QCD challenges at the LHC

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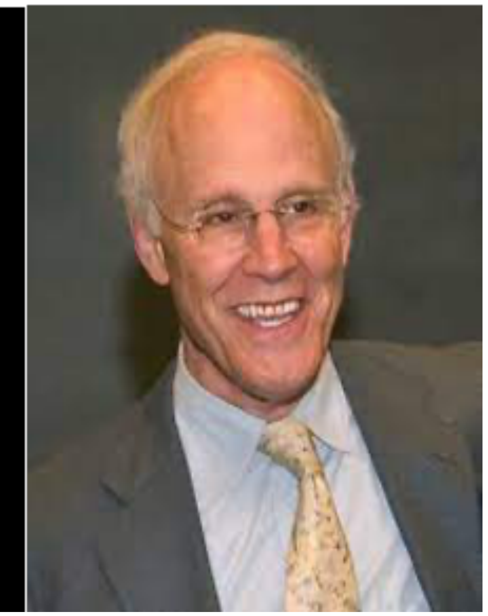
复旦大学

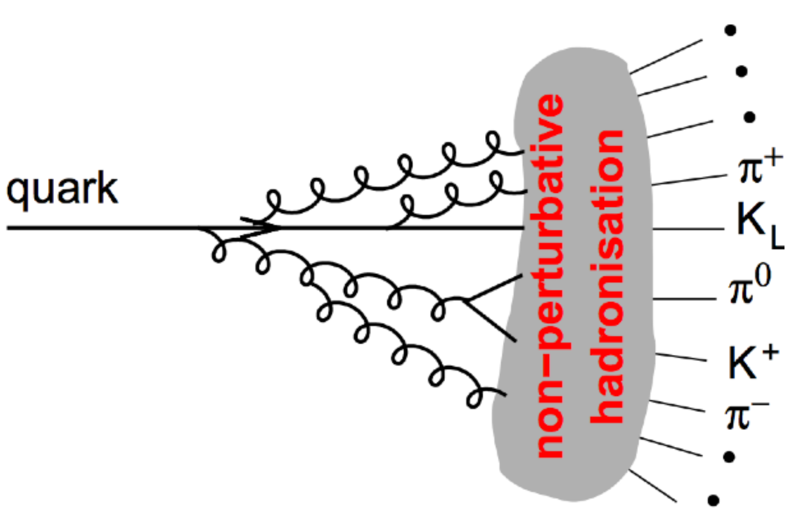
Opportunities and ideas at the QCD Frontier   CCAST   Apr 7 2025

QCD in the future

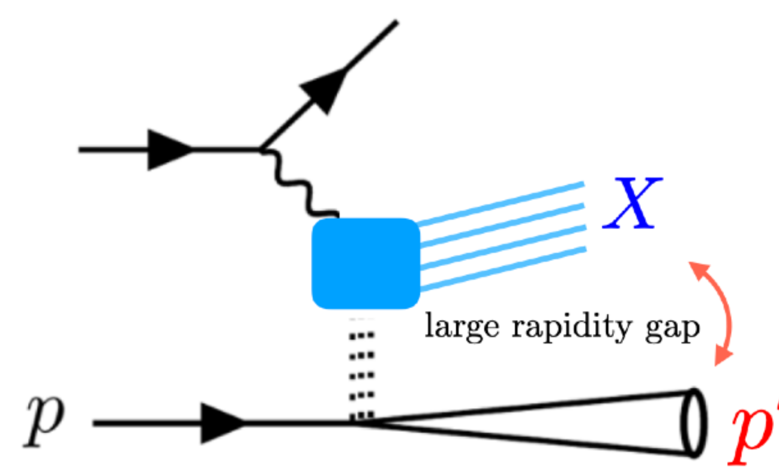


David Gross: Fifty Years of Quantum Chromodynamics

强子的碎裂机制?



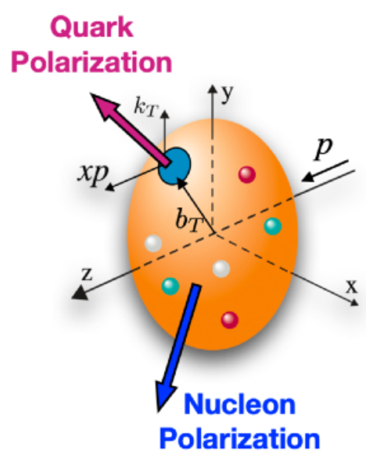
QCD衍射机制?



Polarized fragmentation functions

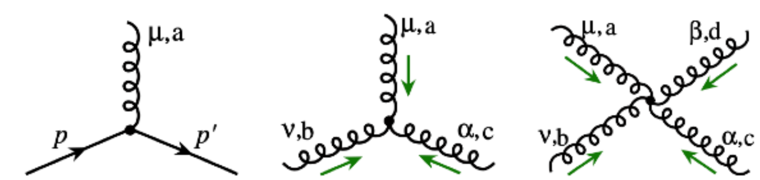
QCD and 3D imaging of nucleon

Transverse momentum distributions (TMDs) encode the quantum correlations between hadron polarization and the motion and polarization of quarks and gluons inside it.



Imaging a hadron would provide insights on QCD

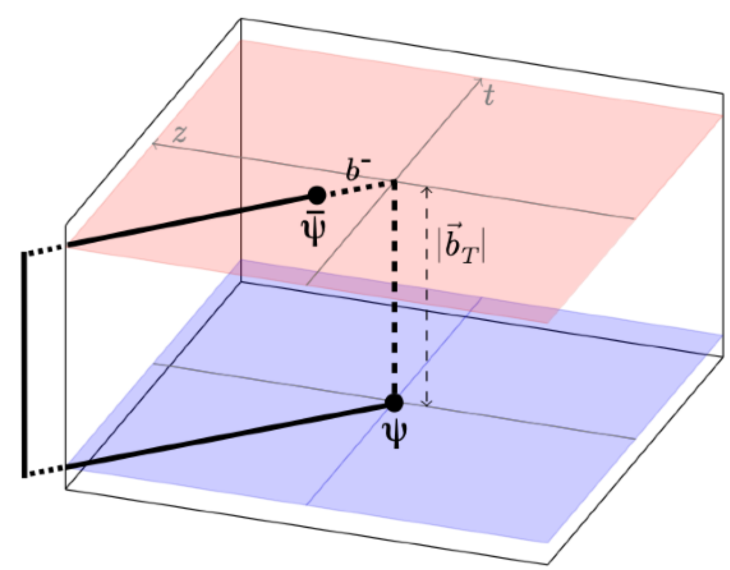
$$-\frac{1}{4}G^2_{\mu\nu,a}[A] + \sum_f \overline{\psi}_f \left(iD_\mu[A]\gamma^\mu - m_f\right)\psi_f$$



- **Both longitudinal and transverse motion**
- **Large Lorentz boost in longitudinal direction, but not in transverse momentum**
- **Correlation between hadron spin with parton(quark, gluon) orbital angular momentum**

$$\tilde{f}^{[\Gamma]0(u)}_{i/p_S}(x, \mathbf{b}_T, \epsilon, \tau, xP^+) =$$

$$\int \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \left\langle p(P,S) \right| \left[\bar{\psi}^i(b^\mu) W_\sqsubset(b^\mu, 0) \frac{\Gamma}{2} \psi^i(0) \right]_\tau \left| p(P,S) \right\rangle$$



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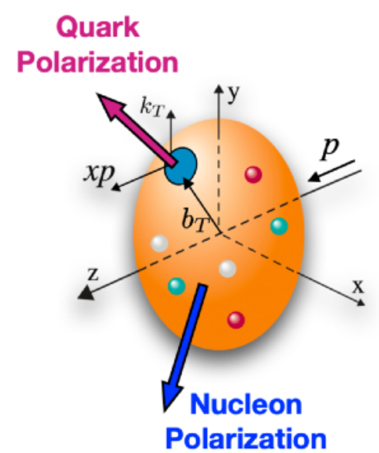

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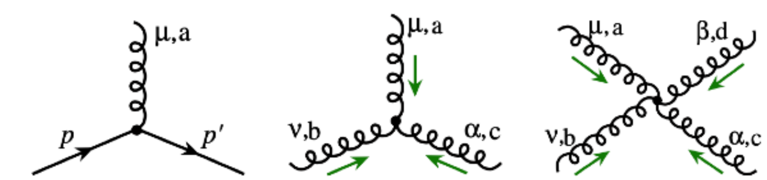

- Both longitudinal and transverse motion
- Large Lorentz boost in longitudinal direction, but not in transverse momentum
- Correlation between hadron spin with parton(quark, gluon) orbital angular momentum

TMDs		Quark Polarization		
		Unpolarized (U)	Longitudinally polarized (L)	Transversely polarized (T)
Nucleon Polarization	U	f_1 unpolarized		$h_1^\perp$ Boer-Mulders
	L		g_{1L} helicity	$h_{1L}^\perp$ longi-transversity
	T	$f_{1T}^\perp$ Sivers	g_{1T} trans-helicity	h_1 transversity $h_{1T}^\perp$ pretzelosity

Nucleon spin   Quark spin

Figure 2.5: The leading-twist quark TMD distributions.

TMD handbook 2304.03302

QCD and 3D imaging of nucleon

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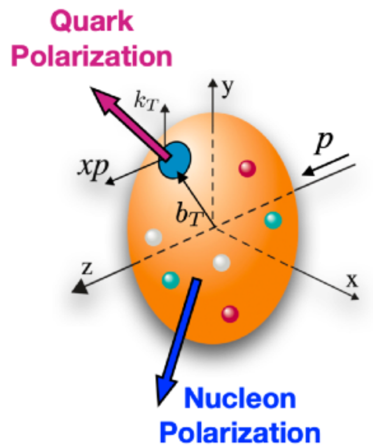
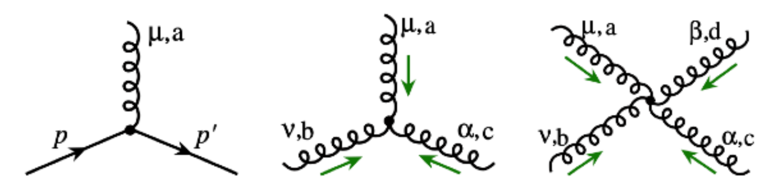


Imaging a hadron would provide insights on QCD

$$-\frac{1}{4}G^2_{\mu\nu,a}[A] + \sum_f \overline{\psi}_f\left(iD_\mu[A]\gamma^\mu - m_f\right)\psi_f$$



- **Both longitudinal and transverse motion**
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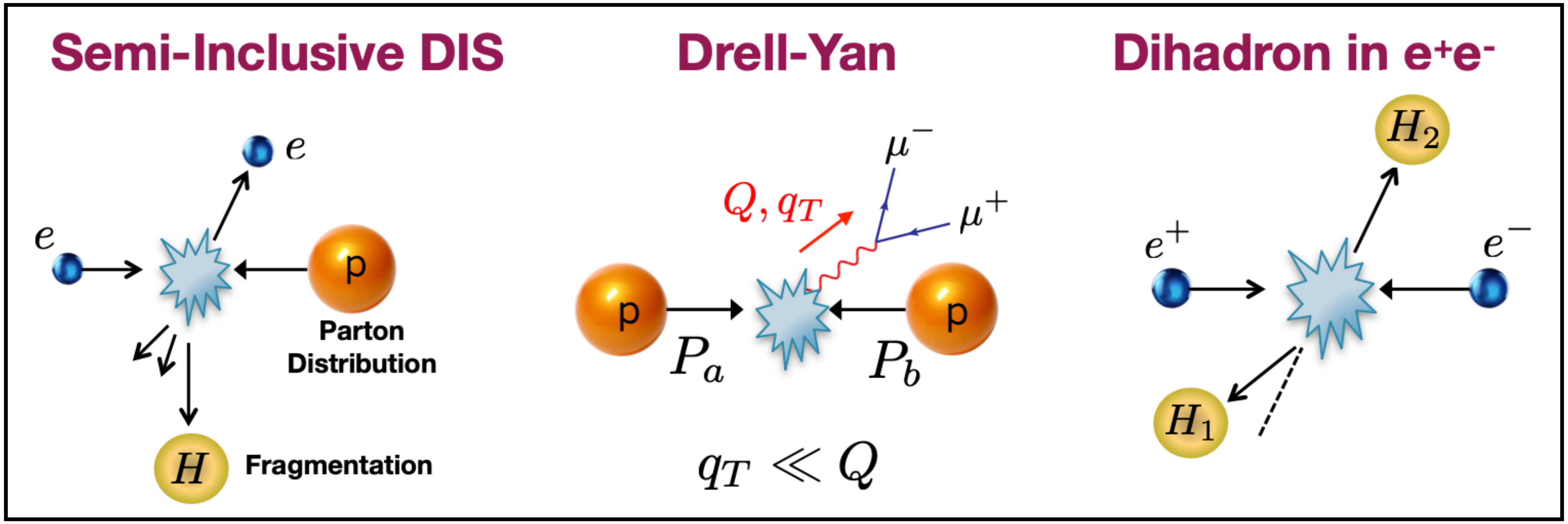
$$f^{[\gamma^+]}_{i/p_S}(x, \mathbf{k}_T, \mu, \zeta) = f_1(x, k_T) - \frac{\epsilon_T^{\rho\sigma} k_{T\rho} S_{T\sigma}}{M}\kappa\, f_{1T}^{\perp}(x, k_T)\,,$$

$$f^{[\gamma^+\gamma_5]}_{i/p_S}(x, \mathbf{k}_T, \mu, \zeta) = S_L\, g_1(x, k_T) - \frac{k_T\cdot S_T}{M} g_{1T}^{\perp}(x, k_T)\,,$$

$$f^{[i\sigma^{\alpha+}\gamma_5]}_{i/p_S}(x, \mathbf{k}_T, \mu, \zeta) = S_T^\alpha h_1(x, k_T) + \frac{S_L k_T^\alpha}{M} h_{1L}^{\perp}(x, k_T)$$
$$- \frac{\mathbf{k}_T^2}{M^2}\left(\frac{1}{2}g_T^{\alpha\rho} + \frac{k_T^\alpha k_T^\rho}{\mathbf{k}_T^2}\right) S_{T\rho} h_{1T}^{\perp}(x, k_T) - \frac{\epsilon_T^{\alpha\rho} k_{T\rho}}{M}\kappa\, h_1^{\perp}(x, k_T)$$

Transverse momentum distributions of quarks

- Three classical processes used to probe quark TMDs



- Typical "two-scale" problem:

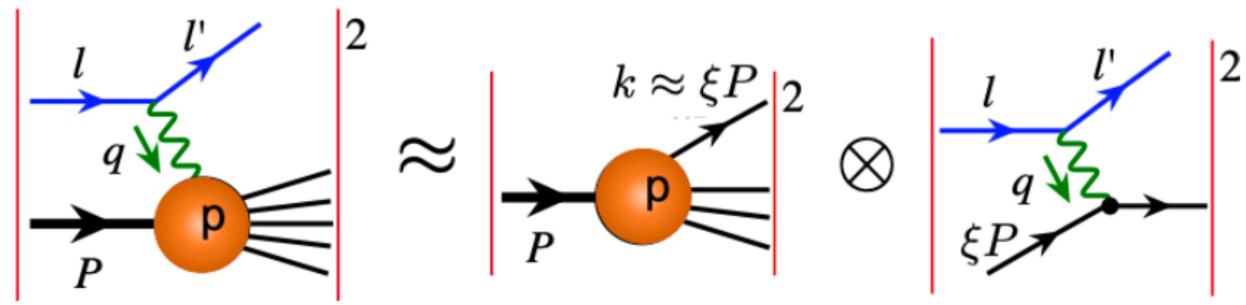
  transverse momentum of final particle (q_T) << scattering energy (Q)

- Theory tools: factorization theorem; renormalization group evolution; effective field theory …

QCD factorization

Collinear factorization

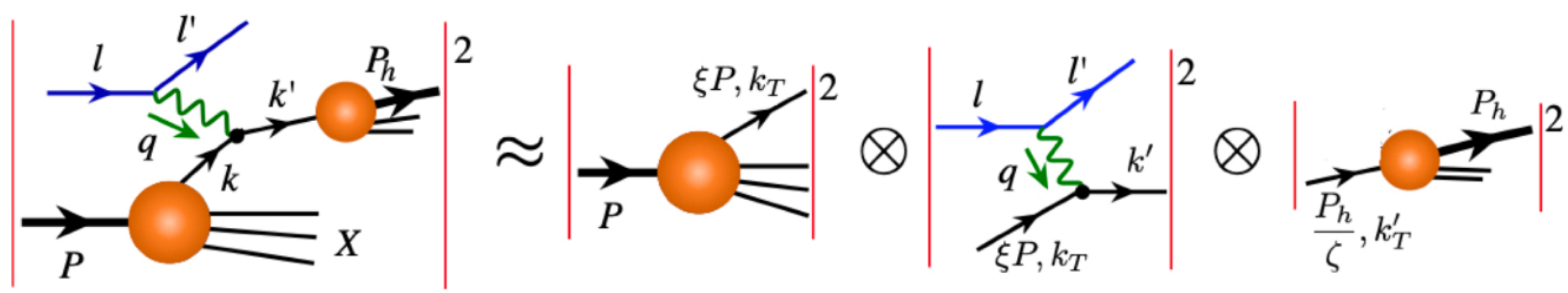
$\sigma_{\mathrm{DIS}} \propto$



$$E'\frac{\mathrm{d}\sigma_{ep\to e'X}}{\mathrm{d}^3l'} \approx \sum_i \int d\xi\, f_{i/p}(\xi)\, E'\frac{d\hat{\sigma}_{ei\to e'X}}{d^3l'} = \sum_i e_i^2 \left\{\frac{2\alpha^2}{Q^2 s}\left[\frac{1+(1-y)^2}{y^2}\right]\right\} f_{i/p}(x)$$
$$\equiv \sum_i f_{i/p}\otimes\hat{\sigma}_i \,.$$

TMD factorization

$\sigma_{\mathrm{SIDIS}} \propto$



$$E'E_h\frac{\mathrm{d}\sigma_{ep\to e'hX}}{\mathrm{d}^3l'\mathrm{d}^3P_h} \approx \hat{\sigma}_{eq\to e'q'} \otimes f_1 \,\widetilde{\otimes}\, D_{h/q'}$$

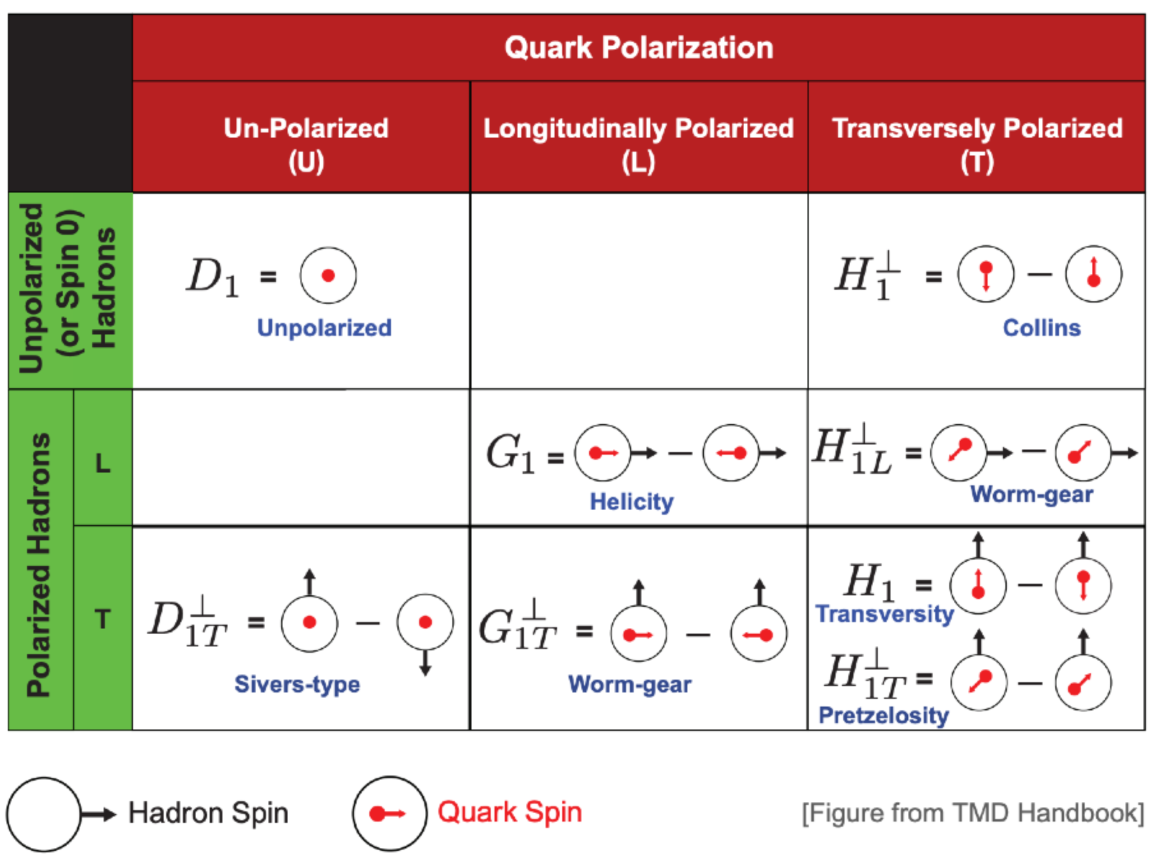
QCD fragmentation functions

In QCD, fragmentation is described by fragmentation functions (FFs) defined via the quark-quark correlator

TMD FFs

$$\tilde{\Delta}_{h/i}^{[\Gamma]0(\mathrm{u})}(z,\mathbf{b}_T,\epsilon,\tau,P^+/z) = \frac{1}{4N_c z}\mathrm{Tr}\int\frac{\mathrm{d}b^-}{2\pi}\sum_X e^{ib^-(P^+/z)}\Gamma^+_{\alpha\alpha'}$$
$$\times\left\langle 0\left|\left[(W_{\lrcorner}\psi_i^{0\alpha})(b)\right]_\tau\right|h(P,S),X\right\rangle\left\langle h(P,S),X\right|\left[(\bar{\psi}_i^{0\alpha'}W_{\neg})(0)\right]_\tau\left|0\right\rangle$$

$$\Delta_{h/i}(z,\mathbf{p}_T=-z\mathbf{p}'_T,\mu,\zeta) = \int\frac{\mathrm{d}^2\mathbf{b}_T}{(2\pi)^2}e^{-i\mathbf{p}'_T\cdot\mathbf{b}_T}\tilde{\Delta}_{h/i}(z,\mathbf{b}_T,\mu,\zeta)$$



[Figure from TMD Handbook]

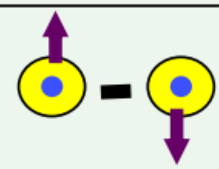
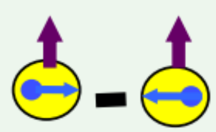
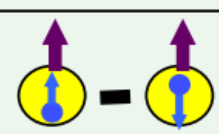
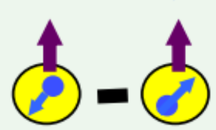
$$\Delta_{h/i}^{[\gamma^+]}(z,-z\mathbf{p}'_T,\mu,\zeta) = D_1(z,zp'_T) - \frac{\epsilon_T^{\rho\sigma}p'_{T\rho}S_{T\sigma}}{M_h}D_{1T}^\perp(z,zp'_T)\,,$$

$$\Delta_{h/i}^{[\gamma^+\gamma_5]}(z,-z\mathbf{p}'_T,\mu,\zeta) = S_L\,G_1(z,zp'_T) - \frac{p'_T\cdot S_T}{M_h}G_{1T}^\perp(z,zp'_T)\,,$$

$$\Delta_{h/i}^{[i\sigma^{\alpha+}\gamma_5]}(z,-z\mathbf{p}'_T,\mu,\zeta) = S_T^\alpha\,H_1(z,zp'_T) + \frac{S_L p_T'^\alpha}{M_h}H_{1L}^\perp(z,zp'_T)$$
$$-\frac{\mathbf{p}_T'^2}{M_h^2}\left(\frac{1}{2}g_T^{\alpha\rho}+\frac{p_T'^\alpha p_T'^\rho}{\mathbf{p}_T'^2}\right)S_{T\rho}H_{1T}^\perp(z,zp'_T) - \frac{\epsilon_T^{\alpha\rho}p'_{T\rho}}{M_h}H_1^\perp(z,zp'_T)$$

Twist-2 TMD FFs defined via quark-quark correlator

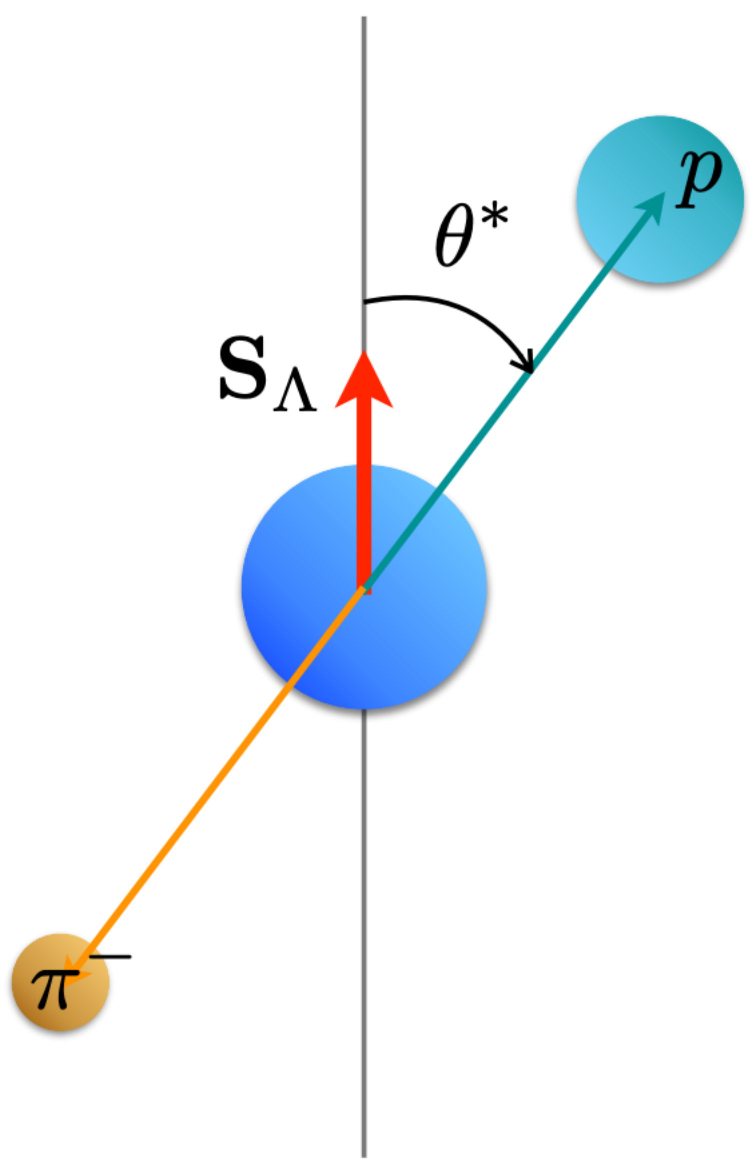
Leading twist (twist 2) D, G, H: quark un-, longitudinally, transversely polarized

polarization: quark	polarization: hadron	polarization: pictorially	TMD FFs (8)	integrated over $k_{F\perp}$	name
U	U		$D_1(z,k_{F\perp})$	$D_1(z)$	number density
	T		$D_{1T}^{\perp}(z,k_{F\perp})$	×	Sivers-type function
L	L		$G_{1L}(z,k_{F\perp})$	$G_{1L}(z)$	spin transfer (longitudinal)
	T		$G_{1T}^{\perp}(x,k_{\perp})$	×	
T	U		$H_1^{\perp}(z,k_{F\perp})$	×	Collins function
	$T(//)$		$H_{1T}(z,k_{F\perp})$	$H_{1T}(z)$	spin transfer (transverse)
	$T(\perp)$		$H_{1T}^{\perp}(z,k_{F\perp})$		
	L		$H_{1L}^{\perp}(z,k_{F\perp})$	×	

ZT Liang's slides

Λ polarization

$$\Lambda \to p\pi^-$$



弱衰变，宇称破坏

$$\frac{dN}{d\Omega^*} = \frac{N}{4\pi}(1 + \alpha_H P_H \cos\theta^*)$$

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		D_1 = Unpolarized		$H_1^\perp$ = − Collins
Polarized Hadrons	L		G_1 = − Helicity	$H_{1L}^\perp$ = − Worm-gear
Polarized Hadrons	T	$D_{1T}^\perp$ = − Sivers-type	$G_{1T}^\perp$ = − Worm-gear	H_1 = − Transversity $H_{1T}^\perp$ = − Pretzelosity

Hadron Spin   Quark Spin

TMD handbook 2304.03302

One of the most important discoveries in QCD and hadron physics over the past decades is the measurements of large spin asymmetries

VOLUME 41, NUMBER 25 PHYSICAL REVIEW LETTERS 18 DECEMBER 1978

Transverse Quark Polarization in Large-p_T Reactions, e^+e^- Jets, and Leptoproduction: A Test of Quantum Chromodynamics

G. L. Kane
Physics Department, University of Michigan, Ann Arbor, Michigan 48109

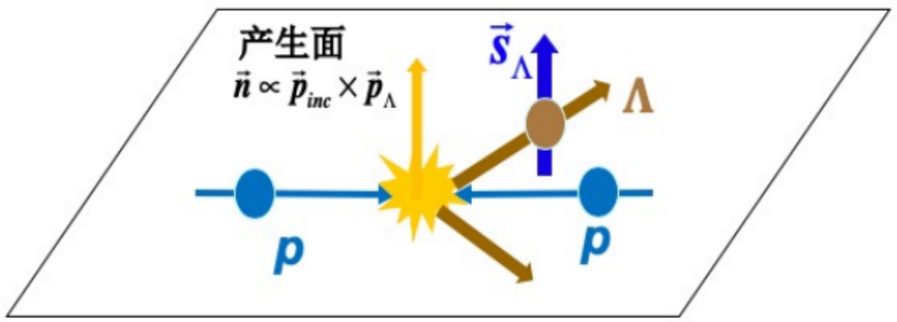
and

J. Pumplin and W. Repko
Physics Department, Michigan State University, East Lansing, Michigan 48823

(Received 5 July 1978)

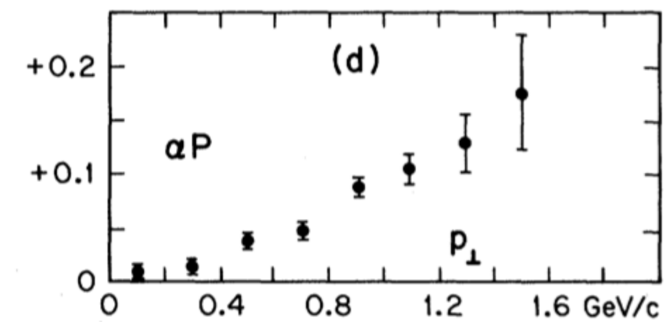
quarks. We discuss how to test the predictions. At least for the cases when P is small, tests should be available soon in large-p_T production [where currently $P(\Lambda) = 25\%$ for $p_T \gtrsim 2$ GeV/c], and e^+e^- reactions. While fragmentation effects could dilute polarizations, they cannot (by parity considerations) induce polarization. Consequently, observation of significant polarizations in the above reactions would contradict either QCD or its applicability.

EXP on transverse Λ polarization



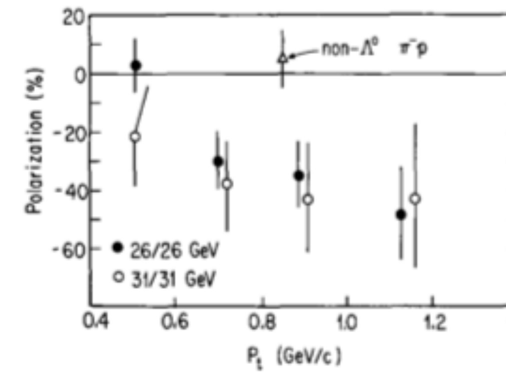
Bunce et.al. '76

$p + Be \to \Lambda^\uparrow + X$

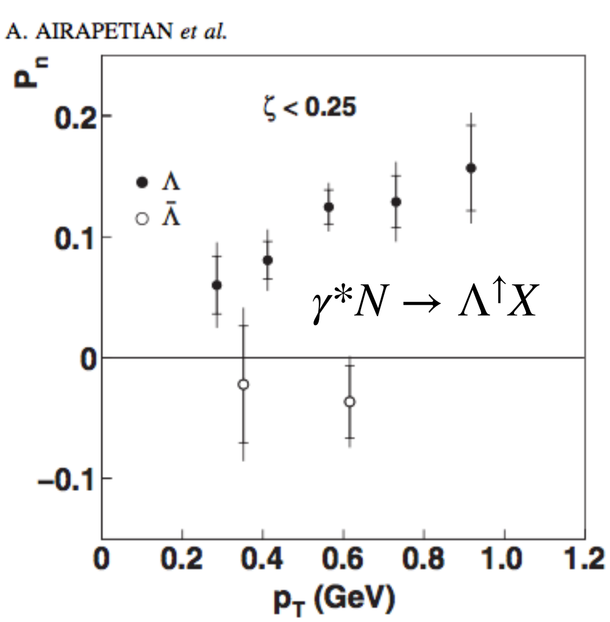


Erhan et.al. '79

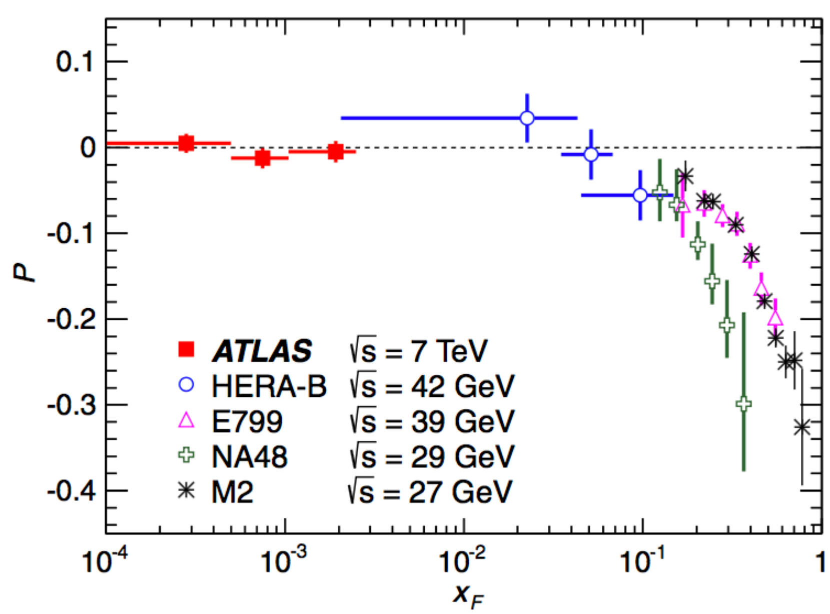
$pp \to \Lambda^\uparrow X$



HERMES '76

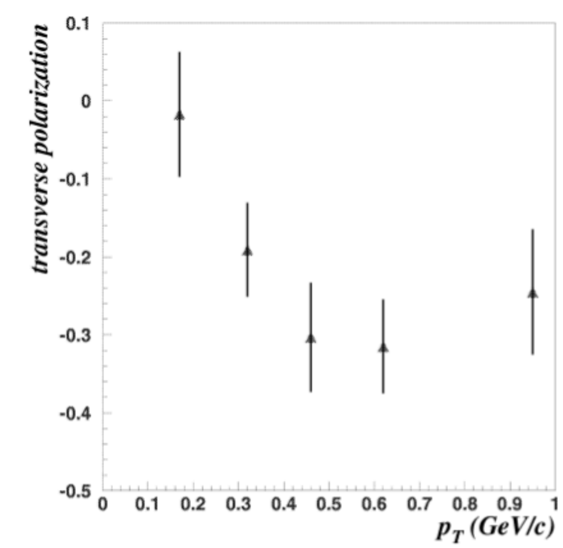


Atlas '15



NOMAD '20

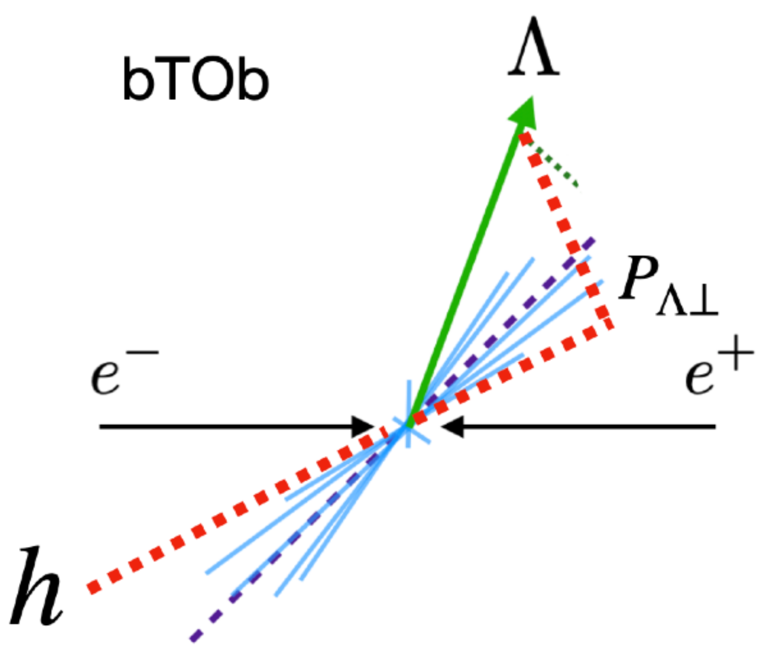
$\nu N \to \Lambda^\uparrow X$



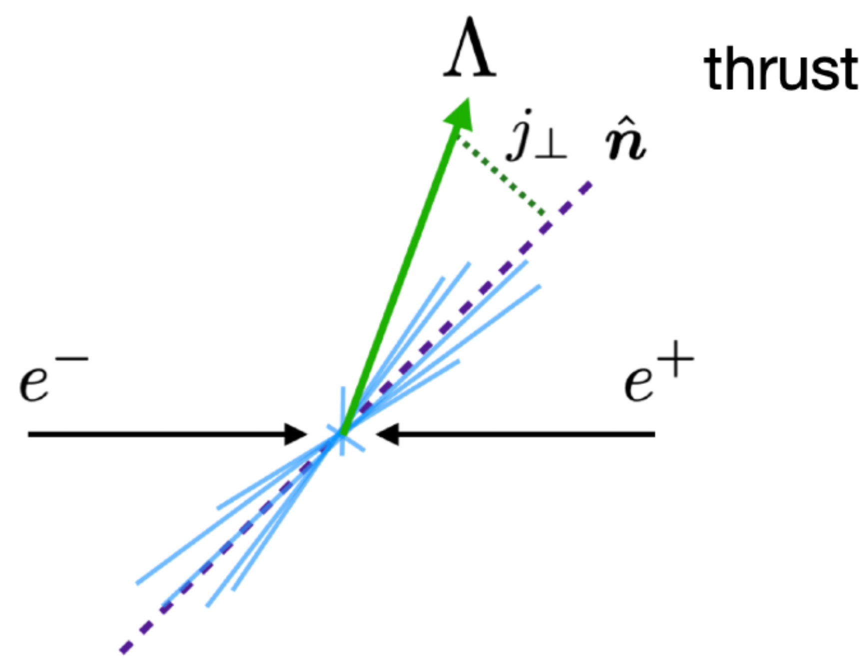
Transverse Λ polarization in electron positron collisions

The cleanest way to access fragmentation functions

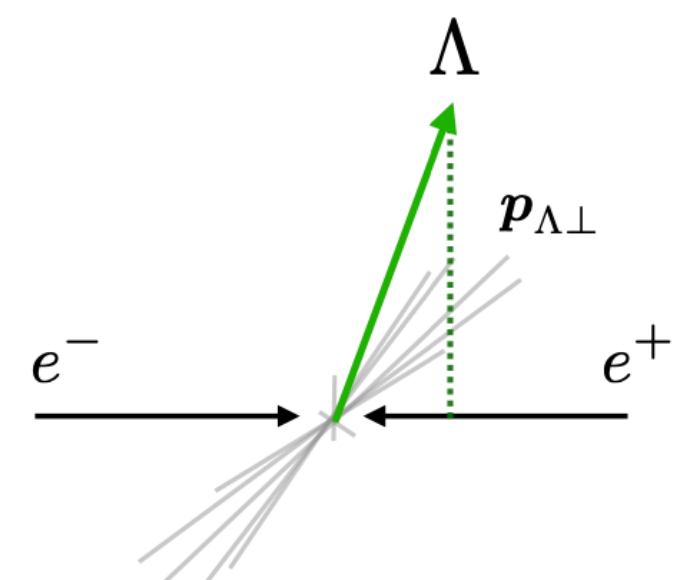
$e^+e^- \to \Lambda^\uparrow h\, X$



$e^+e^- \to \Lambda^\uparrow(\mathrm{Thrust})\, X$



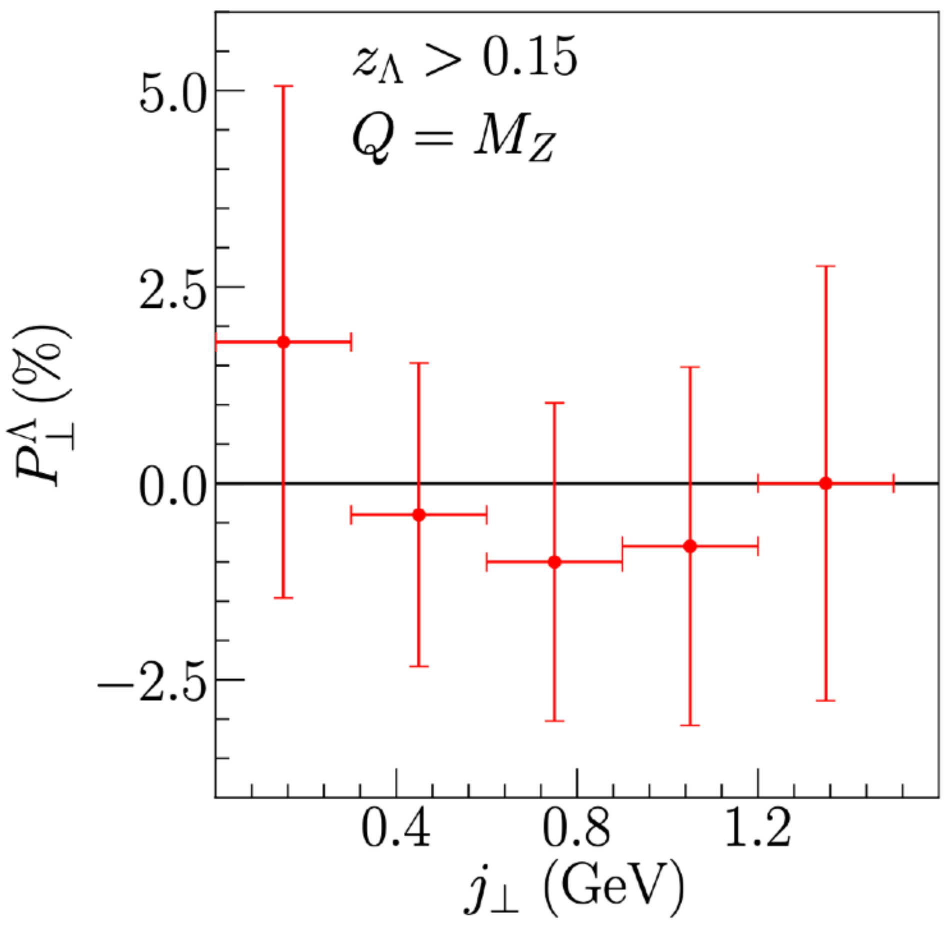
$e^+e^- \to \Lambda^\uparrow X$

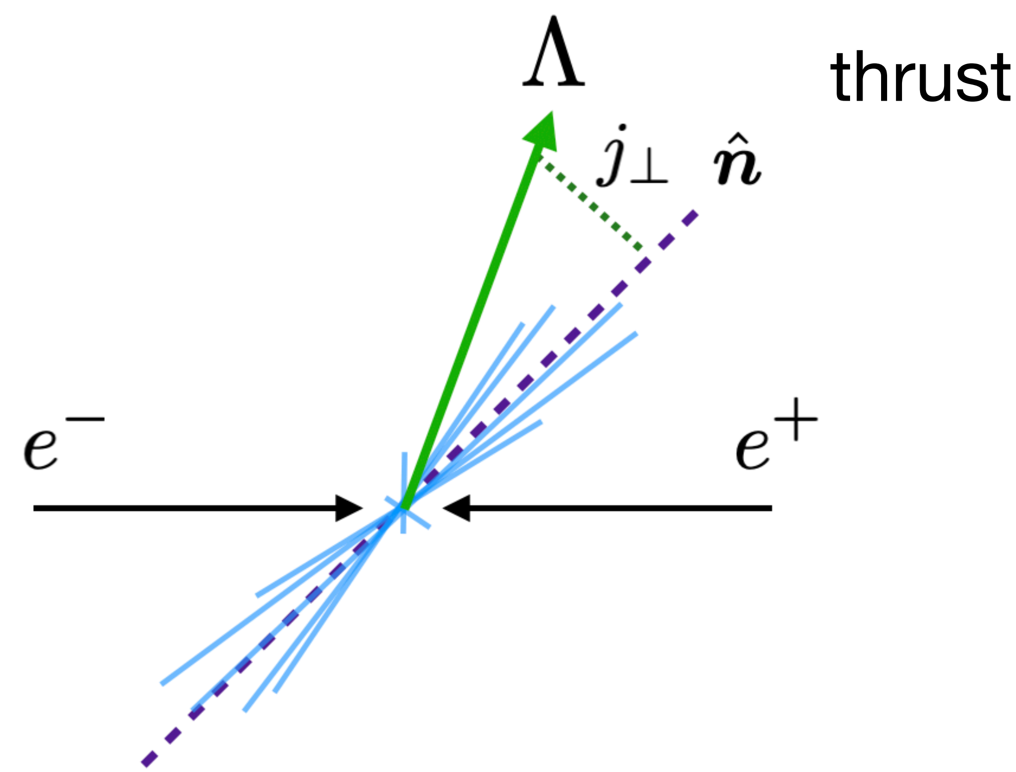


$$T \equiv \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_i |\vec{p}_i|}$$

Transverse Λ polarization at the LEP

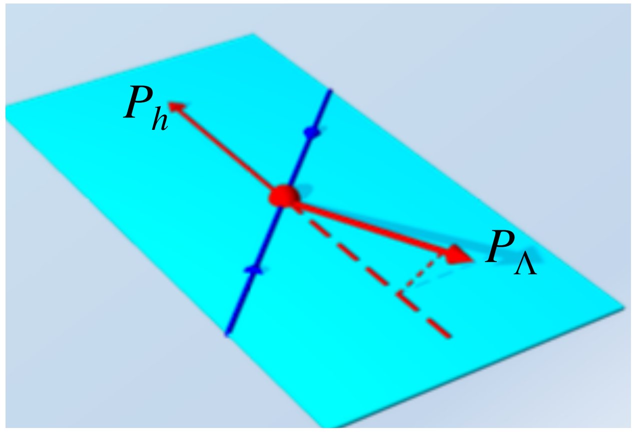
OPAL '97

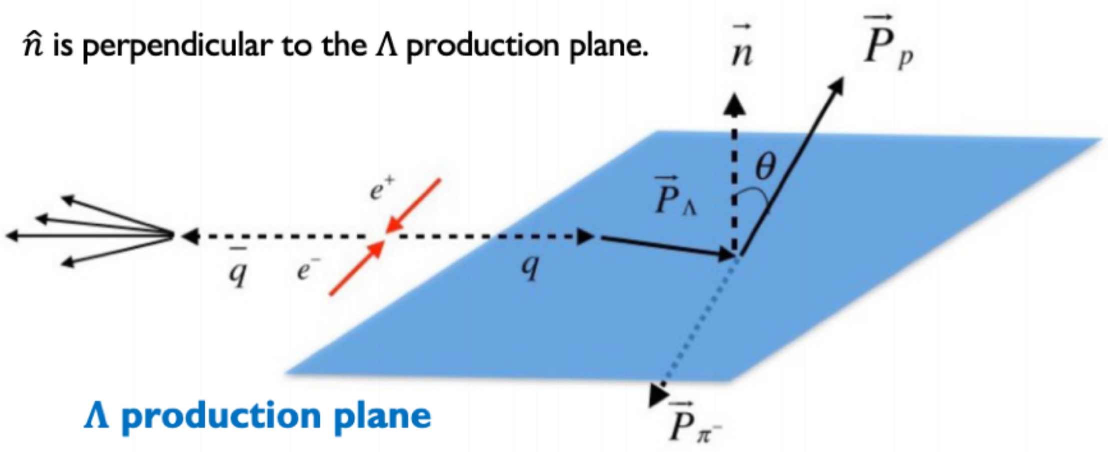




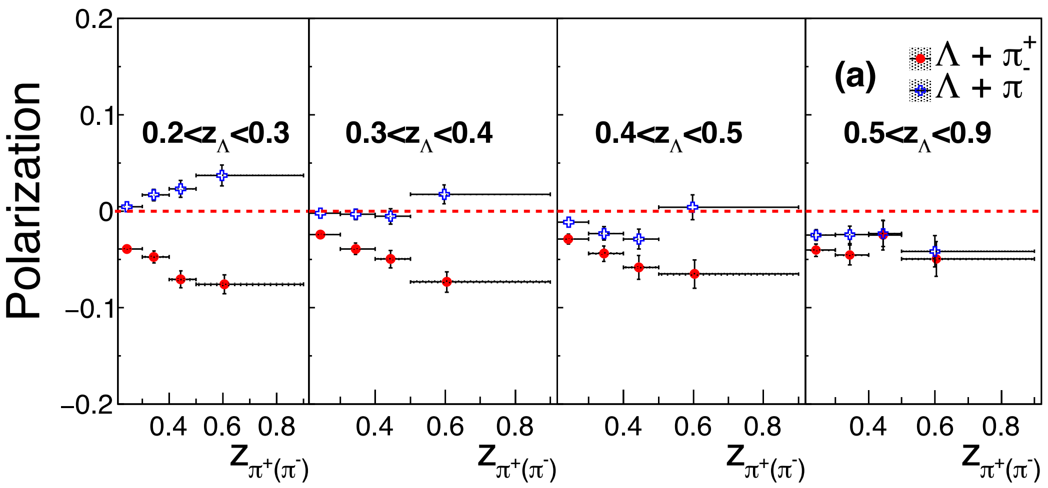
No significant transverse polarization is observed at the LEP

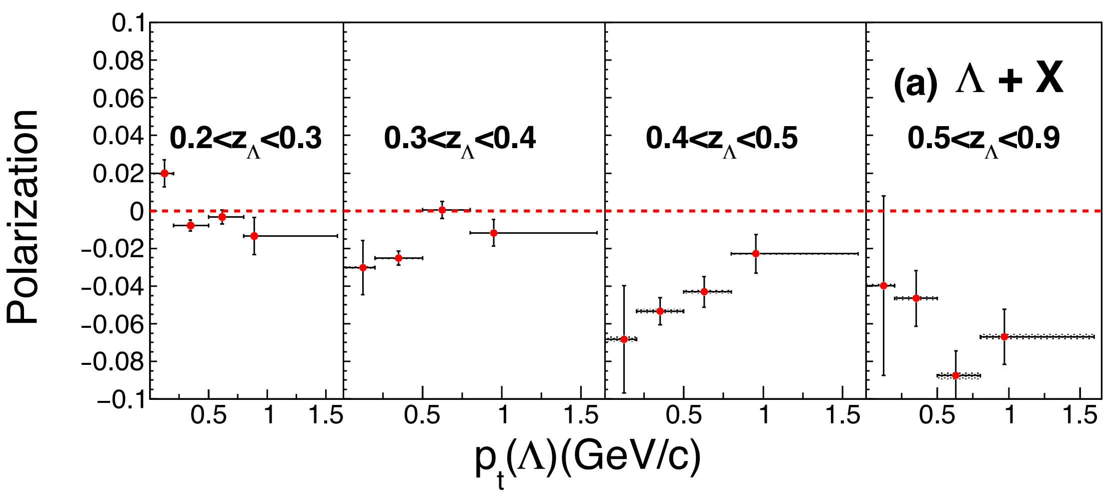
Transverse Λ polarization at the Belle





Belle '18 PRL



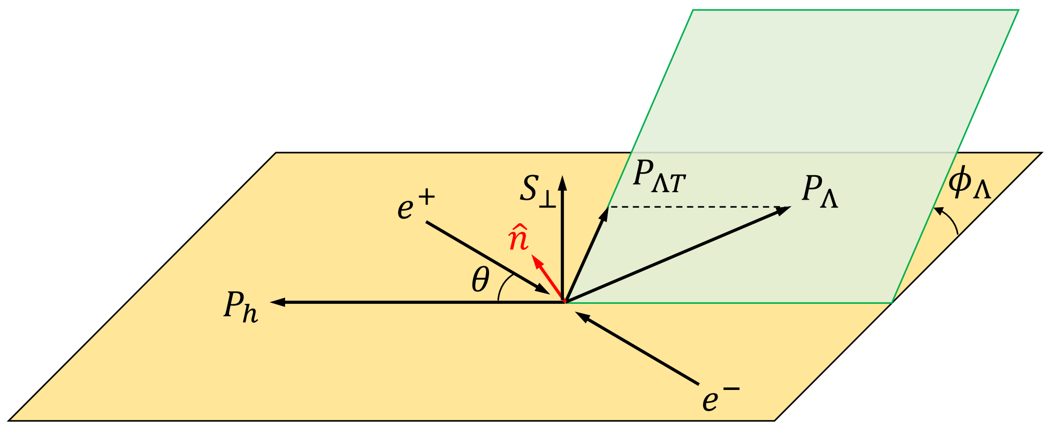


$$e^+e^- \to \Lambda^\uparrow h\, X$$

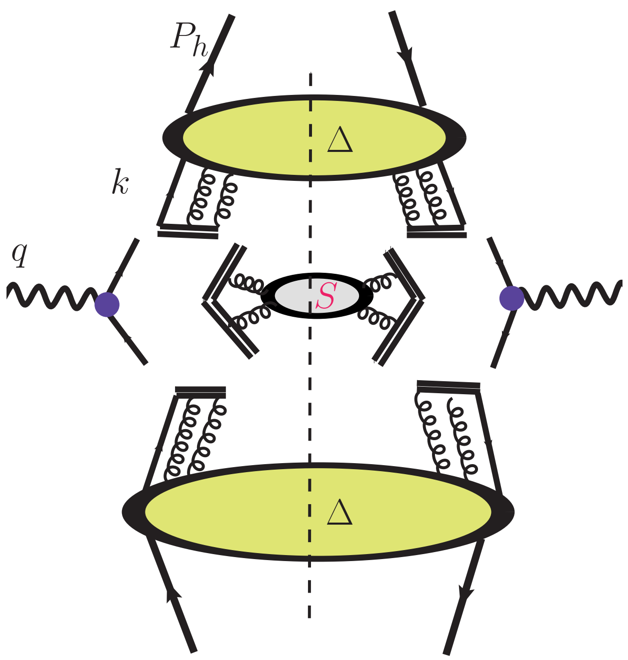
$$e^+e^- \to \Lambda^\uparrow(\mathrm{Thrust})\, X$$

Back-to-back Λ+h

$$e^-(\ell) + e^+(\ell') \to \gamma^*(q) \to h(P_h) + \Lambda(P_\Lambda, \boldsymbol{S}_\perp) + X$$



TMD factorization theorems for back-to-back Λ+h: CSS, JMY, SCET …



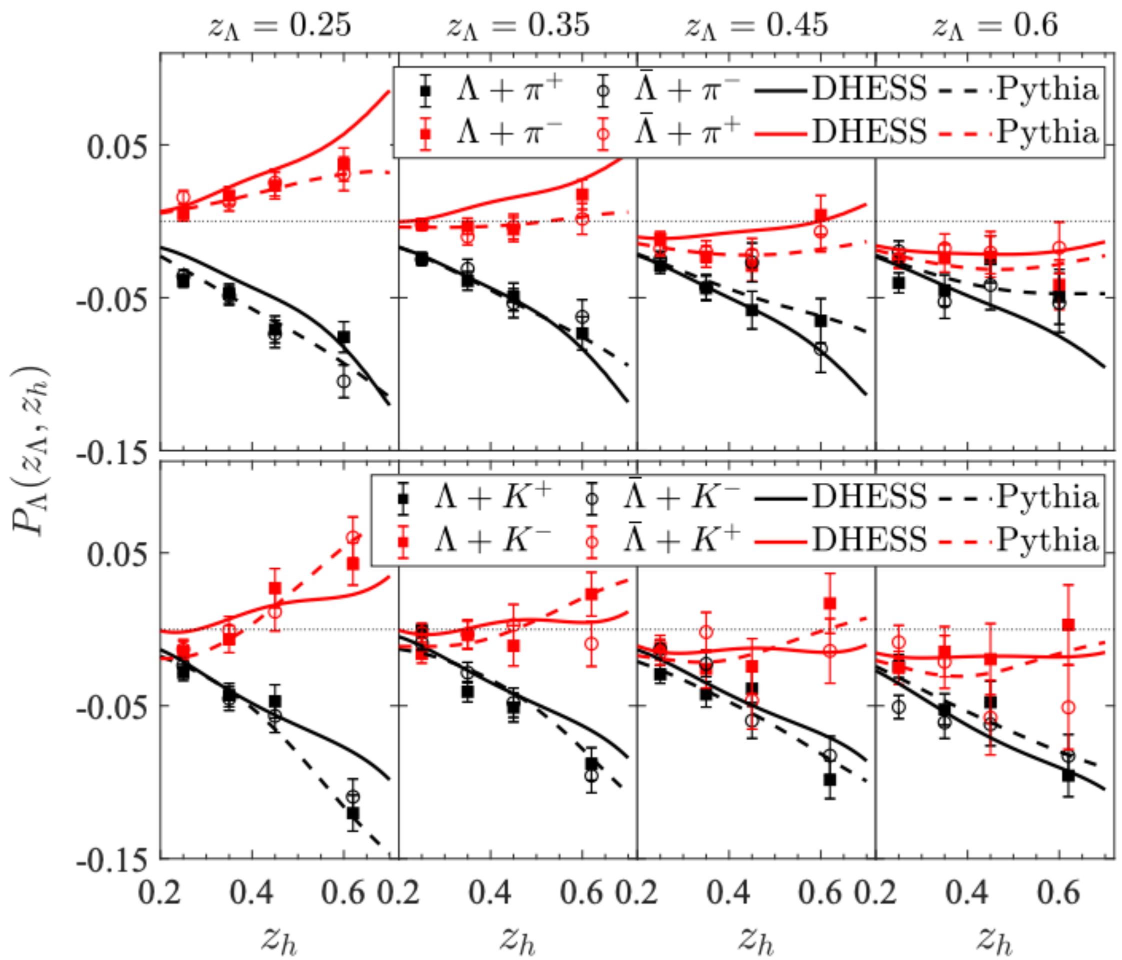
Spin-dependent cross section is factorized as:

$$\frac{d\sigma\left(\boldsymbol{S}_\perp\right)}{d\mathcal{PS}d^2\boldsymbol{q}_\perp} = \sigma_0 \left\{\mathcal{F}\left[D_{\Lambda/q}D_{h/\bar{q}}\right] + |\boldsymbol{S}_\perp| \sin\left(\phi_S - \phi_\Lambda\right) \frac{1}{z_\Lambda M_\Lambda}\mathcal{F}\left[\hat{\boldsymbol{P}}_{\Lambda T}\cdot\boldsymbol{p}_{\Lambda\perp}D_{1T,\Lambda/q}^\perp D_{h/\bar{q}}\right] + \cdots\right\}$$

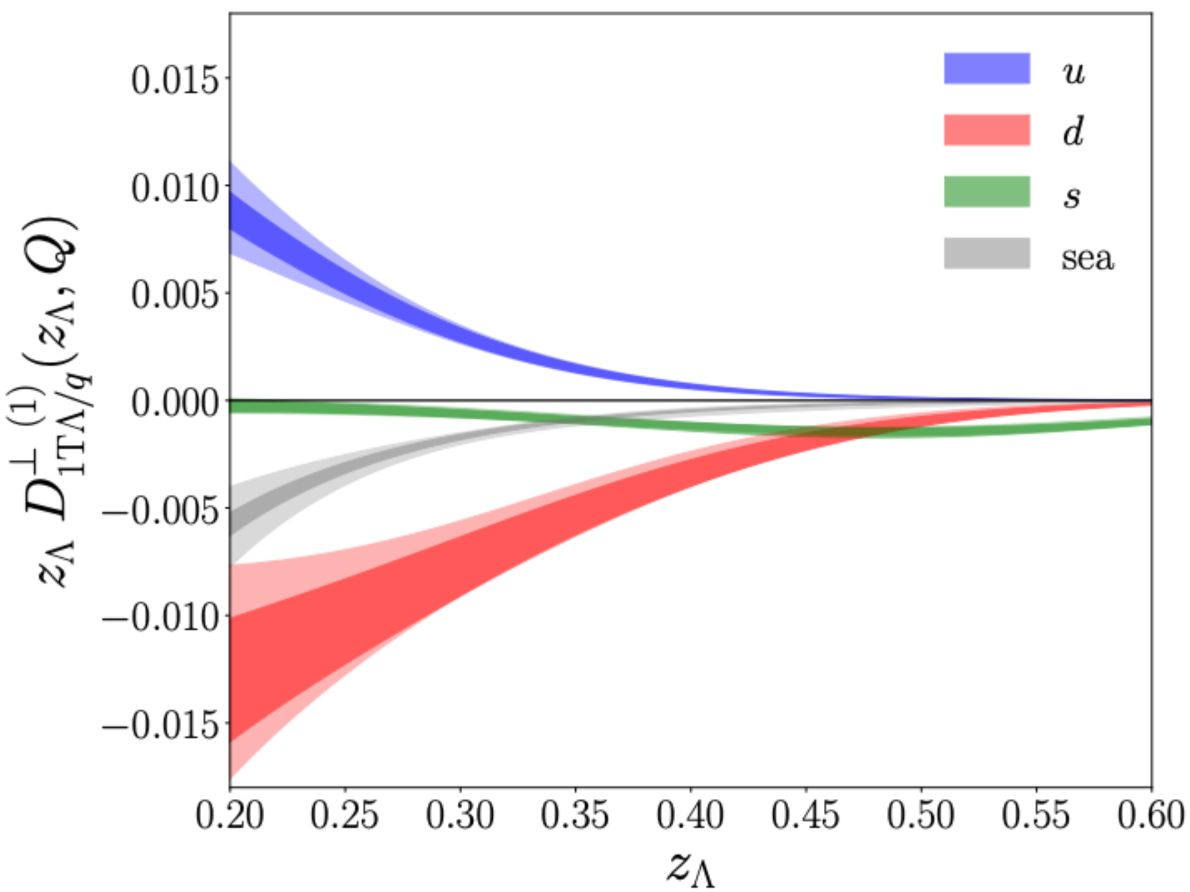
PFFs: Polarizing Fragmentation Functions

Fitting of PFFs from Λ+h data

Chen, Liang, Pan, Song, Wei '21



Kang, Terry, Vossen, Xu, Zhang '21



See also:
D' Alesio, Murgia, Zaccheddu '20
Callos, Kang, Terry '20
… …

Light bands: the uncertainty from the fit to Belle data

Dark bands: the simultaneous fit of the Belle data and the EIC pseudo-data

Theory framework on transverse Λ polarization

$$e^+e^- \to \Lambda^\uparrow h\, X$$

Collins-Soper-Sterman, Ji-Ma-Yuan,
Soft-Collinear Effective Theory… …

$$e^+e^- \to \Lambda^\uparrow(\mathrm{Thrust})\, X$$

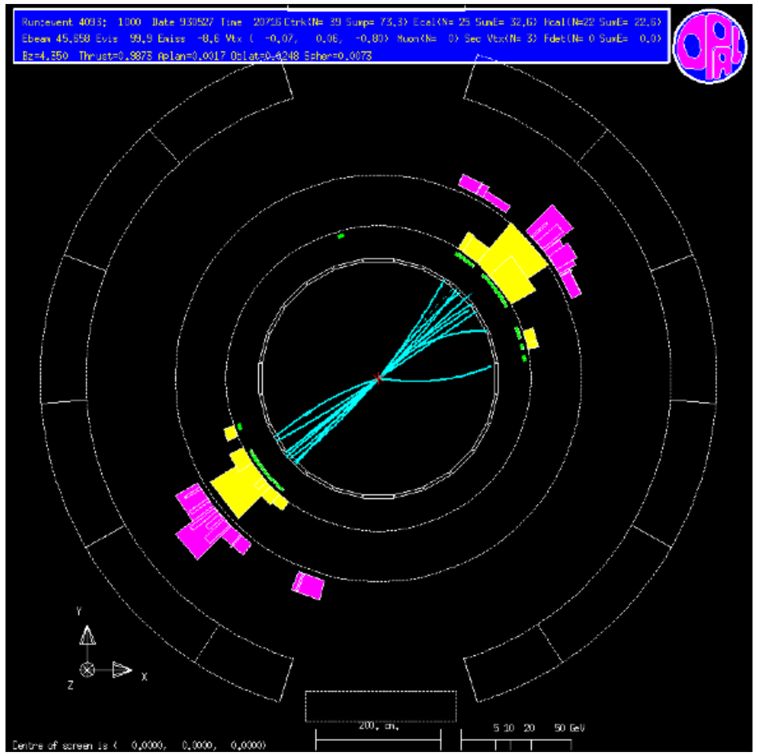
???

TMD factorization two scale problem

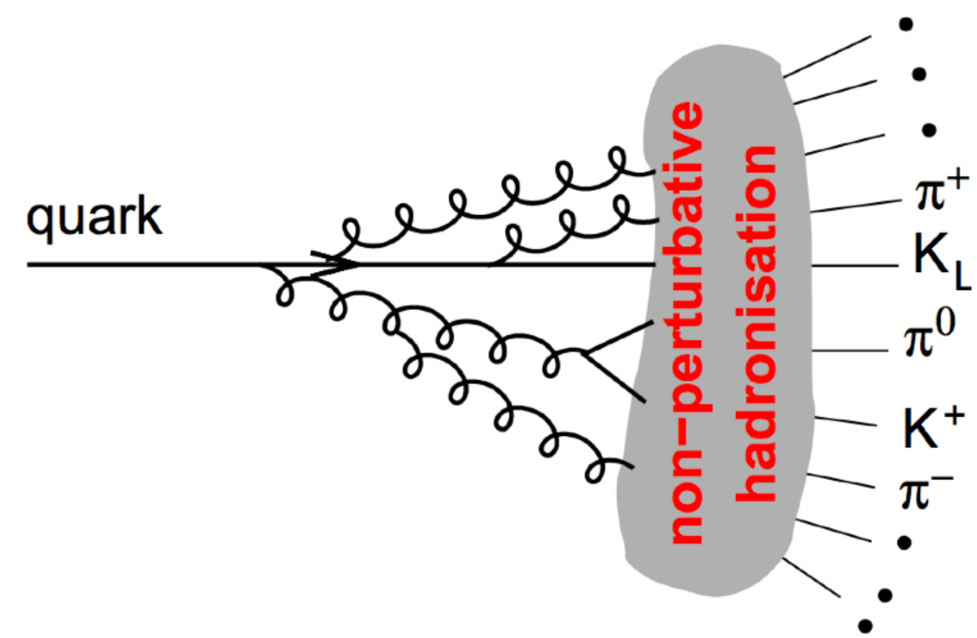
$$\Lambda_{QCD} \lesssim j_\perp \ll Q$$

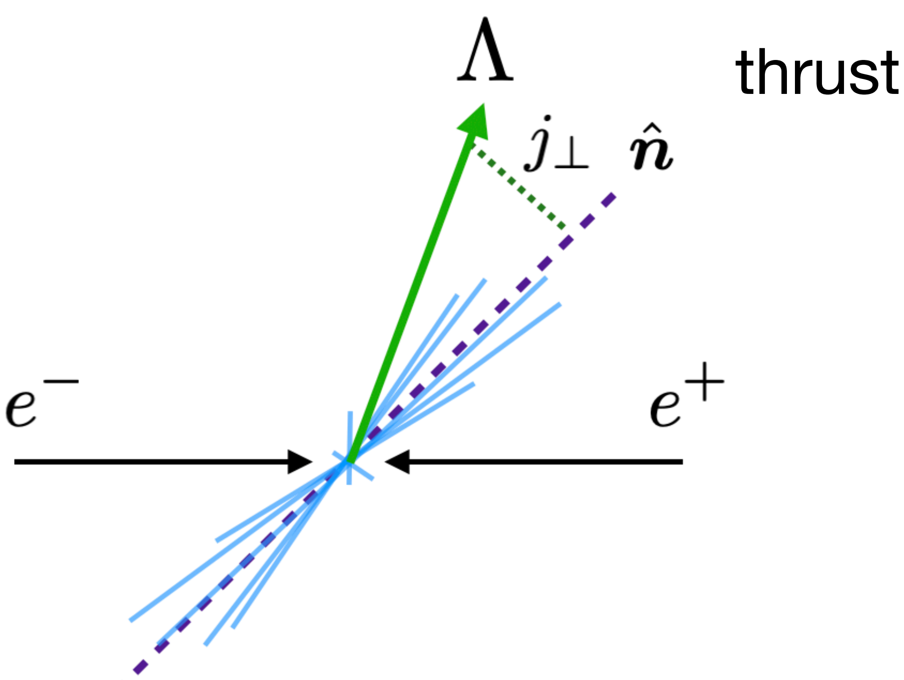
Is it the same (polarizing) fragmentation function in these two measurements ???

TMD factorization for Λ(thrust)



Parton fragmentation and hadronization





From short to long distances in quantum field theory

$$J\,(\text{ scale }\mu_2) \sim J\,(\text{scale}\,\mu_1)\exp\left[\int_{\mu_1}^{\mu_2}\frac{d\mu'}{\mu'}\int dx P\,(x,\alpha_s\,(\mu'))\right]$$

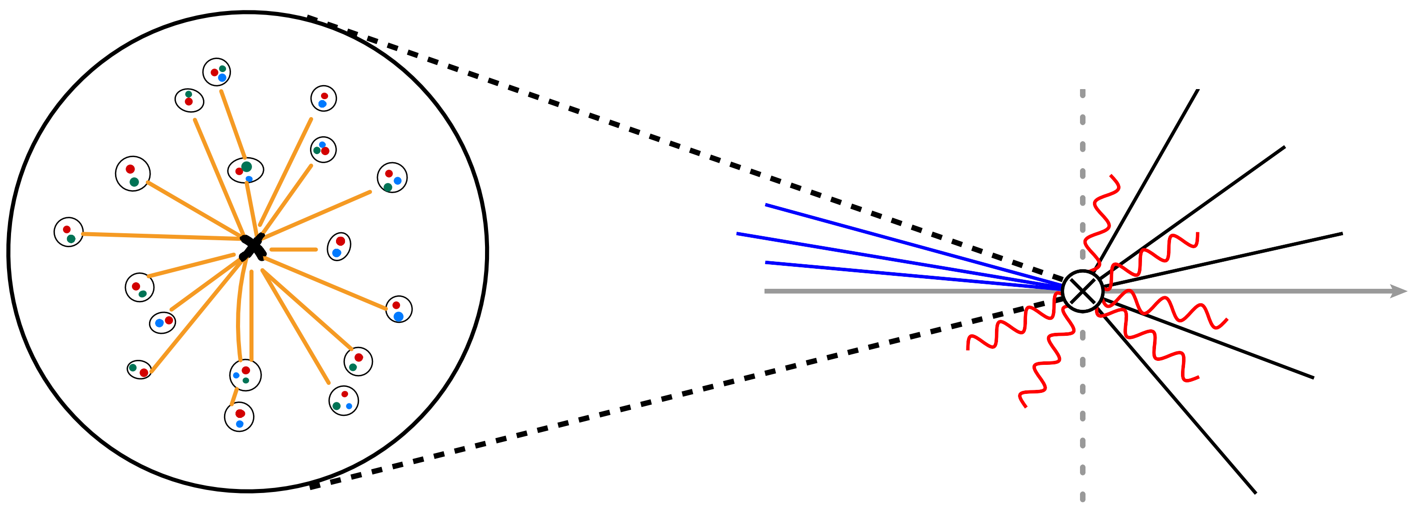
"Jets from Quantum Chromodynamics" Sterman & Weinberg '77

TMD factorization formula on the jet broadening

(Becher, Rahn, DYS '17 JHEP)

Definition of the broadening:

$$b_N = \sum_{i \in \text{jets}} \left| \vec{p}_i^{\perp} \right|$$



Construction of the theory formalism $b_N \ll Q$

- **Two scales in the problem**
- **Rely on effective field theory: SCET + Jet Effective Theory** (Becher, Neubert, Rothen, DYS '16 PRL)

$$\frac{d\sigma}{db_N} = \sum_{f=q,\bar{q},g} \int db_N^s \int d^{d-2}p_N^{\perp} \mathcal{J}_f\left(b_N - b_N^s, p_N^{\perp}\right) \sum_{m=1}^{\infty} \left\langle \mathcal{H}_m^f(\{\underline{n}\}, Q) \otimes \mathcal{S}_m\left(\{\underline{n}\}, b_N^s, -p_N^{\perp}\right) \right\rangle$$

Rapidity divergence cancellation is verified at two-loop order !!!

Factorization on single hadron unpolarized TMDs

Case-I: $e^- e^+ \to h_1 h_2 + X$

$$\frac{d\sigma}{d^2\boldsymbol{q}_T} \sim H \otimes D_{h_1} \otimes D_{h_2} \otimes S$$

Collins, "*Foundations of perturbative QCD*"

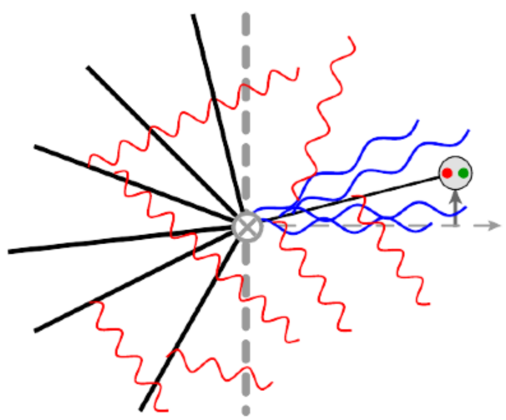
Global observable, standard TMD factorization

Case-II: $e^- e^+ \to h + X$

$$\frac{d\sigma}{d^2\boldsymbol{q}_T} \sim D_h \otimes \sum_m \mathcal{H}_m \otimes \mathcal{S}_m$$

Kang, DYS, Zhao '20 JHEP

Non-global observable; new TMD factorization



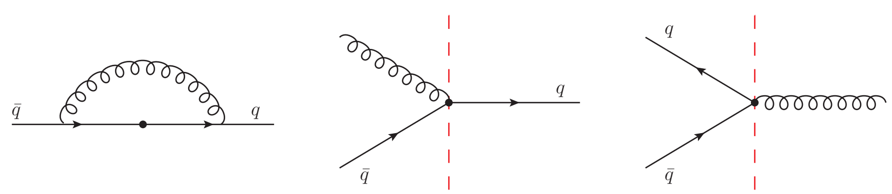
hard: $p_h \sim Q(1,1,1)$

collinear: $p_c \sim Q(\lambda^2, 1, \lambda)$

soft: $p_s \sim Q(\lambda, \lambda, \lambda)$

$$\lambda = j_T/Q \ll 1$$

NLO hard function:



Divergences are half of the hard function in case-I

NLO soft function:

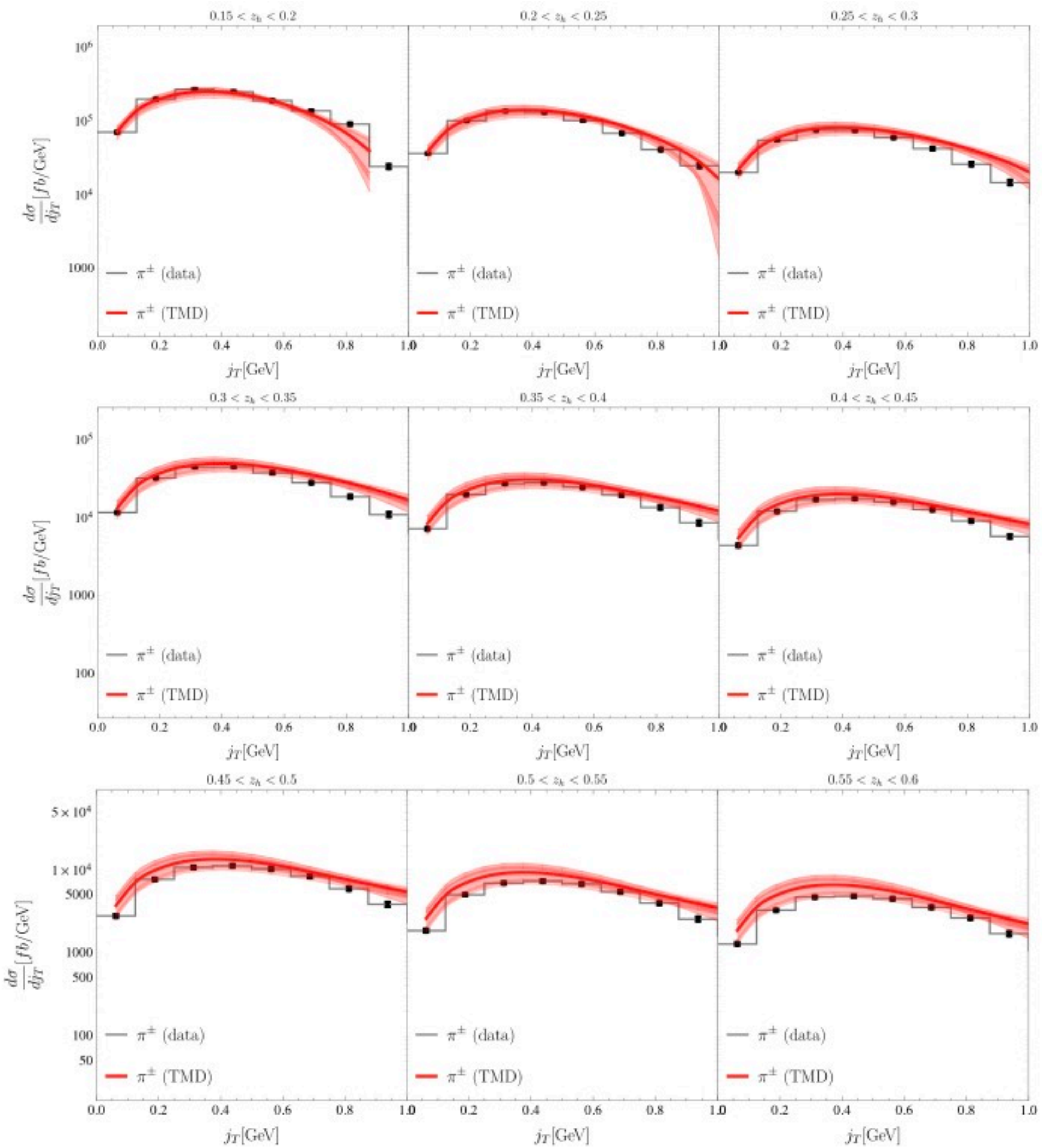
$$\frac{\alpha_s C_F}{2\pi^2}\frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\int \frac{dk^+ dk^-}{2}\left(\frac{\mu^2}{\vec{\lambda}_T^2}\right)^{\epsilon}\frac{2n\cdot\bar{n}}{k^+k^-}\delta^+(k^+k^- - \vec{\lambda}_T^2)\left|\frac{\nu}{2k_z}\right|^{\eta}\theta\left(1-\frac{k^+}{k^-}\right)$$

$$= \frac{\alpha_s}{2\pi}C_F\left[\frac{2}{\eta}\left(-\frac{1}{\epsilon} - \ln\left(\frac{\mu^2}{\mu_b^2}\right)\right) + \frac{1}{\epsilon^2} - \frac{1}{\epsilon}\ln\left(\frac{\nu^2}{\mu^2}\right)\right]$$

Divergences are half of the soft function in case-I

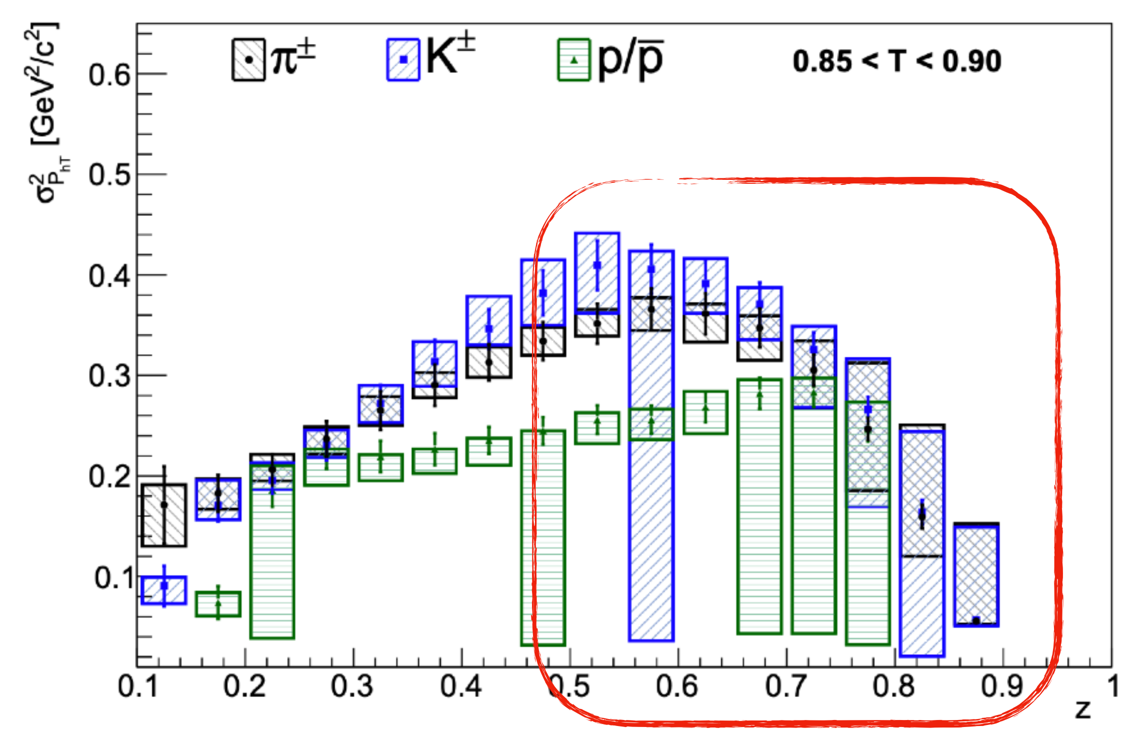
Numerical results

Kang, DYS, Zhao '20 JHEP



- **Our TMD resummation formula gives a good description of the shape of j_T distribution as z_h < 0.65**
- **As z_h > 0.65, one needs to also include threshold resummation effects**

$$\frac{d\sigma}{dz_h d^2\vec{j}_T} \propto \frac{1}{\pi\sigma_{j_T}^2} \exp\left(-j_T^2/\sigma_{j_T}^2\right)$$



Joint threshold and TMD factorization

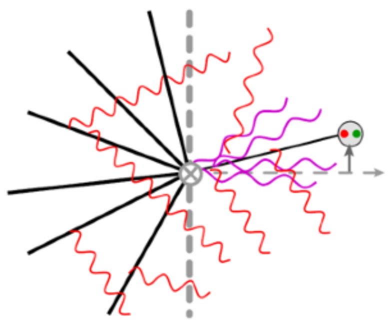
Kang, DYS, Zhao ’20 JHEP

Joint factorization: $z_h \to 1$ **&** $j_T \ll Q$

Joint TMD and threshold resummation is first developed in Li ’98 & Laenen, Sterman, Vogelsang ‘01

in the threshold region, a new mode: collinear-soft (c-soft) modes contribute

Li, Neill, Zhu ’16 & Lustermans, Waalewijn, Zeune '16

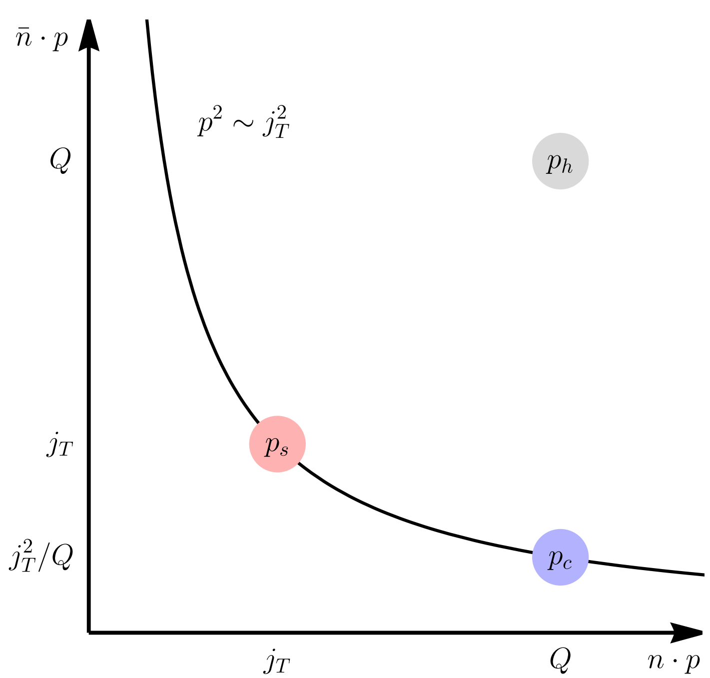


hard: $p_h \sim Q(1,1,1)$

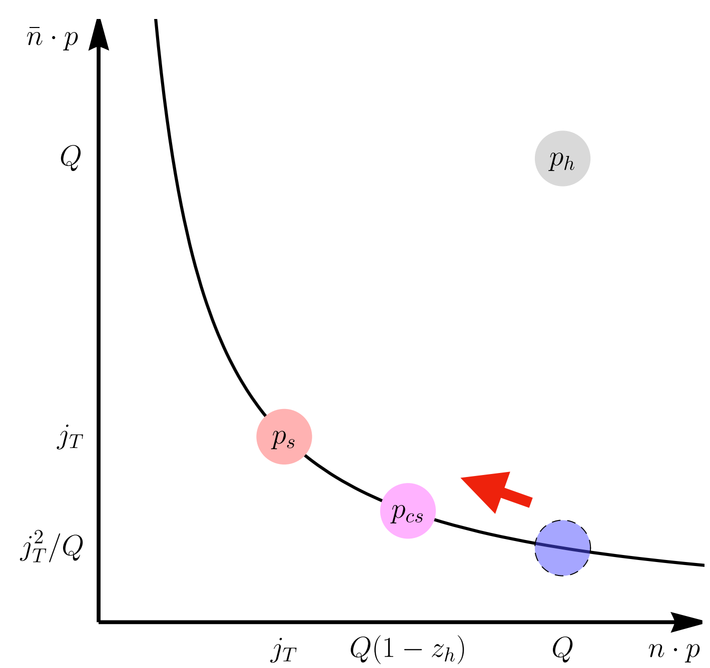
collinear: $p_c \sim Q(\lambda^2, 1, \lambda)$ $\xrightarrow{z_h \to 1}$ **c-soft**: $p_{\mathscr{S}} \sim (j_T^2/(Q(1-z_h)), Q(1-z_h), j_T)$

soft: $p_s \sim Q(\lambda, \lambda, \lambda)$

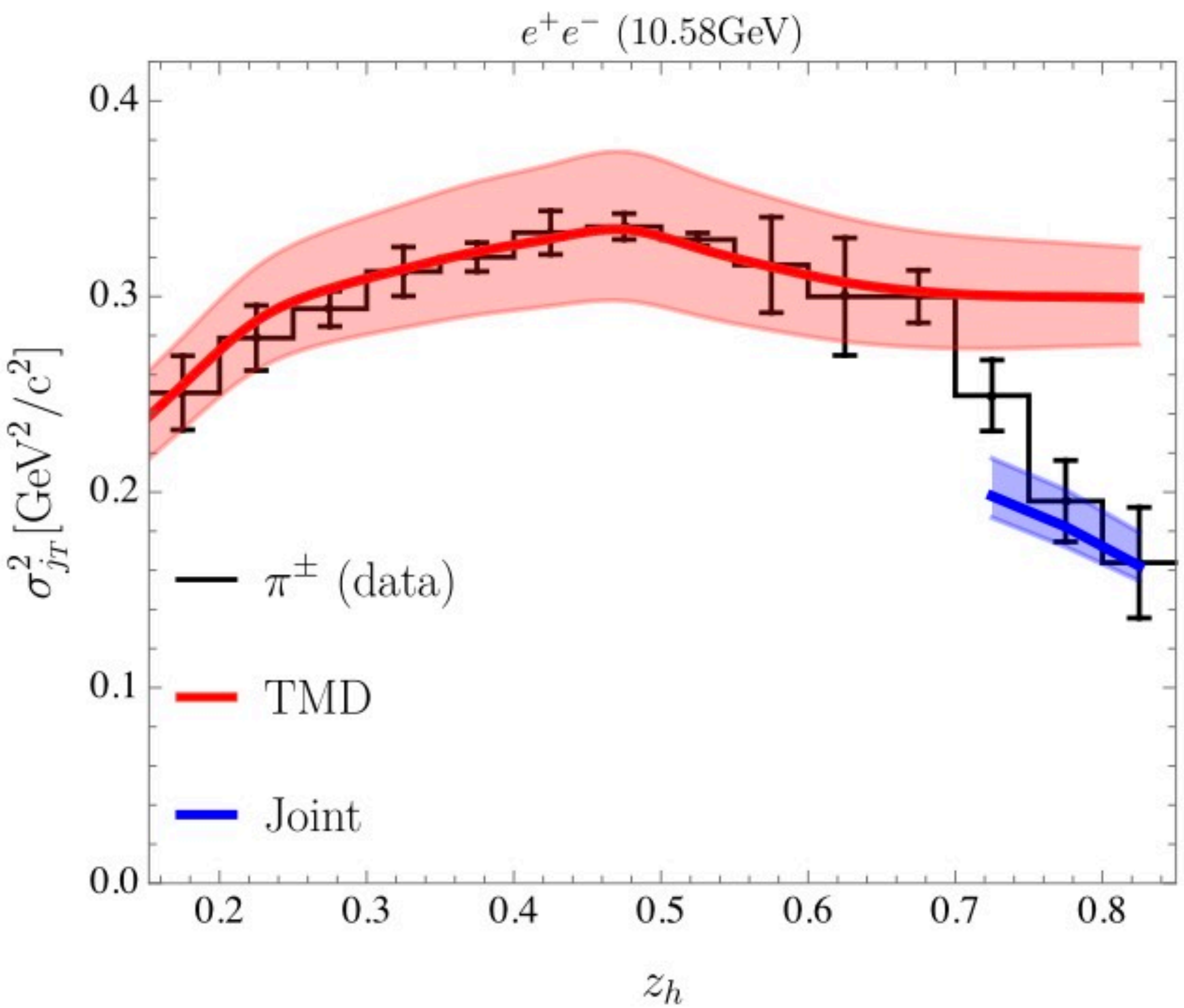
TMD FFs in the threshold limit



$z_h \to 1$



Numerical results

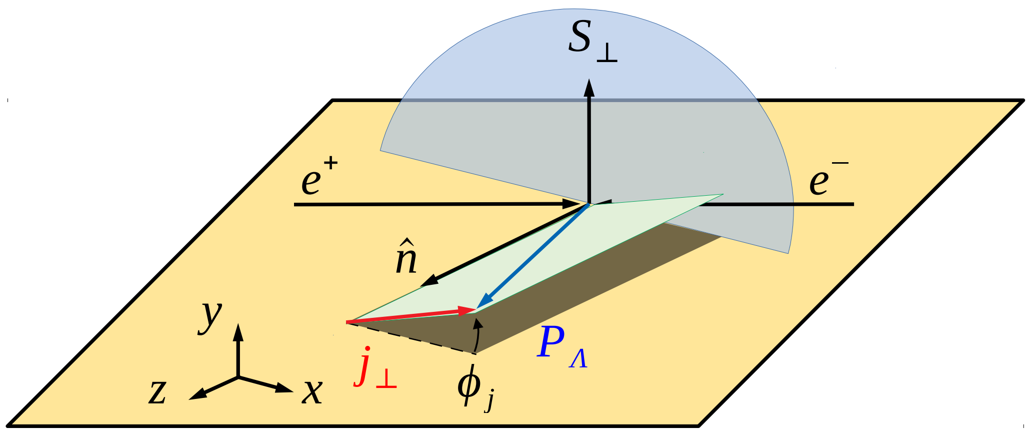


$$\frac{d\sigma}{dz_h d^2\vec{j}_T} \propto \frac{1}{\pi\sigma^2_{j_T}} \exp\left(-j_T^2/\sigma^2_{j_T}\right)$$

- **The Gaussian width of the j_T distribution given by the TMD formalism freeze to a certain value.**
- **After including joint threshold and TMD resummation effects, the theoretical predictions are consistent with the data**

Factorization on transverse polarized Λ hyperon production with the thrust axis

Gamberg, Kang, DYS, Terry, Zhao ’21 PLB

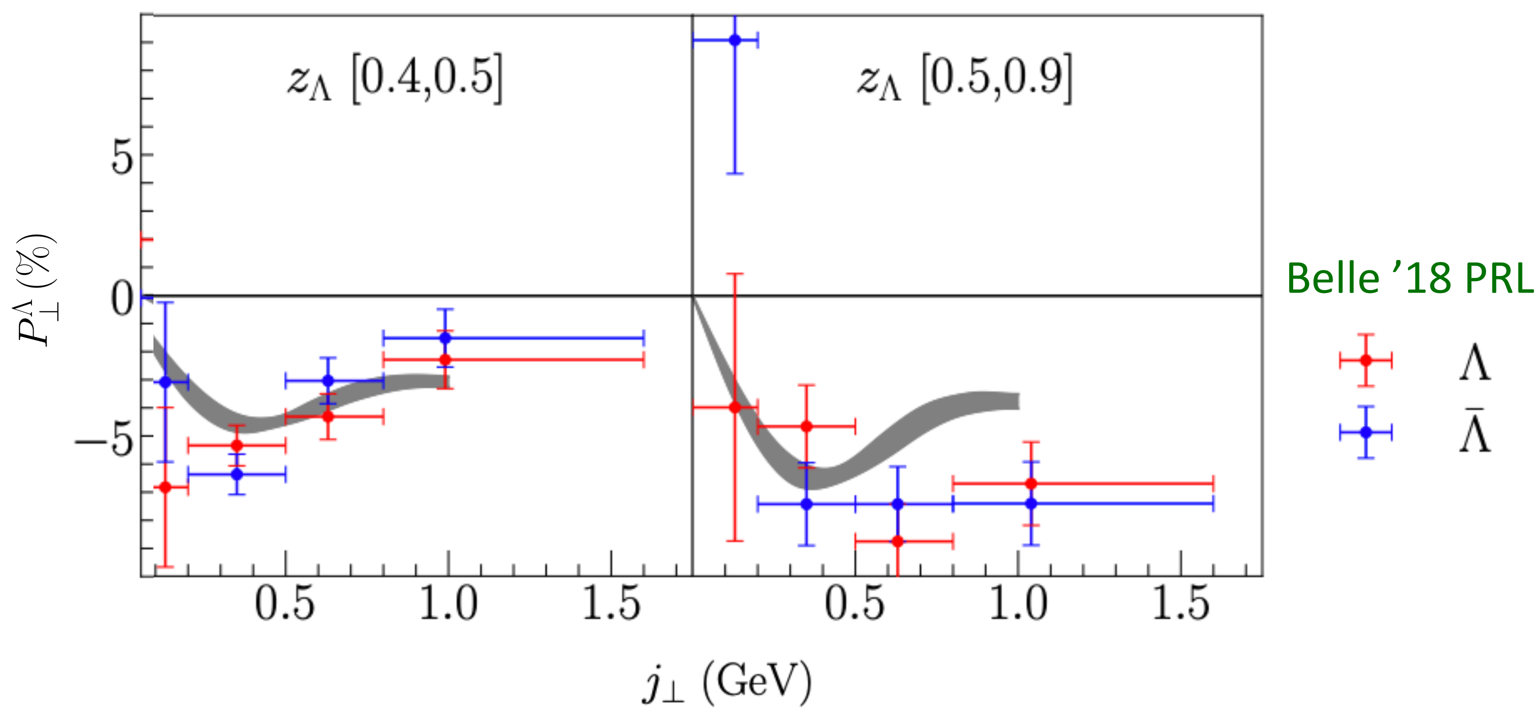


$$P_\perp^\Lambda(z_\Lambda, j_\perp) = \frac{d\Delta\sigma}{dz_\Lambda d^2\boldsymbol{j}_\perp} \Big/ \frac{d\sigma}{dz_\Lambda d^2\boldsymbol{j}_\perp}$$

Theory results are consistent with Belle data

Theory formula including QCD evolution

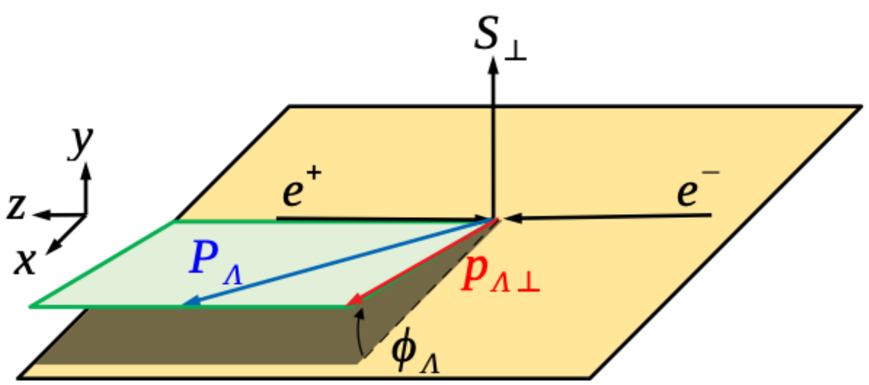
$$\begin{aligned}\frac{d\Delta\sigma}{dz_\Lambda d^2 j_\perp} &= \frac{d\sigma\left(\boldsymbol{S}_\perp\right)}{dz_\Lambda d^2\boldsymbol{j}_\perp} - \frac{d\sigma\left(-\boldsymbol{S}_\perp\right)}{dz_\Lambda d^2\boldsymbol{j}_\perp} \\ &= \sigma_0 \sin\left(\phi_s - \phi_j\right) \sum_q e_q^2 \int_0^\infty \frac{b^2 db}{2\pi} J_1\left(\frac{b j_\perp}{z_\Lambda}\right) \\ &\times \frac{M_\Lambda}{z_\Lambda^2} D_{1T,\Lambda/q}^{\perp(1)}\left(z_\Lambda, \mu_{b_*}\right) e^{-S_{\rm NP}^\perp\left(b, z_\Lambda, Q_0', Q\right) - S_{\rm pert}\left(\mu_{b_*}, Q\right)} \\ &\times U_{\rm NG}\left(\mu_{b_*}, Q\right)\end{aligned}$$

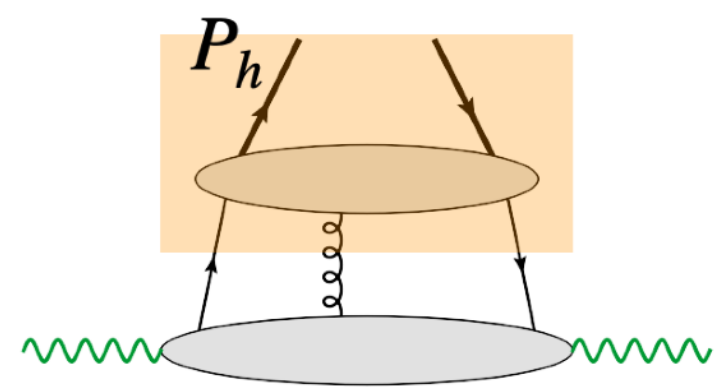


This result provides proof of principle that the experimental data can be described using the factorization and resummation formalism that we have introduced.

Factorization on transverse polarized Λ hyperon production with the thrust axis

Gamberg, Kang, DYS, Terry, Zhao '21 PLB



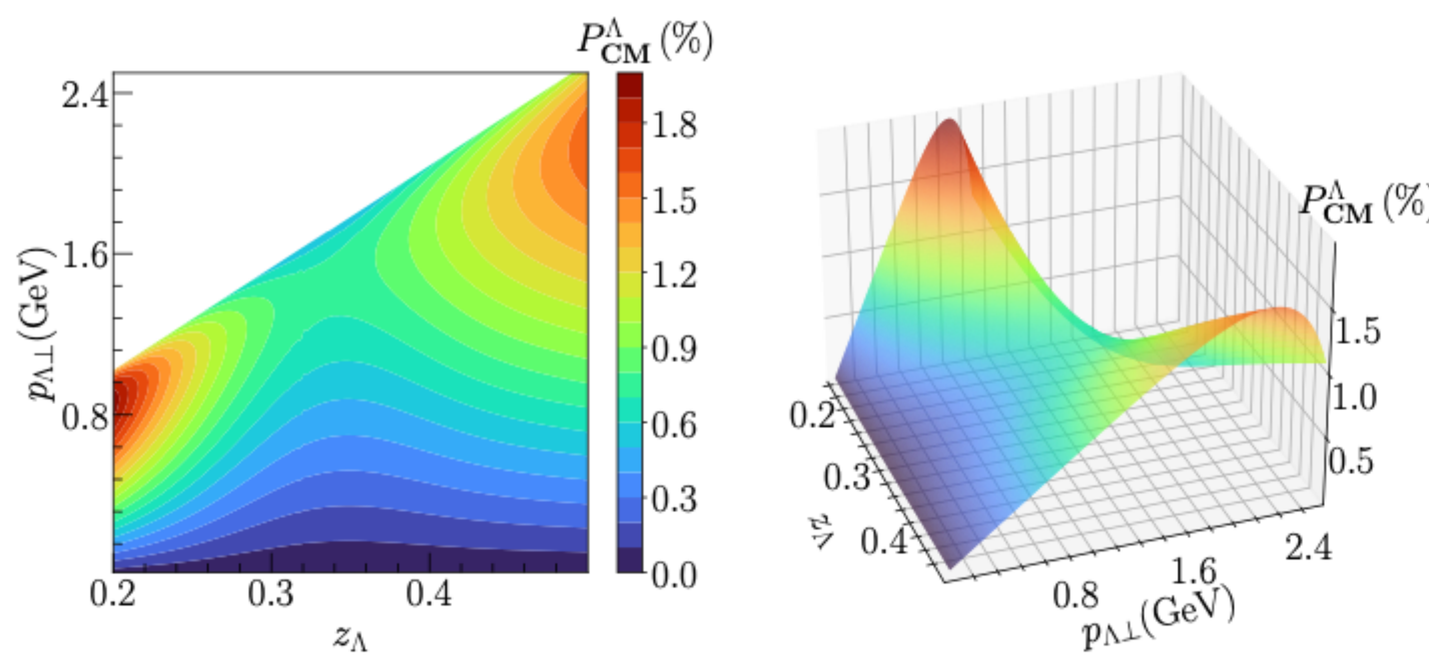


Twist-3 theory formula including QCD evolution

$$\frac{d\Delta\sigma}{dz_\Lambda\, d^2p_{\Lambda\perp}} = -\sin(\phi_s - \phi_\Lambda)\frac{2N_c\alpha_{\rm em}^2}{Q^4 z_\Lambda}\left(\frac{4M_\Lambda}{Q}\right)\frac{p_{\Lambda\perp}}{Q} \times \frac{1}{z_\Lambda^3}\sum_q e_q^2 \frac{D_{T,\Lambda/q}(z_\Lambda, Q)}{z_\Lambda}\,.$$

$$\frac{1}{z_\Lambda}D_{T,\Lambda/q}(z_\Lambda, Q) = -\left(1 - z_\Lambda\frac{d}{dz_\Lambda}\right)D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda, Q) - 2\int_0^1 d\beta\,\frac{\Im\left[\hat{D}_{FT}^{qg}(z_\Lambda, Q, \beta)\right]}{(1-\beta)^2}\,.$$

Theory predictions



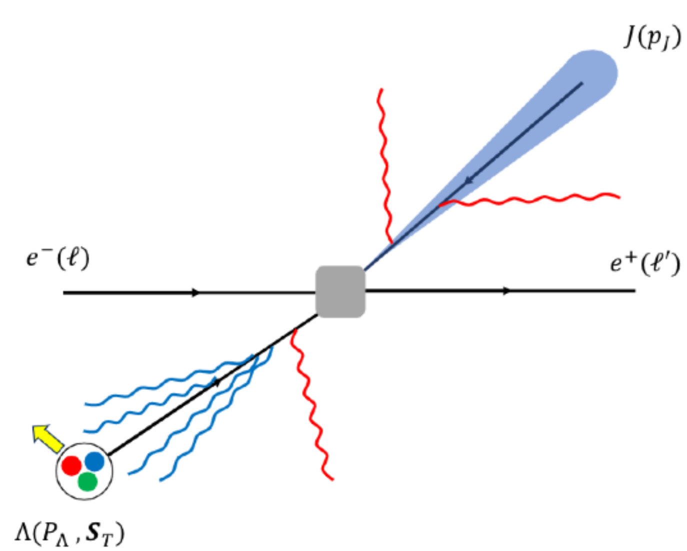
$$P_{\rm CM}^\Lambda(z_\Lambda, p_{\Lambda\perp}) = \frac{d\Delta\sigma}{dz_\Lambda\, d^2p_{\Lambda\perp}}\Bigg/\frac{d\sigma}{dz_\Lambda\, d^2p_{\Lambda\perp}}$$

Transverse Lambda polarization and jet charge

(Gamberg, Kang, DYS, Terry, Zhao in progress)

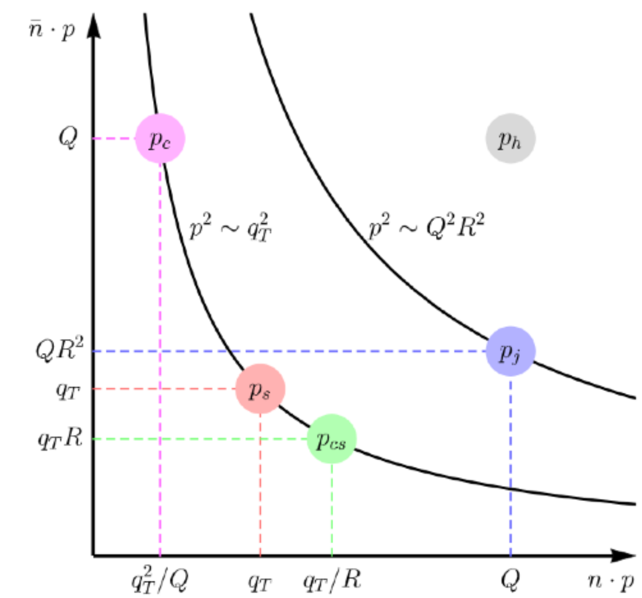
As shown in (Kang, Liu, Mantry, DYS '20 PRL)**, the jet charge observable is a novel probe of flavor structure for the hadron spin**

$$e^-(\ell) + e^+(\ell') \to J(p_J) + \Lambda(P_\Lambda, \boldsymbol{S}_T) + X$$



Dynamic modes:

- hard: $p_h \sim Q(1,1,1)$,
- soft: $p_s \sim q_T(1,1,1)$,
- collinear: $p_c \sim (q_T^2/Q, Q, q_T)$,
- jet: $p_j \sim Q(1, R^2, R)$,
- collinear-soft: $p_{cs} \sim q_T/R(1, R^2, R)$,

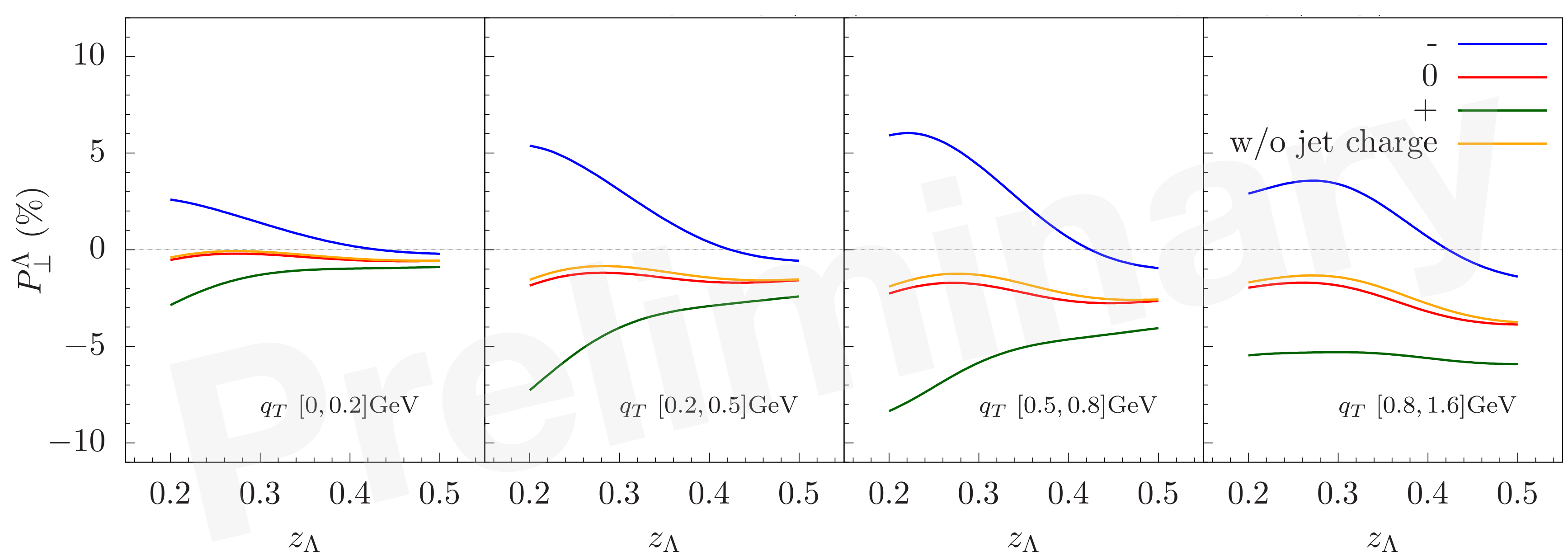


Jet charge definition: $Q_\kappa = \sum_i \left(\frac{p_{i,T}}{p_J}\right)^\kappa Q_i$

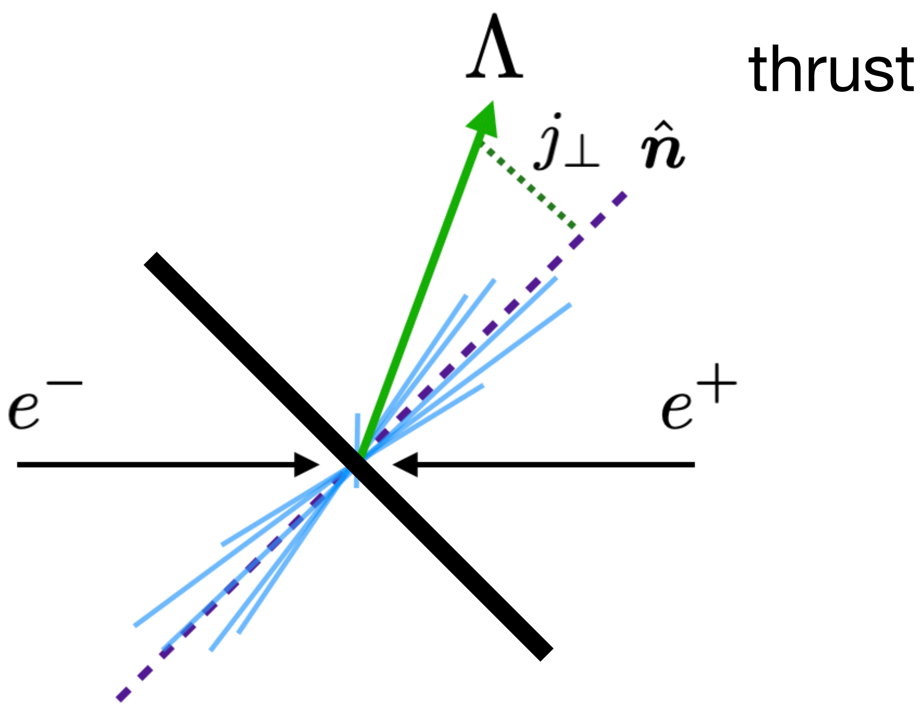
Charge tagged jet function:

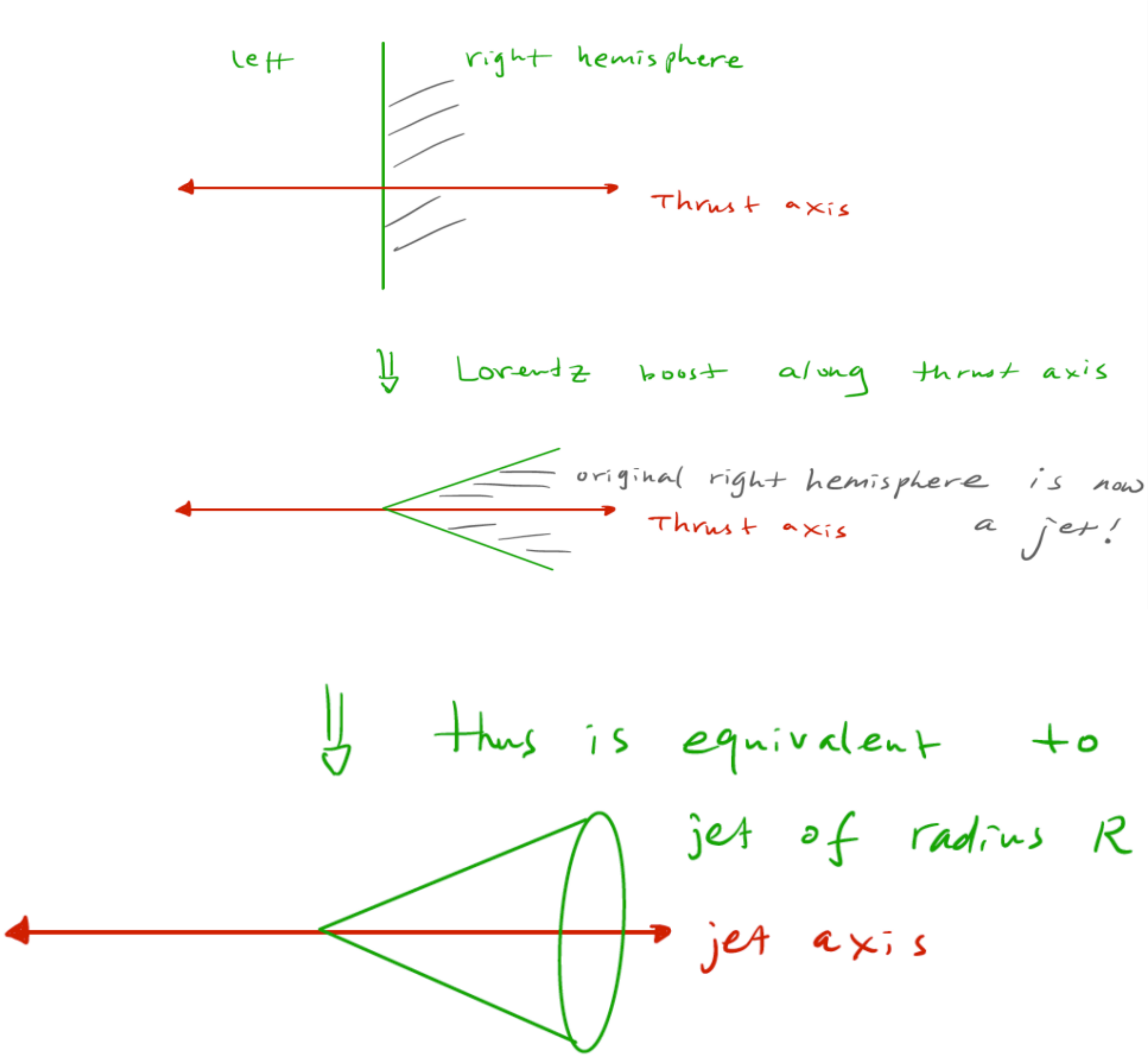
$$\mathcal{G}_i(Q_\kappa, p_T R, \mu)$$

Polarization w/ jet charge



Thrust and Jet measurements are closely related





Drawing by Zhong-Bo Kang

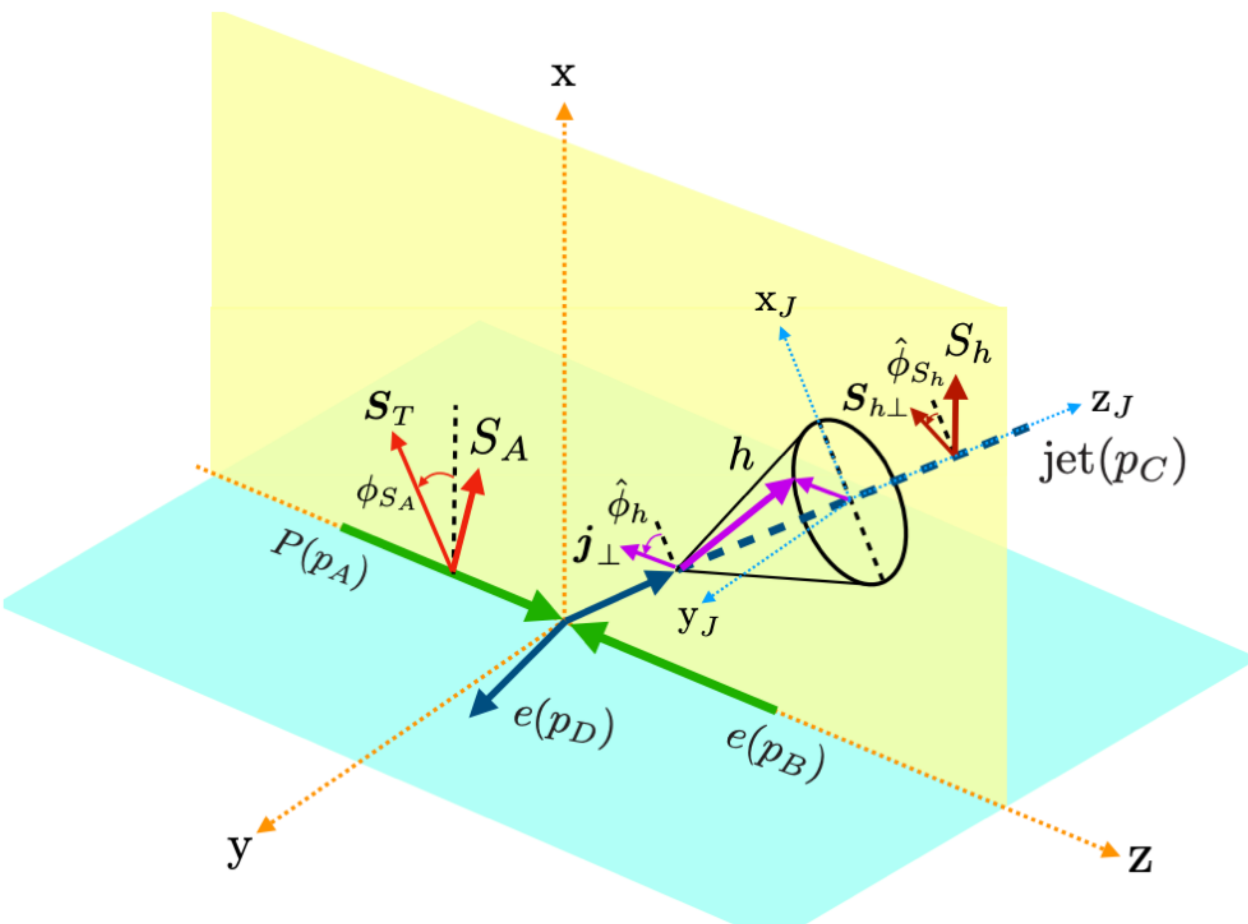
Spin Asymmetries in Electron-jet Production at the EIC

Kang, Lee, DYS, Zhao '21 '22 JHEP

We present a general theoretical framework for the hadron distribution in a jet, where both incoming particles and outgoing hadrons inside the jet have general polarizations.

$$p(p_A, S_A) + e(p_B, \lambda_e) \to \Big[\text{jet}(p_C)\, h\,(z_h, \boldsymbol{j}_\perp, \boldsymbol{S}_{h\perp})\Big] + e(p_D) + X$$

We investigate all possible azimuthal asymmetries.



$$\begin{aligned}
&\frac{d\sigma^{p(S_A)+e(\lambda_e)\to e+(\text{jet}\,h(S_h))+X}}{dp_T^2\,dy_J\,d^2\boldsymbol{q}_T\,dz_h\,d^2\boldsymbol{j}_\perp} = F_{UU,U} + \cos(\phi_q - \hat{\phi}_h) F_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)} \\
&+ \lambda_p \Big\{ \lambda_e F_{LL,U} + \sin(\phi_q - \hat{\phi}_h) F_{LU,U}^{\sin(\phi_q-\hat{\phi}_h)} \Big\} \\
&+ S_T \Big\{ \sin(\phi_q - \phi_{S_A}) F_{TU,U}^{\sin(\phi_q-\phi_{S_A})} + \lambda_e \cos(\phi_q - \phi_{S_A}) F_{TL,U}^{\cos(\phi_q-\phi_{S_A})} \\
&\qquad + \sin(\phi_{S_A} - \hat{\phi}_h) F_{TU,U}^{\sin(\phi_{S_A}-\hat{\phi}_h)} + \sin(2\phi_q - \hat{\phi}_h - \phi_{S_A}) F_{TU,U}^{\sin(2\phi_q-\hat{\phi}_h-\phi_{S_A})} \Big\} \\
&+ \lambda_h \Big\{ \lambda_e F_{UL,L} + \sin(\hat{\phi}_h - \phi_q) F_{UU,L}^{\sin(\hat{\phi}_h-\phi_q)} + \lambda_p \Big[F_{LU,L} + \cos(\hat{\phi}_h - \phi_q) F_{LU,L}^{\cos(\hat{\phi}_h-\phi_q)} \Big] \\
&\qquad + S_T \Big[\cos(\phi_q - \phi_{S_A}) F_{TU,L}^{\cos(\phi_q-\phi_{S_A})} + \lambda_e \sin(\phi_q - \phi_{S_A}) F_{TL,L}^{\sin(\phi_q-\phi_{S_A})} \\
&\qquad\qquad + \cos(\phi_{S_A} - \hat{\phi}_h) F_{TU,L}^{\cos(\phi_{S_A}-\hat{\phi}_h)} + \cos(2\phi_q - \phi_{S_A} - \hat{\phi}_h) F_{TU,L}^{\cos(2\phi_q-\phi_{S_A}-\hat{\phi}_h)} \Big] \Big\} \\
&+ S_{h\perp} \Big\{ \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} + \lambda_e \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UL,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})} \\
&\qquad + \sin(\hat{\phi}_{S_h} - \phi_q) F_{UU,T}^{\sin(\hat{\phi}_{S_h}-\phi_q)} + \sin(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_q) F_{UU,T}^{\sin(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_q)} \\
&\qquad + \lambda_p \Big[\cos(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{LU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})} + \cos(\phi_q - \hat{\phi}_{S_h}) F_{LU,T}^{\cos(\phi_q-\hat{\phi}_{S_h})} \\
&\qquad\qquad + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_q) F_{LU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_q)} + \lambda_e \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{LL,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} \Big] \\
&\qquad + S_T \Big[\cos(\phi_{S_A} - \hat{\phi}_{S_h}) F_{TU,T}^{\cos(\phi_{S_A}-\hat{\phi}_{S_h})} + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_{S_A}) F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_{S_A})} \\
&\qquad\qquad + \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) \sin(\phi_q - \phi_{S_A}) F_{TU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})\sin(\phi_q-\phi_{S_A})} \\
&\qquad\qquad + \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) \cos(\phi_q - \phi_{S_A})) F_{TU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})\cos(\phi_q-\phi_{S_A})} \\
&\qquad\qquad + \cos(2\phi_q - \phi_{S_A} - \hat{\phi}_{S_h}) F_{TU,T}^{\cos(2\phi_q-\phi_{S_A}-\hat{\phi}_{S_h})} \\
&\qquad\qquad + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} + 2\phi_q - \phi_{S_A}) F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}+2\phi_q-\phi_{S_A})} \\
&\qquad\qquad + \lambda_e \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) \sin(\phi_{S_A} - \phi_q) F_{TL,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})\sin(\phi_{S_A}-\phi_q)} \\
&\qquad\qquad + \lambda_e \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) \cos(\phi_{S_A} - \phi_q)) F_{TL,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})\cos(\phi_{S_A}-\phi_q)} \Big] \Big\},
\end{aligned}$$

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We investigate all possible azimuthal asymmetries.

JFF \ PDF	$\tilde{f}_1$	$\tilde{f}_{1T}^{\perp(1)}$	$\tilde{g}_{1L}$	$\tilde{g}_{1T}^{(1)}$
$\mathcal{D}_1$	1	$\sin(\phi_q - \phi_{S_A})$	1	$\cos(\phi_q - \phi_{S_A})$
$\mathcal{D}_{1T}^{\perp}$	$\sin(\hat{\phi}_h - \hat{\phi}_{S_h})$	$\sin(\hat{\phi}_h - \hat{\phi}_{S_h})\sin(\phi_q - \phi_{S_A})$	$\sin(\hat{\phi}_h - \hat{\phi}_{S_h})$	$\sin(\hat{\phi}_h - \hat{\phi}_{S_h})\cos(\phi_q - \phi_{S_A})$
$\mathcal{G}_{1L}$	1	$\sin(\phi_q - \phi_{S_A})$	1	$\cos(\phi_q - \phi_{S_A})$
$\mathcal{G}_{1T}$	$\cos(\hat{\phi}_h - \hat{\phi}_{S_h})$	$\cos(\hat{\phi}_h - \hat{\phi}_{S_h})\sin(\phi_{S_A} - \phi_q)$	$\cos(\hat{\phi}_h - \hat{\phi}_{S_h})$	$\cos(\hat{\phi}_h - \hat{\phi}_{S_h})\cos(\phi_q - \phi_{S_A})$
JFF \ PDF	$\tilde{h}_1^{\perp(1)}$	$\tilde{h}_{1L}^{\perp(1)}$	$\tilde{h}_1$	$\tilde{h}_{1T}^{\perp(2)}$
$\mathcal{H}_1^{\perp}$	$\cos(\hat{\phi}_h - \phi_q)$	$\sin(\phi_q - \hat{\phi}_h)$	$\sin(\phi_{S_A} - \hat{\phi}_h)$	$\sin(2\phi_q - \hat{\phi}_h - \phi_{S_A})$
$\mathcal{H}_{1L}^{\perp}$	$\sin(\hat{\phi}_h - \phi_q)$	$\cos(\phi_q - \hat{\phi}_h)$	$\cos(\phi_{S_A} - \hat{\phi}_h)$	$\cos(2\phi_q - \hat{\phi}_h - \phi_{S_A})$
$\mathcal{H}_1$	$\sin(\hat{\phi}_{S_h} - \phi_q)$	$\cos(\phi_q - \hat{\phi}_{S_h})$	$\cos(\phi_{S_A} - \hat{\phi}_{S_h})$	$\cos(2\phi_q - \hat{\phi}_{S_h} - \phi_{S_A})$
$\mathcal{H}_{1T}^{\perp}$	$\sin(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_q)$	$\cos(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_q)$	$\cos(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_{S_A})$	$\cos(2\hat{\phi}_h - \hat{\phi}_{S_h} + \phi_{S_A} - 2\phi_q)$

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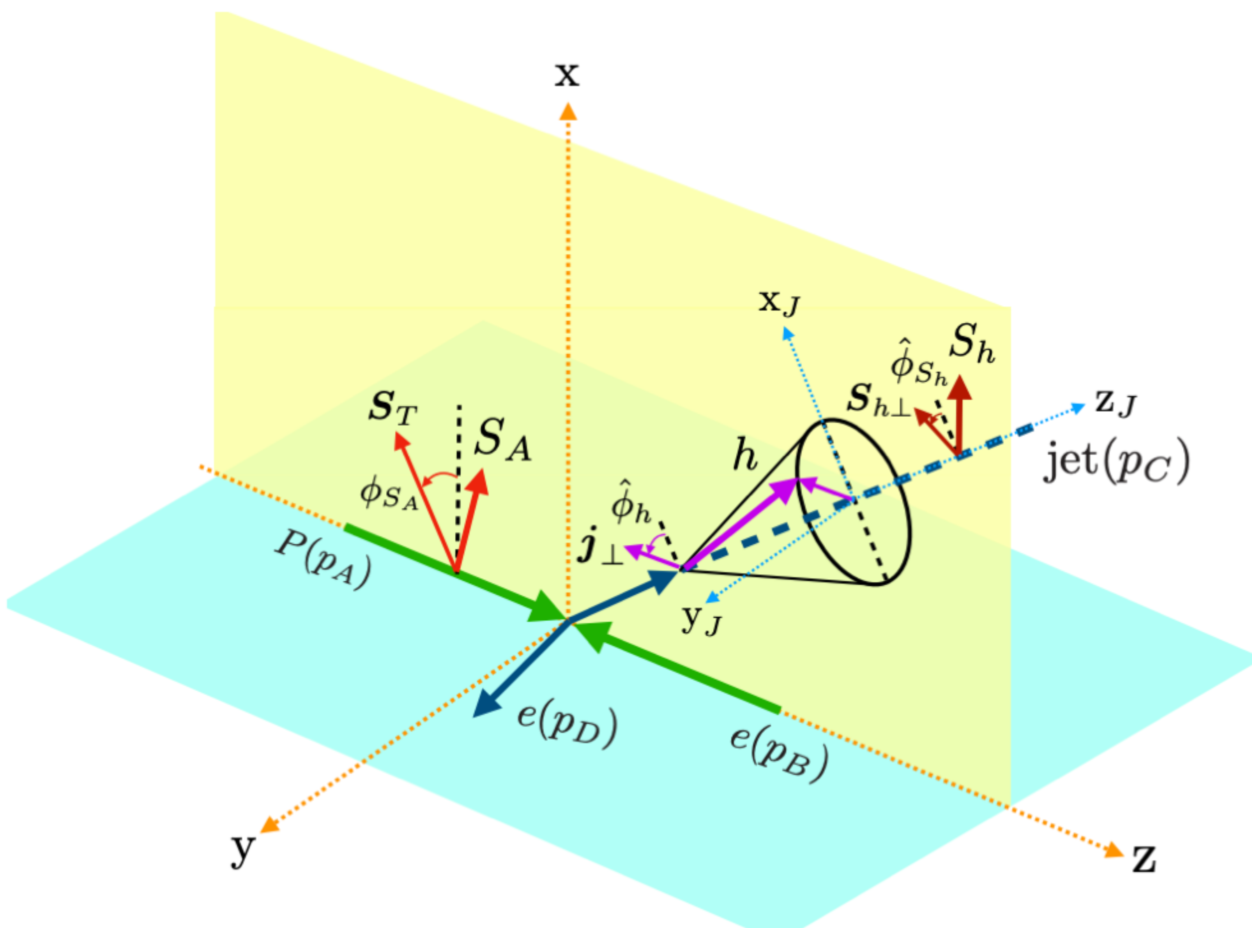
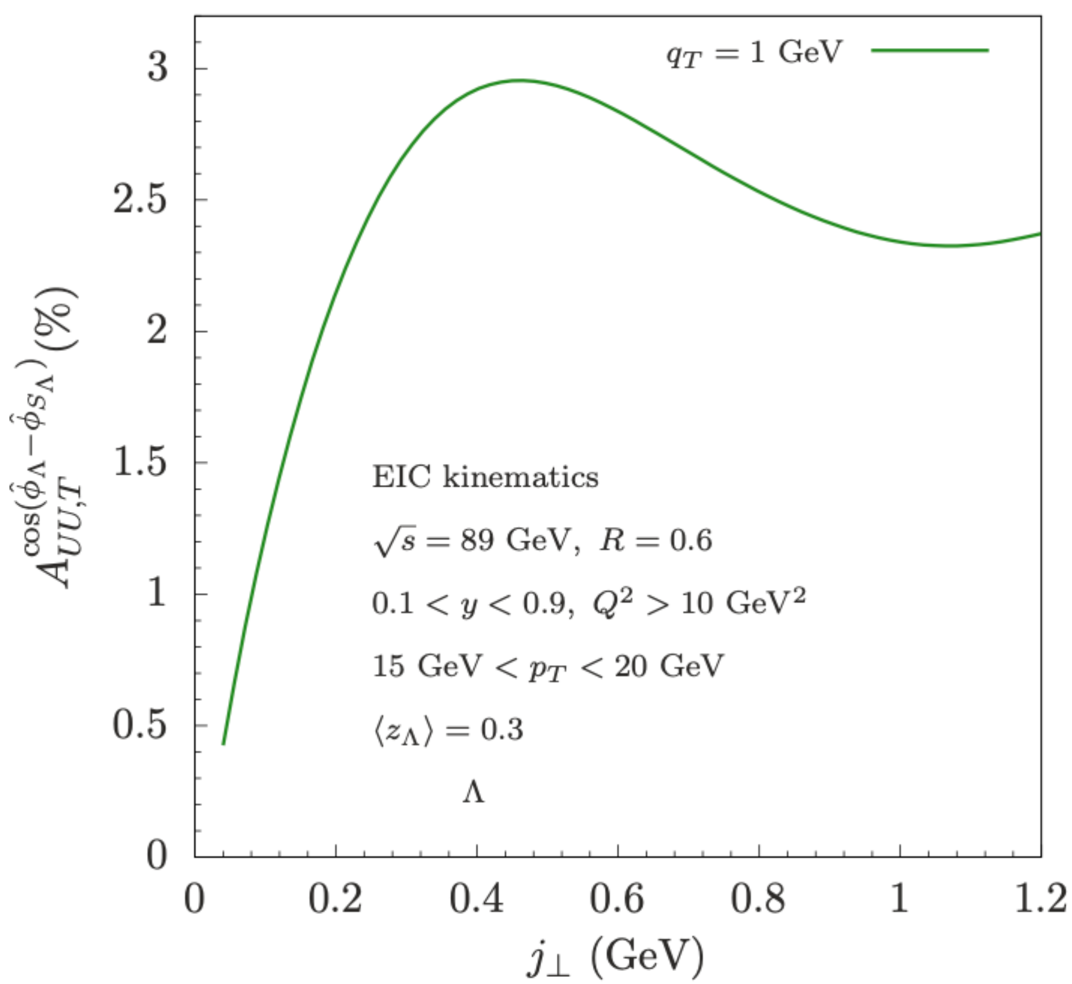
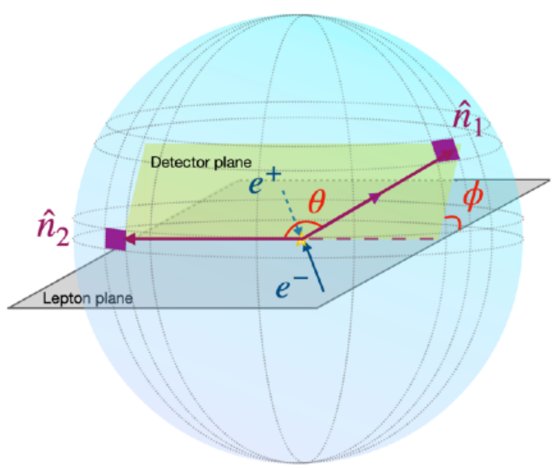




Polarized event shape

Spin asymmetry of EEC in the large angle limit

- **Many spin asymmetries arise from the azimuthal correlations**
- **Azimuthal angle dependence in the small angle limit** Chen, Moult, & Zhu '20; Li, Liu, Yuan, Zhu '23
- **Fracture Functions** Chen, Ma, Tong '24
- **NEEC, long-range azimuthal correlation and entanglement** Guo, Liu, Yuan, Zhu '24
- **We extend the EEC in the back-to-back by considering azimuthal asymmetries associated with the EEC** Kang, Lee, DYS, Zhao '23



$$\frac{1}{\sigma_{\rm tot}}\frac{d\Sigma_{\rm e^+e^-}}{d\tau d\phi} = \frac{\langle \mathcal{O}\mathcal{E}\left(\vec{n}_1\right)\mathcal{E}\left(\vec{n}_2\right)\mathcal{O}^\dagger\rangle}{\langle \mathcal{O}\mathcal{O}^\dagger\rangle}$$

$$\boxed{\mathrm{EEC}_{e^+e^-}(\tau,\phi) \equiv \frac{d\Sigma_{e^+e^-}}{d\tau d\phi} = \frac{1}{2}\sum_{1,2}\int d\sigma\, z_1 z_2\, \delta\left(\tau - \frac{1+\cos\theta_{12}}{2}\right)\delta(\phi-\phi_{12})}$$

Azimuthal dependent EEC in e+e-

- **The standard TMD factorization for the back-to-back di-hadron process**

$$\frac{d\sigma}{dz_i dz_j d^2\boldsymbol{q}_T} = \sigma_0\, H(Q,\mu) \sum_q e_q^2 \int d^2\boldsymbol{p}_{1\perp} d^2\boldsymbol{p}_{2\perp} d^2\boldsymbol{\lambda}_\perp \delta^2\left(\frac{\boldsymbol{p}_{1\perp}}{z_1} + \frac{\boldsymbol{p}_{2\perp}}{z_2} - \boldsymbol{\lambda}_\perp + \boldsymbol{q}_T\right) S(\boldsymbol{\lambda}_\perp^2, \mu, \nu)$$

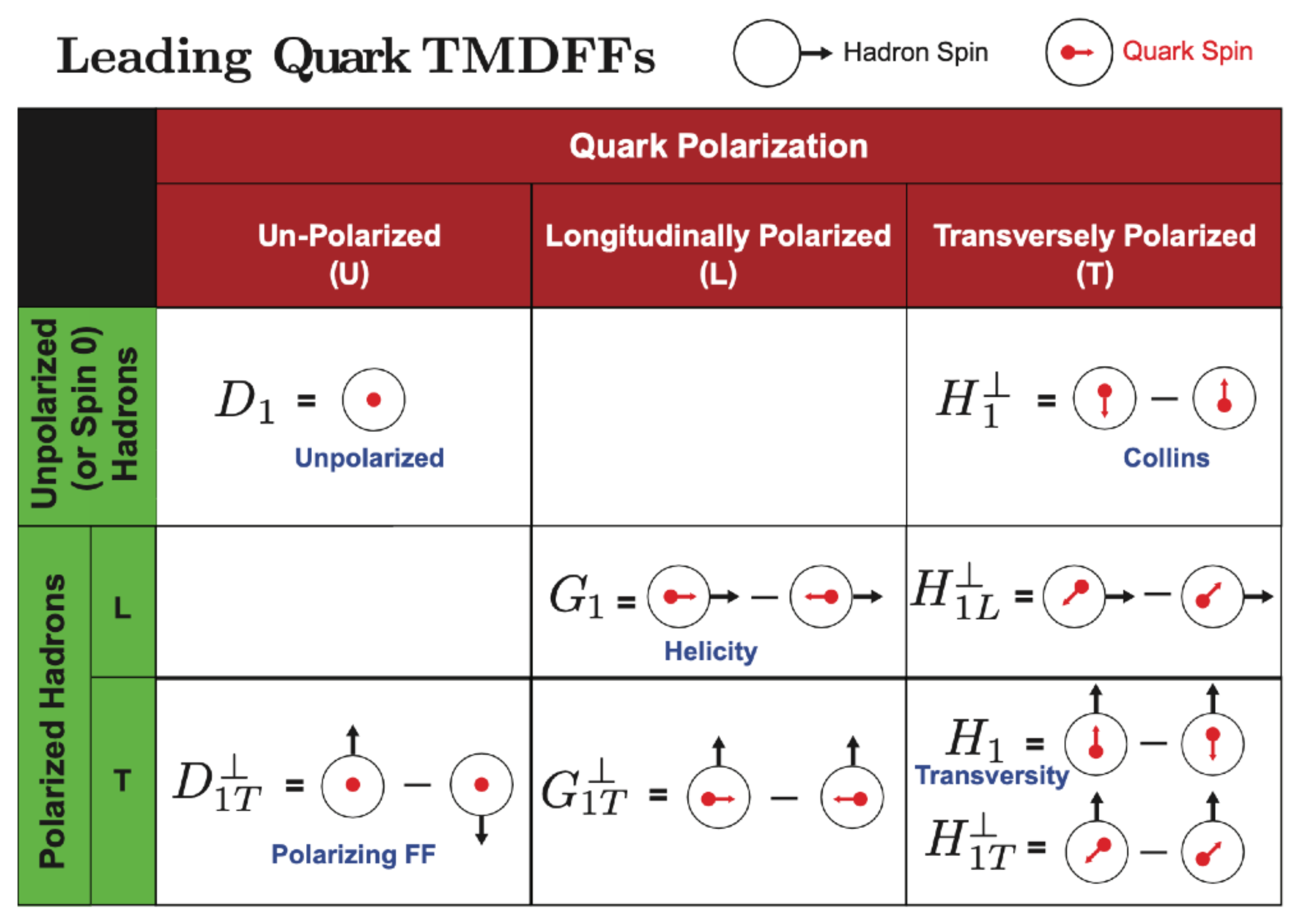
$$\times \left[D_{1,h_1/q}^{(u)}(z_1, \boldsymbol{p}_{1\perp}^2, \mu, \zeta/\nu^2) D_{1,h_2/\bar{q}}^{(u)}(z_2, \boldsymbol{p}_{2\perp}^2, \mu, \zeta/\nu^2) + \cos(2\phi_{12}) \left(\hat{\boldsymbol{q}}_{T,\alpha}\hat{\boldsymbol{q}}_{T,\beta} - \frac{1}{2} g_{\perp,\alpha\beta}\right)\right.$$

$$\left. \times \frac{\boldsymbol{p}_{1\perp}^\alpha}{z_1 M_1} H_{1,h_1/q}^{\perp(u)}(z_1, \boldsymbol{p}_{1\perp}^2, \mu, \zeta/\nu^2) \frac{\boldsymbol{p}_{2\perp}^\beta}{z_2 M_2} H_{1,h_2/\bar{q}}^{\perp(u)}(z_2, \boldsymbol{p}_{2\perp}^2, \mu, \zeta/\nu^2) \right].$$

$$z_i = \frac{2P_{hi}\cdot q}{Q^2} = \frac{2E_i}{Q}$$ **Energy fraction**

$$D_{1,h/q}^{(u)}(z, \boldsymbol{p}_\perp^2, \mu, \zeta/\nu^2)$$ **Unpolarized TMD FF**

$$H_{1,h/q}^{\perp(u)}(z, \boldsymbol{p}_\perp^2, \mu, \zeta/\nu^2)$$ **Collins TMD FF**



Azimuthal dependent EEC in e+e-

- **The TMD factorization for the azimuthal-dependent EEC in the back-to-back limit**

$$\mathrm{EEC}_{e^+e^-}(\tau,\phi) = \frac{1}{2}\sigma_0\, H(Q,\mu) \sum_q e_q^2 \int \frac{b\,db}{2\pi}\Big[J_0(b\sqrt{\tau}Q) J_q(b,\mu,\zeta) J_{\bar{q}}(b,\mu,\zeta) + \cos(2\phi)\,\frac{b^2}{8} J_2(b\sqrt{\tau}Q)\; J_q^{\perp}(b,\mu,\zeta) J_{\bar{q}}^{\perp}(b,\mu,\zeta)\Big]$$

New term: azimuthal asymmetry

"Collins-type" EEC jet functions

A similar structure for Winner-take-All jet function was given in W. Lai, X. Liu, M Wang, H. Xing '21 '22

- **The unpolarized EEC jet function has a close relation to the unpolarized TMD FFs**

$$J_q(b,\mu,\zeta) \equiv \sum_h \int_0^1 dz\, z\, \tilde{D}_{1,h/q}(z,b,\mu,\zeta)$$

- **Collins-type EEC jet functions are closely connected with the Collins FFs**

$$J_q^{\perp}(b,\mu,\zeta) \equiv \sum_h \int_0^1 dz\, z\, \tilde{H}_{1,h/q}^{\perp}(z,b,\mu,\zeta)$$

Collins-type EEC jet function

- **We introduce Collins-type EEC jet function**

$$J_q(\boldsymbol{b},\mu,\zeta) \equiv \sum_h \int_0^1 dz z \tilde{D}_{1,h/q}(z,b,\mu_{b_*},\zeta_i)\, e^{-S_{\rm pert}\,(\mu,\mu_{b_**})-S_{\rm NP}^{D_1}(b,Q_0,\zeta)} \left(\sqrt{\frac{\zeta}{\zeta_i}}\right)^{\kappa(b,\mu_{b_*})}$$

$$J_q^{\perp}(\boldsymbol{b},\mu,\zeta) \equiv \sum_h \int_0^1 dz z \tilde{H}^{\perp}_{1,h/q}(z,b,\mu_{b_*},\zeta_i)\, e^{-S_{\rm pert}(\mu,\mu_{b_*})-S_{\rm NP}^{H_1^\perp}(b,Q_0,\zeta)} \left(\sqrt{\frac{\zeta}{\zeta_i}}\right)^{\kappa(b,\mu_{b_*})}$$

Collins function in *b*-space

- **The OPE of the subtracted unpolarized and Collins TMD FFs gives**

$$\tilde{D}_{1,h/q}(z,b,\mu,\zeta) = \left[C_{j\leftarrow q}\otimes D_{1,h/j}\right](z,b,\mu,\zeta) + \mathcal{O}(b^2\Lambda^2_{\rm QCD}),$$

$$\tilde{H}^{\perp}_{1,h/q}(z,b,\mu,\zeta) = \left[\delta C^{\rm Collins}_{j\leftarrow q}\otimes \hat{H}^{\perp(3)}_{1,h/j} + A_{j\leftarrow q}\tilde{\otimes}\hat{H}_{F,h/j}\right](z,b,\mu,\zeta) + \mathcal{O}(b^2\Lambda^2_{\rm QCD}),$$

twist-3 FFs (H_F is ignored)

$$\delta C^{\rm Collins}_{q'\leftarrow q}(z,b,\mu,\zeta) = \delta_{qq'}\left\{\delta(1-z) + \frac{\alpha_s}{\pi}\left[C_F\delta(1-z)\left(-\frac{L_b^2}{4}+\frac{L_b}{2}\left(\frac{3}{2}+\ln\frac{\mu^2}{\zeta^2}\right)-\frac{\pi^2}{24}\right)\right.\right.$$
$$\left.\left. + \left(\ln z - \frac{L_b}{2}\right)\hat{P}^c_{q\leftarrow q}(z)\right]\right\} + \mathcal{O}(\alpha_s^2),$$

The OPE of the Collins TMD FFs

- **The OPE of the subtracted unpolarized and Collins TMD FFs gives**

$$\tilde{H}^{\perp}_{1,h/q}(z,b,\mu,\zeta) = \left[\delta C^{\text{Collins}}_{j\leftarrow q} \otimes \hat{H}^{\perp(3)}_{1,h/j} + A_{j\leftarrow q}\tilde{\otimes}\hat{H}_{F,h/j}\right](z,b,\mu,\zeta) + \mathcal{O}(b^2\Lambda^2_{\text{QCD}})$$

- **Standard convolution** $\left[C_{j\leftarrow q}\otimes F_{h/j}\right](z,b,\mu,\zeta) = \int_z^1 \frac{dx}{x} C_{j\leftarrow q}\left(\frac{z}{x},b,\mu,\zeta\right) F_{h/j}(x,\mu)$

$$\delta C^{\text{Collins}}_{q'\leftarrow q}(z,b,\mu,\zeta) = \delta_{qq'}\Big\{\delta(1-z) + \frac{\alpha_s}{\pi}\Big[C_F\delta(1-z)\left(-\frac{L_b^2}{4} + \frac{L_b}{2}\left(\frac{3}{2}+\ln\frac{\mu^2}{\zeta^2}\right) - \frac{\pi^2}{24}\right) + \left(\ln z - \frac{L_b}{2}\right)\hat{P}^c_{q\leftarrow q}(z)\Big]\Big\} + \mathcal{O}(\alpha_s^2)\,,$$

$\hat{H}^{\perp(3)}_{1,h/j}(z,\mu)$: twist-3 fragmentation function, related to the first $k_\perp$-moment of the Collins TMD FF

- **Double convolution**

$$\left[A_{j\leftarrow q}\tilde{\otimes}\hat{H}_{F,h/j}\right](z,b,\mu,\zeta) = \int_z^1 \frac{dx}{x}\int \frac{dz_1}{z_1^2}\,\text{PV}\left(\frac{1}{\frac{1}{x}-\frac{1}{z_1}}\right) A_{j\leftarrow q}\left(\frac{z}{x},z_1,b,\mu,\zeta\right)\hat{H}_{F,h/j}(x,z_1,\mu)$$

starts at the order $O(\alpha_S)$ and is ignored in our work

Sum rule

- **The collinear functions in the OPE matching obey the sum rules**

$$\sum_h \int_0^1 dz\, z\, D_{1,h/j}\,(z,\mu) = 1\,,$$

sum over longitudinal momentum fraction carried by the hadron is 1

$$\sum_h \int_0^1 dz\, \hat{H}^{\perp(3)}_{1,h/q}\,(z,\mu) = 0\,.$$

the transverse momentum carried by the final hadron sum to 0 (Schafer-Teryaev sum rule)

- **In the OPE region**

$$\begin{aligned}
J_q^{\perp}(b,\mu,\zeta) &= \sum_h \int_0^1 dz\, z\, \tilde{H}^{\perp}_{1,h/q}(z,b,\mu,\zeta) \\
&= \sum_h \int_0^1 dz \int_z^1 \frac{dx}{x}\, \delta C^{\text{Collins}}_{q\leftarrow q}(\frac{z}{x}, b, \mu_{b_*}, \zeta) \hat{H}^{\perp(3)}_{1,h/q}\,(x,\mu_{b_*})\, e^{-S_{\text{pert}}(\mu,\mu_{b_*})} \\
&= \int_0^1 d\tau \delta C^{\text{Collins}}_{q\leftarrow q}(\tau, b, \mu_{b_*}, \zeta) \underbrace{\left[\sum_h \int_0^1 dx\, \hat{H}^{\perp(3)}_{1,h/q}\,(x,\mu_{b_*})\right]}_{=\,0} e^{-S_{\text{pert}}(\mu,\mu_{b_*})} \\
&= 0\,,
\end{aligned}$$

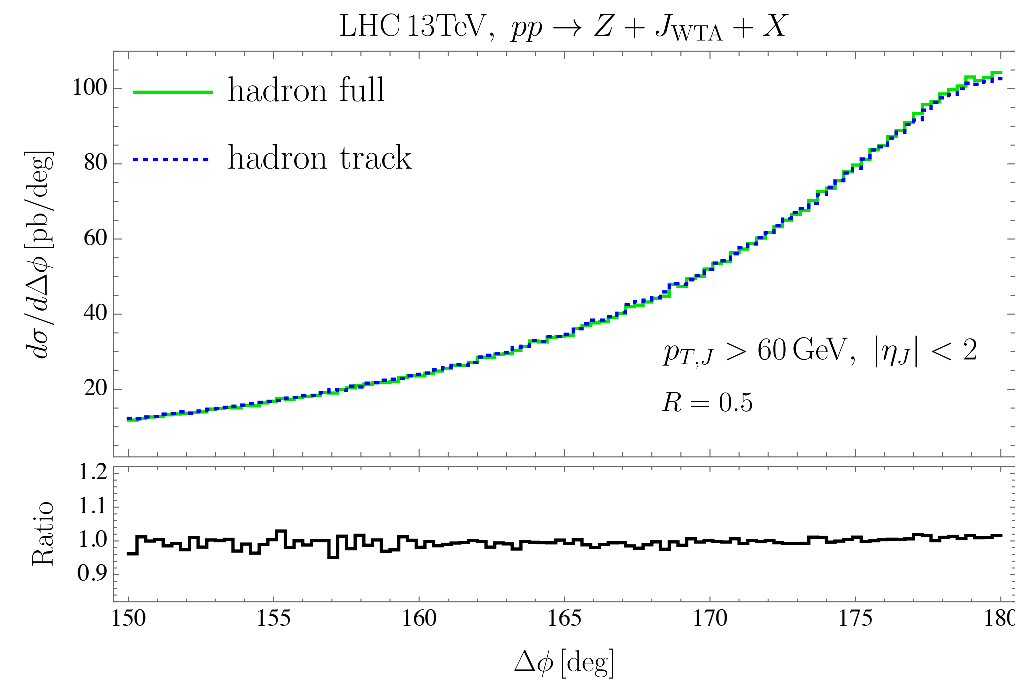
- **We find that the Collins-type EEC jet function becomes zero in the OPE region upon neglecting the off-diagonal matching terms.**

Collins-type EEC with subsets of hadrons

- **In the small angle limit, the track function formalism was used to study energy correlation between hadrons with specific quantum number** $\langle \mathcal{E}_{\mathbb{S}_1}(\hat{n}_1)\mathcal{E}_{\mathbb{S}_2}(\hat{n}_2)\rangle$ Chang, Procura, Thaler, & Waalewijn '13; Y, Li, Moult, Schrijnder, Waalewijn, H. X. Zhu '21; Jaarsma, Y. Li, Moult, Waalewijn, Z. X. Zhua '22, '23 + H. Chen '22 '23
- **In the large angle limit (TMD region), one can also use subset $\mathbb{S}$ of hadrons to define the jet function**
  - **E.g. Tacking jet function for the recoil free jets**

$$\bar{\mathscr{J}}_q^{(1)} = \mathscr{J}_q^{(1)} + 4C_F \int_0^1 \mathrm{d}x\, \frac{1+x^2}{1-x} \ln\frac{x}{1-x} \int_0^1 \mathrm{d}z_1\, T_q(z_1,\mu) \times \int_0^1 \mathrm{d}z_2\, T_g(z_2,\mu)\left[\theta(z_1 x - z_2(1-x)) - \theta(x - \tfrac{1}{2})\right]$$

Chien, Rahn, DYS, Waalewijn & Wu '22 JHEP + Schrignder '21 PLB



- **We explore a less inclusive version of EEC in the back-to-back limit that is only sensitive to the energy flow of subset $\mathbb{S}$ of hadrons**

$$\sum_h \Rightarrow \sum_{h\in\mathbb{S}}$$

E.g. $\mathbb{S}$ = charged particles — Better energy resolution and smaller experimental uncertainties

$\mathbb{S} = h$ — Probe fragmentation function

Collins-type EEC with subsets of hadrons

- **We define the modified jet functions**

$$J_{q/\mathbb{S}}(b,\mu,\zeta) \equiv \sum_{h\in\mathbb{S}} \int_0^1 dz\, z\, \tilde{D}_{1,h/q}(z,b,\mu,\zeta)\,,$$
$$J^{\perp}_{q/\mathbb{S}}(b,\mu,\zeta) \equiv \sum_{h\in\mathbb{S}} \int_0^1 dz\, z\, \tilde{H}^{\perp}_{1,h/q}(z,b,\mu,\zeta)\,.$$

- **In the OPE region**

$$F_{j\to\mathbb{S}} = \sum_{h\in\mathbb{S}} \int_0^1 dz\, z\, D_{1,h/j}\,(z,\mu_{b_*})$$

average fraction of longitudinal momentum carried by $\mathbb{S}$

$$J_{q/\mathbb{S}}(b,\mu,\zeta) = \sum_j F_{j\to\mathbb{S}} \int_0^1 d\tau\, \tau\, C_{j\leftarrow q}(\tau,b,\mu_{b_*},\zeta) e^{-S_{\rm pert}(\mu,\mu_{b_*})}\,,$$
$$J^{\perp}_{q/\mathbb{S}}(b,\mu,\zeta) = \sum_j F^{\perp}_{j\to\mathbb{S}} \int_0^1 d\tau\, \delta C^{\rm Collins}_{j\leftarrow q}(\tau,b,\mu_{b_*},\zeta) e^{-S_{\rm pert}(\mu,\mu_{b_*})}\,.$$

$$F^{\perp}_{j\to\mathbb{S}} = \sum_{h\in\mathbb{S}} \int_0^1 dz\, \hat{H}^{\perp(3)}_{1,h/j}\,(z,\mu_{b_*})$$

average transverse momentum carried by $\mathbb{S}$

- **All the non-perturbative information are captured by the moments of FFs in the OPE region**

Collins-type EEC with subsets of hadrons

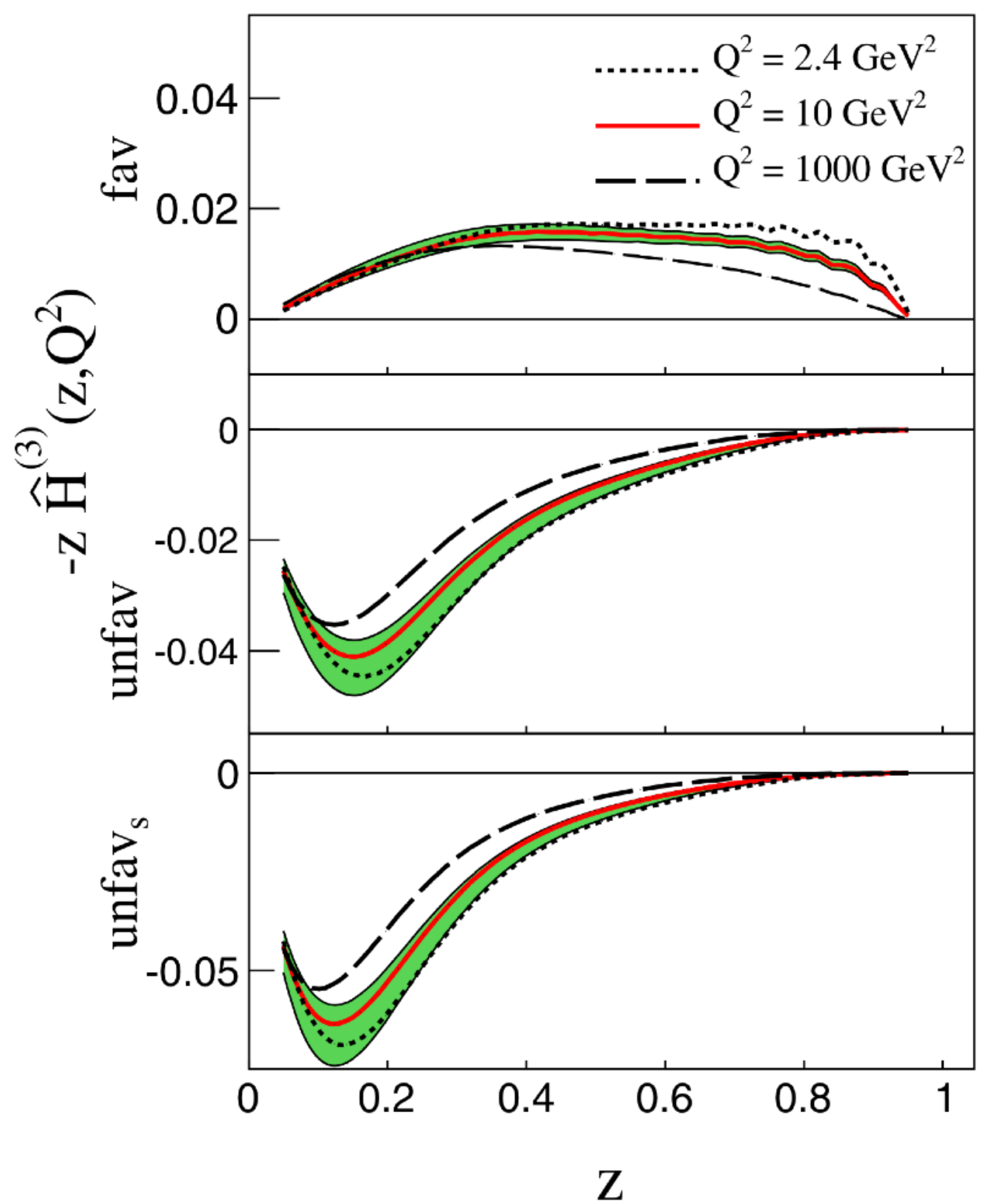
- **From the perspective of Collins-type EEC jet function, we are motivated to consider the so-called favored and unfavored subset**

- **Twist-3 Collins fragmentation functions**

$$\hat{H}^{(3)}_{\pi^+/\bar{u}}(z,Q_0) = \hat{H}^{(3)}_{\pi^-/u}(z,Q_0) = \hat{H}^{(3)}_{\pi^-/\bar{d}}(z,Q_0) = \hat{H}_{\rm unf}\,(z,Q_0)\,,$$

$$\hat{H}^{(3)}_{\pi^+/\bar{d}}(z,Q_0) = \hat{H}^{(3)}_{\pi^-/d}(z,Q_0) = \hat{H}^{(3)}_{\pi^-/\bar{u}}(z,Q_0) = \hat{H}_{\rm fav}\,(z,Q_0)\,.$$

$$\hat{H}^{(3)}_{fav}(z,Q_0) = N_u^c z^{\alpha_u}(\tau)^{\beta_u} D_{1,\pi^+/u}(z,Q_0)\,,$$
$$\hat{H}^{(3)}_{unfav}(z,Q_0) = N_d^c z^{\alpha_d}(\tau)^{\beta_d} D_{1,\pi^+/d}(z,Q_0)\,,$$
$$\hat{H}^{(3)}_{s/\bar{s}}(z,Q_0) = N_d^c z^{\alpha_d}(\tau)^{\beta_d} D_{1,\pi^+/s,\bar{s}}(z,Q_0)\,,$$



Kang, Prokudin, Sun, Yuan `15

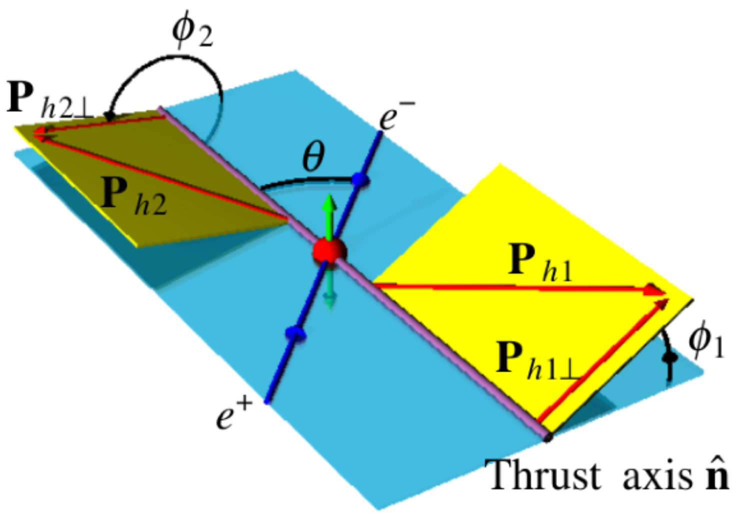
- **The vanishing value of Collins-type EEC jet function in the OPE region can be understood as**

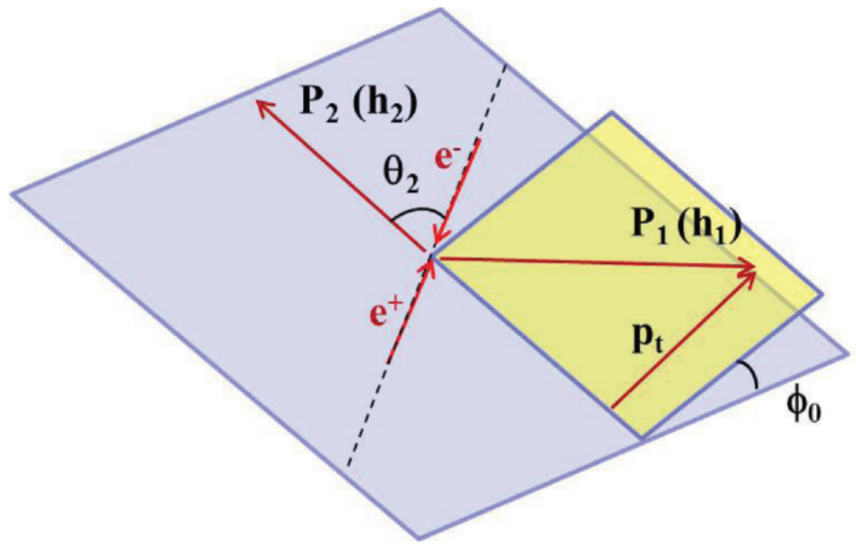
$$F^{\perp}_{j\to{\rm fav}} \approx -F^{\perp}_{j\to{\rm unfav}}$$

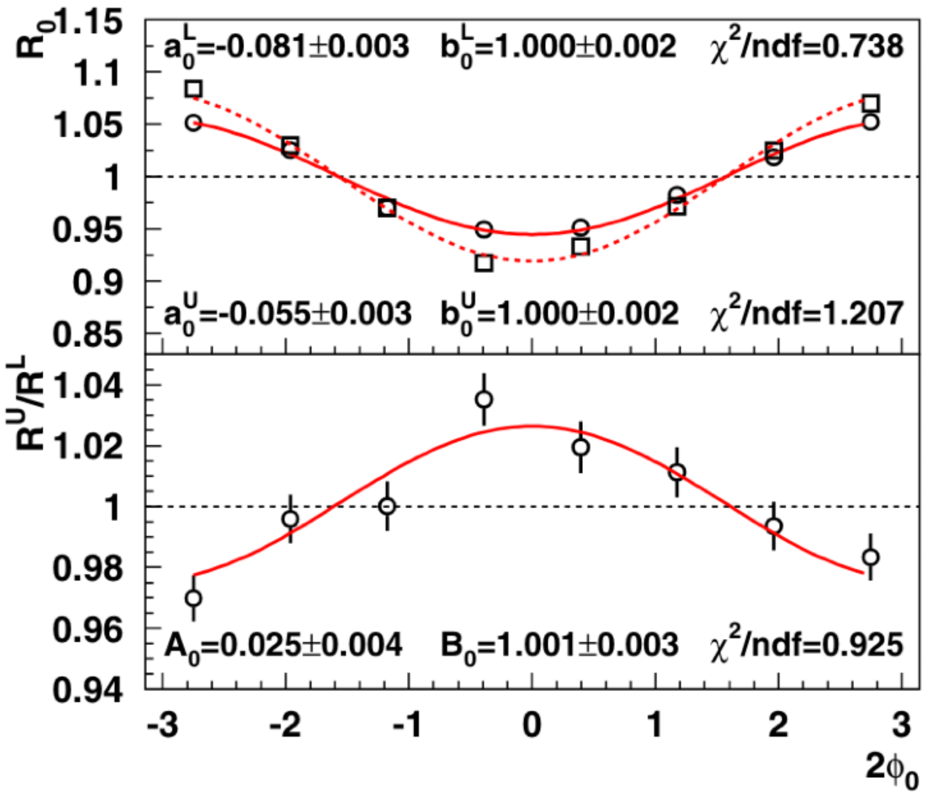
- **In phenomenology we consider**

$$\mathbb{S} = \{\pi^+\},\{\pi^-\},\{\pi^0\},\{\pi^+,\pi^-\},\{\pi^+,\pi^-,\pi^0\}$$

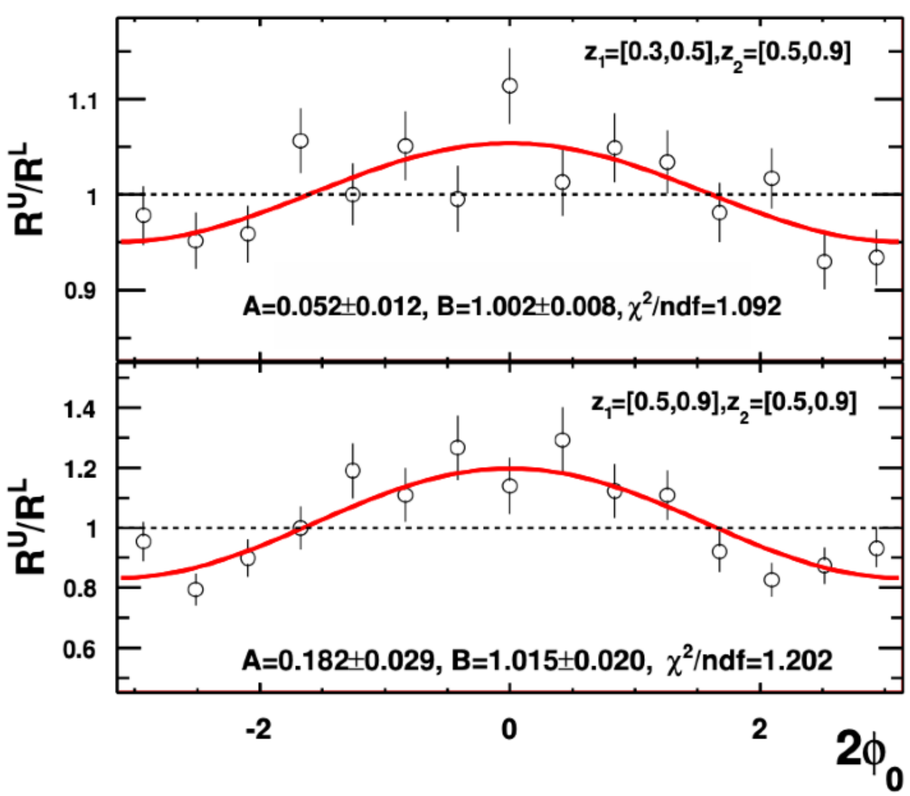
Collins asymmetry in e+e-







Belle 2006

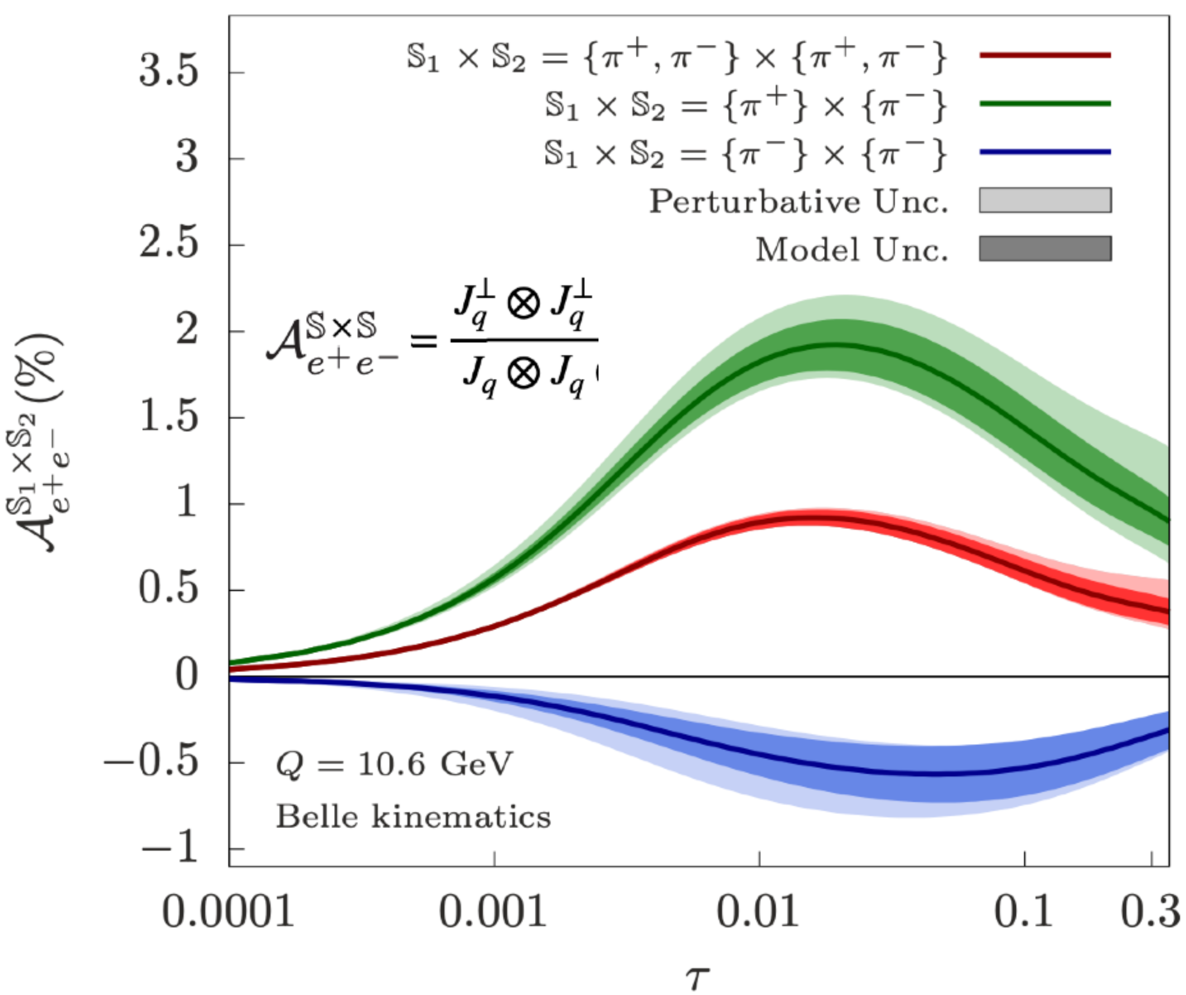


BESIII 2016

EEC in e+e- : Collins asymmetry

- **We provide a prediction for Collins asymmetry at Belle kinematics**

$$\mathrm{EEC}_{e^+e^-}(\tau,\phi) = \frac{d\Sigma_{e^+e^-}}{d\tau d\phi} = \frac{1}{2}\sigma_0 \sum_q e_q^2 \int d\boldsymbol{q}_T^2\, \delta\left(\tau - \frac{\boldsymbol{q}_T^2}{Q^2}\right) Z_{uu} \left[1 + \cos(2\phi)\, \frac{Z_{\mathrm{Collins}}}{Z_{uu}}\right]$$
$$\equiv \frac{1}{2}\sigma_0 \sum_q e_q^2\, Z_{uu} \left[1 + \cos(2\phi)\, A_{e^+e^-}(\tau Q^2)\right],$$



$$Z_{uu} = \int \frac{bdb}{2\pi} J_0(bq_T) J_q(b,\mu,\zeta) J_{\bar{q}}(b,\mu,\zeta)\,,$$
$$Z_{\mathrm{Collins}} = \int \frac{bdb}{2\pi}\frac{b^2}{8} J_2(bq_T) J_q^\perp(b,\mu,\zeta) J_{\bar{q}}^\perp(b,\mu,\zeta)\,.$$

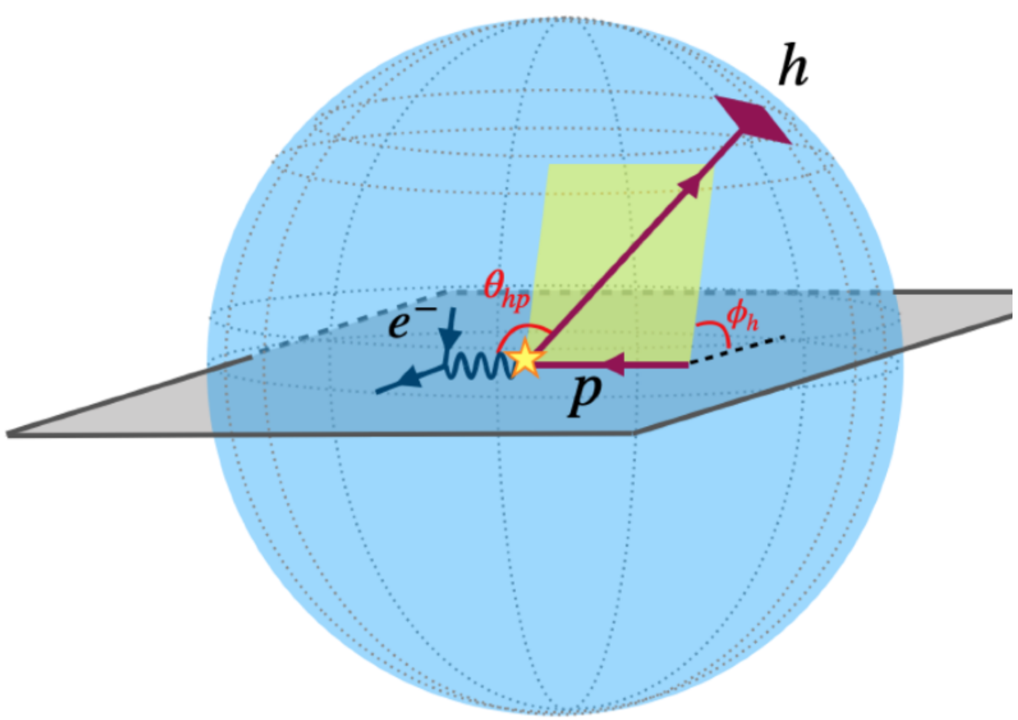
- When choosing a subset of either positively or negatively charged pions detected in EEC, one observes sizable asymmetries, which worth further measurement.
- BESIII collaboration is performing this analysis

EEC in DIS

The definition of EEC in DIS Li, Marks, Vitev `21

$$\mathrm{EEC_{DIS}}\,(\tau) \equiv \frac{1}{2}\sum_a \int d\theta_a dz_a z_a \frac{1}{\sigma}\frac{d\sigma}{d\theta_{ap} d\phi_{ap} dz_a}\delta\left(\tau - \frac{1+\cos\theta_{ap}}{2}\right)$$

We generalize the above definition



$$\begin{aligned}\frac{d\Sigma_{\mathrm{DIS}}}{dx dy d\tau d\phi} &= \sigma_0 \sum_q e_q^2 \int d^2\boldsymbol{q}_T \delta(\tau - \frac{\boldsymbol{q}_T^2}{Q^2})\delta(\phi-\phi_h)\int\frac{db\, b}{2\pi}H(Q,\mu)\\
&\times\Big\{J_0(bq_T)\tilde{f}_{1,q}J_q + \cos(2\phi_h)\frac{2(1-y)}{1+(1-y)^2}J_2(bq_T)\frac{b^2}{2}\tilde{h}_{1,q}^{\perp(1)}J_q^{\perp}\\
&+S_{\parallel}\sin(2\phi_h)\frac{2(1-y)}{1+(1-y)^2}J_2(bq_T)\frac{b^2}{2}\tilde{h}_{1L,q}^{\perp(1)}J_q^{\perp}\\
&+|\boldsymbol{S}_\perp|\Big[-\sin(\phi_h-\phi_s)J_1(bq_T)b\tilde{f}_{1T,q}^{\perp(1)}J_q\\
&+\sin(\phi_h+\phi_s)\frac{2(1-y)}{1+(1-y)^2}\frac{b}{2}J_1(bq_T)\tilde{h}_{1,q}J_q^{\perp}\\
&+\sin(3\phi_h-\phi_s)\frac{2(1-y)}{1+(1-y)^2}\frac{M^2b^3}{8}J_3(bq_T)\tilde{h}_{1T,q}^{\perp(2)}J_q^{\perp}\Big]\\
&+\lambda_e\Big[S_{\parallel}\frac{y(2-y)}{1+(1-y)^2}J_0(bq_T)\tilde{g}_{1L,q}J_q + |\boldsymbol{S}_\perp|\cos(\phi_h-\phi_s)J_1(bq_T)b\tilde{g}_{1T,q}^{\perp(1)}J_q\Big]\Big\}\end{aligned}$$

New probe for all TMDPDFs

Collins and Sivers asymmetry of EEC in DIS

Kang, Lee, DYS, Zhao '23 JHEP

$$\mathcal{A}_{\rm DIS}^{\mathbb{S}} = \frac{2(1-y)}{1+(1-y)^2} \frac{\mathcal{F}_{UT}^{\sin(\phi_{q_T}+\phi_s)}}{\mathcal{F}_{UU}}$$

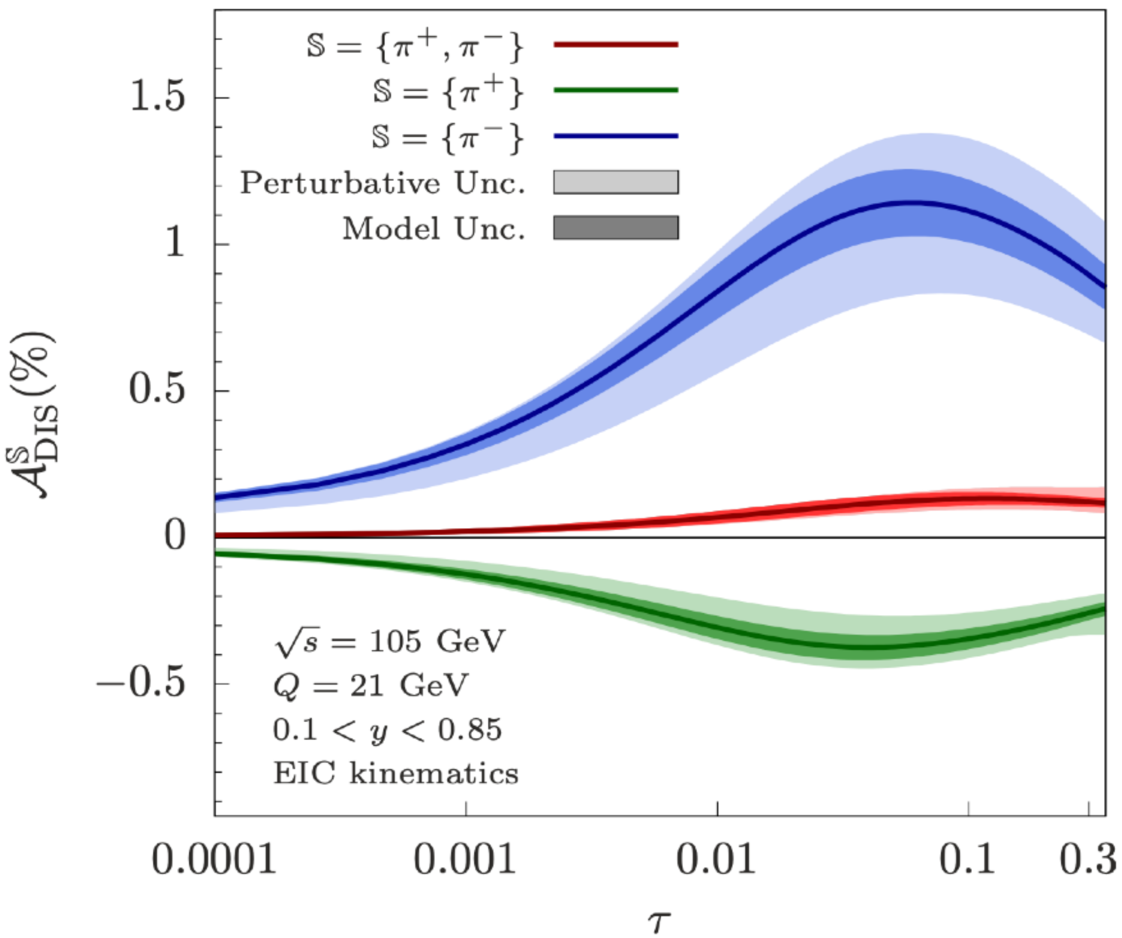
$$\mathcal{F}_{UT}^{\sin(\phi_{q_T}+\phi_s)} \sim h_1 \otimes J_q^{\perp}$$

$$\mathcal{F}_{UU} \sim f_1 \otimes J_q$$

$$\mathcal{A}_{\rm DIS}^{Sivers} = \frac{\mathcal{F}_{UT}^{\sin(\phi_{q_T}-\phi_s)}}{\mathcal{F}_{UU}}$$

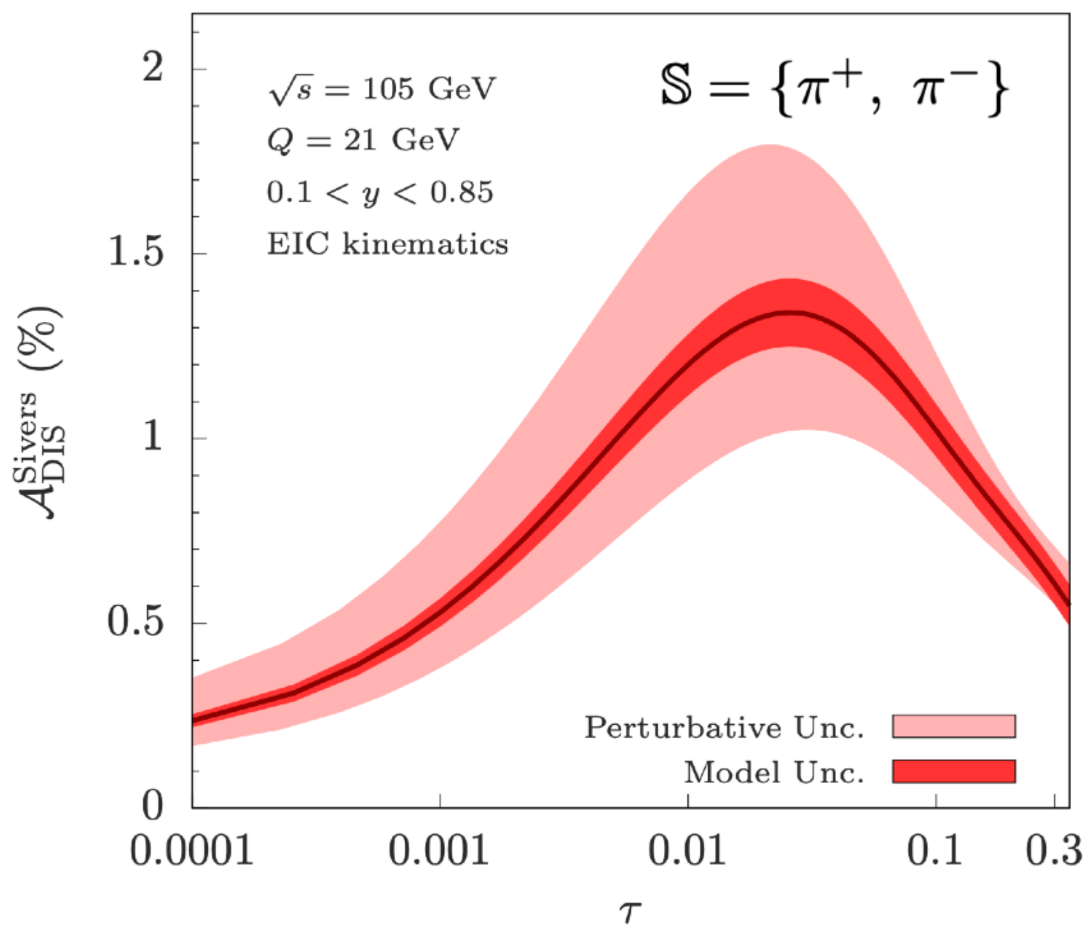
$$\mathcal{F}_{UT}^{\sin(\phi_{q_T}-\phi_s)} \sim f_{1T}^{\perp} \otimes J_q$$

$$\mathcal{F}_{UU} \sim f_1 \otimes J_q$$



Transversity and Collins function

Kang, Prokudin, Sun, Yuan `15



Sivers function:

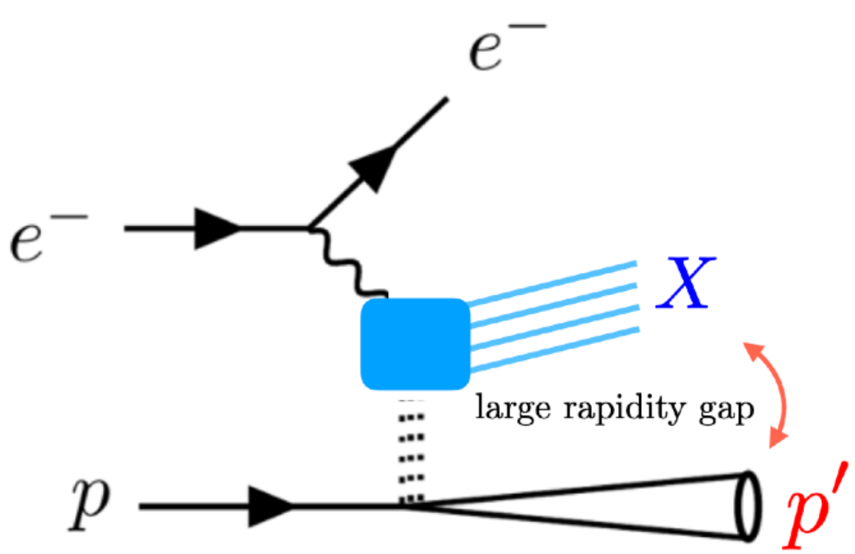
Echevarria, Kang, Terry `20

Diffraction

Diffraction in ep collisions

- **Hadronic diffraction processes and total cross sections have been described using the concept of ‘pomeron exchange’.**

- **The simplest way to introduce the concept of pomeron is within the framework of Regge theory**



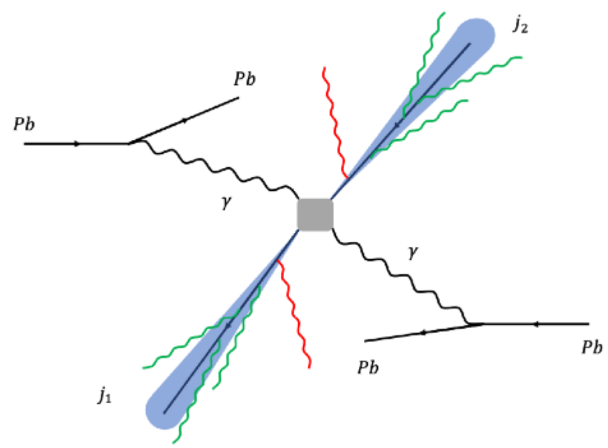
central region	inclusive diffraction $\sum_X X$	exclusive diffraction $X = J/\psi, \mathrm{Jet}_1 + \mathrm{Jet}_2, \cdots$
forward region	coherent diffraction $p' = \mathrm{proton}$	incoherent diffraction $p' = \mathrm{Jet}$

- **Some recent theory progress:**
  - **Fracture Functions:** Chen, Ma, Tong ‘21 ’24 + Chai ‘19
  - **Glauber SCET:** Lee, Schindler, Stewart ’25
  - …..

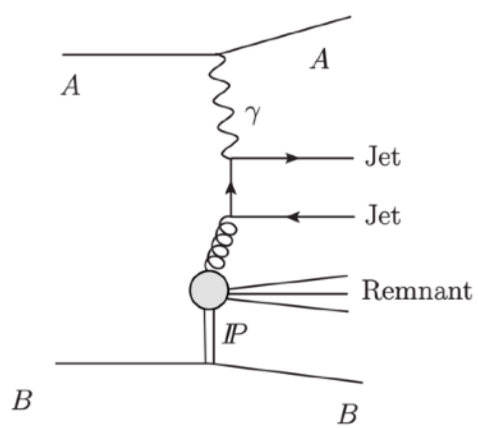
Azimuthal decorrelation of QCD jets in ultra-peripheral collisions

(Zhang, Dai, DYS, '23 JHEP)

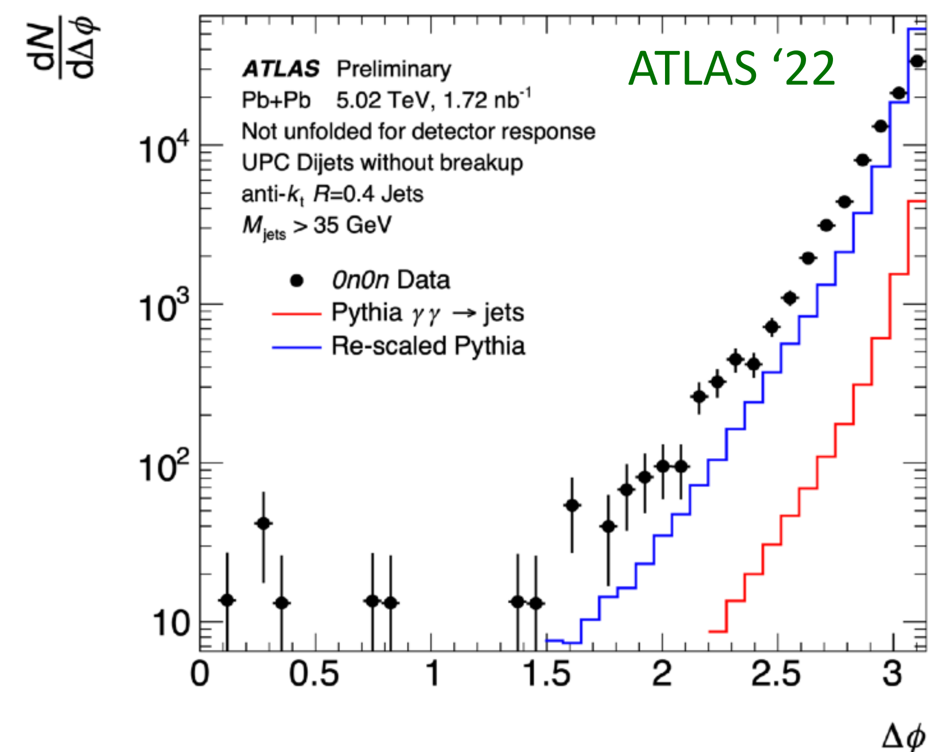
Dijet production with no nuclear breakup



Photon-photon fusion



diffractive photo-production

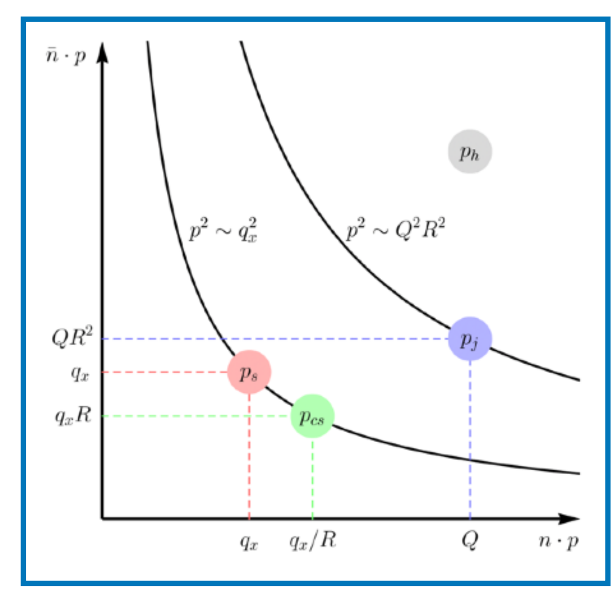


We apply equivalent photon approximation (Fermi 1924; Weizsacker 1934; Williams 1935) **+ SCET**

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}q_x \mathrm{d}p_T \mathrm{d}y_1 \mathrm{d}y_2} = \int_{-\infty}^{+\infty} \frac{\mathrm{d}b_x}{2\pi} e^{iq_x b_x} \tilde{B}(b_x, p_T, y_1, y_2) H(p_T, \Delta y, \mu) \tilde{S}(b_x, y_1, y_2, \mu, \nu)\, \tilde{U}_1(b_x, R, y_1, \mu, \nu) J_1(p_T, R, \mu) \tilde{U}_2(b_x, R, y_2, \mu, \nu) J_2(p_T, R, \mu)$$

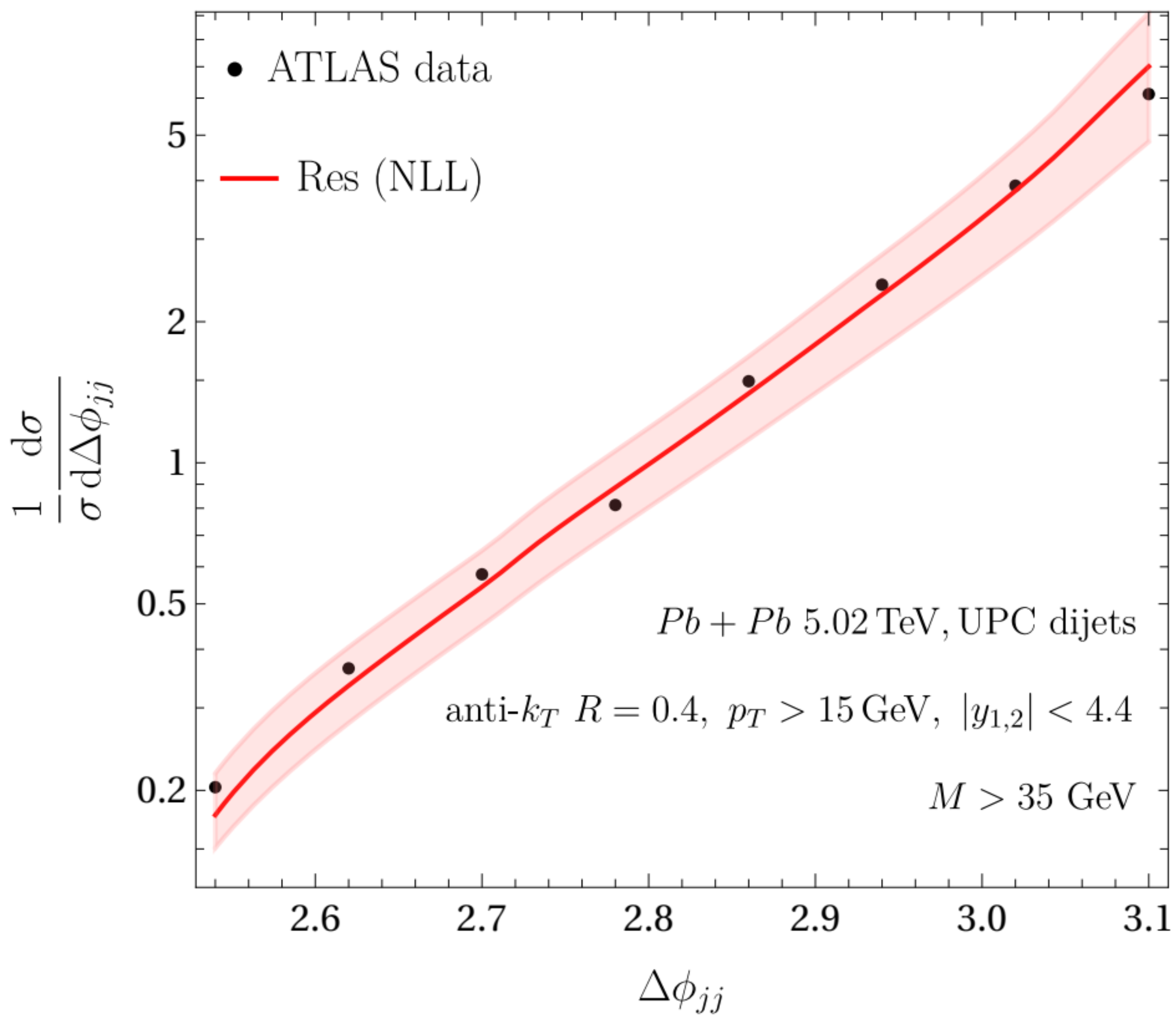
Photon Wigner distribution: (Klein, Mueller, Xiao, Yuan, '20, also see Ji, '03; Belitsky, Ji, Yuan, '04, …)

$$x f_\gamma(x, k_T; b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp \cdot b_\perp} \int \frac{d\xi^- d^2 r_\perp}{(2\pi)^3} e^{ixP^+\xi^- - ik_T\cdot r_\perp} \times \left\langle A, -\frac{\Delta_\perp}{2} \middle| F^{+\perp}\left(0, \frac{r_\perp}{2}\right) F^{+\perp}\left(\xi^-, -\frac{r_\perp}{2}\right) \middle| A, \frac{\Delta_\perp}{2} \right\rangle$$



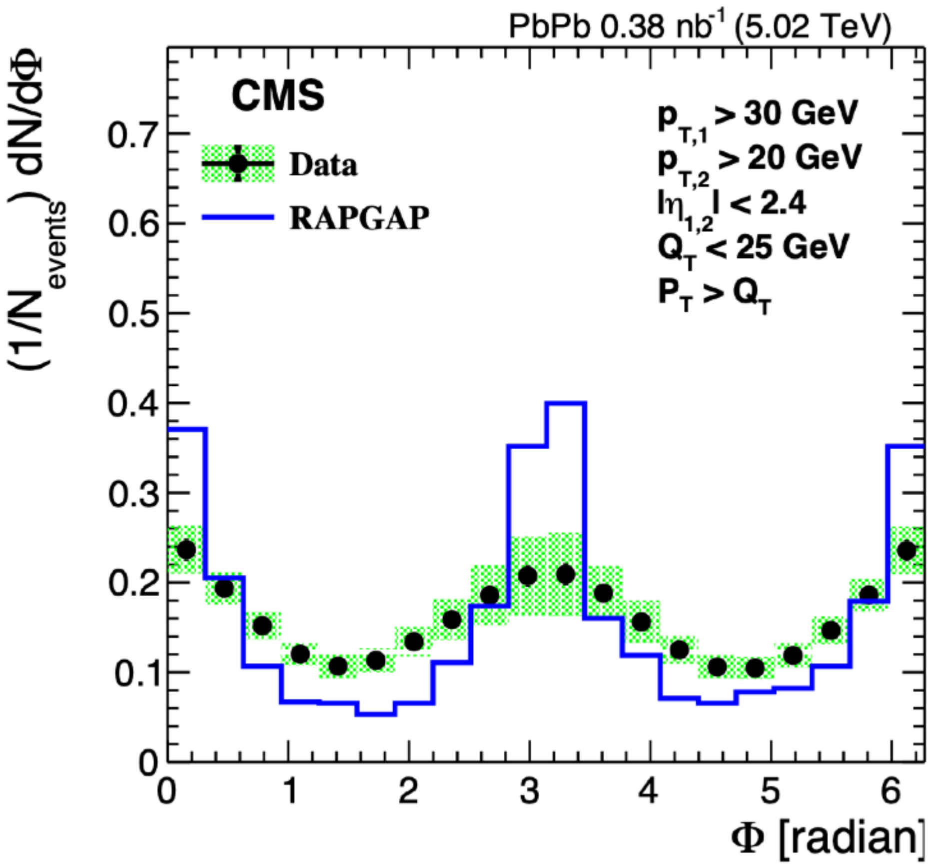
Numerical results

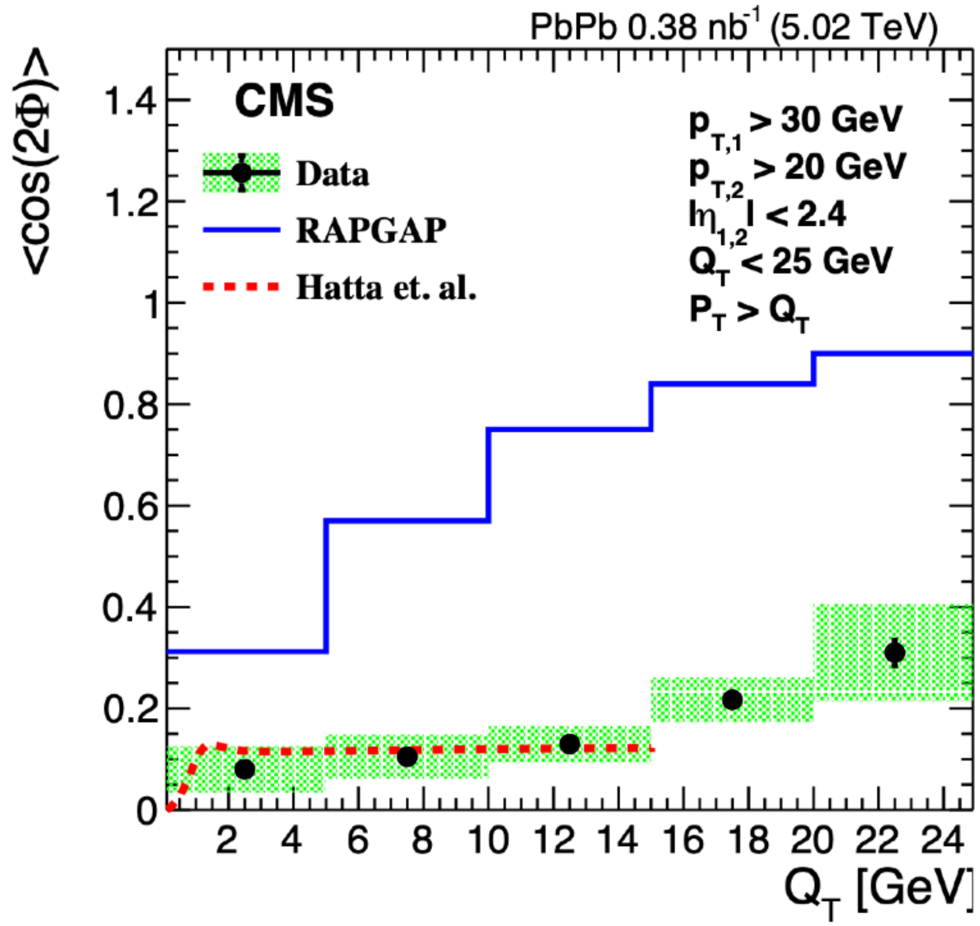
(Zhang, Dai, DYS, '23 JHEP)



- **A good agreement with the ATLAS data in the nearly back-to-back region**
- **Photo-productions may enhance the dijet production rate, but should barely change the shape**

Azimuthal Correlations within Exclusive Dijets with Large Momentum Transfer in Photon-Lead Collisions

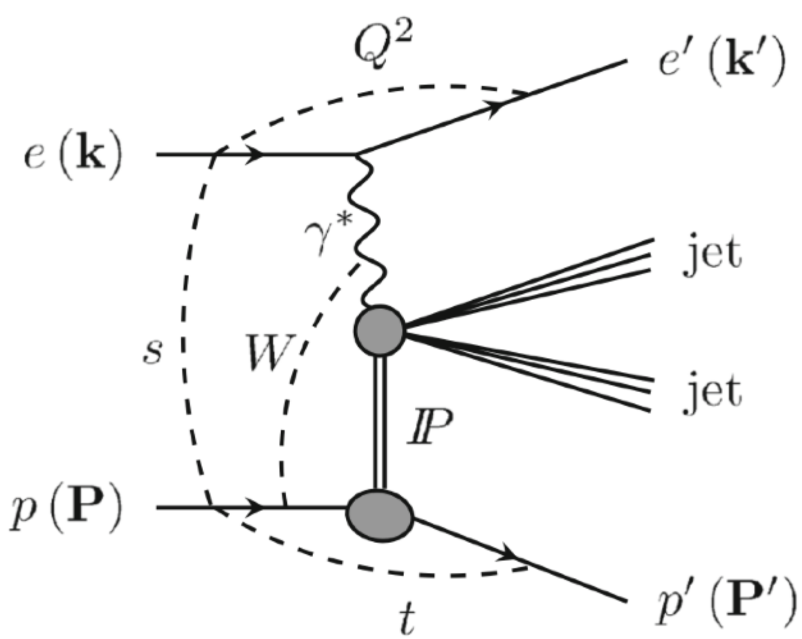




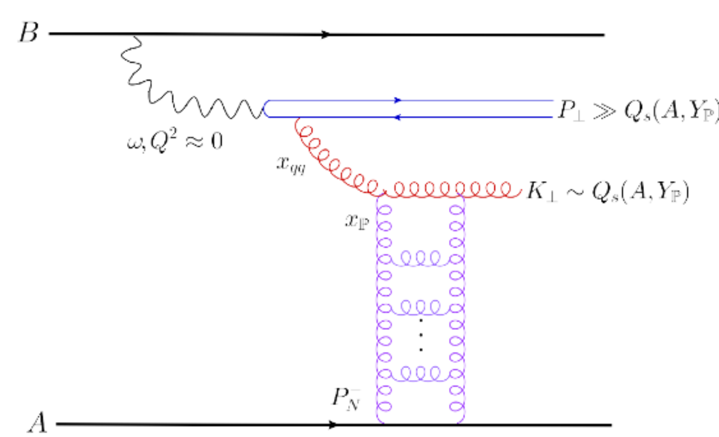
A calculation based on the RAPGAP model, which is tuned to HERA results, predicts that ⟨cos(2Φ)⟩ rises with QT, but overshoots the data by a factor of 3-5.

Diffractive dijets photo-production

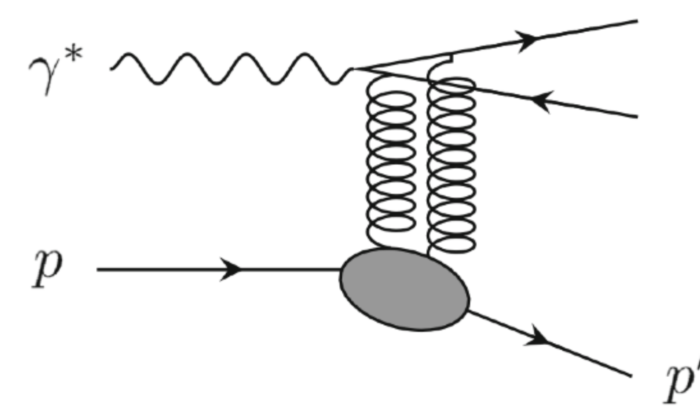
- **Diffractive di-jet production provide rich information on nucleon internal structure.**



diffractive production of (2+1) jets



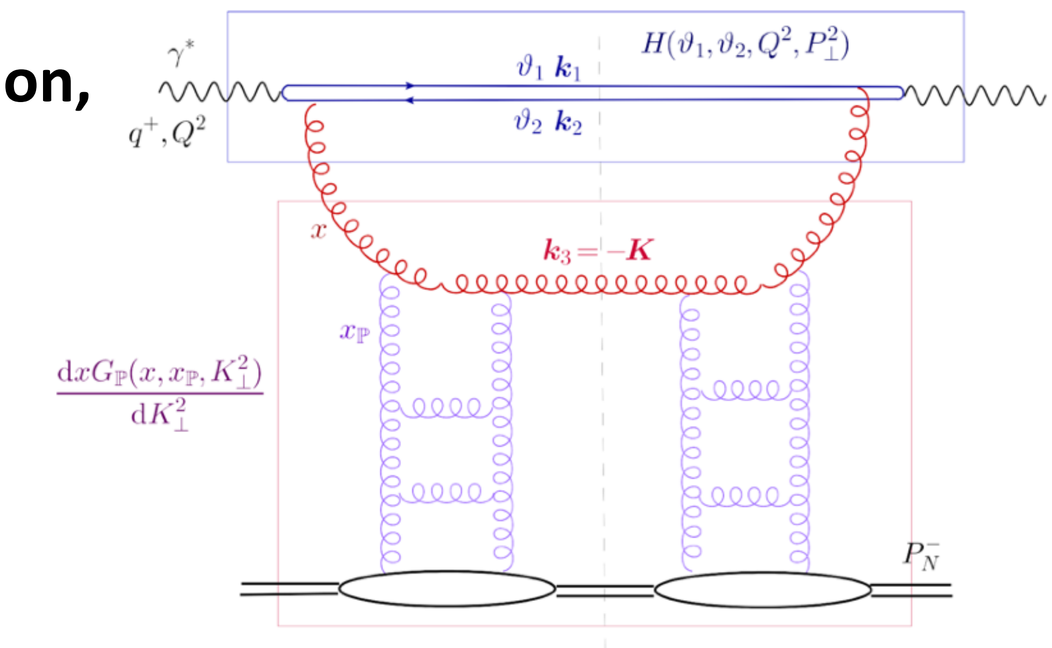
diffractive production of exclusive dijets



- **In cases of diffractive tri-jet production, where a semi-hard gluon is emitted towards the target direction and remains undetected, the experimental signature of this process becomes indistinguishable from that of exclusive di-jet production.**
- **Recent studies have shown that the cross section for coherent tri-jet photo-production significantly surpasses that of exclusive di-jet production** Iancu, Mueller & Triantafyllopoulos '21
- **The production of color octet hard quark-anti-quark dijets enables the emission of soft gluons from the initial state. This mechanism significantly influences the total transverse momentum $q_\perp$ distribution of the dijet.**

Diffractive dijets photo-production

- **The CGC calculation of diffractive di-jet photo-production, accompanied by a semi-hard gluon emission, has been studied in** Iancu, Mueller & Triantafyllopoulos ’21; Iancu, Mueller, Triantafyllopoulos, & S. Y. Wei ’23



$$\gamma(x_\gamma p) + A \to q(k_1) + \bar{q}(k_2) + g(l) + A$$

- **The Born cross section for semi-inclusive diffractive back-to-back dijet production is expressed as**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp} = \sigma_0 x_\gamma f_\gamma(x_\gamma) \int \frac{\mathrm{d}x_{\mathbb{P}}}{x_{\mathbb{P}}} x_g G_{\mathbb{P}}(x_g, x_{\mathbb{P}}, q_\perp)$$

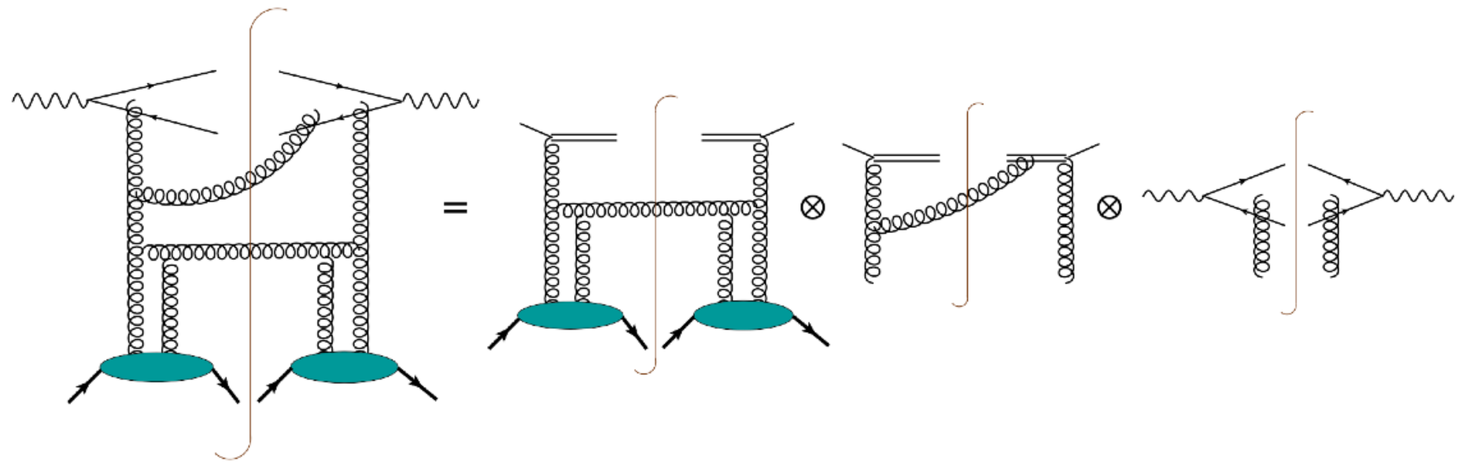
- **Within the CGC formalism, the gluon distribution of the pomeron is related to the gluon-gluon dipole scattering amplitude**

$$x_g G_{\mathbb{P}}(x_g, x_{\mathbb{P}}, q_\perp) = \frac{S_\perp(N_c^2-1)}{8\pi^4(1-x)} \left[\frac{xq_\perp^2}{1-x} \int r_\perp \mathrm{d}r_\perp J_2(q_\perp r_\perp) K_2\left(\sqrt{\frac{xq_\perp^2 r_\perp^2}{1-x}}\right) \mathcal{T}_g(x_{\mathbb{P}}, r_\perp)\right]^2$$

dipole amplitude

Factorizaton and resummation

- **By treating the gluon DTMD as if it were an ordinary TMD, we assume that the standard TMD factorization framework can be used in the back-to-back region**
  Hatta, Xiao & Yuan '22



$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{d}^2\boldsymbol{P}_\perp\,\mathrm{d}^2\boldsymbol{q}_\perp} = \sigma_0 x_\gamma f_\gamma(x_\gamma) H_{\gamma^* g}(P_\perp, R, \mu) \int \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}^2\boldsymbol{\lambda}_\perp \delta^{(2)}(\boldsymbol{q}_\perp - \boldsymbol{k}_\perp - \boldsymbol{\lambda}_\perp)$$

$$\times S(\boldsymbol{\lambda}_\perp, R, \mu) \int \frac{\mathrm{d}x_{\mathbb{P}}}{x_{\mathbb{P}}} x_g G_{\mathbb{P}}^{\mathrm{unsub}}(x_g, x_{\mathbb{P}}, k_\perp, \mu).$$

- **We refactorize the gluon DTMD as the matching coefficients and the integrated pomeron gluon function**

DGLAP evolution of the pomeron gluon DPDF ?

Glauber SCET Rothstein, Stewart, `16

$$G_{\mathbb{P}}(x_g, x_{\mathbb{P}}, k_\perp, \mu, \zeta) = \int_{x_g}^1 \frac{\mathrm{d}z}{z} I_{g\leftarrow g}(z, k_\perp, \mu, \zeta) G_{\mathbb{P}}(x_g/z, x_{\mathbb{P}}, \mu) + G_{\mathbb{P}}(x_g, x_{\mathbb{P}}, k_\perp)$$

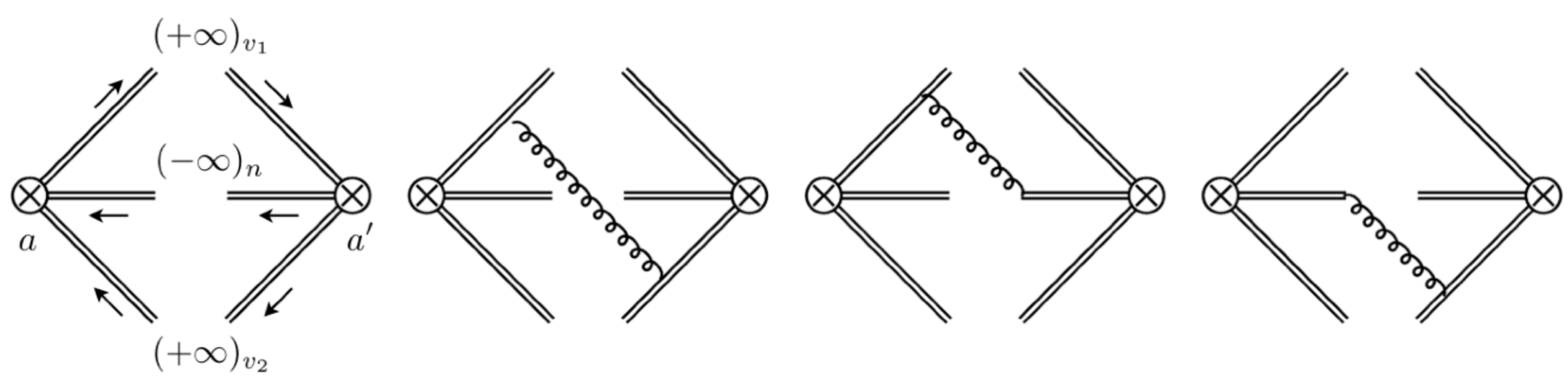
additional static source term in the modified DGLAP equation
Iancu, Mueller, Triantafyllopoulos, & Wei '23

Factorization and resummation

- **Resummation formula**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{d}^2\boldsymbol{P}_\perp\,\mathrm{d}^2\boldsymbol{q}_\perp}=\sigma_0 x_\gamma f_\gamma(x_\gamma)\int\frac{\mathrm{d}^2\boldsymbol{b}_\perp}{(2\pi)^2}e^{i\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}e^{-\mathrm{Sud}_{\mathrm{pert}}(b_\perp)}\tilde{S}^{\mathrm{rem}}(\boldsymbol{b}_\perp,\mu_b)$$
$$\times\int\mathrm{d}^2\boldsymbol{k}_\perp e^{-i\boldsymbol{b}_\perp\cdot\boldsymbol{k}_\perp}\int\frac{\mathrm{d}x_{\mathbb{P}}}{x_{\mathbb{P}}}x_g G_{\mathbb{P}}(x_g,x_{\mathbb{P}},k_\perp),$$

- **NLO azimuthal angle-dependent soft function**

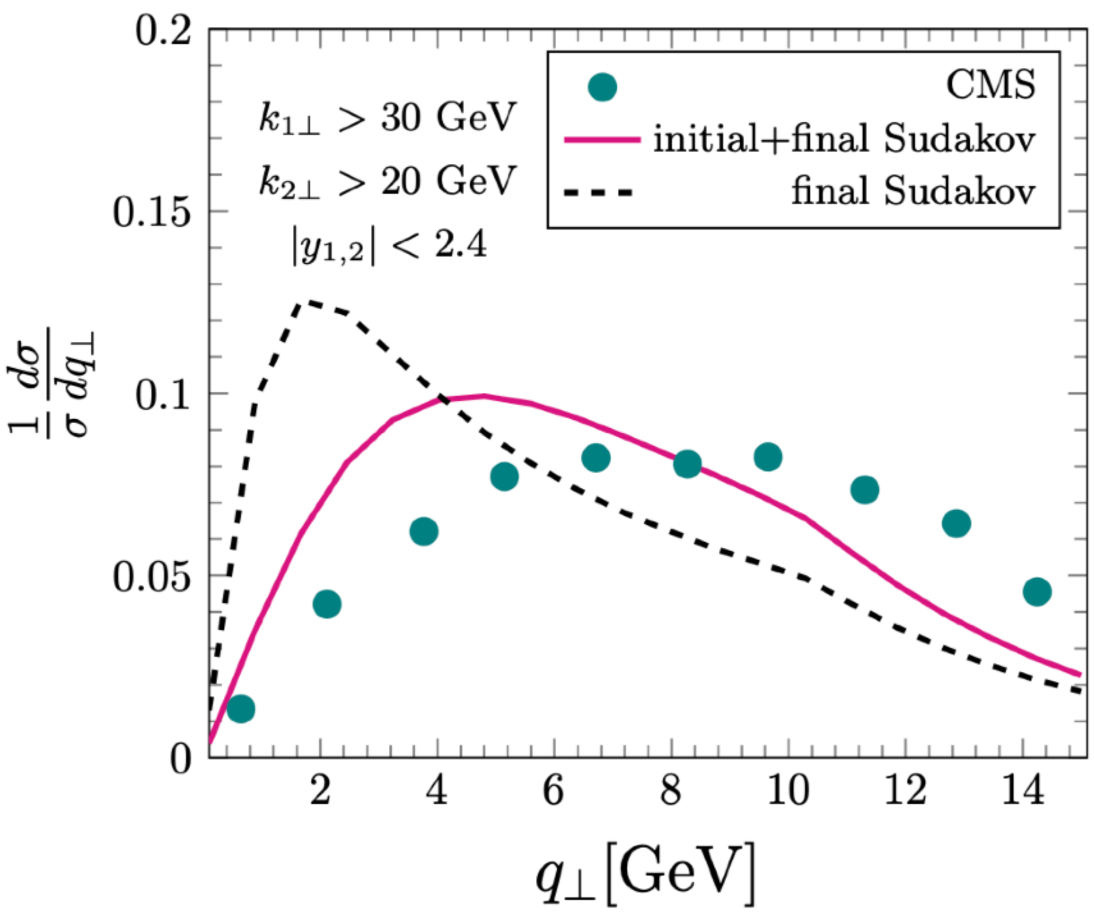


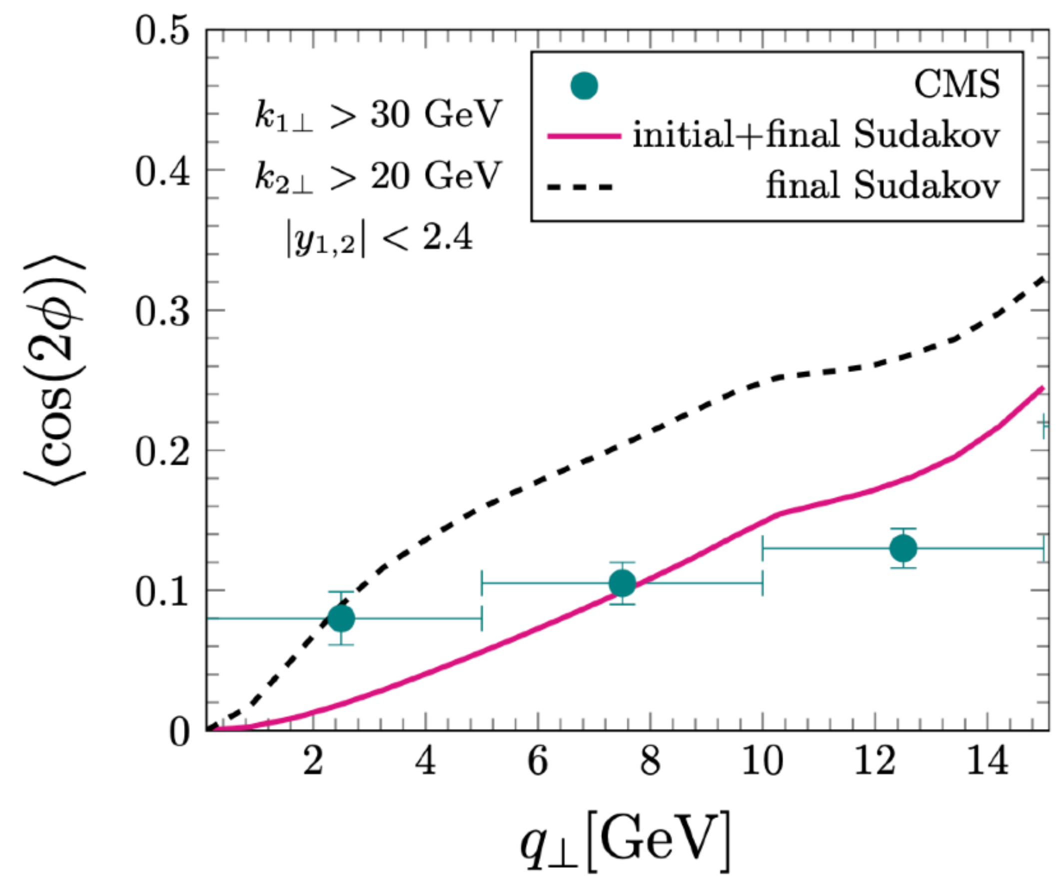
$$\tilde{S}^{\mathrm{NLO}}_{\phi_b-\mathrm{dep}}(\boldsymbol{b}_\perp,R,\mu_b)=\frac{\alpha_s(\mu_b)}{\pi}\Bigg\{C_F\left(-\ln\frac{4}{R^2}\ln c_\phi^2-\frac{1}{2}\ln^2 c_\phi^2\right)+\frac{1}{2N_c}\Bigg[-\frac{1}{2}\ln^2 c_\phi^2+\ln c_\phi^2\ln x$$
$$-\ln c_\phi^2\ln\left(1-\frac{x}{c_\phi^2}\right)+\ln x\log\left(1-\frac{x}{c_\phi^2}\right)+\mathrm{Li}_2\left(\frac{x}{c_\phi^2}\right)\Bigg]\Bigg\}$$

$$\boxed{c_\phi\;=\;\cos\phi_b}$$

Numerical results and measurements in UPCs

DYS, Y. Shi, C. Zhang, J, Zhou, Y. Zhou ’24 JHEP





- **Incorporating the initial state gluon radiation offers a more accurate representation of the CMS data**
- **Difference remains.**

$$\langle\cos(2\phi)\rangle \equiv \frac{\int d\mathcal{P.S.}\cos(2\phi)\frac{d\sigma}{dy_1 dy_2 d^2\boldsymbol{P}_\perp d^2\boldsymbol{q}_\perp}}{\int d\mathcal{P.S.}\frac{d\sigma}{dy_1 dy_2 d^2\boldsymbol{P}_\perp d^2\boldsymbol{q}_\perp}}$$

The azimuthal asymmetry: Our result underestimates the asymmetry at low $q_\perp$ and overshoots it at high

Summary and Outlook

- **We develop the factorization framework to study transverse polarization effects for Λ(thrust) production**
- **We present a comprehensive study of the azimuthal angle dependence of EEC in the back-to-back region**
- **We study azimuthal angular asymmetry in diffractive di-jet production. The production of color octet dijets expands the color space, enabling the emission of soft gluons in the initial state.**

Thank you