Hadronization models in Monte Carlo Event Generators

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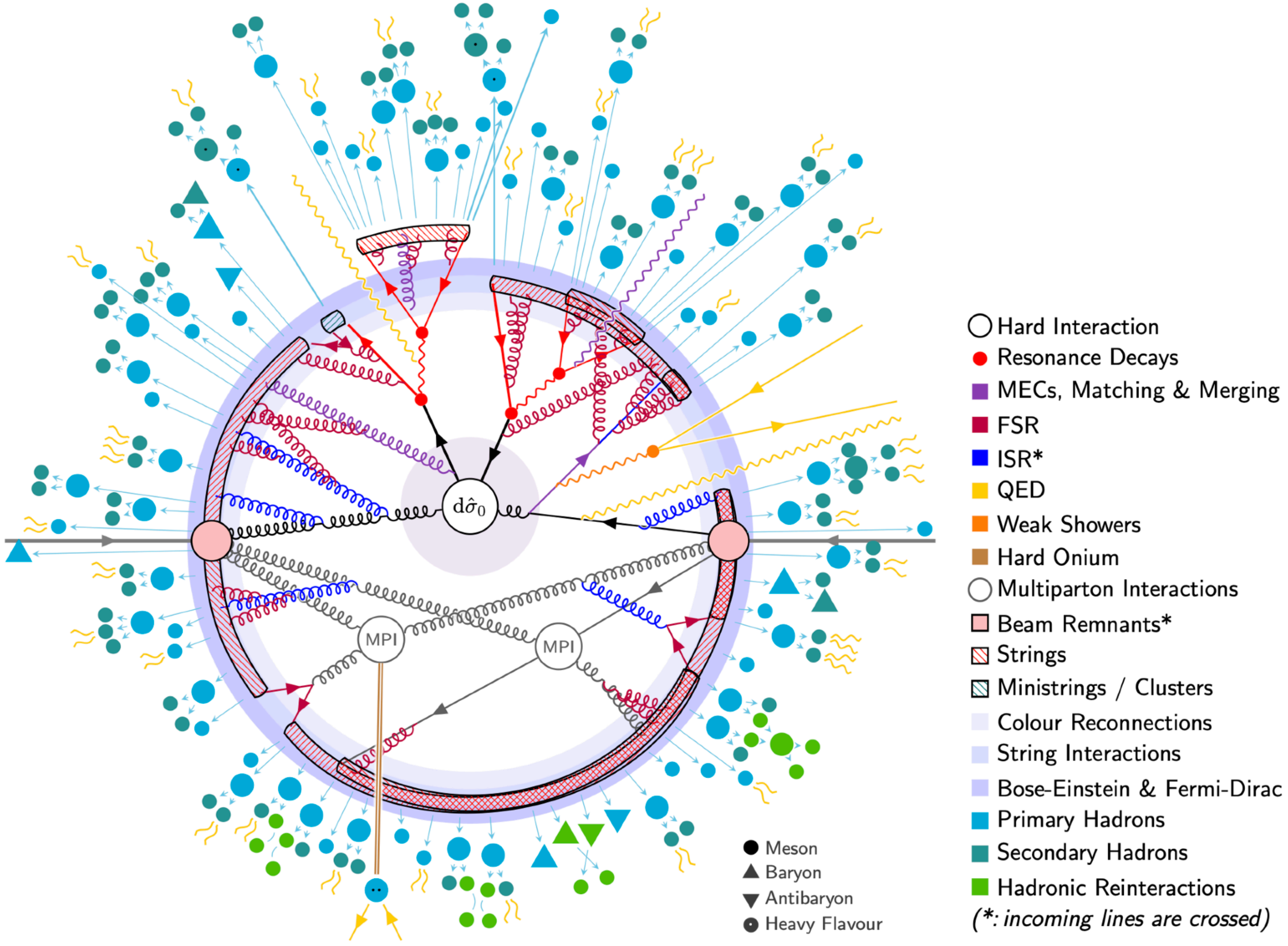
Opportunities and Ideas at the QCD Frontier

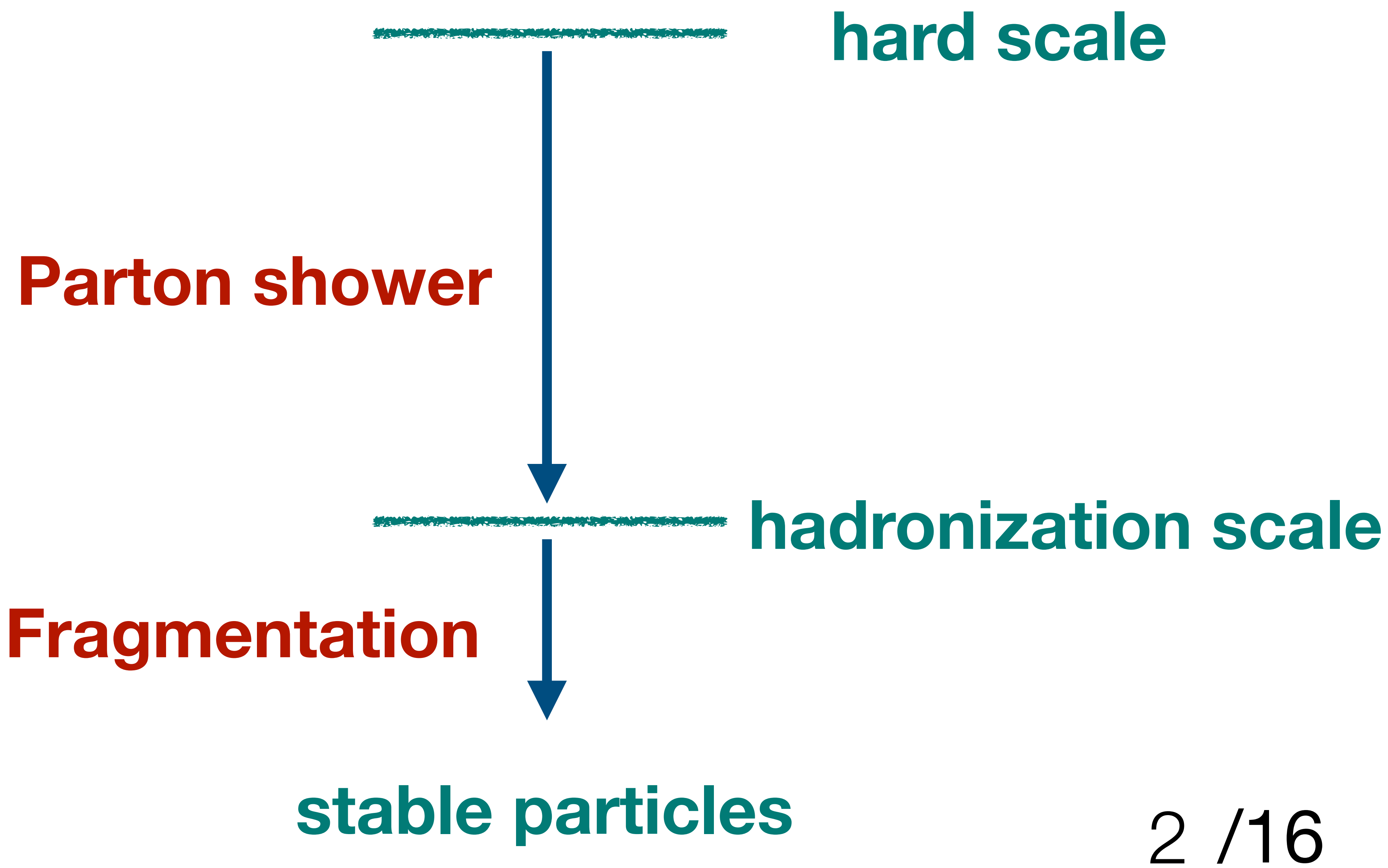
2025-4-8

Monte Carlo Event Generators



From PYTHIA 8.3

- ❑ Hard process in high energy
- ❑ Transition from high energy to low energy —parton shower
- ❑ Low energy soft regime —fragmentation



Monte Carlo Event Generators

Parton Showers:

- generate a cascade of emissions.
- simulate the rich structure of the events or the jets

- start from Sudakov form factor: Non-branching probability

  for example $\Delta(Q^2, q^2) = \exp\left[\int_{Q^2}^{q^2} \mathbf{d}\phi_{\mathbf{n}\to\mathbf{n+1}} \frac{|M_{n+1}|^2}{|M_n|^2}\right]$

- use Monte Carlo Method to generate kinematics variables such as $k_\perp, z, \phi$

- use these variables to construct $\phi_n \to \phi_{n+1}$

- repeat the above algorithm recursively

Monte Carlo Event Generators

Sudakov form factor: Non-branching probability $\exp\left[\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right]$

Probability that there is no branching from Q to q is $\Delta_i\left(Q^2, q^2\right)$

choose kinematic variable as the evolution scale

$$\Delta\left(Q^2, q^2\right) = \exp\left\{\int_{Q^2}^{q^2} d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right\}$$

Probability for one observed branching $1 - \Delta\left(Q^2, q^2\right)$

Probability one branching between the scale q^2 to $q^2 + dq^2$

$$\frac{d}{dq^2}\Delta\left(Q^2, q^2\right) = \Delta\left(Q^2, q^2\right) \times d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}$$

Additional radiations can be added according to the function $\Delta\left(Q^2, q^2\right)$

Monte Carlo Event Generators

Parton Showers:

- generate a cascade of emissions.
- simulate the rich structure of the events or the jets

```
 --------  PYTHIA Event Listing  (complete event)  ---------------------------------------------------------------------------------

    no         id  name            status     mothers   daughters     colours      p_x        p_y        p_z         e          m
     0         90  (system)           -11     0     0     0     0     0     0      0.000      0.000      0.000     91.188     91.188
     1         11  (e-)               -12     0     0     3     0     0     0      0.000      0.000     45.594     45.594      0.001
     2        -11  (e+)               -12     0     0     4     0     0     0      0.000      0.000    -45.594     45.594      0.001
     3         11  (e-)               -21     1     0     5     0     0     0      0.000      0.000     45.594     45.594      0.000
     4        -11  (e+)               -21     2     0     5     0     0     0      0.000      0.000    -45.594     45.594      0.000
     5         23  (Z0)               -22     3     4     6     7     0     0      0.000      0.000      0.000     91.188     91.188
     6          3  (s)                -23     5     0     8     9   101     0    -29.975      1.918     34.298     45.594      0.500
     7         -3  (sbar)             -23     5     0    10    10     0   101     29.975     -1.918    -34.298     45.594      0.500
     8          3  s                   51     6     0     0     0   103     0    -19.155     -0.555     22.389     29.474      0.500
     9         21  (g)                -51     6     0    13    13   101   103    -10.768      2.470     11.849     16.200      0.000
    10         -3  (sbar)             -52     7     7    11    12     0   101     29.923     -1.915    -34.238     45.514      0.500
    11         -3  sbar                51    10     0     0     0     0   105     16.893     -1.870    -20.679     26.772      0.500
    12         21  (g)                -51    10     0    14    15   105   101     13.009     -0.040    -13.535     18.773      0.000
    13         21  (g)                -52     9     9    16    16   101   103    -10.747      2.465     11.825     16.168      0.000
    14         21  (g)                -51    12     0    25    25   105   106     12.378     -0.798    -12.329     17.488      0.000
    15         21  (g)                -51    12     0    19    19   106   101      0.527      0.782     -1.092      1.442      0.000
    16         21  (g)                -52    13    13    17    18   101   103    -10.642      2.441     11.710     16.011      0.000
    17         -2  ubar                51    16     0     0     0     0   103     -4.526      1.098      4.097      6.212      0.330
    18          2  (u)                -51    16     0    22    22   101     0     -6.103      1.363      7.586      9.836      0.330
    19         21  (g)                -52    15    15    20    21   106   101      0.513      0.762     -1.064      1.405      0.000
    20         21  (g)                -51    19     0    23    24   106   107      0.518      0.239     -1.029      1.177      0.000
    21         21  g                   51    19     0     0     0   107   101     -0.390      0.609      0.444      0.849      0.000
    22          2  u                   52    18    18     0     0   101     0     -5.718      1.277      7.107      9.216      0.330
    23         21  g                   51    20     0     0     0   110   107      3.597     -0.501     -4.105      5.481      0.000
    24         21  g                   51    20     0     0     0   106   110      1.334      0.455     -1.320      1.932      0.000
    25         21  g                   52    14    14     0     0   105   106      7.964     -0.514     -7.933     11.253      0.000
                                   Charge sum:  0.000           Momentum sum:      0.000     -0.000     -0.000     91.188     91.188

 --------  End PYTHIA Event Listing  -----------------------------------------------------------------------------------------------
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Monte Carlo Event Generators

hadronization effects

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
27	-2	(ubar)	-71	17	17	28	29	0	103	-4.526	1.098	4.097	6.212	0.330
28	3322	(Xi0)	-83	26	27	46	47	0	0	-16.278	-0.490	18.555	24.723	1.315
29	-3222	(Sigmabar-)	-84	26	27	48	49	0	0	-7.403	1.034	7.932	10.963	1.189
30	2	(u)	-71	22	22	36	45	101	0	-5.718	1.277	7.107	9.216	0.330
31	21	(g)	-71	21	21	36	45	107	101	-0.390	0.609	0.444	0.849	0.000
32	21	(g)	-71	23	23	36	45	110	107	3.597	-0.501	-4.105	5.481	0.000
33	21	(g)	-71	24	24	36	45	106	110	1.334	0.455	-1.320	1.932	0.000
34	21	(g)	-71	25	25	36	45	105	106	7.964	-0.514	-7.933	11.253	0.000
35	-3	(sbar)	-71	11	11	36	45	0	105	16.893	-1.870	-20.679	26.772	0.500
36	111	(pi0)	-83	30	35	50	51	0	0	-3.511	0.738	4.002	5.377	0.135
37	211	pi+	83	30	35	0	0	0	0	0.002	0.218	0.085	0.273	0.140
38	-211	pi-	83	30	35	0	0	0	0	-1.767	-0.071	2.475	3.045	0.140
39	211	pi+	83	30	35	0	0	0	0	-0.182	0.285	0.651	0.747	0.140
40	-211	pi-	83	30	35	0	0	0	0	0.016	0.232	0.209	0.342	0.140
41	211	pi+	83	30	35	0	0	0	0	-0.413	0.450	-0.145	0.643	0.140
42	-211	pi-	84	30	35	0	0	0	0	2.478	-0.473	-2.622	3.642	0.140
43	2212	p+	84	30	35	0	0	0	0	6.374	-0.009	-6.640	9.252	0.938
44	111	(pi0)	-84	30	35	52	53	0	0	0.270	0.111	-0.364	0.486	0.135
45	-3122	(Lambdabar0)	-84	30	35	54	55	0	0	20.414	-2.024	-24.136	31.696	1.116
46	3122	(Lambda0)	-91	28	0	56	57	0	0	-14.222	-0.534	16.090	21.510	1.116
47	111	(pi0)	-91	28	0	58	59	0	0	-2.056	0.043	2.465	3.213	0.135
48	-2112	nbar0	91	29	0	0	0	0	0	-5.613	0.671	6.203	8.445	0.940
49	-211	pi-	91	29	0	0	0	0	0	-1.790	0.363	1.728	2.518	0.140
50	22	gamma	91	36	0	0	0	0	0	-3.222	0.667	3.613	4.887	0.000
51	22	gamma	91	36	0	0	0	0	0	-0.289	0.071	0.388	0.490	0.000
52	22	gamma	91	44	0	0	0	0	0	0.028	-0.020	-0.008	0.036	0.000
53	22	gamma	91	44	0	0	0	0	0	0.242	0.131	-0.356	0.450	0.000
54	-2212	pbar-	91	45	0	0	0	0	0	18.123	-1.732	-21.512	28.198	0.938
55	211	pi+	91	45	0	0	0	0	0	2.291	-0.292	-2.624	3.498	0.140
56	2212	p+	91	46	0	0	0	0	0	-10.893	-0.393	12.398	16.535	0.938
57	-211	pi-	91	46	0	0	0	0	0	-3.329	-0.140	3.692	4.975	0.140
58	22	gamma	91	47	0	0	0	0	0	-0.678	0.003	0.911	1.136	0.000
59	22	gamma	91	47	0	0	0	0	0	-1.378	0.040	1.554	2.077	0.000

Monte Carlo Event Generators

hadronization effects

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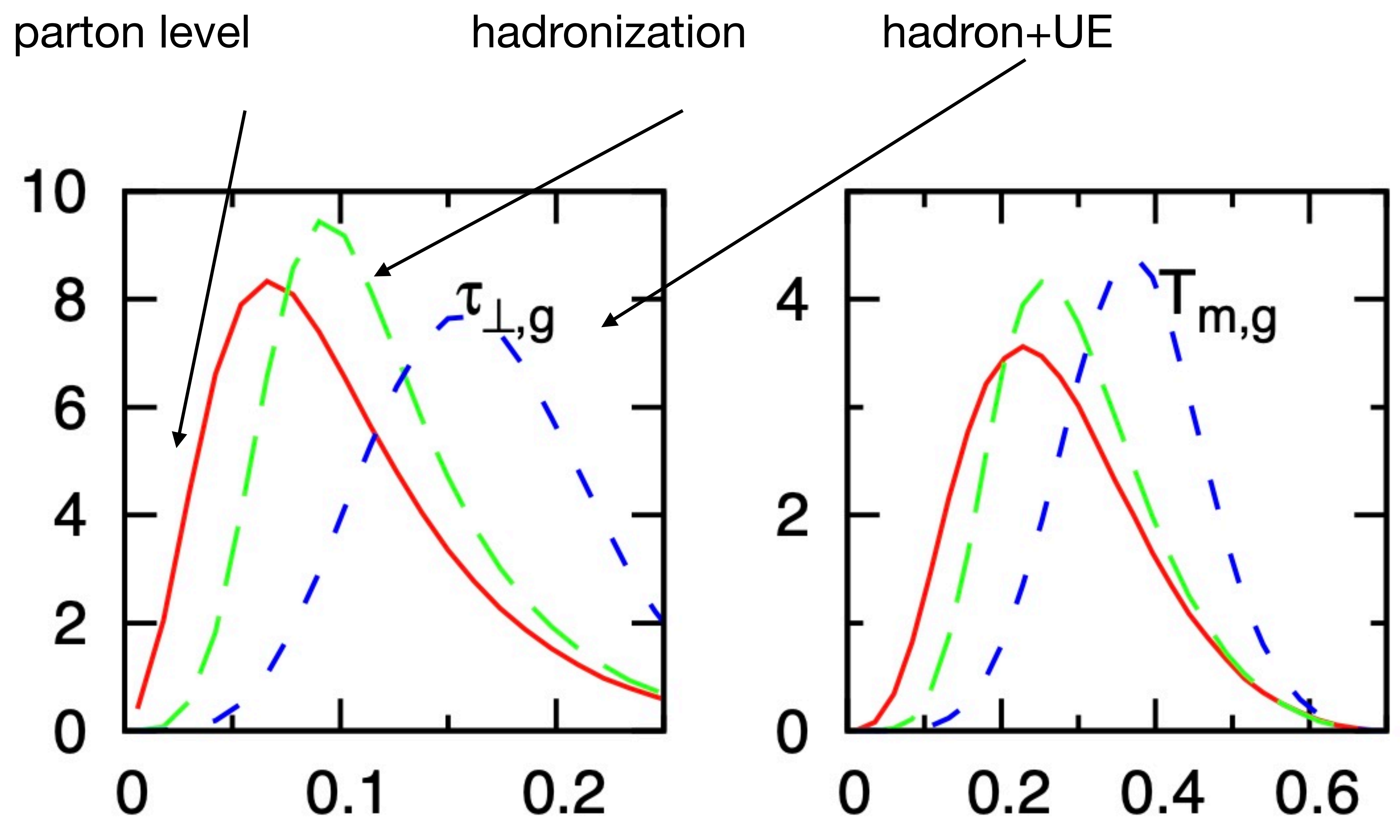
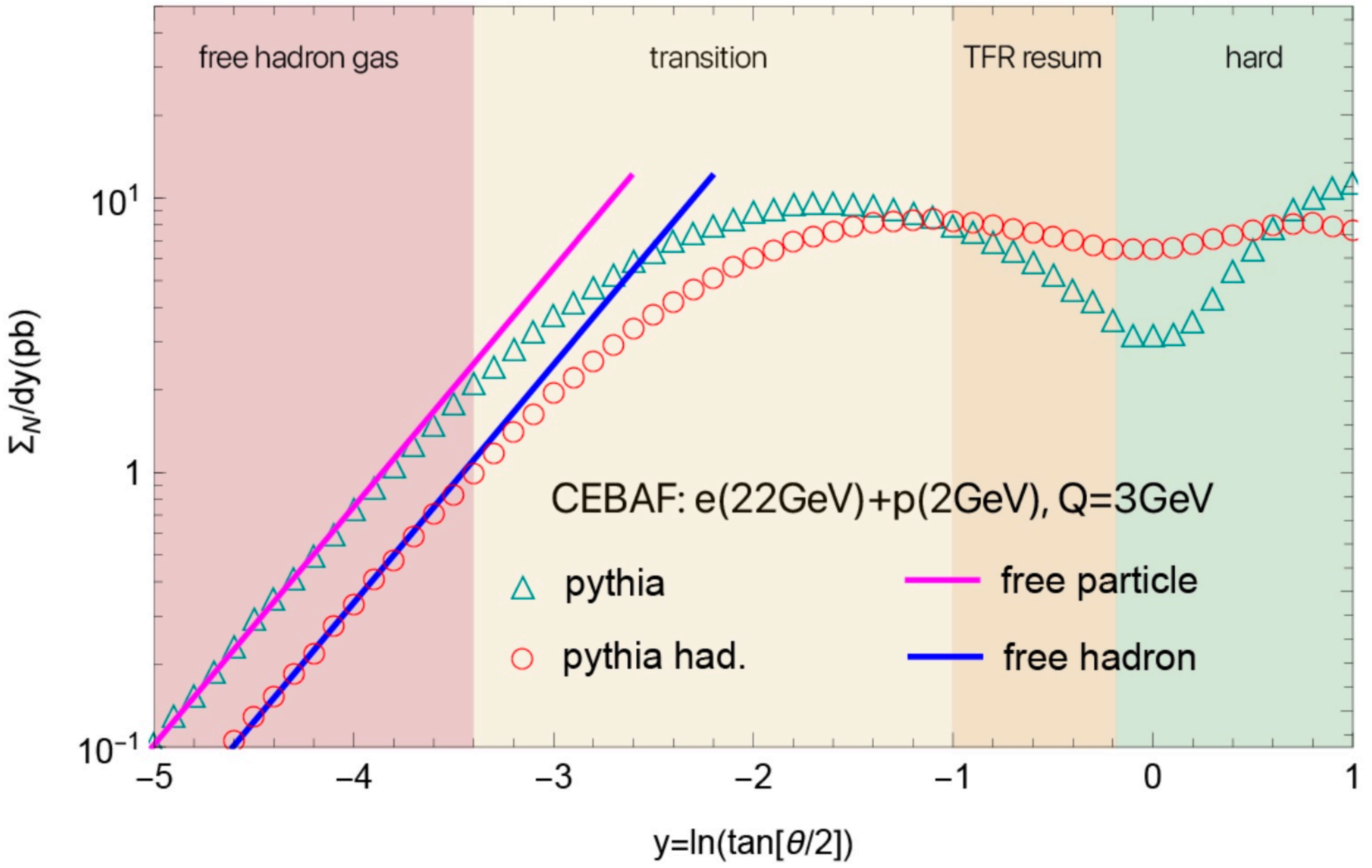


Banfi, Salam and Zanderighi 2010

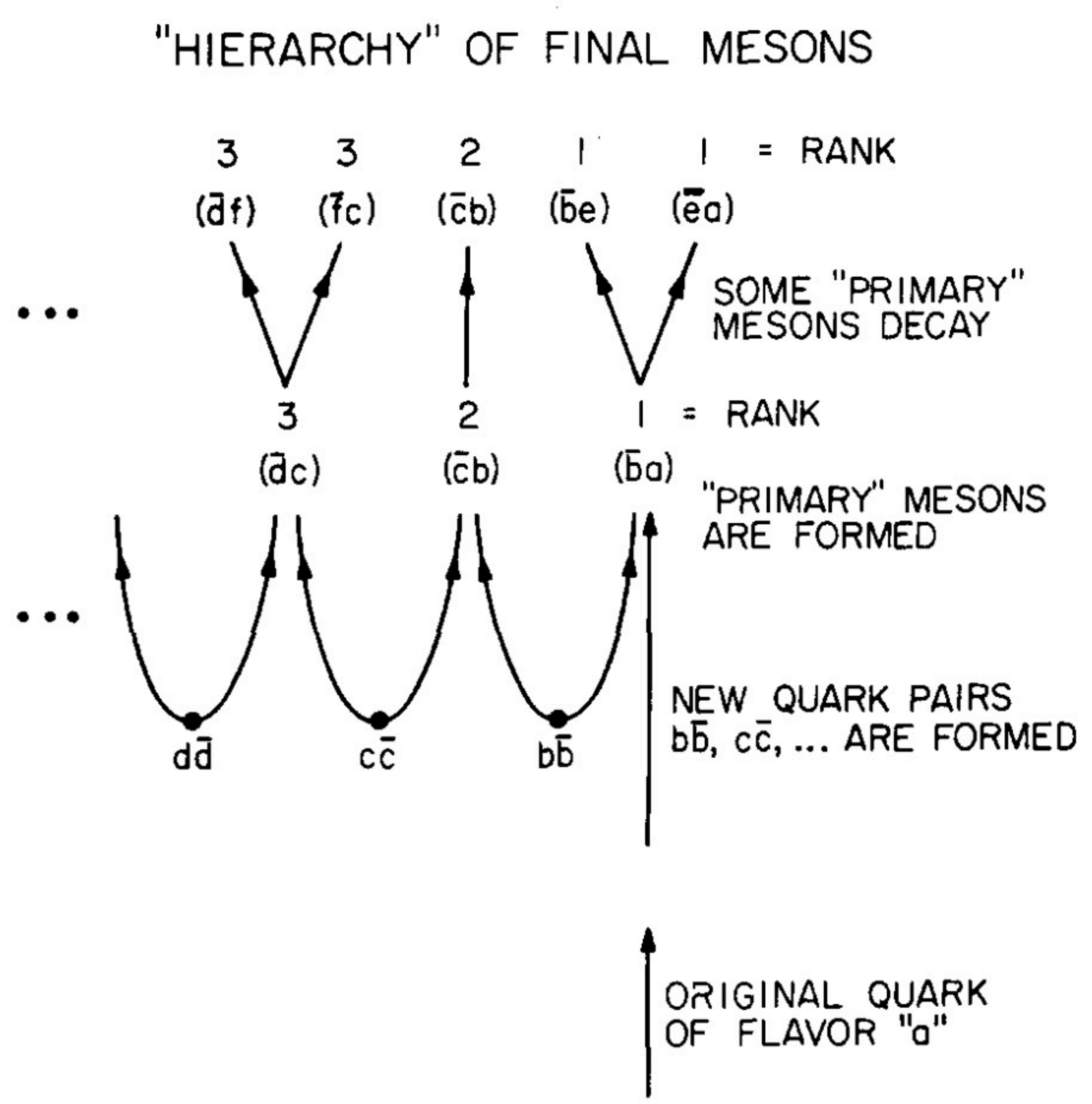


Monte Carlo Event Generators

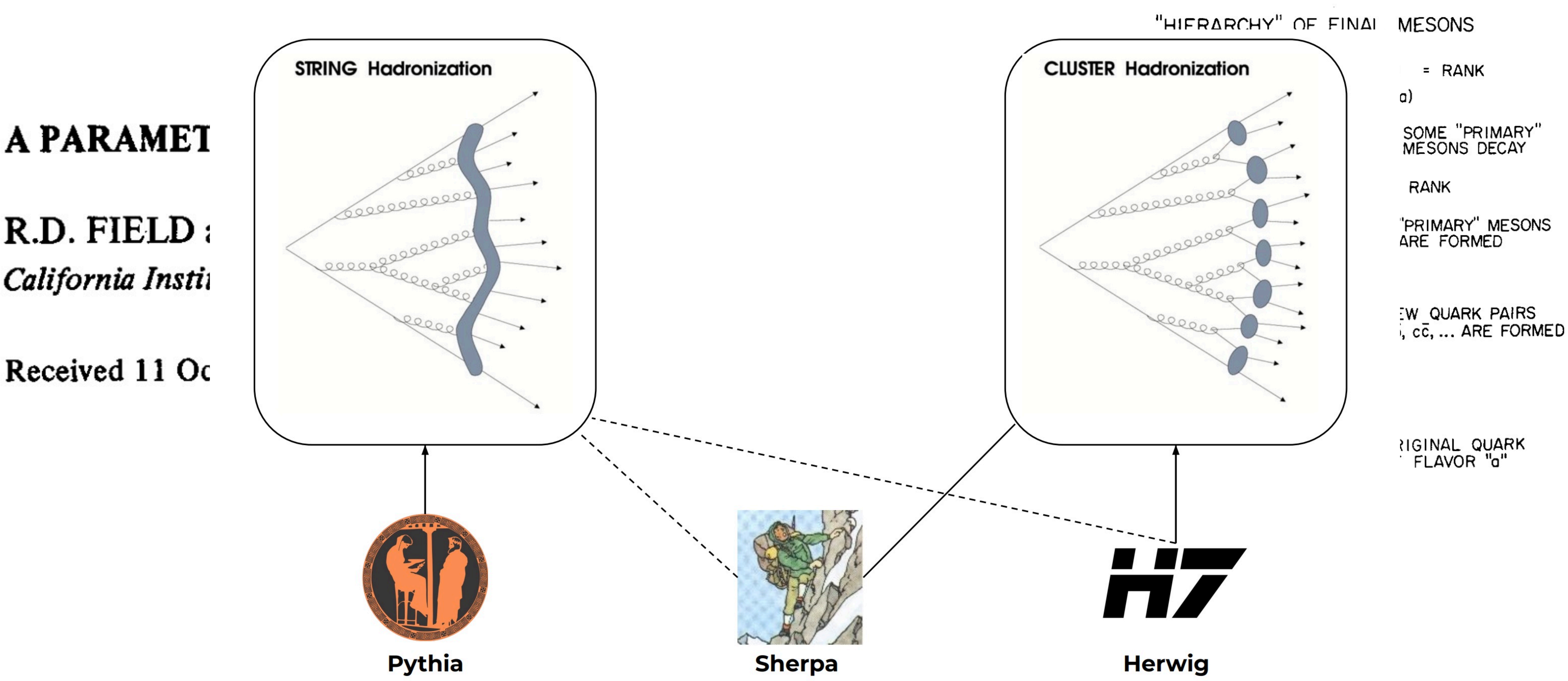
A PARAMETRIZATION OF THE PROPERTIES OF QUARK JE

R.D. FIELD and R.P. FEYNMAN
California Institute of Technology, Pasadena, California 91125, USA

Received 11 October 1977

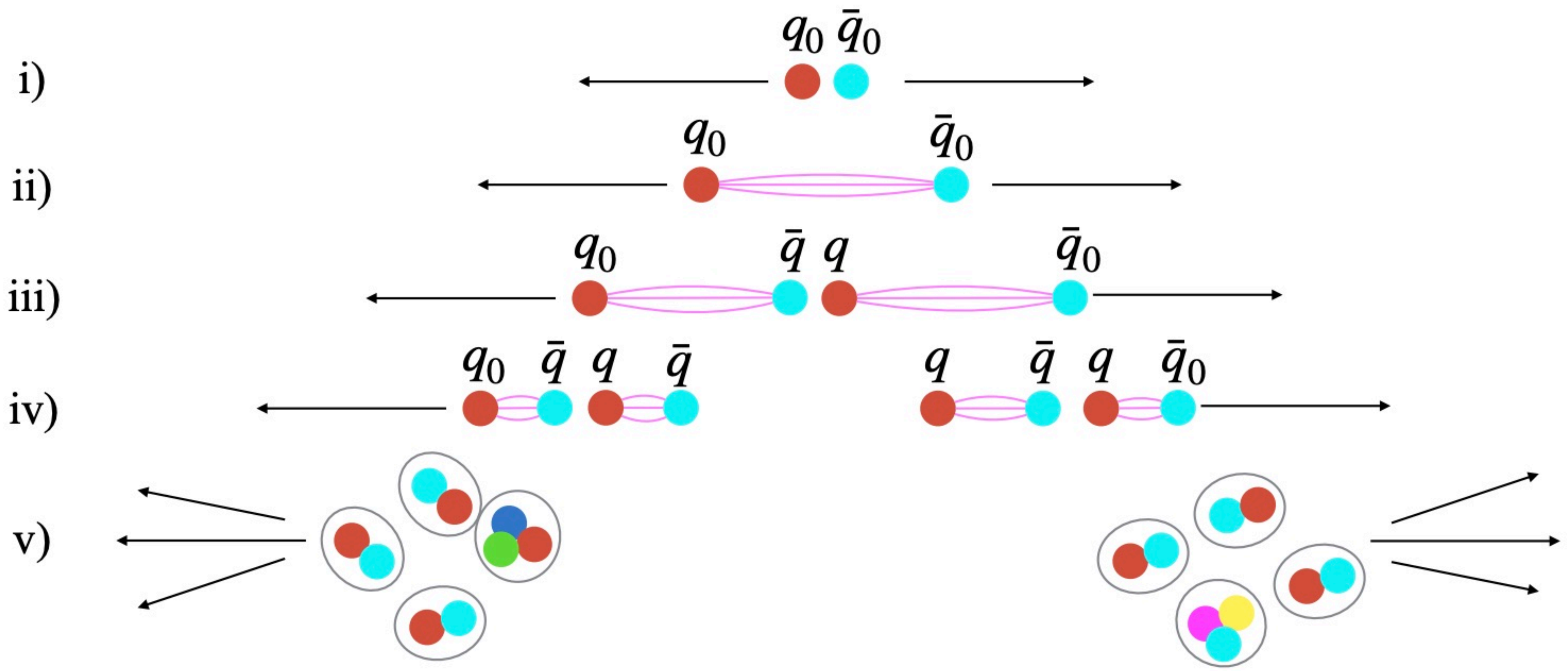


Monte Carlo Event Generators



The two main hadronization classes in current use are the string and cluster ones

Lund String Model



Initial string $q_0\bar{q}_0$

The string begins to form between them

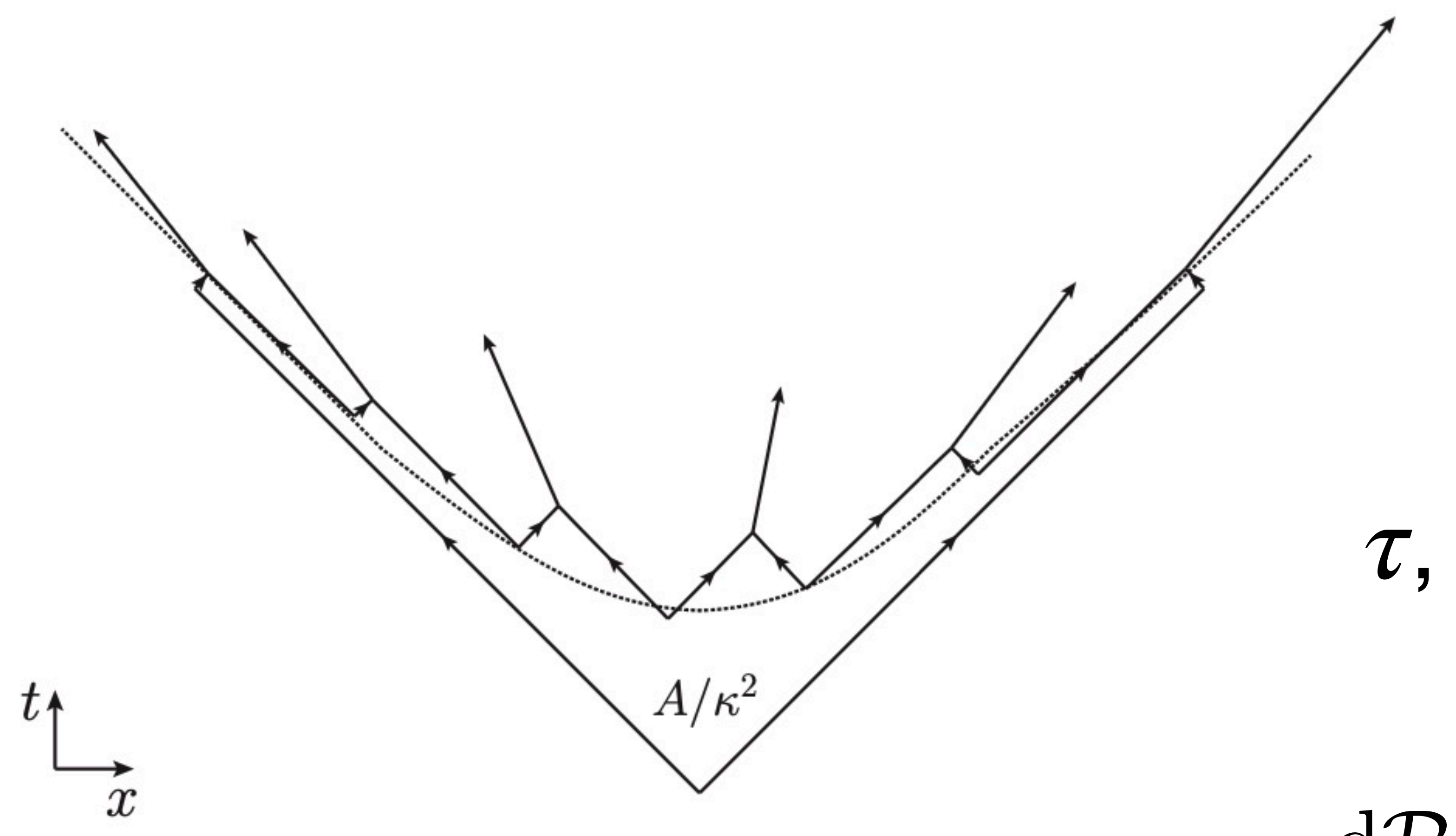
A string break occurs, creating a new $q\bar{q}$ pair.

More string breaks occur.

hadrons from the fragmentation process

string with energy density $\kappa \approx 1\ \mathrm{GeV/fm} \approx 0.1\ \mathrm{GeV}^2$ GeV/fm

Lund String Model



Probability for string beak: a fragmentation function

$$f(z) \propto \frac{1}{z}(1-z)^a \exp\left(-\frac{bm_\perp^2}{z}\right)$$

τ, hadron production time $\quad \Gamma = (\kappa\tau)^2$

$$\mathrm{d}\mathcal{P}/\mathrm{d}\Gamma \propto \Gamma^a \exp(-b\Gamma) \qquad \Gamma_2 = (1-z)\left(\Gamma_1 + \frac{m_\perp^2}{z}\right) \qquad \langle\tau^2\rangle \approx 2\ \mathbf{fm}$$

Heavy quarks, i.e. charm and bottom, are not produced at new string breaks (see below), but may be at the endpoints of a string.

$$f(z) \propto \frac{1}{z^{1+bm_Q^2}}(1-z)^a \exp\left(-\frac{bm_\perp^2}{z}\right)$$

Lund String Model

If the quarks have mass or transverse momentum, they must classically be produced

$$\frac{1}{\kappa}\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}^2 p_\perp} \propto \exp(-\pi m_\perp^2/\kappa) = \exp(-\pi m^2/\kappa)\exp(-\pi p_\perp^2/\kappa)$$

Gaussian broadening $\sigma \sim 0.25$ GeV

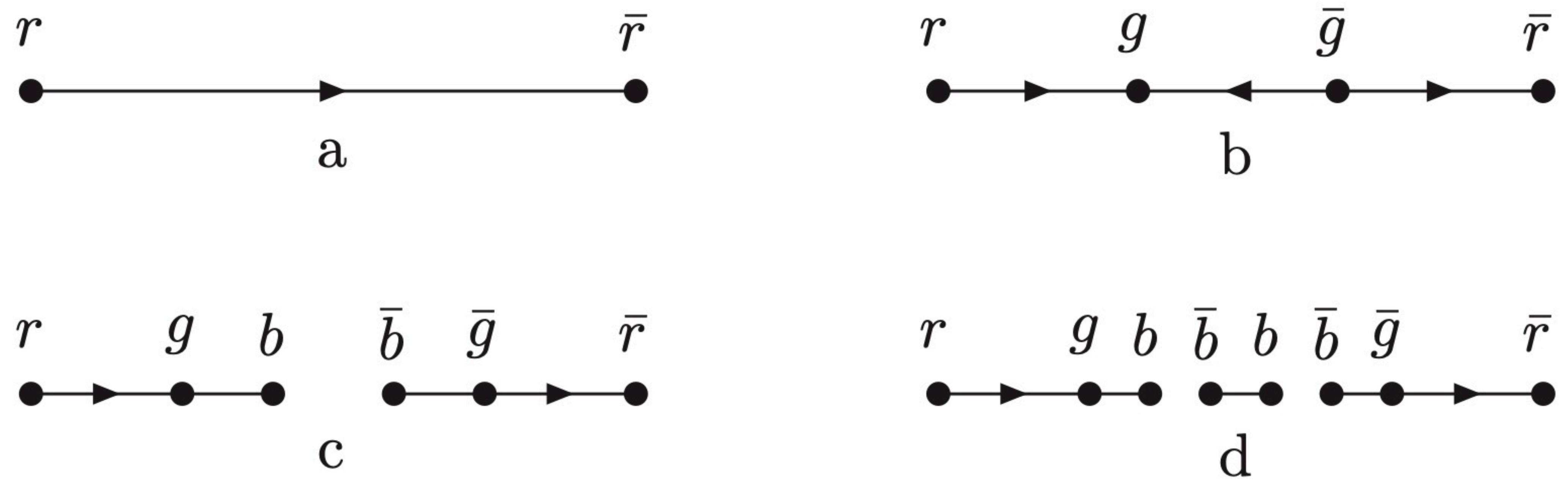
Quark masses lead to strangeness suppression

The suppression is viewed as a free parameter, and tuned to LEP data.

$$u : d : s : c \approx 1 : 1 : 0.3 : 10^{-11}$$

A quark and an antiquark may combine to produce either a pseudoscalar or a vector meson.

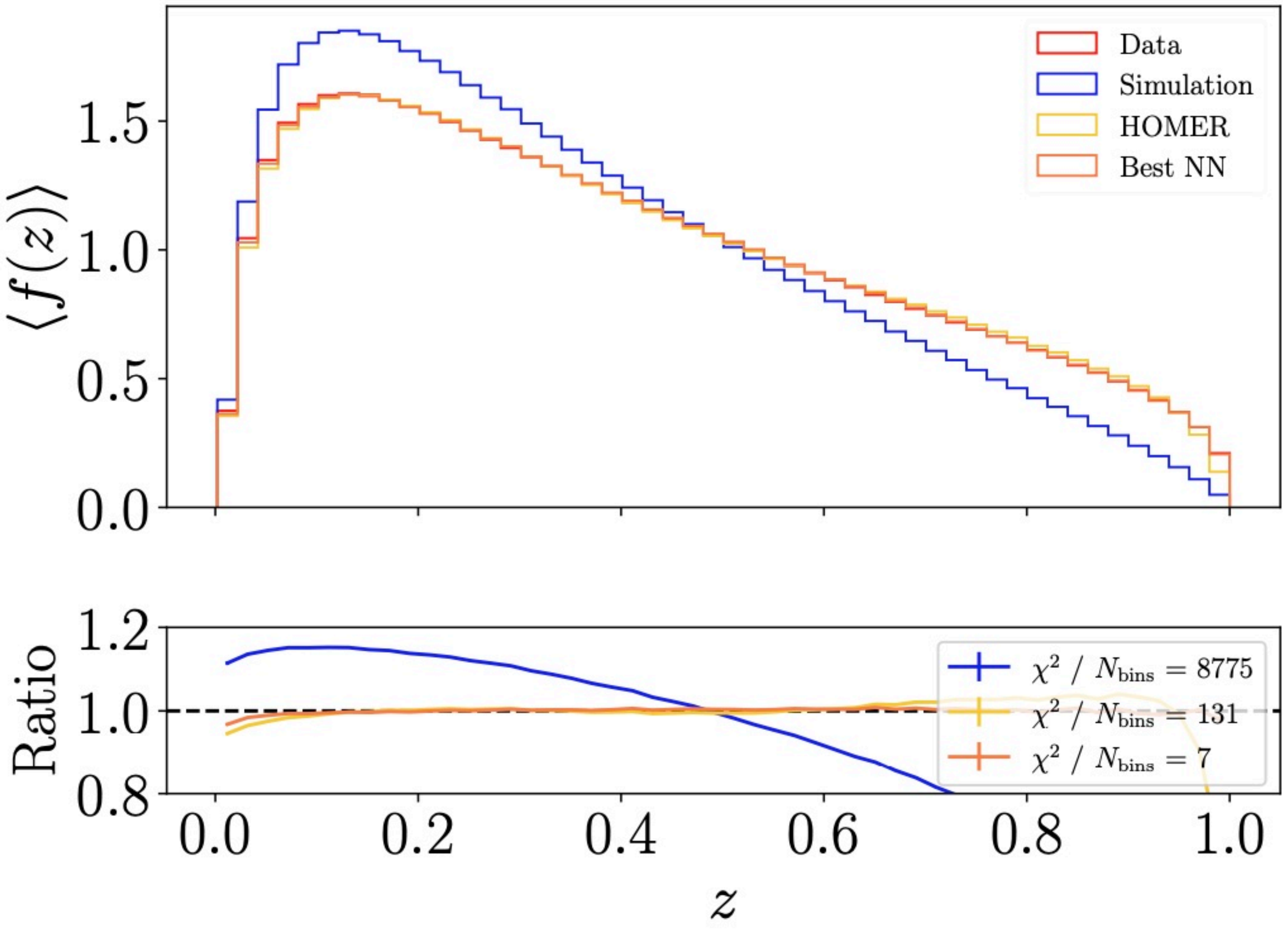
baryon-antibaryon pair example

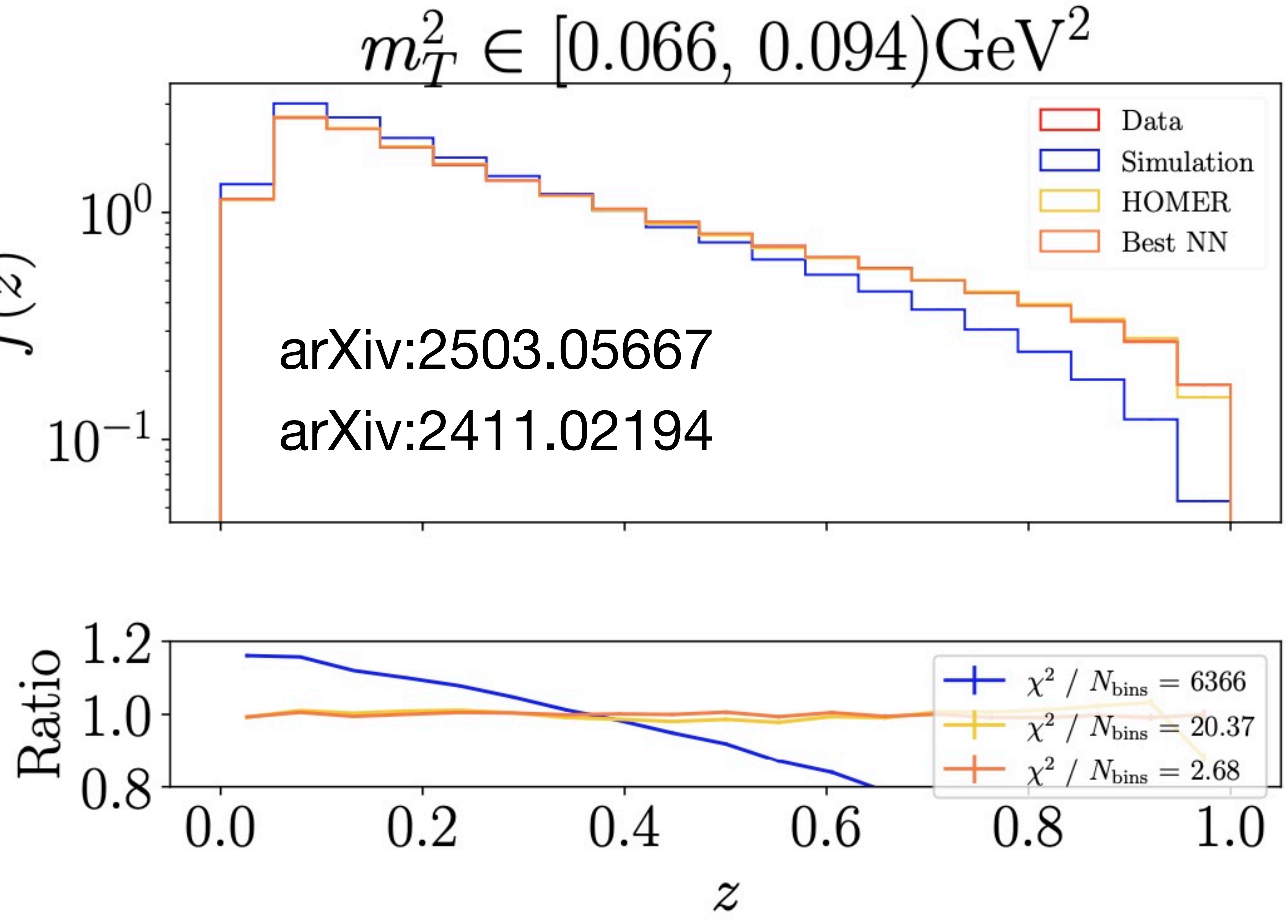


Lund String Model

$$f(z) \propto \frac{1}{z}(1-z)^a \exp\left(-\frac{bm_\perp^2}{z}\right)$$

message passing graph neural network extracting $f(z)$





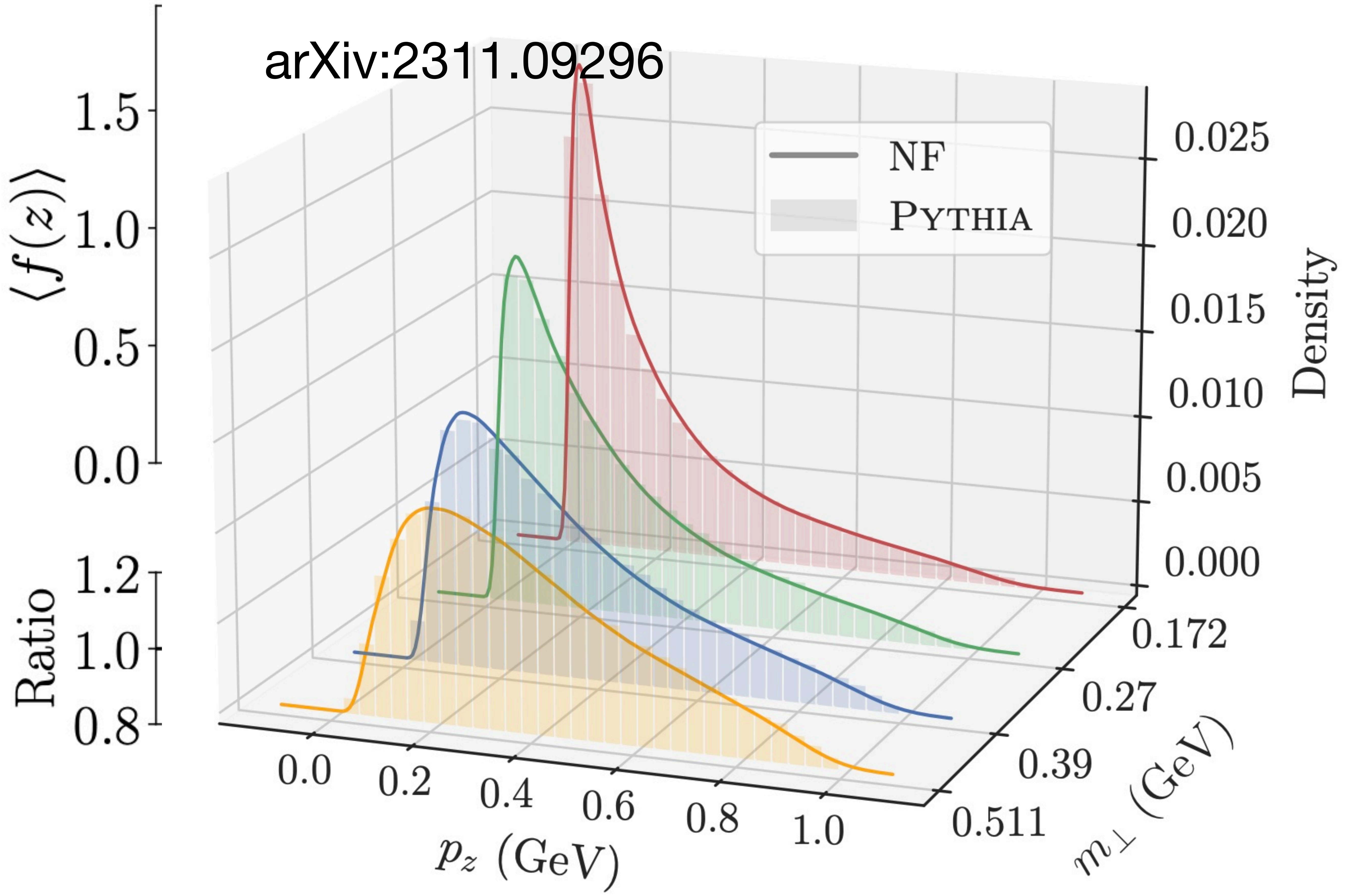
arXiv:2503.05667

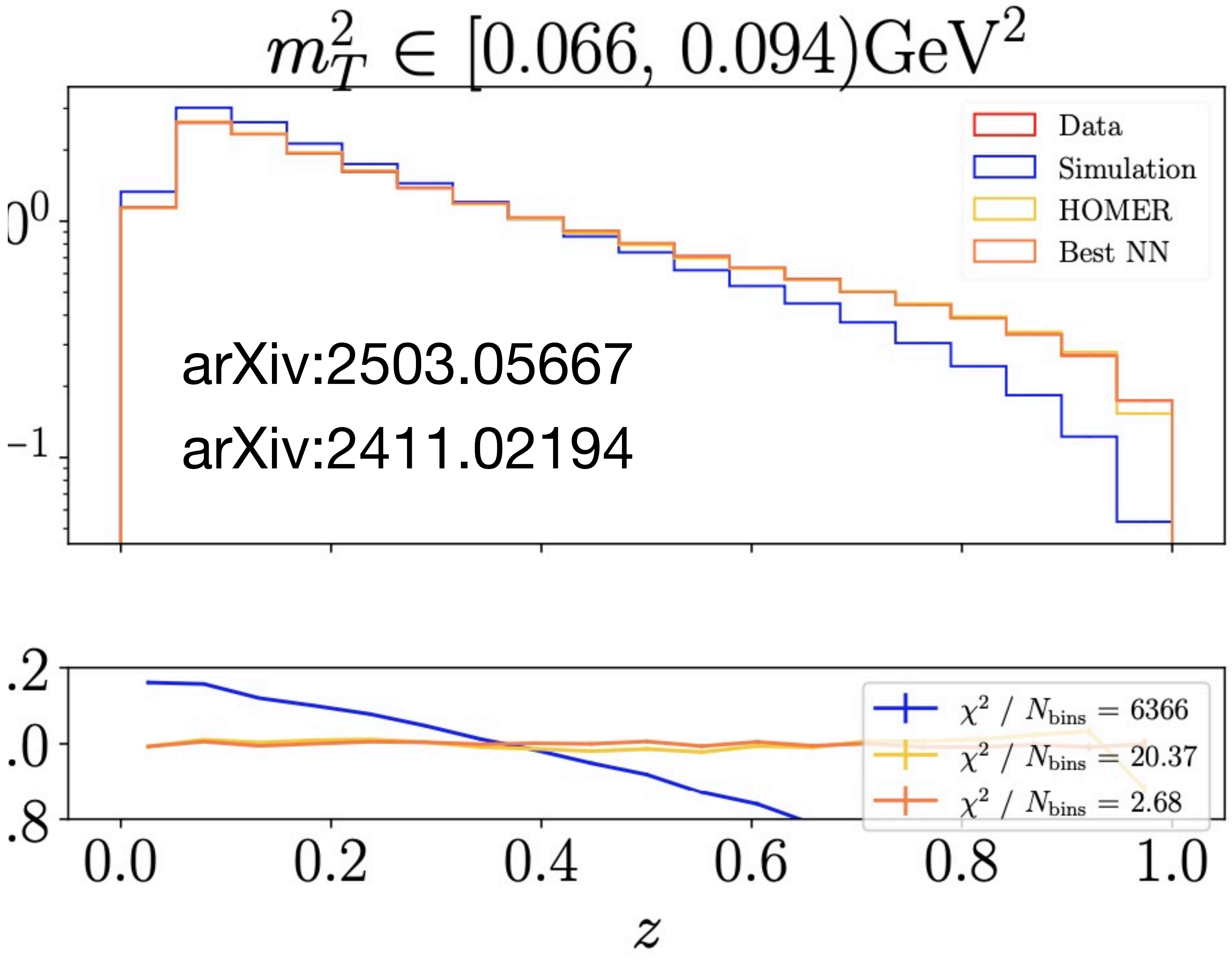
arXiv:2411.02194

Lund String Model

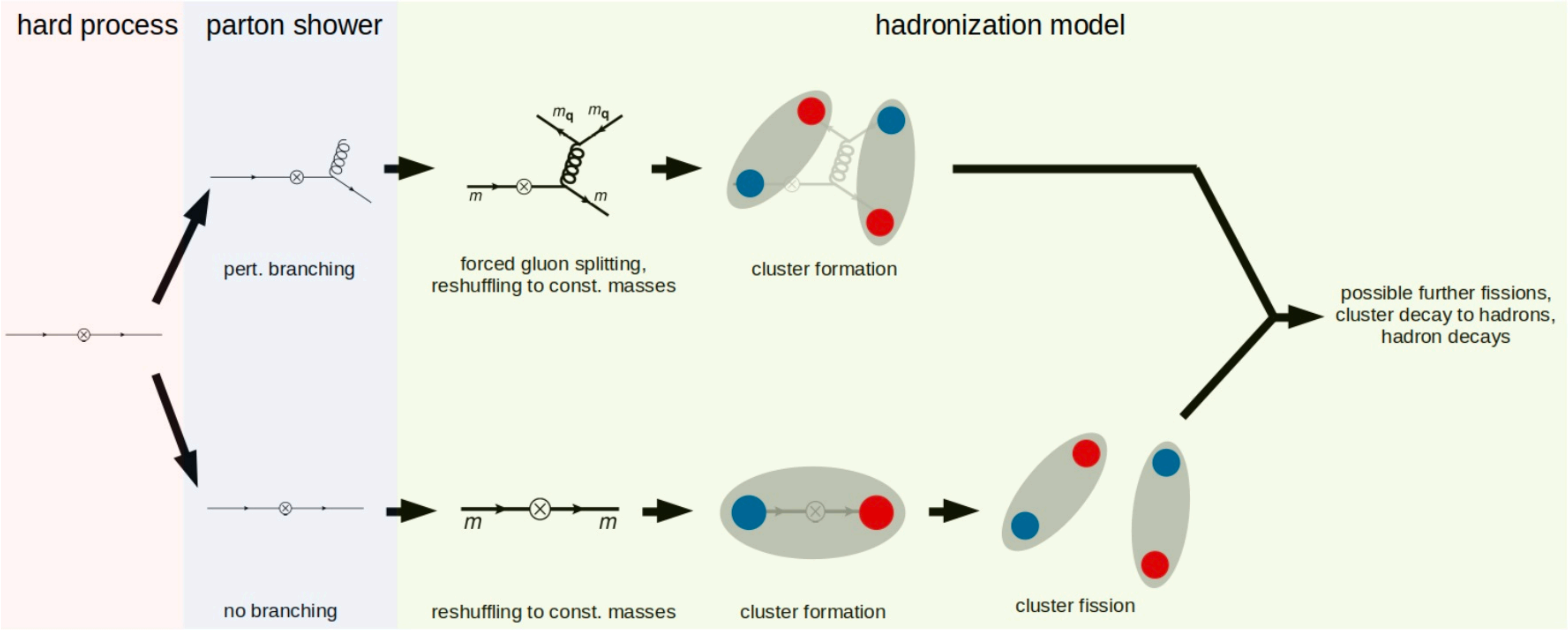
$$f(z) \propto \frac{1}{z}(1-z)^a \exp\left(-\frac{bm_\perp^2}{z}\right)$$

message passing graph neural network extracting $f(z)$

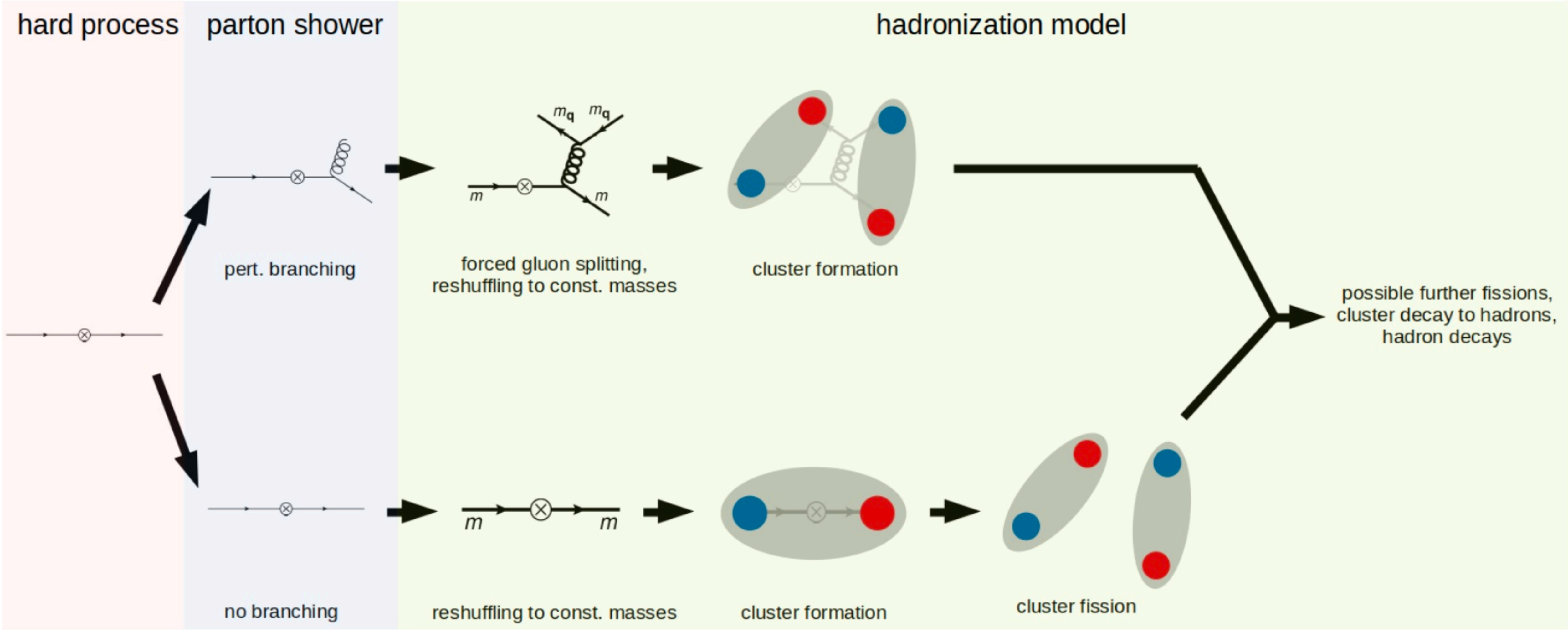




Cluster Hadronization Model



Cluster Hadronization Model



$$m_{u,d} \sim 300\ \mathrm{MeV}$$

$$m_s \sim 450\ \mathrm{MeV}$$

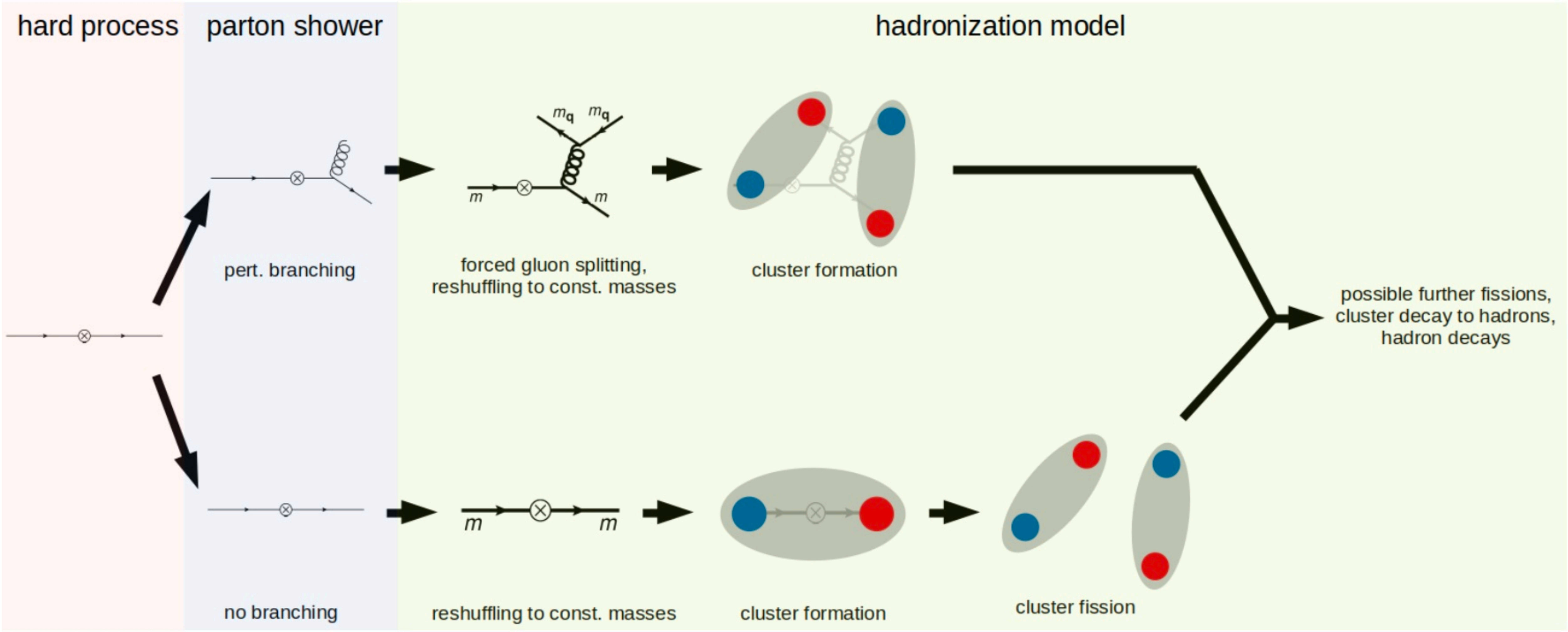
$$m_g \sim Q_0 \sim 1\ \mathrm{GeV}$$

massless quarks and gluons reshuffled to constituent quarks and massive gluons.

gluon is forced to split into a light quark-antiquark pair.

isotropic in the gluon rest frame, and the flavor of ight quark-antiquark pairs are assigned randomly

Cluster Hadronization Model



Heavy clusters

$$M^{\mathrm{Cl_{pow}}} \geq \boxed{\mathrm{Cl_{max}}^{\mathrm{Cl_{pow}}}} + (m_1 + m_2)^{\mathrm{Cl_{pow}}}$$

Cluster Fission

$$\frac{\mathrm{d}^2 P}{\mathrm{d}M_1 \mathrm{d}M_2} \propto \Theta\big(M_0 - M_1 - M_2\big) \times g(M_1) \times g(M_2) \qquad g(M_i) = \frac{\Theta\big(m_i + m_q < M_i < M_0 - m_q\big)}{M_0 - m_1 - m_q} \left[\frac{M_i - m_i}{M_0 - m_q - m_i}\right]^{\boxed{-1+\mathrm{PSplit}}}$$

$$P(a_{(q_1,\bar{q})}, b_{(q,\bar{q}_2)} | q_1, \bar{q}_2) = P_q \frac{1}{N_{(q_1,\bar{q})}} \frac{1}{N_{(q,\bar{q}_2)}} \frac{w_a}{w_{\max(q_1,\bar{q})}} \frac{w_b}{w_{\max(q,\bar{q}_2)}} \frac{s_a s_b p^*_{a,b}}{p^*_{\max}}$$

The decay products for hadrons with the relevant flavour of the cluster

The hadrons produced are then selected according to this weight

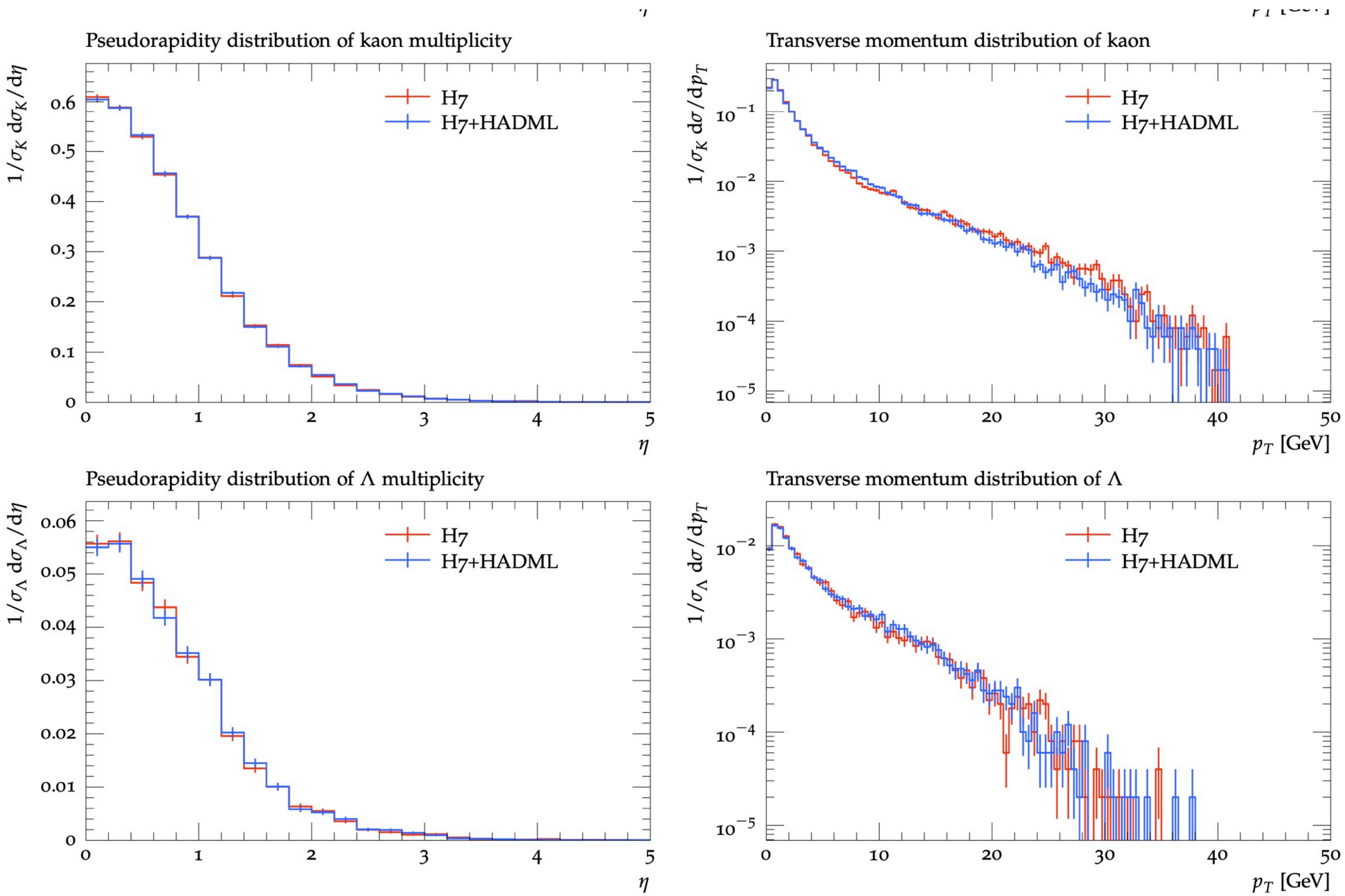
the basic cluster model does not include interaction between clusters,

Cluster Hadronization Model

Replace the part of the cluster decayer inside Herwig with a GAN

arXiv:2203.12660

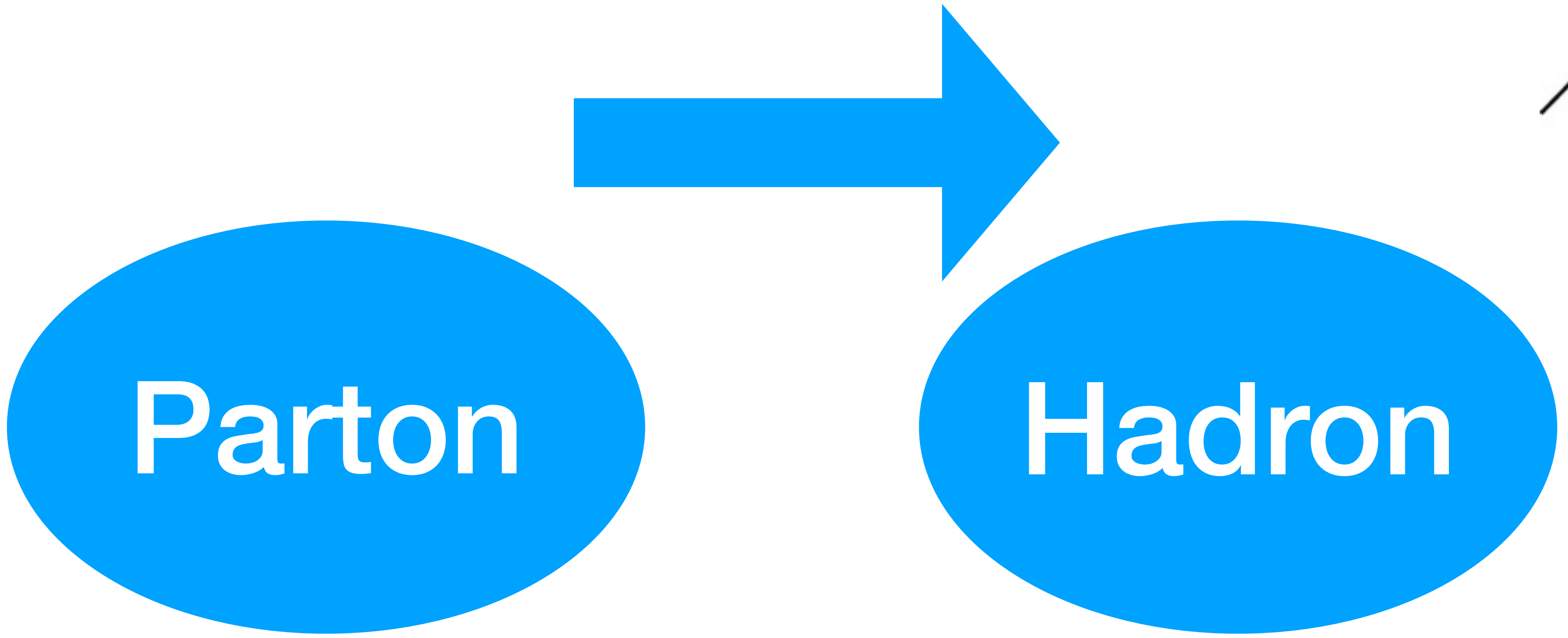
arXiv:2305.17169

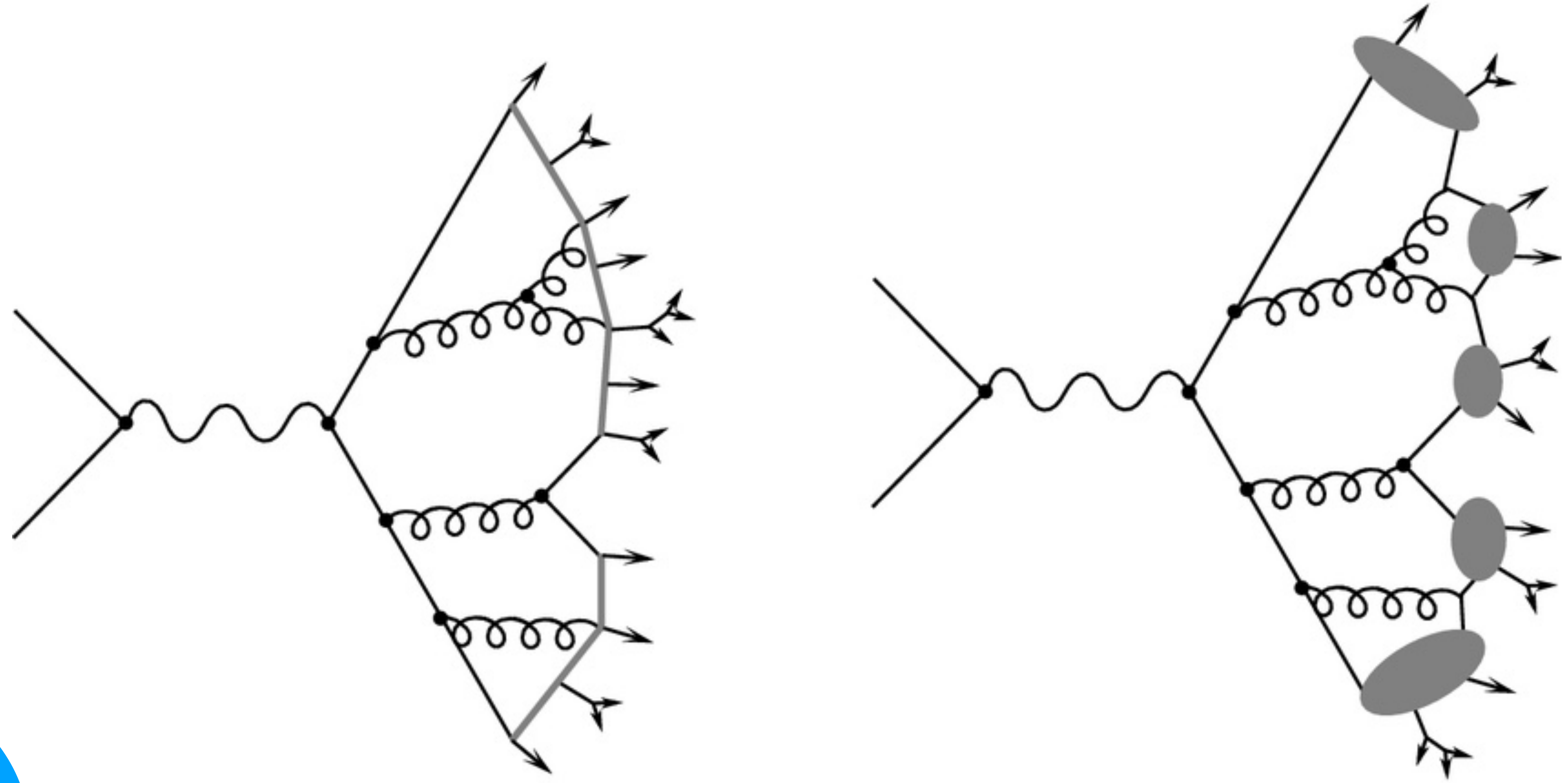


Conclusion

Hadronization Models in

Monte Carlo Event Generators





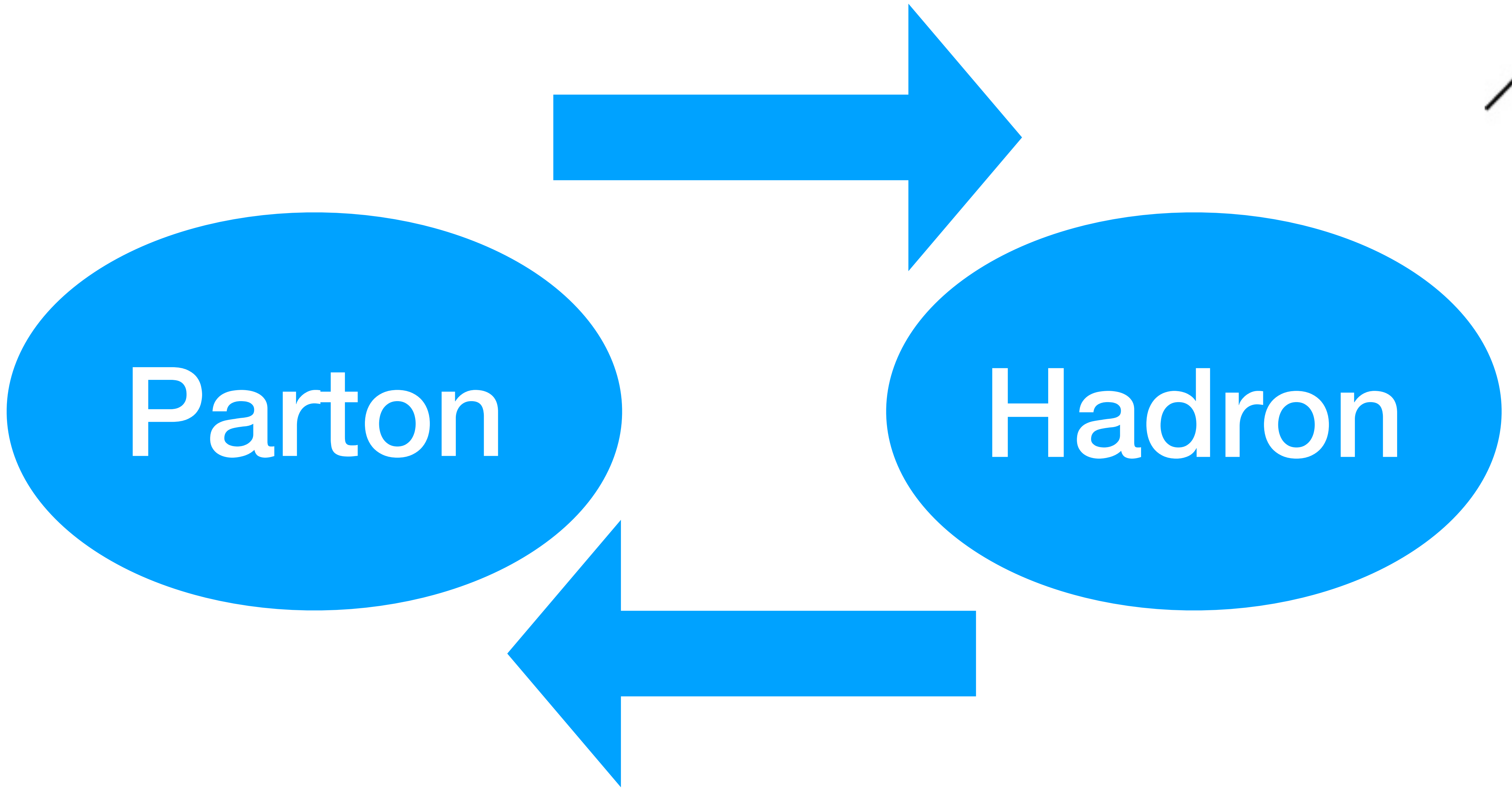
(a) String hadronization

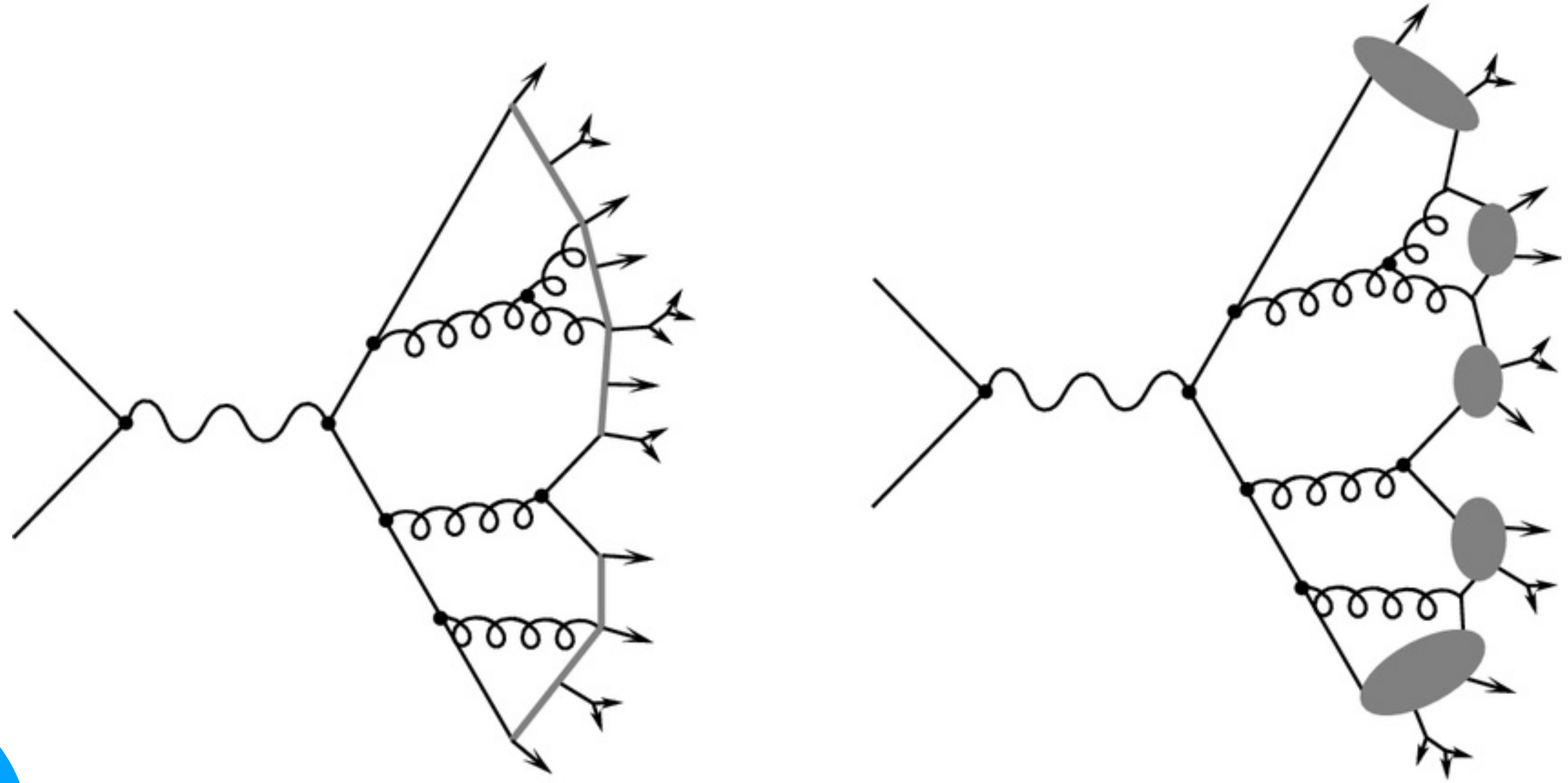
(b) Cluster hadronization

Conclusion

Hadronization Models in

Monte Carlo Event Generators





(a) String hadronization

(b) Cluster hadronization

微扰量子场论及其应用前沿讲习班

Jul 6 – 20, 2025
Asia/Shanghai timezone

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Contact

haitao.li@email.sdu.edu...
jian.wang@email.sdu.ed...

为帮助研究生系统了解粒子物理科研现状、前沿方向、未来发展等专业知识，"微扰量子场论及其应用"前沿讲习活动将于2025年7月6日至20日在山东大学中心校区举办。7月6日报到，7月20日离会。

讲习活动以"微扰量子场论和对撞机物理"为主要内容，由电弱规范理论、量子色动力学理论、对撞机唯象学、微扰量子场论方法、有效场论思想及应用、新物理研究方法等内容组成，同时暑期学校计划安排粒子物理宇宙学、引力波探测、高能宇宙射线等方向的专题讲座。

本次讲习活动主要面向研究生，不收取注册费。**请申请学员认真填写注册表格，并安排导师写一封推荐信，发给会务组，注册截止时间为4月30日。**申请截止后，暑期学校将根据申请人的学术背景和研究兴趣来确定最终名单。**为了保证教学效果，暑期学校限定学员总数为50人，要求全程参加，不接受请假。**

会务组为学员提供餐食，住宿和交通费用自理。如学员需要资助，可联系会务组，择优资助部分费用。

本次讲习活动由国家自然科学基金委理论物理专项、国家自然科学基金委创新群体项目资助。本次学术活动由山东大学主办。

会务组：陈龙，陈暄，蒋军，李海涛，李世渊，刘言锐，路鹏程，司宗国，王健，王耀光，吴群。

讲习班主页：https://indico.ihep.ac.cn/event/25318/

泉城-济南

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Thank You !!