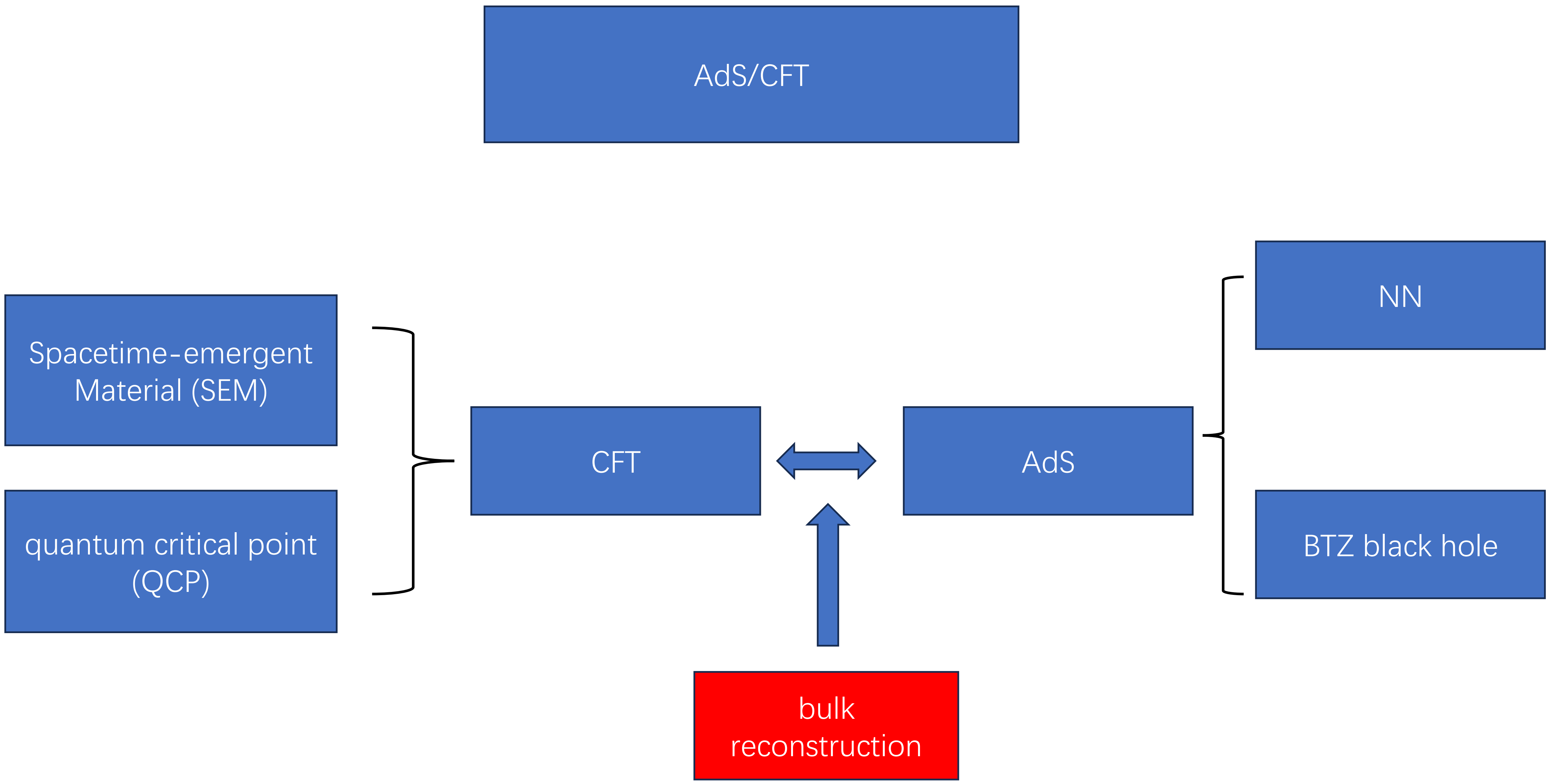
Machine-learning,linear response

Machine-learning emergent spacetime from linear response in future tabletop quantum gravity experiments

Arxiv：2411.16052


AdS/CFT
Spacetime-emergent Material (SEM)
quantum critical point (QCP)
CFT
AdS
NN
BTZ black hole
bulk reconstruction

Klein-Gordon equation of the scalar field

bulk metric

$$\begin{cases} ds^2 = g_{tt}(\xi)dt^2 + g_{\xi\xi}(\xi)d\xi^2 + g_{\theta\theta}(\xi)d\theta^2 \\ \Xi(\xi) = g_{\xi\xi}g^{tt}, \quad \Theta(\xi) = g_{\xi\xi}g^{\theta\theta} \end{cases}$$

Klein-Gordon

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu v}\partial_\nu\Phi\right) = 0$$

Klein-Gordon equation of the scalar field

bulk metric

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Klein-Gordon

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu v}\partial_\nu\Phi\right) = 0$$

$$\Phi(t,\theta,\xi) = \sum_n \Phi_n(\xi)e^{-i\omega t + ik_n\theta}$$

$$\Pi_n(\xi) = \Phi_n'(\xi) \qquad k_n = \frac{2\pi n}{a}$$

$$0 = -\omega^2\Xi(\xi)\Phi_n(\xi) + \partial_\xi\left(\ln\sqrt{-g}g^{\xi\xi}\right)\Pi_n(\xi) + \Pi_n'(\xi) - k_n^2\Theta(\xi)\Phi_n(\xi).$$

Klein-Gordon equation of the scalar field

bulk metric

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$$\mathbf{Z}_n(\xi+\Delta\xi) = \begin{pmatrix} 1 & \Delta\xi \\ \Delta\xi\left(\omega^2\Xi(\xi) + k_n^2\Theta(\xi)\right) & 1 - \Delta\xi/\xi \end{pmatrix}\mathbf{Z}_n(\xi)$$

$$\mathbf{Z}_n = (\Phi_n, \Pi_n)^{\mathrm{T}}$$

Klein-Gordon equation of the scalar field

bulk metric

$$\begin{cases} ds^2 = g_{tt}(\xi)dt^2 + g_{\xi\xi}(\xi)d\xi^2 + g_{\theta\theta}(\xi)d\theta^2 \\ \Xi(\xi) = g_{\xi\xi}g^{tt}, \quad \Theta(\xi) = g_{\xi\xi}g^{\theta\theta} \end{cases}$$

Klein-Gordon

$$\mathbf{Z}_n(\xi + \Delta\xi) = \begin{pmatrix} 1 & \Delta\xi \\ \Delta\xi\left(\omega^2\Xi(\xi) + k_n^2\Theta(\xi)\right) & 1 - \Delta\xi/\xi \end{pmatrix} \mathbf{Z}_n(\xi)$$

$$\mathbf{Z}_n = (\Phi_n, \Pi_n)^{\mathrm{T}}$$

BTZ black hole

$$ds^2_{\mathrm{BTZ}} = -\frac{r_h^2 \xi}{L^2(1-\xi)} dt^2 + \frac{L^2}{4\xi(1-\xi)^2} d\xi^2 + \frac{r_h^2}{L^2(1-\xi)} d\theta^2$$

$$\xi \in [0, 1]$$

double Wick rotation

AdS soliton metric

$$ds^2_{\mathrm{Sol}} = -\frac{r_s^2}{L^2(1-\xi)} dt^2 + \frac{L^2}{4\xi(1-\xi)^2} d\xi^2 + \frac{r_s^2 \xi}{L^2(1-\xi)} d\theta^2.$$

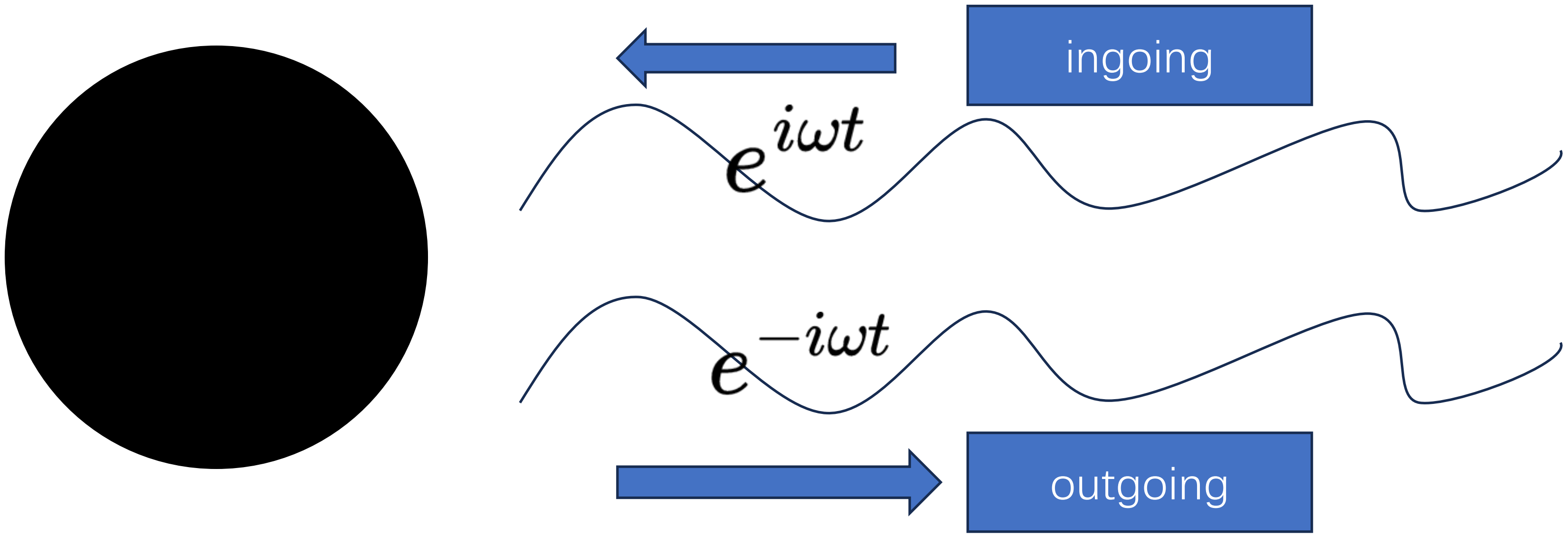
8.6	AdS/CFT Duality User Guide

$$Z_{\mathrm{gauge}} \simeq e^{-\mathrm{S_E}} + e^{-\mathrm{S'_E}}$$

$$\Delta F = F_{\mathrm{BH}} - F_{\mathrm{soliton}} = -\frac{V_3 L^3}{16\pi G_5} \pi^4 \left(T^4 - \frac{1}{l^4} \right)$$

high temperature	black hole
low temperature	AdS soliton

14.2	AdS/CFT Duality User Guide



The ingoing boundary condition corresponds to the retarded propagation.

Klein-Gordon

ingoing

$$\Phi_n = \xi^{\frac{\alpha_n+\beta_n}{2}} F\left(\alpha_n, \beta_n, \gamma_n; \xi\right)$$

Hypergeometric function

BTZ black hole

$$\alpha_n := -i\frac{L^2}{2r_h}(\omega + k_n), \beta_n := -i\frac{L^2}{2r_h}(\omega - k_n)$$

$$\gamma_n := 1 + \alpha_n + \beta_n$$

AdS soliton metric

$$\alpha_n := \frac{L^2}{2r_s}(|k_n| - \omega), \beta_n := \frac{L^2}{2r_s}(|k_n| + \omega)$$

linear response

$$\delta S = \int d^4x \phi^{(0)}(t, \boldsymbol{x}) O(\boldsymbol{x})$$

9.1.2	AdS/CFT Duality User Guide

$$\delta\langle O(t, \boldsymbol{x})\rangle = -\int_{-\infty}^{\infty} d^4x' G_R^{OO}(t - t', \boldsymbol{x} - \boldsymbol{x}') \phi^{(0)}(t', \boldsymbol{x}').$$

AdS/CFT

In many examples $\langle O\rangle = 0$, so $\delta\langle O\rangle = \langle O\rangle_s - \langle O\rangle = \langle O\rangle_s$.

$$\Phi(z, x) = A(x) z^{d-\Delta} + B(x) z^{\Delta} + \cdots$$

$$\Delta = \frac{d}{2} + v \quad v = \sqrt{\frac{d^2}{4} + m^2 R^2}$$

$$A(x) \Longleftrightarrow \int d^d x \phi_0(x) \mathcal{O}_s(x), \phi_0(x) = A(x)$$

$$B(x) \Longleftrightarrow \mathcal{O}_s(x) (B(x) \sim \langle \mathcal{O}_s(x)\rangle)$$

external source $\phi^{(0)}$	$\rightarrow$ response to O
magnetic field H	magnetization m
gauge potential μ	charge density ρ
vector potential $A_i^{(0)}$	current J^i
spacetime fluctuation $h_{\mu\nu}^{(0)}$	energy-momentum tensor $T^{\mu\nu}$

Hong Liu	String Theory and Holographic Duality, Lec19

ingoing

$$\Phi_n(\xi) = C_n^1 \xi^{\frac{\alpha_n+\beta_n}{2}} F(\alpha_n, \beta_n, \gamma_n; \xi)$$
$$+ C_n^2 \xi^{-\frac{\alpha_n+\beta_n}{2}} F(-\beta_n, -1-\alpha_n, 1-\alpha_n-\beta_n; \xi).$$

outgoing

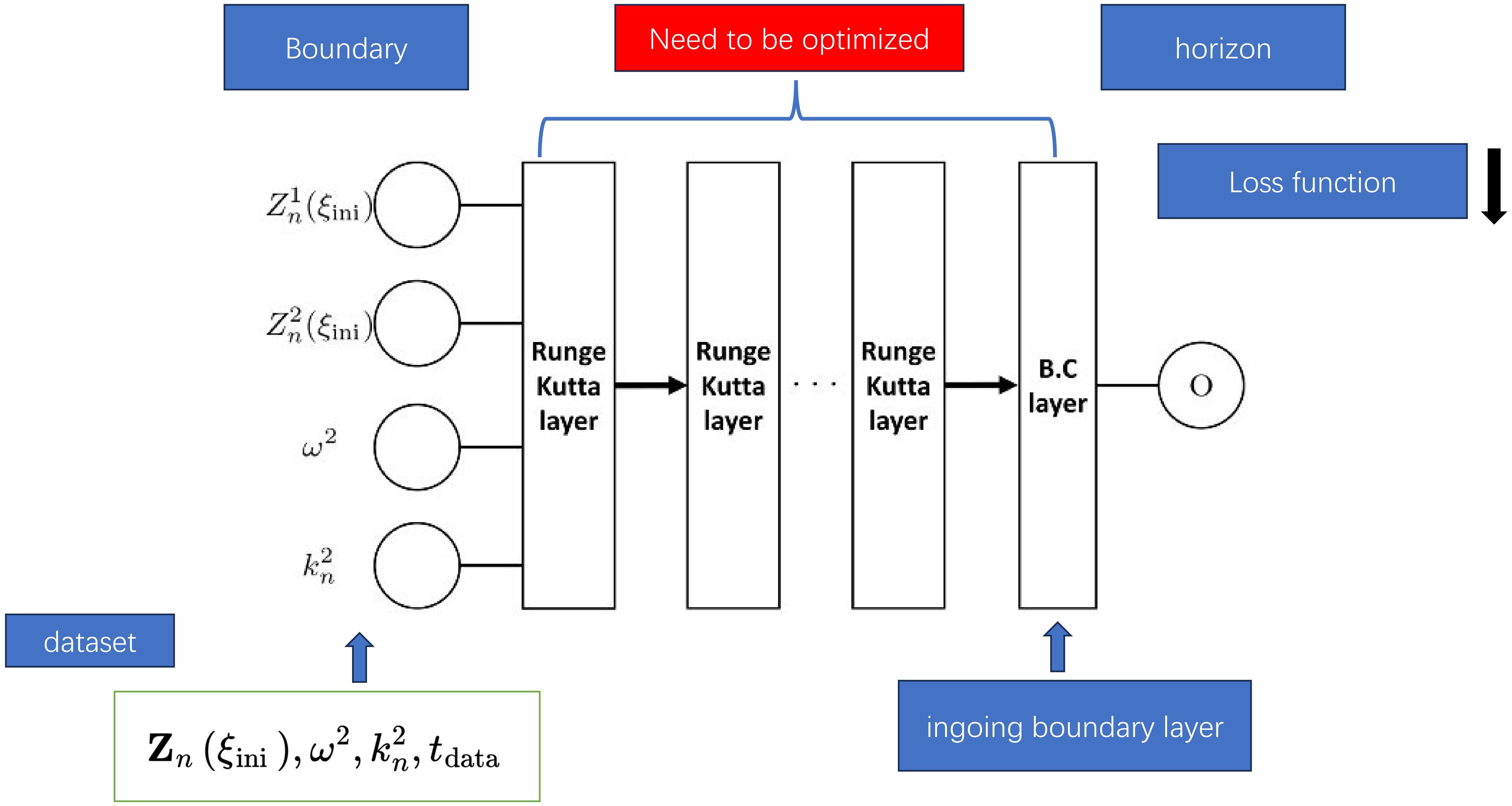
$$\xi = 1$$

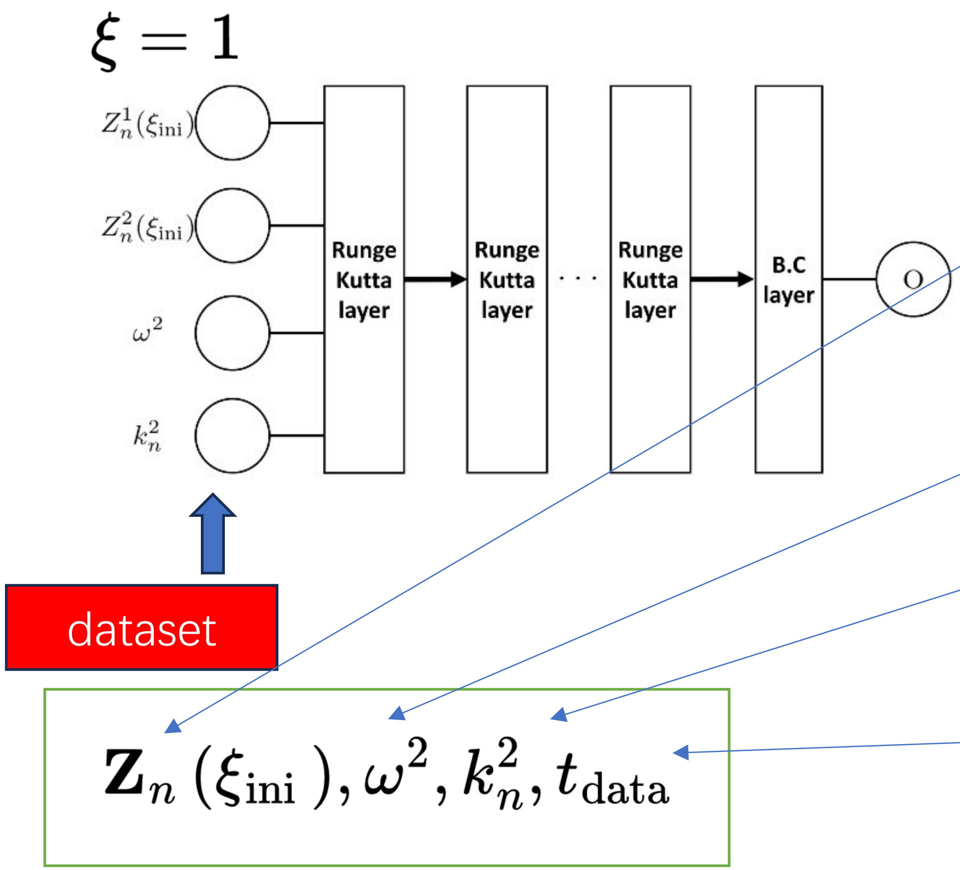
source

$$\Phi_n(\xi) \sim D_n^1 \left(1 + \alpha_n\beta_n(1-\xi)\ln(1-\xi)\right.$$
$$\left. - (1+\alpha_n\beta_n)(1-\xi)\right) + D_n^2(1-\xi)/r_h^2,$$
$$\Pi_n(\xi) \sim D_n^1 \left(1 - \alpha_n\beta_n \ln(1-\xi)\right) - D_n^2/r_h^2,$$

response

Can the metric of bulk be reconstructed using boundary data?


Boundary
Need to be optimized
horizon
Loss function
$Z_n^1(\xi_{\text{ini}})$
$Z_n^2(\xi_{\text{ini}})$
ω^2
k_n^2
Runge Kutta layer
Runge Kutta layer
Runge Kutta layer
B.C layer
O
dataset
$\mathbf{Z}_n(\xi_{\text{ini}}), \omega^2, k_n^2, t_{\text{data}}$
ingoing boundary layer

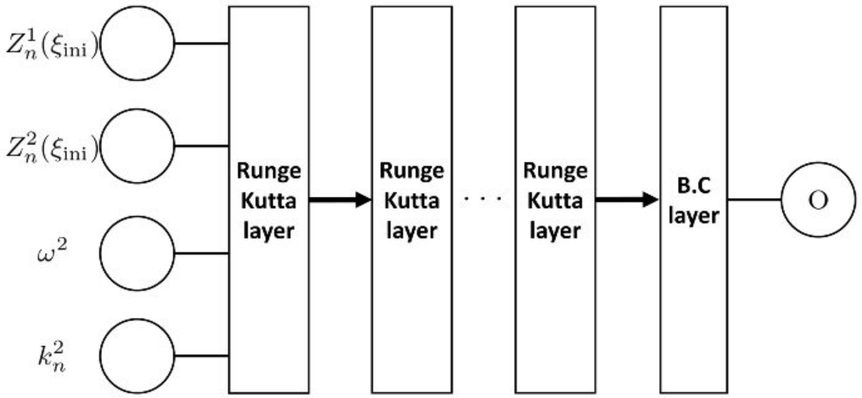


C_n^1	$\mathcal{U}(0.01, 1.0)$	$0.5 < \lvert C_n^1 \rvert < 1.2$	ingoing
C_n^2	$\mathcal{U}(0.001, 0.5)$	outgoing	
ω	$\mathcal{U}(0.0, 3.0)$		
k_n	$\mathcal{U}(0.0, \omega)$		

$$t_{\rm data} = \begin{cases} 0 & (\lvert C_n^2/C_n^1 \rvert < 0.01) : \text{positive} \\ 1 & (\lvert C_n^2/C_n^1 \rvert > 1.00) : \text{negative} \end{cases}$$

$0.01 \leq \lvert C_n^2/C_n^1 \rvert \leq 1.0$	discard

1000 examples for $t_{\rm data} = 0$ and $t_{\rm data} = 1$



$$\mathbf{Z}_n(\xi+\Delta\xi)=\begin{pmatrix} 1 & \Delta\xi \\ \Delta\xi\left(\omega^2\,\Xi(\xi)+k_n^2\,\Theta(\xi)\right) & 1-\Delta\xi/\xi \end{pmatrix}\mathbf{Z}_n(\xi)$$

parameter

Runge-Kutta

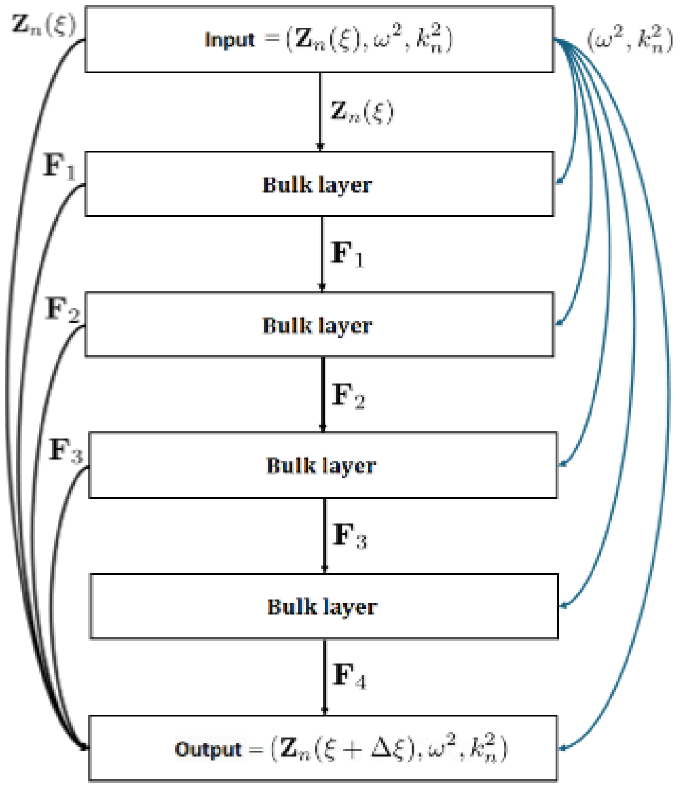
$$\mathbf{Z}_n(\xi+\Delta\xi)=\mathbf{Z}_n(\xi)+\frac{\Delta\xi\left(\mathbf{F}_1+2\mathbf{F}_2+2\mathbf{F}_3+\mathbf{F}_4\right)}{6}$$

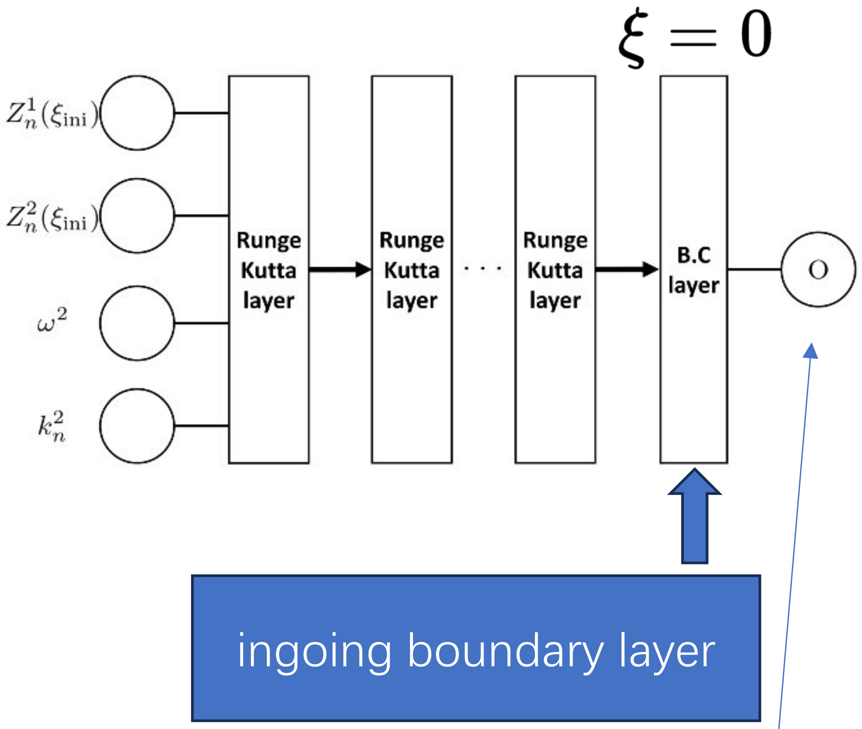
$$\mathbf{F}_1=\mathbf{F}\left(\xi,\mathbf{Z}_n(\xi)\right)$$

$$\mathbf{F}_2=\mathbf{F}\left(\xi+\frac{\Delta\xi}{2},\mathbf{Z}_n(\xi)+\mathbf{F}_1\frac{\Delta\xi}{2}\right)$$

$$\mathbf{F}_3=\mathbf{F}\left(\xi+\frac{\Delta\xi}{2},\mathbf{Z}_n(\xi)+\mathbf{F}_2\frac{\Delta\xi}{2}\right)$$

$$\mathbf{F}_4=\mathbf{F}\left(\xi+\Delta\xi,\mathbf{Z}_n(\xi)+\mathbf{F}_3\Delta\xi\right)$$





$$\lim_{\xi\to 0}\left(\xi\Pi_n(\xi) + i\omega\frac{L^2}{2r_h}\Phi_n(\xi)\right) = 0.$$

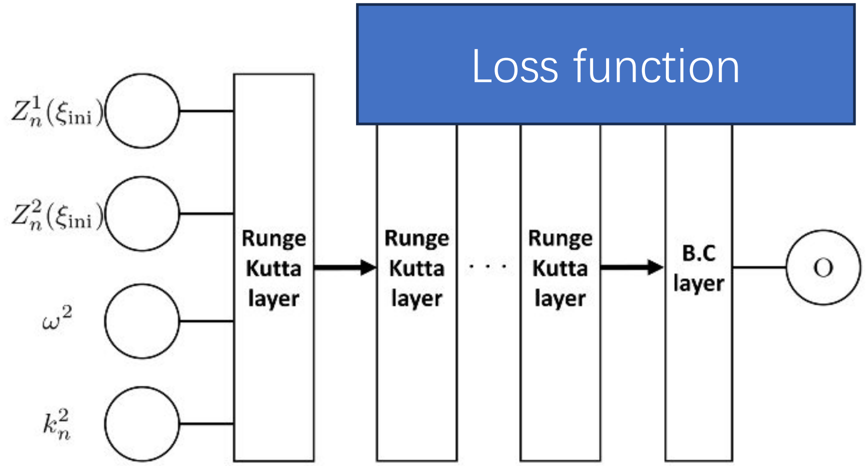
$$\frac{\xi_{\rm fin}\,\Pi_n(\xi_{\rm fin}) + \rho_n\Phi_n(\xi_{\rm fin})}{\sqrt{|\xi_{\rm fin}\,\Pi_n(\xi_{\rm fin})|^2 + |\rho_n\Phi_n(\xi_{\rm fin})|^2 + \epsilon}} = 0.$$

ρ_n is a learnable parameter

$$\rho_n = i\omega a_n + |k_n| b_n$$

- BTZ black hole: $a_n = \frac{L^2}{2r_h}, \quad b_n = 0$
- AdS soliton metric: $a_n = 0, b_n = -\frac{L^2}{2r_s}$

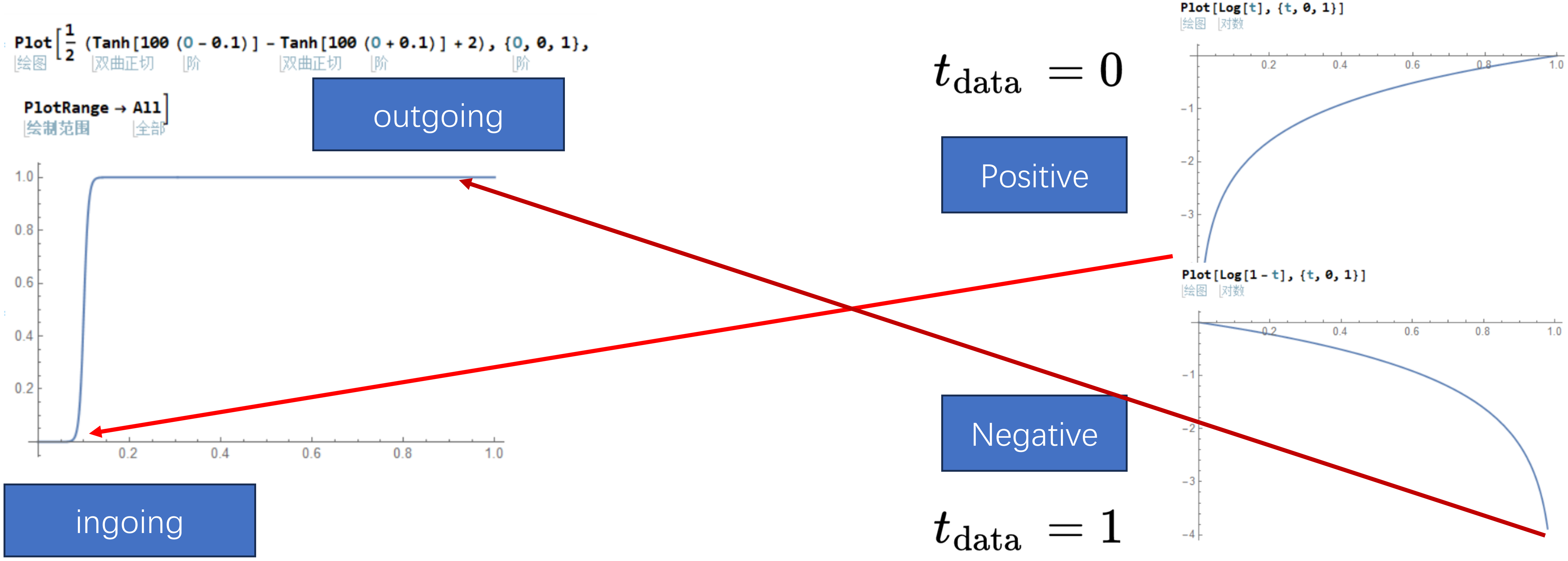
$$\mathrm{O} = \left|\frac{\xi_{\rm fin}\,\Pi_n(\xi_{\rm fin}) + \rho_n\Phi_n(\xi_{\rm fin})}{\sqrt{|\xi_{\rm fin}\,\Pi_n(\xi_{\rm fin})|^2 + |\rho_n\Phi_n(\xi_{\rm fin})|^2 + \epsilon}}\right|,$$



$$L(t) = -t_{\rm data}\,\log t - (1 - t_{\rm data})\log(1-t),$$

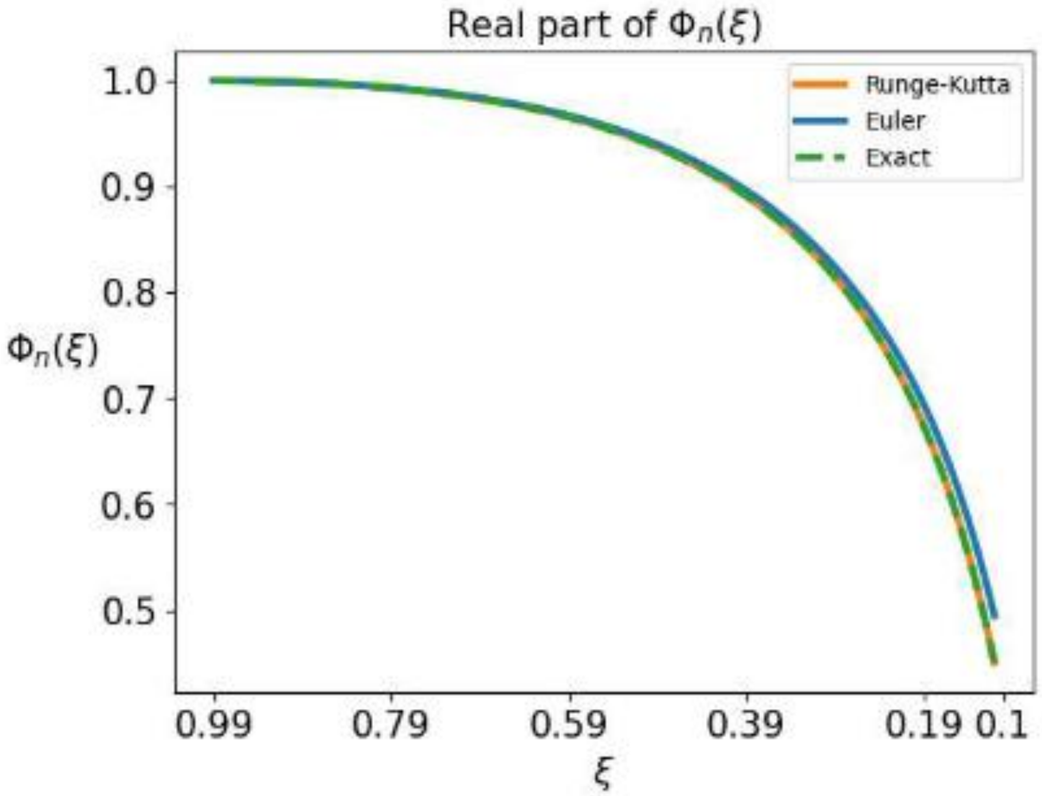
$$t = \frac{1}{2}[\tanh(100(\mathrm{O}-0.1)) - \tanh(100(\mathrm{O}+0.1)) + 2],$$

$$\mathrm{O} = \left|\frac{\xi_{\rm fin}\,\Pi_n(\xi_{\rm fin}) + \rho_n\Phi_n(\xi_{\rm fin})}{\sqrt{|\xi_{\rm fin}\,\Pi_n(\xi_{\rm fin})|^2 + |\rho_n\Phi_n(\xi_{\rm fin})|^2 + \epsilon}}\right|,$$

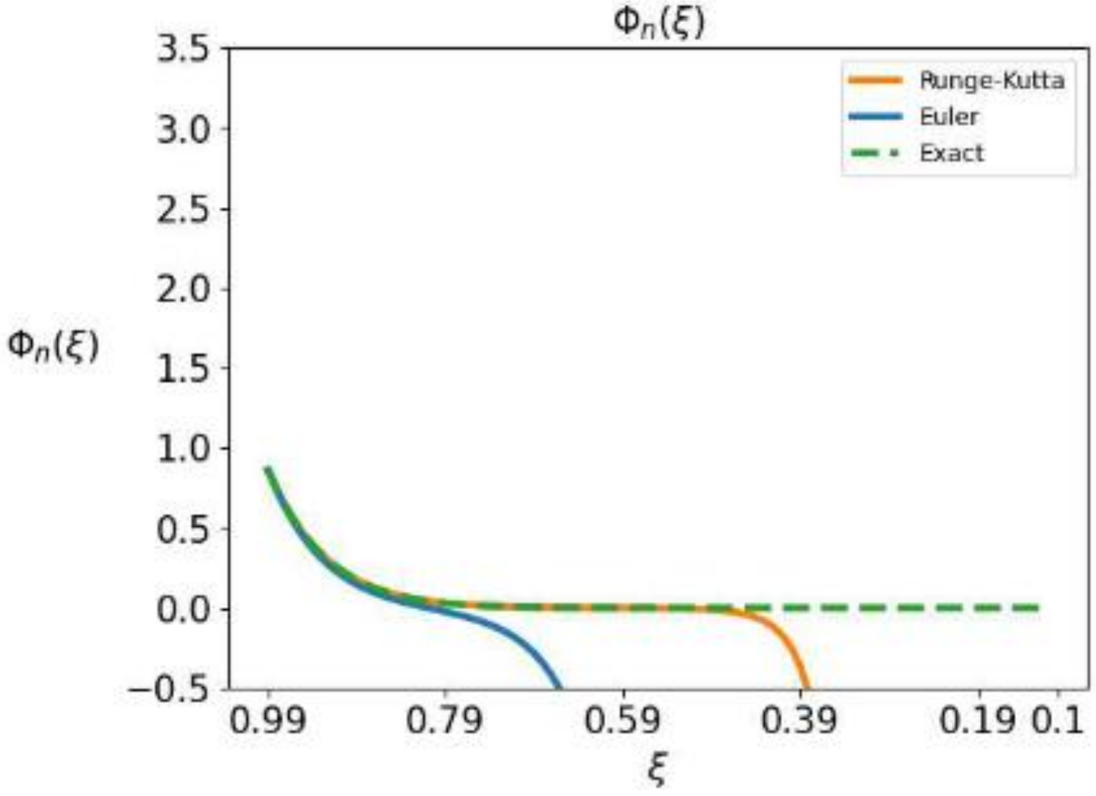


Result

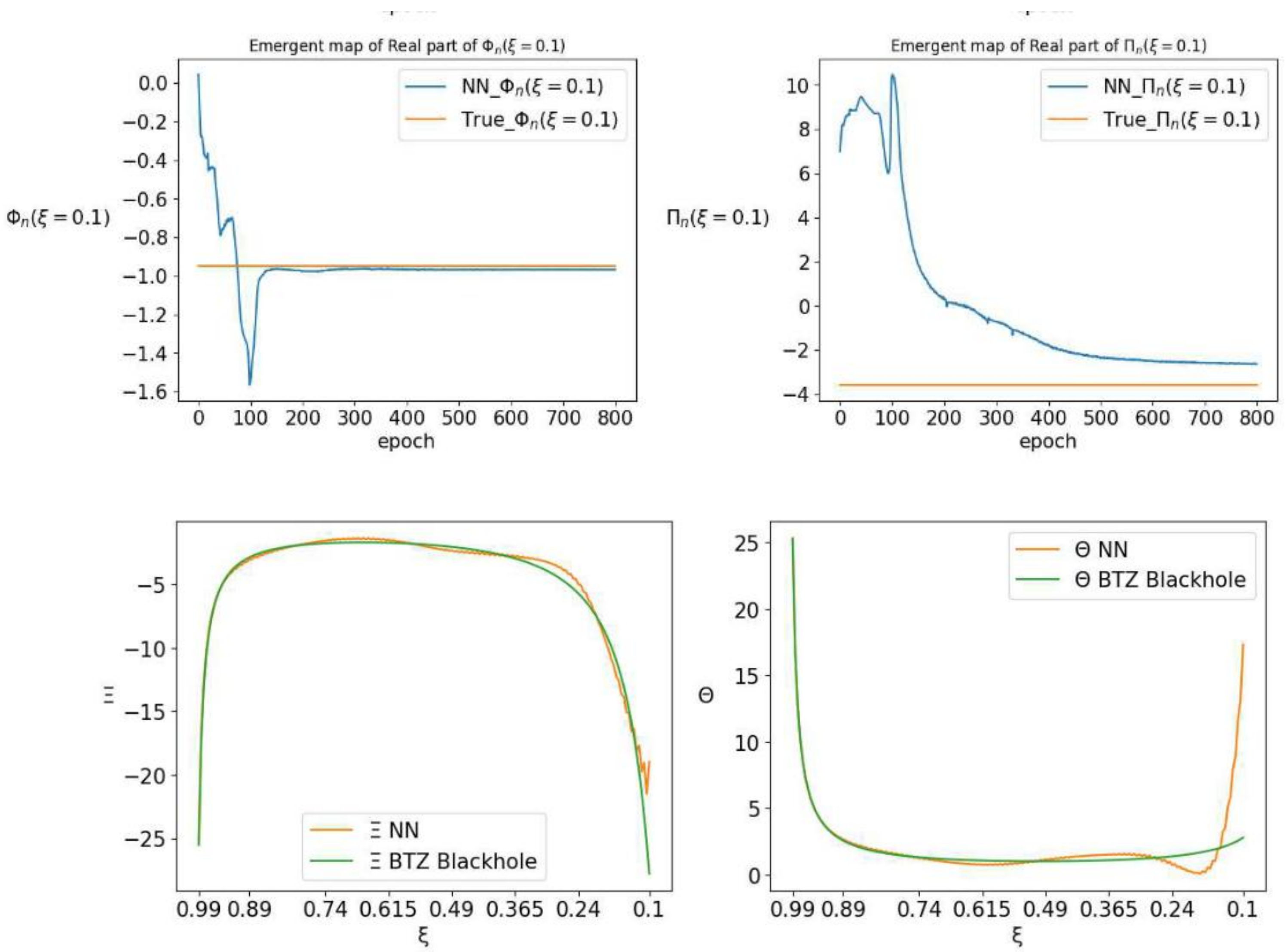
BTZ



AdS soliton



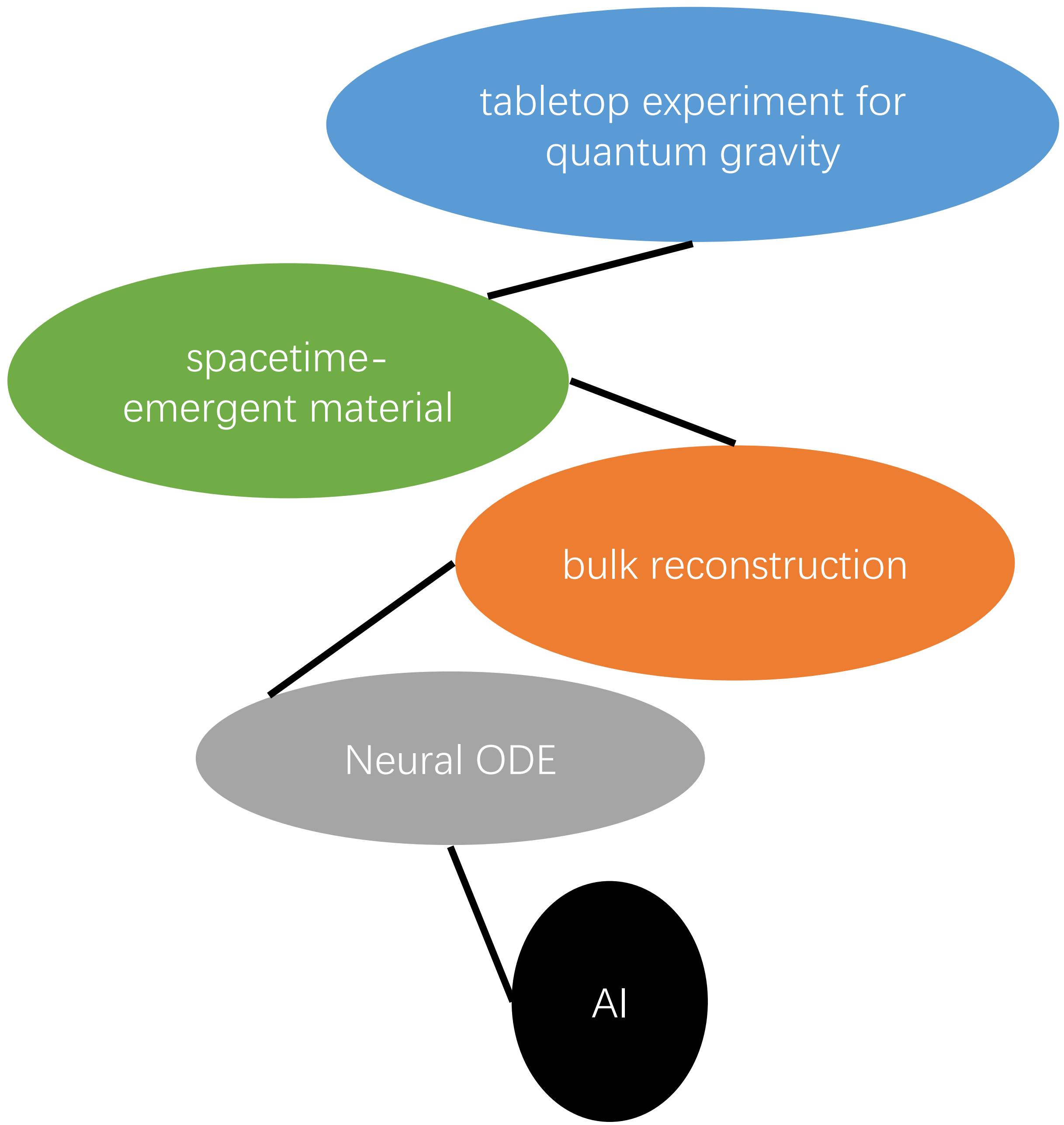
Result



(a_n, b_n) being $(0.50, 0.00)$

BTZ black hole

$$a_n = \frac{L^2}{2r_h}, \quad b_n = 0$$


tabletop experiment for
quantum gravity
spacetime-
emergent material
bulk reconstruction
Neural ODE
AI

The ingoing boundary condition corresponds to the retarded propagation.

Appendix

Minkowski-space correlators in AdS/CFT correspondence: recipe and applications	Dam T. Son and Andrei O. Starinets	标量场频率方程

$$\frac{1}{\sqrt{-g}}\partial_z\left(\sqrt{-g}g^{zz}\partial_z f_k\right)-\left(g^{\mu\nu}k_\mu k_\nu+m^2\right)f_k=0$$

$$\phi(z,x)=\int\frac{d^4k}{(2\pi)^4}e^{ik\cdot x}f_k(z)\phi_0(k)$$

with unit boundary value at the boundary, $f_k(z_B)=1$ and satisfying the incoming-wave boundary condition at $z=z_H$.

$$S=\int\frac{d^4k}{(2\pi)^4}\phi_0(-k)\mathcal{F}(k,z)\phi_0(k)\Big|_{z=z_B}^{z=z_H}$$

$$\mathcal{F}(k,z)=K\sqrt{-g}g^{zz}f_{-k}(z)\partial_z f_k(z)$$

推迟格林函数被证明为

$$G^R(k)=-2\mathcal{F}(k,z)|_{z_B}.$$

9.1.2	AdS/CFT Duality User Guide

$$\delta S=\int d^4x\phi^{(0)}(t,\boldsymbol{x})O(\boldsymbol{x})$$

$$\delta\langle O(t,\boldsymbol{x})\rangle=-\int_{-\infty}^{\infty}d^4x' G_R^{OO}\left(t-t',\boldsymbol{x}-\boldsymbol{x}'\right)\phi^{(0)}\left(t',\boldsymbol{x}'\right).$$

10.2	AdS/CFT Duality User Guide	边界场若写为如下形式

$$\phi\sim\phi^{(0)}\left(1+\phi^{(1)}u^4\right),\quad (u\to 0).$$

$$\underline{\mathrm{S}}\left[\phi^{(0)}\right]=\int d^4x 2\phi^{(0)2}\phi^{(1)}\Longleftrightarrow G_R^{OO}(k=0)=-4\phi^{(1)}$$

如果有AdS/CFT对应，应该有推迟格林函数和边界展开的对应

综上：场在边界展开的算符项在取incoming边界条件的时候对应推迟格林函数和线性响应真空扰动的推迟格林函数一致，所以要线性扰动就需要incoming边界条件

如果有incoming边界条件则可以证明这两个就是对应：因为红色的对应都是在引力里面计算的