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Determination of the Strong Coupling Constant α_s from Inclusive Semi-leptonic b and c Meson Decays

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See more details in:
https://arxiv.org/abs/2412.02480
https://iopscience.iop.org/article/10.1088/1674-1137/ad8baf

Opportunities and Ideas at the QCD Frontier, CCAST, 2025

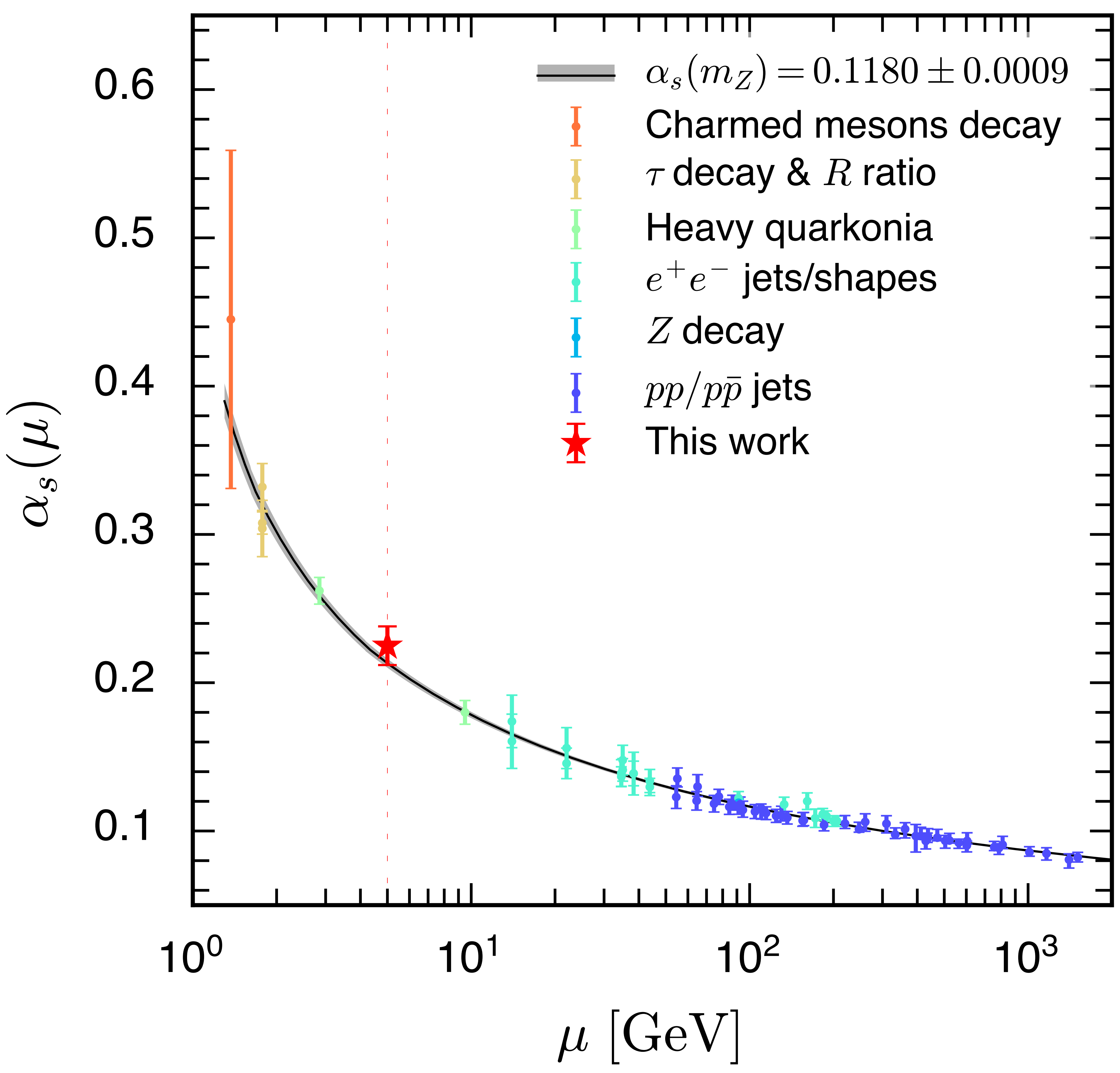
Motivation

- Strong coupling constant α_s is a fundamental parameter in the Standard Model (SM) and in Quantum Chromodynamics (QCD).
- The running behavior of α_s reflects the fundamental properties of asymptotic freedom and color confinement.
- The measurement of $\alpha_s(\mu)$ across the entire range of energy scale is crucial for understanding and testing of QCD.

A. Theoretical input in Higgs/EW sector,

B. Testing RGE,

C. Low-energy QCD, such as emergent hadron mass (EHM)



(Left) The α_s measurements at different energy scales.

Section No.
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Inclusive semi-leptonic B decay

- **Heavy Quark Expansion (HQE)**

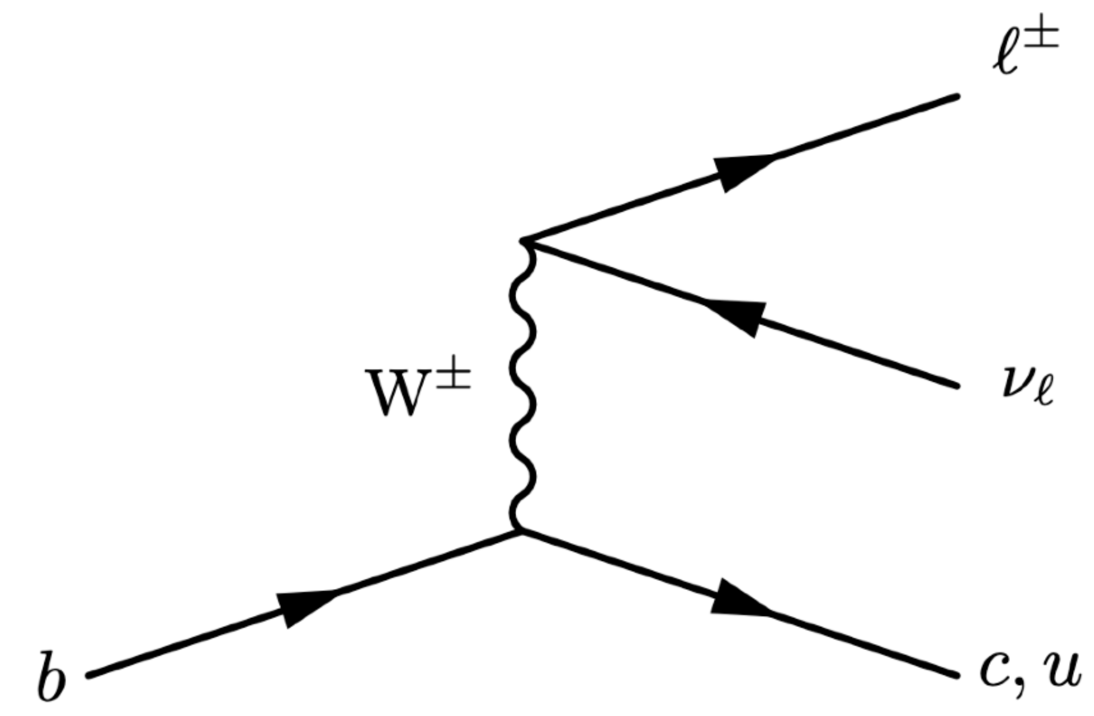
- $$\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right) = \Gamma_0 \left[C_0 - C_{\mu_\pi} \frac{\mu_\pi^2}{2m_b^2} + C_{\mu_G} \frac{\mu_G^2}{2m_b^2} + \ldots \right]$$

  - $$\Gamma_0 \equiv \frac{G_F^2 |V_{cb}|^2 m_b^5 A_{\text{ew}}}{192\pi^3}$$

  - C_0: perturbative correction, expanded as a series in α_s,

  $$C_0 = \mathbf{c}_0 + \mathbf{c}_1 \frac{\alpha_s}{\pi} + \mathbf{c}_2 \left(\frac{\alpha_s}{\pi}\right)^2 + \mathbf{c}_3 \left(\frac{\alpha_s}{\pi}\right)^3 + \mathcal{O}(\alpha_s^4)$$

  - $-C_{\mu_\pi} \frac{\mu_\pi^2}{2m_b^2} + C_{\mu_G} \frac{\mu_G^2}{2m_b^2} + \ldots$: higher-order power corrections, including non-perturbative parameters



- Under HQE, $B \to X_c \ell \bar{\nu}_\ell$ was used to fit:[1,2]
  - CKM elements: $|V_{cb}|$
  - Quark masses: m_b, m_c
  - HQE parameters: μ_π^2, μ_G^2, etc.

  with α_s fixed at the world average.

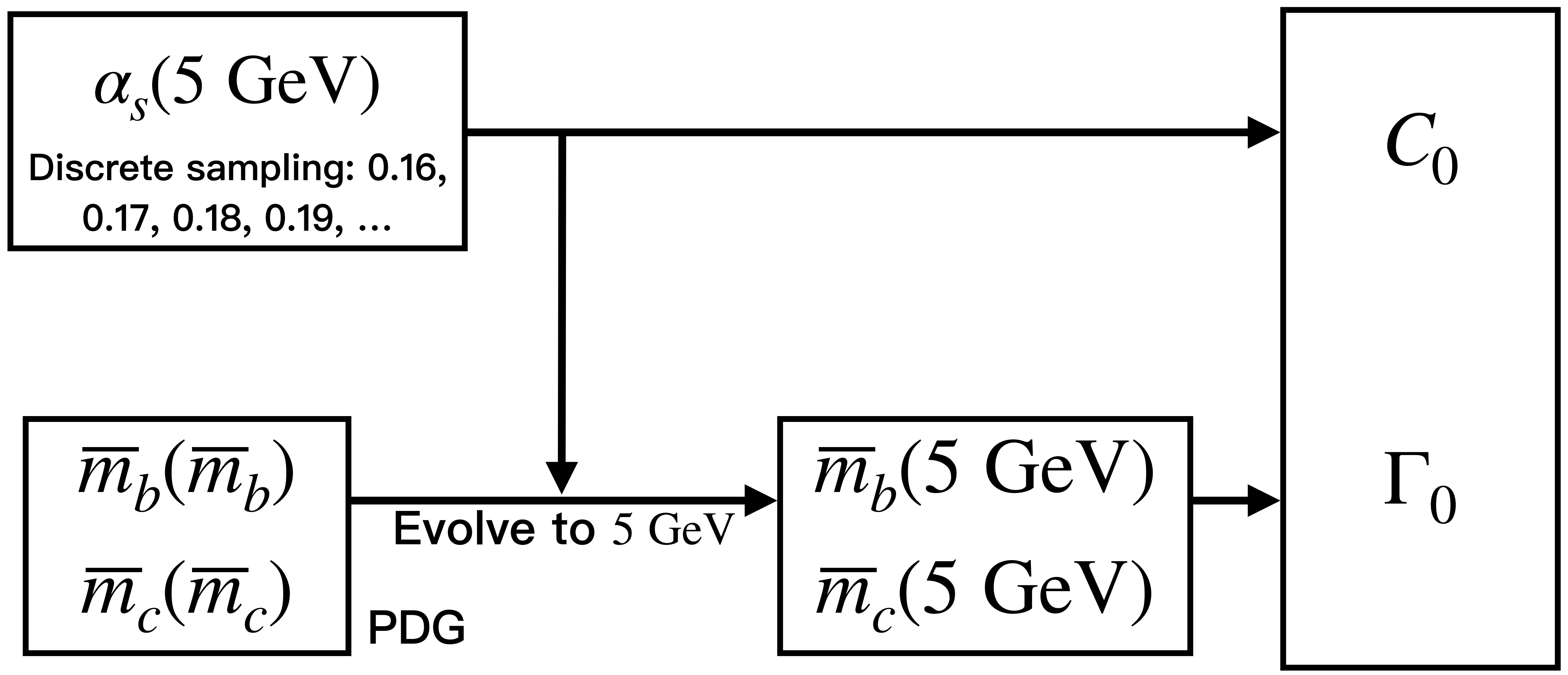
- ***Is it possible to extract α_s from inclusive $B \to X_c \ell \bar{\nu}_\ell$***
  1. constrain other parameters using independent determinations,
  2. *or perform simultaneously fit?*

[1] 10.1103/PhysRevD.81.032003
[2] 10.1016/j.physletb.2021.136679

- $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right) = \Gamma_0 \left[C_0 + C_{\mu_\pi} \frac{\mu_\pi^2}{2m_b^2} + C_{\mu_G} \frac{\mu_G^2}{2m_b^2} + \ldots \right]$

- $C_0 = \mathbf{c}_0 + \mathbf{c}_1 \frac{\alpha_s}{\pi} + \mathbf{c}_2 \left(\frac{\alpha_s}{\pi}\right)^2 + \mathbf{c}_3 \left(\frac{\alpha_s}{\pi}\right)^3 + \mathcal{O}(\alpha_s^4)$
  - $\mathbf{c}_i$: calculated to 4th order, depending on m_b, m_c.[1,2]

- **Reform C_0:**
  - in $\overline{\text{MS}}$ scheme (transformed from the calculation in OS scheme.)
  - at scale $\mu = 5$ GeV
  - as a function of $\overline{m}_b(5\text{ GeV})$, $\overline{m}_c(5\text{ GeV})$ & $\alpha_s(5\text{ GeV})$



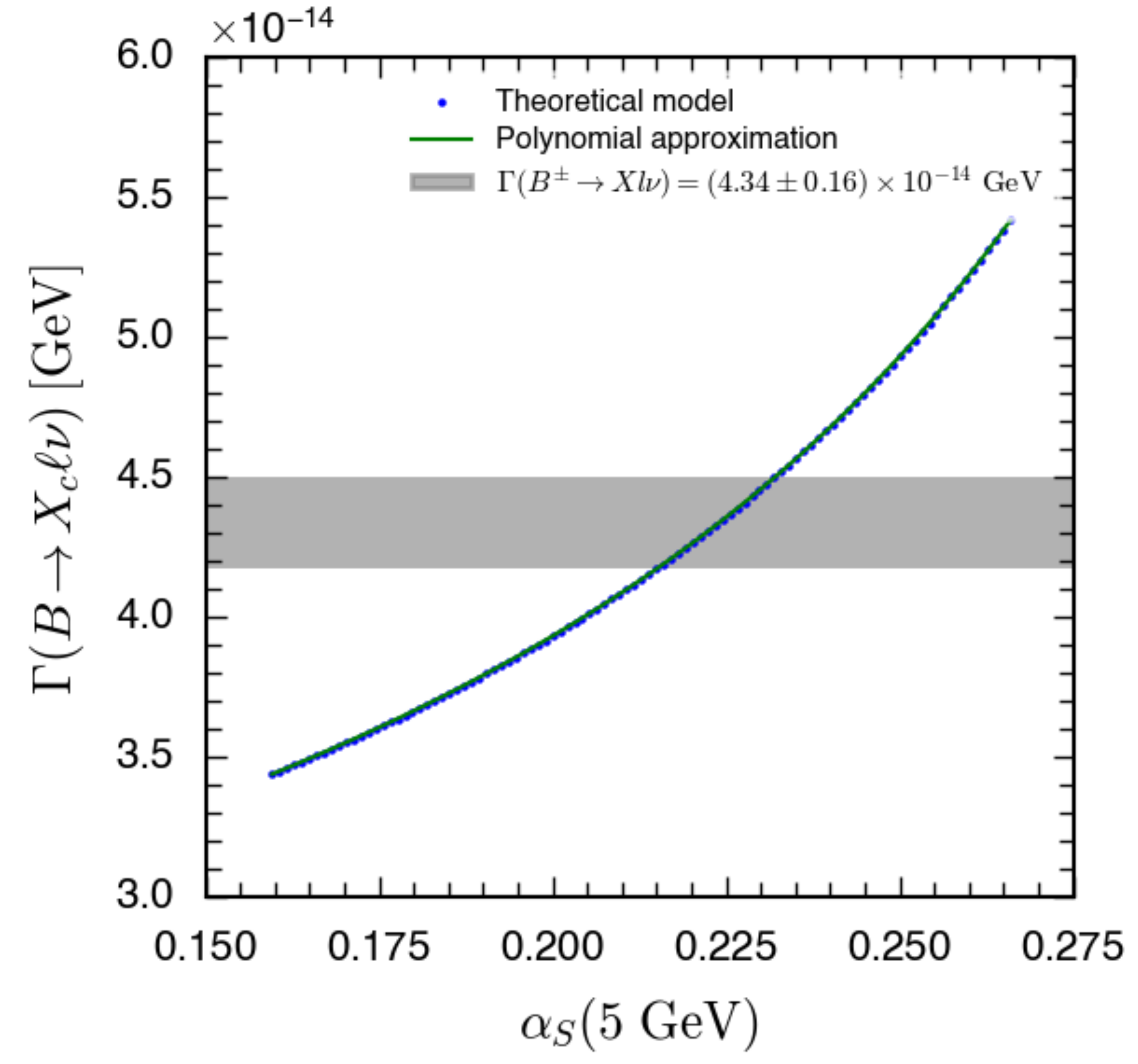
- **External input (fixed parameters):**
  - $\overline{m}_b(\overline{m}_b)$, $\overline{m}_c(\overline{m}_c)$ *at PDG world averages*
- **Floating variable:**
  - α_s *in perturbative expansion +* m_b, m_c *evolution.*

[1] https://doi.org/10.1103/PhysRevD.78.114015
[2] https://doi.org/10.1103/PhysRevD.104.016003

Higher order power correction

- $$\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right) = \Gamma_0 \left[C_0 - C_{\mu_\pi} \frac{\mu_\pi^2}{2m_b^2} + C_{\mu_G} \frac{\mu_G^2}{2m_b^2} + \ldots \right]$$

- The numerical calculations for the coefficients C_{μ_π} and C_{μ_G} have been provided in the **kinetic scheme**, [1]
  - $$C_{\mu_\pi} = 2\mathbf{c}_0 \left(\frac{1}{2} - 0.99\frac{\alpha_s}{\pi}\right),\ C_{\mu_G} = -2\mathbf{c}_0 \left(1.94 + 3.46\frac{\alpha_s}{\pi}\right)$$
- The non-perturbative parameters: [2]
  - $\mu_\pi^2 = 0.477 \pm 0.056\,\mathrm{GeV}^2,\ \mu_G^2 = 0.306 \pm 0.050\,\mathrm{GeV}^2,\ m_b^{\mathrm{kin}} = 4.573 \pm 0.012\,\mathrm{GeV}$
- The higher order power corrections are estimated around 4%, with C_{μ_π} and C_{μ_G} estimated using their first terms.
- **Truncation error:** sub-leading terms of C_{μ_π} and C_{μ_G}, ~ 0.2%
- **Error from parameters μ_π, μ_G, m_b^{kin}:** ~ 0.5%



The numerical function of $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$ versus $\alpha_s(5\ \mathrm{GeV})$, compared with $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right) = (4.34 \pm 0.16) \times 10^{-14}\ \mathrm{GeV}$. The numerical function is parameterized by a polynomial function in the $\alpha_s(5\ \mathrm{GeV})$ range from 0.16 to 0.26.

[1] https://doi.org/10.1007/JHEP01(2014)147
[2] https://doi.org/10.1016/j.physletb.2021.136679

Parameters and result

- **External parameters:**
  - **CKM element $|V_{cb}|$:**
    - derived from exclusive B decays with form factor calculated from lattice QCD.[1]
  - **Quark masses $\overline{m}_b(\overline{m}_b)$, $\overline{m}_c(\overline{m}_c)$:**
    - Fixed at the world averages.
    - Evolve to $\overline{m}_b(5\ \text{GeV})$, $\overline{m}_c(5\ \text{GeV})$ with varying $\alpha_s(5\ \text{GeV})$
  - **HQE parameters μ_π^2, μ_G^2, m_b^{kin}:**
    - Fixed at the values derived from spectral moments of inclusive semi-leptonic B decay.

Parameter	Notation	Value & error	Note
Fermi coupling constant	G_F	$1.16637886 \times 10^{-5}$ GeV^{-2}	Workman et al. (2022)
Electroweak correction factor	A_{ew}	1.014	Sirlin (1982)
CKM matrix element	$\|V_{cb}\|$	0.0410 ± 0.0007	Prim et al. (2023)
b-quark mass in $\overline{\text{MS}}$	$\overline{m}_b(\overline{m}_b)$	$4.18^{+0.03}_{-0.02}$ GeV	Workman et al. (2022)
c-quark mass in $\overline{\text{MS}}$	$\overline{m}_c(\overline{m}_c)$	1.27 ± 0.02 GeV	Workman et al. (2022)
HQE parameters	μ_π^2	0.477 ± 0.056 GeV^2	Bordone et al. (2021)
	μ_G^2	0.306 ± 0.050 GeV^2	Bordone et al. (2021)
b-quark mass in kinetic scheme	m_b^{kin}	4.573 ± 0.012 GeV	Bordone et al. (2021)

- **Decay width $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$**: derived from experimental measurements of the life time and branching ratio:
  - $\tau_{B^\pm} = 1.638 \pm 0.004\ \text{ps}, \quad \mathcal{B}(B^\pm \to X_c \ell \bar{\nu}) = 10.8 \pm 0.4\,\%$,

    $\Longrightarrow \Gamma(B^\pm \to X_c \ell \bar{\nu}_\ell) = (4.34 \pm 0.16) \times 10^{-14}\ \text{GeV}$
  - $\tau_{B^0} = 1.517 \pm 0.004\ \text{ps}, \quad \mathcal{B}(B^0 \to X_c \ell \bar{\nu}) = 10.1 \pm 0.4\,\%$,

    $\Longrightarrow \Gamma(B^0 \to X_c \ell \bar{\nu}_\ell) = (4.38 \pm 0.17)) \times 10^{-14}\ \text{GeV}$
- Perform a fit of $\alpha_s(5\ \text{GeV})$ to $\Gamma(B \to X_c \ell \bar{\nu}_\ell)$,
  - $\alpha_s(5\,\text{GeV}) = 0.224 \pm 0.017$
  - $\alpha_s(5\,\text{GeV}) = 0.226 \pm 0.017$
- **Combined result:** $\alpha_s(5\,\text{GeV}) = 0.225 \pm 0.012$
- **Evolve to m_Z:** $\alpha_s(m_Z) = 0.121 \pm 0.003$ [2]

[1] arXiv:2310.20286v1
[2] The $\alpha_s(\mu)$ evolution is performed using RunDec: 10.1016/S0010-4655(00)00155-7

Uncertainties

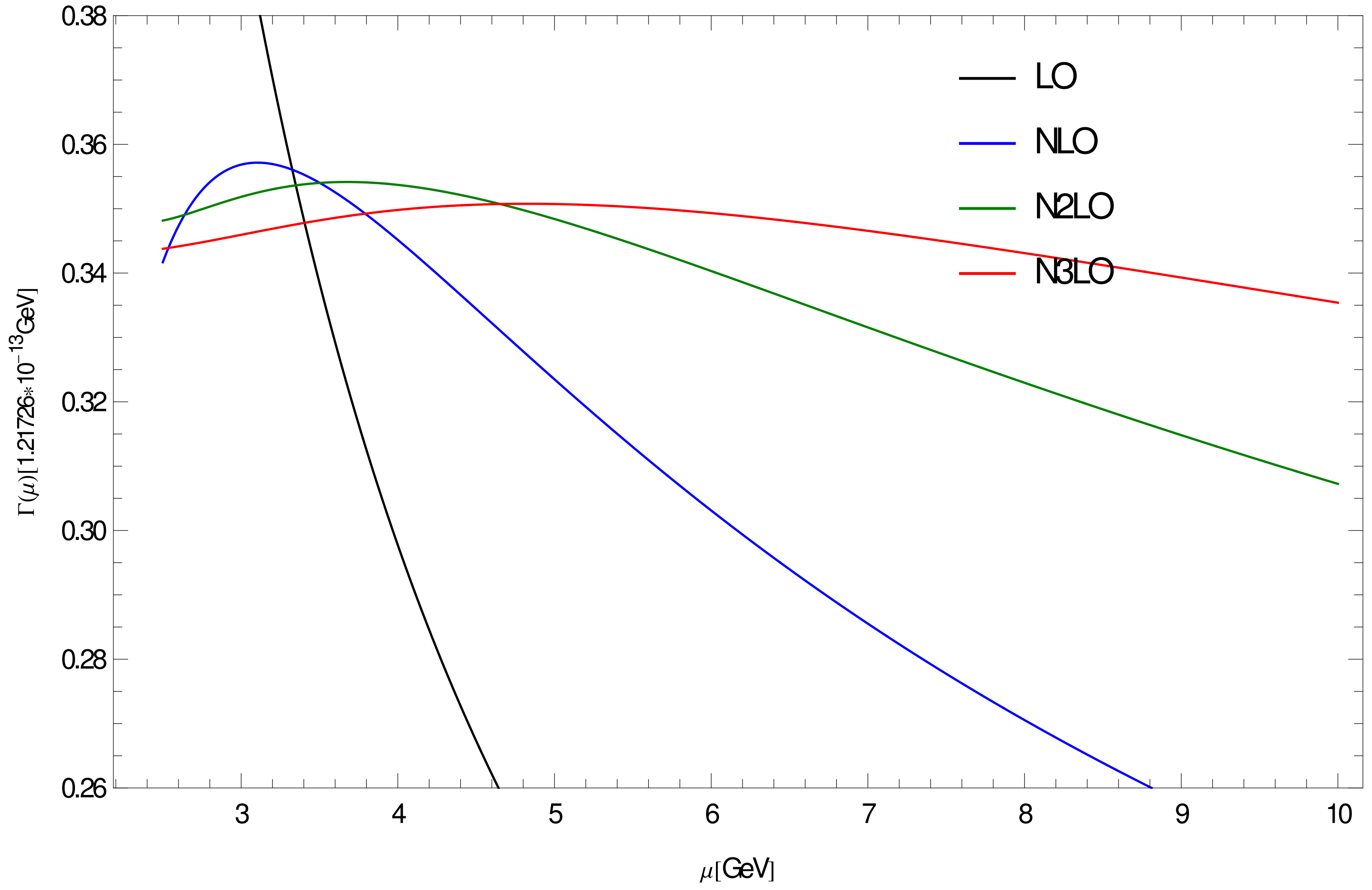
- The **uncertainty from $|V_{cb}|$, B life-time and branching ratios:** error propagation.

- The errors induced by the uncertainties on the **input values of $\overline{m}_b(\overline{m}_b)$ and $\overline{m}_c(\overline{m}_c)$**:
  - Estimated by floating them within their errors and taking the largest Γ_{sl} deviations.

- The uncertainty due to the **remnant renormalization scale** dependence:
  - estimated by varying the RG-scale μ from 2.5 GeV to 10 GeV, and taking the largest deviations conservatively ($\pm 4.4\ \%$).

- The uncertainty of **non-perturbative** terms:
  - Truncation error: sub-leading terms of C_{μ_π} and C_{μ_G}, ~ 0.2%
  - Error from HQE parameters μ_π, μ_G, m_b^{kin}: ~ 0.5%

	Γ_{sl} prediction [%]	$\alpha_s(5\,\text{GeV})$ [%]
$\lvert V_{cb}\rvert = 0.0410 \pm 0.0007$	3.4 (1.4)	3.1 (1.3)
$\overline{m}_b(\overline{m}_b) = 4.18^{+0.03}_{-0.02}$ GeV	3.0 (1.1)	2.7 (1.0)
$\overline{m}_c(\overline{m}_c) = 1.27 \pm 0.02$ GeV	2.1 (1.4)	1.8 (1.2)
R-scale $\mu = 5^{+5}_{-2.5}$ GeV	4.4 (2.2)	4.0 (2.0)
High-order power corrections	0.5	0.5
$\tau_{B^\pm} = 1.638 \pm 0.004$ ps	-	0.2
$\mathcal{B}(B^\pm \to X_c \ell \nu) = 10.8 \pm 0.4$ %	-	2.4 (1.8)
Sum	6.7(3.2)	6.5 (3.4)



The scale dependence of the fixed-order results in $\overline{\text{MS}}$ for $\Gamma(b \to c\ell\overline{\nu}_\ell)$ in $\mu \in [2.5, 10]$ GeV, using on-shell results in Ref.[1-2]

[1] https://doi.org/10.1103/PhysRevD.78.114015
[2] https://doi.org/10.1103/PhysRevD.104.016003

- **Remnant renormalization scale** dependence:
  - Based on the scaling behavior of perturbative uncertainty from $\mathcal{O}(\alpha_s^2)$ to $\mathcal{O}(\alpha_s^3)$, we expect the next order result may halve the uncertainty.
- **CKM element** $|V_{cb}|$:
  - is expected to achieve the accuracy around 0.7% on the future e^+e^- collider: W boson decays.
- **Branching ratio** $\mathscr{B}(B \to X_c \ell \bar{\nu}_\ell)$:
  - accuracy around 0.3% is expected with $50\ \mathrm{ab}^{-1}$ Belle II data, assuming systematic error remains at the same level.
- **Quark masses**:
  - will be improved by lattice QCD.
- **Taking into these advantages, $\Delta\alpha_s(m_Z) \sim 0.0018$ is anticipated, become competitive with other methods.**

	Γ_{sl} prediction [%]	$\alpha_s(5\,\mathrm{GeV})$ [%]
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$\mathcal{B}(B^\pm \to X_c \ell \nu) = 10.8 \pm 0.4$ %	-	2.4 (1.8)
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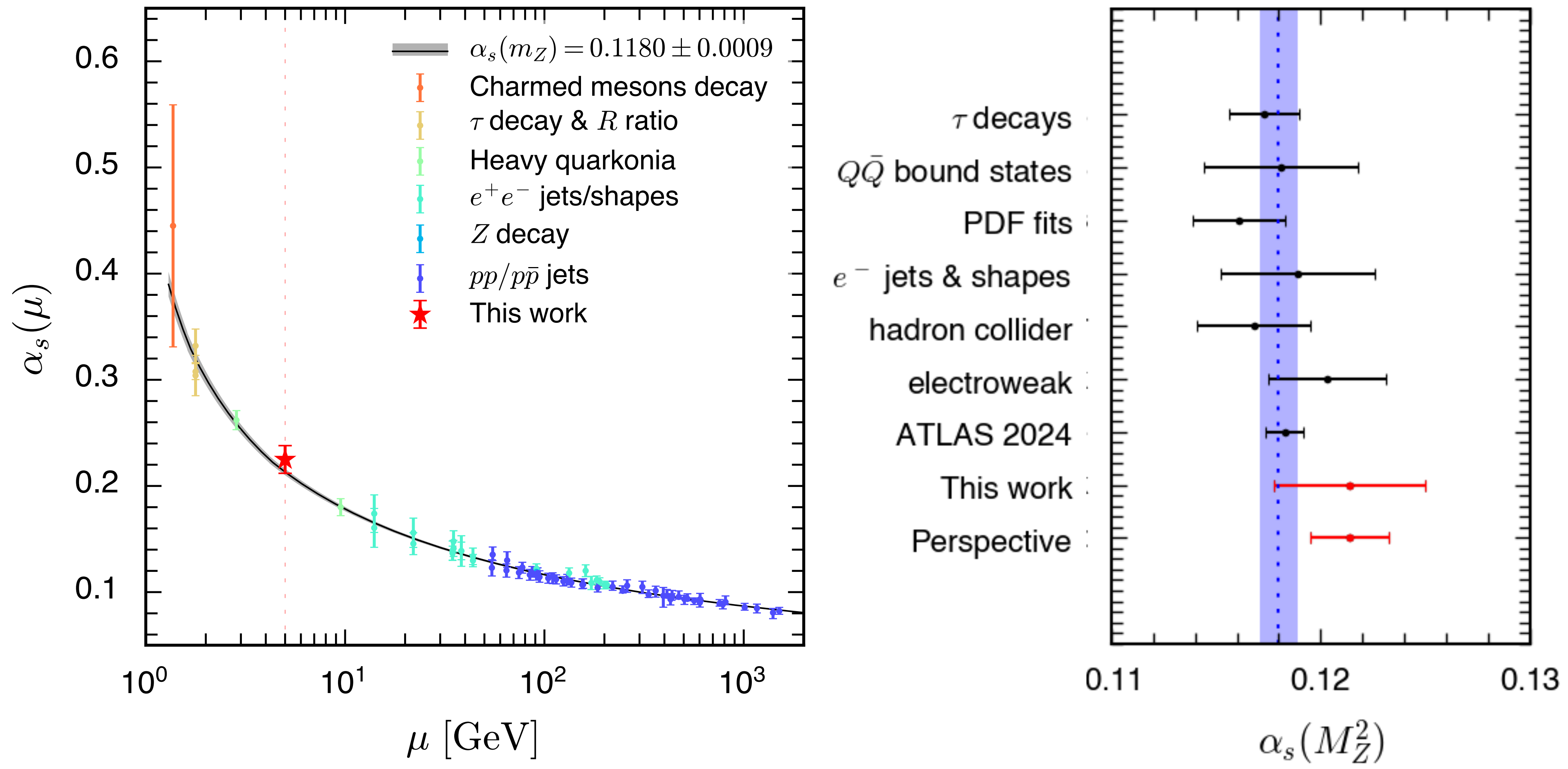


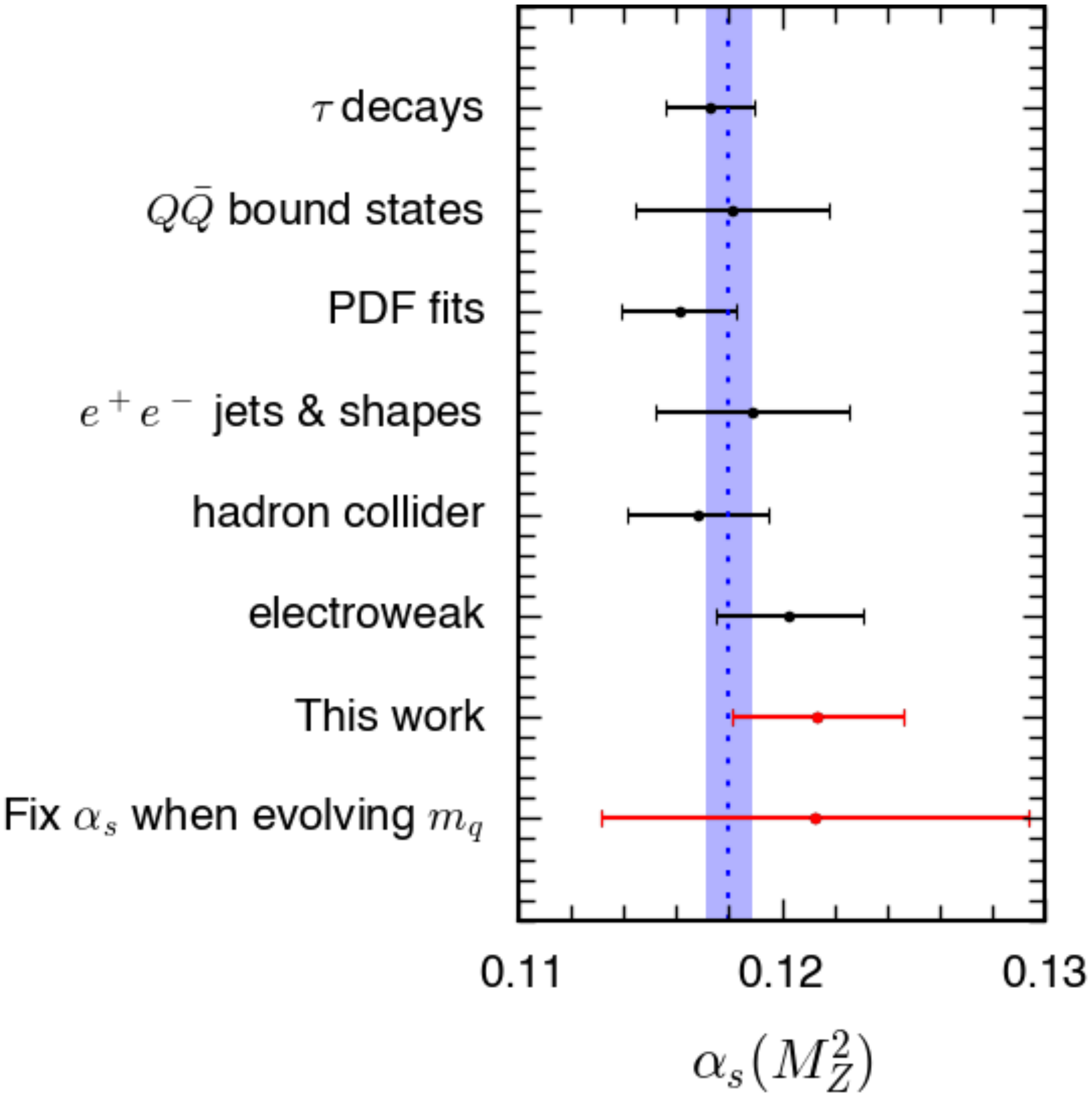
(Left) The α_s measurements at different energy scales.

(Right) The comparison of the $\alpha_s(m_Z)$ pre-averages from six experimental sources in PDG and the extrapolation value of this work. The α_s evolving is performed using the RunDec package.[1]

[1] https://arxiv.org/abs/hep-ph/0004189

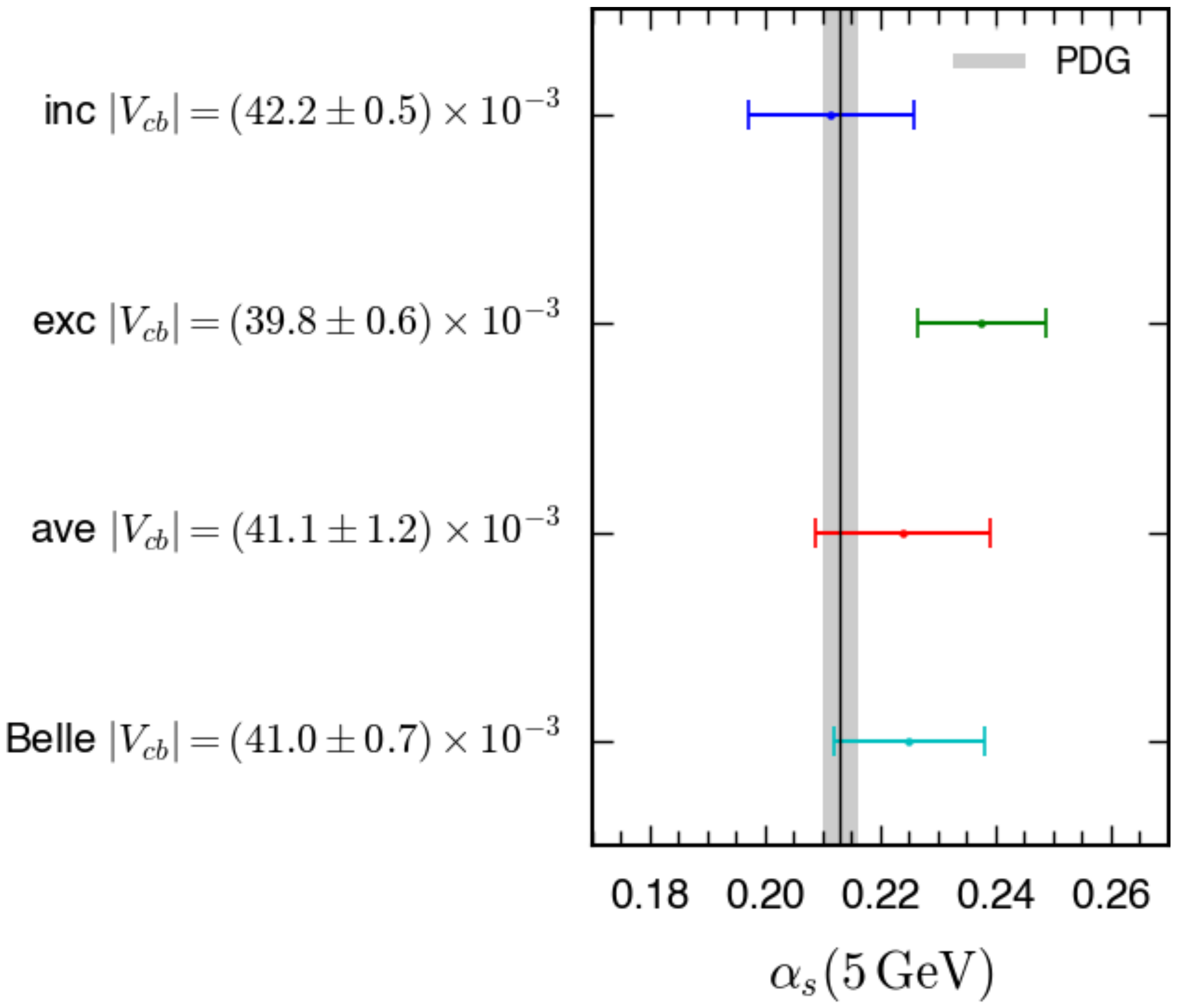
Discussion: α_s in running quark mass

- The α_s value used in the evolution of the quark masses from $\overline{m}_b(\overline{m}_b)$, $\overline{m}_c(\overline{m}_c)$ to $\overline{m}_b(5\ \mathrm{GeV})$, $\overline{m}_c(5\ \mathrm{GeV})$ is floating.
  - This treatment enlarges the sensitivity of the $\overline{\mathrm{MS}}$ prediction on α_s.
  - ***Does it make sense to obtain α_s sensitivity from quark masses?***
- The external $\overline{m}_b(\overline{m}_b)$, $\overline{m}_c(\overline{m}_c)$, from PDG averages, depend on the assumptions of the perturbative α_s.
  - ***Global fit of quark masses & α_s using more observables, such as spectral moments of $B \to X_c \ell \overline{\nu}_\ell$ and masses of B mesons, etc.***



Discussion: impact from $|V_{cb}|$ puzzle

- This method relies on an external input of $|V_{cb}|$ from the exclusive B meson decays.
  - However, the present exclusive $|V_{cb}|$ keeps a tension with the inclusive $|V_{cb}|$.
  - The distance between the inclusive $|V_{cb}|$ and exclusive $|V_{cb}|$ corresponds to a relative uncertainty of 14%.
  - ***Fit α_s, $|V_{cb}|$ and HQE parameters from moments of inclusive B decays?***



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Theoretical predictions

The Γ_{SL} has been calculated using heavy quark expansion(HQE), which is expand as a series of $\alpha_s(m_c^2)$ and r (the ratio of m_s^2/m_c^2).

Nucl.Phys.B 840 (2010) 424-437

$$\Gamma_{SL} = \frac{G_F^2 m_c^5}{192\pi^3} \times |V_{cs}|^2 \times [f_0(r) + \frac{\alpha_S}{\pi} f_1(r) + \frac{\alpha_S^2}{\pi^2} f_2(r) + \frac{\mu_\pi^2}{m_c^2} f_\pi(r) + \frac{\mu_G^2}{m_c^2} f_G(r) + \frac{\rho_{LS}^3}{m_c^3} f_{LS}(r) + \frac{\rho_D^3}{m_c^3} f_D(r) + \frac{32\pi^2}{m_c^3} B_{WA}]$$

- Strong dependence of m_c
- Can extract the $|V_{cs}|$

- $\alpha_S = \alpha_S(m_c^2)$, **can be extracted from** Γ_{SL}

- $\mu^2_{\pi,\ G}$:the kinetic and chromo-magnetic dimension-five operators.
- $\rho^3_{LS,\ D}$: Darwin and spin-orbital (LS) dimension-six operators.
- B_{WA}: weak annihilation (WA).

- We had tried to extract the value of α_S by fitting the Γ_{SL} with the experimental and theoretical results.

Extraction of α_S

A χ^2 minimization method is employed to determine $\alpha_s(m_c^2)$ from the Γ_{SL},

$$\chi^2(\alpha_s, \theta_j) = \sum_i \frac{[\Gamma_{SL,D_i} - \hat{\Gamma}_{SL}(\alpha_s, \theta_j)]^2}{\sigma^2_{\Gamma_{SL,D_i}}} + \sum_j \frac{(\theta_j - \theta'_j)^2}{\sigma^2_{\theta'_j}}$$

- $\Gamma_{SL,\ D_i}$: measured Γ_{SL} for D_i meson.
- $\sigma_{\Gamma_{SL,\ D_i}}$: uncertainty of Γ_{SL} for D_i meson.
- $\hat{\Gamma}_{SL,\ D_i}$: predicational Γ_{SL} for D_i meson.
- θ'_j and $\sigma_{\theta'_j}$: the value and uncertainty of constrained parameters in the fit.

- The uncertainty caused by theoretical prediction is estimated by varying $\hat{\Gamma}_{SL}$ with 10% uncertainty, (might be a conservative estimation)
  - **High order perturbative corrections,** need **more precise calculation**.
  - Miss **Cabibbo suppressed processes,** need **more measurements**.

Extraction of α_S

The parameters involved in the Γ_{SL} of charmed mesons are listed in the table,

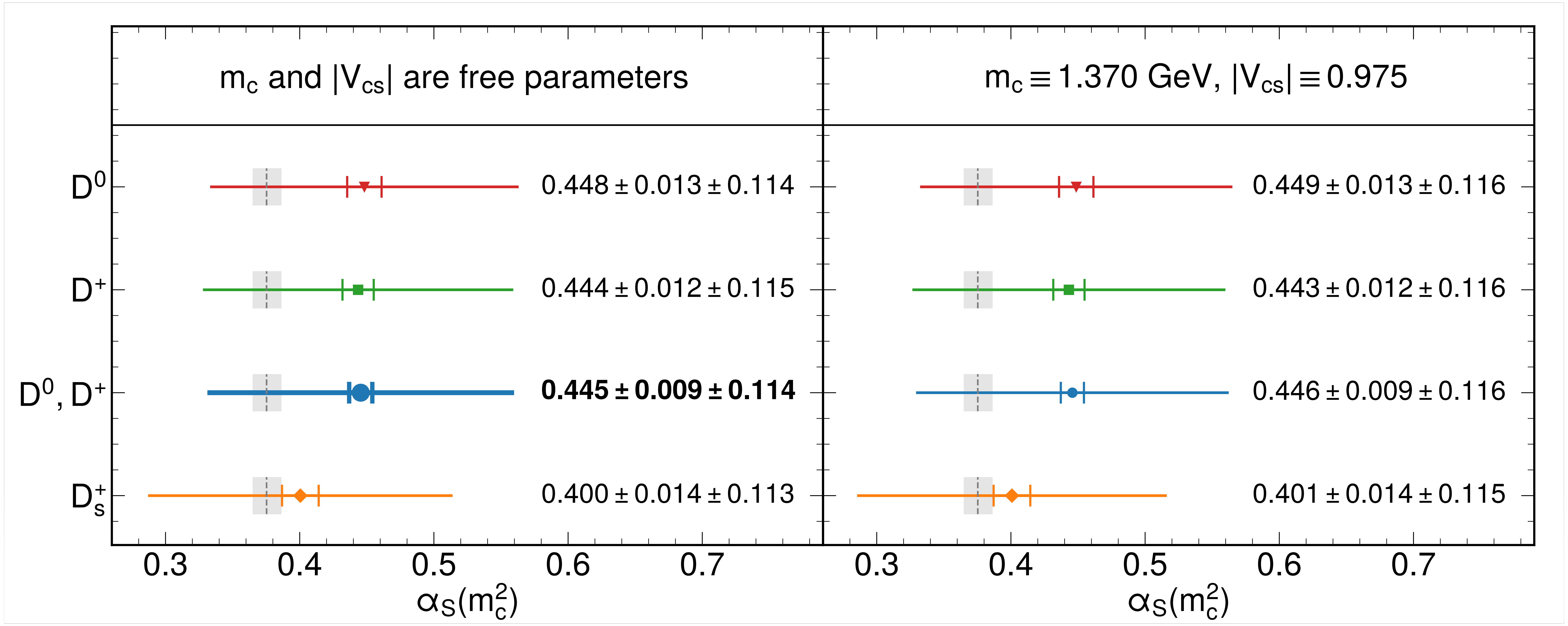
- The values are not from the measurements related to the Γ_{SL} of D mesons.
- The **kinetic scheme** is used to avoid the bad convergence behavior.

Parameter	Value
G_F	1.1663788×10^{-5}
$\lvert V_{cs}\rvert$	0.975 ± 0.006
$m_c(0.5\ \mathrm{GeV})$	$(1.370 \pm 0.034)\ \mathrm{GeV}$
$m_s(0.5\ \mathrm{GeV})$	$(93.4 \pm 8.6)\ \mathrm{MeV}$
$\mu_G^2(0.5\ \mathrm{GeV})$	$(0.288 \pm 0.049)\ \mathrm{GeV}^2$
$\mu_\pi^2(0.5\ \mathrm{GeV})$	$(0.26 \pm 0.06)\ \mathrm{GeV}^2$
$\rho_D^3(0.5\ \mathrm{GeV})$	$(0.05 \pm 0.04)\ \mathrm{GeV}^3$
$\rho_{LS}^3(0.5\ \mathrm{GeV})$	$(-0.113 \pm 0.090)\ \mathrm{GeV}^3$
$B_{WA,D^{+,0}}$	$-0.001\ \mathrm{GeV}^3$
B_{WA,D_s^+}	$-0.002\ \mathrm{GeV}^3$

- The **uncertainties of these parameters** are dominate uncertainty sources in the extraction of α_s.
- More precise measurements may help to reduce the systematic uncertainties.

Extraction of α_S

- The values of $\alpha_S(m_c^2)$ are extracted for D^0, D^+, and D_s^+.



Shade : world average.

Marker : fitted α_s

Inner error bar : experimental uncertainty,

Outer error bar : total uncertainty,

- The values of $\alpha_s(m_c^2)$ are **consistent** within 1σ **among different D mesons**, and with **the world average** (running from $\alpha_s(m_Z^2)$).

- The consistence among different D mesons demonstrate the robustness of this method.

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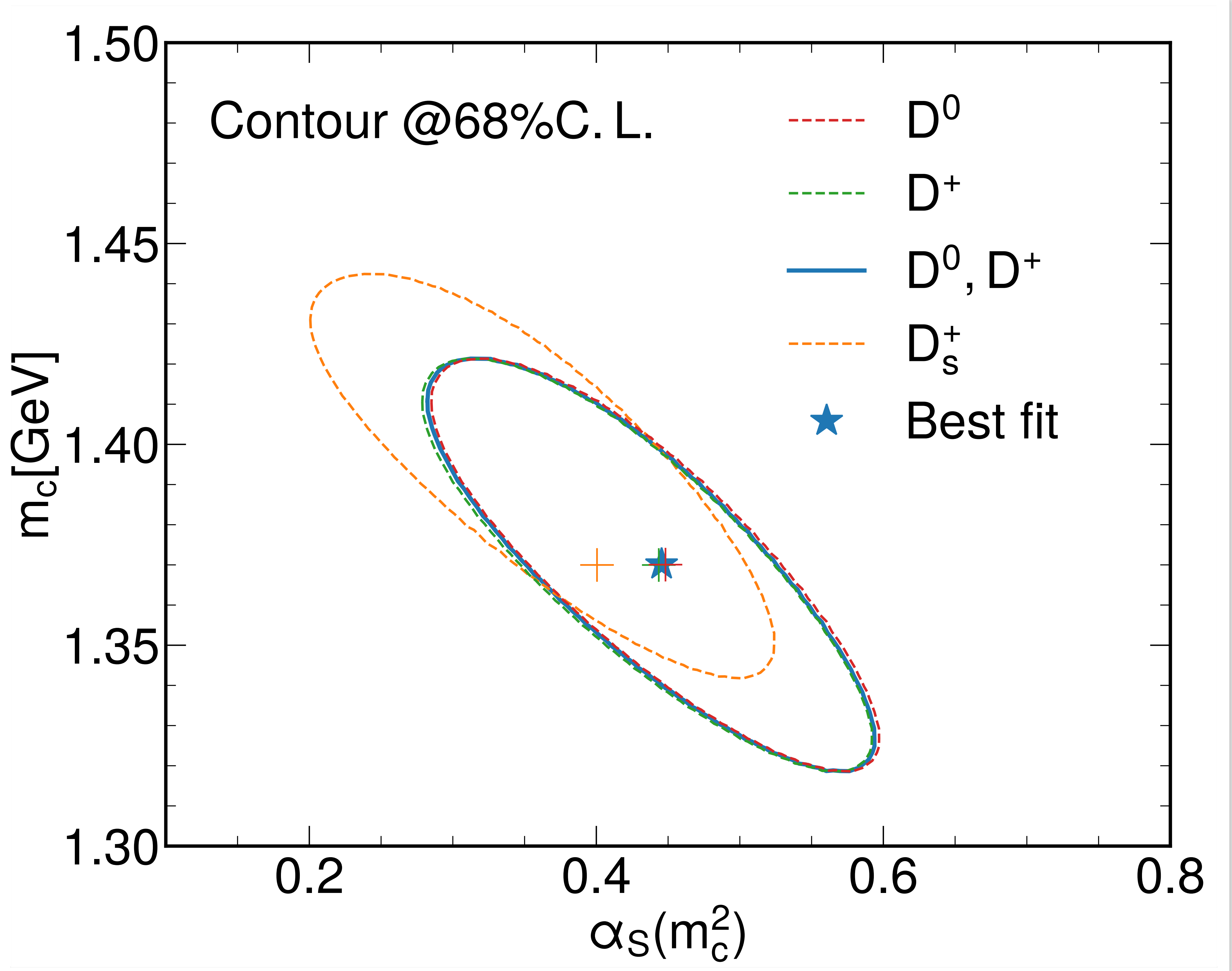
Extraction of α_S

- The results of different charmed mesons,

Sample	D^0	D^+	$\boldsymbol{D^+, D^0}$	D_s^+
m_c[GeV]	1.3701 ± 0.0339	1.3699 ± 0.0340	**1.3701 ± 0.0338**	1.3699 ± 0.0340
$\alpha_S(m_c^2)[10^{-3}]$	$448 \pm 13 \pm 114$	$444 \pm 12 \pm 115$	**$445 \pm 9 \pm 114$**	$400 \pm 14 \pm 113$



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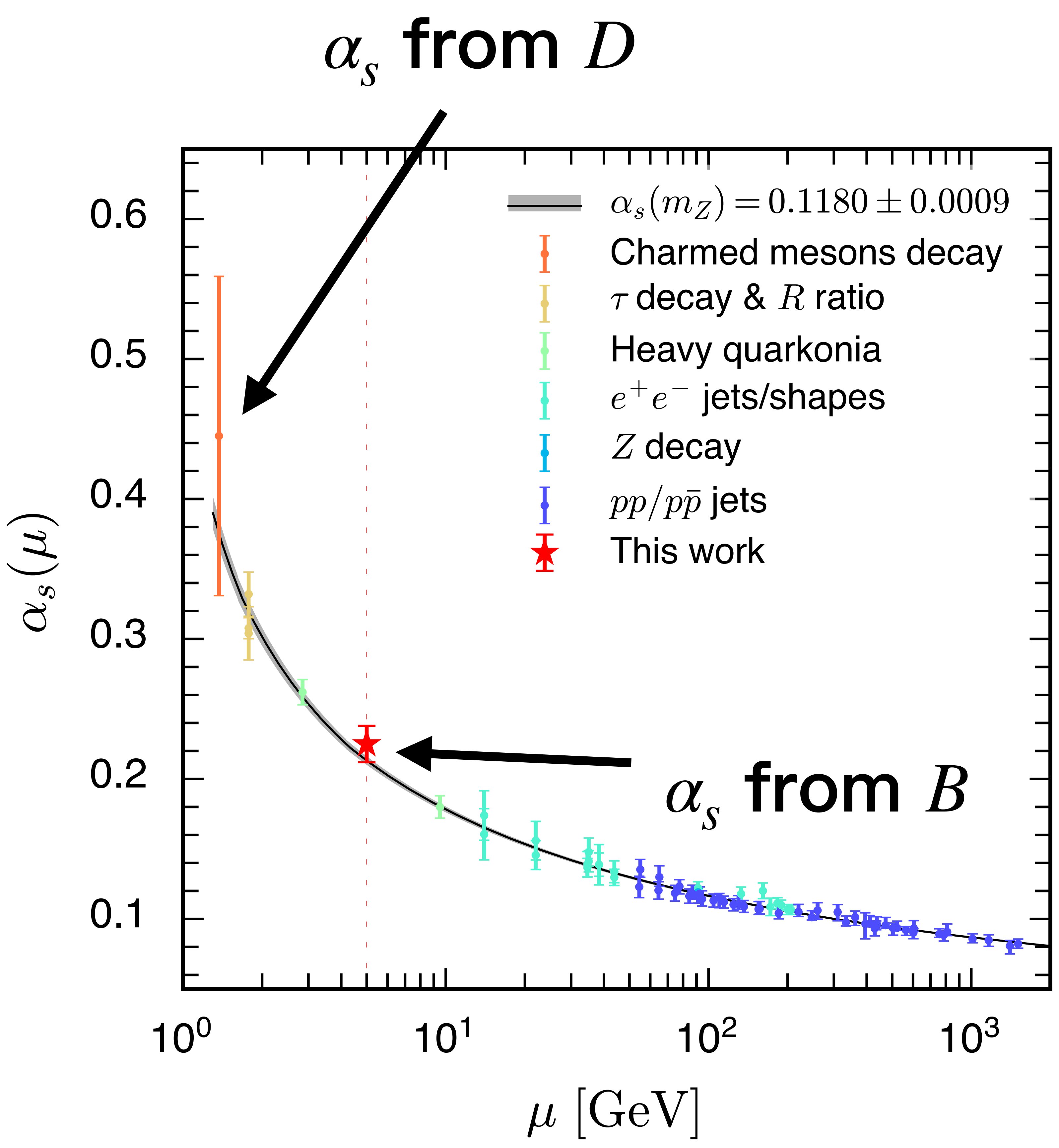
- The profile contours of $\alpha_s(m_c^2)$ vs. m_c,
  - The consistence among different D mesons,
  - The strong correlation between m_c and $\alpha_s(m_c^2)$,
  - New observables are needed to reduce the correlation.

Summary

- **Summary:**
  - We discuss a methodology determining α_s from the inclusive semi-leptonic B/D decay width.
    - Similar expression in HQE: double expansion of α_s and heavy quark masses.
    - By matching theoretical prediction and experimental measurements on $\Gamma(B \to X_c \ell \bar{\nu}_\ell)$ and $\Gamma(D \to X_s \ell \bar{\nu}_\ell)$, with $|V_{cb}|$, quark masses, HQE parameters constrained with external determinations.
- **Discussions:**
  - RG scheme: balance between convergence and sensitivity.
  - Obtain α_s sensitivity from running quark masses.
  - Potential improvement by using invariant mass and momentum spectra of final states.
  - Problems about α_s running in low energy region.



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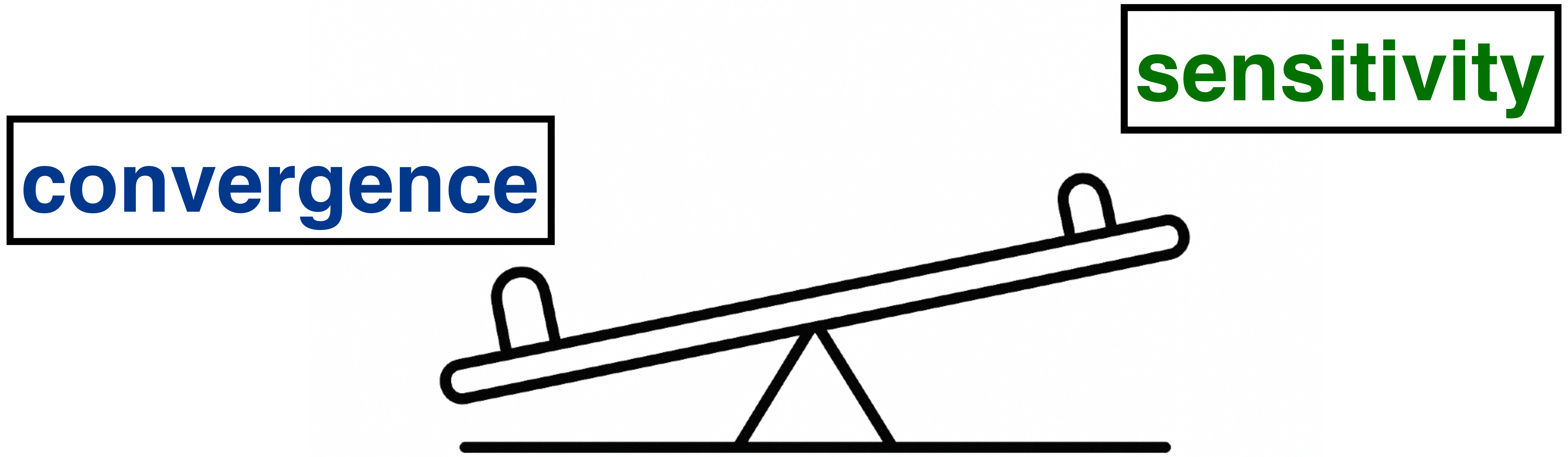
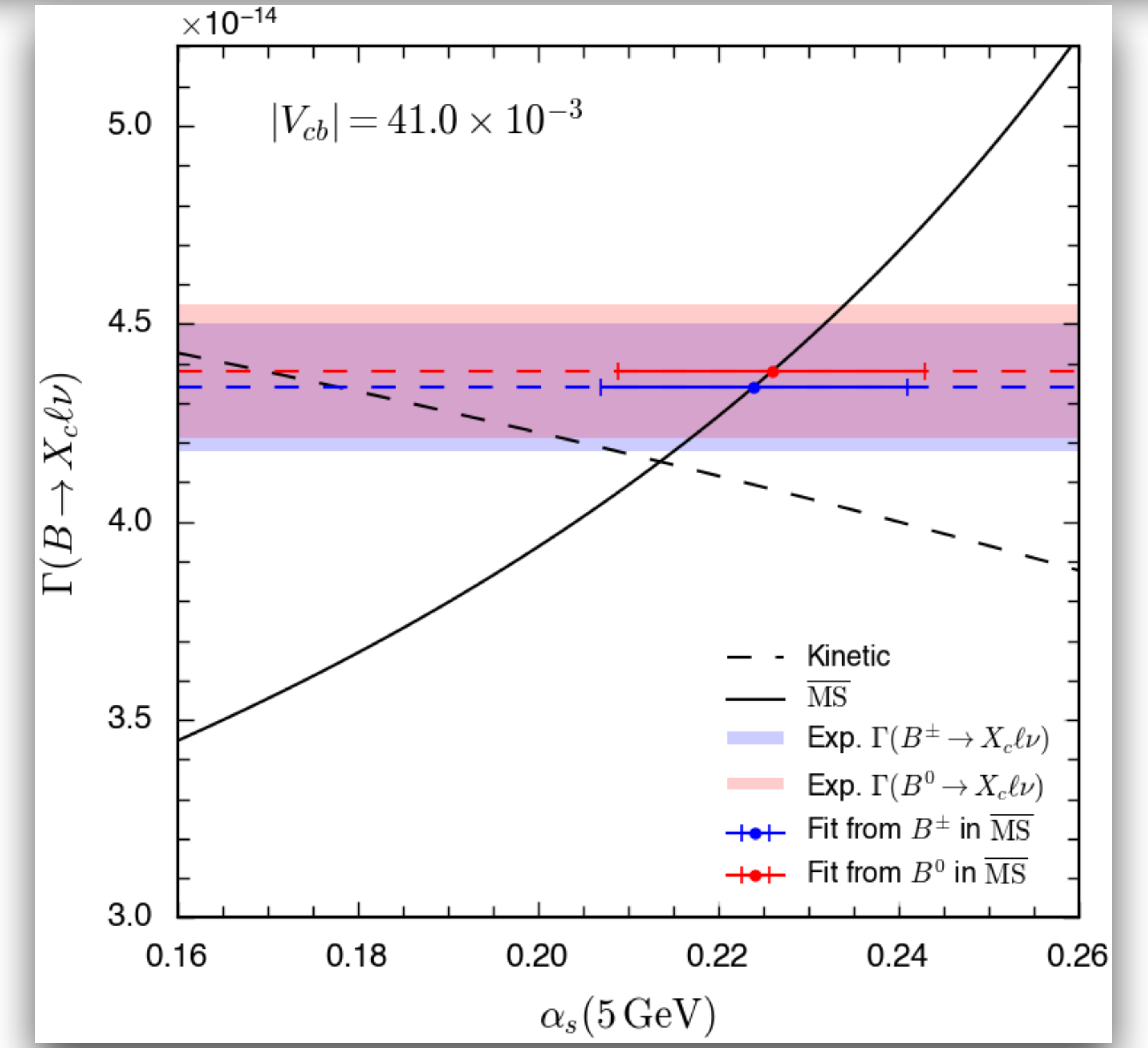
Renormalization Scheme: Convergence or Sensitivity?

- **In principle, physics observables are understand as scheme independent.**
  - The kinetic mass scheme was widely used in the semi-leptonic decays of b/c mesons, for better convergence.
  - Kinetic mass scheme appears to provide better **convergence** but lower α_s **sensitivity**.
  - Different schemes for the same observable: ***Does good convergence imply poor sensitivity?***



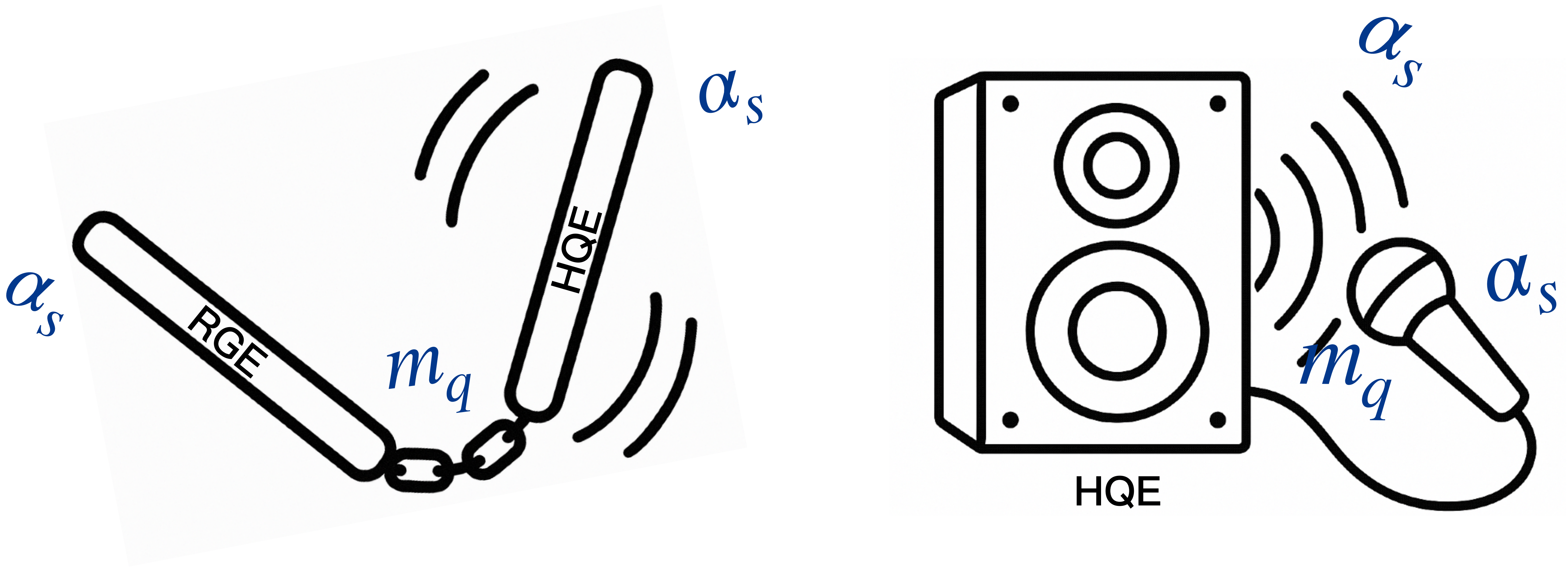
Phys. Rev. Lett. 125, 052003 (2020)

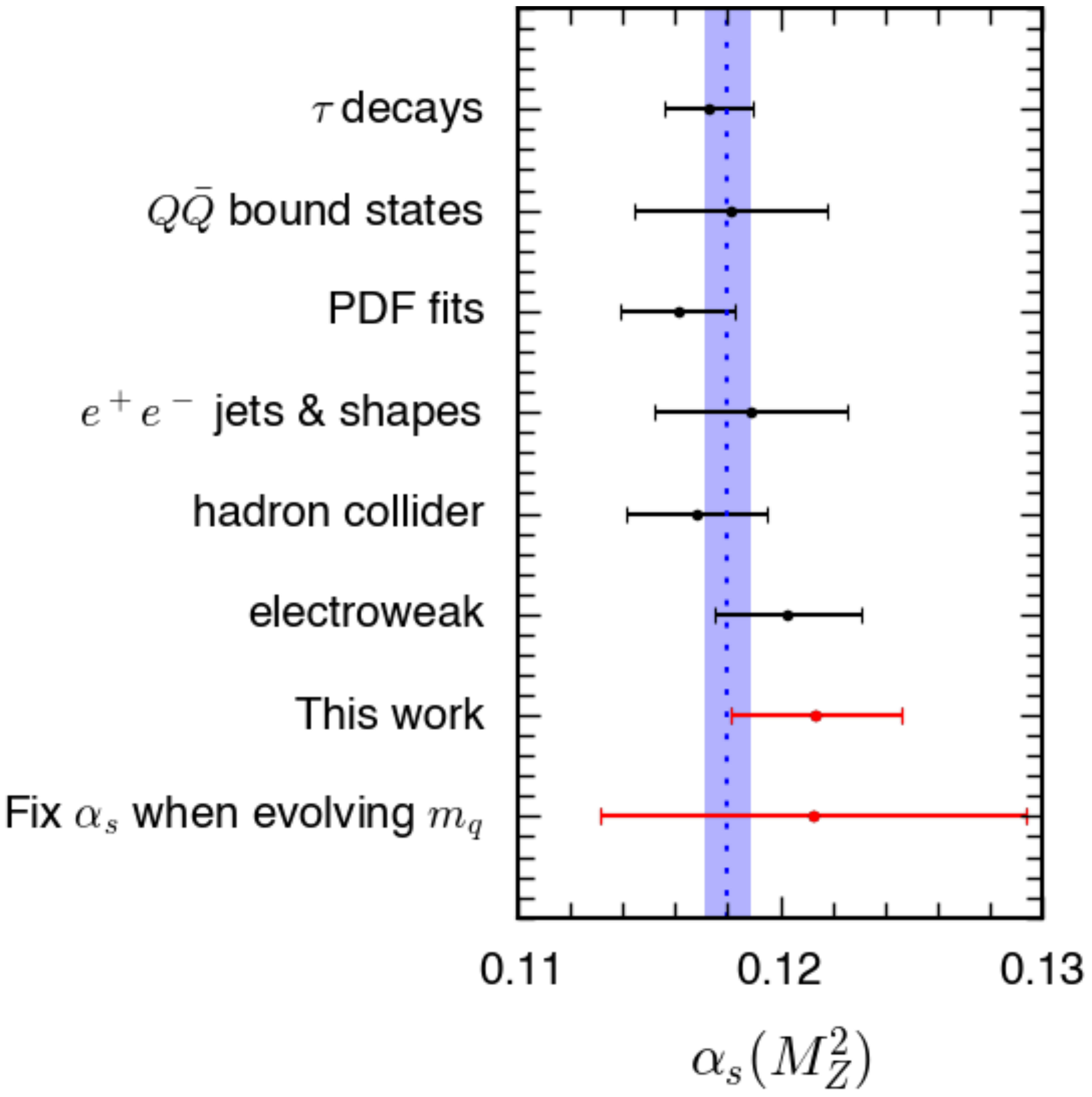
The rate and the moments of $B \to X_c \ell \nu$ strongly depend on the mass definition of the heavy quark, the choice of which is closely intertwined with the size of the QCD corrections. Perturbative calculations using the on-shell mass scheme are affected by the renormalon ambiguity, which manifests itself through bad behavior of the perturbative series [17,18]. However, QCD corrections to the semileptonic rates also exhibit a bad convergence in the $\overline{\rm MS}$ scheme [9,19]. In fact, large $(n\alpha_s)^k$ terms, with $n = 5$, arise from the $m^{\rm OS}$–$\bar{m}$ conversion of the overall factor $\Gamma \simeq G_F^2 m_b^5 |V_{cb}|^2/(192\pi^3)$.



RG scale dependency: Obtain α_s sensitivity from m_q?

- **In principle, physics observables are understand as scheme independent.**
  - In HQE, observables are sensitive to α_s through perturbative corrections, whose coefficients are sensitive to m_q.
  - The heavy quark masses are sensitive to α_s through RGE evolution.
  - Consequently, ***the α_s fit obtains additional sensitivity from quark mass running.***
  - ***is this only a technical trick or does it have physical meaning?***





Presently used: semi-leptoinc decay width.

- **Measurement**: Indirect, with lepton momenta threshold
- **CKM elements**: High correlation

Possible improvement: Moments of distributions or other optimized options?

- **Measurement**: Indirect, with lepton momenta threshold
- **CKM elements**: High correlation
- **Sensitivity on α_s**: weak in kinetic scheme.

JHEP05(2024)287

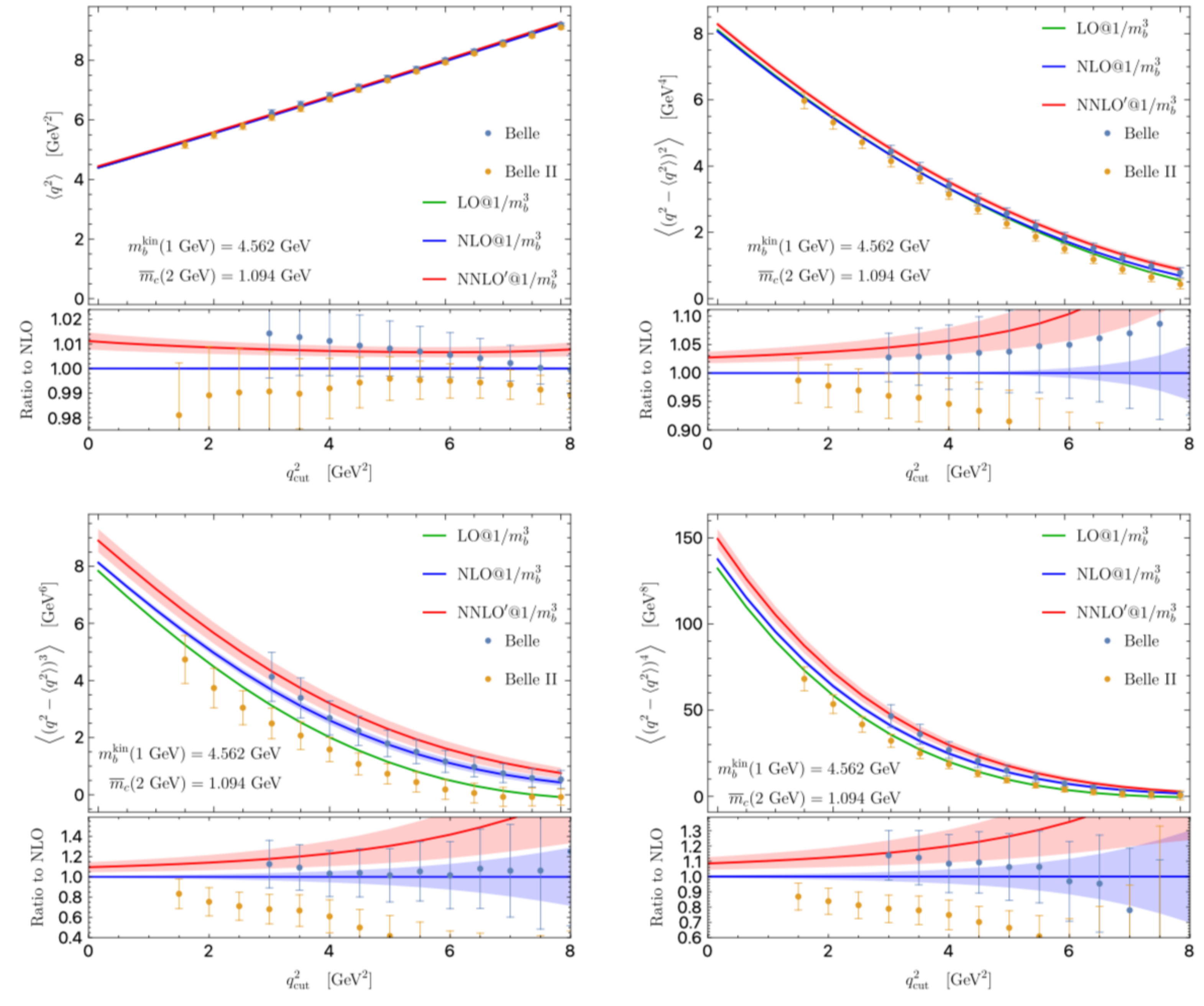


Figure 3. The first four q^2 moments of $B \to X_c \ell \bar{\nu}_\ell$ as a function of the lower cut q^2_{cut}. The heavy quark masses are $m_b^{\text{kin}}(1\text{GeV}) = 4.562\,\text{GeV}$ and $\overline{m}_c(2\,\text{GeV}) = 1.094\,\text{GeV}$. For the HQE parameter we adopt the RPI basis up to $1/m_b^3$ [15, 16] and values from the fit in ref. [17]. Measurements from Belle [18] and Belle II [19].

Phys. Rev. D 104, 012003

We report a measurement of the inclusive electron energy spectrum for charmed semileptonic decays of B mesons in a 140 fb^{-1} data sample collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric energy e^+e^- collider. We determine the first four moments of the electron energy spectrum for threshold values of the electron energy between 0.4 and 2.0 GeV. In addition, we provide values of the partial branching fraction (zeroth moment) for the same electron threshold energies, and independent measurements of the B^+ and B^0 partial branching fractions at 0.4 GeV and 0.6 GeV electron threshold energies. We measure the independent B^+ and B^0 partial branching fractions with electron threshold energies of 0.4 GeV to be $\Delta\mathcal{B}(B^+ \to X_c e\nu) = (10.79 \pm 0.25(\text{stat.}) \pm 0.27(\text{sys.}))\%$ and $\Delta\mathcal{B}(B^0 \to X_c e\nu) = (10.08 \pm 0.30(\text{stat.}) \pm 0.22(\text{sys.}))\%$. Full correlations between all measurements are evaluated.

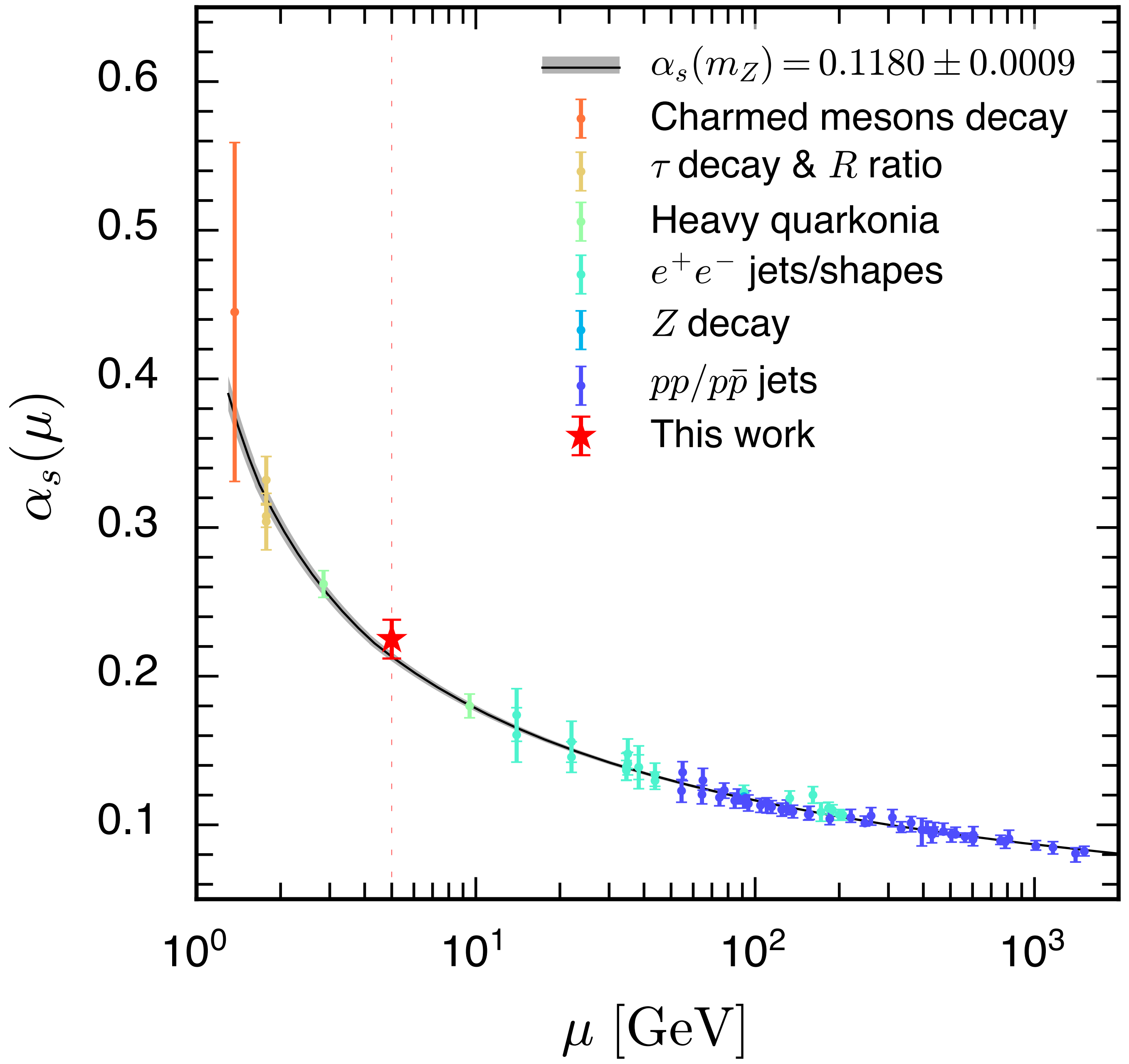
Phys. Rev. D 104, 012003 (2021)

We measure the inclusive semielectronic decay branching fraction of the D_s^+ meson. A double-tag technique is applied to e^+e^- annihilation data collected by the BESIII experiment at the BEPCII collider, operating in the center-of-mass energy range 4.178–4.230 GeV. We select positrons from $D_s^+ \to Xe^+\nu_e$ with momenta greater than 200 MeV/c and determine the laboratory momentum spectrum, accounting for the effects of detector efficiency and resolution. The total positron yield and semielectronic branching fraction are determined by extrapolating this spectrum below the momentum cutoff. We measure the D_s^+ semielectronic branching fraction to be $(6.30 \pm 0.13(\text{stat.}) \pm 0.09(\text{syst.}) \pm 0.04(\text{ext.}))\%$, showing no evidence for unobserved exclusive semielectronic modes. We combine this result with external data taken from literature to determine the ratio of the D_s^+ and D^0 semielectronic widths, $\frac{\Gamma(D_s^+ \to Xe^+\nu_e)}{\Gamma(D^0 \to Xe^+\nu_e)} = 0.790 \pm 0.016(\text{stat.}) \pm 0.011(\text{syst.}) \pm 0.016(\text{ext.})$. Our results are consistent with and more precise than previous measurements.

What are we measuring when we measure α_s at low Q^2 scale?

- 😷 **Limitation of pQCD**: As the QCD energy scale decreases,
  - non-perturbative effects strengthen,
  - perturbation theory tends to break down.
- 💊 **Supplementary strategy**: The non-perturbative effects are captured by high-order power corrections of HQE (OPE) theory.
  - with parameters corresponding to local operators determined from experiments.
  - therefore theory models can explain data (with PDG world average α_s as input).

- ❓ **Question**: Using HQE model consistent with data,
  - can we get an α_s deviating from RGE?
  - how do we interpret the deviation of measured α_s and RGE extrapolation?
    - the incorrect understanding of nature in QCD theory?
    - bad modeling of non-perturbative effect?
- ***The α_s^{pQCD} is a model dependent parameter in QCD.***



Model independent α_s?

- Is there any model independent definition for α_s at low energy region (α_s^{IR})?
  - Effective charge

$$\Gamma_1^{\mathrm{p-n}}(Q^2) = \frac{g_A}{6}\left[1 - \frac{\alpha_s^{\mathrm{pQCD}}(Q^2)}{\pi} - 3.58\left(\frac{\alpha_s^{\mathrm{pQCD}}(Q^2)}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s^{\mathrm{pQCD}}(Q^2)}{\pi}\right)^3 - 175.7\left(\frac{\alpha_s^{\mathrm{pQCD}}(Q^2)}{\pi}\right)^4 + \right.$$
$$\left. \sim -893.38\left(\frac{\alpha_s^{\mathrm{pQCD}}(Q^2)}{\pi}\right)^5 + \mathcal{O}\left(\left(\alpha_s^{\mathrm{pQCD}}\right)^6\right)\right] + \sum_{n>1}\frac{\mu_{2n}(Q^2)}{Q^{2n-2}}.$$

$$\Gamma_1^{p-n}(Q^2) =: \frac{g_A}{6}\left(1 - \frac{\alpha_{g_1}(Q^2)}{\pi}\right), \qquad \alpha_{g_1}(Q^2) = \pi\left(1 - \frac{6}{g_A}\Gamma_1^{p-n}(Q^2)\right)$$

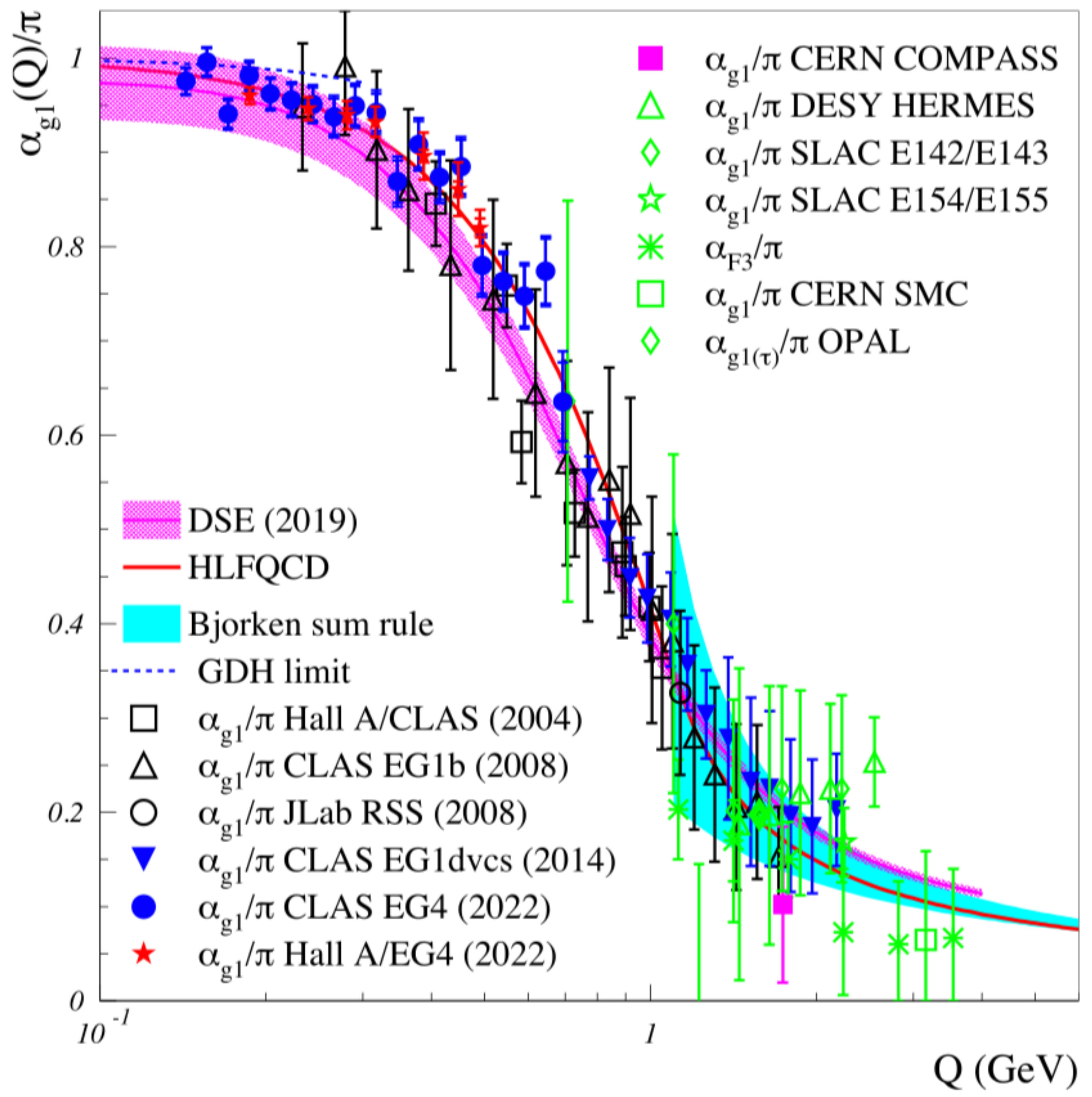


Fig. 4.1. Effective charge $\alpha_{g_1}(Q)/\pi$. The most recent extractions from data [119] are shown by the filled blue circles and red stars. Their inner error bars give the statistical uncertainties and the outer ones represent the quadratic sum of the systematic and statistical uncertainties. The open symbols show earlier extractions [369,370], with the error bars being the quadratic sum of the systematic and statistical uncertainties. Recent theoretical predictions are also shown: CSM [35,36] (magenta curve and shaded band); and HLFQCD [373] (red line, using $\kappa = 0.534$ GeV). The cyan band and associated dashed curve are computed using the Bjorken and GDH sum rules, respectively.

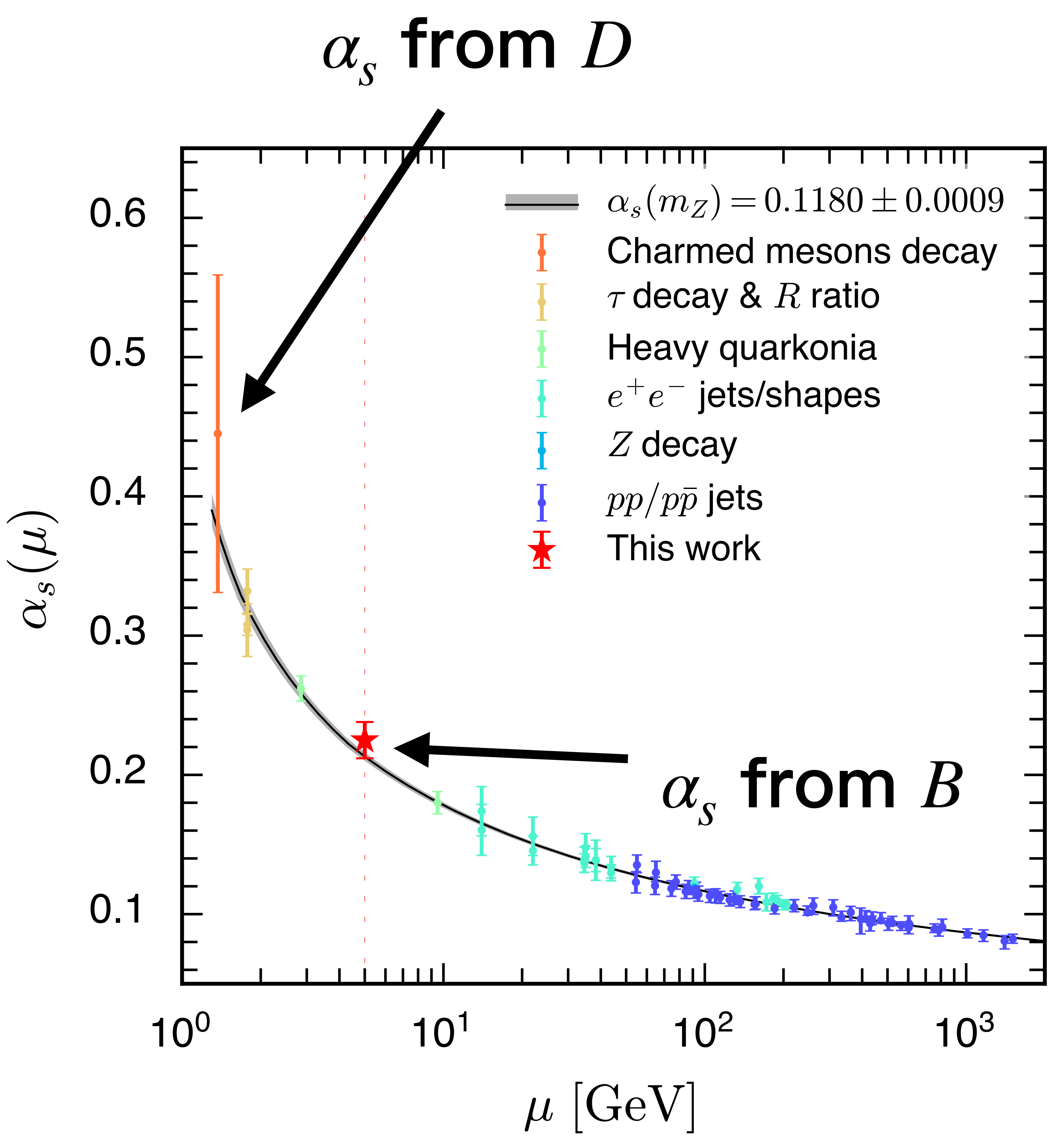
- **How to compare:**
  - Effective charges defined from different processes?
  - Effective charges and $\overline{\mathrm{MS}}$ α_s?
  - …

See 10.1016/j.ppnp.2023.104081

Summary

- **Summary:**
  - We discuss a methodology determining α_s from the inclusive semi-leptonic B/D decay width.
    - Similar expression in HQE: double expansion of α_s and heavy quark masses.
    - By matching theoretical prediction and experimental measurements on $\Gamma(B \to X_c \ell \bar{\nu}_\ell)$, with $|V_{cb}|$, quark masses, HQE parameters constrained with external determinations.

- **Discussions:**
  - RG scheme: balance between convergence and sensitivity.
  - Obtain α_s sensitivity from running quark masses.
  - Potential improvement by using invariant mass and momentum spectra of final states.
  - Problems about α_s running in low energy region.



Looking forward your comments

Yuzhi Che

Back Up

Perturbation expansion of C_0 in $\overline{\text{MS}}$ scheme

$$\Gamma_{\text{pert}} \equiv \Gamma_0 C_0 = 1.217 \times 10^{-13}\ \text{GeV} \cdot \left(a_0 + a_1 \left(\frac{\alpha_s}{\pi}\right) + a_2 \left(\frac{\alpha_s}{\pi}\right)^2 + a_3 \left(\frac{\alpha_s}{\pi}\right)^3 \right)$$

α_S	$\overline{m}_b(5\,\text{GeV})$	$\overline{m}_c(5\,\text{GeV})$	a_0	a_1	a_2	a_3	$\frac{\Gamma_{\text{pert}}}{1.217\times10^{-13}\,\text{GeV}}$
0.1596	4.0877	1.0265	0.2299	1.0173	4.0834	-1.8285	0.2918
0.1628	4.0855	1.0196	0.2305	1.0250	4.1460	-1.4167	0.2946
0.1660	4.0833	1.0125	0.2312	1.0328	4.2103	-0.9918	0.2974
0.1692	4.0810	1.0054	0.2319	1.0409	4.2762	-0.5531	0.3002
0.1724	4.0788	0.9981	0.2326	1.0491	4.3439	-0.0999	0.3032
0.1755	4.0765	0.9906	0.2333	1.0576	4.4135	0.3684	0.3062
0.1787	4.0742	0.9830	0.2340	1.0662	4.4851	0.8527	0.3093
0.1819	4.0718	0.9753	0.2348	1.0751	4.5586	1.3538	0.3126
0.1851	4.0695	0.9674	0.2355	1.0841	4.6343	1.8726	0.3159
0.1883	4.0671	0.9594	0.2363	1.0934	4.7123	2.4100	0.3193
0.1915	4.0648	0.9512	0.2371	1.1030	4.7925	2.9670	0.3228
0.1947	4.0624	0.9428	0.2379	1.1127	4.8752	3.5447	0.3265
0.1979	4.0600	0.9342	0.2388	1.1228	4.9605	4.1443	0.3302
0.2011	4.0575	0.9254	0.2396	1.1331	5.0484	4.7670	0.3341
0.2043	4.0551	0.9165	0.2405	1.1437	5.1392	5.4142	0.3381
0.2075	4.0526	0.9073	0.2415	1.1545	5.2329	6.0873	0.3423
0.2107	4.0501	0.8979	0.2424	1.1657	5.3298	6.7879	0.3466

α_S	$\overline{m}_b(5\,\text{GeV})$	$\overline{m}_c(5\,\text{GeV})$	a_0	a_1	a_2	a_3	$\frac{\Gamma_{\text{pert}}}{1.217\times10^{-13}\,\text{GeV}}$
0.2138	4.0476	0.8883	0.2434	1.1772	5.4299	7.5178	0.3510
0.2170	4.0451	0.8784	0.2444	1.1890	5.5336	8.2787	0.3556
0.2202	4.0425	0.8684	0.2454	1.2012	5.6408	9.0727	0.3604
0.2234	4.0400	0.8580	0.2464	1.2138	5.7519	9.9019	0.3654
0.2266	4.0374	0.8474	0.2475	1.2267	5.8671	10.7686	0.3706
0.2298	4.0348	0.8365	0.2486	1.2400	5.9866	11.6755	0.3759
0.2330	4.0322	0.8253	0.2498	1.2538	6.1107	12.6253	0.3815
0.2362	4.0295	0.8137	0.2509	1.2680	6.2396	13.6210	0.3873
0.2394	4.0269	0.8019	0.2522	1.2826	6.3736	14.6660	0.3934
0.2426	4.0242	0.7896	0.2534	1.2978	6.5130	15.7639	0.3997
0.2458	4.0215	0.7771	0.2547	1.3134	6.6583	16.9187	0.4063
0.2490	4.0188	0.7641	0.2560	1.3296	6.8096	18.1346	0.4132
0.2522	4.0160	0.7507	0.2574	1.3464	6.9675	19.4167	0.4204
0.2553	4.0133	0.7368	0.2588	1.3638	7.1323	20.7699	0.4279
0.2585	4.0105	0.7225	0.2603	1.3817	7.3044	22.2003	0.4358
0.2617	4.0077	0.7077	0.2618	1.4004	7.4845	23.7140	0.4441
0.2649	4.0048	0.6923	0.2633	1.4197	7.6729	25.3181	0.4528

Γ_{SL} of charmed meson in theory

- Since the Γ_{SL} **has strong dependence on** m_c, the reasonable definition of m_c can simplify this study.

- To avoid the bad convergence behavior, we use the **kinetic scheme** to perform this study.

- The relation of m_c between the $\overline{MS}$ and kinetic had been studied to N^{3}LO. The $m_c^{kin}(0.5\ GeV)$ is calculated from different scale in $\overline{MS}$.

$$m_c^{kin}(0.5\ \text{GeV}) = 1336\ \text{MeV from } \overline{m}_c(\mu_s = 3\ \text{GeV})$$
$$m_c^{kin}(0.5\ \text{GeV}) = 1372\ \text{MeV from } \overline{m}_c(\mu_s = 2\ \text{GeV})$$
$$m_c^{kin}(0.5\ \text{GeV}) = 1404\ \text{MeV from } \overline{m}_c(\mu_s = \overline{m}_c)$$

$$\overline{m_c^{kin}(0.5\ GeV)} = 1370 \pm 34\ MeV$$

The μ^{kin} is set to 0.5 GeV.

- The **average among 3 options** is taken into account for m_c in this study.

Nucl.Phys.B 426 (1994) 301-343 Phys.Rept. 494 (2010) 197-414 PhysRevLett.125.052003

The prediction of Γ_{SL}

- The P. Gambino and J. F. Kamenik calculated the Γ_{SL} using the framework of HQE[4].

$$f_0(r) = 1 - 8r + 8r^3 - r^4 - 12r^2 \cdot \log(r)$$

$$f_1(r) = 2.86\sqrt{r} - 3.84r \cdot \log(r)$$

$$f_2(r) = \beta_0[8.16\sqrt{r} - 1.21r \cdot \log(r) - 3.38]$$

$$f_G(r) = \frac{1}{2} f_0(r) - 2(1-r)^4$$

$$f_\pi(r) = -f_0(r)/2$$

$$f_{LS}(r) = -f_G(r)$$

$$f_D(r) = \frac{77}{6} + \mathcal{O}(r) + 8\log(\frac{\mu_{WA}^2}{m_c^2})$$

[4] Nucl.Phys.B 840 (2010) 424-437

α_S from charm data

- In the prediction of Γ_{SL}, two parts are missed :
  - The high order α_S correction,
  - The absence of Cabibbo suppressed processes of $c \to dl\bar{\nu}$ in the calculation.
- The high order α_S correction in $b \to cl\bar{\nu}$ is less than 1%. We take **5 times larger than** $b \to cl\bar{\nu}$ as the high order correction in $c \to sl\bar{\nu}$, PhysRevD.104.016003
  - 5% is taken.
- The absence of $c \to dl\bar{\nu}$ causes the 5% uncertainty on Γ_{SL}, that is proportional to $|V_{cd}|^2/(|V_{cd}|^2 + |V_{cs}|^2) = 5\,\%$.
- In total, we take **10%** as the uncertainty of theoretical Γ_{SL}.