The Quantum Road to Thermalization: Entropy Production in Heavy-Ion Collision

Chenxi Liang

Institute of Modern Physics, Fudan Univerity

QCD Phase Transition and Relativistic Heavy-Ion Physics (QPT2025)

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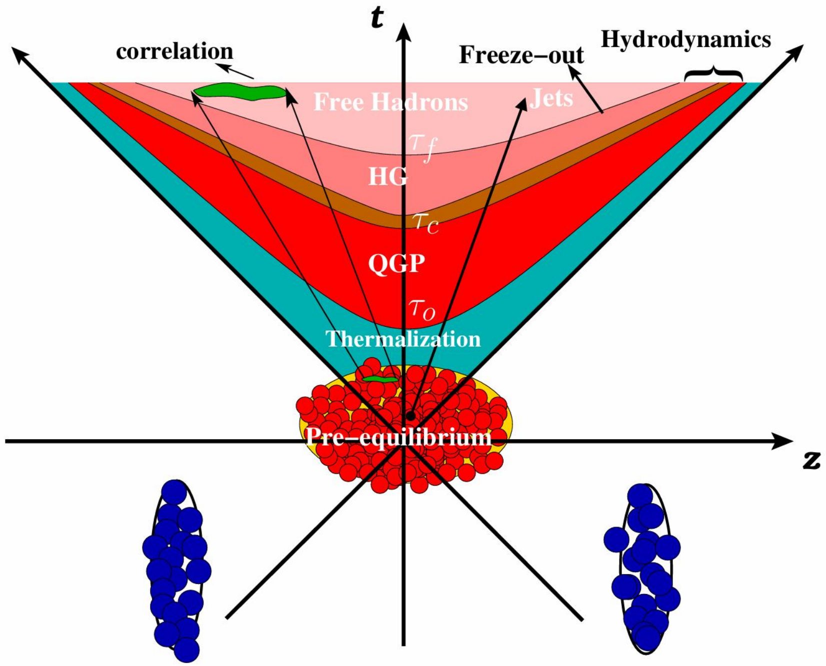
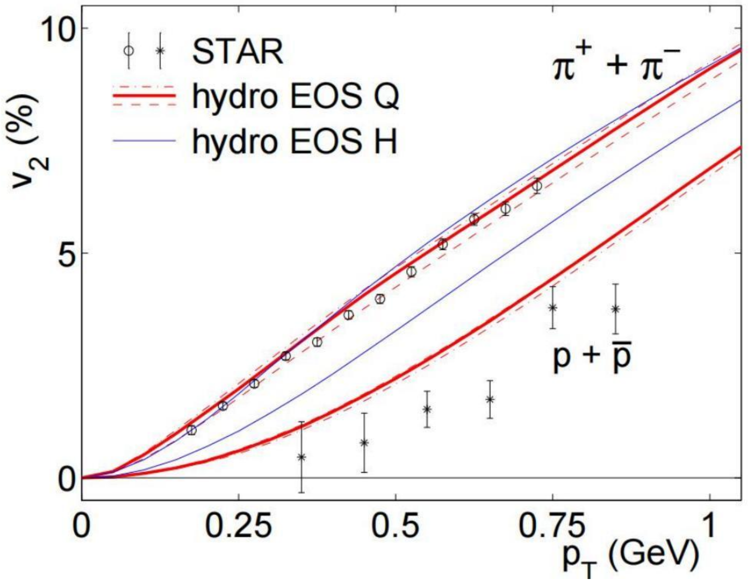
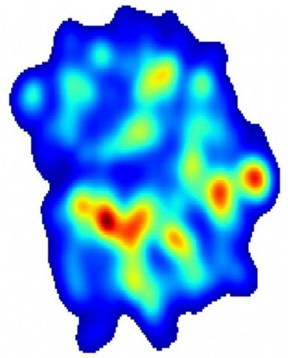
with Li Yan, Xiaojian Du

Thermalization assumption in QGP





[P. Kolb and U. Heinz, nucl-th/0305084]



i.c.

$$\begin{cases} \partial_\mu T^{\mu\nu} = 0 \\ T^{\mu\nu} = e u^\mu u^\nu + (P - \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} \\ (\tau_\pi u \cdot \partial + 1)\pi^{\mu\nu} = \eta\sigma^{\mu\nu} + \ldots \\ (\tau_\Pi u \cdot \partial + 1)\Pi = -\zeta\nabla \cdot u + \ldots \end{cases}$$

$$+ \quad \text{LEOS: } e(P)\,, e(T)\,, P(T)\,, \ldots$$

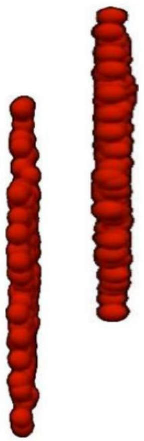
freeze-out

hadron transport

Entropy production $\leftrightarrow$ Thermalization

Entropy production in different stages

- Evolution stages and effective model characterization

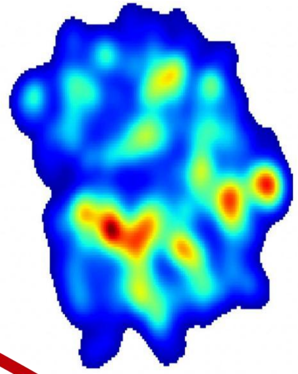


I

late stages →

II

S_{hydro}



III

S_{total}

?

I. Entanglement Entropy and Quantum Thermalization.

?

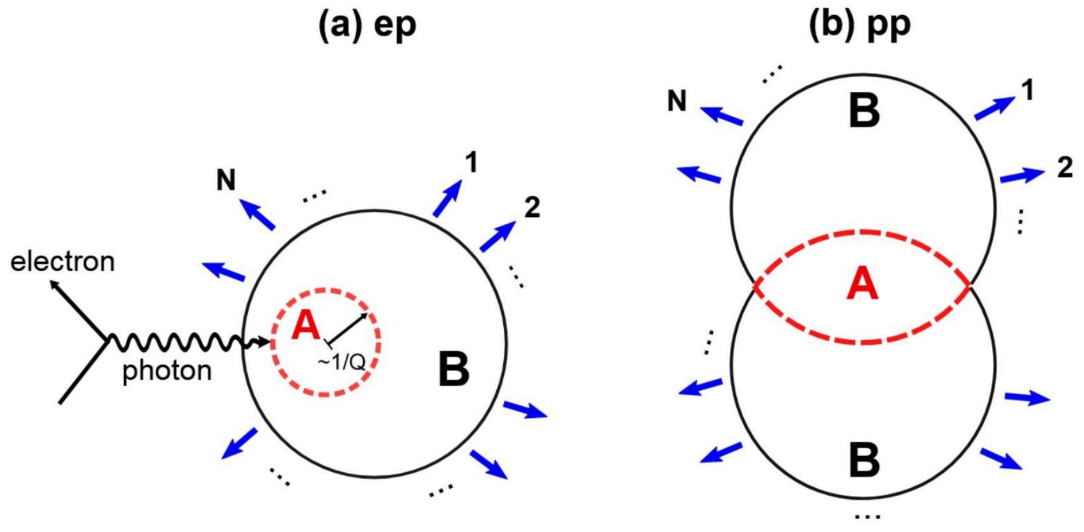
II. Pre-equilbrium Entropy: e.g., attractor, KOMPOST

III. Fluid Entropy production: dissipative fluid:

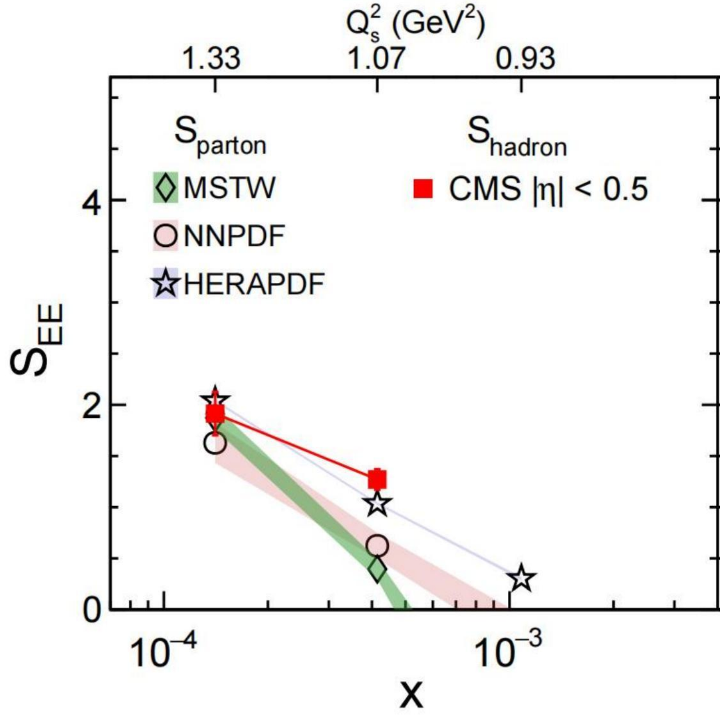
$$\partial_\mu s^\mu = \pi^{\alpha\beta}\langle\nabla_\alpha u_\beta\rangle/T + \dots$$

Entanglement entropy production in high-Q^2 inclusive scatterings

[D. Kharzeev and E. Levin, PRD 95, 114008 (2017),
Z. Tu, D. Kharzeev and T. Ulirich, PRL19]



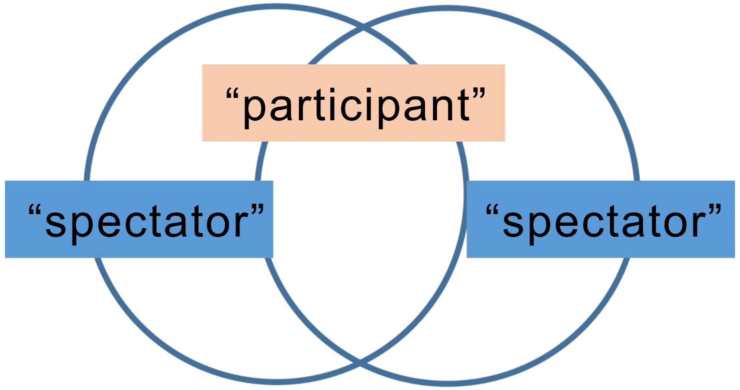
$$S_{\mathrm{A}} = -tr[\hat{\rho}_{\mathrm{A}} \ln \hat{\rho}_{\mathrm{A}}] = S_{\mathrm{B}} = -tr[\hat{\rho}_{\mathrm{B}} \ln \hat{\rho}_{\mathrm{B}}] \overset{?}{=} S_{\mathrm{hadron}} = -\sum \mathrm{P(N)} \ln \mathrm{P(N)}$$



- In high-Q^2 inclusive scatterings no interference among states of different n_{parton} , in small x limit.
- Region involved in collision approaches maximally entanglement.
- Entanglement entropy is calculable and related to the parton distribution function (pdf).

$$S_{En} = \log(xG(x, Q^2))$$

Quantum thermalization in inclusive scatterings



- “Spectators” are traced away → reduced density matrix

$$|\psi\rangle_p = \sum_n \frac{1}{\sqrt{\mathcal{D}}} |n, x, Q^2\rangle \,, \qquad \mathcal{D}\text{: dimension of Fock space}$$

Renormalization scale

$$\rho_p = |\psi\rangle_{pp}\langle\psi|$$

$$\downarrow$$

$$S_{En} = -\mathrm{tr}[\rho_p \log \rho_p]$$

$$\downarrow$$

$$\log \mathcal{D} \approx \log(xG(x, Q^2))$$

$$Q \gg \Lambda_{QCD} \,, \quad x \ll 1$$

$\ln\frac{1}{x}$++

t++

time scale evolution

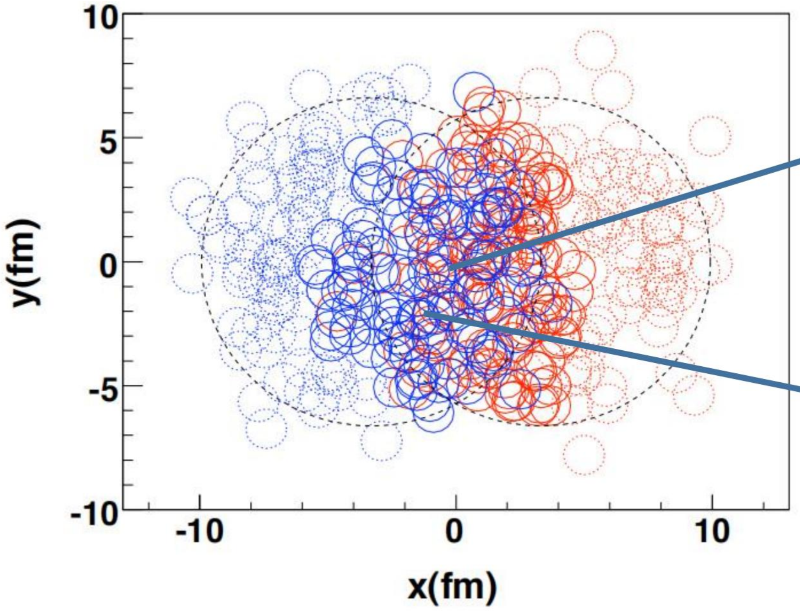
$$N_{mn} = N(E)\delta_{mn} + e^{-S} r_{mn}$$

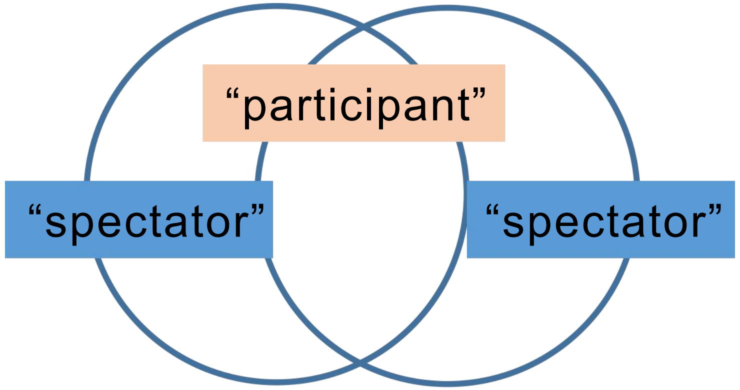
$$\Big\downarrow \text{ ETH}$$

$$xG(x, Q^2) \equiv \langle\psi|\hat{N}|\psi\rangle = \frac{\mathcal{D}}{2}$$

no interference among states of different n_{parton}

Generalization to heavy-ion collisions





$$S_{En} = N_{part} \times \log[xG_N(x, Q^2)]$$

- Generalization: (pdf $\rightarrow$ npdf) + nuclear geometry from Trento [2112.12462]
- Consider central rapidity and Q depends on collision energy and collision system, e.g.,

$$Q \sim A^{1/3}, \qquad Q \sim \sqrt{s}^{\lambda}, \qquad x \sim Q/\sqrt{s}$$

[L.McLerran and R. Venugopalan, PRD 1994]

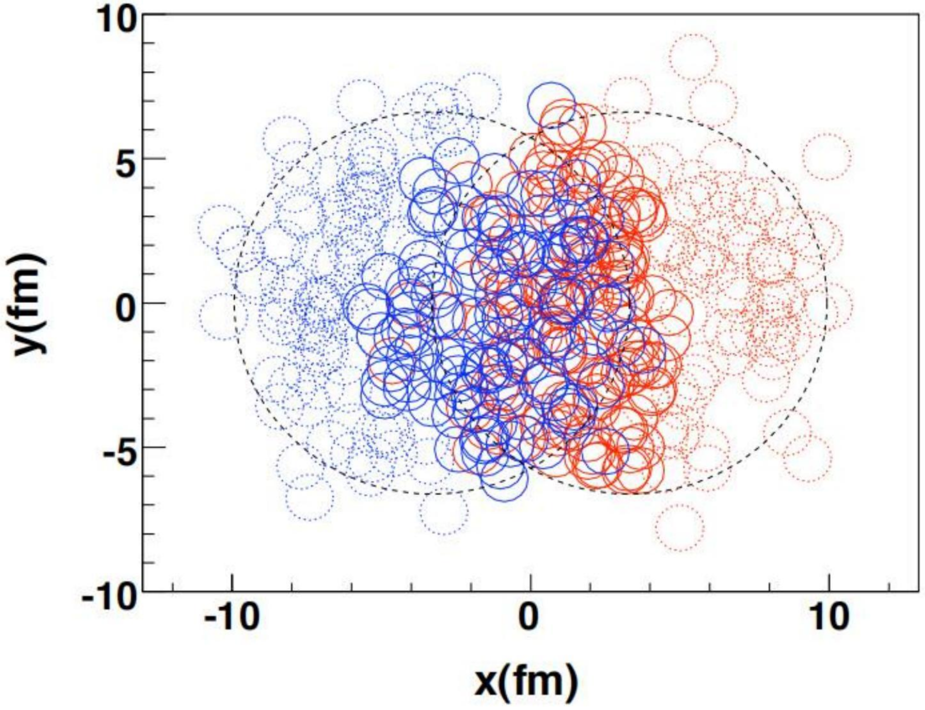
- A more sophisticated/realistic formulation:

$$S_{En} = \sum_{i}^{N_{part}} \log[xG_N(x, Q^2(\vec{x}^{\,i}_{\perp}, b))]$$

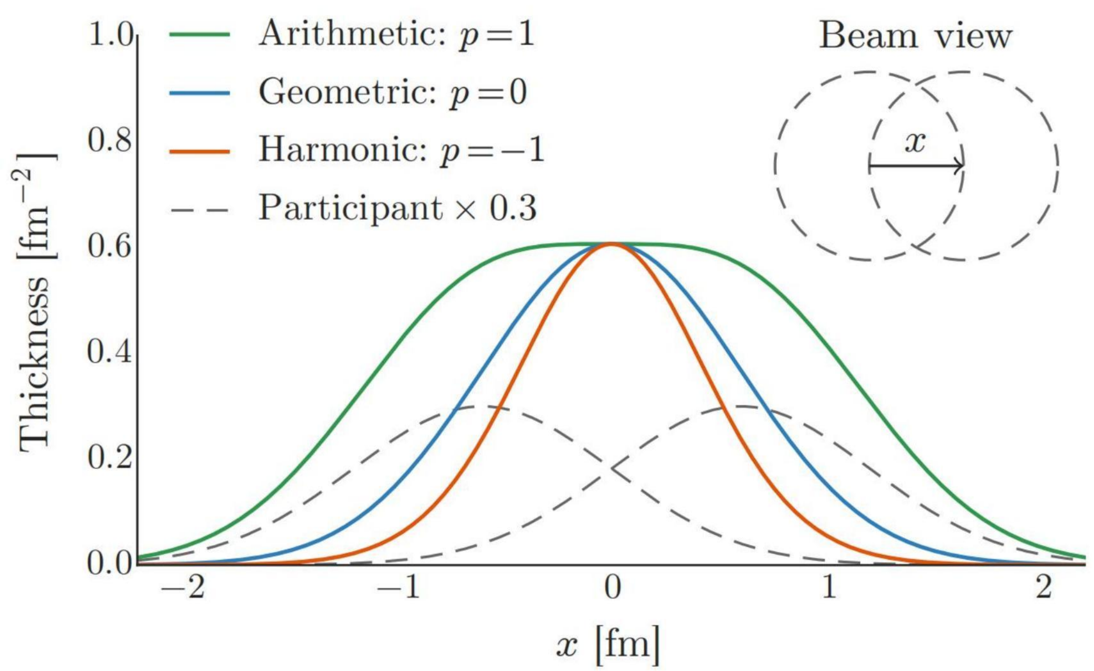
[H. Kowalski and D. Teaney, PRD 2003]

Thermalization assumption and entropy production in models

[MC-Glauber, 0805.4411]



[J. Moreland, J. Bernhard, and S.Bass, 1412.4708]

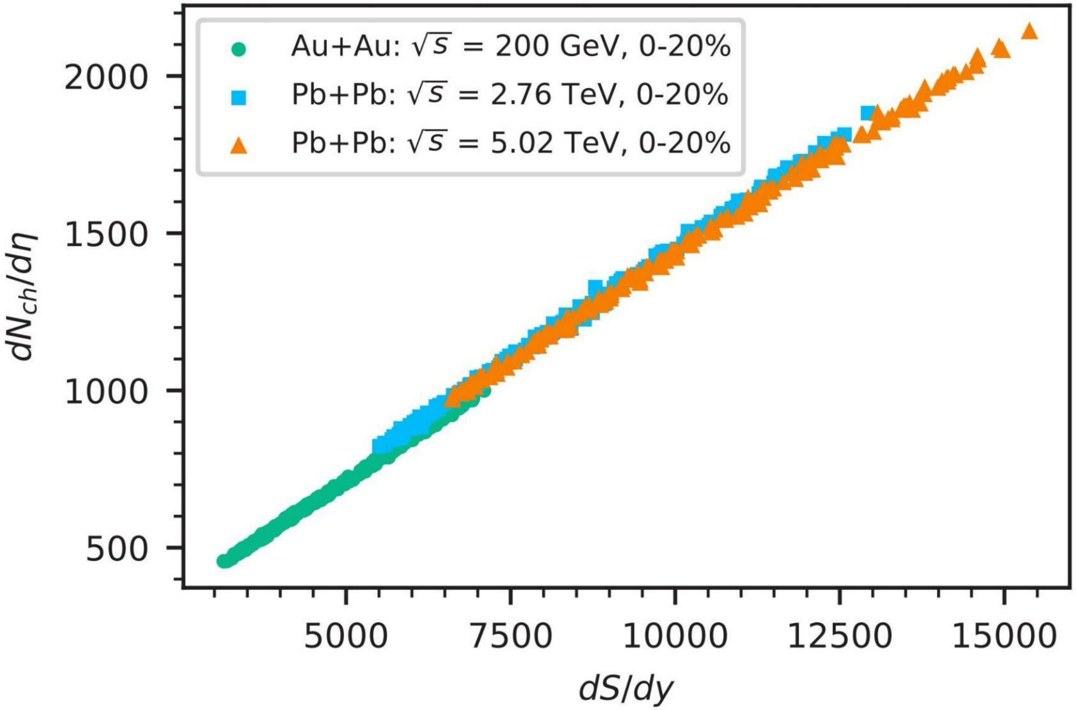


- Determine where entropy is produced.
- Determine whether entropy is produced.
- How much entropy is produced?

$$\frac{dS}{dy}\Big|_{\tau_0} = N_{norm}\underbrace{T_R(T_A, T_B)}_{\text{nucleon-nucleon collision}} \propto \left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}$$

Estimate entropy production from $dN_c/d\eta$

- Linearity between $dN_c/d\eta$ and dS/dy



$$\frac{dN_c}{d\eta} \approx \frac{1}{7.14}\frac{dS}{dy}\Big|_{hyd}$$

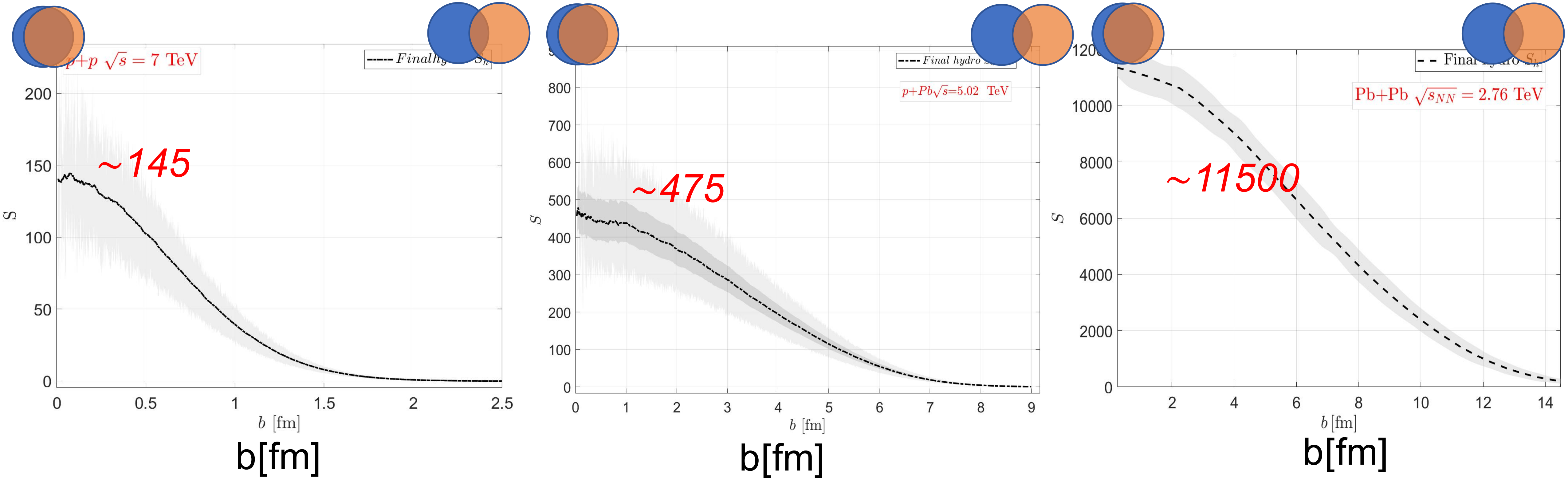
[J. Churchill, LY, S. Jeon and C. Gale, PRC 21]

[P. Hanus, K. Reygers and A. Mazeliauskas, 1908.02792]

system	dS/dy	$(dS/dy)/(dN_{ch}/dy)$
Pb–Pb, 0–10%	$11\,335 \pm 1188$	6.7 ± 0.8
pp minimum bias	37.8 ± 3.7	5.2 ± 0.5
pp high mult.	135.7 ± 17.9	5.4 ± 0.7

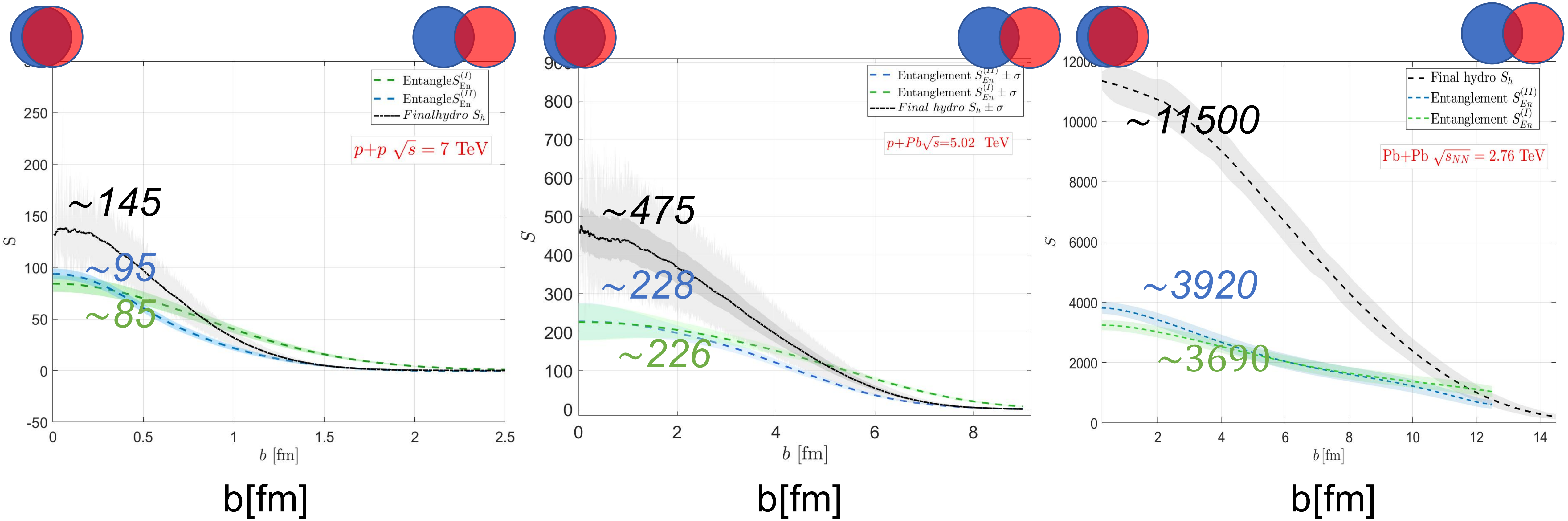
$$\frac{dN_c}{d\eta} \propto \frac{dS}{dy}\Big|_{total} \gtrsim \frac{dS}{dy}\Big|_{hyd}$$

Empirical estimate of the total entropy S_{total} from Trento



- Calibrate with respect to $N_c \propto S_{total}$ in ultra-central events.
- Determine collision centrality using EbE Trento, estimate S_{total} for all centralities.
- Empirical based on hydro, less reliable at peripheral collisions.

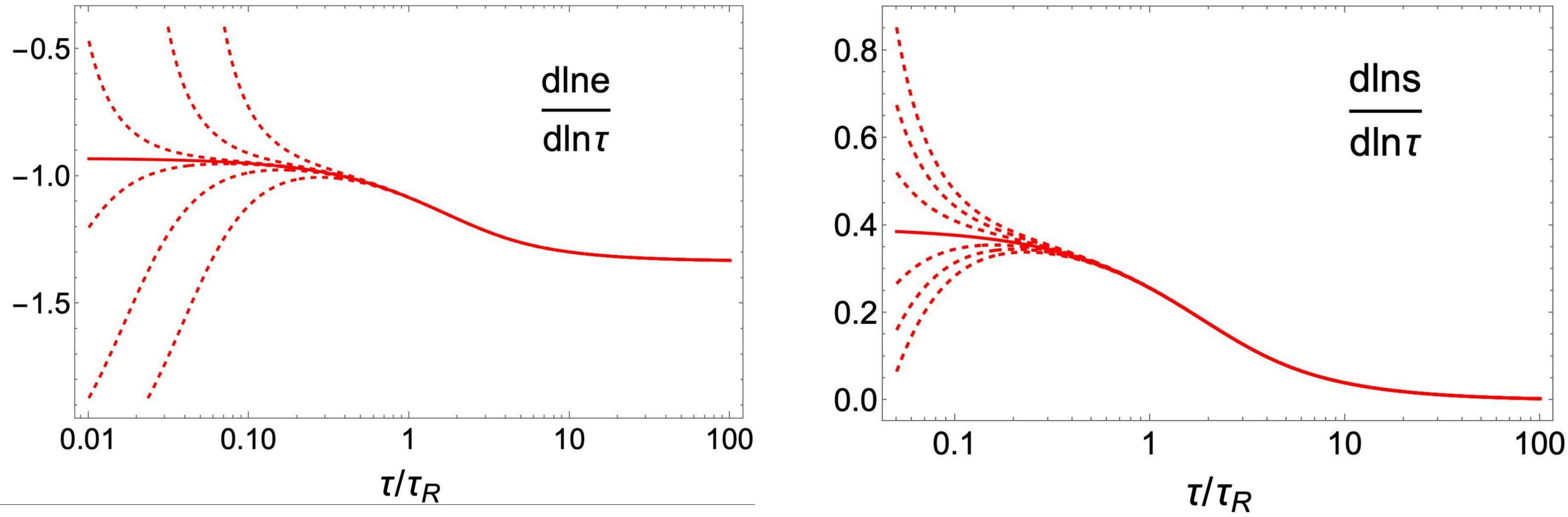
Entanglement entropy production



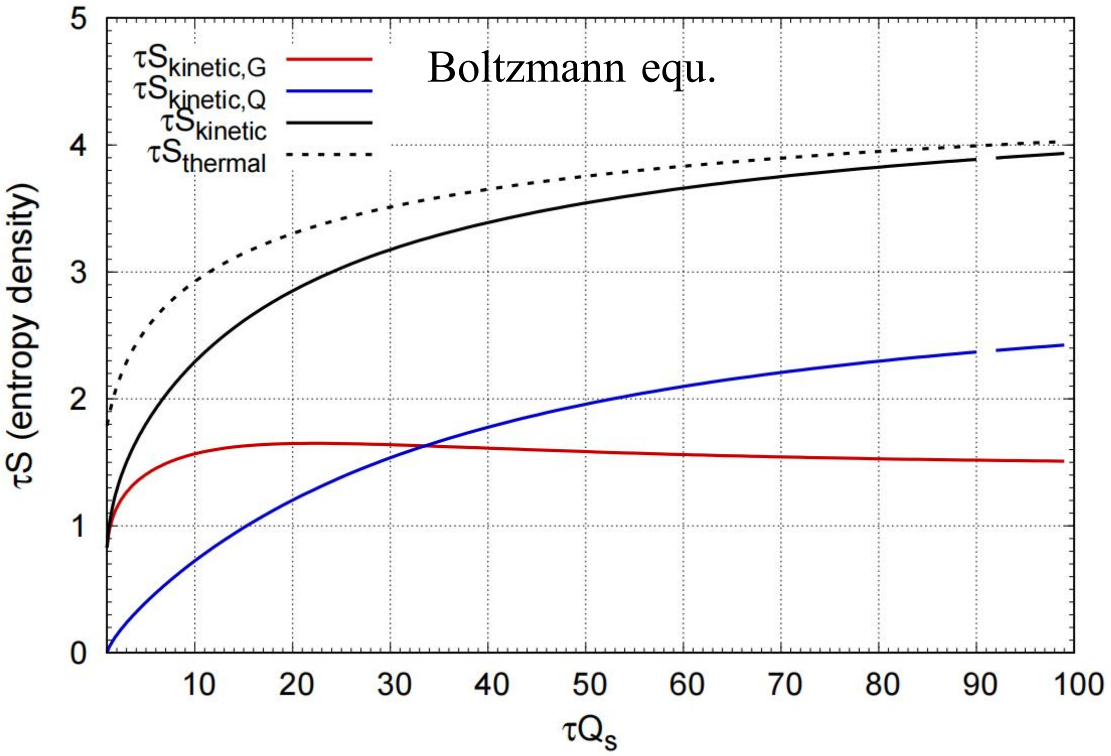
Exclude S_{En}, one does notice an increase of entropy production from late stages, from pp to PbPb.

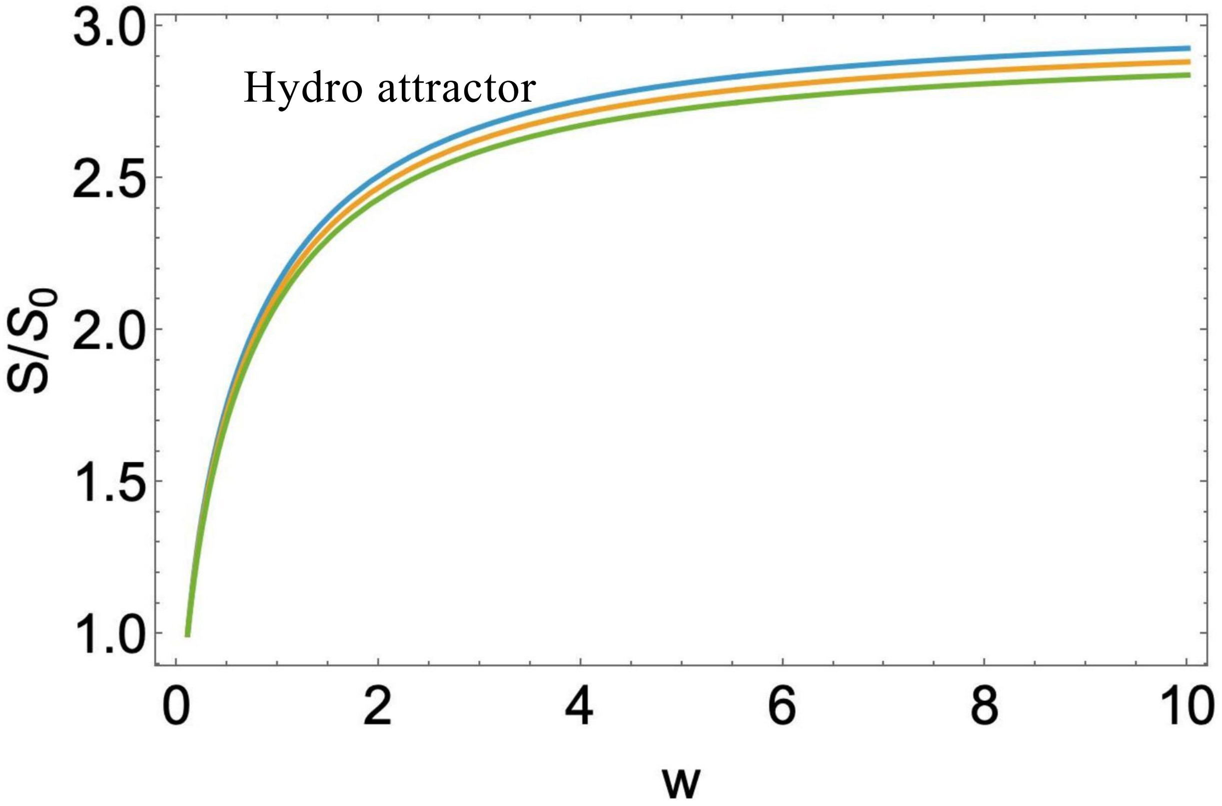
Non-equilibrium entropy production

- Hydro attractor



Non-equilibrium entropy production



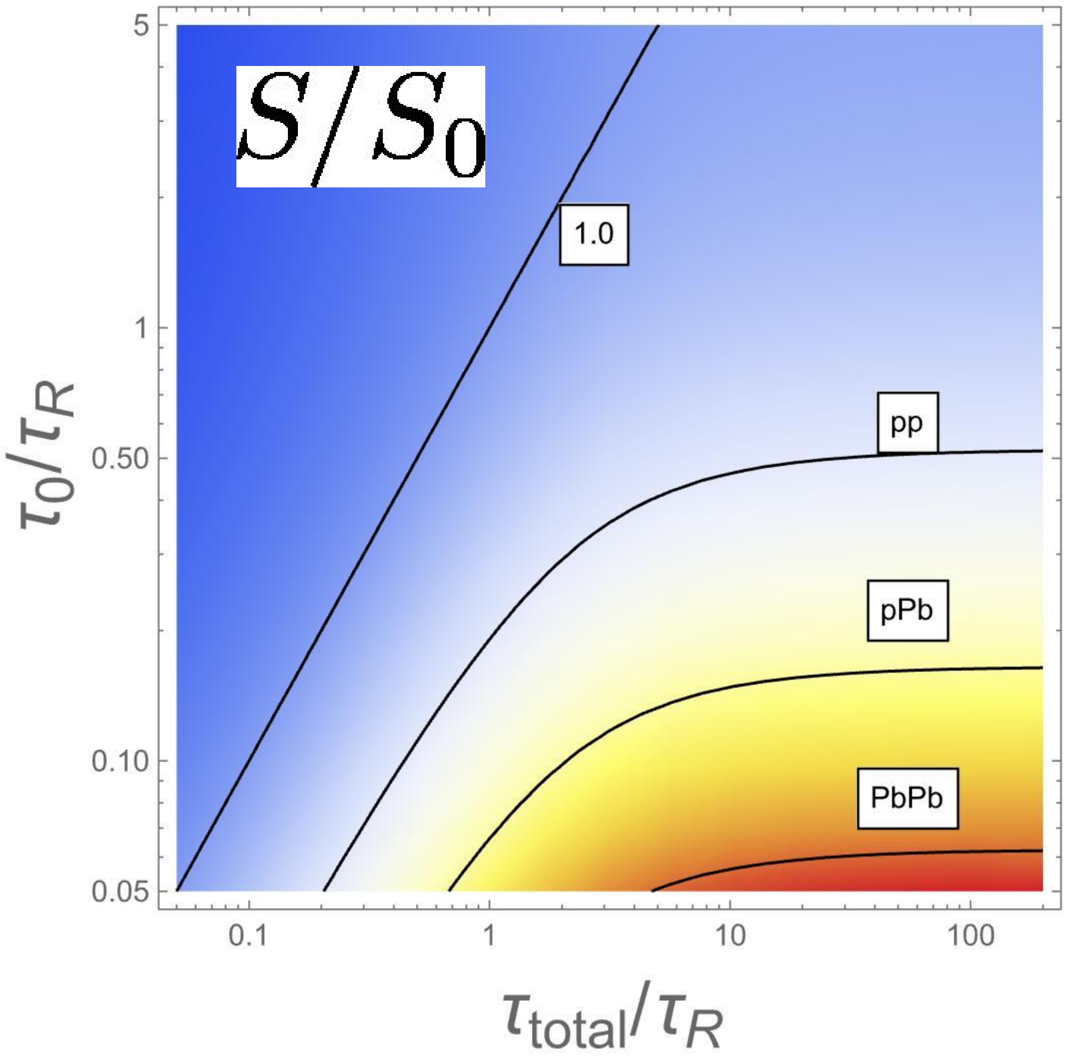


- More rigorous results from kinetic theory including 1-2, 2-2 kernel.

$$\left[\frac{\partial}{\partial \tau} - \frac{p_{\parallel}}{\tau}\frac{\partial}{\partial p_{\parallel}}\right] f_a(\tau, p_T, p_{\parallel}) = -C_a^{2\leftrightarrow 2}[f](\tau, p_T, p_{\parallel}) - C_a^{1\leftrightarrow 2}[f](\tau, p_T, p_{\parallel})$$

- 2nd law: Thermal entropy increases as system approaching equilibrium,

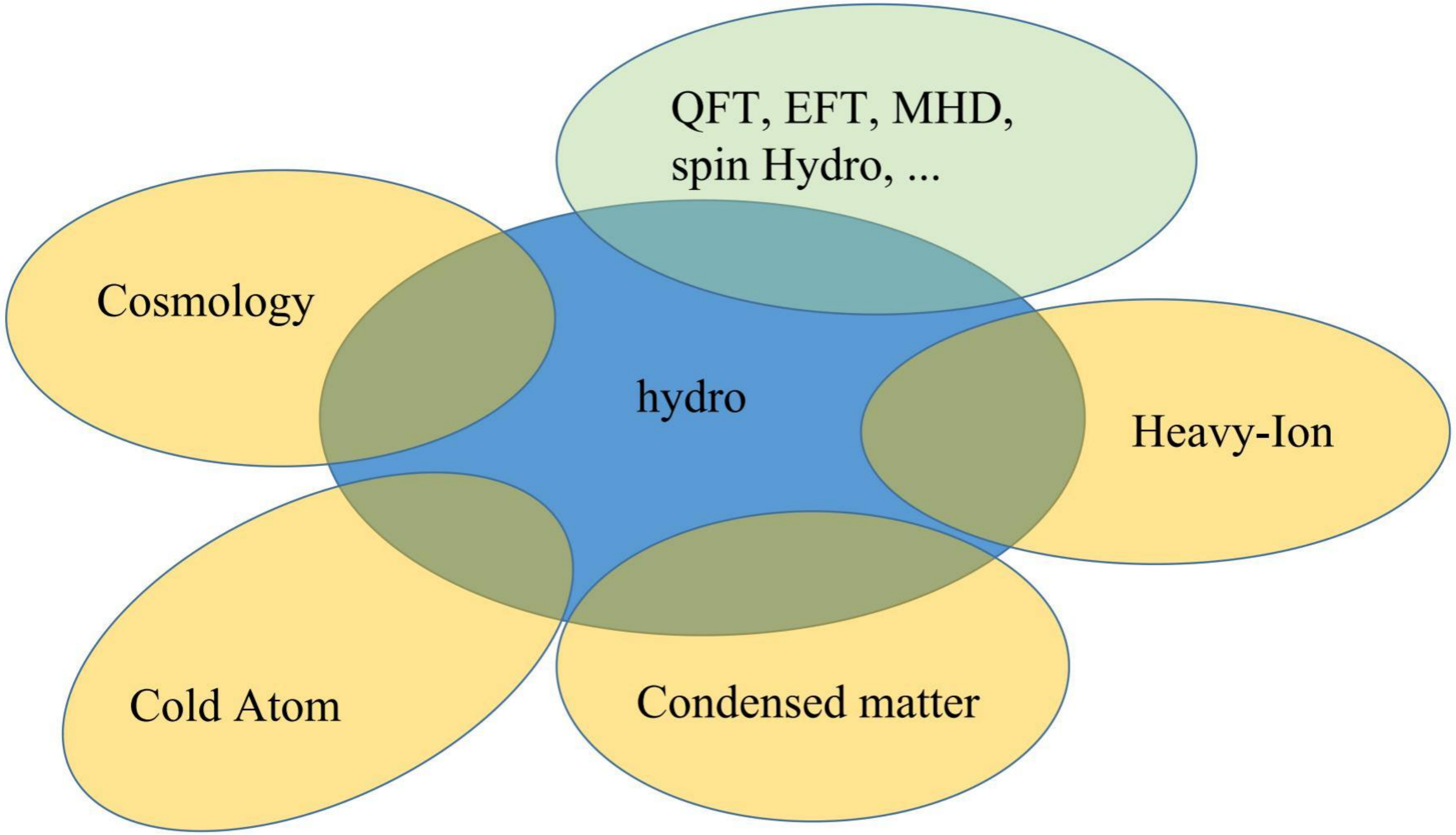
Entropy production and thermalization



- Structure of thermalization plateau.
- Initial time (~ 1/Q) is bounded from above in different systems: earlier in larger systems.
- Total evolution time is longer in larger systems.
- pp is unlikely thermalized …

Summary

- Entanglement entropy can be estimated in heavy-ion collisions.
- Thermalization can be studied in terms of late-stage entropy production.



Thanks