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Anisotropic Hydrodynamics Expands the Domain of Applicability of Hydrodynamics

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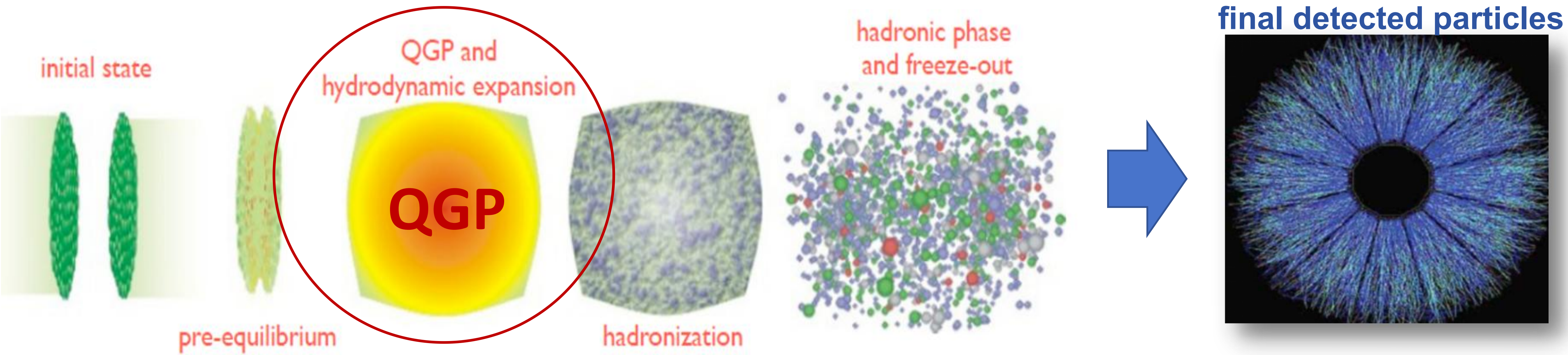
S. Zhao, Y. Peng, H. Heinz and H. Song, arXiv: 2509.03841
Y. Peng, V. Ambrus, C. Werthmann, S. Schlichting, U. Heinz and H. Song, arXiv: 2509.04431

Quark Gluon Plasma (QGP) and Hydrodynamics



Conservation laws:

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N_i^\mu = 0.$$

2nd order I-S eq:

$$\dot{\Pi} = -\frac{1}{\tau_\Pi}\left[\Pi + \zeta\theta - l_{\Pi q}\nabla_\mu q^\mu + \Pi\zeta T\partial_\mu\left(\frac{\tau_\Pi u^\mu}{2\zeta T}\right)\right],$$

$$\dot{q}^{\langle\mu\rangle} = -\frac{1}{\tau_q}\left[q^\mu + \lambda\frac{nT^2}{e+p}\nabla^\mu\frac{\nu}{T} + l_{q\pi}\nabla_\nu\pi^{\mu\nu} + l_{q\Pi}\nabla^\mu\Pi - \lambda T^2 q^\mu\partial_\nu\left(\frac{\tau_q u^\nu}{2\lambda T^2}\right)\right],$$

$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle} - l_{\pi q}\nabla^{\langle\mu}q^{\nu\rangle} + \pi^{\mu\nu}\eta T\partial_\alpha\left(\frac{\tau_\pi u^\alpha}{2\eta T}\right)\right].$$

Input: EoS $\mathcal{E} = \mathcal{E}(\mathcal{P})$; Initial condition

Hydrodynamics as an Long-range Effective Theory

$$f(x,p)$$
$$p\cdot\partial f = C[f]$$

Kinetic theory

energy density, velocity, thermal pressure, shear stress

$$T^{\mu\nu} = \langle p^\mu p^\nu\rangle_f = \mathcal{E}u^\mu u^\nu - (\mathcal{P}+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

bulk pressure

$$\partial_\mu T^{\mu\nu} = 0$$

Hydrodynamics

Applicability: small Knudsen number and inversed Reynolds number,

$$\boxed{\mathrm{Kn} = \frac{l_{\mathrm{micr}}}{L_{\mathrm{macr}}} \ll 1}, \quad \boxed{\mathrm{R}^{-1}_{\pi,\Pi} = \frac{|\pi^{\mu\nu}|}{\mathcal{P}}, \frac{|\Pi|}{\mathcal{P}} \ll 1}$$

coarse-graining — local equilibrium

Gradient expansion:

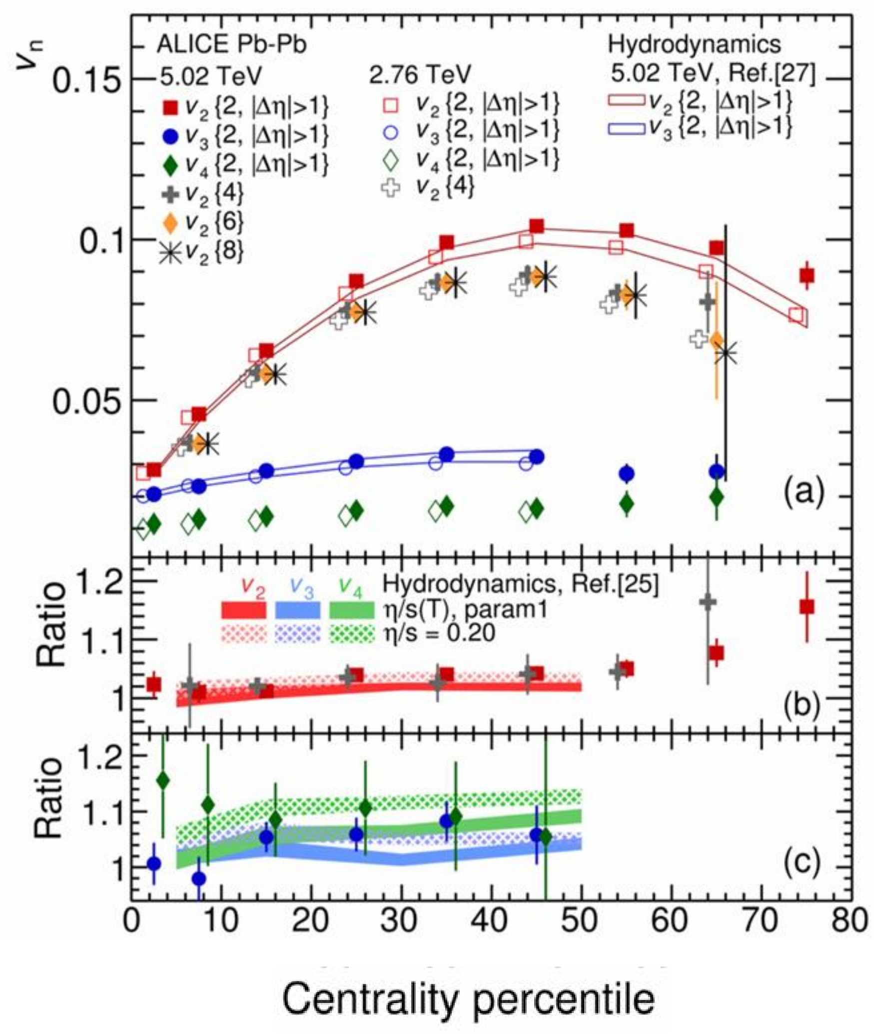
$$f \approx f_{eq} + \underset{\mathcal{O}(\mathrm{Kn})}{\underset{\wr}{f^{(1)}}} + \underset{\mathcal{O}(\mathrm{Kn}^2)}{\underset{\wr}{f^{(2)}}} + \cdots + \underset{\mathcal{O}(\mathrm{Kn}^n)}{\underset{\wr}{f^{(n)}}}$$

Truncate to 2nd order:

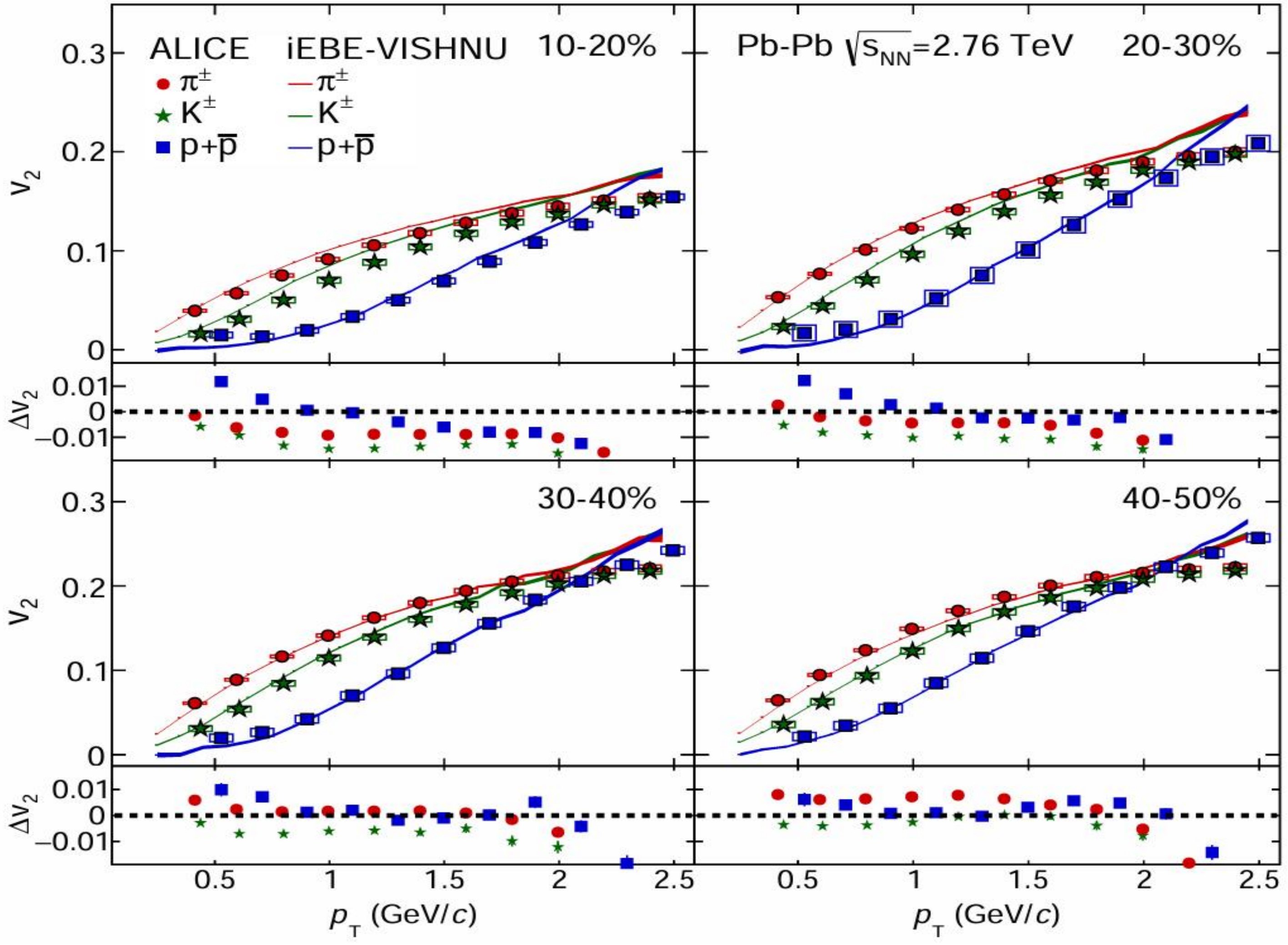
$$\dot{\pi}^{\mu\nu} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + \ldots, \quad \dot{\Pi} = -\frac{\Pi}{\tau_\Pi} + \ldots$$

Success in Large Systems

Hydrodynamics nice description of integrated and differential flow of charged and identified hadrons.



Phys. Rev. Lett. 116(2016) 132302

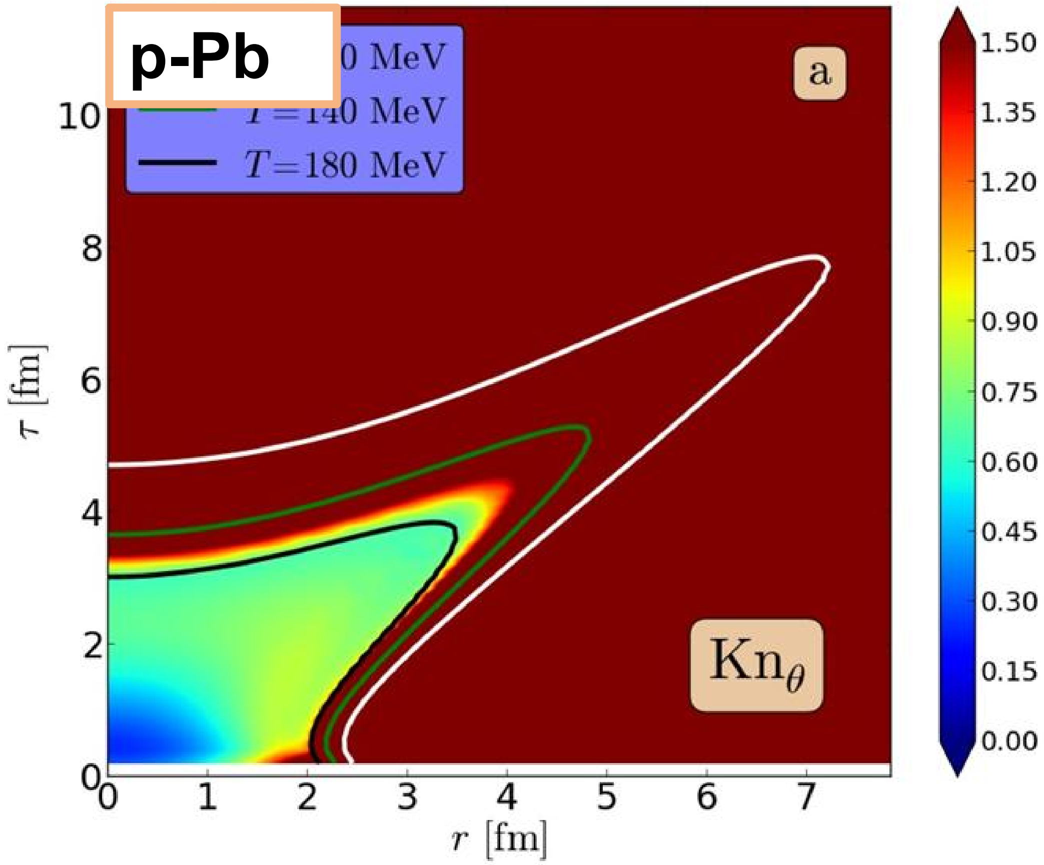


ALICE Paper, JHEP 1609 164 (2016)

Challenge in Small Systems

Applicability of hydro challenged in small systems(p-Pb, p-Au, O-O, p-p, ...):

Large Knudsen number:

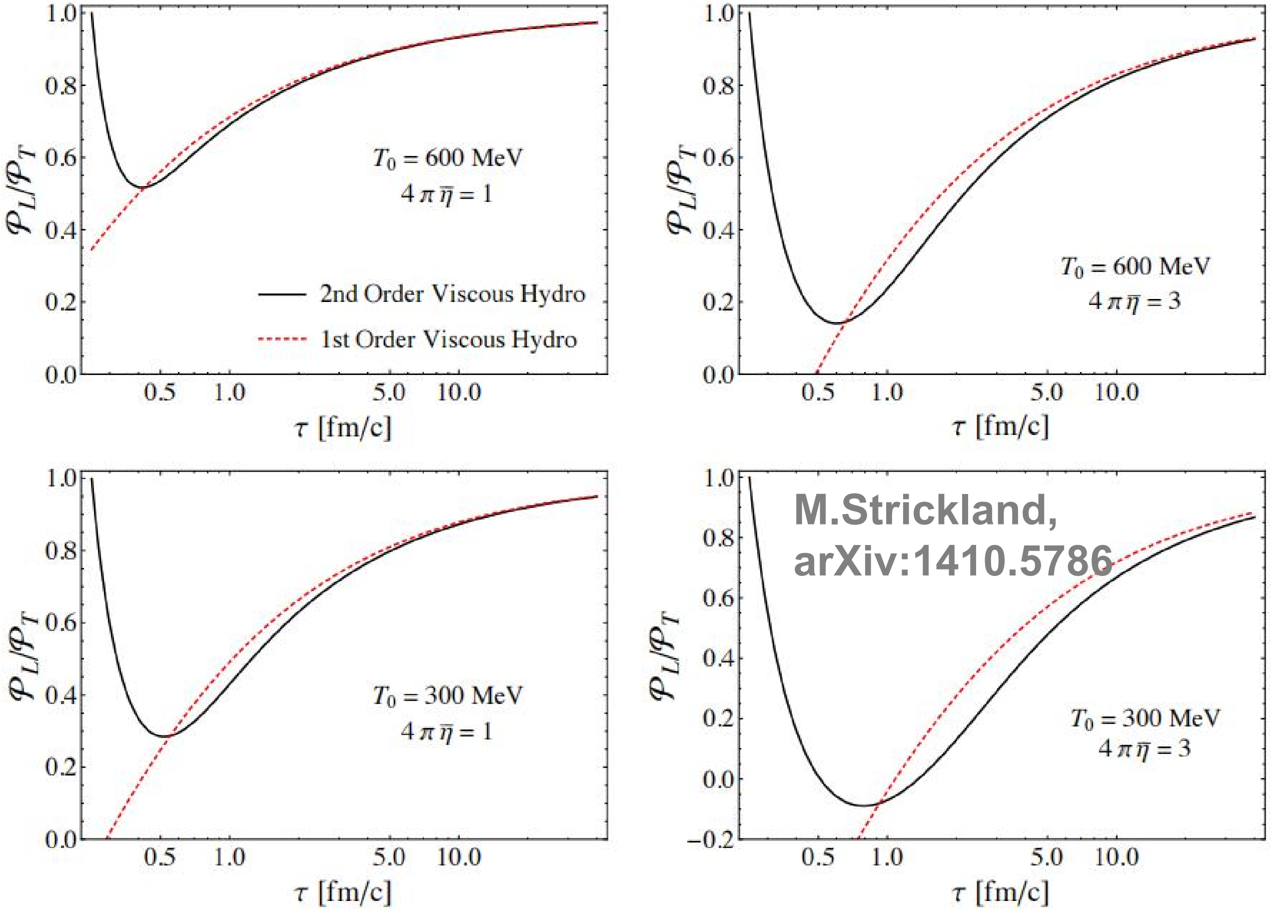


H. Niemi, G.S. Denicol, arXiv:1404.7327

Early-time pressure anisotropy (large inversed Reynolds number):

Longitudinal pressure: $\mathcal{P}_L = \mathcal{P} + \Pi + \tau^2 \pi^{\eta_s \eta_s}$

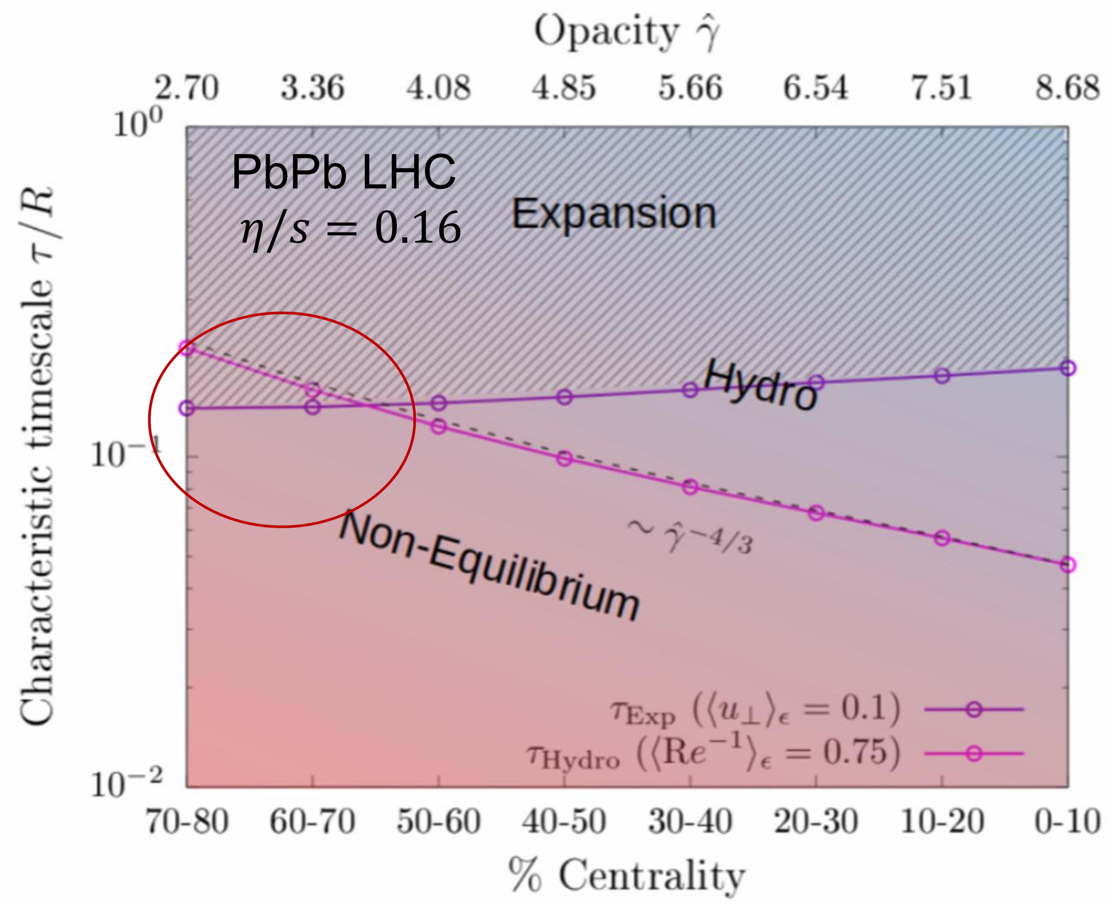
Transverse pressure: $\mathcal{P}_T = \mathcal{P} + \Pi + (\pi^{xx} + \pi^{yy})/2$



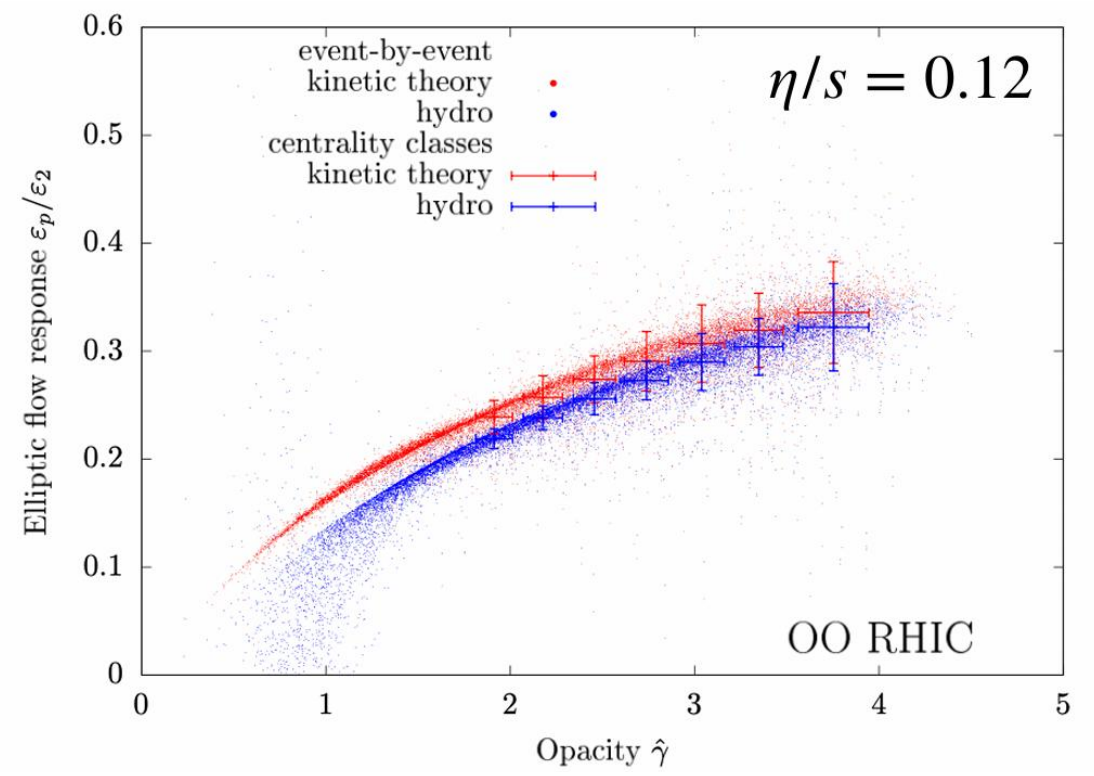
M.Strickland, arXiv:1410.5786

Non-hydro Effects on Final-state Observables

Transverse expansion before hydrodynamization in dilute systems, leading to overestimated dissipative effects on collective flow in hydrodynamics simulation.



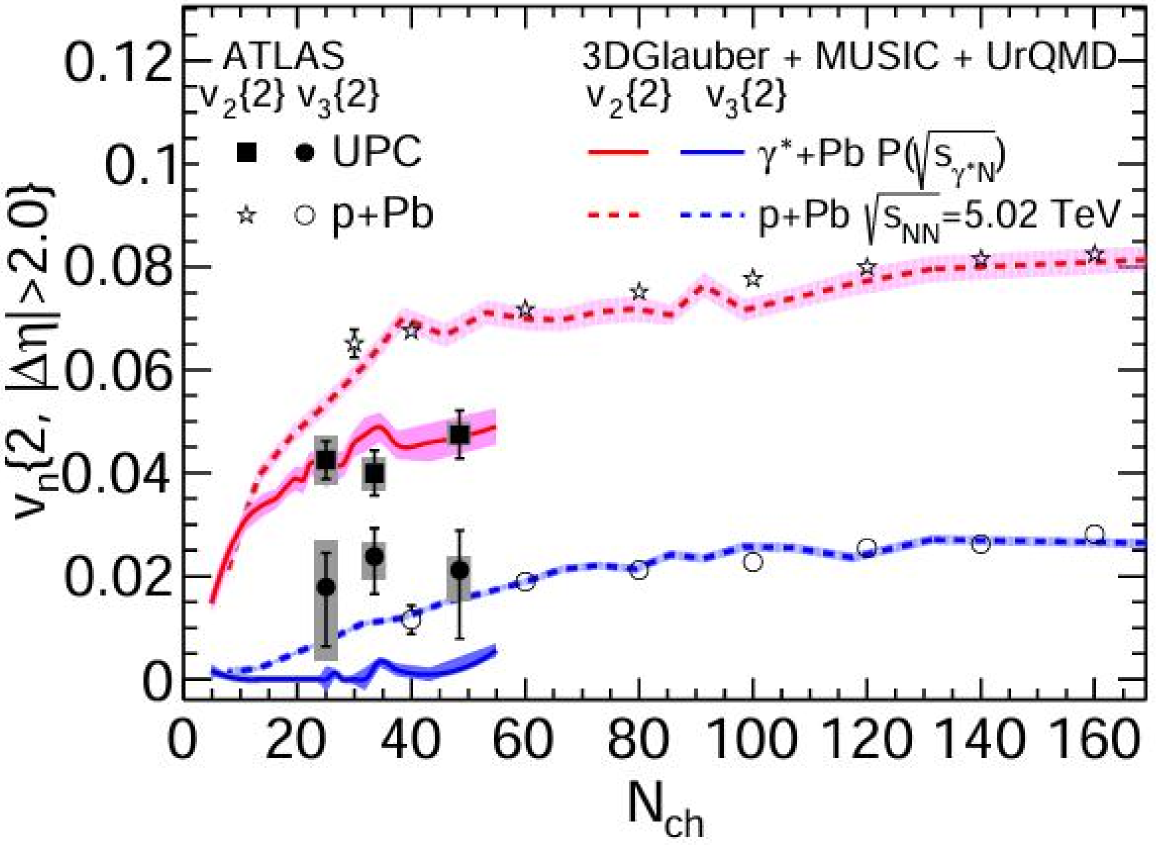
V. Ambrus, S. Schlichting and C. Werthmann, Phys. Rev. Lett. 130, 152301 (2023)



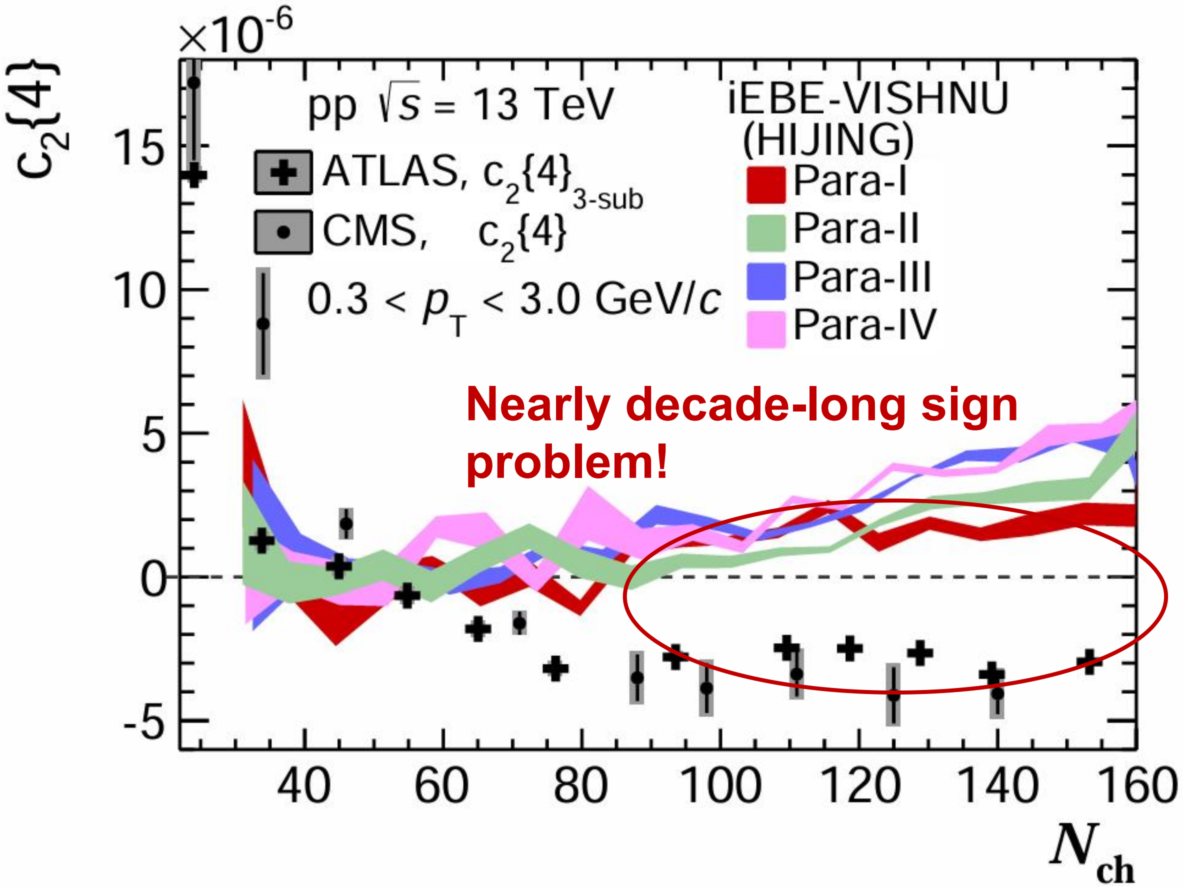
V. Ambrus, S. Schlichting and C. Werthmann, Phys. Rev. D 111, 054024 (2025)

Hydro Simulations in Small Systems

Imperfect description of 2-particle cumulants ($v_n\{2\}$) and even wrong sign of 4-particle cumulant $c_2\{4\}$:



W.Zhao, C.Shen and B.Shenke, Phys.Rev.Lett. 129 (2022) 252302



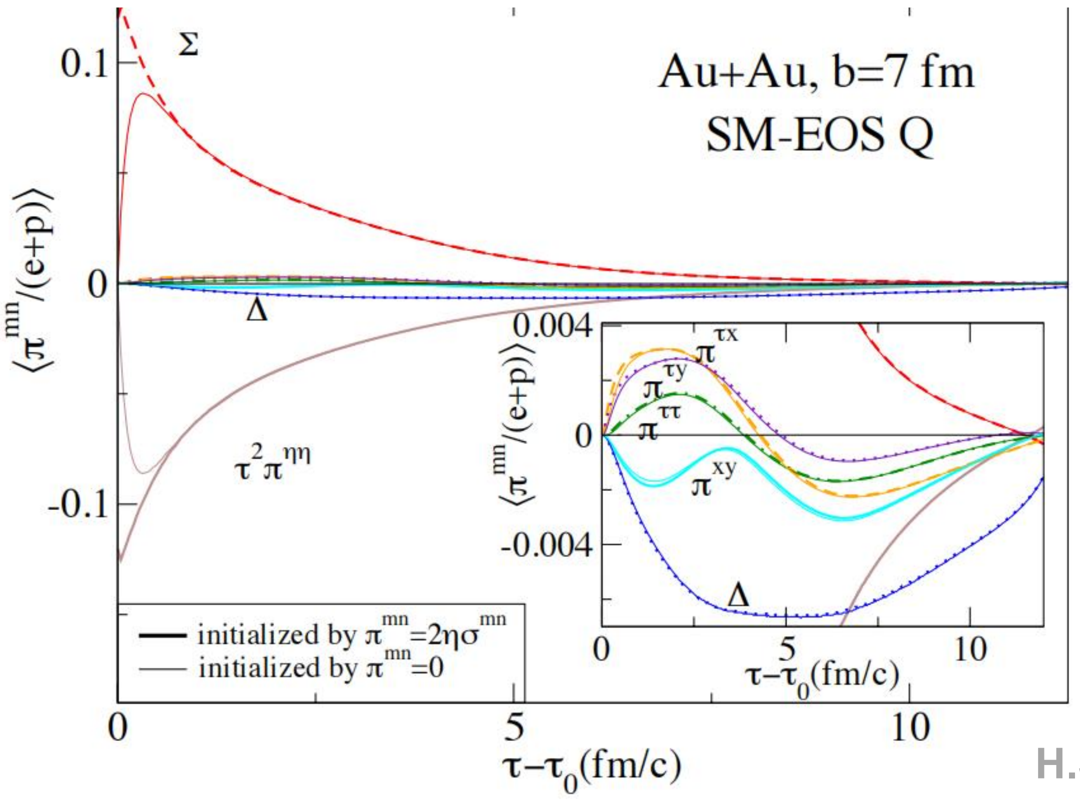
W.Zhao et al., Phys.Lett.B 780 (2018) 495-500

Can we improve viscous hydrodynamics?

Dorminant Dissipative Terms

- **Rapid longitudinal v.s. moderate transverse expansion**

$\Rightarrow$ $\tau^2\pi^{\eta\eta},\ \Sigma \equiv \pi^{xx}+\pi^{yy} \gg \Delta \equiv \pi^{xx}-\pi^{yy},\ \pi^{\mu\nu}|_{\mu\nu=\tau\tau,\tau x,\tau y,xy}$



H.Song, arXiv: 0908.3656

Only $\mathcal{P}_L - \mathcal{P}_{\rm eq}$ and $\mathcal{P}_T - \mathcal{P}_{\rm eq}$ are considerable

$\Rightarrow$ **How to describe these terms better?**

Viscous Anisotropic Hydrodynamics (VAH)

- **Difference between VAH and traditional hydro:**

Traditional hydro:

$$T^{\mu\nu} = \boxed{\mathcal{E}u^\mu u^\nu - (\mathcal{P}_{eq}} \boxed{+ \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}.}$$

dominant terms **small corrections**

$$f = f_{\mathrm{eq}} + \delta f \qquad \boxed{f_{\mathrm{eq}} = \left[\exp\left(-\sqrt{m^2+\mathbf{p}^2}/T\right) + \Theta\right]^{-1}} \qquad \boxed{\delta f = f_{\mathrm{eq}}(1-\Theta f_{\mathrm{eq}})\left[\frac{p^i p^j \pi^{ij}}{2E_{\mathbf{p}}\beta_\pi} - \frac{\Pi}{E_{\mathbf{p}} T \beta_\Pi}(m^2 - (1-3c_s^2)E_{\mathbf{p}}^2)\right]}$$

VAH:

$$T^{\mu\nu} = \boxed{\mathcal{E}\, u^\mu u^\nu + \mathcal{P}_L\, z^\mu z^\nu - \mathcal{P}_\perp\, \Xi^{\mu\nu}} \boxed{+ 2\, W_{\perp z}^{(\mu} z^{\nu)} + \pi_\perp^{\mu\nu}}$$

dominant terms **small corrections**

$$f(x,p) = f_a(x,p) + \delta\tilde{f}(x,p) \qquad \boxed{f_a = \left[\exp\left(-\frac{1}{\Lambda}\sqrt{m^2 + \frac{\mathbf{p}_\perp^2}{\alpha_\perp^2} + \frac{p_z^2}{\alpha_L^2}}\right) + \Theta\right]} \qquad \boxed{\delta\tilde{f} = f_a(1-\Theta f_a)\left(\frac{p_z \mathbf{p}_\perp \cdot \mathbf{W}_{\perp z}}{\mathcal{J}_{4210}} + \frac{p_\perp^i p_\perp^j \pi_\perp^{ij}}{2\mathcal{J}_{4020}}\right)}$$

- **Relation between different decomposition:**

$$\pi^{\mu\nu} = \pi_\perp^{\mu\nu} + 2W_{\perp z}^{(\mu} z^{\nu)} + \frac{1}{3}(\mathcal{P}_L - \mathcal{P}_\perp)\left(2z^\mu z^\nu - \Xi^{\mu\nu}\right), \quad \Pi = \frac{2\mathcal{P}_\perp + \mathcal{P}_L}{3} - \mathcal{P}_{\mathrm{eq}}.$$

- **The evolution more reasonable with large** $\mathcal{P}_{L,\perp} - \mathcal{P}_{\mathrm{eq}}$ **.**
- **Return to traditional hydro with small** $\mathcal{P}_{L,\perp} - \mathcal{P}_{\mathrm{eq}}$ **.**

M. McNelis, D. Bazow, and U. Heinz, Com. Phys. Comm 267, 108077 (2021)

Viscous Anisotropic Hydrodynamics (VAH)

Relaxation equations of VAH:

$$\dot{\mathcal{P}}_L = -\frac{\bar{\mathcal{P}} - \mathcal{P}_{\rm eq}}{\tau_\Pi} - \frac{\mathcal{P}_L - \mathcal{P}_\perp}{3\tau_\pi/2} + \bar{\zeta}^L_z z_\mu D_z u^\mu + \bar{\zeta}^L_\perp \theta_\perp - 2W^\mu_{\perp z}\dot{z}_\mu + \bar{\lambda}^L_{Wu} W^\mu_{\perp z} D_z u_\mu$$
$$+ \bar{\lambda}_{W\perp} W^\mu_{\perp z} z_\nu \nabla_{\perp\mu} u^\nu - \bar{\lambda}^L_\pi \pi^{\mu\nu}_\perp \sigma_{\perp,\mu\nu},$$

Longitudinal Pressure

$$\dot{\mathcal{P}}_\perp = -\frac{\bar{\mathcal{P}} - \mathcal{P}_{\rm eq}}{\tau_\Pi} + \frac{\mathcal{P}_L - \mathcal{P}_\perp}{3\tau_\pi} + \bar{\zeta}^\perp_z z_\mu D_z u^\mu + \bar{\zeta}^\perp_\perp \theta_\perp + W^\mu_{\perp z}\dot{z}_\mu + \bar{\lambda}^\perp_{Wu} W^\mu_{\perp z} D_z u_\mu$$
$$- \bar{\lambda}^\perp_{W\perp} W^\mu_{\perp z} z_\nu \nabla_{\perp\mu} u^\nu + \bar{\lambda}^\perp_\pi \pi^{\mu\nu}_\perp \sigma_{\perp,\mu\nu}.$$

Transverse Pressure

$$\dot{W}^{\{\mu\}}_{\perp z} = -\frac{W^\mu_{\perp z}}{\tau_\pi} + 2\bar{\eta}^W_u \Xi^{\mu\nu} D_z u_\nu - 2\bar{\eta}^W_\perp z_\nu \nabla^\mu_\perp u^\nu - \left(\bar{\tau}^W_z \Xi^{\mu\nu} + \pi^{\mu\nu}_\perp\right)\dot{z}_\nu + \bar{\delta}^W_W W^\mu_{\perp z}\theta_\perp$$
$$-\bar{\lambda}^W_{Wu} W^\mu_{\perp z} z_\nu D_z u^\nu + \bar{\lambda}^W_{W\perp}\sigma^{\mu\nu}_\perp W_{\perp z,\nu} + \omega^{\mu\nu}_\perp W_{\perp z,\nu} + \bar{\lambda}^W_{\pi u}\pi^{\mu\nu}_\perp D_z u_\nu - \bar{\lambda}^W_{\pi\perp}\pi^{\mu\nu}_\perp z_\alpha \nabla_{\perp\nu} u^\alpha,$$

Longitudinal momentum diffusion

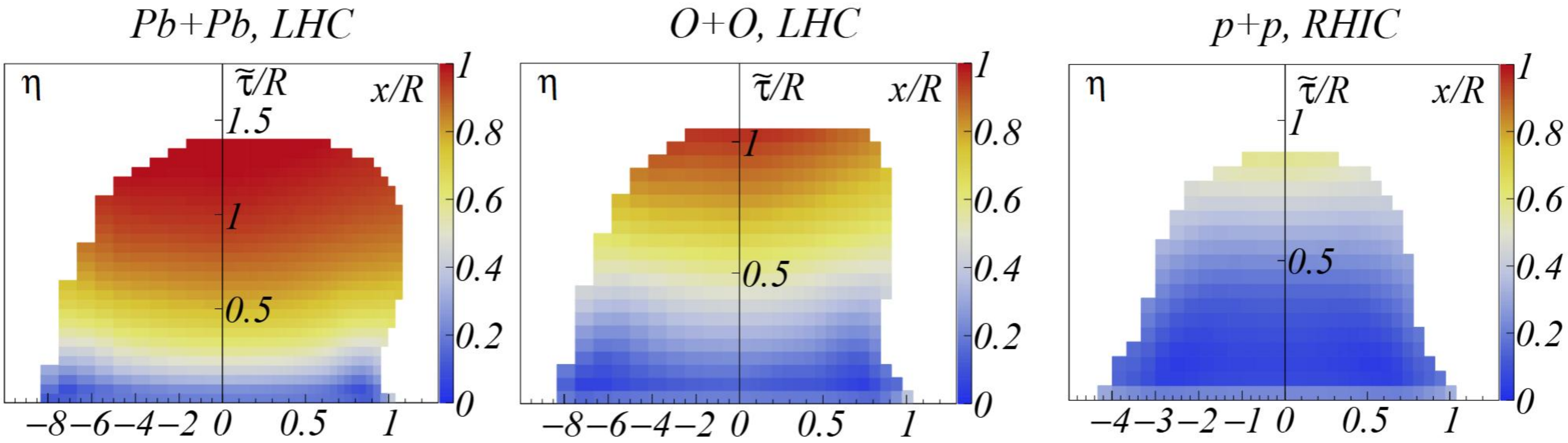
$$\dot{\pi}^{\{\mu\nu\}}_\perp = -\frac{\pi^{\mu\nu}_\perp}{\tau_\pi} + 2\bar{\eta}_\perp \sigma^{\mu\nu}_\perp - 2W^{\{\mu}_{\perp z}\dot{z}^{\nu\}} - \bar{\delta}^\pi_\pi \pi^{\mu\nu}_\perp \theta_\perp - \bar{\tau}^\pi_\pi \pi^{\alpha\{\mu}_\perp \sigma^{\nu\}}_{\perp,\alpha} + 2\pi^{\alpha\{\mu}_\perp \omega^{\nu\}}_{\perp,\alpha} + \bar{\lambda}^\pi_\pi \pi^{\mu\nu}_\perp z_\alpha D_z u^\alpha$$
$$-\bar{\lambda}^\pi_{Wu} W^{\{\mu}_{\perp z} D_z u^{\nu\}} + \bar{\lambda}^\pi_{W\perp} W^{\{\mu}_{\perp z} z_\alpha \nabla^{\nu\}}_\perp u^\alpha.$$

Transverse shear stress tensor

Evaluating Pressure Anisotropy

Evolution of $P_L/P_\perp(\tau, x, \eta)$:

- **Large anisotropy in the early stage and peripheral area.**
- **It's hard for smaller system to reach isotropization.**



S. Zhao, Y. Peng, U. Heinz and H. Song, paper in preparation

Self-consistency of VAH

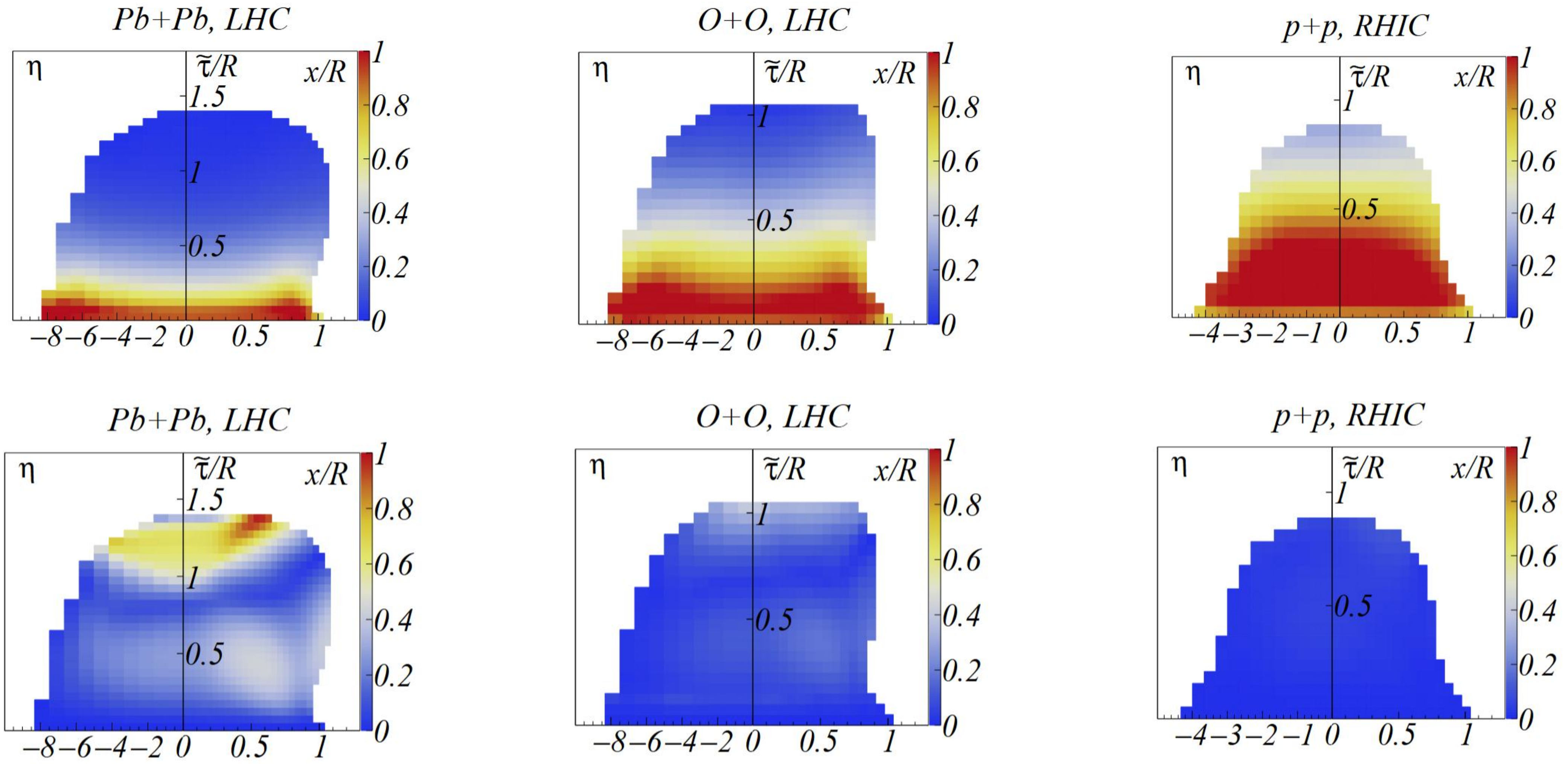
Evolution of Inversed Reynolds number:

$$\mathrm{Re}_{\pi}^{-1} = \sqrt{\pi_{\mu\nu}\pi^{\mu\nu}}/\mathcal{P}_{eq}, \quad \mathrm{Re}_{\pi_\perp W}^{-1} = \sqrt{\pi_{\perp}^{\mu\nu}\pi_{\perp\mu\nu} - W_{\perp z\mu}W_{\perp z}^{\mu}}/\mathcal{P}_{eq},$$

- **Small $\pi_{\perp}^{\mu\nu}, W_{\perp z}^{\mu}$ indicate self-consistency of VAH from large to small systems.**

Re_{π}^{-1} :

$\dfrac{\mathrm{Re}_{\pi_\perp W}^{-1}}{\mathrm{Re}_{\pi}^{-1}}$:

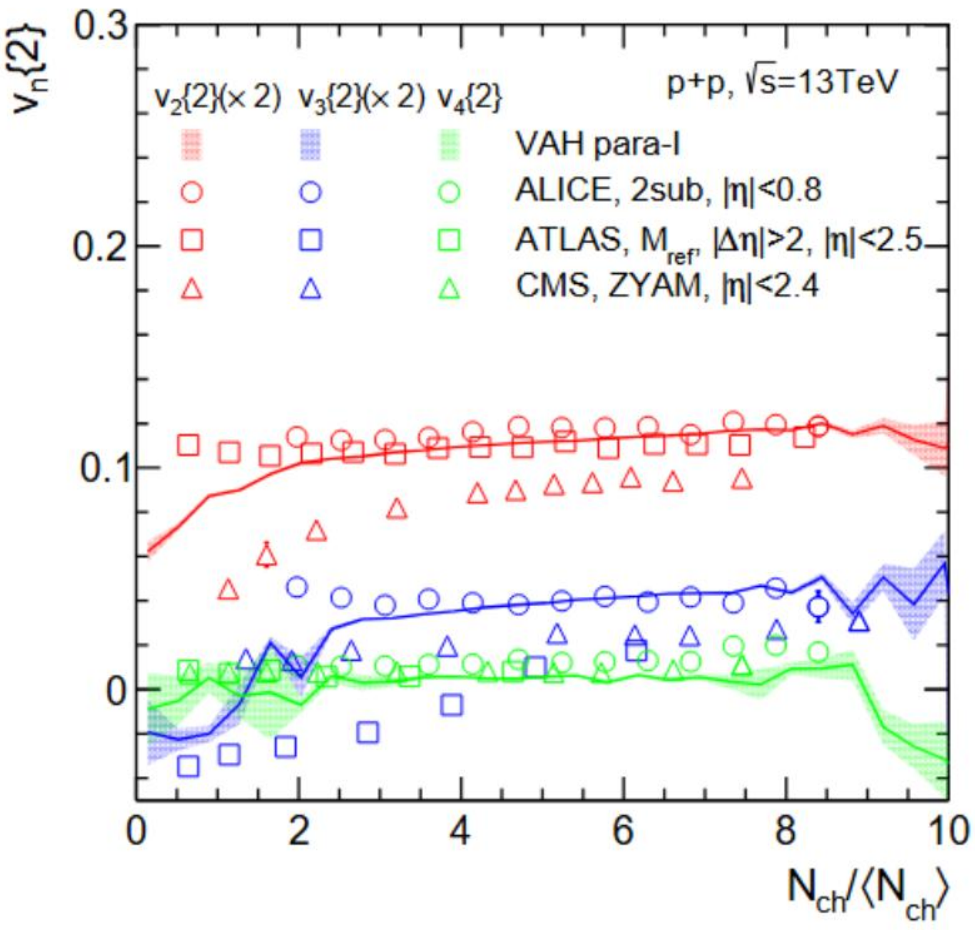


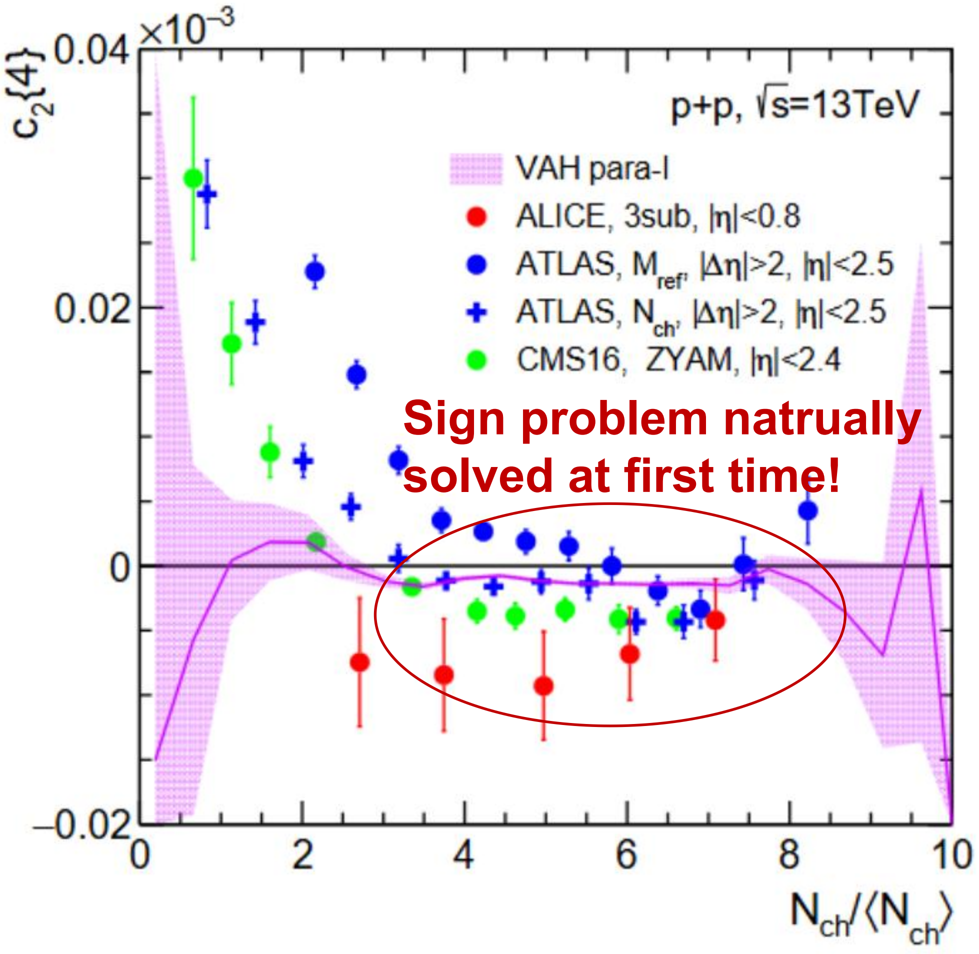
Apply VAH in p+p at 13 TeV

(2+1)-D simulation in the TRENTo3D+VAH+SMASH framework:

- **Good description of $v_n\{2\},\ n = 2, 3, 4$.**
- **Produce correct sign of $c_2\{4\}$.**

S. Zhao, Y. Peng, U. Heinz and H. Song, arXiv: 2509.03841

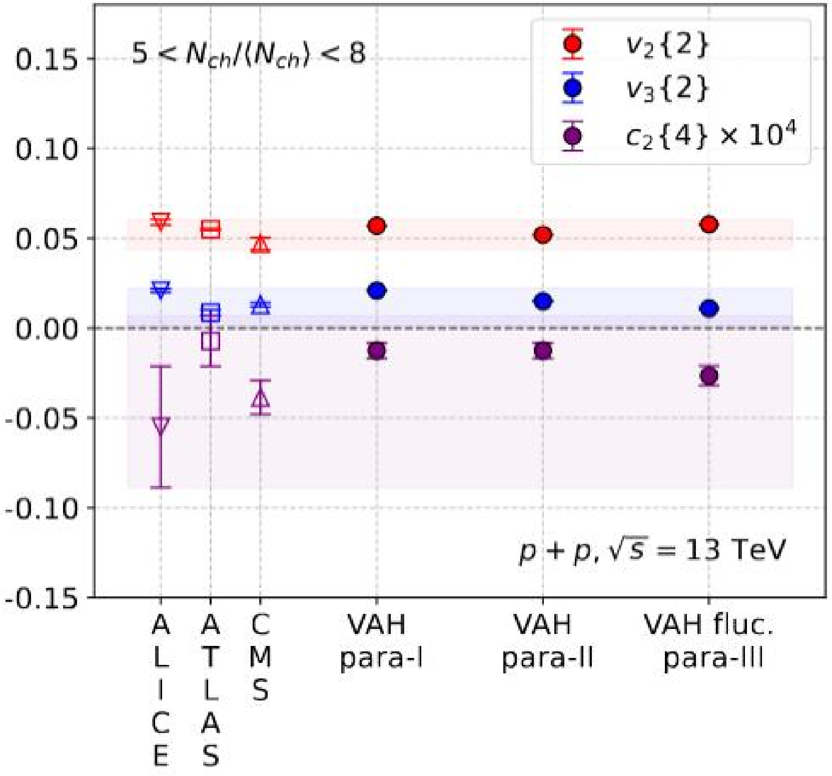


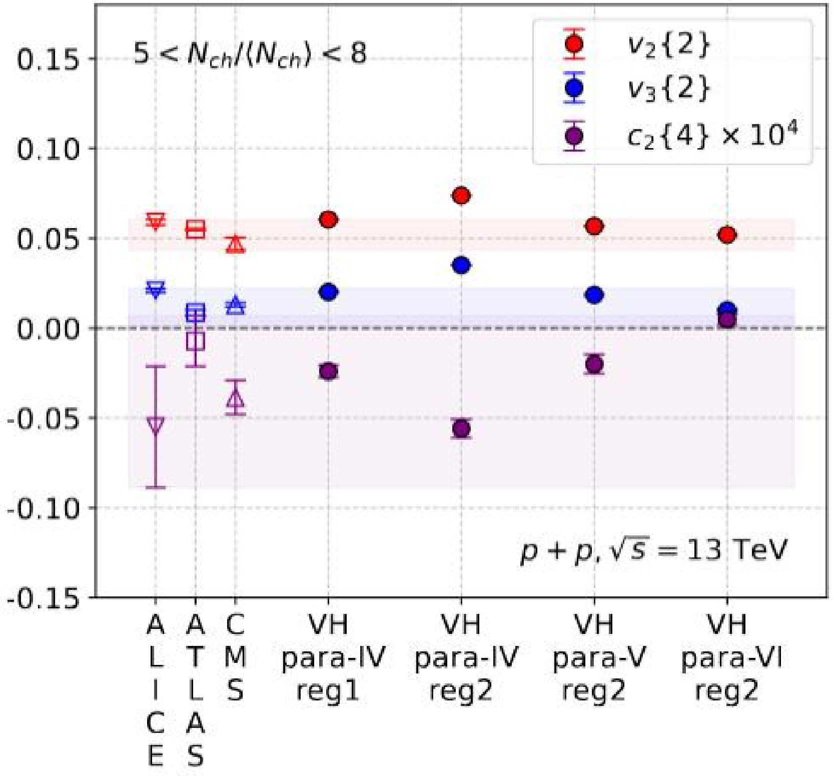


Apply VAH in p+p at 13 TeV

VAH and viscous hydro (VH) with different parameters and regulation schemes:

- **Robustness of $c_2\{4\}$ sign for VAH.**
- **For VH, $c_2\{4\}$ sign has significant parameters and regulation dependence.**





VAH, Hydro and Kinetic Theory Comparison

Assumptions:

- Finite transverse extent with longitudinal boost invariance ((2+1)-D simulation)
- Systems consist of massless particles (conformal symmetry).

$$\mathcal{P}_L + 2\mathcal{P}_\perp = 3\mathcal{P} = \mathcal{E}, \quad \Pi = 0$$

Boltzmann equation in the relaxation time approximation (RTA):

$$\boxed{p \cdot \partial f(x,p) = -\frac{u \cdot p}{\tau_R(x)}[f(x,p) - f_{eq}(x,p)], \quad \tau_R = 5\eta/(sT)}$$

→ Benchmark

Traditional viscous hydro:

Y. Peng, V. Ambrus, C. Werthmann, S. Schlichting, U. Heinz and H. Song, arXiv: 2509.04431

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + 2\tau_\pi \pi_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} - \delta_{\pi\pi}\pi^{\mu\nu}\theta - \tau_{\pi\pi}\pi^{\lambda\langle\mu}\sigma_\lambda^{\nu\rangle} + \phi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha},$$

VAH:

$$\dot{P}_L = -\frac{P_L - P_\perp}{3\tau_\pi/2} + \bar{\zeta}_z^L \theta_L + \bar{\zeta}_\perp^L \theta_\perp - \bar{\lambda}_\pi^L \pi_\perp^{\mu\nu}\sigma_{\perp,\mu\nu},$$

$$\dot{\pi}_\perp^{\{\mu\nu\}} = -\frac{\pi_\perp^{\mu\nu}}{\tau_\pi} + 2\bar{\eta}_\perp \sigma_\perp^{\mu\nu} - \bar{\lambda}_\pi^\pi \pi_\perp^{\mu\nu}\theta_L - \bar{\delta}_\pi^\pi \pi_\perp^{\mu\nu}\theta_\perp - \bar{\tau}_\pi^\pi \pi_\perp^{\lambda\{\mu}\sigma_{\perp,\lambda}^{\nu\}} + 2\pi^{\lambda\{\mu}\omega_{\perp,\lambda}^{\nu\}},$$

Coefficients in consistency with RTA (e.g., $\tau_\pi = \tau_R$)

VAH, Hydro and Kinetic Theory Comparison

Quantities of interest:

Transverse energy $$\frac{dE_\perp}{d\eta} = \tau \int_{\mathbf{x}_\perp} (T^{xx} + T^{yy})$$ ➡ Cooling behavior

Inversed Reynolds number $$\langle \mathrm{Re}^{-1} \rangle_\mathcal{E} = \left\langle \left(\frac{\pi^{\mu\nu}\pi_{\mu\nu}}{\mathcal{E}^2} \right)^{1/2} \right\rangle_\mathcal{E}$$ ➡ Hydrodynamization

Average transverse velocity $$\langle u_\perp \rangle_\mathcal{E} = \left\langle \left(u_x^2 + u_y^2 \right)^{1/2} \right\rangle_\mathcal{E}$$ ➡ Radial expansion

Momentum anisotropy $$\varepsilon_p = \frac{\int_{\mathbf{x}_\perp} (T^{xx} - T^{yy})}{\int_{\mathbf{x}_\perp} (T^{xx} + T^{yy})}$$ ➡ Response to initial spatial anisotropy (resemble with v_2)

Radial expansion and response to initial spatial anisotropy: Transverse dynamics (comparison never made before!)

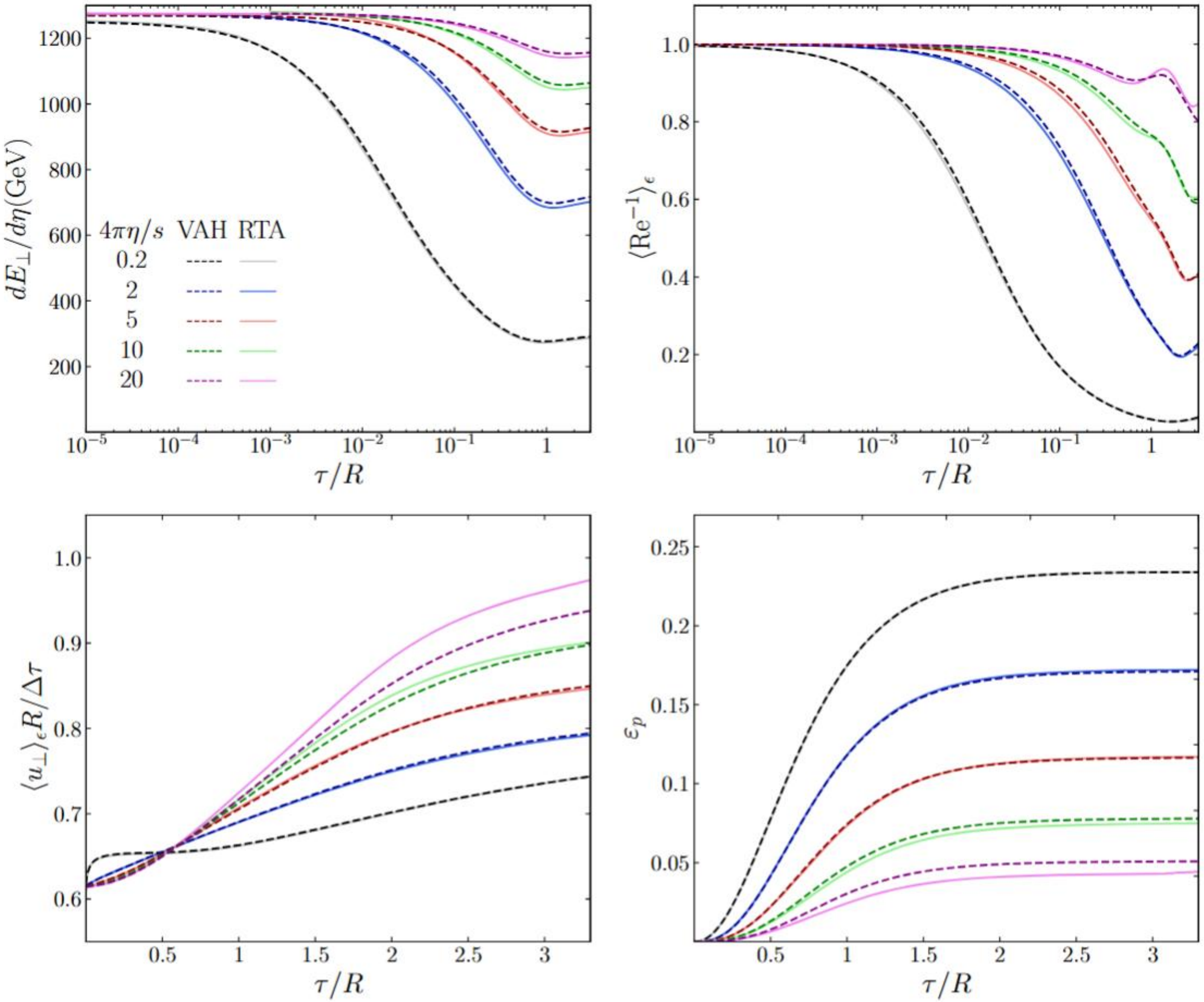
Criterion of diluteness (Opacity):

$$\hat{\gamma} = \frac{1}{5\eta/s} \left(\frac{R}{\pi a} \frac{dE_\perp^0}{d\eta} \right)^{1/4}$$

- The only parameter that determines the evolution.
- Encoding system size, viscosity and energy dependence.
- Transition from free-streaming (ultra-dilute) to ideal Hydro (ultra-dense) with opacity increasing from 0 to ∞ (shear viscosity from ∞ to 0).

Evolution Behaviours

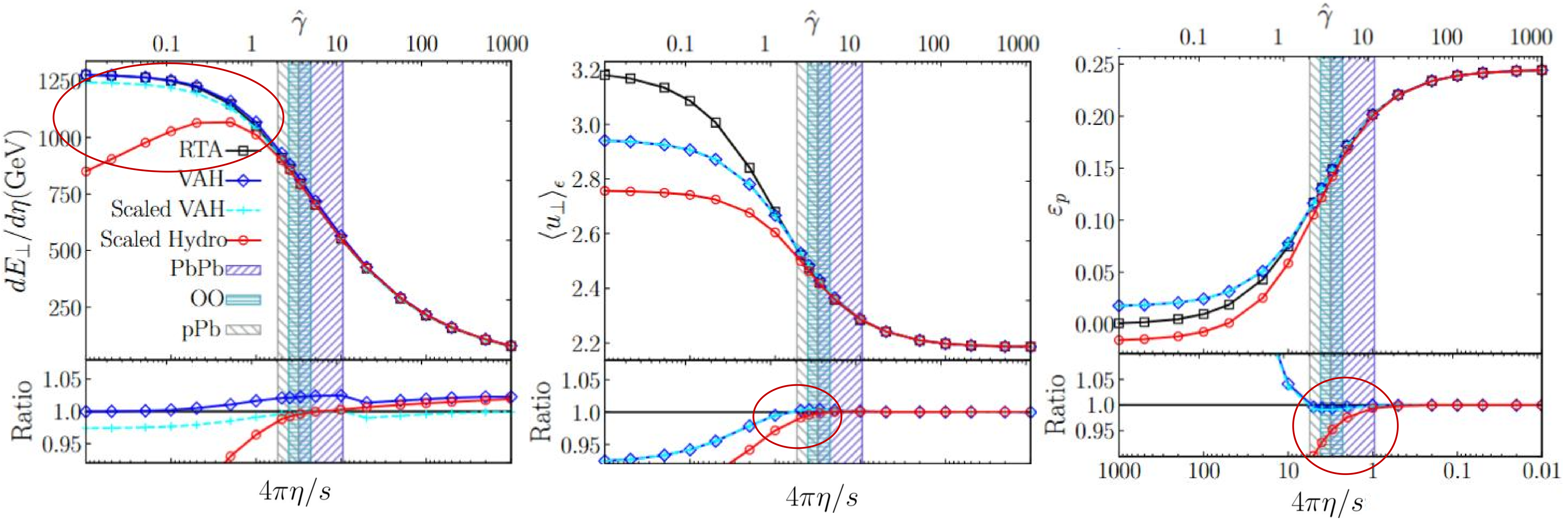
- **VAH nearly converges to RTA in early-time evolution.**
- **Good agreement with RTA at smal or miderate shear viscosities ($4\pi\eta/s \leq 5$)**
- **Discrepancies emerge at large shear viscosity ($4\pi\eta/s \geq 10$), corresponding to very small opacity ($\hat{\gamma} \lesssim 1$)**



Final-state Results

Final state results ($\tau = 3R$) as functions of opacity:

- **Traditional hydro gradually deviates from RTA in O+O and p+Pb collisions, while VAH maintains good agreement.**
- **VAH fails to capture transverse dynamics at exceptionally large shear viscosity (small opacity).**



Summary

- **Viscous anisotropic hydrodynamics (VAH) is an extended version of viscous hydrodynamics with self-consistent assumptions.**
- **Properly capture the sign of 4-particle cumulant $c_2\{4\}$ in p+p collisions at 13 TeV.**
- **Good agreements with microscopic theory in dilute systems when traditional hydro fails; extending the applicability of hydrodynamics.**