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Dynamical fluctuations near the QCD critical point

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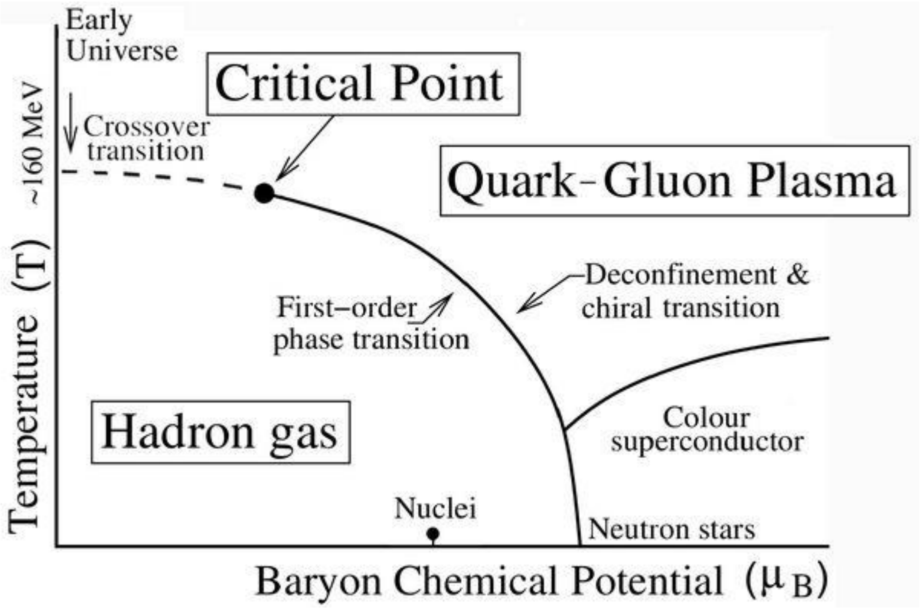
QCD phase diagram

- **Lattice QCD** (small μ_B finite T): Y. Aoki et al, Nature 443, 675 (2006).
  - Crossover (2nd order phase transition)
- **Effective models** (large μ_B) W.J.Fu et al., PRD 77, 014006
  S.X.Qin et al., PRL. 106, 172301
  K. Fukushima et al, PPNP. 72, 99
  - 1st order phase transition

→ Critical End Point

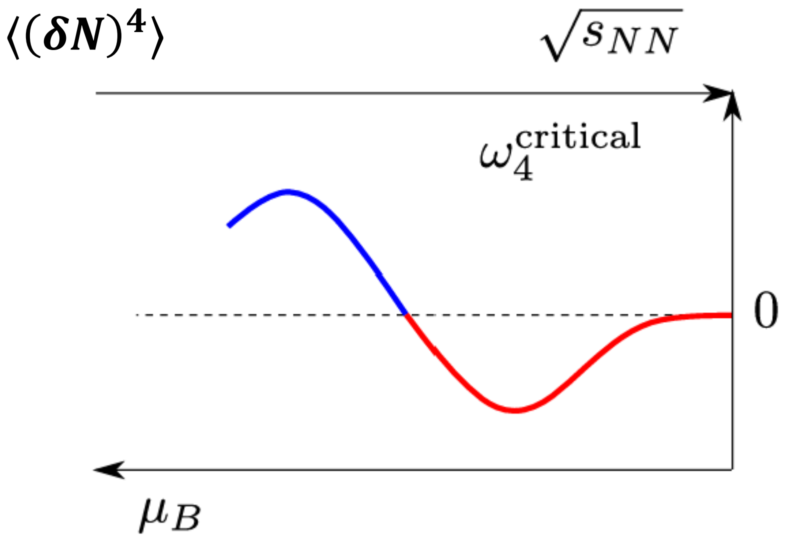


→ **Heavy-ion collisions :**

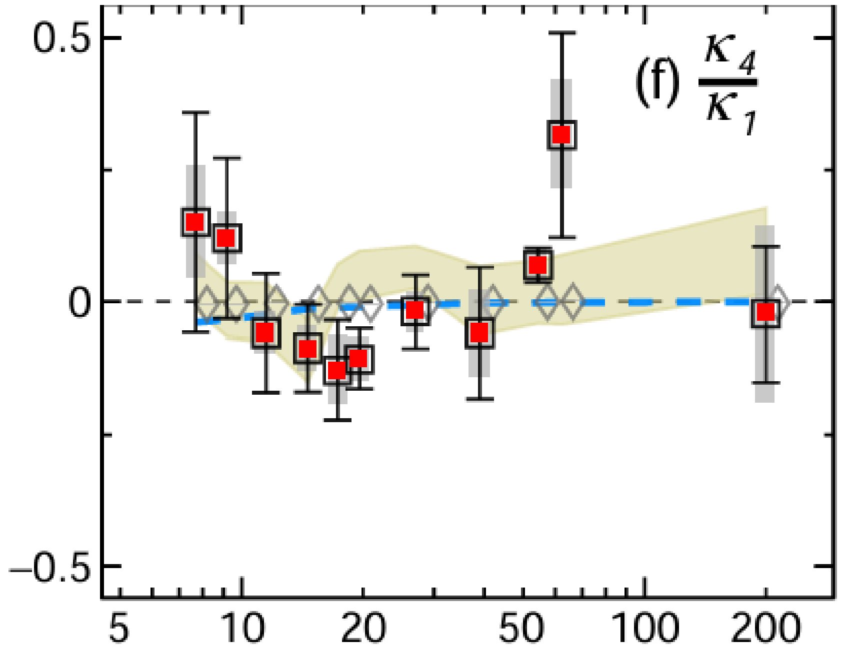
- tuning $\sqrt{s_{NN}}$,mapping $T-\mu$ phase diagram: RHIC(BES),NICA,FAIR,J_PARC,HIAF….

Net-proton fluctuations with Beam Energy Scan

- Characteristic feature of critical point:
  - long range correlation
  - large fluctuations
- **Non-monotonicity** of factorial Cumulant



M.Stephanov, 2410.02861



STAR, Phys.Rev.Lett. 135, 142301

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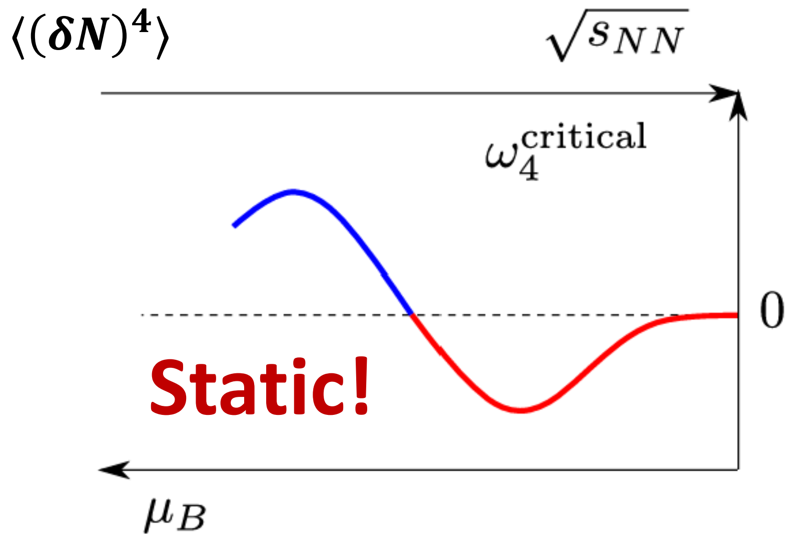

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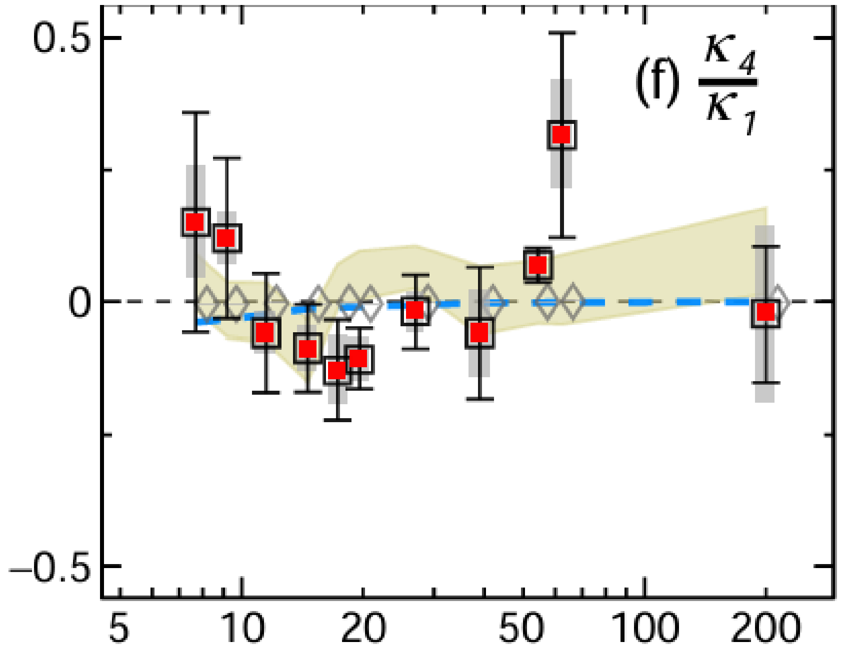

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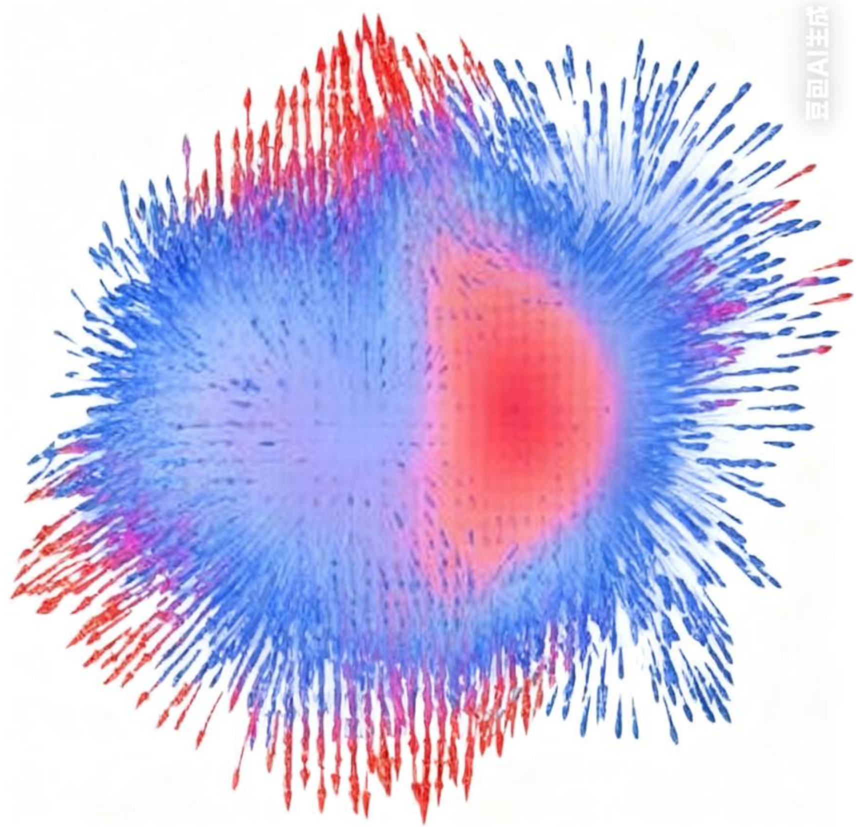
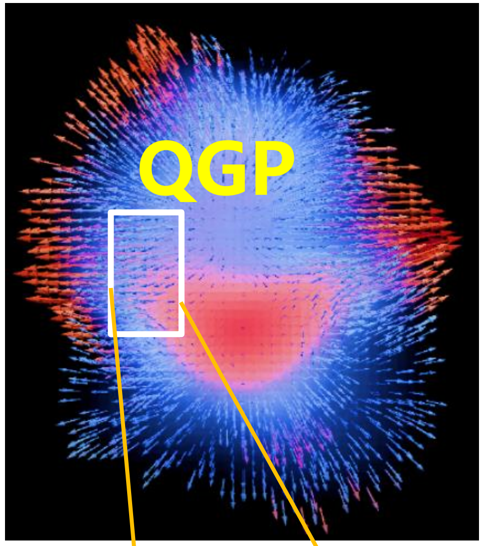


QGP fireball is an expanding system

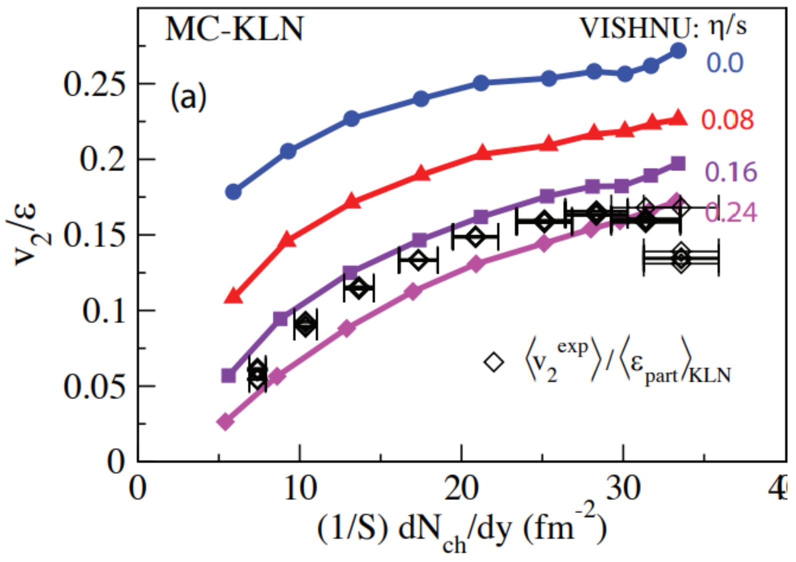
Theory far away from QCD critical point

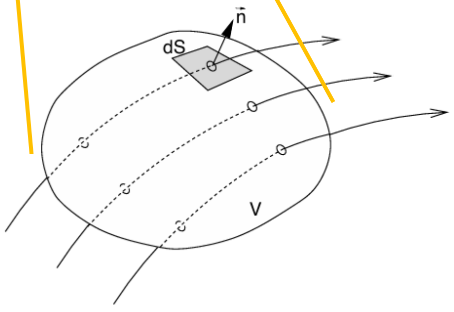
- Hydrodynamics(d.o.f: fluid cell with $e(x,t), n(x,t), u^{\mu}(x,t)$)



  - Energy-momentum conservation:
  $$\partial_{\mu} T^{\mu\nu} = 0$$
  - Mass conservation:
  $$\partial_{\mu} N^{\mu} = 0$$

H.Song et al, Phys.Rev.Lett. 106, 192301





$e(x,t), n(x,t), u^{\mu}(x,t)$

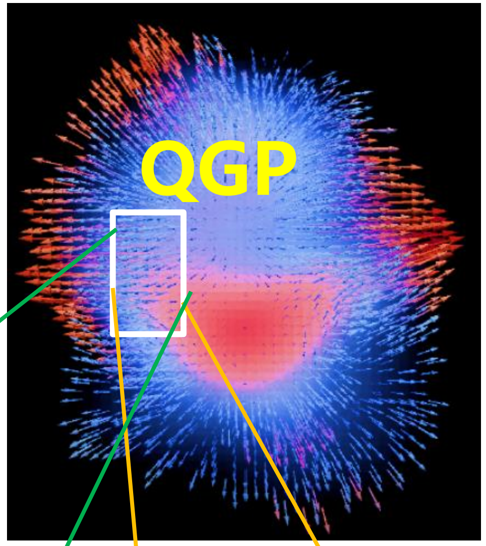
Great success in heavy-ion collisions

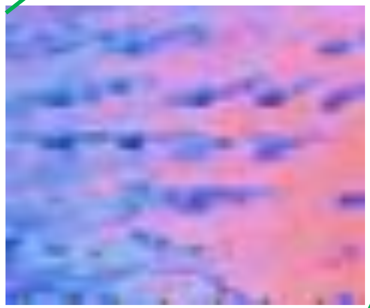
Theory CLOSE to QCD critical point

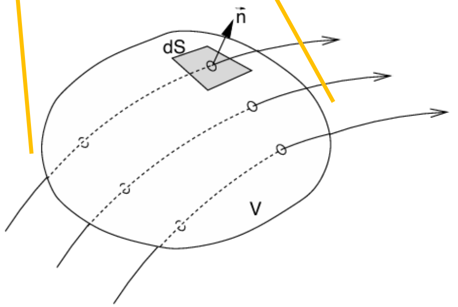
- Hydrodynamics + **critical** (d.o.f: fluid cell with $e(x,t) + \boldsymbol{\delta e}, n(x,t) + \boldsymbol{\delta n}, u^{\mu}(x,t) + \boldsymbol{\delta u}$)

  - Energy-momentum conservation:
$$\partial_{\mu}(T^{\mu\nu} + \boldsymbol{\delta T^{\mu\nu}}) = 0$$
  - Charges conservation:
$$\partial_{\mu}(N^{\mu} + \boldsymbol{\delta N^{\mu}}) = 0$$

Critical Fluctuations needs to be taken into account







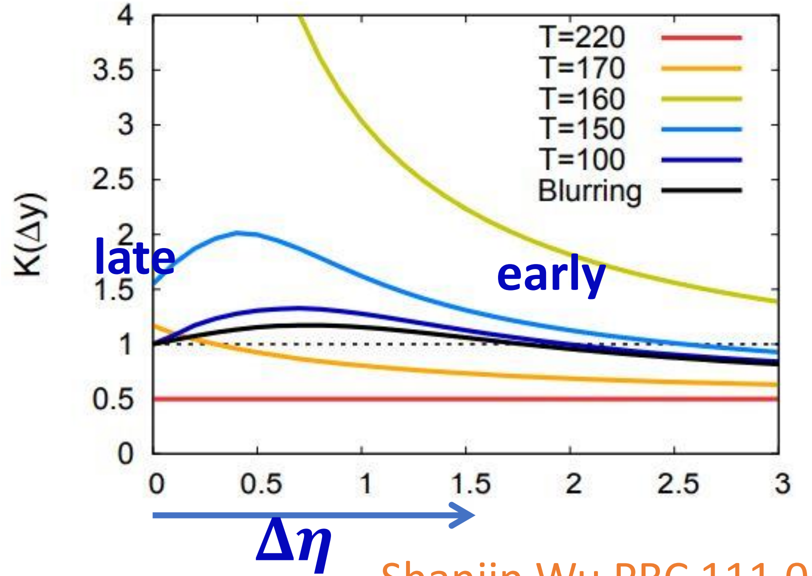
$e(x,t), n(x,t), u^{\mu}(x,t)$

Example: conserved charges near the QCD critical point

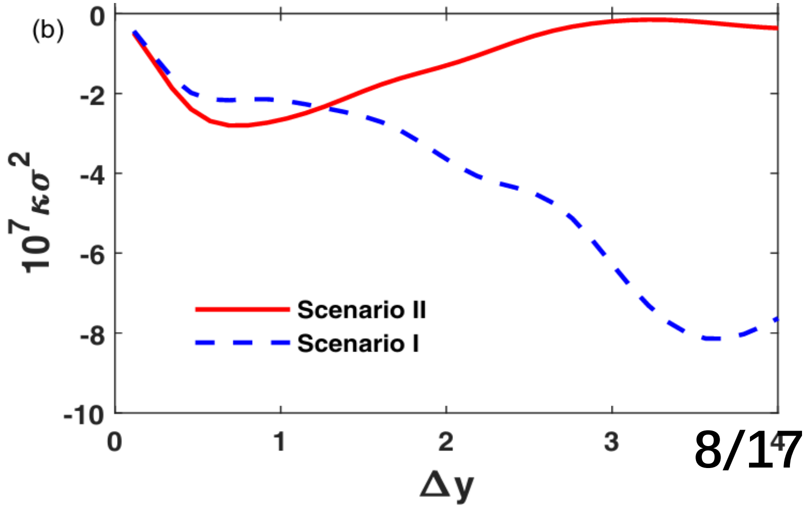
- **Diffusion equation(from $\partial_\mu N^\mu = 0$):**
  - Diffusion equations:
    $$\partial_t n = D\nabla^2 n$$
- **Diffusion equation near critical point**
  - Stochastic diffusion(model B):
    $$\partial_t n = D\nabla^2 n + noise$$
  - Stochastic diffusion with non-Gaussian fluctuations
  - $\partial_\tau n = D\nabla^2(\#n + \#n^2 + \#\, n^3) +$ noise

Diffusion+critical effects induces rapidity dependence

Sakaida et al, PRC.95.064905(2017)



Shanjin Wu PRC 111,014915



Question: diffusion equation works near critical point?

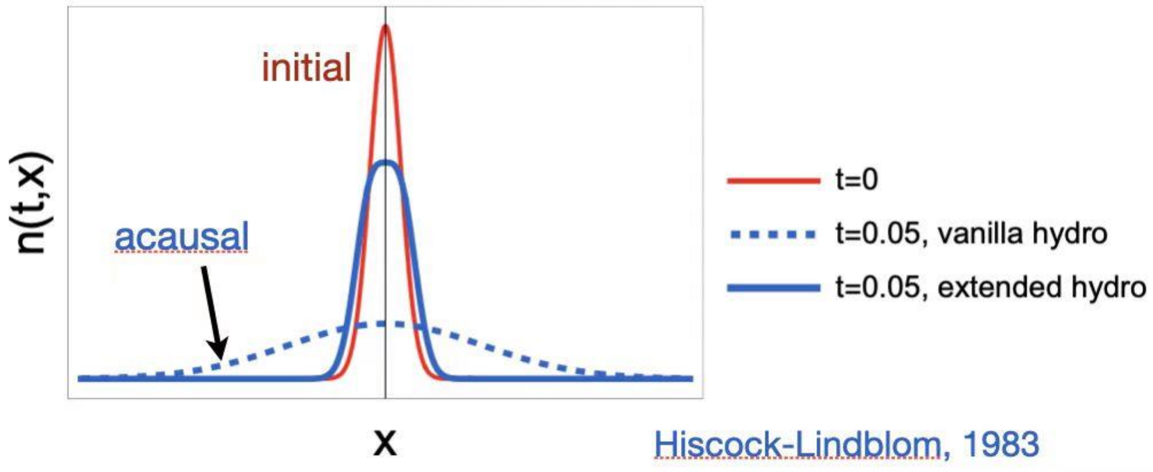
- **Conventional diffusion:**

$$\partial_\tau n = D\nabla^2 \mathrm{n}$$

Speed of diffusion:

$$v = \frac{\partial \omega}{\partial q} = 2iDq$$

Will diverge and become acausal when $q \to \infty$.



- **Maxwell-Catteneo diffusion:**

$$(1 + \tau\partial_\tau)\partial_\tau n = D\nabla^2 \mathrm{n}$$

Then the speed of diffusion(causal):

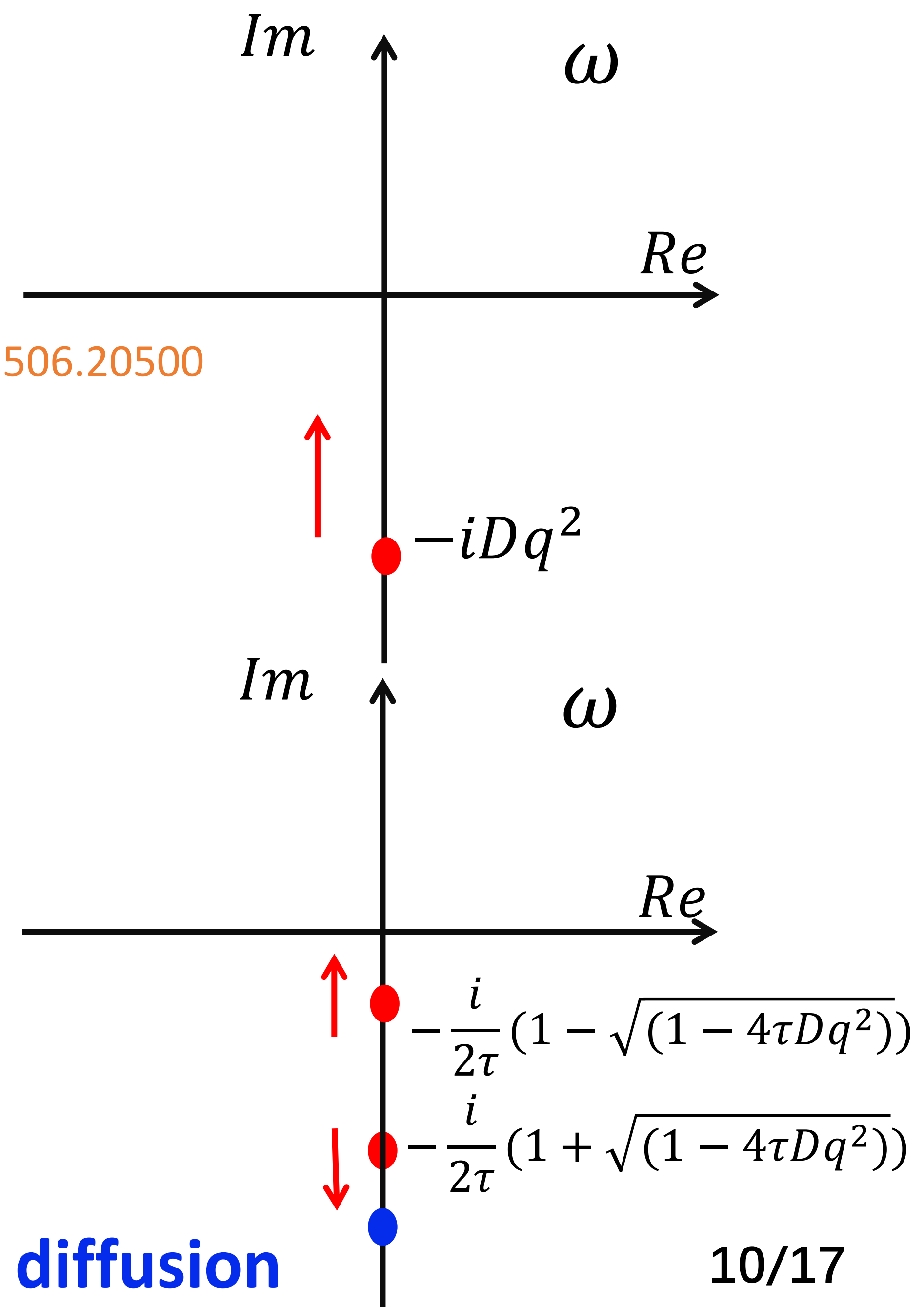
$$v = \frac{\partial \omega}{\partial q} \approx (D/\tau)^{1/2}$$

Naïve diffusion breaks Causality

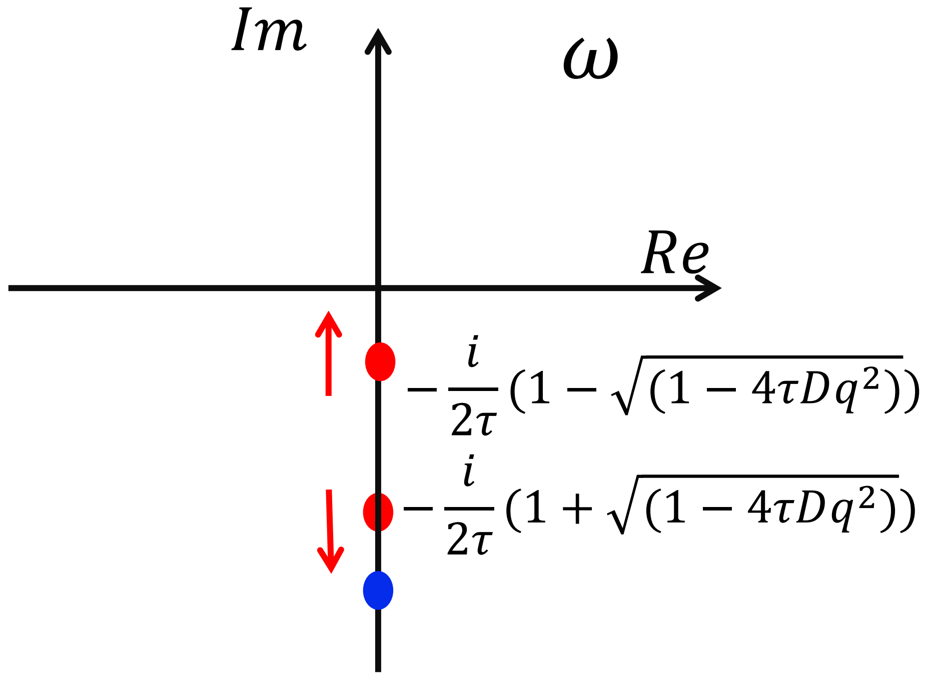
Hydro and Non-hydro mode

- **Hydro mode (e.g. $\boldsymbol{\partial_\mu N^\mu = 0}$):**
  - Diffusion equations:
    $$\partial_t n = D\nabla^2 n$$
  - Dispersion relation:
    $$\omega = -iDq^2$$
- **Non-hydro mode:**
  - Causal diffusion:
    $$\boldsymbol{\tau\partial_t^2 n} + \partial_t n = D\nabla^2 n$$
  - Dispersion relation:
    $$\omega_{1,2} = -\frac{i}{2\tau}\left(1 \mp \sqrt{(1-4\tau Dq^2)}\right)$$

N.Abbasi et al., 2506.20500



Non-hydro modes already exist in causal diffusion



What's the Fate of this non-hydro mode near QCD critical point?

Diffusion equation with non-Gaussian fluctuations

N.Abbasi, X.An, S.Wu, in preparation

- **Causal diffusion in deterministic form:**

  Based on X.An et al, Phys.Rev.Lett.127,072301

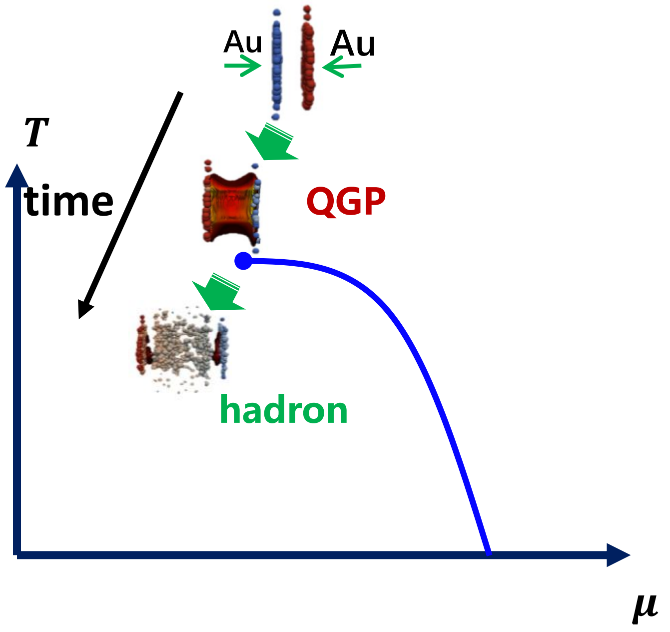
  - $(1+\tau_R\partial_\tau)\,\partial_\tau C_2(q) = -2Dq^2[C_2(q) - T\chi_2]$
  - $(1+\tau_R\partial_\tau)\,\partial_\tau C_3(q) = -3Dq^2[C_3(q) + \#C_2(q)^2 + \#C_2(q)]$
  - $(1+\tau_R\partial_\tau)\partial_\tau C_4(q) = -4Dq^2[C_4(q) + \#C_2(q)C_3(q) + \#C_2(q)^3 + \cdots]$

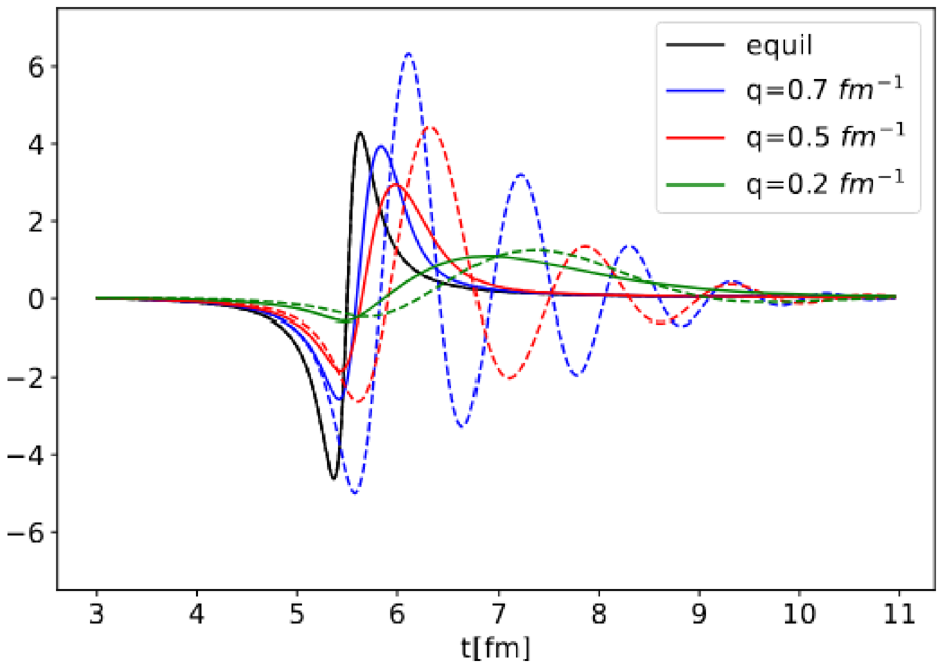
- EoS from Ising model

Causal diffusion with non-Gaussian fluctuations

- **Causal** **Model B:**
- $(1 + \tau_R \partial_\tau)\, \partial_\tau C_3(q) = -3Dq^2[C_3(q) + \#C_2(q)^2 + \#C_2(q)]$

N.Abbasi, X.An, S.Wu, in preparation



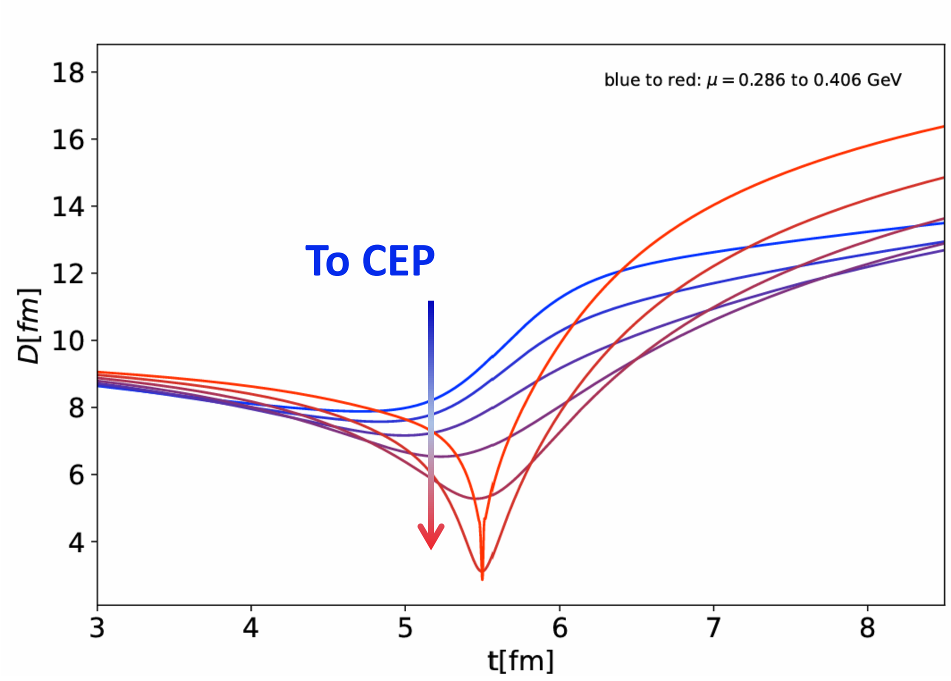


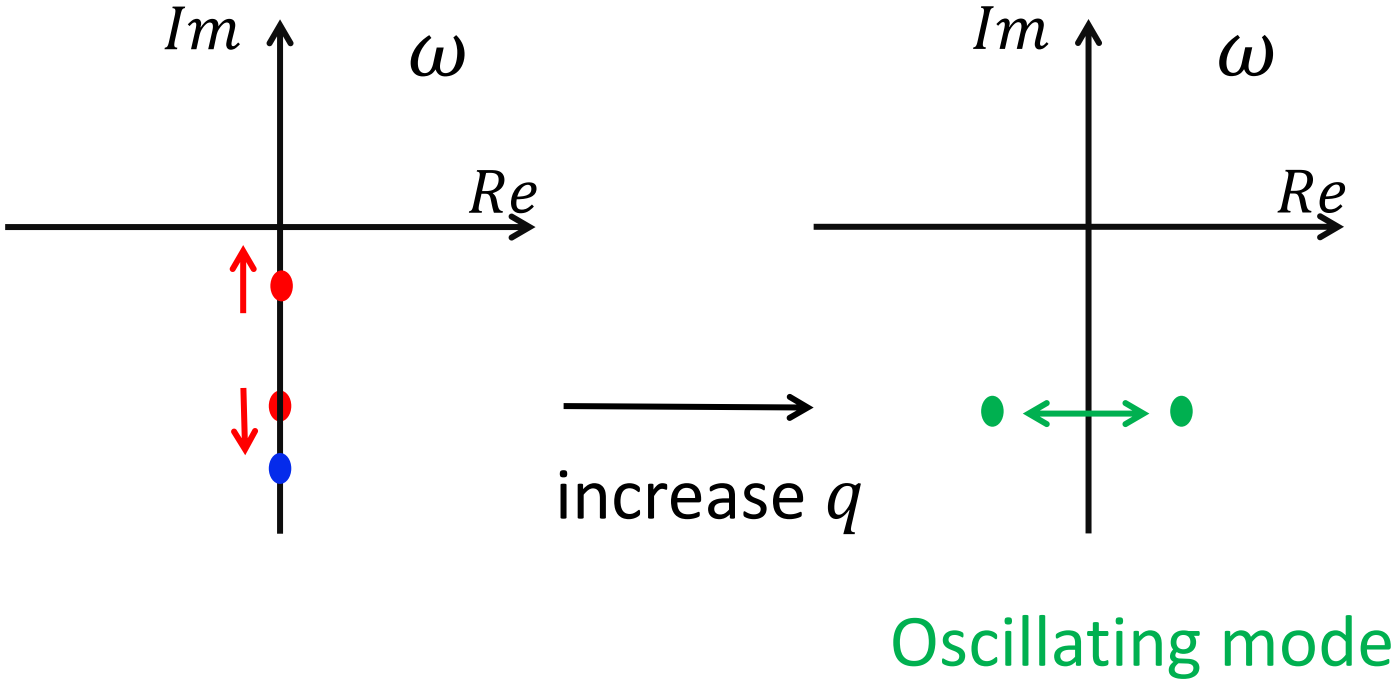
Causality induce: enhancement of amplitude, flip the sign

Correlators approaching to QCD critical point

- Mode related to τ_R: $q_{MC} \sim 1/\sqrt{D\tau_R}$

N.Abbasi, X.An, S.Wu, in preparation

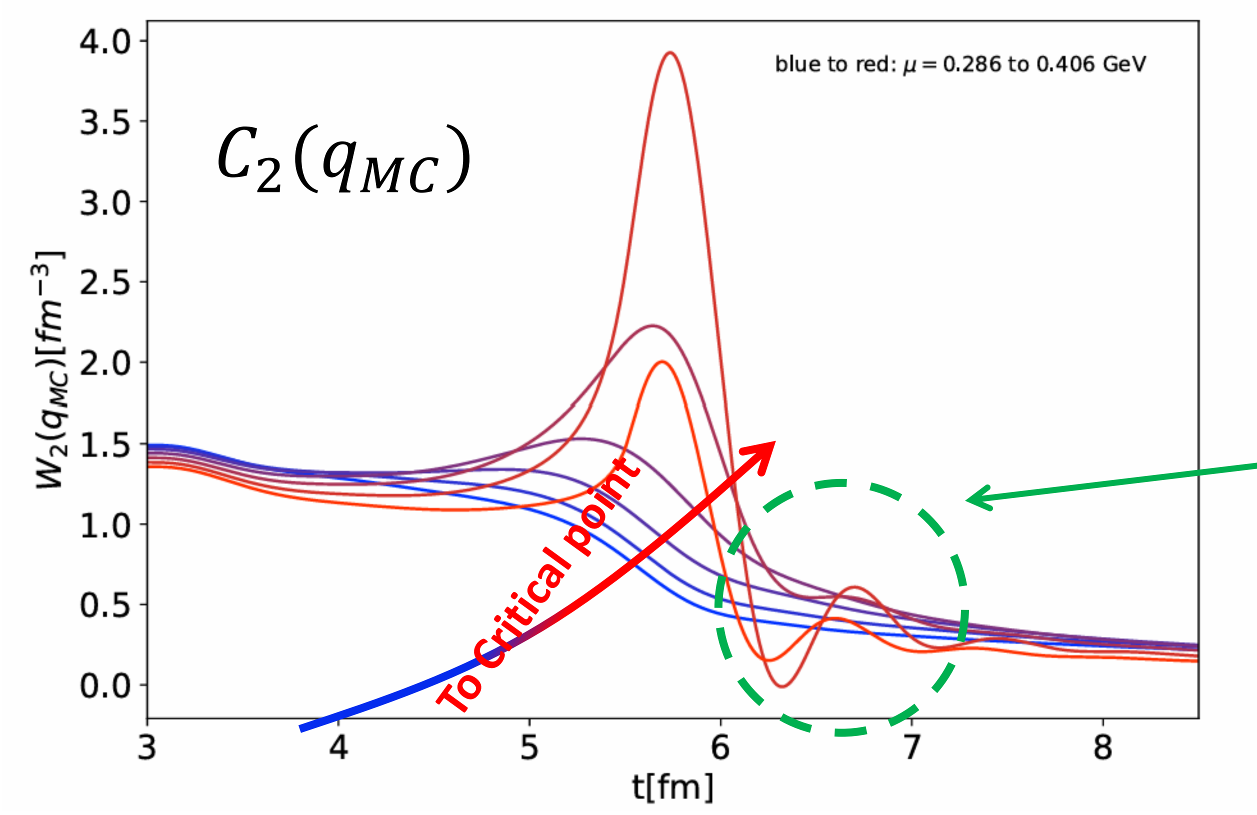


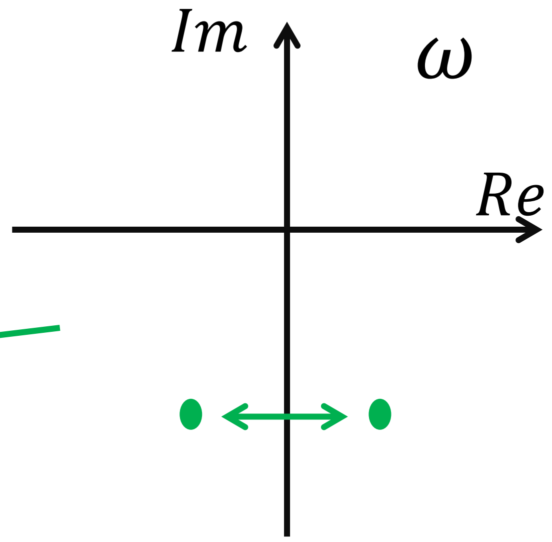


Oscillating behavior as indicator of non-hydro mode

Non-hydro model approaching to QCD critical point

N.Abbasi, X.An, S.Wu, in preparation





Oscillation easier to appear

Non-hydro mode become relevant near CEP, hydro needs to be extended to consider the UV contribution

Summary

- **Dynamical modeling** the QGP evolution near the QCD critical point is essential for the study of fluctuations in heavy-ion experiments;

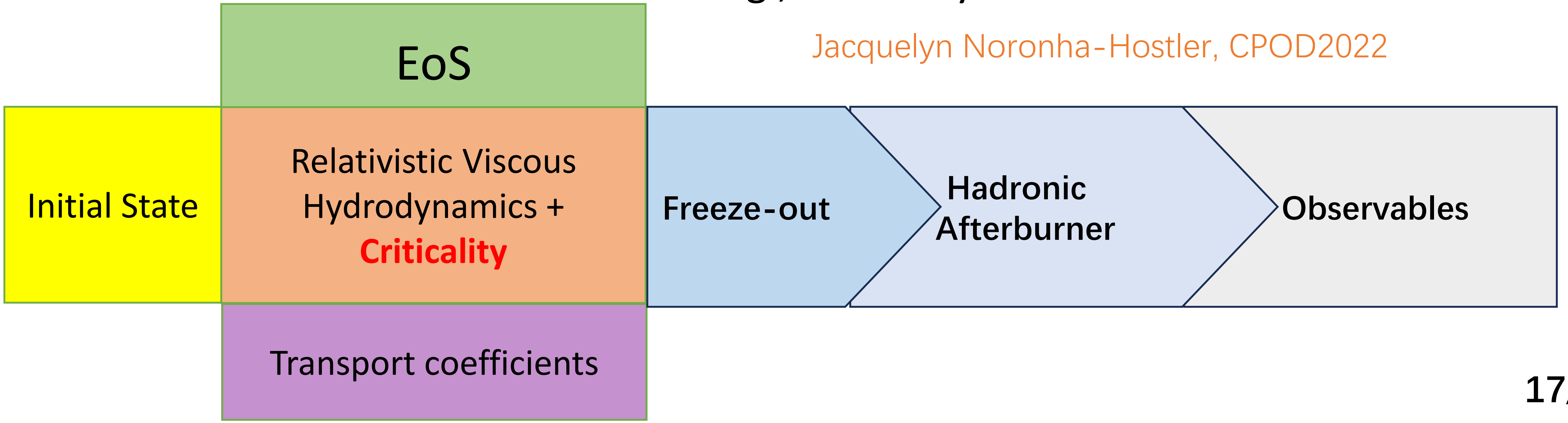
- Conventional stochastic diffusion (model B) predict the rapidity dependence near the QCD critical point

- **Non-hydro mode** become essential approaching critical point, hydro model needed to be extended to taken into account the non-hydro degree of freedom

Outlook

- QGP fireball system in heavy-ion experiments is not an ideal system:
  - Fast expanding
  - Finite size
  - Inhomogeneous temperature and chemical potential
  - Volume fluctuation and initial fluctuations
  - Conservation contamination
  - Correction from detector: e.g., efficiency correction

See e.g., M.Asakawa M. Kitazawa, PPNP, arXiv: 1512.05038
X.Luo, N.Xu, NST, arXiv: 1701.02105
S.Wu, C.Shen, H.Song, CPL, arXiv: 2104.13250



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Thank You!