Differentiating Energy-Energy Correlators with Charged Particle Multiplicities within a Jet

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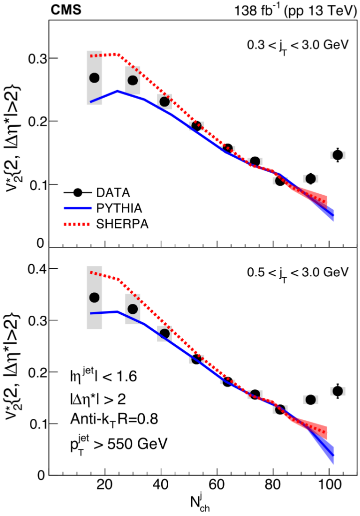
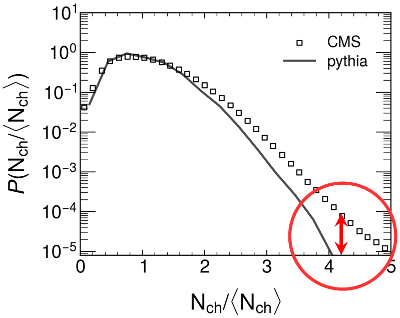
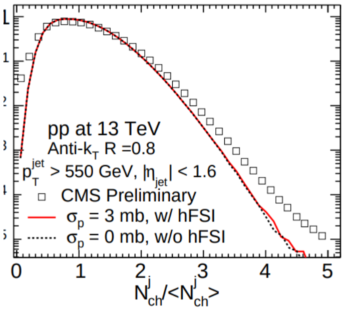
Based on:arXiv:2510.04895 and a paper in preparation.

Why study jet substructure conditioned on multiplicity?





Out of the box Pythia8.



CP5+FS interaction 2401.13137

- Pythia8 underestimates the fraction of very high multiplicity jets by order of magnitude.
- Pythia8 & Sherpa cannot explain sudden enhancement of v_2 at large N_{ch}.

The tool: generating function

- The generating function (or partition function) for a multiplicity probability distribution $P(n) \equiv P(n; E, R)$

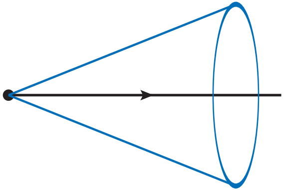
$$Z(s) = \sum_{n=0}^{\infty} P(n) e^{-ns} \Longleftrightarrow P(n) = \frac{1}{2\pi i} \int_{s_0 - i\pi}^{s_0 + i\pi} Z(s) e^{ns} \mathrm{d}s$$

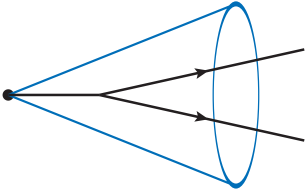
- Nice properties to unpack convolution

$$P_a(n) = \sum_{m=0}^{n} P_b(m) P_c(n-m) \Longleftrightarrow Z_a(s) = Z_b(s) Z_c(s)$$

Multiplicity distribution of exclusive jet

Radiations inside the jet and assume independent fragmentations of daughter partons:





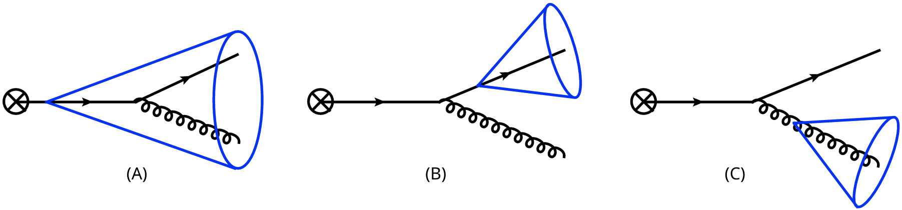
LO given by the $Z_i(s)$ of a single parton.

NLO given by the product of $Z_j(s)Z_k(s)$ of each branch.

$$Z_g^{(1)}(s, \omega_J, \mu) \supset \frac{\alpha_s(\mu)}{2\pi^2} \int_0^1 dx \int \frac{d^{2-2\epsilon} k_\perp}{(2\pi)^{-2\epsilon} k_\perp^2} \Theta_{\mathrm{alg}}^{\mathrm{jet}} P_{gg}(x, \epsilon) Z_g^{(0)}(s, x\omega_J, \mu) Z_g^{(0)}(s, (1-x)\omega_J, \mu)$$

- $\int \frac{\mathrm{d}^2 k_\perp}{k_\perp^2}$ gives the collinear logarithm to be summed.
- Multiplicity is soft sensitive, implement **angular ordering** for soft radiations.

From exclusive jet to semi-inclusive jet



(A)

(B)

(C)

- Out-of-cone radiations cause a **change of the energy** $\omega_J = z\omega$ or **flavor** of the jet.
- Jet multiplicity function factorizes: $M_i^j(z,s) = J_{ji}(z) \times Z_j(s)$.
- Semi-inclusive jet function with flavor identification

$$\frac{\partial}{\partial \ln \mu^2} \begin{bmatrix} J_{jq}(z,\omega_J,\mu) \\ J_{jg}(z,\omega_J,\mu) \end{bmatrix} = \frac{\alpha_s(\mu^2)}{2\pi} \begin{bmatrix} P_{qq}(z) & P_{gq}(z) \\ 2n_f P_{qg}(z) & P_{gg}(z) \end{bmatrix} \otimes_z \begin{bmatrix} J_{jq}(z,\omega_J,\mu) \\ J_{jg}(z,\omega_J,\mu) \end{bmatrix}$$

The evolution equations for internal multiplicity (Leading Log)

- Perform the **non-linear equations** in the angular ordered form $\zeta = 1 - \cos\theta$:

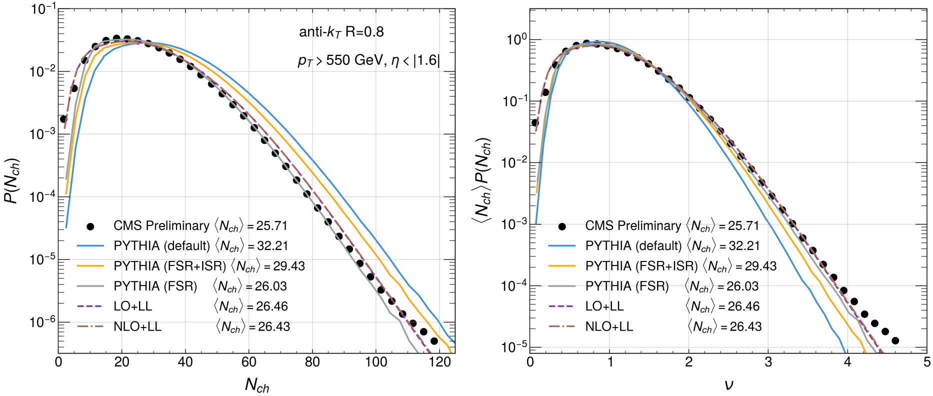
$$\frac{\partial Z_i(s, \omega_J, \zeta)}{\partial \ln \zeta} = \int_0^1 \mathrm{d}x \frac{\alpha_s(k_\perp^2)\Theta(k_\perp^2 - Q_0^2)}{2\pi} \frac{1}{2} \sum_{j,k} \left\{ P_{ji}(x) Z_j(x) Z_k(1-x) + P_{ki}(x) Z_k(x) Z_j(1-x) \right\}$$

  Q_0 severs as a infrared cutoff to regularize the calculation. $Z_j(x) = Z_j(s, x\omega_J, \zeta)$.

- Factorized formula to get final **cross section level information**:

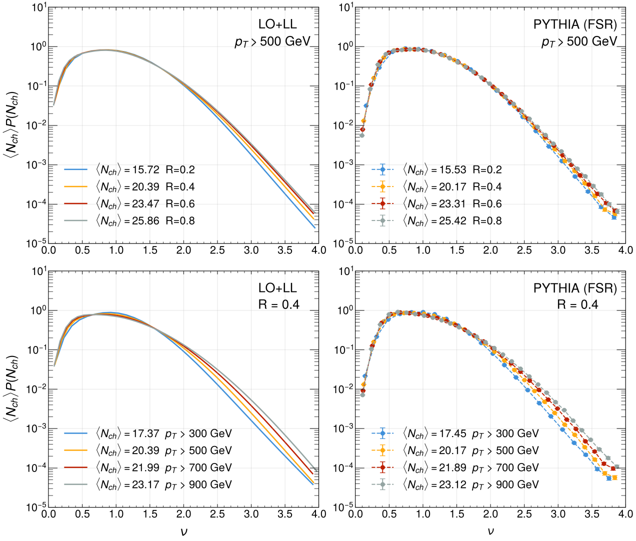
$$\frac{\mathrm{d}\sigma_{pp\to J(n)+X}}{\mathrm{d}p_{T,J}\mathrm{d}\phi_J\mathrm{d}\eta_J} = \sum_{ij} \mathrm{d}\sigma_{pp\to i}(p_{T,J}/z, \mu_H) \otimes_z J_{ji}(z, \mu_H, \mu_J) \times P_j(n, \zeta_R, \zeta_0)$$

Comparison to CMS data $\nu = N/\langle N \rangle$



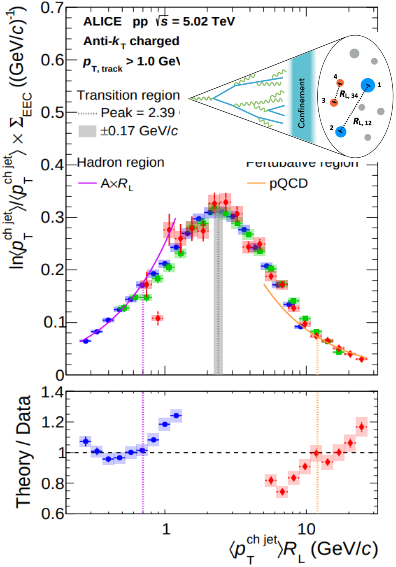
- Both our calculation and PYTHIA8 (FSR) simulations reproduce the overall shape of the multiplicity distribution well.
- However, they systematically underestimate the very high-multiplicity region ($\nu \gtrsim 4$) (Lack of higher-order corrections? Such as intrinsic $1 \to 3$?).

KNO scaling and violation



- KNO scaling holds for $\nu \lesssim 2$, with mild violations at larger ν.
- Deviations arise from nonlinear branching and the mixture of quark- and gluon-initiated jets.

Energy-energy correlator as a starting example



- v_2 in jet is very complicated to calculated.
- Start with something simpler: energy correlators.
- Statistical correlation of the asymptotic energy flux:

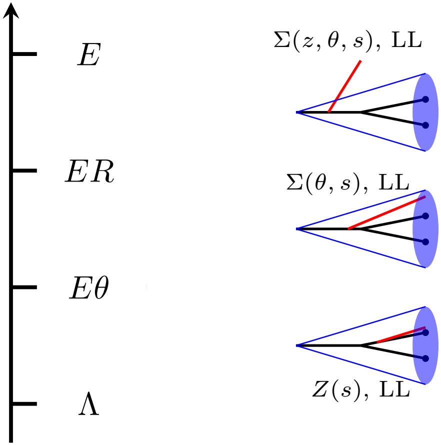
$$\mathrm{ENC} = \langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\cdots\mathcal{E}(\vec{n}_N)\rangle$$

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_{-\infty}^{\infty} dt n^i\, T_{0i}(t, r\vec{n}),$$

Or define using particle-level info:

$$\frac{\mathrm{d}\Sigma(\theta, E_{\mathrm{jet}})}{\mathrm{d}\theta} = \frac{1}{N_{\mathrm{event}}} \sum_{\mathrm{event}} \sum_{i,j\in\mathrm{jet}} \frac{E_i E_j}{E_{\mathrm{jet}}^2}\delta(\theta - \theta_{ij})$$

Hierarchy of scales with multiplcity conditioning



One can map the jet multiplicity–conditioned EEC contributions as follows:

- Emissions below θ do not modify EEC at leading power, just changes multiplicity.
- Emissions from θ to R modifies EEC and multiplicity.
- Emissions larger than R, causes semi-inclusive jet function evolution.

The evolution equations for EEC conditioned on multiplicity

- $\Sigma(s,\theta)$ as the joint multiplicity EEC distribution. The initial condition are:

$$\frac{\mathrm{d}\Sigma_i^{\text{ini}}(s,\theta,\omega_J)}{\mathrm{d}\theta} = \frac{1}{\theta}\frac{\alpha_s}{\pi}\int \mathrm{d}x \frac{1}{2}\sum_{j,k}[x(1-x)]\Big\{P_{ji}(x)Z_j(x)Z_k(1-x) + P_{ki}(x)Z_k(x)Z_j(1-x)\Big\}$$

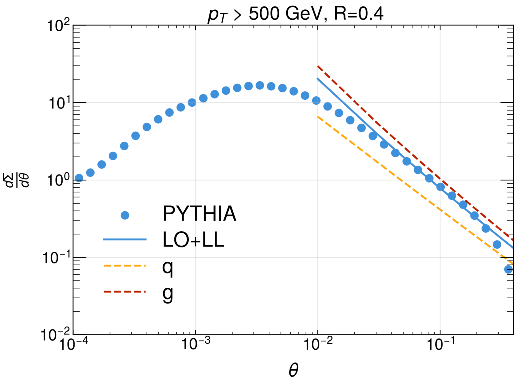
- The anomalous dimension thus depend on $Z(s)$:

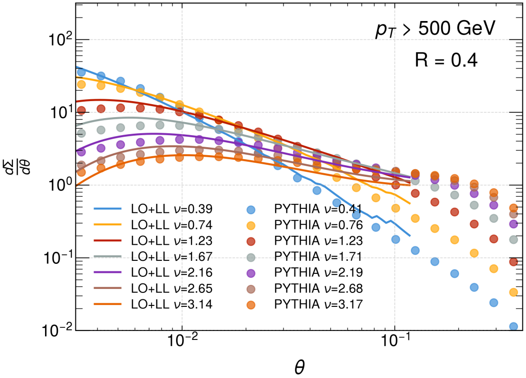
$$\frac{\partial\left(\frac{\mathrm{d}\Sigma_i(s,\theta,\omega_J,\zeta)}{\mathrm{d}\theta}\right)}{\partial \ln\zeta} = \int \mathrm{d}x \frac{\alpha_s(k_\perp^2)\Theta(k_\perp^2 - Q_0^2)}{2\pi}\frac{1}{2}\sum_{j,k}\left[P_{ji}(x)\frac{\mathrm{d}\Sigma_j(x)}{\mathrm{d}\theta}Z_k(1-x) + P_{ki}(x)\frac{\mathrm{d}\Sigma_k(1-x)}{\mathrm{d}\theta}Z_j(x)\right]$$

$Z_j(x) = Z_j(s, x\omega_J, \zeta)$ and $\frac{d\Sigma_j(x)}{\mathrm{d}\theta} = \frac{d\Sigma_j(s,\theta,x\omega_J,\zeta)}{\mathrm{d}\theta}$

- $Z(s)$ evolves independently, since EEC measurement does not affect multiplicity.
- Jet joint multiplicity EEC function factorizes: $\Sigma_i^j(z,s,\theta) = J_{ji}(z)\times\Sigma_j(s,\theta)$.

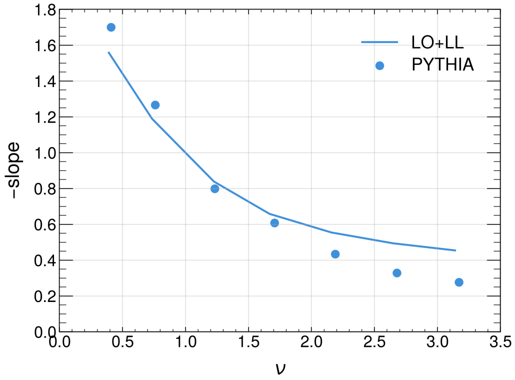
EEC conditioned on normalized multiplicity $\nu = N/\langle N \rangle$

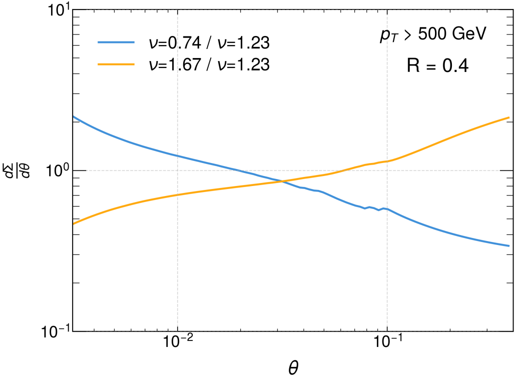




- The observed increase of the EEC anomalous dimension agrees with PYTHIA and remains RGE-governed even under multiplicity selection.
- Conditioning on multiplicity biases the amount of radiation, thereby altering the EEC anomalous dimension.

Why study EEC conditioned on multiplicity?





- Studying the ν-dependence of the EEC anomalous dimension can also provide information on the jet multiplicity distribution itself.
- EEC patterns vary with multiplicity; the ratio of EEC between different multiplicities may resemble medium-like modifications, offering a way to compare *pA* vs *pp* while accounting for multiplicity effects.

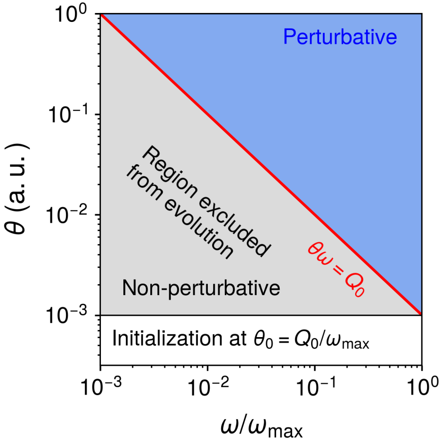
Summary

- Tuned at one CMS data, the model captures $\mathrm{PYTHIA8}$ (FSR) trends in jet multiplicity across p_T and R, yet underestimates the high-ν tail ($\nu > 4$).
- Conditioning jet substructure on multiplicity reveals how radiation evolves with event activity.
- The EEC anomalous dimension changes with multiplicity, indicating mutual influence between substructure and multiplicity (v_2 in jet next?).
- Similar approach can be extend to small system to study soft-hard correlations (pA next?).

Thanks for your listening!

Backup

A non-perturbative model for initial data of $Z(s, \omega)$



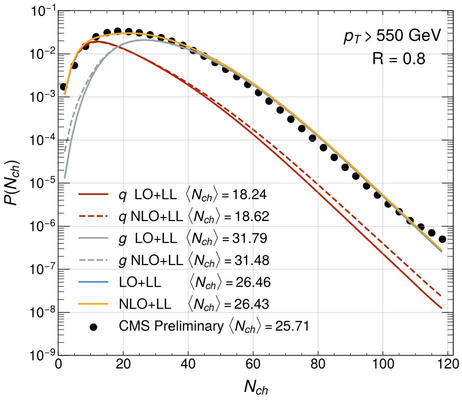
- Parameterize the average particle multiplicity:

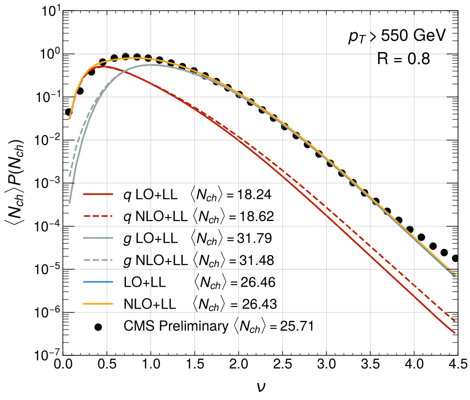
$$\langle n\rangle(\omega, Q_0) = \frac{n_0}{1 + cQ_0^2/\omega^2}$$

- It approaches n_0 at high energy ($\omega \gg Q_0$), while providing a smooth suppression near the production threshold ($\omega \sim Q_0$).
- Assume binomial (or Poisson) distribution, then the initial condition model is:

$$Z^{\mathrm{IC}}(s, \omega, Q_0) = \begin{cases} \exp\left[\langle n\rangle(\omega, Q_0)\left(e^{-s} - 1\right)\right] & \text{Poisson}\,, \\ \left[1 + \left(e^{-s} - 1\right)\frac{\langle n\rangle(\omega, Q_0)}{n_{\max}}\right]^{n_{\max}} & \text{Binomial}\,. \end{cases}$$

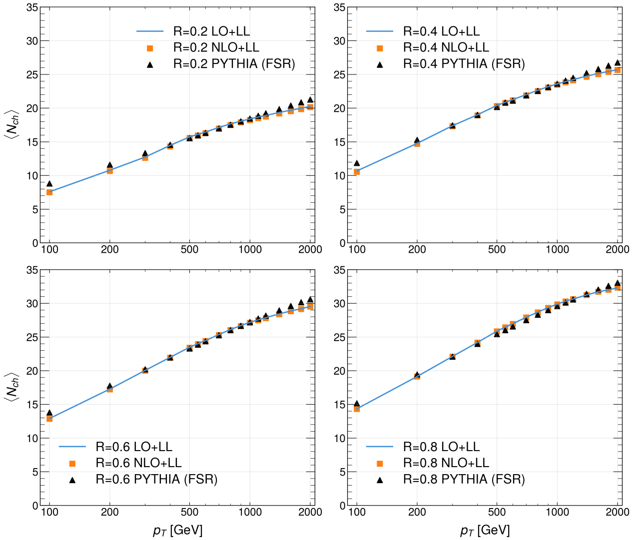
Decomposition into quark and gluon jets





- Quark-initiated jets dominate the low-multiplicity region.
- Gluon-initiated jets dominate the high-multiplicity tail.

$\langle N_{\mathrm{ch}} \rangle$ dependence on jet p_T and R



- $\langle N_{\mathrm{ch}} \rangle$ grows with $p_T R$, and quantitatively agrees with PYTHIA8 simulations.
- The same set of nonperturbative initial condition is used for all cases.

Dependence on jet p_T and R