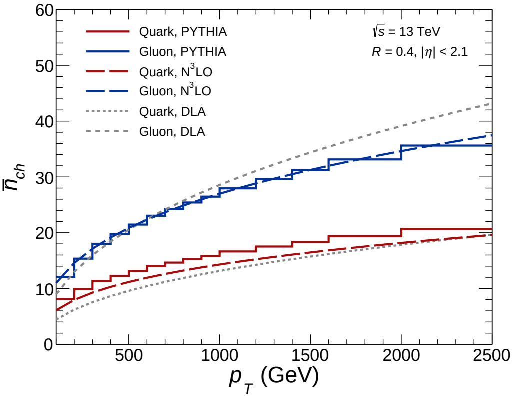
with the relation $n = x\bar{n} = x_q \bar{n}_q = x_g \bar{n}_g$

Mean charged-particle multiplicities: ATLAS vs. DLA vs. N^{3}LO

- **ATLAS:** two leading jets (anti-k_t, $R = 0.4$), $|\eta| < 2.1$, and $p_T^{\text{lead}}/p_T^{\text{sublead}} < 1.5$
- **DLA:** $\bar{n}_{\text{ch}} = K_{\text{LPHD}} \times \bar{n}_{\text{parton}}$, with $K_{\text{LPHD}} = 0.8$ (fit for running α_s)
- **N^{3}LO:** normalization constant $K = 0.0353$

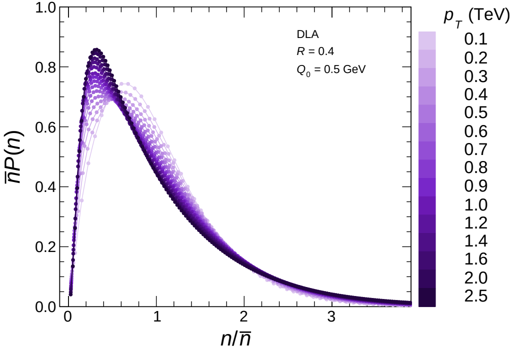
ATLAS, PRD **100**, 052011 (2019)
Dremin and Gary, PLB **459**, 341 (1999)
Capella *et al.*, PRD **61**, 074009 (2000)



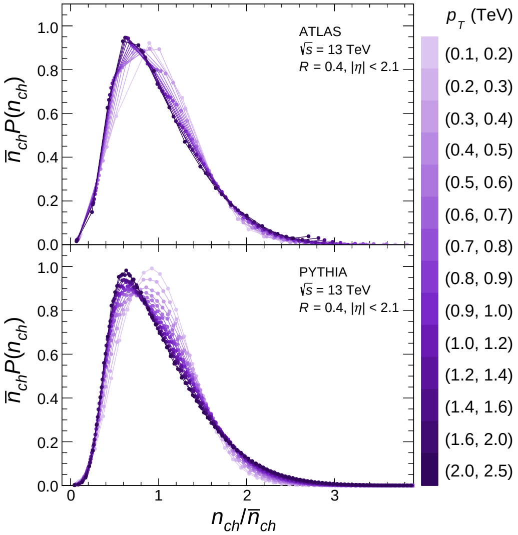


KNO functions in inclusive jets: ATLAS vs. PYTHIA vs. DLA

- **No universal KNO scaling in inclusive jets observed across a wide p_T range**
- **KNO shapes are driven by jet composition:**
  - Dominated by gluon jets at low p_T
  - Dominated by quark jets at high p_T



ATLAS, PRD **100**, 052011 (2019)



Evolution equations in QCD jets within MDLA

- **Modified DLA (MDLA) = DLA + energy conservation:** Dokshitzer, PLB **305**, 295 (1993)

$$\frac{\partial}{\partial \ln Q} Z_a(u,Q) = c_a \int \frac{dz}{z} \gamma_0^2 \left[Z_g(u,zQ) Z_a(u,(1-z)Q) - Z_a(u,Q) \right]$$

- By equaling the following two expansions

$$\Phi_a(\beta) \equiv \int_0^\infty \mathrm{d}x \ \Psi_a(x) e^{-\beta x} = \sum_{k=0}^{\infty} \frac{(-\beta)^k}{k!} \int_0^\infty \mathrm{d}x \ x^k \Psi_a(x) = \sum_{k=0}^{\infty} \frac{(-\beta)^k}{k!} f_a^{(k)}$$

$$\Phi_a(\beta) = \lim_{Q\to\infty} \sum_{k=0}^{\infty} \frac{(e^{-\frac{\beta}{\bar{n}_a(Q)}} - 1)^k}{k!} n_a^{(k)}(Q) = \sum_{k=0}^{\infty} \frac{(-\beta)^k}{k!} \lim_{Q\to\infty} \frac{n_a^{(k)}(Q)}{[\bar{n}_a(Q)]^k}$$

where the second expression corresponds to the expansion of Z_a at $u=1$:

$$Z_a(u,Q) = \sum_{k=0}^{\infty} \frac{(u-1)^k}{k!} n_a^{(k)}(Q)$$

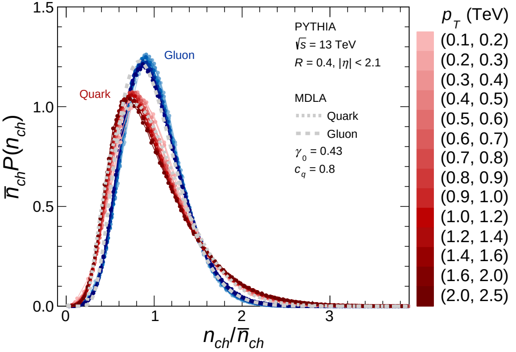
KNO functions in quark and gluon jets within MDLA

- The evolution equations at $u = 1$:

$$f_g^{(m)} = \frac{\gamma_0 m}{m^2 - 1} \sum_{k=1}^{m-1} \frac{m!}{k!(m-k)!} \frac{\Gamma(\gamma_0 k)\Gamma(\gamma_0(m-k)+1)}{\Gamma(\gamma_0 m + 1)} f_g^{(k)} f_g^{(q-k)}$$

$$f_q^{(m)} = \frac{c_q^{1-m}}{m^2} f_g^{(m)} + \gamma_0 \sum_{k=1}^{m-1} \frac{(m-1)!}{k!(m-k)!} \frac{\Gamma(\gamma_0 k)\Gamma(\gamma_0(m-k)+1)}{\Gamma(\gamma_0 m + 1)} c_q^{1-k} f_g^{(k)} f_q^{(m-k)}$$

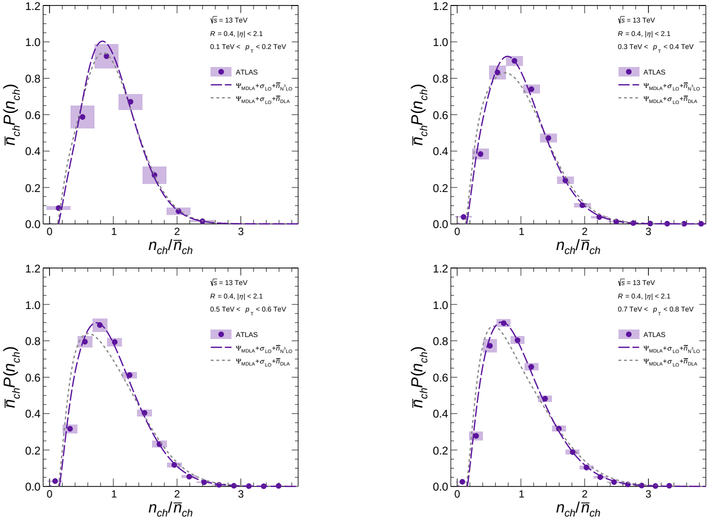
  with $f_a^{(0)} = f_a^{(1)} = 1$ for $m \geq 2$
- **PYTHIA** samples are generated with A14 tune
- Quark-gluon discrimination based on proximity to the initiating parton in a $2 \to 2$ hard scattering
- **MDLA** uses two parameters:
  - $\gamma_0 = 0.43$
  - $c_q = 0.8$



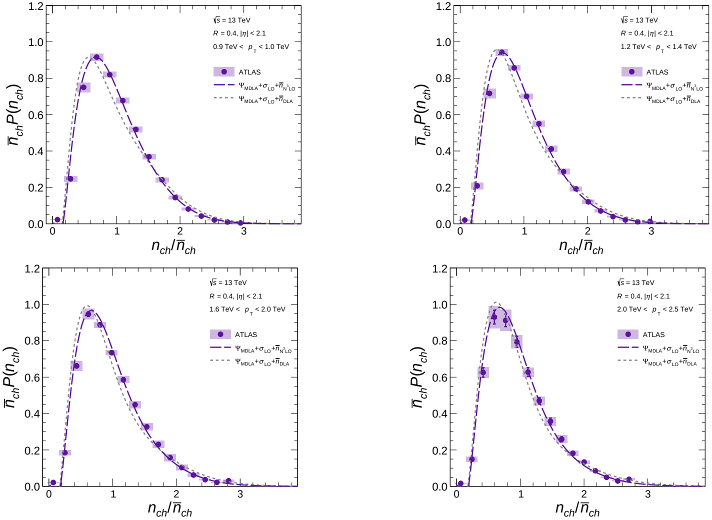
KNO functions in inclusive jets: ATLAS vs. MDLA

ATLAS, PRD **100**, 052011 (2019)

- **KNO functions from MDLA show excellent agreement with ATLAS data**



KNO functions in inclusive jets: ATLAS vs. MDLA



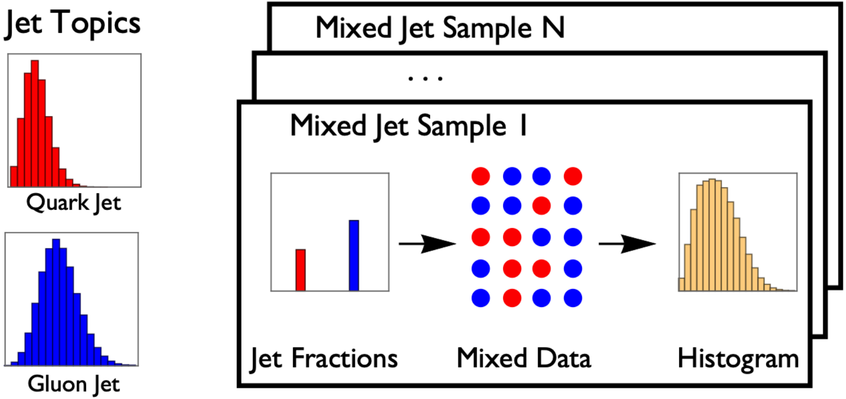
$\Psi_{\rm MDLA}$: the MDLA KNO functions, $\sigma_{\rm LO}$: the LO cross section, and $\bar{n}_{\rm DLA/N^3LO}$: the DLA/N^{3}LO mean multiplicities

Jet Topics

- **Topic modeling:** an unsupervised machine learning approach
- **Applied to separate quark and gluon jets**

Metodiev and Thaler, PRL **120**, 241602 (2018)

Komiske, Metodiev, and Thaler, JHEP **11**, 059 (2018)



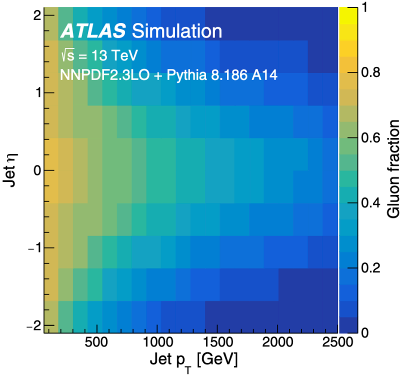
Jet Topics in ATLAS

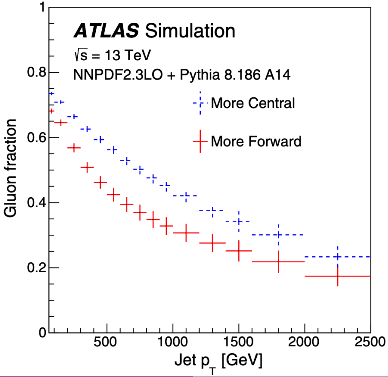
- More forward (f) and more central (c) jet samples are used

ATLAS, PRD **100**, 052011 (2019)

- Multiplicity distributions extracted for **Topic 1 (≈ quark)** and **Topic 2 (≈ gluon)**:

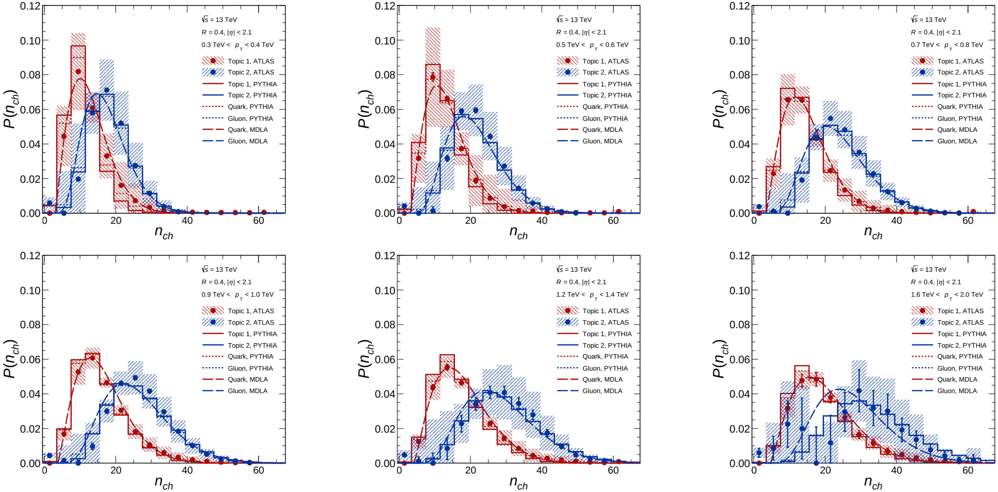
$$h_i^{T_1} = \frac{h_i^f - \min_j\{h_j^f/h_j^c\}\, h_i^c}{1 - \min_j\{h_j^f/h_j^c\}}, \qquad h_i^{T_2} = \frac{h_i^c - \min_j\{h_j^c/h_j^f\}\, h_i^f}{1 - \min_j\{h_j^c/h_j^f\}}$$





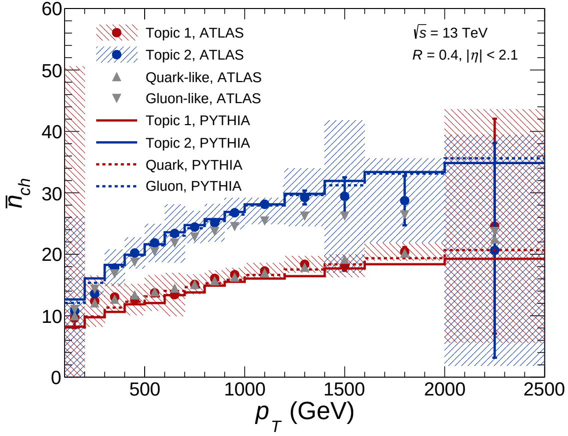
Jet Topics for $P(n_{ch})$: ATLAS vs. PYTHIA vs. MDLA

- More precise data selection: $\sigma_{r_{a/b}}/r_{a/b} < 25\%$ (ATLAS), $h_j^{f,c} > \sqrt{N}/N$ (PYTHIA)
- Our **ATLAS** results consistent with **PYTHIA** and **MDLA**



Jet Topics for $\bar{n}_{ch}$: ATLAS vs. PYTHIA

- **ATLAS** results agree with **PYTHIA** predictions within uncertainties across the full p_T range
- Consistent with the alternative method using nominal quark/gluon fractions from **PYTHIA** provided by ATLAS collaboration



Summary

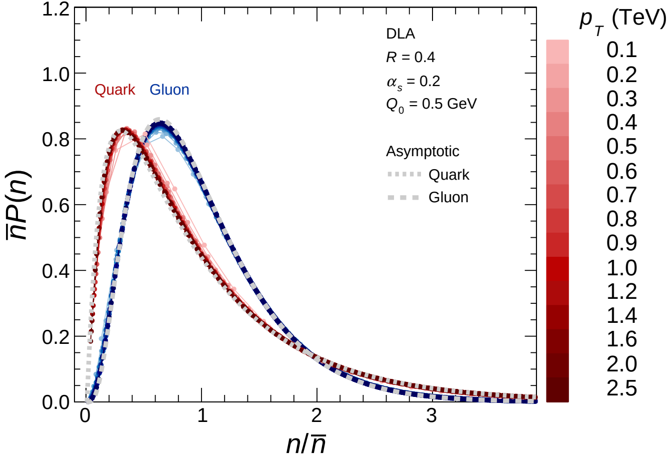
- **Within the DLA framework**
  - Studied multiplicity probability distributions and KNO scaling in QCD jets
  - Fitted mean charged-particle multiplicities to ATLAS data
- **Within the MDLA framework**
  - Extended KNO scaling analysis beyond DLA with energy conservation
  - MDLA KNO functions show excellent agreement with ATLAS data over $p_T = 0.1 - 2.5$ TeV
- **Jet Topics**
  - Separated quark and gluon jets using more forward and more central jet samples
  - MDLA and PYTHIA results are consistent with jet topics extracted from ATLAS

Thank you!

Backup

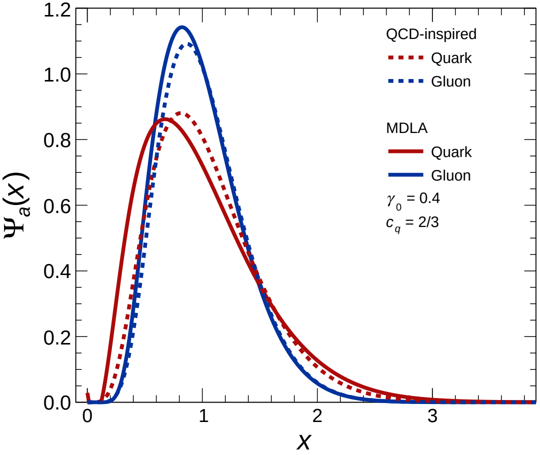
Test KNO scaling in QCD jets within DLA

- Fixed α_s



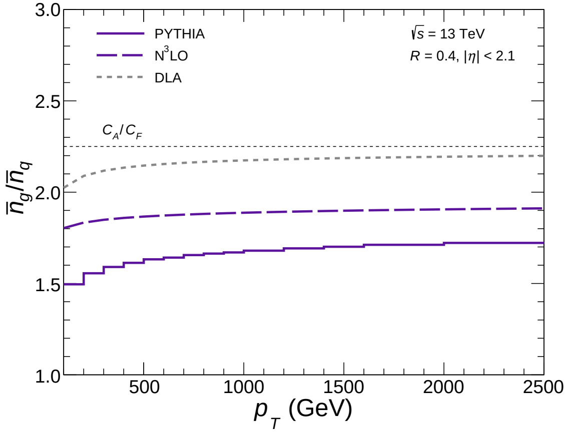
Comparison with QCD-inspired forms provided by Dokshitzer and Webber

- Derived from the relation: $\Phi_q(\beta) = [\Phi_g(\beta/c_q)]^{c_q}$:



Ratio of mean multiplicities: PYTHIA vs. DLA vs. N^{3}LO

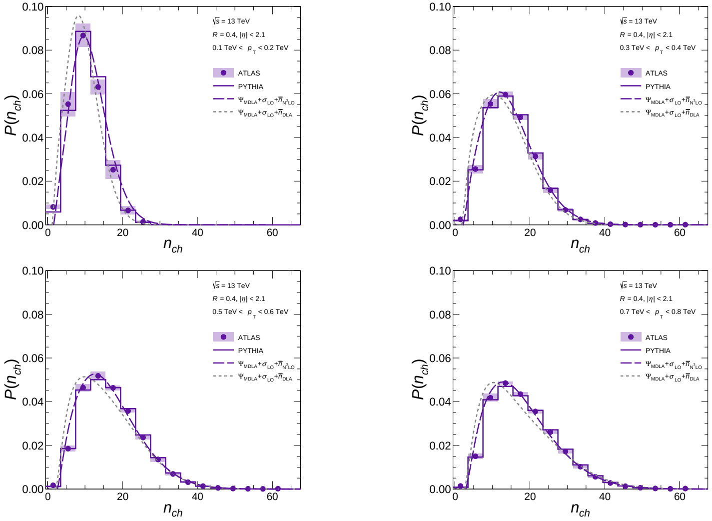
- Ratio of mean multiplicities between gluon and quark jets



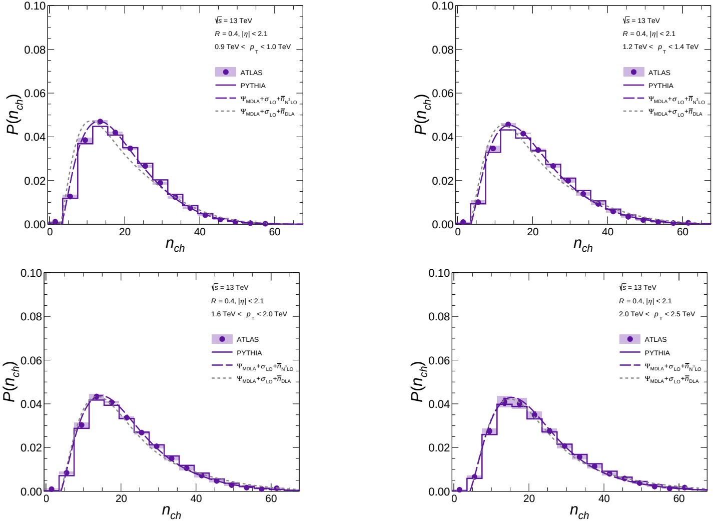
Multiplicity in inclusive jets: ATLAS vs. PYTHIA vs. MDLA

ATLAS, PRD **100**, 052011 (2019)

- **Multiplicity distributions in inclusive jets from MDLA show good agreement with data**



Multiplicity in inclusive jets: ATLAS vs. PYTHIA vs. MDLA



Ψ_{MDLA}: the MDLA KNO functions, σ_{LO}: the LO cross section, and $\bar{n}_{\mathrm{DLA/N^3LO}}$: the DLA/N^{3}LO mean multiplicities