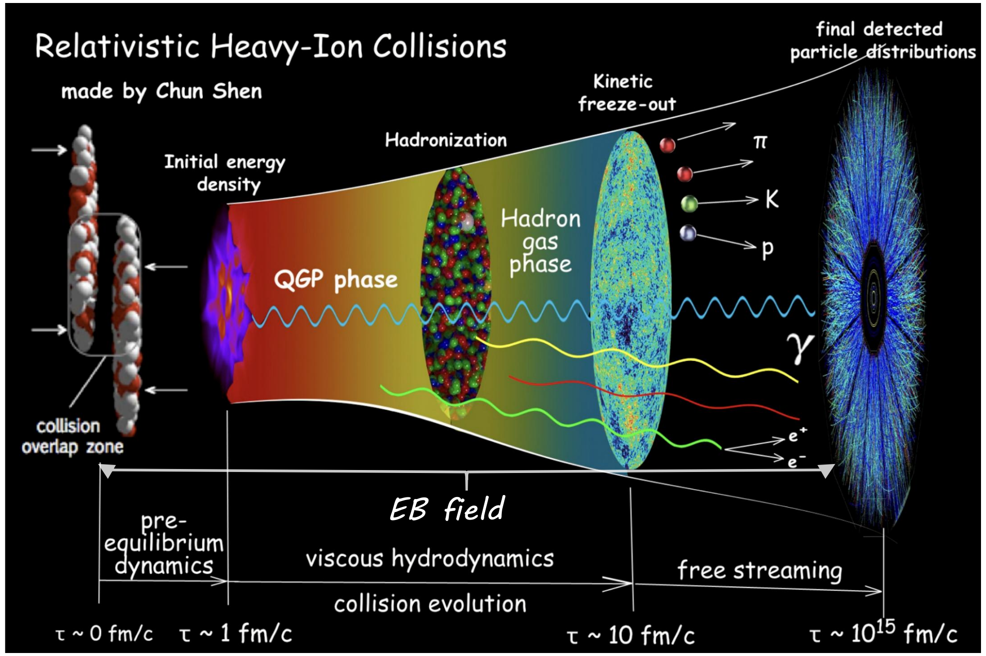
Left-right splitting of elliptic flow in heavy ion collisions: TRENTo-3D + CLVisc
相对论重离子碰撞中椭圆流分裂行为的研究

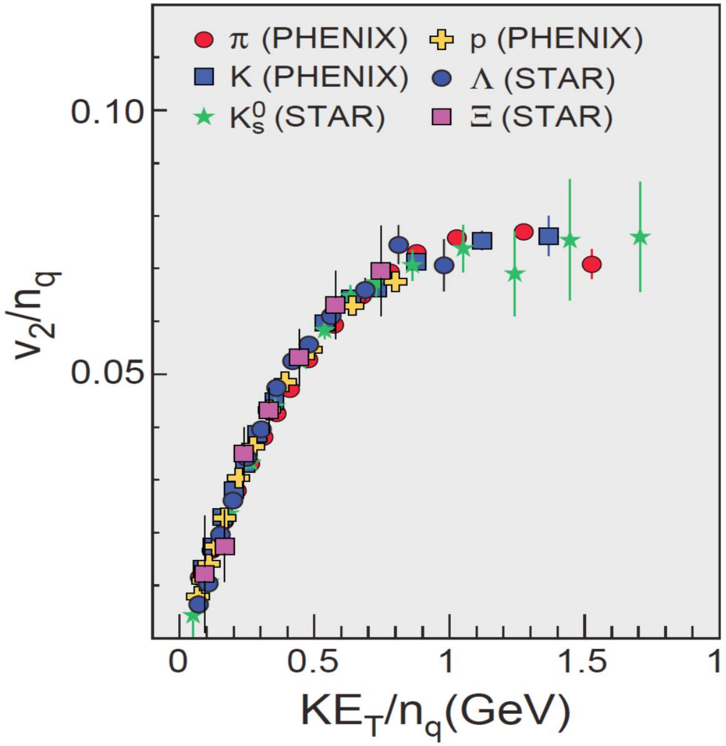
Ze-Fang Jiang
江泽方

arXiv: 2505.14637, Accepted by PRC

Presented at the 16th Workshop on QCD Phase Transition and Relativistic Heavy Ion Physics, Guilin, Oct 26th 2025

Anisotropic Flow in Heavy-Ion Collisions



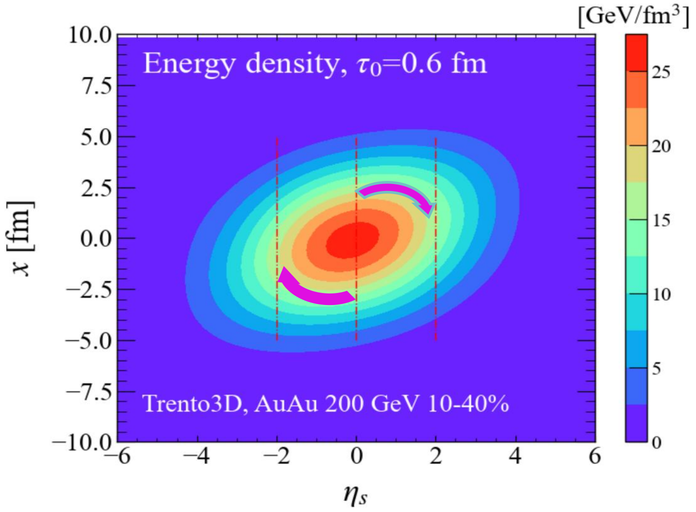


$$\frac{dN}{d\phi} = \frac{1}{2\pi}[1 + 2\sum_{n=1}^{\infty}(v_n \cos(n(\phi - \psi_{\mathrm{RP}}))]$$

$$v_n = \langle \cos\left(n\left[\varphi - \Psi_{RP}\right]\right)\rangle \quad v_1 = \left\langle \frac{p_x}{p_T} \right\rangle, \quad v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle \quad \ldots$$

U. Heinz and R. Snellings. Ann. Rev. Nucl. Part. Sci., 63:123–151, 2013

Why does Δv_2 arise?

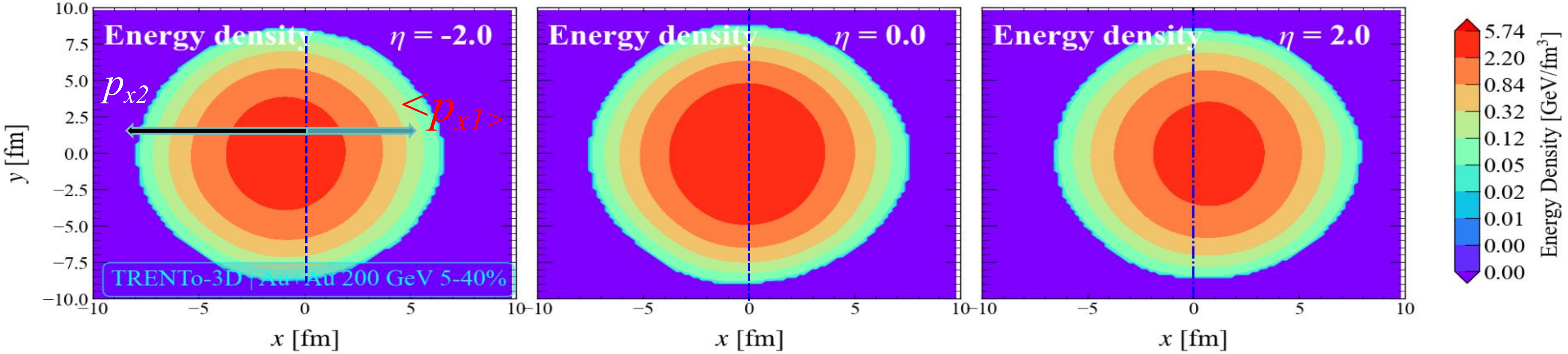


1.Transverse energy-density asymmetry of the QGP；

2. Initial vorticity

Phys.Rev.C 106 (2022) 5, 054910； Phys.Rev.C 106 (2022) 4, 044907；
arXiv: 2108.12735

$$\Delta v_2^{\mathrm{RP}} \equiv v_2^R - v_2^L = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle_{p_x>0} - \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle_{p_x<0}$$



Three questions:

1.How can Δv_2 be calculated in momentum space?
2.How can a realistic three-dimensional **initial QGP distribution** be generated?
3.What physical aspects of the QGP initial state does Δv_2 probe?

Question 1: Theoretical calculation of Δv_2

S. A. Voloshin, A. M. Poskanzer, and R. Snellings, Landolt-Bornstein, 23:293–333, 2010.

Particle spectrum:

$$\boxed{\frac{dN}{d\phi} = \frac{1}{2\pi}\left(1 + 2\sum_{n=1}^{\infty}\left(v_n \cos(n(\phi-\psi_{\mathrm{RP}})) + s_n \sin(n(\phi-\psi_{\mathrm{RP}}))\right)\right)}$$

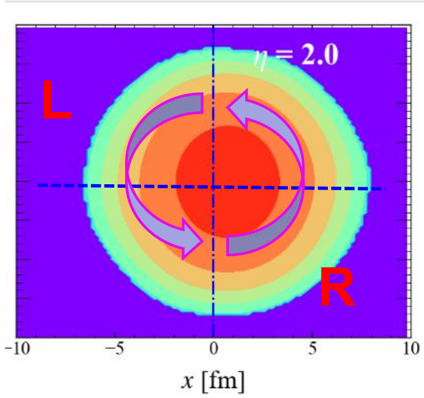
Elliptic flow v_2:

$$\begin{aligned} v_2 &= \frac{\int_{\psi_{\mathrm{RP}}}^{\psi_{\mathrm{RP}}+2\pi} \cos\left[2(\phi-\psi_{\mathrm{RP}})\right] \frac{dN}{d\phi} d\phi}{\int_{\psi_{\mathrm{RP}}}^{\psi_{\mathrm{RP}}+2\pi} \frac{dN}{d\phi} d\phi} \\ &= \frac{\int_{\psi_{\mathrm{RP}}}^{\psi_{\mathrm{RP}}+2\pi} \cos\left[2(\phi-\psi_{\mathrm{RP}})\right] \frac{1}{2\pi}\left(1+2\sum_{n=1}^{\infty}\left(v_n \cos(n(\phi-\psi_{\mathrm{RP}}))\right)\right) d\phi}{\int_{\psi_{\mathrm{RP}}}^{\psi_{\mathrm{RP}}+2\pi} \left(1+2\sum_{n=1}^{\infty}\left(v_n \cos(n(\phi-\psi_{\mathrm{RP}}))\right)\right) d\phi} \end{aligned}$$

v_2 from the left and right sides of the reaction plane:

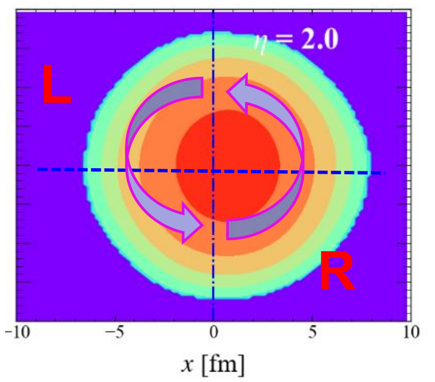
$$\begin{aligned} v_2^R &= \frac{\int_{\psi_{\mathrm{RP}}-\frac{\pi}{2}}^{\psi_{\mathrm{RP}}+\frac{\pi}{2}} \cos\left[2(\phi-\psi_{\mathrm{RP}})\right] \frac{dN}{d\phi} d\phi}{\int_{\psi_{\mathrm{RP}}-\frac{\pi}{2}}^{\psi_{\mathrm{RP}}+\frac{\pi}{2}} \frac{dN}{d\phi} d\phi} \\ &\approx \frac{v_2 + \frac{4}{3\pi}v_1 + \frac{12}{5\pi}v_3 - \frac{20}{21\pi}v_5}{1 + \frac{4}{\pi}v_1 - \frac{4}{3\pi}v_3 + \frac{4}{5\pi}v_5} \\ &= v_2\left[\frac{1 + \frac{4v_1}{3\pi v_2} + \frac{12 v_3}{5\pi v_2} - \frac{20 v_5}{21\pi v_2}}{1 + \frac{4}{\pi}v_1 - \frac{4}{3\pi}v_3 + \frac{4}{5\pi}v_5}\right], \end{aligned}$$

$$\begin{aligned} v_2^L &= \frac{\int_{\psi_{\mathrm{RP}}+\frac{\pi}{2}}^{\psi_{\mathrm{RP}}+\frac{3\pi}{2}} \cos\left[2(\phi-\psi_{\mathrm{RP}})\right] \frac{dN}{d\phi} d\phi}{\int_{\psi_{\mathrm{RP}}+\frac{\pi}{2}}^{\psi_{\mathrm{RP}}+\frac{3\pi}{2}} \frac{dN}{d\phi} d\phi} \\ &\approx \frac{v_2 - \frac{4}{3\pi}v_1 - \frac{12}{5\pi}v_3 + \frac{20}{21\pi}v_5}{1 - \frac{4}{\pi}v_1 + \frac{4}{3\pi}v_3 - \frac{4}{5\pi}v_5} \\ &= v_2\left[\frac{1 - \frac{4v_1}{3\pi v_2} - \frac{12 v_3}{5\pi v_2} + \frac{20 v_5}{21\pi v_2}}{1 - \frac{4}{\pi}v_1 + \frac{4}{3\pi}v_3 - \frac{4}{5\pi}v_5}\right]. \end{aligned}$$



STAR, The STAR Event Plane Detector, arXiv: 1912.05243

Question 1: Theoretical calculation of Δv_2



$$v_2^R = \frac{\int_{\psi_{\mathrm{RP}}-\frac{\pi}{2}}^{\psi_{\mathrm{RP}}+\frac{\pi}{2}} \cos\left[2(\phi-\psi_{\mathrm{RP}})\right]\frac{dN}{d\phi}d\phi}{\int_{\psi_{\mathrm{RP}}-\frac{\pi}{2}}^{\psi_{\mathrm{RP}}+\frac{\pi}{2}} \frac{dN}{d\phi}d\phi} \approx \frac{v_2+\frac{4}{3\pi}v_1+\frac{12}{5\pi}v_3-\frac{20}{21\pi}v_5}{1+\frac{4}{\pi}v_1-\frac{4}{3\pi}v_3+\frac{4}{5\pi}v_5} = v_2\left[\frac{1+\frac{4v_1}{3\pi v_2}+\frac{12v_3}{5\pi v_2}-\frac{20v_5}{21\pi v_2}}{1+\frac{4}{\pi}v_1-\frac{4}{3\pi}v_3+\frac{4}{5\pi}v_5}\right],$$

$$v_2^L = \frac{\int_{\psi_{\mathrm{RP}}+\frac{\pi}{2}}^{\psi_{\mathrm{RP}}+\frac{3\pi}{2}} \cos\left[2(\phi-\psi_{\mathrm{RP}})\right]\frac{dN}{d\phi}d\phi}{\int_{\psi_{\mathrm{RP}}+\frac{\pi}{2}}^{\psi_{\mathrm{RP}}+\frac{3\pi}{2}} \frac{dN}{d\phi}d\phi} \approx \frac{v_2-\frac{4}{3\pi}v_1-\frac{12}{5\pi}v_3+\frac{20}{21\pi}v_5}{1-\frac{4}{\pi}v_1+\frac{4}{3\pi}v_3-\frac{4}{5\pi}v_5} = v_2\left[\frac{1-\frac{4v_1}{3\pi v_2}-\frac{12v_3}{5\pi v_2}+\frac{20v_5}{21\pi v_2}}{1-\frac{4}{\pi}v_1+\frac{4}{3\pi}v_3-\frac{4}{5\pi}v_5}\right].$$

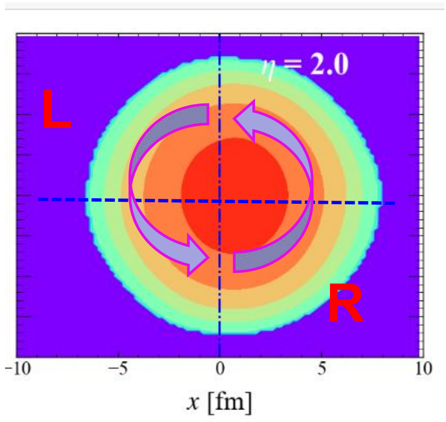
$$\begin{aligned}\Delta v_2^{\mathrm{RP}} &= v_2^R - v_2^L \\ &= \frac{\frac{8}{3\pi}\left[v_1(1-3v_2)+(\frac{9}{5}+v_2)v_3-(\frac{5}{7}+\frac{3}{5}v_2)v_5\right]}{(1-\frac{4}{\pi}v_1+\frac{4}{3\pi}v_3-\frac{4}{5\pi}v_5)(1+\frac{4}{\pi}v_1-\frac{4}{3\pi}v_3+\frac{4}{5\pi}v_5)} \\ &= \frac{\frac{8}{3\pi}\left[v_1(1-3v_2)+(\frac{9}{5}+v_2)v_3-(\frac{5}{7}+\frac{3}{5}v_2)v_5\right]}{1-\frac{16v_1^2}{\pi^2}+\frac{32v_1v_3}{3\pi^2}-\frac{16v_3^2}{9\pi^2}-\frac{32v_1v_5}{5\pi^2}+\frac{32v_3v_5}{15\pi^2}-\frac{16v_5^2}{25\pi^2}}.\end{aligned}$$

$$\Delta v_{2,\mathrm{1st}}^{\mathrm{RP}} = \frac{\frac{8}{3\pi}\left[v_1(1-3v_2)\right]}{1-\frac{16v_1^2}{\pi^2}} \approx \boxed{\frac{8}{3\pi}v_1(1-3v_2)} \approx \frac{8}{3\pi}v_1$$

Leading order contribution

Chao Zhang and Zi-Wei Lin. Left-right splitting of elliptic flow due to directed flow in heavy ion collisions. Phys. Rev. C, 106(5):054910, 2022. （AMPT）
T. Parida and S. Chatterjee. Splitting of elliptic flow in a tilted fireball. Phys. Rev. C, 106(4):044907, 2022. (Optical Glauber Model + Music)

Question 1: Theoretical calculation of Δv_2



$$v_2^R = \frac{\int_{\psi_{\rm RP}-\frac{\pi}{2}}^{\psi_{\rm RP}+\frac{\pi}{2}} \cos[2(\phi-\psi_{\rm RP})]\frac{dN}{d\phi}d\phi}{\int_{\psi_{\rm RP}-\frac{\pi}{2}}^{\psi_{\rm RP}+\frac{\pi}{2}} \frac{dN}{d\phi}d\phi} \approx \frac{v_2+\frac{4}{3\pi}v_1+\frac{12}{5\pi}v_3-\frac{20}{21\pi}v_5}{1+\frac{4}{\pi}v_1-\frac{4}{3\pi}v_3+\frac{4}{5\pi}v_5} = v_2\left[\frac{1+\frac{4v_1}{3\pi v_2}+\frac{12v_3}{5\pi v_2}-\frac{20v_5}{21\pi v_2}}{1+\frac{4}{\pi}v_1-\frac{4}{3\pi}v_3+\frac{4}{5\pi}v_5}\right],$$

$$v_2^L = \frac{\int_{\psi_{\rm RP}+\frac{\pi}{2}}^{\psi_{\rm RP}+\frac{3\pi}{2}} \cos[2(\phi-\psi_{\rm RP})]\frac{dN}{d\phi}d\phi}{\int_{\psi_{\rm RP}+\frac{\pi}{2}}^{\psi_{\rm RP}+\frac{3\pi}{2}} \frac{dN}{d\phi}d\phi} \approx \frac{v_2-\frac{4}{3\pi}v_1-\frac{12}{5\pi}v_3+\frac{20}{21\pi}v_5}{1-\frac{4}{\pi}v_1+\frac{4}{3\pi}v_3-\frac{4}{5\pi}v_5} = v_2\left[\frac{1-\frac{4v_1}{3\pi v_2}-\frac{12v_3}{5\pi v_2}+\frac{20v_5}{21\pi v_2}}{1-\frac{4}{\pi}v_1+\frac{4}{3\pi}v_3-\frac{4}{5\pi}v_5}\right].$$

$$\Delta v_2^{\rm RP} = v_2^R - v_2^L = \frac{\frac{8}{3\pi}\left[v_1(1-3v_2)+(\frac{9}{5}+v_2)v_3-(\frac{5}{7}+\frac{3}{5}v_2)v_5\right]}{(1-\frac{4}{\pi}v_1+\frac{4}{3\pi}v_3-\frac{4}{5\pi}v_5)(1+\frac{4}{\pi}v_1-\frac{4}{3\pi}v_3+\frac{4}{5\pi}v_5)} = \frac{\frac{8}{3\pi}\left[v_1(1-3v_2)+(\frac{9}{5}+v_2)v_3-(\frac{5}{7}+\frac{3}{5}v_2)v_5\right]}{1-\frac{16v_1^2}{\pi^2}+\frac{32v_1v_3}{3\pi^2}-\frac{16v_3^2}{9\pi^2}-\frac{32v_1v_5}{5\pi^2}+\frac{32v_3v_5}{15\pi^2}-\frac{16v_5^2}{25\pi^2}}.$$

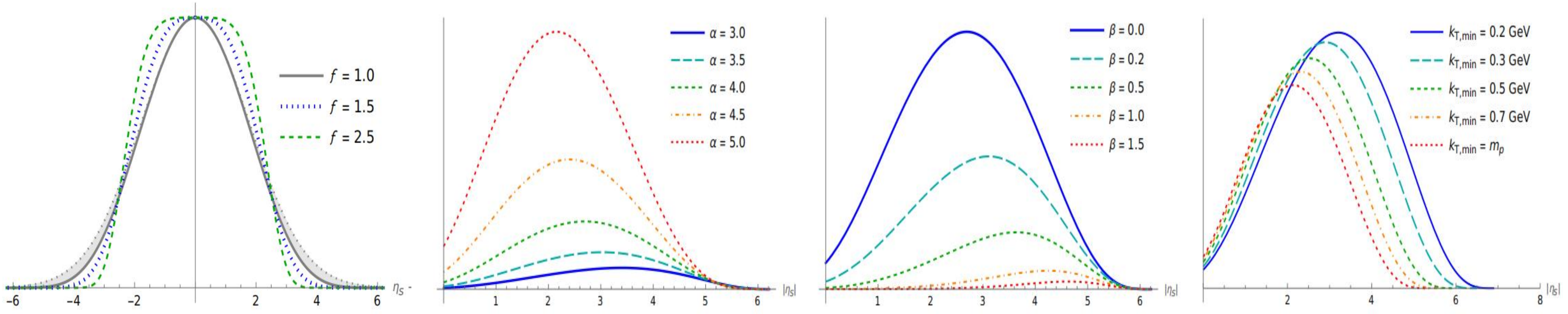
$$\Delta v_{2,\rm 5th}^{\rm RP} = \frac{\frac{8}{3\pi}\left[v_1(1-3v_2)+(\frac{9}{5}+v_2)v_3-(\frac{5}{7}+\frac{3}{5}v_2)v_5\right]}{1-\frac{16v_1^2}{\pi^2}+\frac{32v_1v_3}{3\pi^2}-\frac{16v_3^2}{9\pi^2}-\frac{32v_1v_5}{5\pi^2}+\frac{32v_3v_5}{15\pi^2}-\frac{16v_5^2}{25\pi^2}} \approx \frac{8}{3\pi}\left[\boxed{v_1}(1-3v_2)+(\frac{9}{5}+v_2)\boxed{v_3}-(\frac{5}{7}+\frac{3}{5}v_2)\boxed{v_5}\right].$$

$v_1,\ v_3,\ v_5\ \ldots$

Ze-Fang Jiang, Xiang Fan, Duan She, Ben-Wei Zhang et.al , arXiv: 2505.14637, Accepted by PRC

Question 2: TRENTo-3D initial condition

Derek Soeder, Weiyao Ke, J. F. Paquet, and Steffen A. Bass. arXiv: 2306.08665.



Initial energy density：$\varepsilon_{\mathrm{IC}}(\vec{x}_\perp, \eta_s) = \underbrace{N_{\mathrm{fb}}\sqrt{T_A(\vec{x}_\perp)T_B(\vec{x}_\perp)}f_{\mathrm{fb}}(\eta_s)}_{\text{Central fireball}} + \sum_{X=A,B} \underbrace{\frac{\boxed{k_{\mathrm{T}}}}{2N_{\mathrm{frag}}} F_X(\vec{x}_\perp) f^X_{\mathrm{frag}}(\eta_s)}_{\text{Fragmentation regions}}$

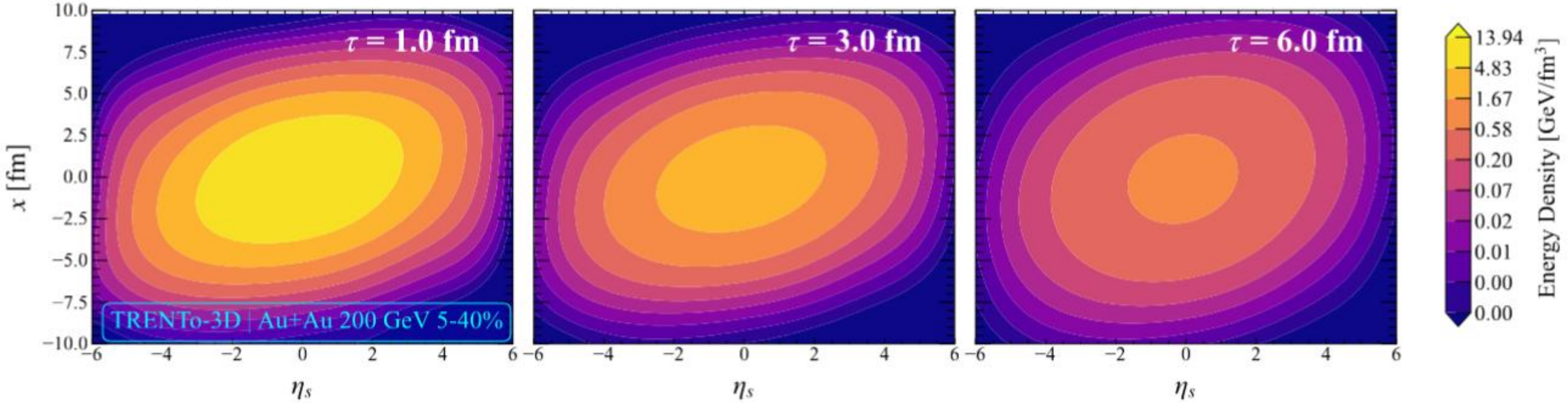
Includes nucleon thickness functions：$T_X(\vec{x}_\perp) = \sum_{p\in X} \boxed{\frac{1}{n_c}} \sum_{c\in p} \gamma_c \frac{\exp\left(-\frac{|\vec{x}_\perp - \vec{x}_p - \vec{s}_c|^2}{2v^2}\right)}{2\pi v^2}$

central region：$f_{\mathrm{fb}}(\eta_s) = \exp\left(-\frac{|\eta_s - \eta_{\mathrm{cm}}|^{\boxed{f}}}{2\Delta\eta^2}\right)\left[1 - \left(\frac{\eta_s - \eta_{\mathrm{cm}}}{\eta_{\max}}\right)^4\right]^4$

fragmentation region：$f^X_{\mathrm{frag}}(x) = \boxed{(-\ln x)^\alpha x^{\beta+1}} \exp\left(-\frac{2k_{\mathrm{T}}}{x\sqrt{s_{\mathrm{NN}}}}\right)$

Question 2: TRENTo-3D initial condition

Derek Soeder, Weiyao Ke, J. F. Paquet, and Steffen A. Bass. arXiv: 2306.08665.
Xiang-Yu Wu, Guang-You Qin, Long-Gang Pang, and Xin-Nian Wang. Phys. Rev. C, 105(3):034909, 2022.



Initial energy density：$$\varepsilon_{IC}(\vec{x}_\perp, \eta_s) = \underbrace{N_{fb}\sqrt{T_A(\vec{x}_\perp)T_B(\vec{x}_\perp)}f_{fb}(\eta_s)}_{\text{Central fireball}} + \underbrace{\sum_{X=A,B} \frac{\boxed{k_T}}{2N_{frag}} F_X(\vec{x}_\perp) f^X_{frag}(\eta_s)}_{\text{Fragmentation regions}}$$

Includes nucleon thickness functions：$$T_X(\vec{x}_\perp) = \sum_{p\in X} \boxed{\frac{1}{n_c}} \sum_{c\in p} \gamma_c \frac{\exp\left(-\frac{|\vec{x}_\perp - \vec{x}_p - \vec{s}_c|^2}{2v^2}\right)}{2\pi v^2}$$

central region：$$f_{fb}(\eta_s) = \exp\left(-\frac{|\eta_s - \eta_{cm}|^{\boxed{f}}}{2\Delta\eta^2}\right)\left[1 - \left(\frac{\eta_s - \eta_{cm}}{\eta_{max}}\right)^4\right]^4$$

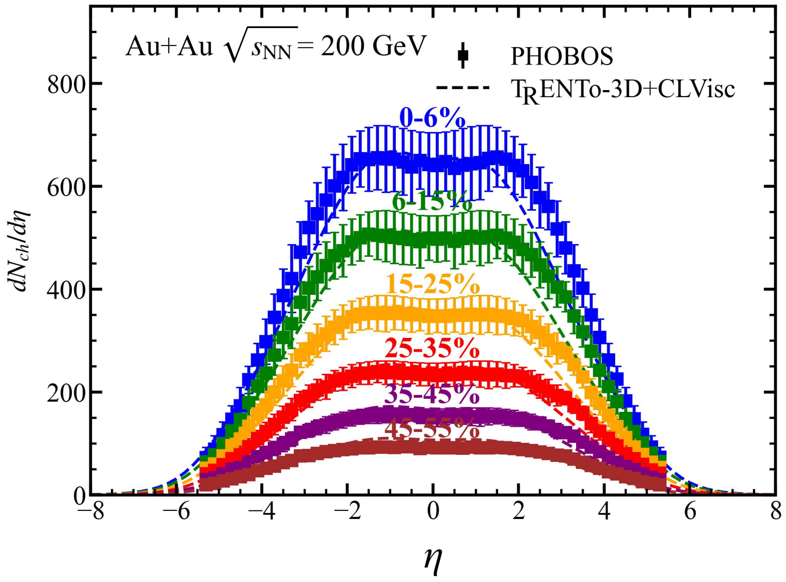
fragmentation region：$$f^X_{frag}(\eta_s) = \boxed{(-\ln x)^\alpha x^{\beta+1}} \exp\left(-\frac{2k_T}{x\sqrt{s_{NN}}}\right)$$

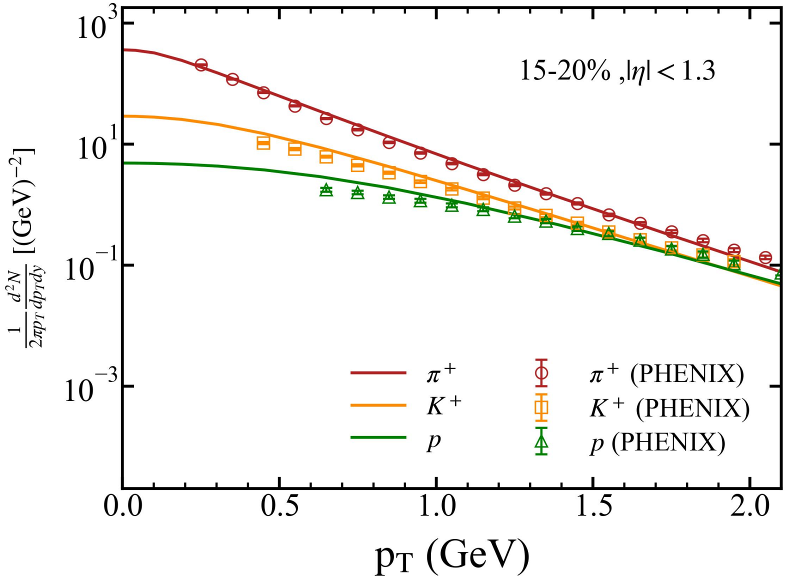
TABLE I: Parameter values used in this study [35].

Parameter	Value	Influence
n_c	16.4	Tunes initial subnucleonic degrees of freedom
w [fm]	1.3	Sets initial source size $\propto \sqrt{\langle r^2 \rangle}$
$\chi = v/(wn_c^{1/4})$	0.5	Tunes subnucleon correlation length
f	1.0	Controls the central fireball profile
k_T [GeV]	0.33	Determines η_{max} and tilted geometry
α	4.6	Controls the shape of the fragmentation profile
β	0.19	Controls the shape of the fragmentation profile

TRENTo-3D+ CLVisc Hydrodynamic

Derek Soeder, Weiyao Ke, J. F. Paquet, and Steffen A. Bass. arXiv: 2306.08665.
Xiang-Yu Wu, Guang-You Qin, Long-Gang Pang, and Xin-Nian Wang. Phys. Rev. C, 105(3):034909, 2022.

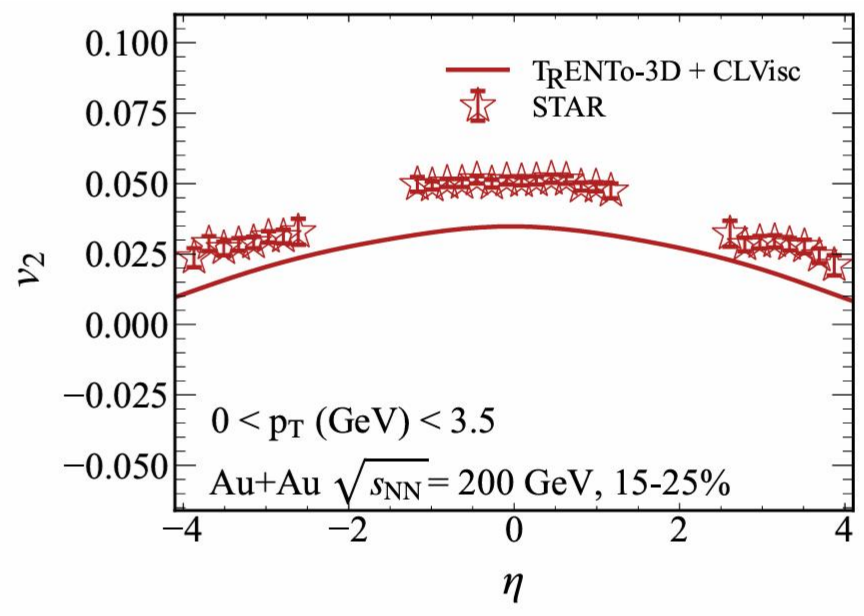


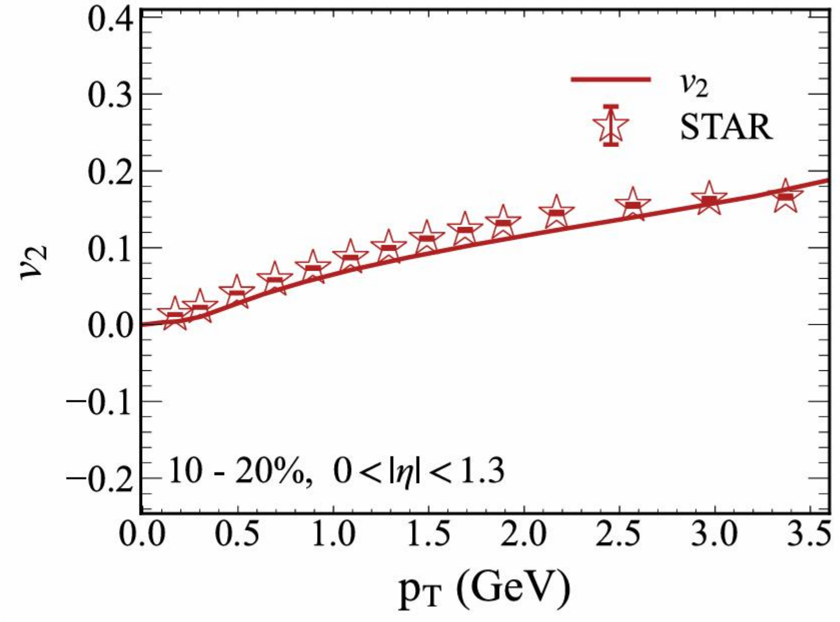


Hydrodynamic simulations with Bayesian-calibrated TRENTo3D parameters and CLVisc evolution provide a good description of the PHOBOS and PHENIX data.

TRENTo-3D+ CLVisc Hydrodynamic

Derek Soeder, Weiyao Ke, J. F. Paquet, and Steffen A. Bass. arXiv: 2306.08665.
Xiang-Yu Wu, Guang-You Qin, Long-Gang Pang, and Xin-Nian Wang. Phys. Rev. C, 105(3):034909, 2022.
STAR, Phys. Rev. C, 72:014904, 2005.





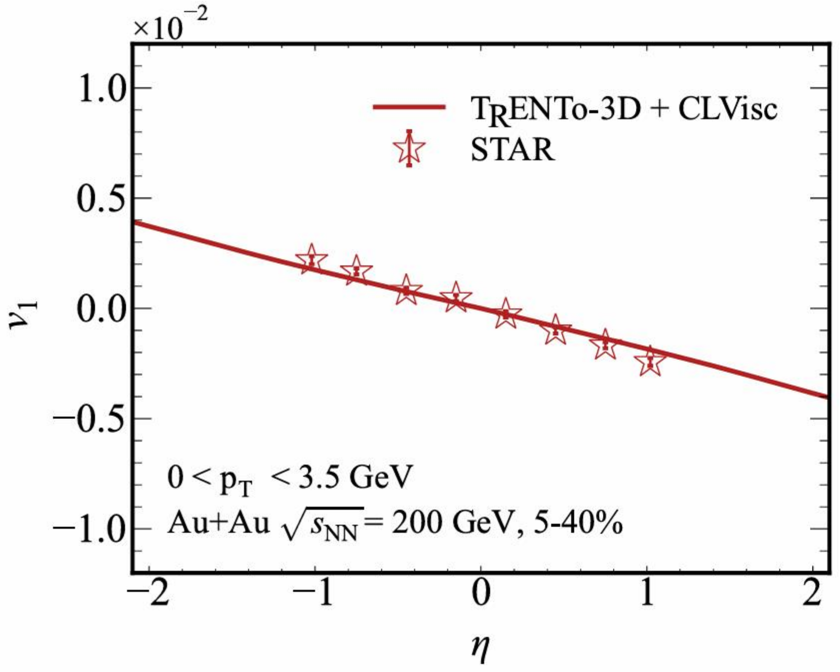
Hydrodynamic simulations using Bayesian-calibrated
TRENTo-3D parameters combined with CLVisc evolution

TRENTo-3D+ CLVisc Hydrodynamic

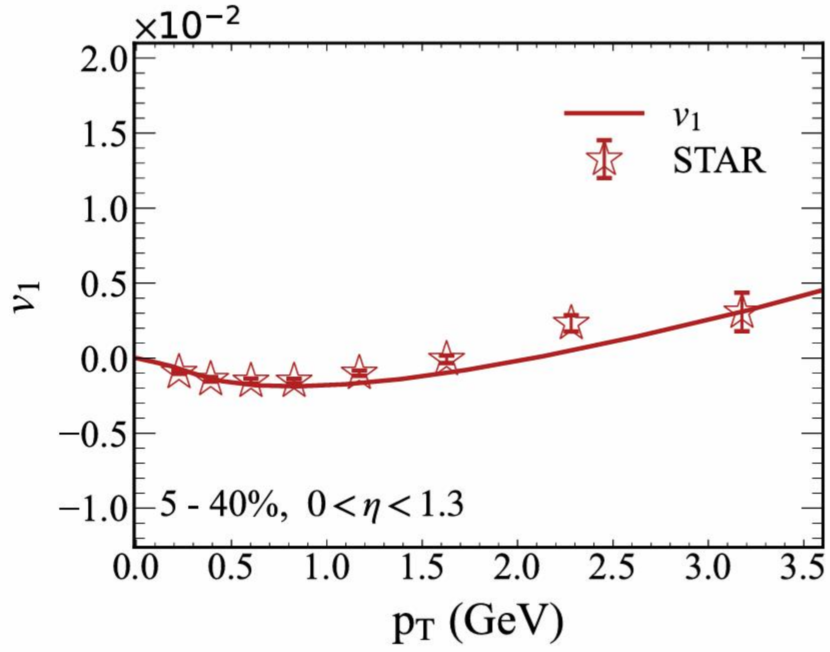
$$\Delta v_{2,\mathrm{1st}}^{\mathrm{RP}} = \frac{\frac{8}{3\pi}\left[v_1(1-3v_2)\right]}{1-\frac{16v_1^2}{\pi^2}} \approx \boxed{\frac{8}{3\pi}v_1(1-3v_2) \approx \frac{8}{3\pi}v_1}$$

STAR, Phys. Rev. Lett., 112(16):162301, 2014.

directed flow $v_1(\eta)$

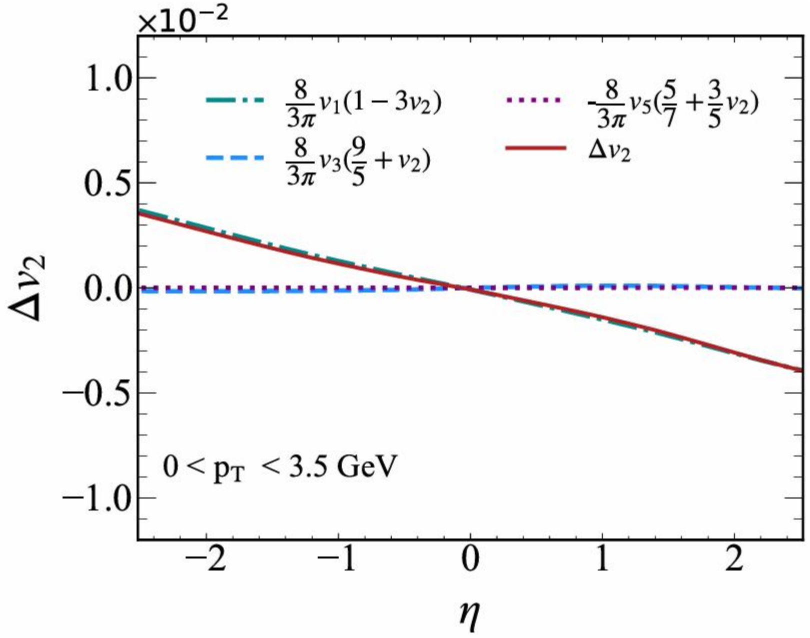


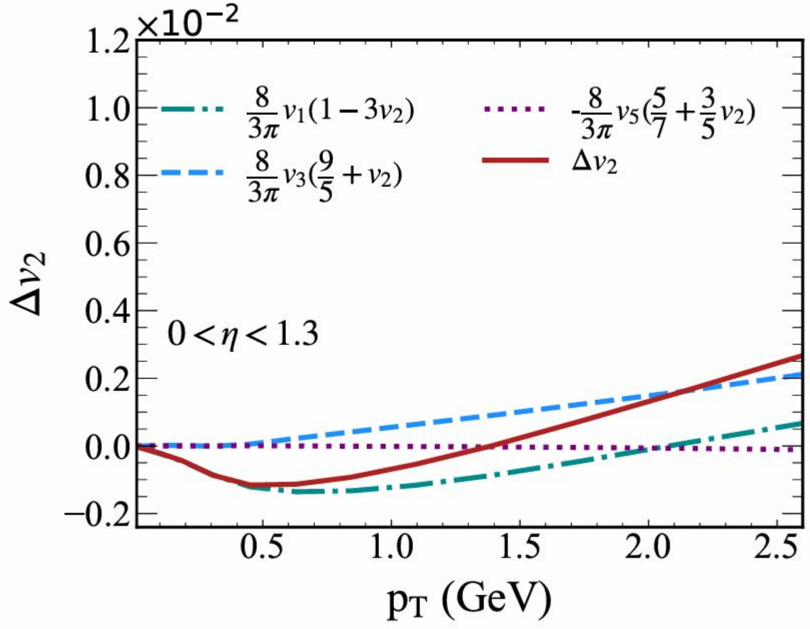
directed flow $v_1(p_T)$



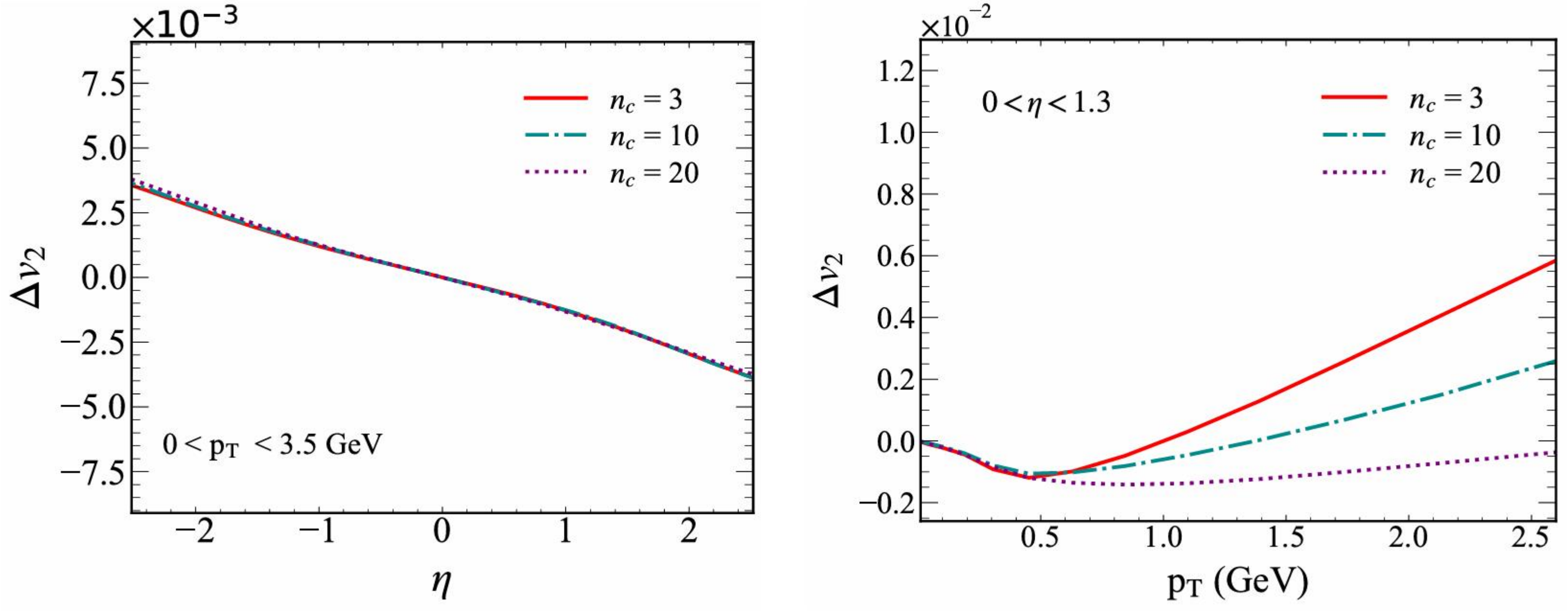
The developed TRENTo-3D+ CLVisc framework successfully describes directed flow distributions.

$$\Delta v_{2,5\text{th}}^{\text{RP}} = \frac{\frac{8}{3\pi}\left[v_1(1-3v_2)+(\frac{9}{5}+v_2)v_3-(\frac{5}{7}+\frac{3}{5}v_2)v_5\right]}{1-\frac{16v_1^2}{\pi^2}+\frac{32v_1v_3}{3\pi^2}-\frac{16v_3^2}{9\pi^2}-\frac{32v_1v_5}{5\pi^2}+\frac{32v_3v_5}{15\pi^2}-\frac{16v_5^2}{25\pi^2}} \approx \frac{8}{3\pi}\left[v_1(1-3v_2)+(\frac{9}{5}+v_2)v_3-(\frac{5}{7}+\frac{3}{5}v_2)v_5\right].$$





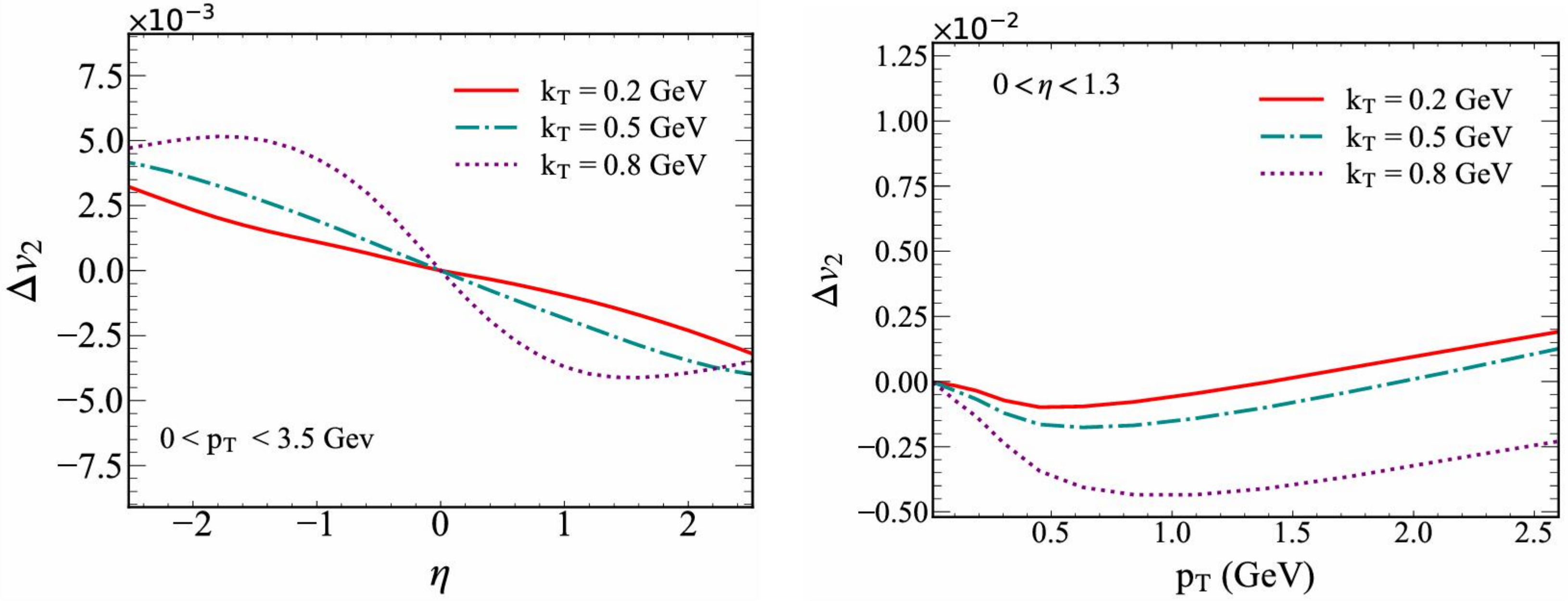
1. directed flow v_1 contribution dominates the rapidity dependence
2. triangular flow v_3 contributes significantly to $\Delta v_2(p_T)$, especially when initial fluctuations are included.



Effect of sub-nucleonic fluctuations on Δv_2:
More hotspots change $\Delta v_2(p_T)$ but little in $\Delta v_2(\eta)$, revealing the role of sub-nucleonic structure.

O. G. Montero, S. Schlichting, and Jie Zhu. Effects of sub-nucleonic fluctuations on the longitudinal structure of heavy-ion collisions. arXiv: 2501.14872

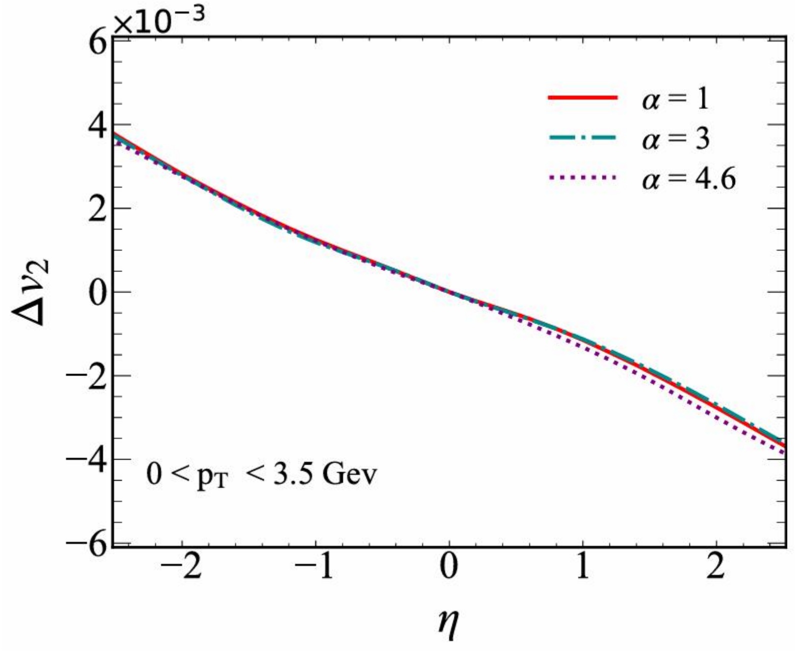
Question 3: Effect of transverse momentum scale on Δv_2

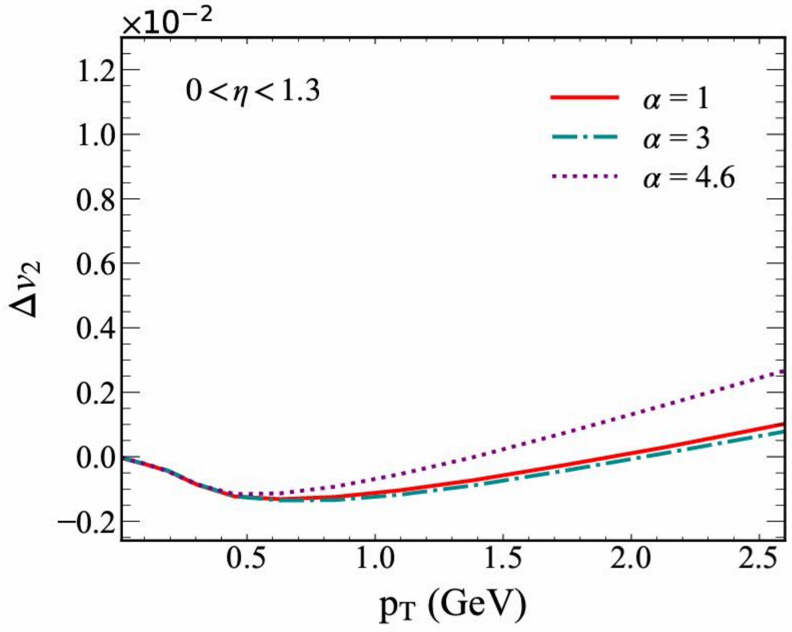


The transverse momentum scale determines the tilt of the fireball.
Δv_2 shows clear dependence on both pseudo-rapidity and p_T.
Measuring Δv_2 thus helps constrain the geometry of the QGP fireball.

Question 3: Effect of the fragmentation region on Δv_2

$$f^X_{\mathrm{frag}}(\eta_s) = (-\ln x)^{\alpha} x^{\beta+1} \exp\left(-\frac{2k_T}{x\sqrt{s_{NN}}}\right)$$



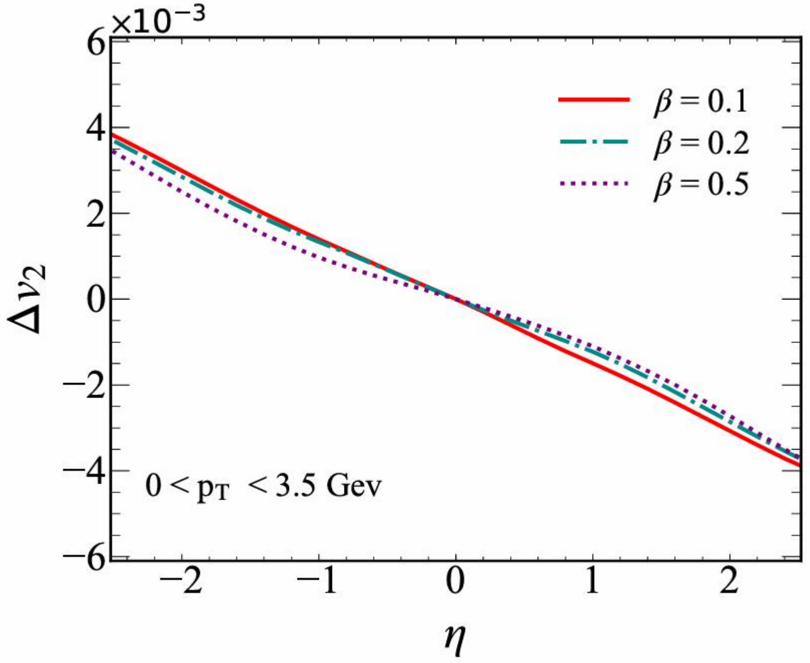


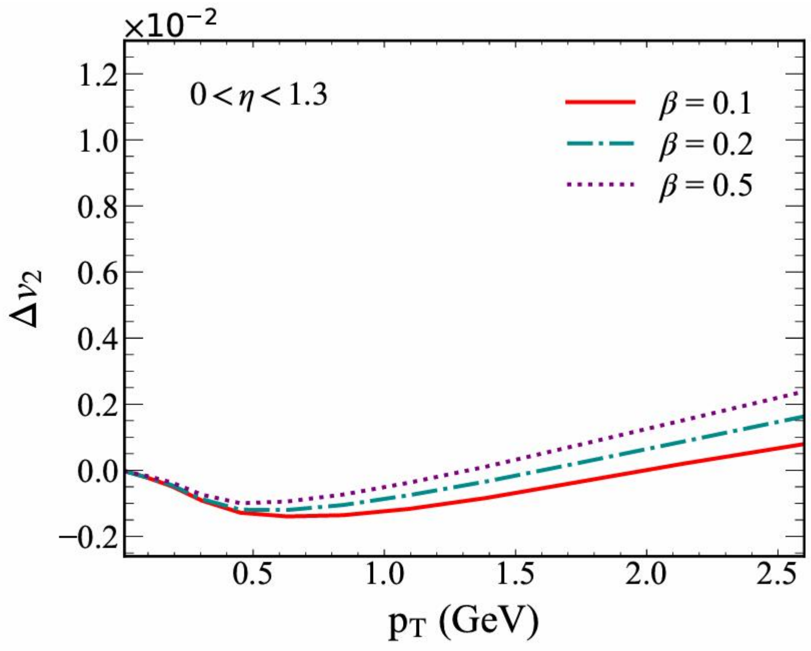
Effect of the fragmentation region on Δv_2:

Its impact is weak in rapidity but shows a threshold-like behavior in the transverse momentum distribution.

Question 3: Effect of the fragmentation region on Δv_2

$$f^{X}_{\mathrm{frag}}(\eta_s) = (-\ln x)^{\alpha} x^{\beta+1} \exp\left(-\frac{2k_T}{x\sqrt{s_{NN}}}\right)$$





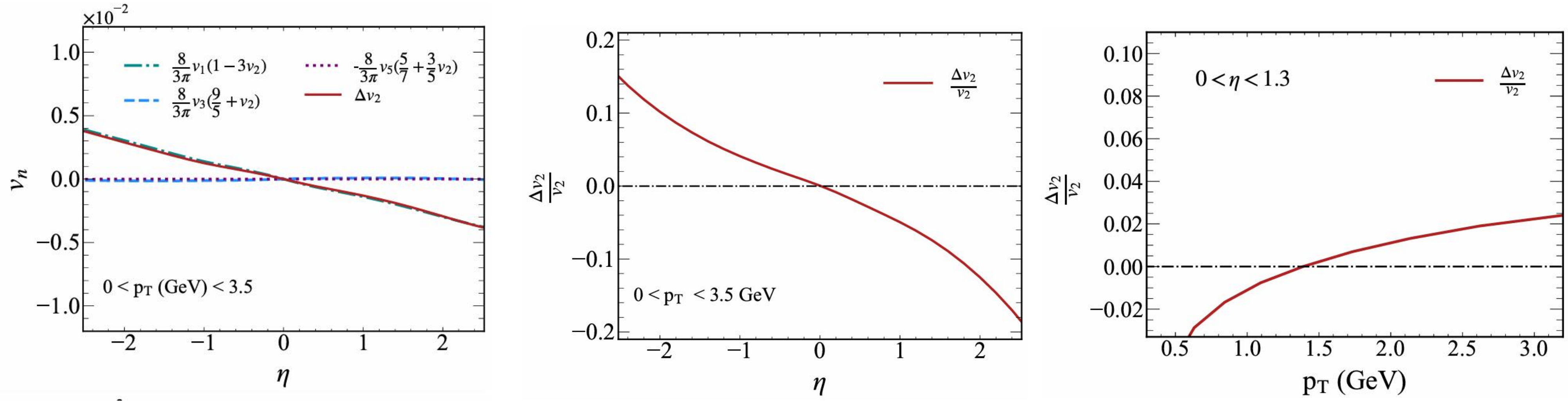
Effect of the fragmentation region on Δv_2 :

$\Delta v_2(\eta)$ is weakly affected, but $\Delta v_2(p_T)$ shows clear variation.

This observable helps probe the exponential profile of the fragmentation region.

Question 3: Proposed observable: elliptic-flow-splitting ratio $\Delta v_2/v_2$

arXiv: 2505.14637, Accepted by PRC



In η-distribution: compare directed and elliptic flows to constrain model parameters — theory predicts a sizable ratio.

In p_T-distribution: the zero-crossing point provides constraints on fluctuation effects.

Measuring Δv_2 thus offers insights into the QGP initial structure and internal dynamics.

Summary and outlook

Summary

arXiv: 2505.14637, Accepted by PRC

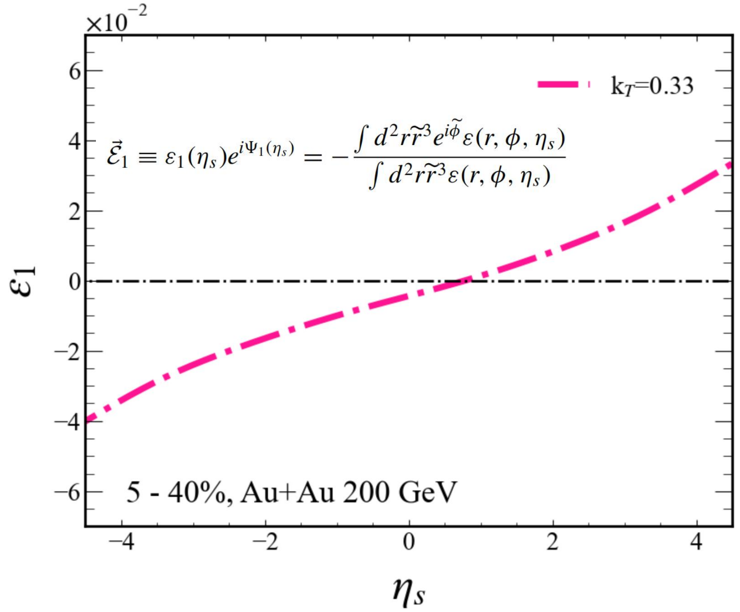
- ✓ Developed the TRENTo3D + CLVisc framework to explore correlations among flow harmonics.
- ✓ Investigated key factors affecting elliptic flow splitting to understand the QGP initial-state structure and nuclear properties.

Outlook

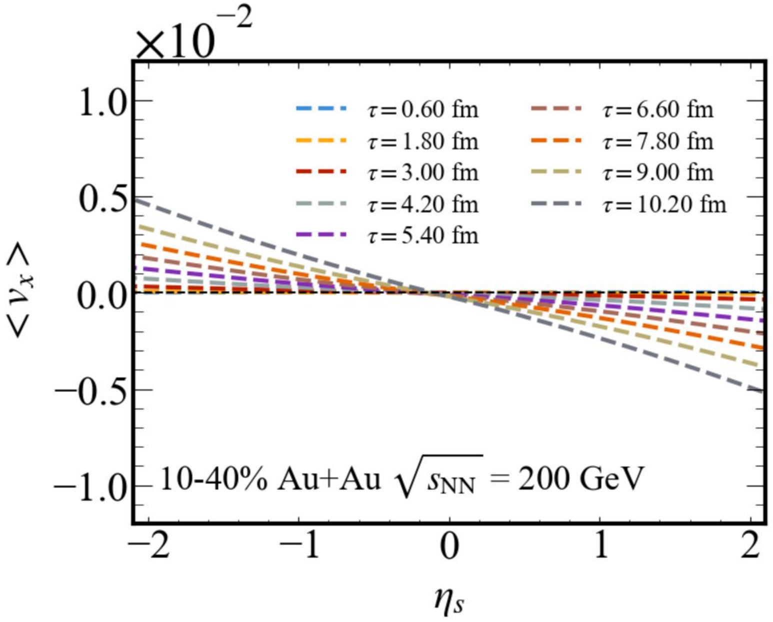
- ➢ Apply this framework to study global polarization of hyperons.
- ➢ Extend the analysis to heavy-flavor particle elliptic-flow splitting using transport models.

Thank you

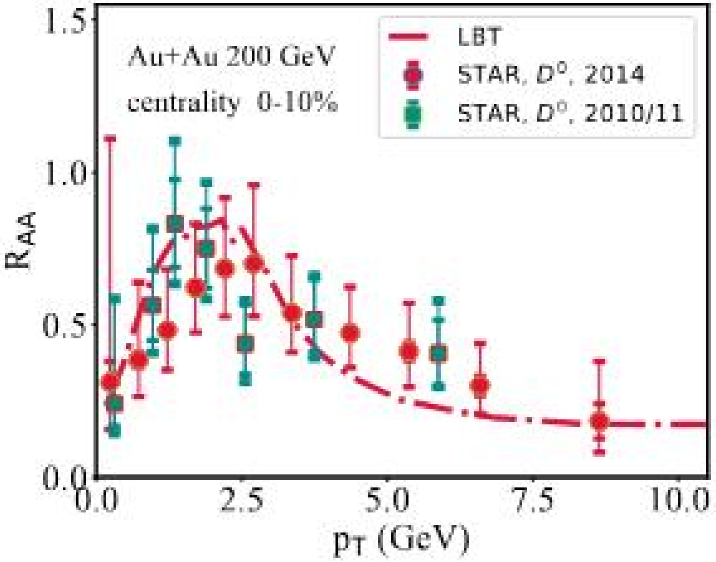
First-order eccentricity of the medium

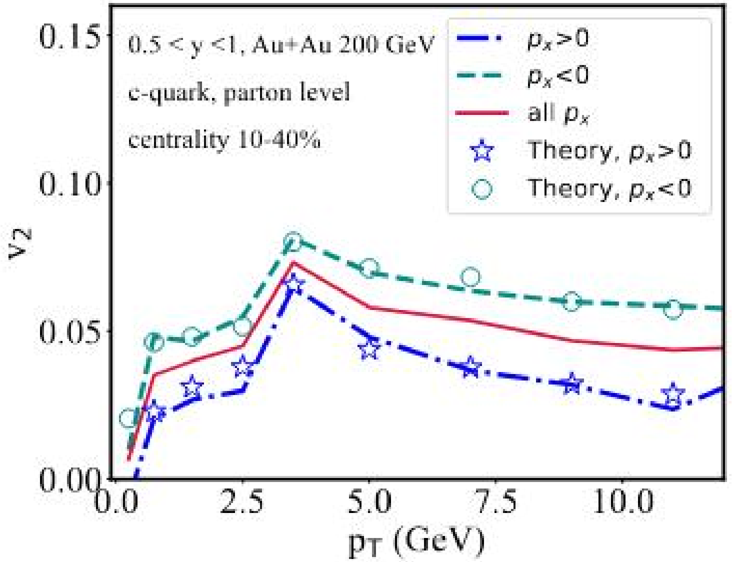


Evolution of the velocity field <v_x>



The energy density from TRENTo3D initialization reproduces the behavior responsible for directed flow.


Au+Au 200 GeV
centrality 0-10%
LBT
STAR, D^0, 2014
STAR, D^0, 2010/11
R_{AA}
p_T (GeV)


0.5 < y <1, Au+Au 200 GeV
c-quark, parton level
centrality 10-40%
p_x>0
p_x<0
all p_x
Theory, p_x>0
Theory, p_x<0
v_2
p_T (GeV)