Opportunities and Ideas at the QCD Frontier
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中国高等科学技术中心

Chiral Perturbation Theory with Axions

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- FKG, U.-G. Meißner, *Cumulants of the QCD topological charge distribution*, PLB 749, 278 (2015);
- Z.-Y. Lu, M.-L. Du, FKG, U.-G. Meißner, T. Vonk, *QCD θ-vacuum energy and axion properties*, JHEP 05, 001 (2020);
- T. Vonk, FKG, U.-G. Meißner, *Precision calculation of the axion-nucleon coupling in chiral perturbation theory*, JHEP 03, 138 (2020);
- T. Vonk, FKG, U.-G. Meißner, *The axion-baryon coupling in SU(3) heavy baryon chiral perturbation theory*, JHEP 08, 024 (2021)
- T. Vonk, FKG, U.-G. Meißner, *Pion axioproduction: The Δ resonance contribution*, PRD 105, 054029 (2022)
- C.-C. Li, T.-R. Hu, FKG, U.-G. Meißner, *Pion axioproduction revisited*, PRD 109, 075050 (2024)

Nucleon EDMs

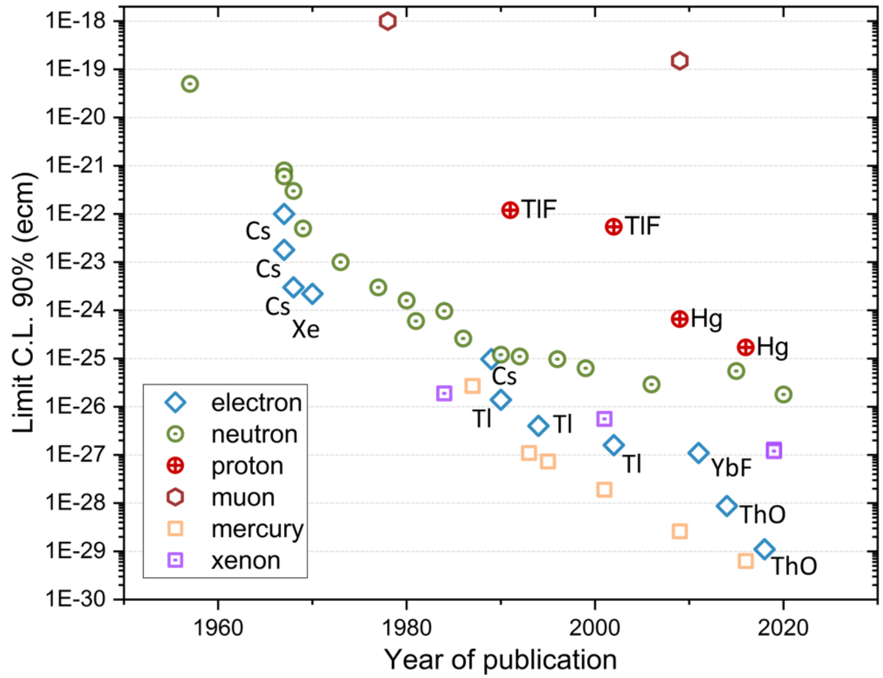
- Nucleon electric dipole moments (EDMs): CP odd, highly suppressed in the Standard Model with CKM: no EDM at the first order of weak interaction; $|d_n(\text{CKM})| \lesssim 10^{-31} e$ cm.

  X.-G. He, McKellar, Pakvasa, IJMPA 4, 5011 (1989) [E: *ibid*, 6, 1063 (1991)]

- Sensitive to the physics beyond the SM

- Experimental upper limits on hadron EDMs:
  - $|d_n| < 1.8 \times 10^{-26} e$ cm C. Abel, et al., PRL 124, 081803 (2020)
  - $|d_p| < 2.1 \times 10^{-25} e$ cm B.K. Sahoo, PRD 95, 013002 (2017), based on the measurement in W.C. Griffith, et al., PRL 102, 101601 (2009)
  - $|d_\Lambda| < 1.5 \times 10^{-16} e$ cm, based on 3×10^6 $\Lambda \to p\pi^-$ events

    L. Pondrom, et al., PRD 23, 814 (1981)



From talk by P. Schmidt-Wellenburg (2022)

Theta term and strong CP problem

- Another source of CP violation in SM: the θ term of QCD

$$\mathcal{L}_{\text{QCD},0} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \mathcal{L}_{\text{quarks}}$$

  - ☐ Instanton solutions to the classical EoM — Belavin et al., PLB 59, 85 (1975)
  - ☐ Non-trivial vacuum structure — Callan, Dashen, Gross, PLB 63, 334 (1976); see e.g, Donoghue et al., Dynamics of the Standard Model

    $|n\rangle \to |n+Q\rangle$, gauge invariant vac. must be superposition of all topological classes

$$|\theta\rangle = \sum_n e^{-in\theta} |n\rangle$$

    Topological charge (integer): $Q = \frac{g^2}{32\pi^2} \int d^4x \tilde{G}^a_{\mu\nu}(x) G^{a,\mu\nu}(x)$, $\tilde{G}^a_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{a,\rho\sigma}$

  - ☐ QCD Lagrangian with a θ term

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{QCD},0} + \frac{\theta\, g^2}{32\pi^2} \tilde{G}^a_{\mu\nu} G^{a,\mu\nu}$$

- The theta term violates P and CP:

$$G_{\mu\nu} \tilde{G}^{\mu\nu} \propto \mathbf{E} \cdot \mathbf{B} \xrightarrow{\text{CP}} -\mathbf{E} \cdot \mathbf{B}$$

- Strong CP problem: neutron EDM constrains $\bar{\theta} \lesssim 10^{-10}$, with $\bar{\theta} = \theta + \arg(\det M_q)$ the measurable quantity

QCD axion

For an excellent recent review, L. Di Luzio et al., The landscape of QCD axion models, Phys.Rept. 870, 1 (2020)

- A possible solution to the strong CP problem
  - Peccei-Quinn mechanism, hidden $\mathrm{U}(1)_A$ symmetry — Peccei, Quinn (1977)
  - Nambu-Goldstone boson: pseudoscalar axion — Weinberg (1978); Wilczek (1978)

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \mathcal{L}(\partial_\mu a, \psi) + \frac{g_s^2}{32\pi^2}\frac{a}{f_a} G^c_{\mu\nu}\tilde{G}^{c,\mu\nu}$$

  - Its VEV cancels the theta term $\theta + \frac{\langle a \rangle}{f_a} = 0$, thus solves the strong CP problem

$$\mathcal{L}_{G\tilde{G}} = \left(\theta + \frac{a}{f_a}\right)\frac{g_s^2}{32\pi^2} G^c_{\mu\nu}\tilde{G}^{c,\mu\nu}$$

    - Expanding around $\langle a \rangle \Rightarrow$ the θ vacuum energy gives the axion potential, $\theta \to a/f_a$
    - $\chi_t \Rightarrow m_a, c_4 \Rightarrow$ axion self-interaction

$$m^2_{a,\mathrm{LO}} = \frac{F_\pi^2 M_{\pi^+}^2 \bar{m}}{2 f_a^2 \hat{m}} \qquad \text{isospin limit: } m^2_{a,\mathrm{LO}} f_a^2 = \frac{1}{4} F_\pi^2 M_\pi^2$$

Georgi, Kaplan, Randall (1986)

- Dark matter candidate — Preskill, Wise, Wilczek (1983); Abbott, Sikivie (1983); Dine, Fischler (1983); …
- Axion decay constant window from astrophysical and cosmological data:

$$10^9\,\mathrm{GeV} \lesssim f_a \lesssim 10^{12}\,\mathrm{GeV}$$

e.g., J.E. Kim, G. Carosi, RMP 82, 557 (2010)

For compilations of various constraints, see L. Di Luzio et al., Phys.Rept. 870, 1 (2020)

Cumulants of the QCD topological distribution

- The QCD partition function in a θ vacuum

$$Z(\theta) = \int [DG][Dq][D\bar{q}]e^{-S_{\mathrm{QCD},0}[G,q,\bar{q}]-i\theta Q}$$

- For large Euclidean time τ, dominated by the ground state (vacuum) energy

$$Z(\theta) = e^{-\tau E_{\mathrm{vac}}(\theta)} = e^{-V e_{\mathrm{vac}}(\theta)}$$

V: space-time volume; $e_{\mathrm{vac}}(\theta)$: vacuum energy density

- In terminology of statistics
  - $Z(\theta)$: moment-generating function for the distribution of Q, $Z(\theta) = \sum_n m_n \frac{\theta^n}{n!}$
  - $e_{\mathrm{vac}}(\theta)$: cumulant-generating function, $e_{\mathrm{vac}}(\theta) = \sum_n c_n \frac{\theta^n}{n!}$
  - Cumulants

$$c_2 = \chi_t = \frac{1}{V}\langle Q^2\rangle_{\theta=0}, \quad c_4 = -\frac{1}{V}\left(\langle Q^4\rangle - 3\langle Q^2\rangle^2\right)_{\theta=0}$$

topological susceptibility

For a general relation between cumulants and moments, see FKG, U.-G. Meißner, PLB 749, 278 (2015)

Theta vacuum energy in ChPT

- The θ term can be built into chiral perturbation theory (ChPT) with a complex quark mass matrix: $\chi_\theta = 2B_N \mathcal{M}_q \exp(i\theta/N)$
- Leading order (LO) vacuum energy density in SU(N) ChPT

$$e_{\rm vac}^{(2)}(\theta) = -\frac{F_N^2}{4}\langle \chi_\theta U_0^\dagger + \chi_\theta^\dagger U_0 \rangle$$

- Vacuum alignment for 2-flavor ChPT: $U_0 = \mathrm{diag}\{e^{i\varphi}, e^{-i\varphi}\}$, minimizing $e_{\rm vac}^{(2)}(\theta)$ gives

$$\tan\varphi = -\epsilon \tan\frac{\theta}{2}, \quad \epsilon = \frac{m_d - m_u}{m_d + m_u}$$

LO results: R. Brower et al., PLB 560, 64 (2003)

- ☐ Vacuum energy density: $e_{\rm vac}^{(2)}(\theta) = -F^2 \mathring{M}^2(\theta)$
- ☐ Pion mass:

$$\mathring{M}^2(\theta) = 2B\hat{m}\cos\frac{\theta}{2}\sqrt{1+\epsilon^2\tan^2\frac{\theta}{2}} \qquad \hat{m} = \frac{m_d + m_u}{2}$$

- ☐ Topological susceptibility and the 4th cumulant ($\Rightarrow$ axion mass and self-interaction)

$$\chi_t^{(2)} = \frac{1}{2}F^2 B\hat{m}(1-\epsilon^2) \qquad \Rightarrow \; m_{a,\rm LO}^2 f_a^2 = \frac{1}{4}F^2 M_\pi^2$$

$$c_4^{(2)} = -\frac{1}{8}F^2 B\hat{m}(1+2\epsilon^2-3\epsilon^4)$$

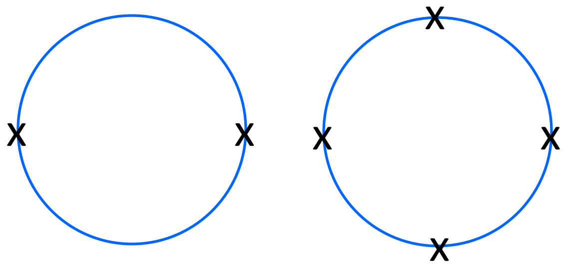
See also H. Leutwyler, A.V. Smilga, PRD 46, 5607 (1992); S. Aoki, H. Fukaya, PRD 81, 034022 (2010); Y.Y. Mao, T.W. Chiu, PRD 80, 034502 (2009)

Theta vacuum energy at NLO

FKG, U.-G. Meißner, PLB 749, 278 (2015)

- χ_t, c_4 were calculated up to the next-to-leading order (NLO) in ChPT

  Y.Y. Mao, T.W. Chiu, PRD 80, 034502 (2009); V. Bernard, S. Descotes-Genon, G. Toucas, JHEP 12, 080 (2012)



obtained as an expansion in powers of θ

- ➢ complicated derivation for each cumulant

1-loop effective action: $$Z_{1\text{-loop}} = \frac{i}{2}\ln(\det D)$$

with the differential operator $$D = D_0(\theta) + \hat{V}(\theta)$$

$$D_0(\theta) = \delta_{PQ}\left(\Box + \mathring{M}_P^2(\theta)\right)$$

$$\hat{V}(\theta) = \sigma^\chi + \underbrace{\sigma^\Delta + \left\{\hat{\Gamma}_\mu, \partial^\mu\right\} + \hat{\Gamma}_\mu\hat{\Gamma}^\mu}_{\text{derivatives/external fields}}$$

$$\sigma^\chi_{PQ} = \frac{1}{8}\left\langle\left\{\lambda_P, \lambda_Q^\dagger\right\}\left(\chi_\theta^\dagger U + U^\dagger\chi_\theta\right)\right\rangle - \delta_{PQ}\mathring{M}_P^2(\theta)$$

contributes to the vacuum energy

expanded as $\mathring{M}_P^2(0) + \cdots$

$$Z_{1\text{-loop}} = \frac{i}{2}\,\mathrm{Tr}\ln D_0 + \underbrace{\frac{i}{2}\,\mathrm{Tr}\left(D_0^{-1}\hat{V}\right)}_{\text{tadpoles}} - \underbrace{\frac{i}{4}\,\mathrm{Tr}\left(D_0^{-1}\hat{V}D_0^{-1}\hat{V}\right)}_{\text{2-point loops}} + \cdots.$$

Typo in Eq.(8.5) in the GL (1985) paper: $i/2$ in the second term was written as $i/4$

Theta vacuum energy at NLO

FKG, U.-G. Meißner, PLB 749, 278 (2015)

- A much easier way to get all cumulants (thus the axion potential at NLO) at once:

Notice $\frac{1}{8}\left\langle\left\{\lambda_P, \lambda_Q^\dagger\right\}\left(\chi_\theta^\dagger U_0 + \chi_\theta U_0^\dagger\right)\right\rangle = \delta_{PQ}\mathring{M}_P^2(\theta) \quad \Rightarrow \sigma_{PQ}^\chi\Big|_{U\to U_0} = 0$

The 1-loop contribution to the vacuum energy density is then given by

$$e_{\rm vac}^{(4,\,\rm loop)}(\theta) = -\frac{i}{2V}\ln\det D_0(\theta) = -\frac{i}{2V}\,{\rm Tr}\ln D_0(\theta)$$

$$= -\frac{i}{2}\left(N^2-1\right)\int\frac{d^dp}{(2\pi)^d}\ln\left[-p^2+\mathring{M}^2(\theta)\right]$$

$$= \frac{i}{2}\left(N^2-1\right)\int\frac{d^dp}{(2\pi)^d}\int_0^\infty\frac{d\tau}{\tau}e^{-\tau\left[-p^2+\mathring{M}^2(\theta)\right]}$$

$$= \left(N^2-1\right)\mathring{M}^4(\theta)\left\{\frac{\lambda}{2} - \frac{1}{128\pi^2}\left[1-2\ln\frac{\mathring{M}^2(\theta)}{\mu^2}\right]\right\}$$

$$\lambda \equiv \frac{\mu^{d-4}}{16\pi^2}\left[\frac{1}{d-4}-\frac{1}{2}\left(\ln(4\pi)+\Gamma'(1)+1\right)\right]$$

UV divergence

holds for SU(2) as well as SU(N) with degenerate quark flavors

Theta vacuum energy at NLO

FKG, U.-G. Meißner, PLB 749, 278 (2015)

- NLO tree-level contribution from the LECs and HECs

$$\begin{aligned} e_{\rm vac}^{(4,\ {\rm tree})}(\theta) =& -\frac{l_3}{16}\left\langle \chi_\theta^\dagger U_0 + \chi_\theta U_0^\dagger \right\rangle^2 + \frac{l_7}{16}\left\langle \chi_\theta^\dagger U_0 - \chi_\theta U_0^\dagger \right\rangle^2 \\ & - \frac{h_1+h_3}{4}\left\langle \chi_\theta^\dagger \chi_\theta \right\rangle - \frac{h_1-h_3}{2}\,{\rm Re}\left(\det \chi_\theta\right) \\ =& -\mathring{M}^4(\theta)\left\{ l_3 + l_7\left[\frac{\left(1-\epsilon^2\right)\tan(\theta/2)}{1+\epsilon^2\tan^2(\theta/2)}\right]^2 \right\} \\ & - 2B^2\hat{m}^2\left[(h_1+h_3)\left(1+\epsilon^2\right) + (h_1-h_3)\left(1-\epsilon^2\right)\cos\theta\right] \end{aligned}$$

- UV divergences from 1-loop and from counterterms cancel, we get the NLO vacuum energy density in a closed, simple form:

$$\begin{aligned} e_{\rm vac}(\theta) =& -F^2\mathring{M}^2(\theta) - \mathring{M}^4(\theta)\left\{ \frac{3}{128\pi^2}\left[1 - 2\ln\frac{\mathring{M}^2(\theta)}{\mu^2}\right] \right. \\ & \left. + l_3^r + h_1^r - h_3 + l_7\left[\frac{\left(1-\epsilon^2\right)\tan(\theta/2)}{1+\epsilon^2\tan^2(\theta/2)}\right]^2 \right\} \end{aligned}$$

- A similar expression can be derived for SU(N) in the symmetric limit
- Any topological cumulants can now be easily computed up to NLO!

Theta vacuum energy at NLO

FKG, U.-G. Meißner, PLB 749, 278 (2015)

- Following the suggestion in V. Bernard, S. Descotes-Genon, G. Toucas, JHEP 12, 080 (2012):
  - Extracting SU(N) symmetric quark condensate from LEC-free combination of cumulants
  - Easy to construct combinations

$$\chi_t + \frac{N^2}{4}c_4 = \frac{3F_N^2 B_N m}{4N} + \frac{3\left(N^2-1\right)B_N^2 m^2}{32\pi^2 N^2} + \mathcal{O}\left(p^6\right)$$

$$\chi_t - \frac{N^4}{16}c_6 = \frac{15F_N^2 B_N m}{16N} + \frac{15\left(N^2-1\right)B_N^2 m^2}{64\pi^2 N^2} + \mathcal{O}\left(p^6\right)$$

- Sum rule between the topological sector and quark condensate free of NLO correction:

$$\Sigma_N = \frac{N}{m}\left(\frac{8}{5}\chi_t + \frac{2N^2}{3}c_4 + \frac{N^4}{15}c_6\right) + \mathcal{O}\left(p^6\right)$$

Theta vacuum energy at NLO

Z.-Y. Lu, M.-L. Du, FKG, U.-G. Meißner, T. Vonk, JHEP 05, 001 (2020)

- SU(N) ($N \geq 3$) with non-degenerate quark flavors

$$e_{\rm vac} = -F_0^2 B_0 \sum_f m_f \cos\phi_f - \sum_P \frac{\mathring{A}_P^4(\theta)}{128\pi^2}\left[1 - 2\ln\frac{\mathring{A}_P^2(\theta)}{\mu^2}\right]$$
$$- 16 B_0^2 \left[L_6^r \left(\sum_f m_f \cos\phi_f\right)^2 + N\left(N L_7^r + L_8^r\right) m_1^2 \cos^2\phi_1\right]$$

- No closed expression is possible; relation between the vacuum alignment angle ϕ_f and θ is complicated

$$\phi_f = \sum_{n=0}^{\infty} C_{f,2n+1}\theta^{2n+1}$$

- Recursion relation:

$$C_{f,2n+1} = \sum_{t=1}^{n} \sum_{(k_1,\ldots,k_t)} s_{K_t} \binom{K_t}{k_1,\ldots,k_t} \left[\frac{\bar{m}}{m_f} \sum_{i=1}^{N} \prod_{j=1}^{t} C_{i,2j-1}^{k_j} - \prod_{j=1}^{t} C_{f,2j-1}^{k_j}\right]$$

and $C_{f,1} = \bar{m}/m_f$ $\qquad \frac{1}{\bar{m}} = \sum_i \frac{1}{m_i}$

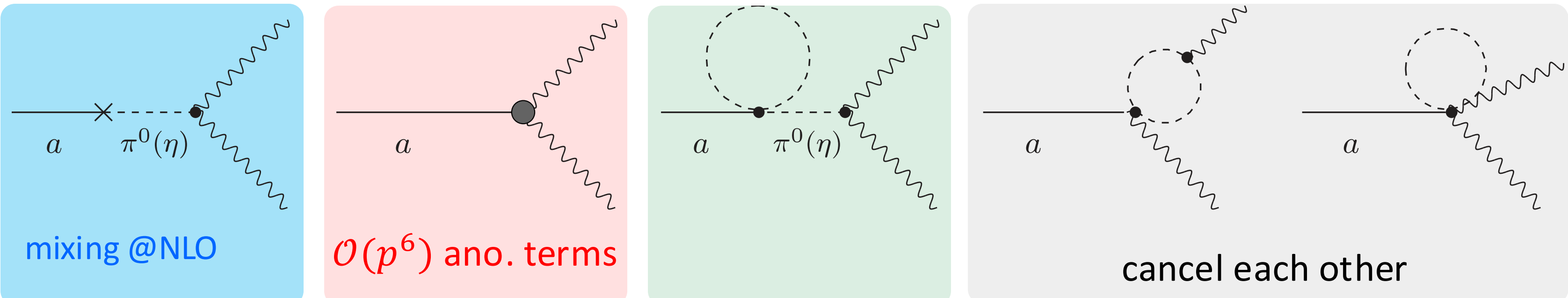
- The θ vacuum energy density with $\theta \to a/f_a$ is the QCD axion potential!

Axion mass and photon coupling at NLO

Z.-Y. Lu, M.-L. Du, FKG, U.-G. Meißner, T. Vonk, JHEP 05, 001 (2020)

- Axion mass and self-coupling at the NLO
- Axion-photon coupling $\mathcal{L}_{a\gamma\gamma} = \frac{1}{4} g_{a\gamma\gamma} a F^{\mu\nu} \tilde{F}_{\mu\nu}$, $\quad g_{a\gamma\gamma} = \frac{\alpha_{\rm em}}{2\pi f_a} \frac{\mathcal{E}}{\mathcal{C}} + g_{a\gamma\gamma}^{\rm QCD}$

depending on high-energy model



$$g_{a\gamma\gamma} = \frac{\alpha_{\rm em}}{2\pi f_a} \left\{ \frac{\mathcal{E}}{\mathcal{C}} - \overbrace{\frac{2}{3} \frac{m_u + 3\bar{m}}{m_u}}^{\text{sizable } \not{I}} - \underbrace{\frac{1024\pi^2}{3\hat{m}} \bar{m} M_{\pi^0}^2 \left(C_7^W + 3 C_8^W \right)}_{\mathcal{O}(p^6)\text{ anomalous terms}} + \frac{2\bar{m} M_{\pi^0}^2}{3\hat{m}} \overbrace{\left[\frac{f_+(\cos,\sin)}{\sqrt{3} M_\eta^2} + \frac{f_-(\sin,\cos)}{M_{\pi^0}^2} \right]}^{\text{Loop corrections}} \right\}$$

J. Bijnens, L. Girlanda, P. Talavera, EPJC 23, 539 (2002)

Axion mass and photon coupling at NLO

- Axion properties at NLO:

[53]: G. Grilli di Cortona et al., JHEP 01, 034 (2016)

N	$m_a\,\left[\mu\text{eV}\cdot\frac{10^{12}\,\text{GeV}}{f_a}\right]$	$(-\lambda_4)^{1/4}\,[10^{-2}\text{GeV}/f_a]$	$g_{a\gamma\gamma}^{\text{QCD}}\,\left[\frac{\alpha_{\text{em}}}{2\pi f_a}\right]$	$\chi_t^{1/4}\,[\text{MeV}]$	b_2
2 [53]	5.70(7)	5.79(10)	−1.92(4)	75.5(5)	−0.029(2)
3	5.71(9)	5.77(18)	−2.05(3)	75.6(6)	−0.028(3)

5.691(51)

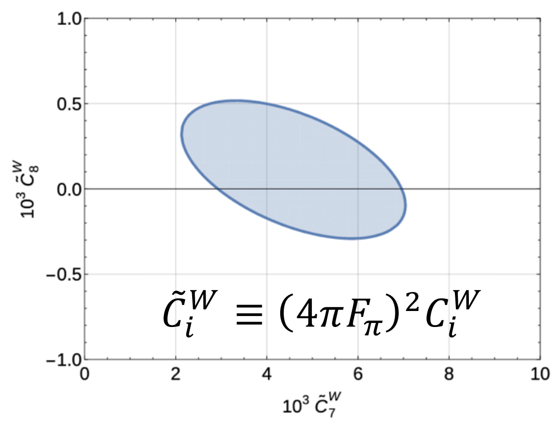
Early NLO calculation: M. Spalinski, EPJC 41, 87 (1988)

w/ NNLO + QED corrections M. Gorghetto, G. Villadoro, JHEP 03, 033 (2019)

Differences in $g_{a\gamma\gamma}^{\text{QCD}}$: loops +

➢ C_7^W neglected based on the estimate $C_7^W \ll C_8^W$ in Kampf, Moussallam, PRD 79, 076005 (2009); C_8^W: from $\eta \to \gamma\gamma$

➢ $C_7^W > C_8^W$ in G. Grilli di Cortona et al., JHEP 01, 034 (2016)



➢ $g_{a\gamma\gamma}^{\text{QCD}}$ in U(3) ChPT: $-1.63(1)$ R. Gao, Z.-H. Guo, J. Oller, H.-Q. Zhou, JHEP 04, 022 (2023)

neglected isospin breaking, which is sizable here

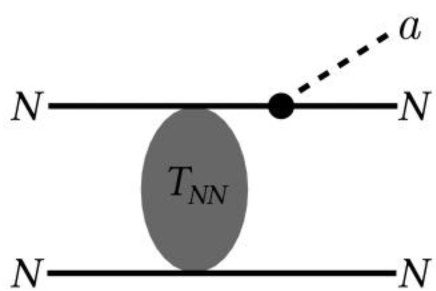
Axion-baryon couplings

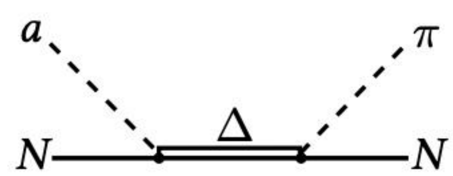
- Why?
  - nuclear bremsstrahlung processes $NN \to NNa$ in massive stellar objects

    e.g., Turner, Phys.Rept. 197, 67 (1990); Raffelt, Phys.Rept. 197, 67 (1990); G. Raffelt, D. Seckel, Phys. Rev. D 52 (1995) 1780; Hanhart, Phillips, Reddy, PLB 499, 9 (2005); …
  - $N\pi \to Na, \pi\pi \to \pi a$ also for axion production during nucleosynthesis

    S. Chang, K. Choi, PLB 316, 51 (1993)
  - novel perspectives in experimental axion (of meV scale) searches?

    P. Carenza et al., PRL 126, 071102 (2021)





  - Impact of hyperons in neutron stars see, e.g., L. Tolos, L. Fabbietti, PPNP 112, 103770 (2020)

Many interesting applications of ChPT to ALP in mesonic processes, too, e.g.,

M. Bauer et al., PRL 127, 081803 (2021); C. Cornella et al., JHEP 06, 029 (2024); D.S. Alves, S. Gonzàlez-Solís, JHEP 07, 264 (2024); …

QCD with axion

- QCD Lagrangian with axion

$$\mathcal{L}_{\rm QCD+a} = \mathcal{L}_{\rm QCD,0} - \bar{q}\mathcal{M}_q q + \frac{a}{f_a}\left(\frac{g}{4\pi}\right)^2 {\rm Tr}\left[G_{\mu\nu}\tilde{G}^{\mu\nu}\right] + \frac{\partial^\mu a}{2f_a} J_\mu^{\rm PQ}$$

PQ current: $J_\mu^{\rm PQ} = f_a \partial_\mu a + \bar{q}\gamma_\mu\gamma_5 \mathcal{X}_q q$

last term: model-dependent

here assuming diagonal axion-quark coupling matrix: $\mathcal{X}_q = {\rm diag}\{X_q\}$

$$X_q^{\rm KSVZ} = 0$$

$$X_{u,d,s}^{\rm DFSZ} = \frac{1}{3}\sin^2\beta$$

$$X_{c,b,t}^{\rm DFSZ} = \frac{1}{3}\cos^2\beta = \frac{1}{3} - X_{u,d,s}^{\rm DFSZ}$$

typical invisible axion models

$\tan\beta \in [0.25, 170]$: ratio of VEVs of the two Higgs doublets in the DFSZ model

L. Di Luzio et al., Phys.Rept. 870, 1 (2020)

KSVZ: Kim, PRL 43, 103 (1979); Shifman, Vainshtein, Zakharov, NPB 166, 493 (1980)
DFSZ: Dine, Fischler, Srednicki, PLB 104, 199 (1981); Zhitnitsky, SJNP 31, 260 (1980)

QCD with axion

T. Vonk, FKG, U.-G. Meißner, JHEP 03, 138 (2020); JHEP 08, 024 (2021)

- With an axial rotation, so that

$$\mathcal{L}_{aq} = -\left(\bar{q}_L \mathcal{M}_a q_R + \text{ h.c. }\right) + \bar{q}\gamma^\mu\gamma_5 \frac{\partial_\mu a}{2f_a}\left(\mathcal{X}_q - \mathcal{Q}_a\right) q$$

$$\mathcal{M}_a = \exp\left(i\frac{a}{f_a}\mathcal{Q}_a\right)\mathcal{M}_q, \quad \mathcal{Q}_a \approx \frac{1}{1+\underbrace{z}_{m_u/m_d}+\underbrace{w}_{m_u/m_s}} \operatorname{diag}(1,z,w,0,0,0)$$

- Rewrite in the form of coupling to external currents $s, p, a_\mu, a_{\mu,i}^{(s)}$:

$$\left(\bar{q}\gamma^\mu\gamma_5 \frac{\partial_\mu a}{2f_a}\left(c^{(1)} + c^{(3)}\lambda_3 + c^{(8)}\lambda_8\right) q\right)_{q=(u,d,s)^{\mathrm{T}}} + \sum_{q=\{c,b,t\}}\left(\bar{q}\gamma^\mu\gamma_5 \frac{\partial_\mu a}{2f_a} X_q q\right)$$

$$c^{(1)} = \frac{1}{3}\left(X_u + X_d + X_s - 1\right)$$

$$c^{(3)} = \frac{1}{2}\left(X_u - X_d - \frac{1-z}{1+z+w}\right)$$

$$c^{(8)} = \frac{1}{2\sqrt{3}}\left(X_u + X_d - 2X_s - \frac{1+z-2w}{1+z+w}\right)$$

$$s + ip = \mathcal{M}_a$$

$$a_\mu = \frac{\partial_\mu a}{2f_a}\left(c^{(3)}\lambda_3 + c^{(8)}\lambda_8\right)$$

$$a_{\mu,i}^{(s)} = c_i \frac{\partial_\mu a}{2f_a}, \quad i = 1,\ldots,4$$

$$c_1 = c^{(1)},\ c_2 = X_c,\ c_3 = X_b,\ c_4 = X_t$$

- Building blocks with axion:

$$u_\mu = i\left[u^\dagger \partial_\mu u - u\partial_\mu u^\dagger - iu^\dagger a_\mu u - iua_\mu u^\dagger\right]$$

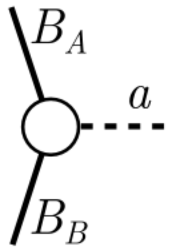
$$u_{\mu,i} = i\left[-iu^\dagger a_{\mu,i}^{(s)} u - iua_{\mu,i}^{(s)} u^\dagger\right] = 2a_{\mu,i}^{(s)}$$

$$\chi_\pm = 2B_0\left[u^\dagger (s+ip) u^\dagger \pm u(s+ip)^\dagger u\right]$$

Axion-baryon couplings

T. Vonk, FKG, U.-G. Meißner, JHEP 03, 138 (2020); JHEP 08, 024 (2021)

- General form of axion-baryon couplings:

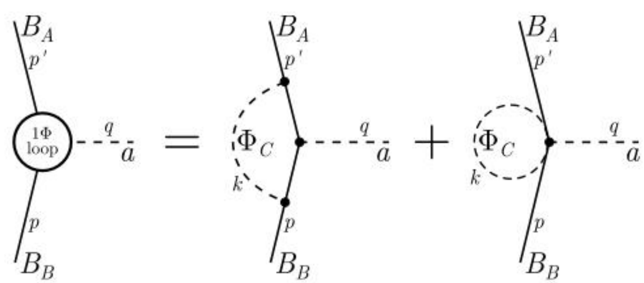


$$= G_{aAB}\,(S\cdot q) \quad \text{with } G_{aAB} = -\frac{1}{f_a}g_{aAB} + \mathcal{O}\left(\frac{1}{f_a^2}\right)$$

- Expansion in the chiral power counting

$$g_{aAB} = \underbrace{g_{aAB}^{(1)}}_{\text{LO}} + \underbrace{g_{aAB}^{(2)}}_{1/m_B} + \underbrace{g_{aAB}^{(3)}}_{\text{NNLO},1/m_B^2,\text{1-loop}} + \ldots$$

$$\begin{aligned}\mathcal{L}^{(3)} &= d_{36}(\lambda)\langle(v\cdot\partial\bar{B})\{(S\cdot u),(v\cdot\partial B)\}\rangle\\ &+ d_{37}(\lambda)\langle(v\cdot\partial\bar{B})[(S\cdot u),(v\cdot\partial B)]\rangle\\ &+ d_{38}(\lambda)\langle\bar{B}\{[(v\cdot\partial),[(v\cdot\partial),(S\cdot u)]],B\}\rangle\\ &+ d_{39}(\lambda)\langle\bar{B}[[(v\cdot\partial),[(v\cdot\partial),(S\cdot u)]],B]\rangle\\ &+ d_{36}^{i}(\lambda)\left\langle(v\cdot\partial\bar{B})\,(S\cdot u_i)\,(v\cdot\partial B)\right\rangle\\ &+ d_{38}^{i}(\lambda)\left\langle\bar{B}\,[(v\cdot\partial),[(v\cdot\partial),(S\cdot u_i)]]\,B\right\rangle\\ &+ \ldots\end{aligned}$$



new terms

N. Fettes, U.-G. Meißner, S. Steininger, NPA 640, 199 (1998); M. Frink, U.-G. Meißner, EPJA 29, 255 (2006)

Axion-baryon couplings

T. Vonk, FKG, U.-G. Meißner, JHEP 03, 138 (2020); JHEP 08, 024 (2021)

- LO couplings (for aNN, see D. Kaplan, NPB 260, 215 (1985); G. Grilli di Cortona et al., JHEP 01, 034 (2016); …)

$$g_{app}^{(1)} = -\frac{\Delta u + z\Delta d + w\Delta s}{1+z+w} + \Delta u X_u + \Delta d X_d + \Delta s X_s$$

$$g_{ann}^{(1)} = -\frac{\Delta d + z\Delta u + w\Delta s}{1+z+w} + \Delta d X_u + \Delta u X_d + \Delta s X_s$$

$$g_{a\Sigma^+\Sigma^+}^{(1)} = -\frac{\Delta u + z\Delta s + w\Delta d}{1+z+w} + \Delta u X_u + \Delta s X_d + \Delta d X_s$$

$$g_{a\Sigma^0\Sigma^0}^{(1)} = -\frac{\frac{\Delta u+\Delta s}{2}(1+z) + w\Delta d}{1+z+w} + \frac{\Delta u + \Delta s}{2}(X_u + X_d) + \Delta d X_s$$

$$g_{a\Lambda\Lambda}^{(1)} = -\frac{\frac{\Delta u+4\Delta d+\Delta s}{6}(1+z) + \frac{2\Delta u-\Delta d+2\Delta s}{3}w}{1+z+w} + \frac{\Delta u + 4\Delta d + \Delta s}{6}(X_u + X_d) + \frac{2\Delta u - \Delta d + 2\Delta s}{3}X_s,$$

$$g_{a\Sigma^0\Lambda}^{(1)} = -\frac{\frac{\Delta u-2\Delta d+\Delta s}{2\sqrt{3}}(1-z)}{1+z+w} + \frac{\Delta u - 2\Delta d + \Delta s}{2\sqrt{3}}(X_u - X_d)$$

$$\ldots$$

LECs D, F and D^1 [SU(3) singlet axial coupling] has been matched to $s^\mu \Delta q = \langle p\,|\bar{q}\gamma^\mu\gamma_5 q|\,p\rangle$

$$\Delta u = 0.847(50), \quad \Delta d = -0.407(34), \quad \Delta s = -0.035(13)$$ FLAG19

quark contributions to the nucleon spin

Axion-baryon couplings

T. Vonk, FKG, U.-G. Meißner, JHEP 08, 024 (2021)

- Results w/ naturalness assumption for the unknown LECs at $\mathcal{O}(p^3)$ (dominant uncertainty) in HBChPT

Process	KSVZ	DFSZ
$\Sigma^+ \to \Sigma^+ + a$	$-0.547(84)$	$-0.709(94) + 0.446(54)\sin^2\beta$
$\Sigma^- \to \Sigma^- + a$	$-0.245(80)$	$-0.113(92) - 0.142(54)\sin^2\beta$
$\Sigma^0 \to \Sigma^0 + a$	$-0.399(78)$	$-0.417(87) + 0.158(43)\sin^2\beta$
$p \to p + a$	$-0.432(86)$	$-0.589(96) + 0.436(53)\sin^2\beta$
$\Xi^- \to \Xi^- + a$	$0.166(79)$	$0.299(91) - 0.161(52)\sin^2\beta$
$n \to n + a$	$0.003(83)$	$0.271(94) - 0.400(53)\sin^2\beta$
$\Xi^0 \to \Xi^0 + a$	$0.303(81)$	$0.570(92) - 0.409(52)\sin^2\beta$
$\Lambda \to \Lambda + a$	$0.138(87)$	$0.314(96) - 0.228(47)\sin^2\beta$
$\Sigma^0 \to \Lambda + a$, $\Lambda \to \Sigma^0 + a$	$-0.161(24)$	$-0.323(33) + 0.309(32)\sin^2\beta$

Pion axioproduction

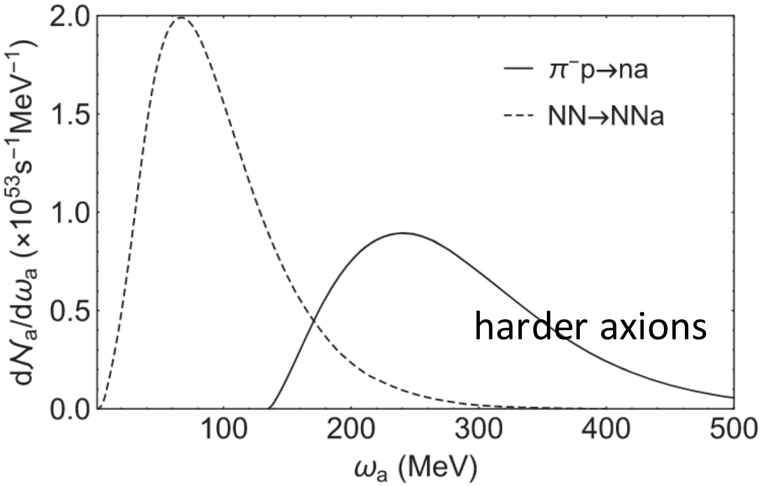
- Effects of pions in supernova, $\pi^- p \to na$, found to be important
  - surpasses bremsstrahlung $NN \to NNa$, increases axion emission
  - leads to harder axions
    - possible detection with megaton water Cherenkov detectors via $aN \to \pi N$
  - $aN \to \pi N$ was estimated as

assumed to be ~ 100 mb

$$\sigma(aN \to \pi N) \approx \frac{F_\pi^2}{f_a^2}\sigma(\pi N \to \pi N) \sim 1\ \text{mb}\left(\frac{\text{GeV}}{f_a}\right)^2$$

and Δ was argued to be important;

10^3 pions estimated for a SN at 1 kiloparsec



P. Carenza et al., PRL 126, 071102 (2021)

- However, $aN \to \Delta \to \pi N$ breaks isospin!
  - Is the suppression factor $\sim 10^{-2}$ as given by $\mathcal{O}(\frac{m_d - m_u}{m_s, \Lambda_{\rm QCD}})$ or $\mathcal{O}(\alpha)$?

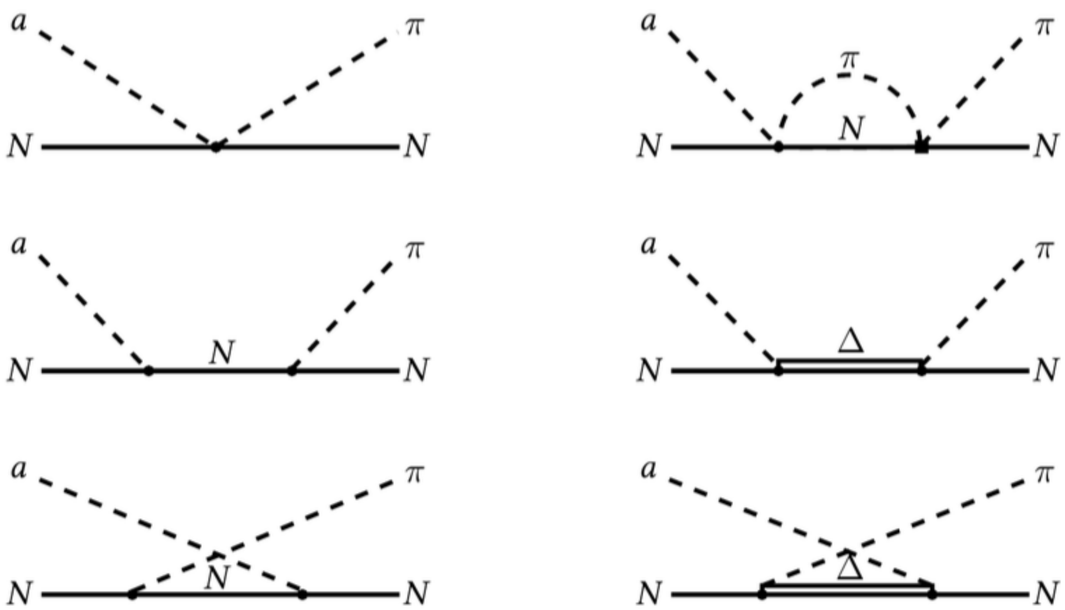
Pion axioproduction

T. Vonk, FKG, U.-G. Meißner, PRD 105, 054029 (2022)

- Mild isospin breaking in $aN \to \Delta \to \pi N$: $\propto \frac{m_d - m_u}{m_d + m_u} \approx 0.34$

$$c^{(3)} = \frac{1}{2}\left(X_u - X_d - \frac{1-z}{1+z+w}\right)$$

- Real calculations reveal at least one order of magnitude suppression (depending on the axion models) ⇒ at most $\mathcal{O}(100)$ pions



Supernova axion emission with Δ contribution estimated in S.-Y. Ho et al., PRD 107, 075002 (2023)

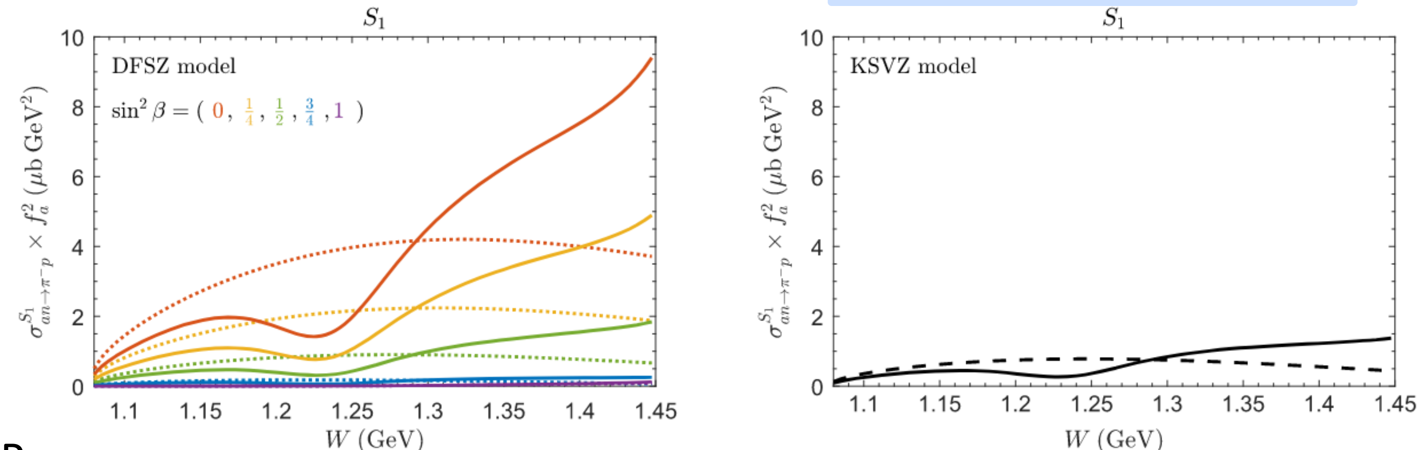
Pion axioproduction

C.-C. Li, T.-R. Hu, FKG, U.-G. Meißner, PRD 109, 075050 (2024)

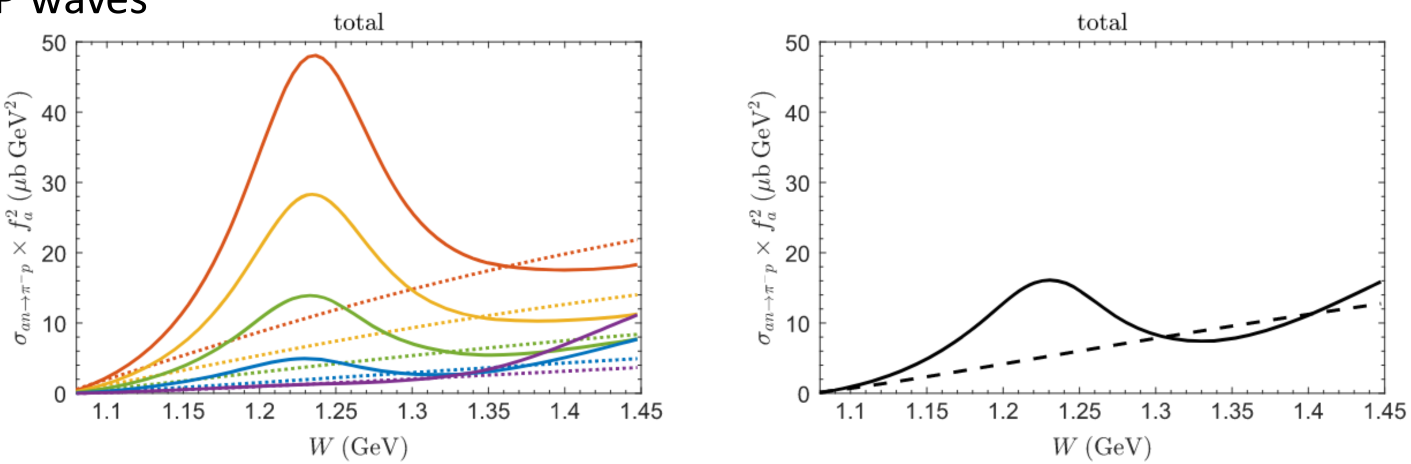
- Including both the $\Delta(1232)$ and the Roper resonance $N^*(1440)$ that strongly couples to πN, depending on $\sin^2\beta$, about 20 to 1000 smaller than the naïve estimate of $\sim 1\ \mathrm{mb}\left(\frac{\mathrm{GeV}}{f_a}\right)^2$
  - $an \to \pi^- p$ cross sections

solid: w/ resonances
dashed: w/o resonances

✓ S waves

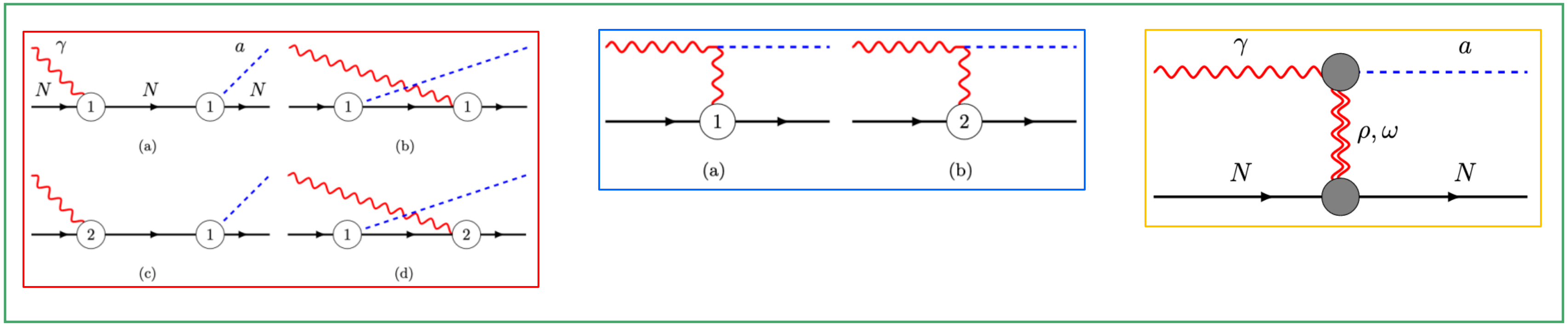


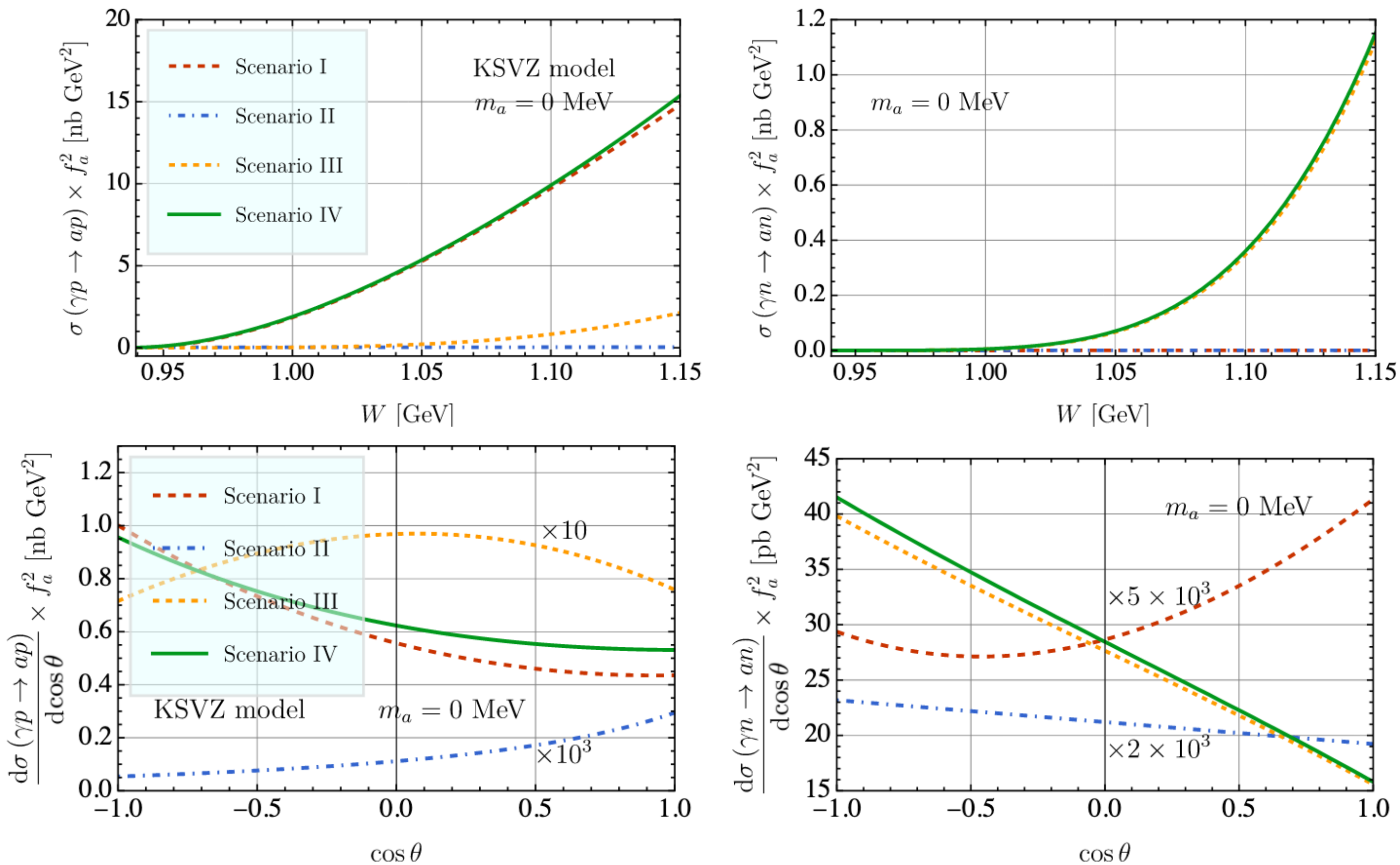
✓ Total: S+P waves



$\gamma N \to aN$

X.-H. Cao, Z.-H. Guo, arXiv: 2408.15825





Summary

- Derived a closed form of the θ-vacuum energy density (QCD axion potential) at NLO
  - ✓ NLO-correction free relation between quark condensate and top. cumulants
- Precision calculation of the axion properties
  - ✓ mass, self-couplings, $a\gamma\gamma$ coupling
  - ✓ axion-baryon couplings
    - ➢ $g_{a\Lambda\Lambda}$ could be much larger than g_{ann}, impact on the axion emissivity of dense stellar objects such as neutron stars?
  - ✓ Pion axioproduction cross section much smaller than naïve estimate
  - ☐ Consequences remain to be explored

Thank you for your attention!