Orbital dynamics in magnetovortical matter

Kenji Fukushima, KH, Kazuya Mameda, PRL 135 (2025) [2409.18652 [hep-ph]]
"Preponderant Orbital Polarization in Relativistic Magnetovortical Matter"

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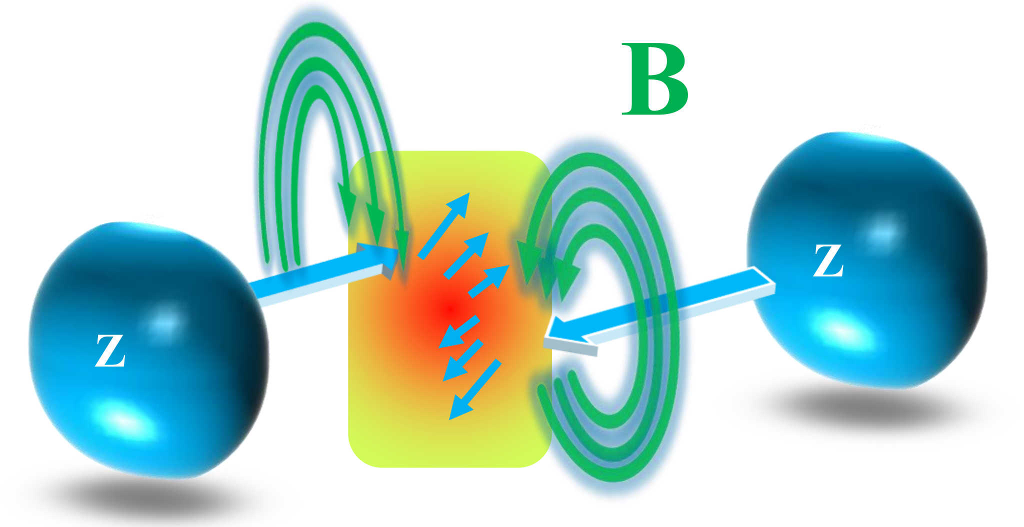
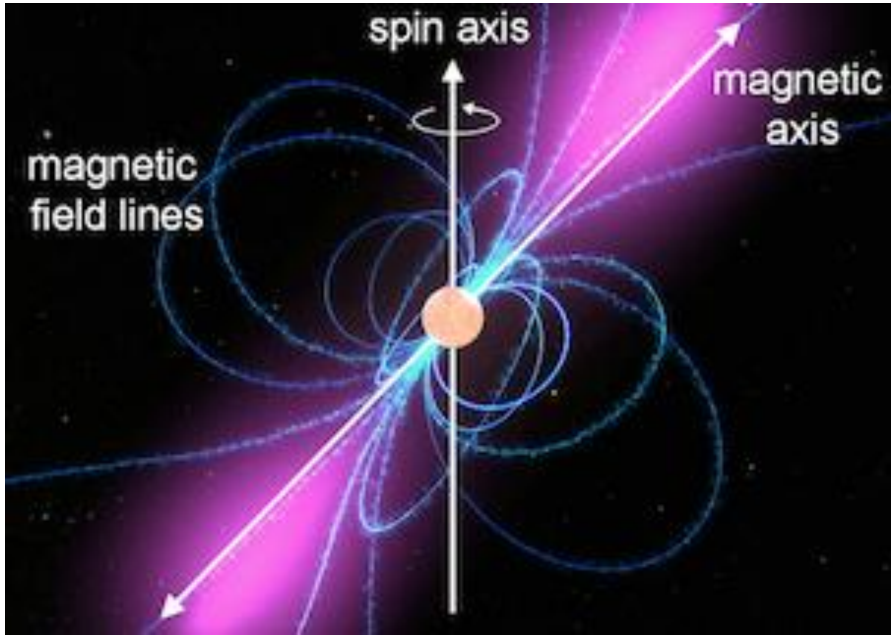
Magneto-vortical matter



$$\omega = \nabla \times v$$



Anomalous quantum transport phenomena

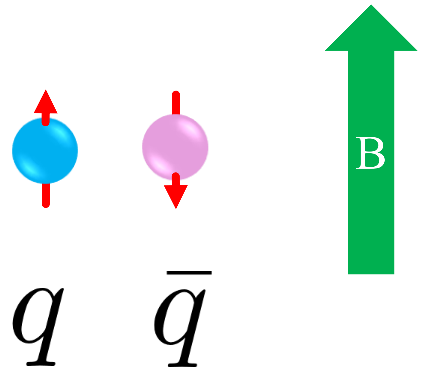
Dirac equation in the chiral representation
→ Chirality-spin-momentum locked in the massless limit

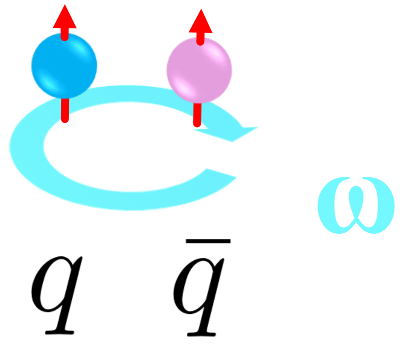
{Chirality, Spin, Momentum}

$$\begin{pmatrix} -m & p_\mu \sigma^\mu \\ p_\mu \bar{\sigma}^\mu & -m \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

Spin polarization in B or ω

Chirality-momentum locking
→ Finite currents when one of chirality is favored.





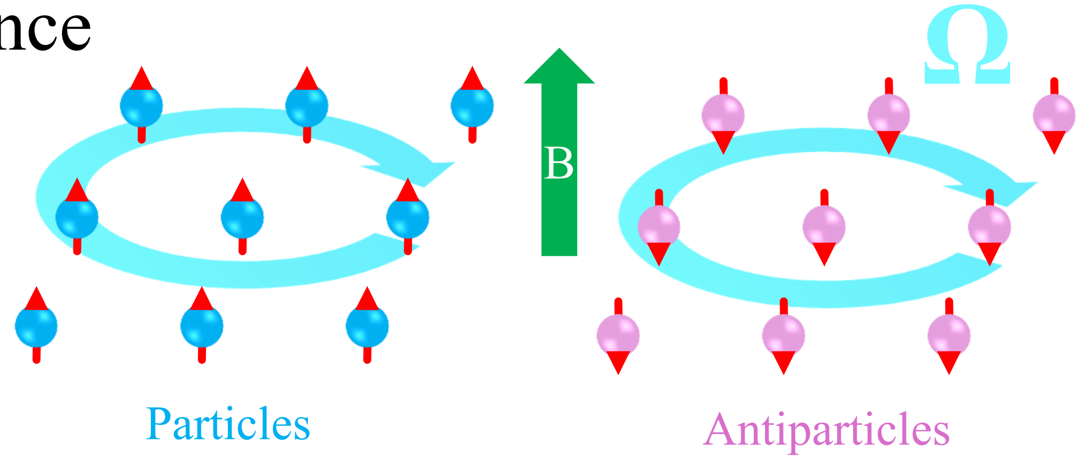
$$\boldsymbol{j}_{\mathrm{CME}} \propto \mu_A \boldsymbol{B} \qquad \boldsymbol{j}_{\mathrm{CVE}} \propto \mu \mu_A \boldsymbol{\omega}$$

$$\mu_A = (\mu_R - \mu_L)/2, \quad \mu = (\mu_R + \mu_L)/2$$

Anomalous transport in magneto-vortical matter w/o chirality imbalance

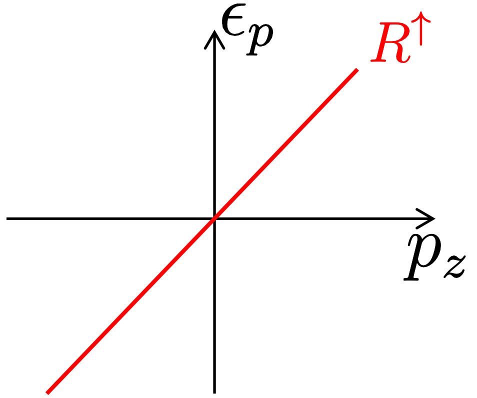
$$n_V = \frac{C_A}{2}\boldsymbol{B}\cdot\boldsymbol{\Omega}$$

KH, Yin, PRL [1607.01513 [hep-th]]



Charge-dependent energy shift: $\Delta E = \boldsymbol{S}\cdot\boldsymbol{\Omega} = \left(\pm\frac{1}{2}\hat{\boldsymbol{B}}\right)\cdot\boldsymbol{\Omega}$

→ Effective chemical potential $\quad f_R^{\pm} = \dfrac{1}{e^{\pm\beta(p_z - \boldsymbol{S}\cdot\boldsymbol{\Omega} - \mu)} + 1}$



$$n_R = \frac{|q_f B|}{2\pi}\left[\int_0^{\infty}\frac{dp_z}{2\pi}f_R^{+}(p_z) + \int_{-\infty}^{0}\frac{dp_z}{2\pi}f_R^{-}(p_z)\right] = \frac{C_A}{4}\boldsymbol{B}\cdot\boldsymbol{\Omega}$$

The integral is independent of T and μ.

Kubo-formula calculation is tied to the chiral anomaly in the Schwinger model KH, Yin

- Effective theory as the Chern-Simons current
Yamamoto and Yang, 2103.13208

- Chiral kinetic theory
Yang, et al., 2003.04517; Lin, Yang, 2103.11577; Mameda, 2305.02134; Yang, et al., 2409.00456

Another approach with explicit solution for the Dirac eq.

Ebihara, Fukushima, Mameda, PLB [1608.00336 [hep-ph]]

1. Solving the Dirac equation under B and global rotation Ω (in a cylinder) by the use of the symmetric gauge.
2. Computing the charge density with the explicit solution.

$$F = -\frac{1}{\pi R^2} \sum_{q=\pm} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \sum_{n=0}^{\infty} \alpha_n \sum_{\ell=-n}^{N-n} \left\{ \frac{\varepsilon + q\Omega j + q\mu}{2} + T \ln\left[1 + e^{-(\varepsilon + q\Omega j + q\mu)/T}\right] \right\},$$

where $\alpha_n = 2 - \delta_{n,0}$, $j = \ell + 1/2$, $N = eBR^2/2$, and $\varepsilon = \sqrt{p_z^2 + 2neB}$.

$$n_{\text{total}} = \frac{\Omega}{\pi^2 R^2} \sum_{\ell=0}^{N} (\ell + 1/2) = \frac{eB\Omega}{4\pi^2}(N+1)\,.$$

ℓ: “Canonical” angular momentum

There is an orbital angular momentum contribution in addition to spin!

However, this orbital angular momentum is a gauge-dependent quantity!

Private communication among us.

Analytic and quantum mechanics in B

KH, Itakura, Ozaki,
Review article in PPNP [2305.03865 [hep-ph]]

$$\hat{H} = \frac{\hat{\boldsymbol{\pi}}_\perp \cdot \hat{\boldsymbol{\pi}}_\perp}{2m} + \frac{\hat{p}_z^2}{2m}$$

$$\hat{\boldsymbol{\pi}}_\perp = \hat{\boldsymbol{p}}_\perp - e\boldsymbol{A}_\perp(\hat{\boldsymbol{x}}_\perp)$$

$\boldsymbol{A}_\perp$ is for a constant B_z.

Canonical momentum (p) $\quad p = \frac{\partial \mathcal{L}}{\partial \dot{x}} \quad \longrightarrow \quad [\hat{x}^i,\, \hat{p}^j] = i\hbar\delta^{ij}$

A(x) breaks translation generated by **p**.
(Whether or not **p** is conserved depends on gauge.)

Kinetic momentum (π) $\quad \dot{\boldsymbol{x}} = (i\hbar)^{-1}[\boldsymbol{x},\, H] = \frac{\pi}{m}$

Heisenberg eq.

Pseudo-momentum (k) $\quad 0 = \frac{d}{dt}(m\dot{\boldsymbol{x}} - e\boldsymbol{x} \times \boldsymbol{B}) =: \frac{d\boldsymbol{k}}{dt}$

k is a generator of translation in a constant B.
(Called magnetic translation, Conserved in any gauge)

Orbital AM in B

KH, Itakura, Ozaki,
Review article in PPNP [2305.03865 [hep-ph]]

To define orbital angular momentum (AM), we need to specify

1. one of the three linear momenta (canonical, kinetic, pseudo)
2. a reference coordinate point

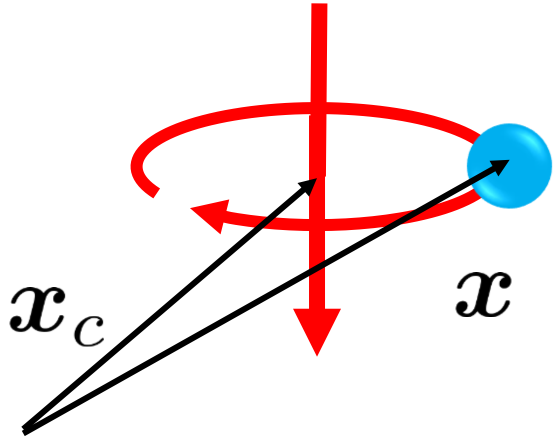
Relevant AM in this talk:
Kinetic AM around the guiding center (x_c)

$$\boldsymbol{\Lambda} := (\boldsymbol{x} - \boldsymbol{x}_c) \times \boldsymbol{\pi}$$

Nonzero expectation value in the Landau level (n) $\langle n|\hat{\Lambda}_z|n\rangle = -\mathrm{sgn}(eB)\hbar(2n+1)$

	Kinetic momentum $\hat{\boldsymbol{\pi}}$	Canonical momentum $\hat{\boldsymbol{p}}$	Pseudo-momentum $\hat{\boldsymbol{k}}$	Kinetic AM $(\hat{\boldsymbol{x}} - \hat{\boldsymbol{x}}_c) \times \hat{\boldsymbol{\pi}}$	Canonical AM $\hat{\boldsymbol{x}} \times \hat{\boldsymbol{p}}$	Pseudo-AM $\hat{\boldsymbol{x}}_c \times \hat{\boldsymbol{k}}$
Gauge invariance	✔	✘	✔	✔	✘	✔
Conservation	✘	Up to gauges	✔	✔	Up to gauges	✔

Fig. 2. Properties of the three linear momenta and the associated angular momenta (AM).



Starting over from the very beginning to resolve the puzzle

Fukushima, KH, Mameda, PRL [2409.18652 [hep-ph]]

1. Thermodynamic stability: Thermodynamic partition function should be constructed for static thermal states. → Corrigendum to Ebihara, Fukushima, Mameda (2016)

2. Identifying the static state automatically leads to a gauge-invariant formulation with the kinetic orbital AM.

3. As a consequence, the orbital contribution overwhelms the spin contribution found earlier. → Corrigendum to KH and Yin (2016)

$$n_V = C_A\left(\frac{1}{2} - 1\right)\boldsymbol{B}\cdot\boldsymbol{\Omega} = -\frac{C_A}{2}\boldsymbol{B}\cdot\boldsymbol{\Omega}$$

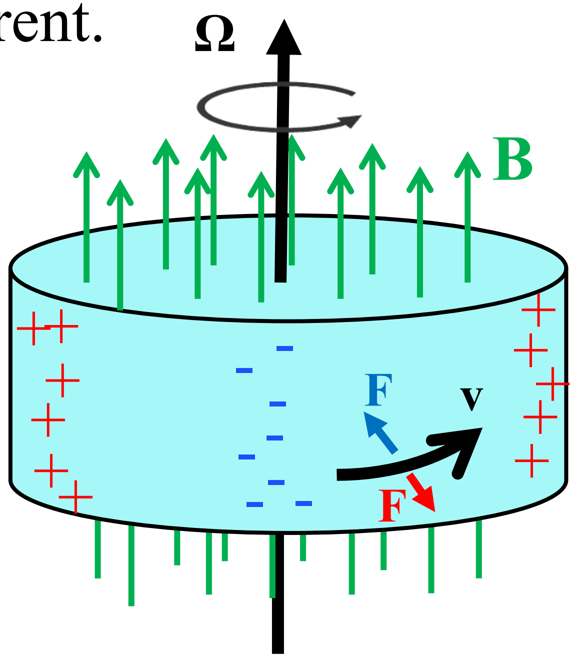
Spin - Orbital

Thermodynamic stability in B and Ω

Condition: No electric current in static states.

However, when a system starts rotating in B, there is a radial Hall current.

"Rotational Hall effect" (Unipolar induction)



$$v_\theta = r\Omega$$

Cf. Goldreich-Julian, ApJ (1969)

$$F_{\text{Lorentz}}(r) = \pm|e|v_\theta B$$

The Lorentz force driven by rotation depends on electric charges.

Charge separation occurs in a steady state so that an induced E offsets the Lorentz force (and thus the Hall current) in the lab frame; I.e., there is E in the lab frame.
⇔ No E in the comoving frame.

$$0 = \gamma^{-1}\boldsymbol{E}_{\text{com}} = \boldsymbol{E}_{\text{lab}} + (\boldsymbol{x} \times \boldsymbol{\Omega}) \times \boldsymbol{B}_{\text{lab}}$$

Gauge-invariant partition function for the static thermal state

$$Z = \det\left[\gamma^0\partial_\tau - i\gamma^i D_i + M - \gamma^0\Omega(\underline{L_{\rm kin}} + \frac{i}{2}\gamma^1\gamma^2)\right]$$

Kinetic orbital AM   Spin

This work: No E in the comoving frame

$$A^\mu_{\rm com} = (0, -\frac{B}{2}y, \frac{B}{2}x, 0)$$

Previous works by Chen, Fukushima, Huang, Mameda; Ebihara, Fukushima, Mameda:

E $\neq 0$ in the comoving frame ⇔ E = 0 in the lab frame

$$A^\mu_{\rm com} = (-\frac{B}{2}\Omega r^2, -\frac{B}{2}y, \frac{B}{2}x, 0)$$

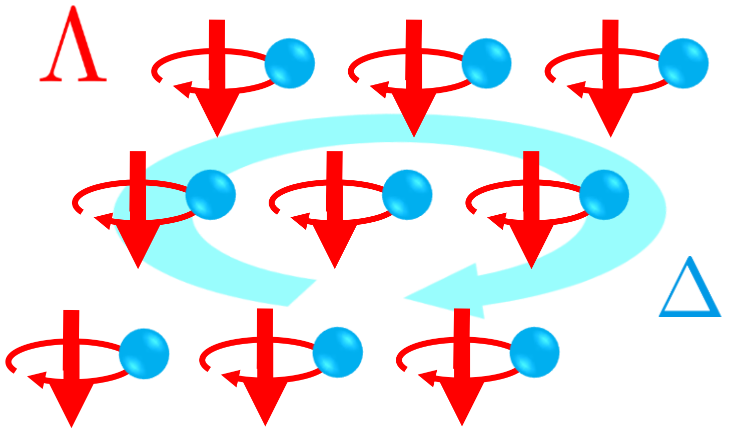
$$L_{\rm kin} = x\Pi_y - y\Pi_x = \Lambda + \Delta$$

Local AM around a guiding center of each cyclotron orbit.

$$\Lambda = (x - x_c)\Pi_y - (y - y_c)\Pi_x$$

Global AM of the guiding center around the rotation axis.

$$\Delta = x_c\Pi_y - y_c\Pi_x$$



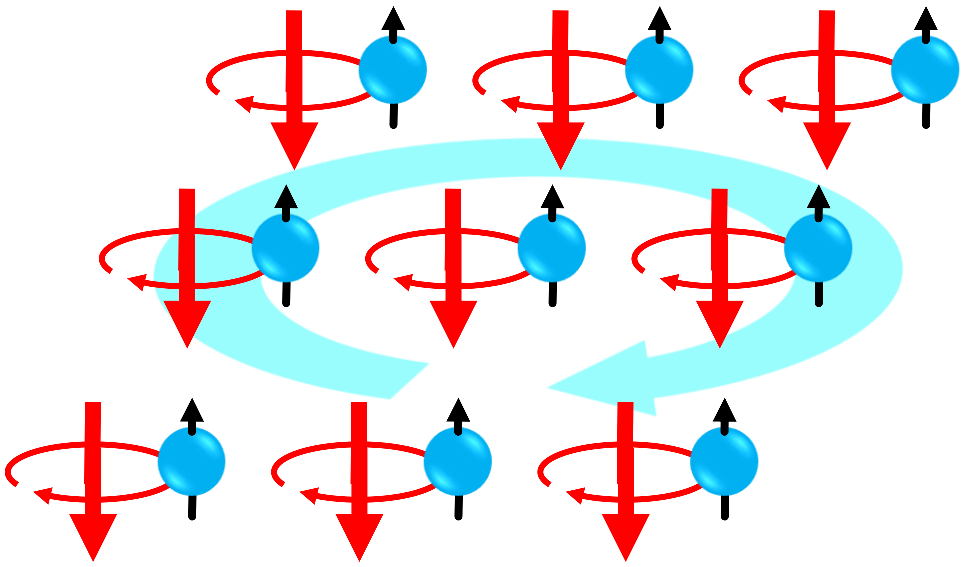
Sign inversion by the orbital contribution in the strong-field regime

Fukushima, KH, Mameda 2409.18652

$$n_V = C_A\left(\frac{1}{2} - 1\right)\boldsymbol{\omega}\cdot\boldsymbol{B} = -\frac{C_A}{2}\boldsymbol{\omega}\cdot\boldsymbol{B}$$

Spin - Orbital

Each cyclotron orbit has a kinetic angular momentum
that is additive to spin but with an opposite sign (Lenz's law, Landau diamagnetism).

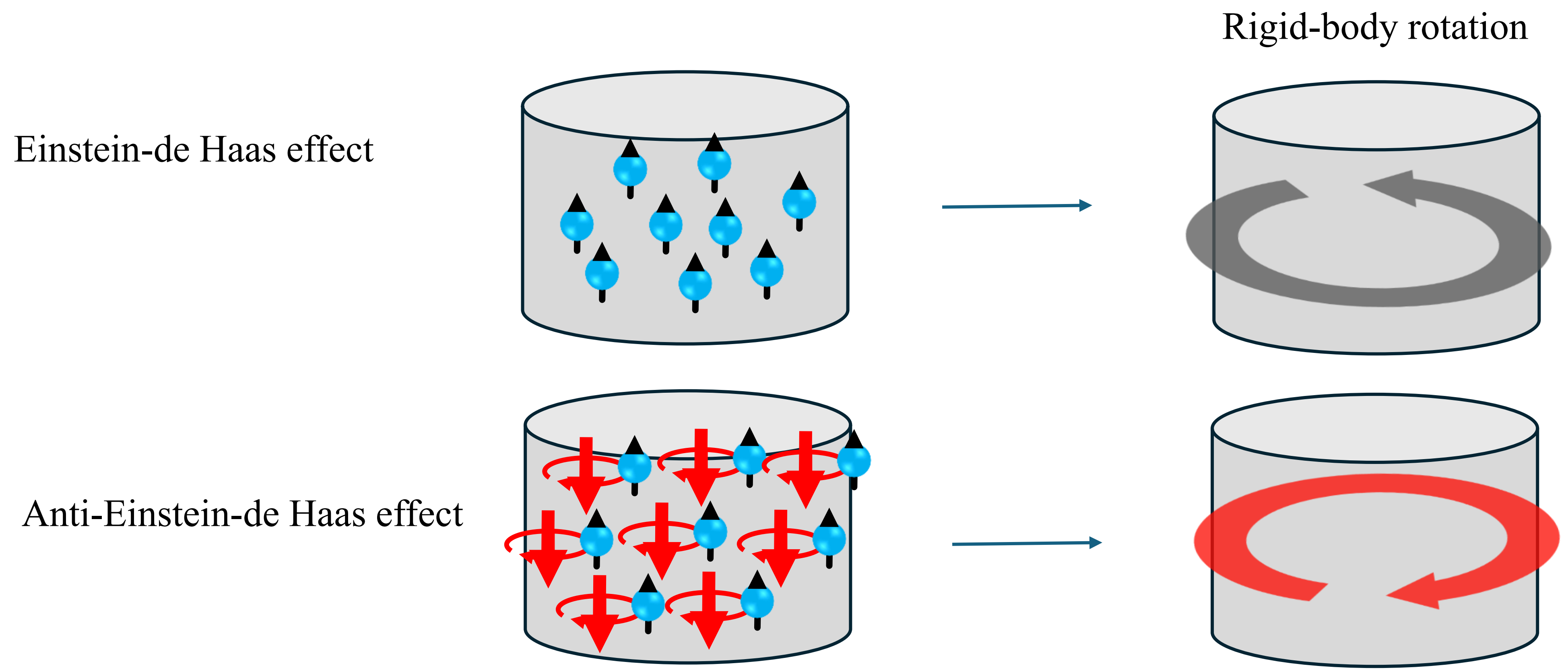


$$\langle n|\Lambda_{\mathrm{kin}}|n\rangle = -\mathrm{sgn}(eB)(2n+1)$$

Cf. KH, Itakura, Ozaki, 2305.03865

Anti-Einstein-de Haas effect
--- Storage of negative angular momentum

$$J := \left.\frac{\partial P}{\partial \Omega}\right|_{\Omega=0} = \mu \frac{eB}{2\pi^2}\left(\frac{1}{2} - 1\right)$$



Summary

- We examined thermodynamic stability and a gauge-invariant formulation.
- Magnetic field B induces orbital AM (associated with cyclotron orbits) as well as spin polarization.
- Their coupling to rotation Ω induces spectral shift.

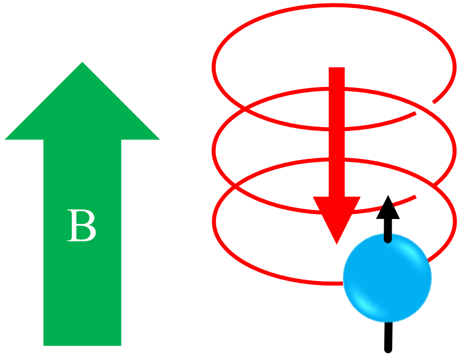
$$\text{Charge-dependent energy shift: } \Delta E = \boldsymbol{S}\cdot\boldsymbol{\Omega} = \pm\left(\tfrac{1}{2} - 1\right)\hat{\boldsymbol{B}}\cdot\boldsymbol{\Omega}$$

(Spin) – (Kinetic orbital AM)

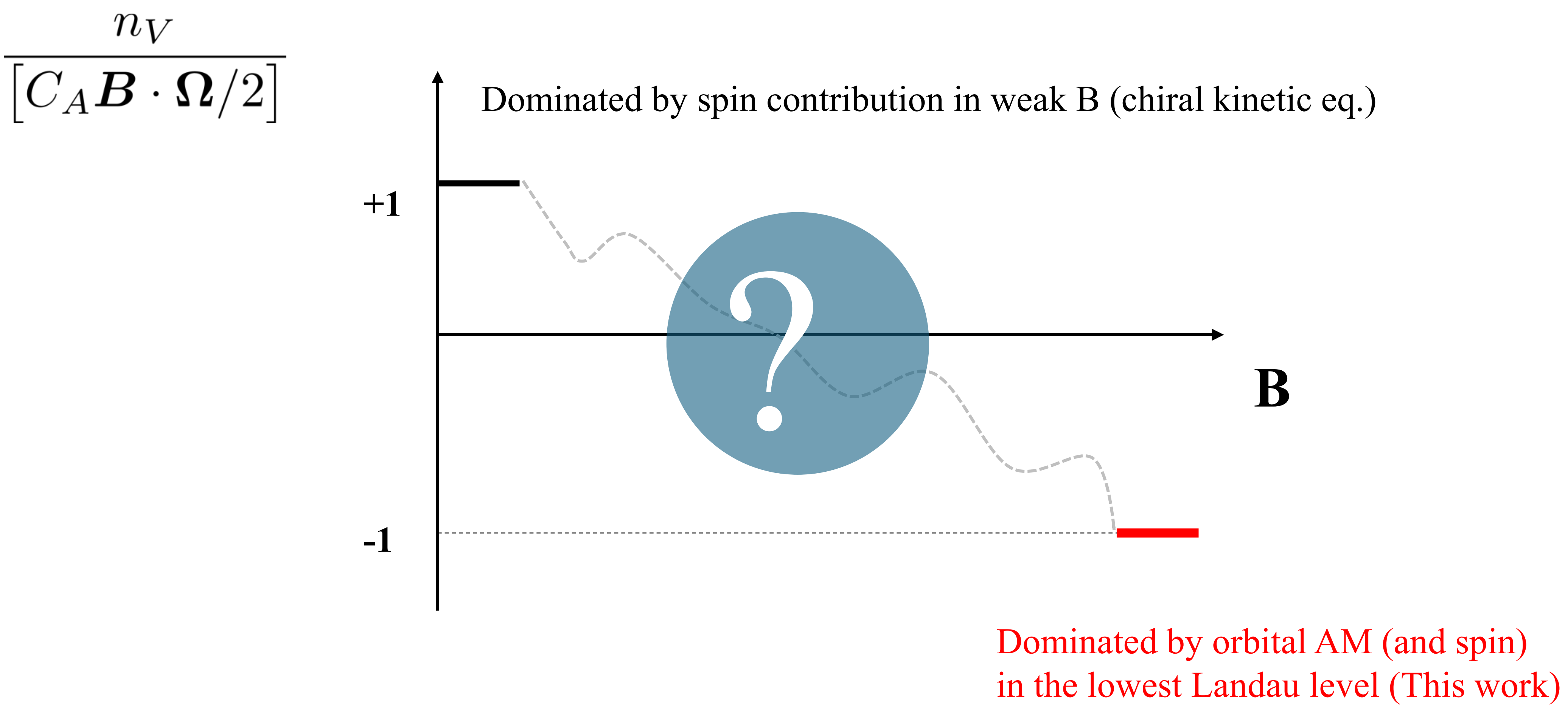
- The new orbital contribution inverts the overall sign.

$$n_V = C_A\left(\frac{1}{2} - 1\right)\boldsymbol{\omega}\cdot\boldsymbol{B} = -\frac{C_A}{2}\boldsymbol{\omega}\cdot\boldsymbol{B}$$

Kinetic orbital AM



Work in progress: Transition from the strong to weak fields

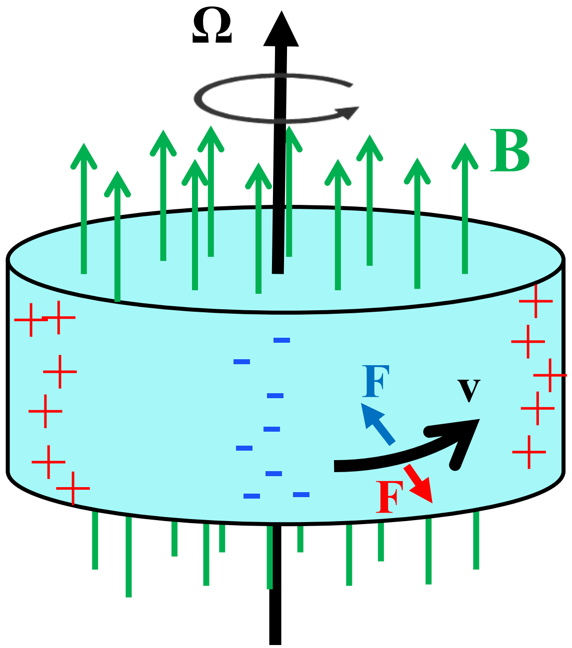


Azimuthal current

When the charge density $n_V = \kappa \boldsymbol{B}_{\rm lab} \cdot \boldsymbol{\Omega}$ is rotating with $v_\theta = r\Omega$,

$$j_\theta = n_V v_\theta = \kappa(\boldsymbol{B}_{\rm lab} \cdot \boldsymbol{\Omega})\ (\boldsymbol{x} \times \boldsymbol{\Omega})_\theta$$

$$= \kappa(\boldsymbol{E}_{\rm lab} \times \boldsymbol{\Omega})_\theta$$



Steady-state condition

$$0 = \gamma^{-1}\boldsymbol{E}_{\rm com} = \boldsymbol{E}_{\rm lab} + (\boldsymbol{x} \times \boldsymbol{\Omega}) \times \boldsymbol{B}_{\rm lab}$$

When $\kappa = \frac{C_A}{2}$ for the spin contribution, agrees with Yamamoto and Yang, 2103.13208.

Including the orbital contribution, $\kappa = -\frac{C_A}{2}$, and the azimuthal current is reversed due to the opposite charge density.

Fukushima, KH, Mameda, PRL [2409.18652 [hep-ph]]

$$\boldsymbol{E}_{\text{lab}} \times \boldsymbol{\Omega} = -\{(\boldsymbol{x} \times \boldsymbol{\Omega}) \times \boldsymbol{B}_{\text{lab}}\} \times \boldsymbol{\Omega}$$

$$(\boldsymbol{A} \times \boldsymbol{B}) \times \boldsymbol{C} = \boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) + \boldsymbol{B} \times (\boldsymbol{C} \times \boldsymbol{A})$$

$$= -(\boldsymbol{x} \times \boldsymbol{\Omega}) \times (\boldsymbol{B}_{\text{lab}} \times \boldsymbol{\Omega}) - \boldsymbol{B}_{\text{lab}} \times \{\boldsymbol{\Omega} \times (\boldsymbol{x} \times \boldsymbol{\Omega})\}$$

$$\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{A} \cdot \boldsymbol{C})\boldsymbol{B} - (\boldsymbol{A} \cdot \boldsymbol{B})\boldsymbol{C}$$

$$= -(\boldsymbol{x} \times \boldsymbol{\Omega}) \times \cancel{(\boldsymbol{B}_{\text{lab}} \times \boldsymbol{\Omega})} - \{\cancel{\boldsymbol{B}_{\text{lab}} \cdot (\boldsymbol{x} \times \boldsymbol{\Omega})}\}\boldsymbol{\Omega} + \underline{\boldsymbol{B}_{\text{lab}} \cdot \boldsymbol{\Omega}\, (\boldsymbol{x} \times \boldsymbol{\Omega})}$$

$$\boldsymbol{B}_{\text{lab}} \ // \ \boldsymbol{\Omega}$$