QCD at high densities: progress & challenges

Jan M. Pawlowski

Universität Heidelberg & ExtreMe Matter Institute

Dalian, October 8th 2025

fqcd-collaboration.github.io

fqcd-collaboration.github.io

Beijing, Dalian, Darmstadt, Heidelberg, Gießen

Braun, Chen, Fu, Gao, Geissel, Huang, Ihssen, Kockler, Lu, Pawlowski, Rennecke, Sattler, Schallmo, Tan, Töpfel, Wang, Wen, Wessely, Yamada, Yin, Zheng, Zorbach

Selected talks

Workshop Confinement and symmetry from vacuum to QCD phase diagram:
The QCD phase structure from first principles

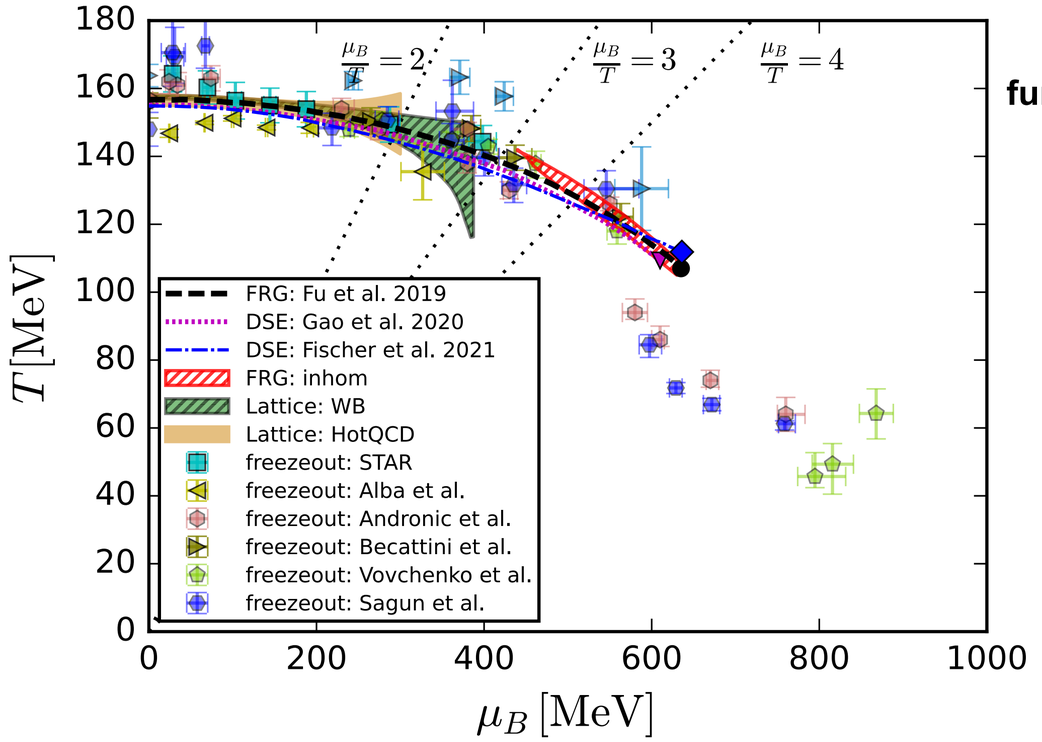
Satellite meeting Quark Matter 2025: Physics at high net-baryon densities:
New Insights on the Phase Structure with Functional QCD

PhD school xQCD 2024:
Functional QCD and the QCD phase structure

Outline

- **Critical end point and new phases: where do we stand?**
- **Strong correlations above the chiral crossover**
- **The physics of soft modes**
- **Ripples of the critical and point and new phases**
- **QCD moat and inhomogeneous phases**

Critical end point and new phases: where do we stand?



funQCD: $\mu_u = \mu_d = \mu_s$

$$\frac{\kappa_{n_s=0}}{\kappa_{\mu_i=\mu_q}} = 0.974$$

Lu, Gao, Liu, JMP, in preparation

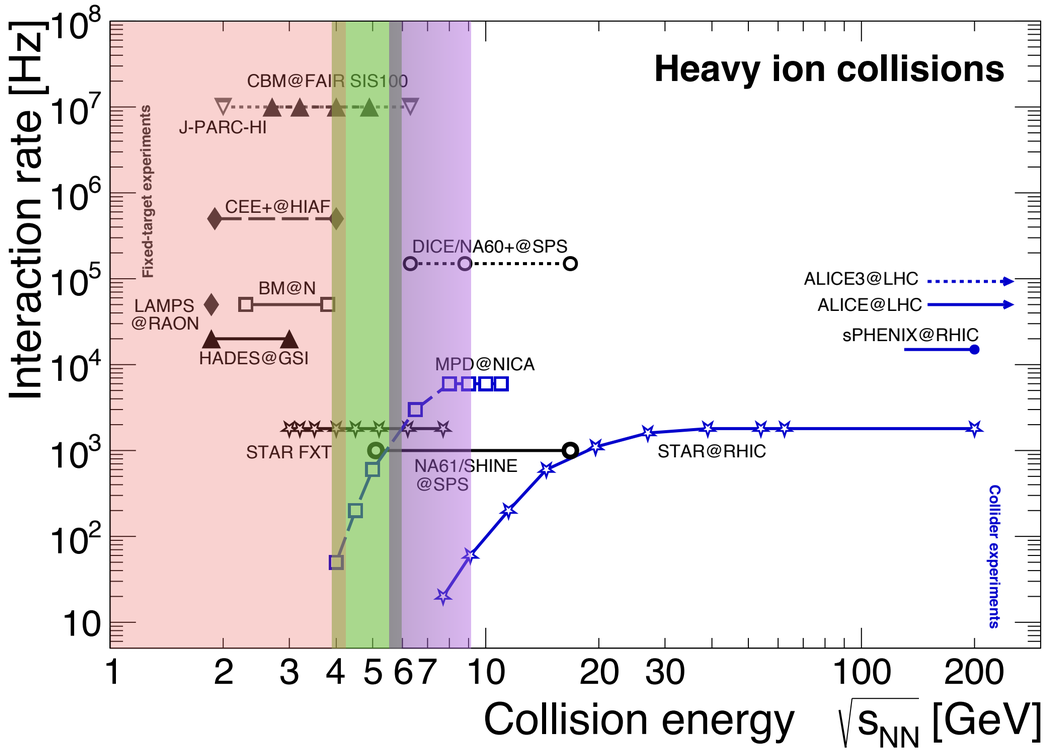
Experimental & Theoretical Landscape

Experimental landscape

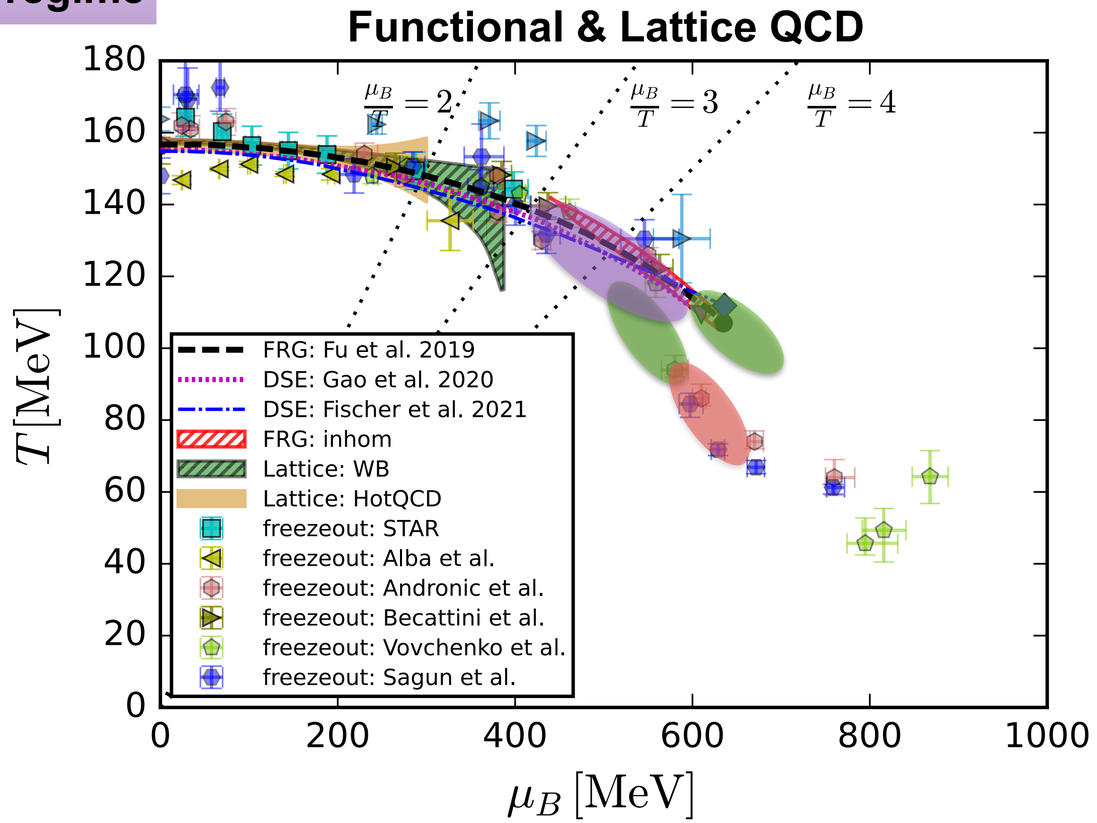
Theoretical landscape

Onset of new phases on the chiral transition line (CEP)

New phases



Transition regime



Peak of kurtosis on the freeze-out line

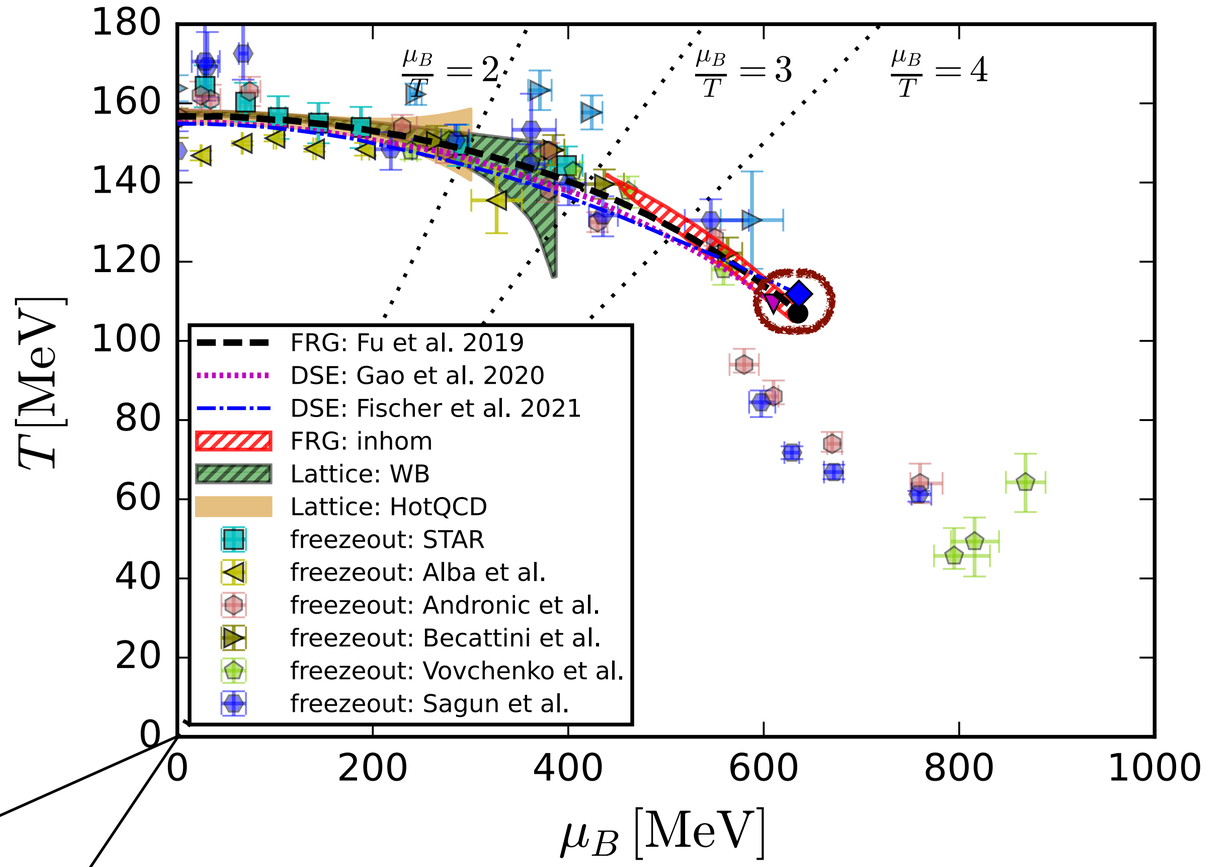
Galatyuk, A982 (2019) update 2025; CBM, EPJA 53 3 (2017) 60

Fu, JMP, Rennecke, PRD 101 (2020) 054032

Gao, JMP, PLB 820 (2021) 136584

Gunkel, Fischer, PRD 104 (2021) 054022

Comprehensive QCD phase structure



$\mu_{B,\mathrm{Im}}$

m_q

μ_{Iso} B

Collect all possible information/structure for physics understanding & extrapolations

Functional QCD: CEP estimate

fRG: **Fu, JMP, Rennecke, PRD 101 (2020) 054032**

DSE: **Gao, JMP, PLB 820 (2021) 136584**

Gunkel, Fischer, PRD 104 (2021) 054022

$$(\mu_B\,,\,T)_{\mathrm{CEP}} \sim (600-650\,,\,105-115)\ \mathrm{MeV}$$

Lattice:

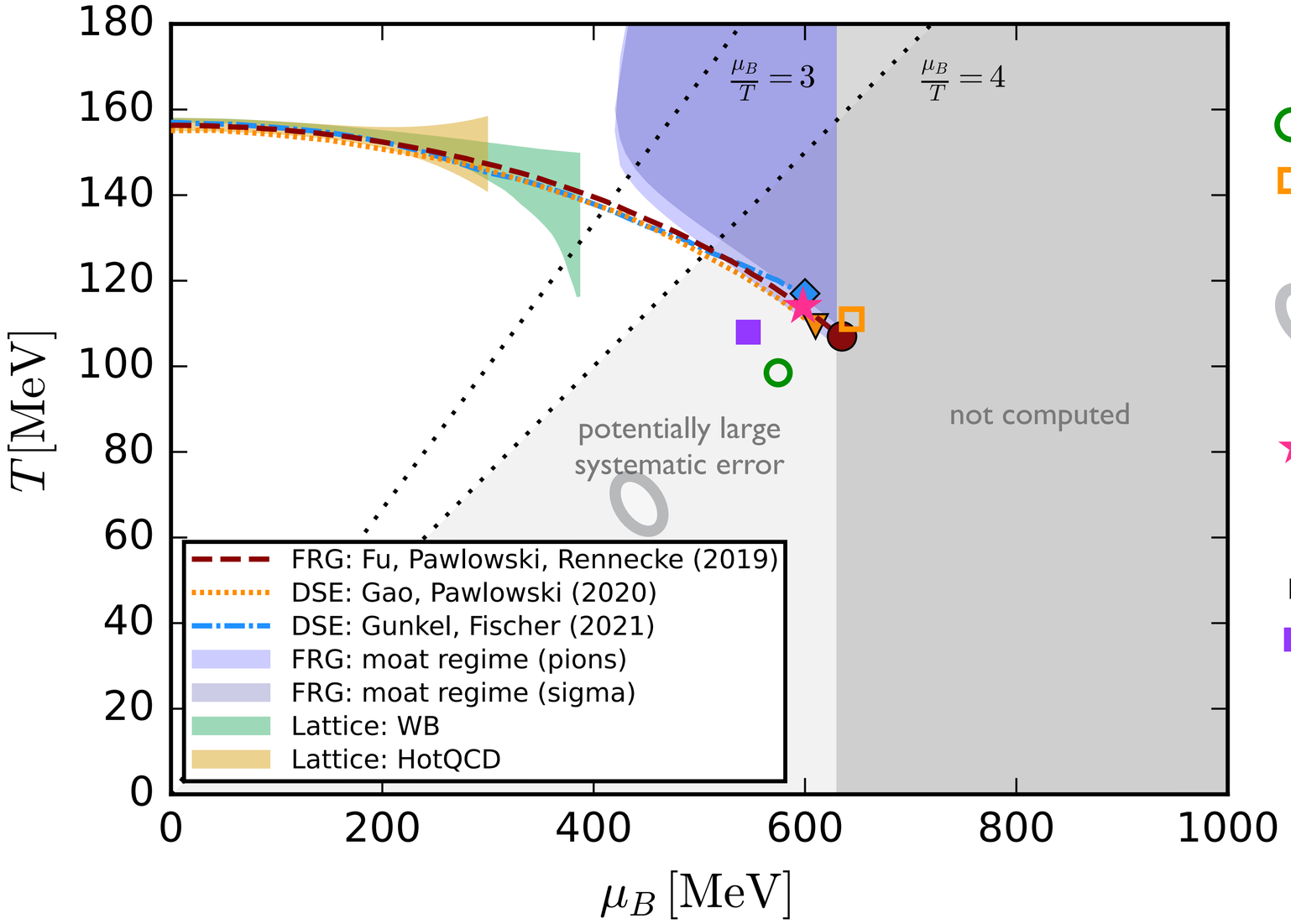
HotQCD:
Bazavov, Ding, Hegde, Kaczmarek, Karsch, Karthik, Laermann, Lahiri, Larsen, Li, Mukherjee, Ohno, Petreczky, Sandmeyer, Schmidt, Sharma, Steinbrecher,
PLB 795 (2019)

WB:
Borsányi, Fodor, Guenther, Kara, Katz, Parotto, Pasztor, Ratti, Szabo,
PRL 125 (2020) 052001

QCD PHASE DIAGRAM & THE CEP

Modified from Fabian Rennecke's plenary talk @ QM 2025

FRG & DSE results ? corroborated ? by subsequent "extrapolations" of lattice data



Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

using Yang-Lee edge singularities:

- conformal Padé [Basar, 2312.06952]
- multi-point Padé [Clarke et al., 2405.10196]
  $N_\tau = 6, 8$ results + continuum estimate
  [Schmidt, 2504.00629]
- **very recent lattice extrapolation (16^3 x 8)**
  [Adam et al, 2507.13254]

using thermodynamics:

- constant entropy density [Shah et al. 2410.16206]

model-based extrapolation (tiny selection):

- holography [Hippert et al., 2309.00579]
  (agrees with [Cai et al., 2201.02004])

CEP location well constrained by now. And it's **in FAIR range!**

$\sqrt{s_{NN}} \approx 3.6 - 4.1\,\mathrm{GeV}$

Direct computations and the minimal point of view

Those are my interpretations, and if you don't like them…. well, I have others

- **Self-consistent truncations to functional relations define analytic functions in μ_B , eg:**

$$\partial_t \Big\langle q(x)\bar{q}(y)\Big\rangle(\mu_B) = \mathrm{Loop}\left[\Big\langle q(x)\bar{q}(y)\Big\rangle(\mu_B),\ \Big\langle q(x)A_\mu(y)\bar{q}(z)\Big\rangle(\mu_B), \cdots;\ \mu_B\right]$$

Unique: QCD-based analytic continuations that satisfy the lattice benchmarks at small chemical potential.

- **?Corroborated?**

By definition, LEFTs (including extrapolations) cannot assess the existence or absence of emergent phenomena!

See also talk of Szabolcs Borsanyi

Minimal point of view: CEP exclusion regime

No chiral critical end point for $\mu_B \lesssim 600\,\mathrm{MeV}$

Lattice QCD

$$\mu_B \lesssim 450\,\mathrm{MeV}$$

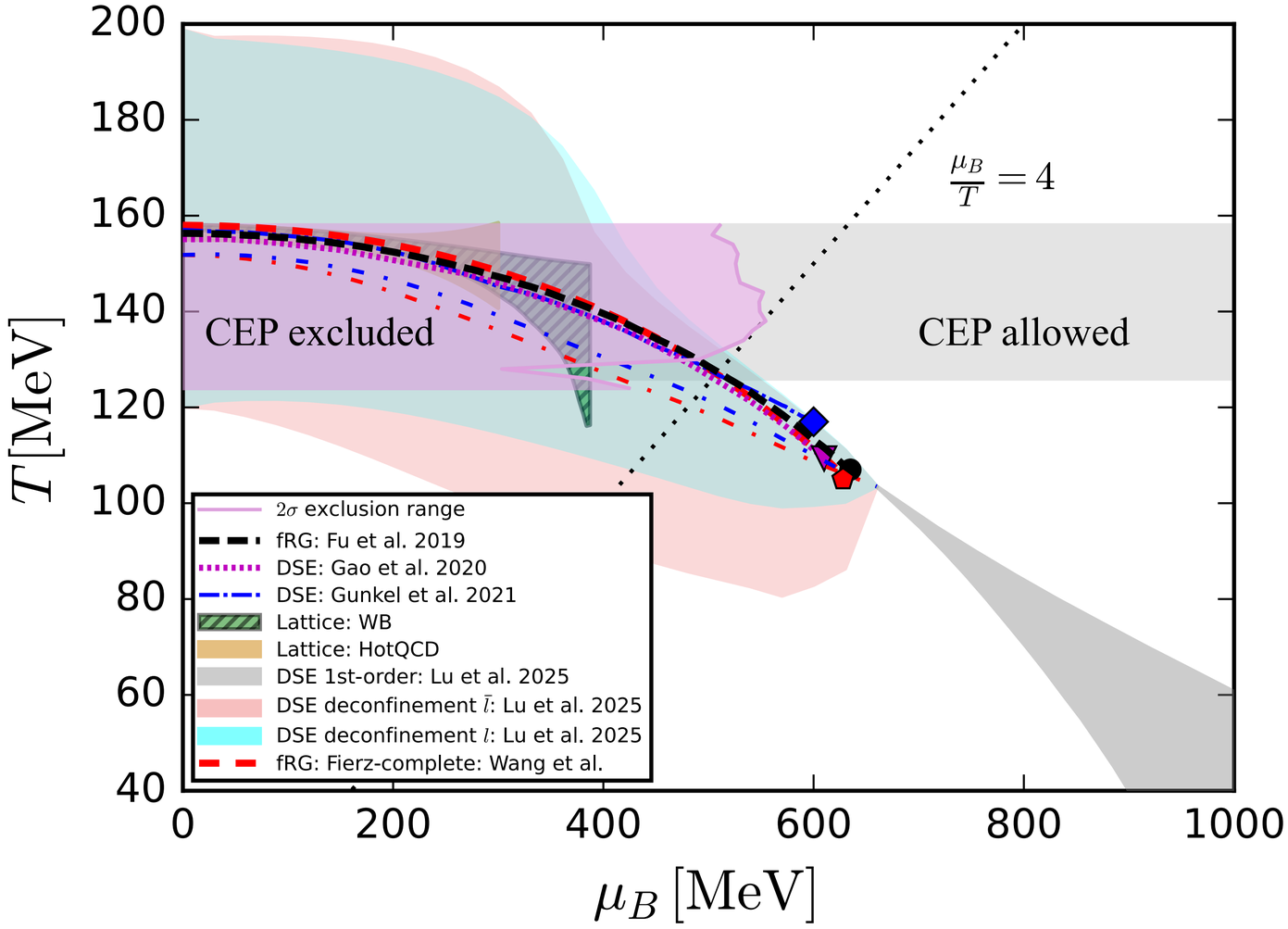
Borsanyi, Fodor, Guenther, Parotto, Pasztor, Ratti, Vovchenko, Wong, arxiv: 2502.10267

Functional QCD

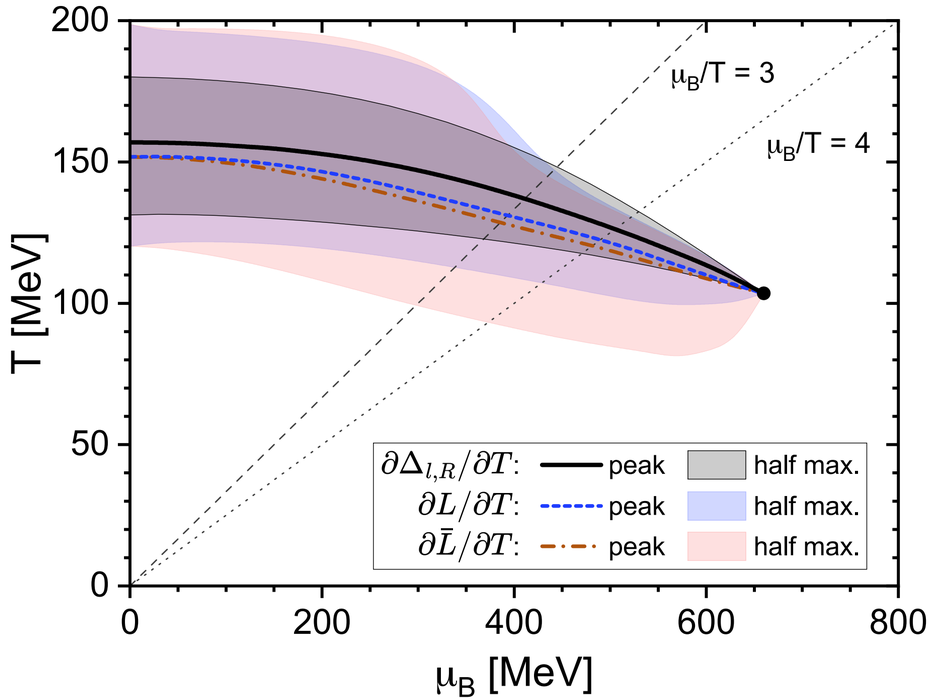
Fu, JMP, Rennecke, PRD 101 (2020) 054032
Gao, JMP, PLB 820 (2021) 136584
Gunkel, Fischer, PRD 104 (2021) 054022

Lu, Gao, Liu, JMP, arXiv:2504.05099
Lu, Gao, Liu, arXiv:2509.02974
Wang, Zhou, Huang, Wen, Yin, Fu, in preparation

Functional QCD CEP+ lattice QCD exclusion plot



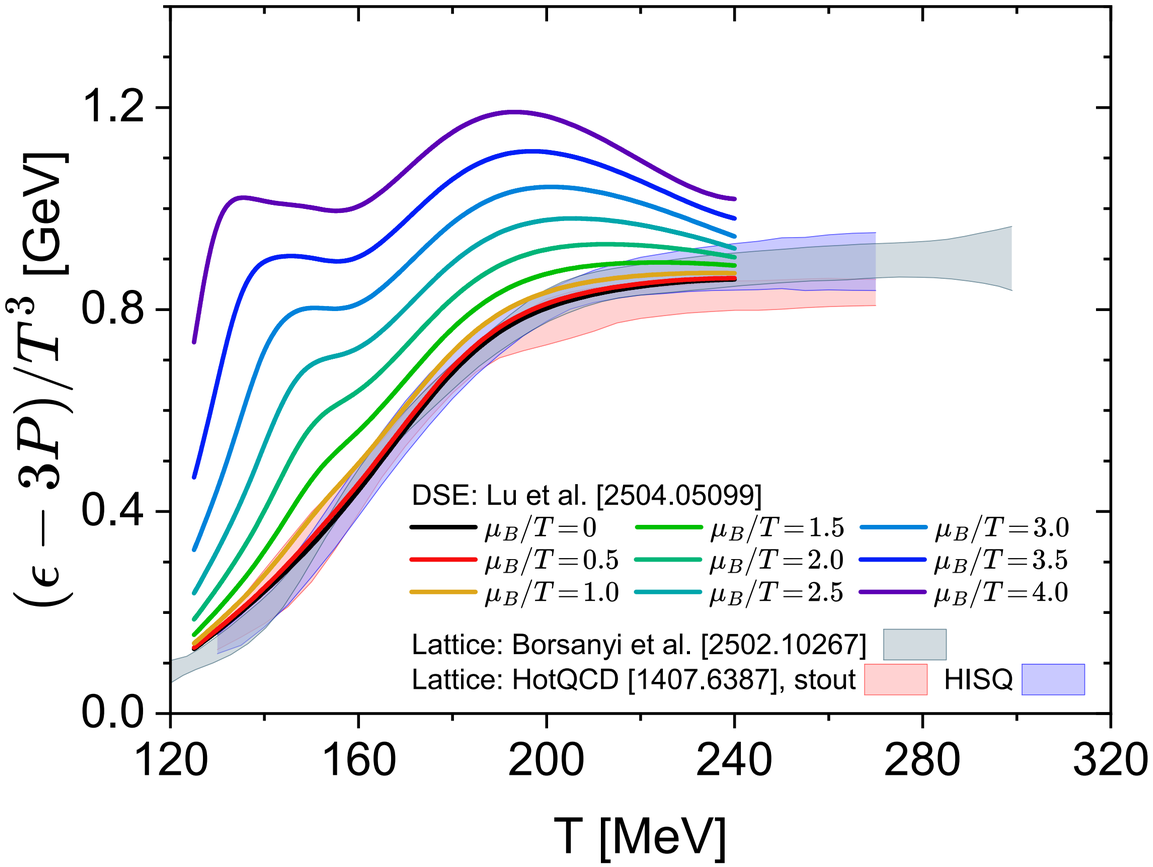
Strong correlation above the chiral crossover



See talk of Yi Lu

Strongly correlated physics

Strongly correlated phase for temperatures $T_\chi \lesssim T \lesssim 2\,T_\chi$



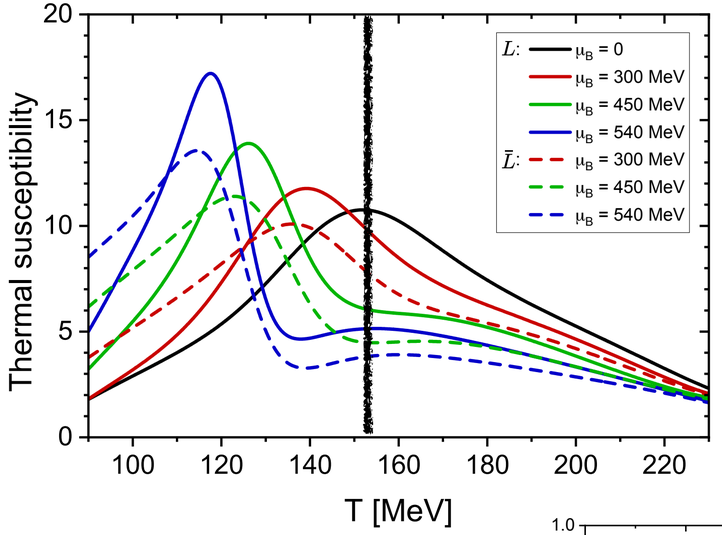
Lu, Gao, Liu, JMP, arXiv:2504.05099

See talk of Yi Lu

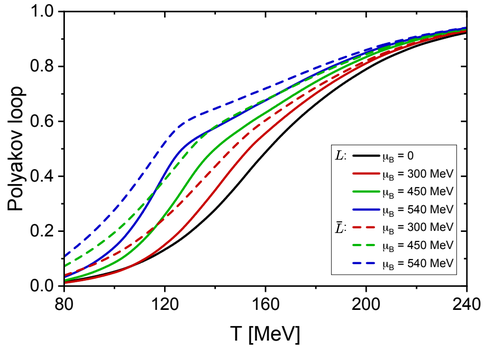
Strongly correlated physics

‘Confinement’ related crossovers at $T_\chi \lesssim T_{\rm conf} \lesssim 2\,T_\chi$

Functional QCD



$$\propto P\big(\langle \mathrm{ev}(\varphi)\rangle\big)$$



Braun, Gies, JMP, PLB 684 (2010) 262-267

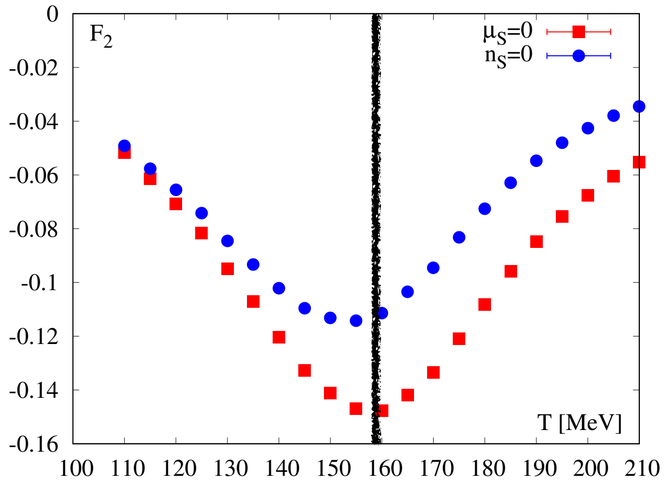
⋮

Lu, Gao, Liu, JMP, arxiv:2504.05099

Variants of the traced Polyakov loop

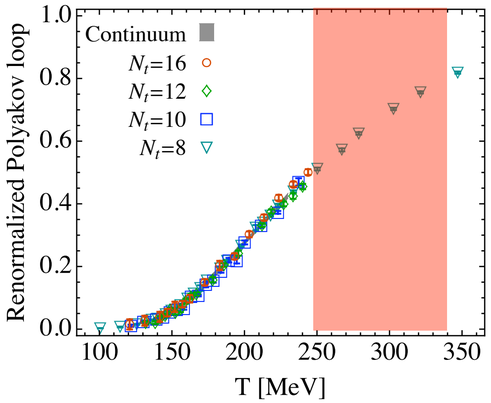
$$P = \frac{1}{N_c}\mathrm{tr}\, e^{2\pi i\varphi}$$

Lattice QCD



$$\propto T\frac{\partial^2 \log|\langle P\rangle|}{(\partial\mu_B/T)^2}$$

$$\propto \langle P\rangle$$



⋮

Borsanyi, Fodor, Hoelbling, Katz, Krieg, Ratti, Szabo, JHEP 09, 073 (2010)

⋮

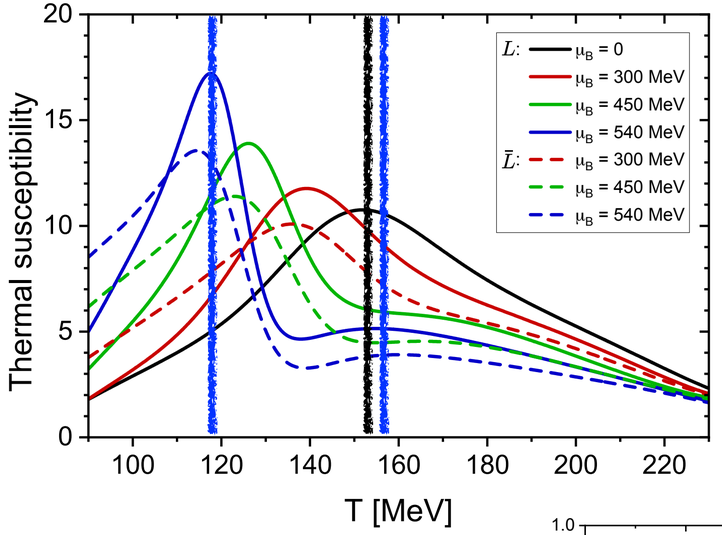
Borsányi, Fodor, Guenther, Kara, Paolo Parotto, Pásztor, Pirelli, Wong, PRD 110 (2024) 114507

See talk of Yi Lu

Strongly correlated physics

‘Confinement’ related crossovers at $T_\chi \lesssim T_{\rm conf} \lesssim 2\,T_\chi$

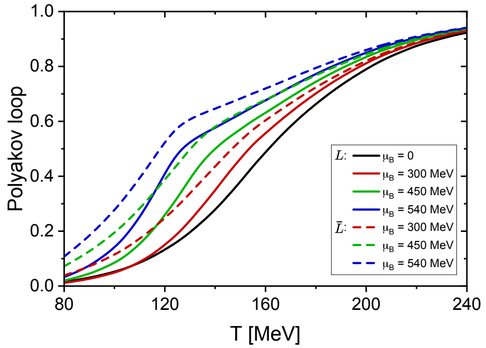
Functional QCD



Variants of the traced Polyakov loop

$$P = \frac{1}{N_c}\,\mathrm{tr}\, e^{2\pi i \varphi}$$

$$\propto P\big(\langle \mathrm{ev}(\varphi)\rangle\big)$$

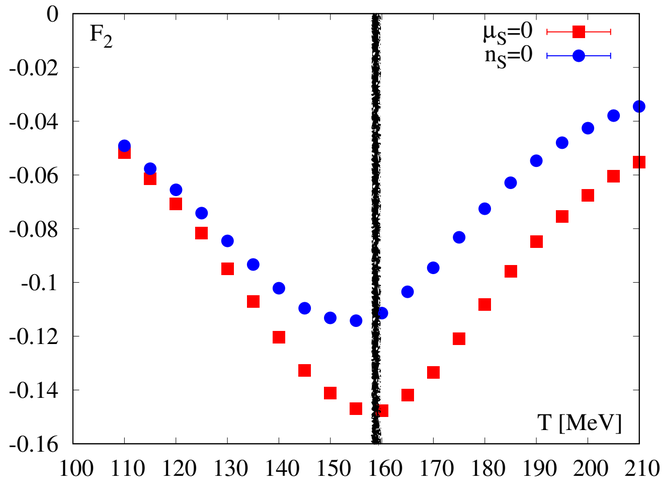


Braun, Gies, JMP, PLB 684 (2010) 262-267

⋮

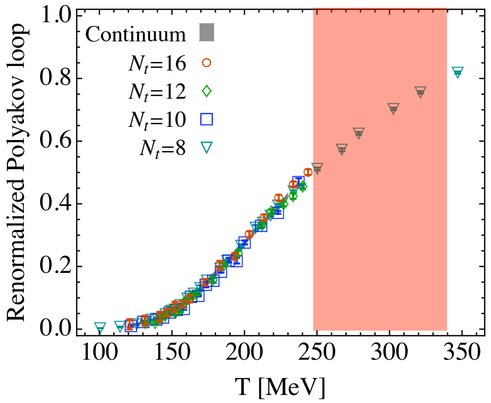
Lu, Gao, Liu, JMP, arxiv:2504.05099

Lattice QCD



$$\propto T\frac{\partial^2 \log|\langle P\rangle|}{(\partial \mu_B/T)^2}$$

$$\propto \langle P\rangle$$



⋮

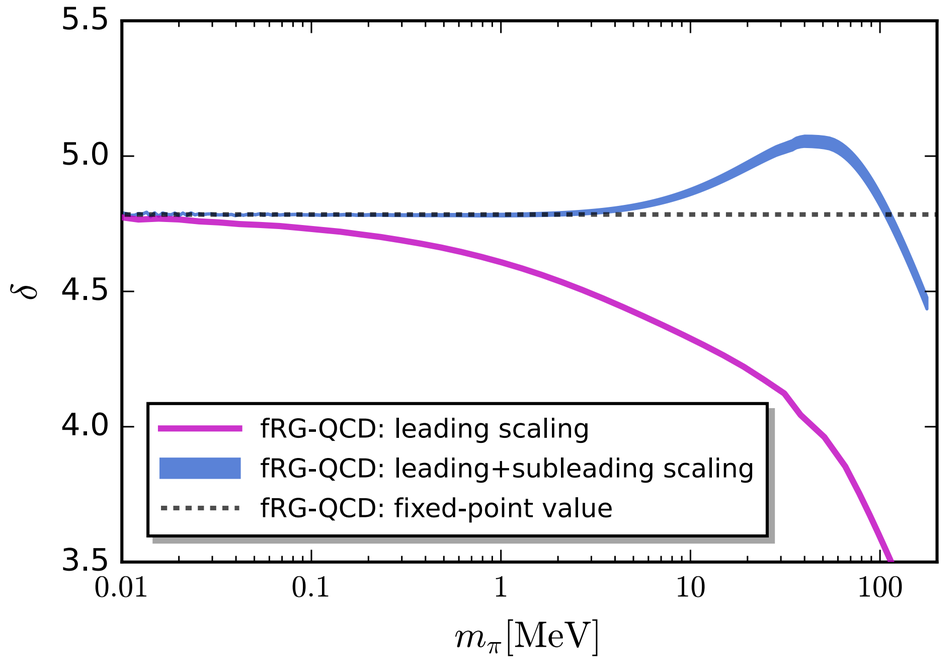
Borsanyi, Fodor, Hoelbling, Katz, Krieg, Ratti, Szabo, JHEP 09, 073 (2010)

⋮

Borsányi, Fodor, Guenther, Kara, Paolo Parotto, Pásztor, Pirelli, Wong, PRD 110 (2024) 114507

The physics of soft modes

$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta}\left[1 + a_m m_\pi^{2\theta_H} + \cdots\right]$$



Chiral dynamics & quasi-massless modes in the chiral limit

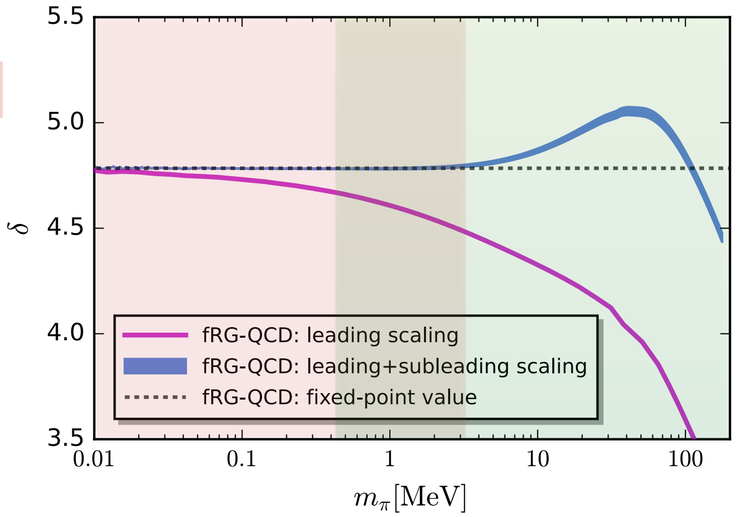
Scaling coefficient as function of the pion mass

Critical O(4) scaling

'chiral scaling'

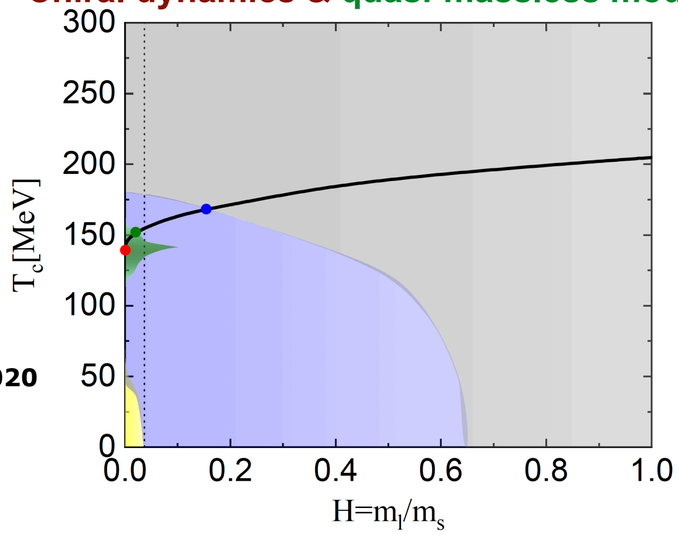
Trivial $\Delta_l^{1+\delta}$ scaling

$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta}\left[1 + a_m m_\pi^{2\theta_H} + \cdots\right]$$



Small chiral scaling regime

Chiral dynamics & quasi-massless modes



Critical scaling

Gao, JMP, PRD 105 (2022) 094020

'Non-critical chiral scaling'

Far away from the critical regime for $m_\pi \gtrsim 1\,\mathrm{MeV}$

$$\Delta_l(T,H) \approx \Delta_{l,\chi}(0)\left(c_0 + c_{\frac{1}{5}} H^{\frac{1}{5}} + c_{\frac{1}{3}} H^{\frac{1}{3}} + c_1 H\right)$$

$$V_\chi(\Delta_l) \propto \quad \Delta_l^6 \quad \Delta_l^4 \quad \Delta_l^2$$

Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, PRD 111 (2025) 094010

Criticality & quasi-massless modes around the CEP

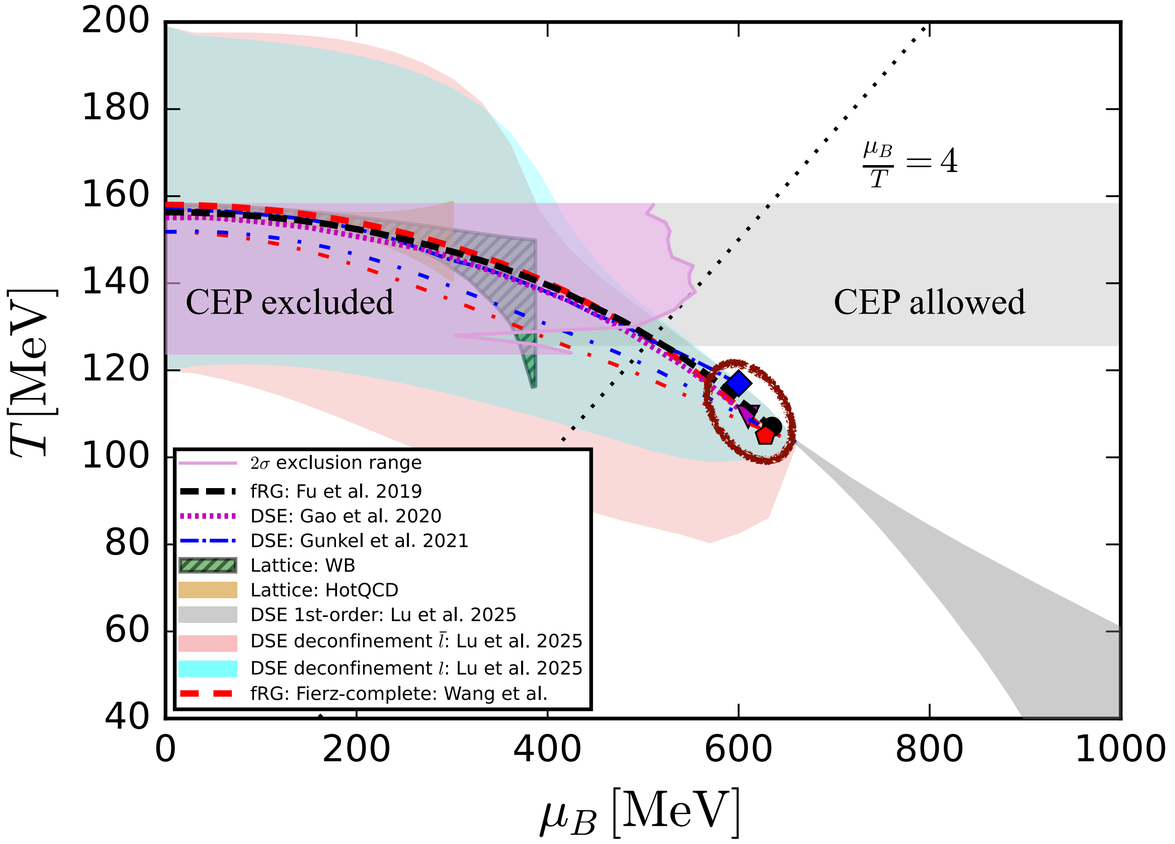
Fu, JMP, Rennecke, PRD 101 (2020) 054032

Wang, Zhou, Huang, Wen, Yin, Fu, in preparation

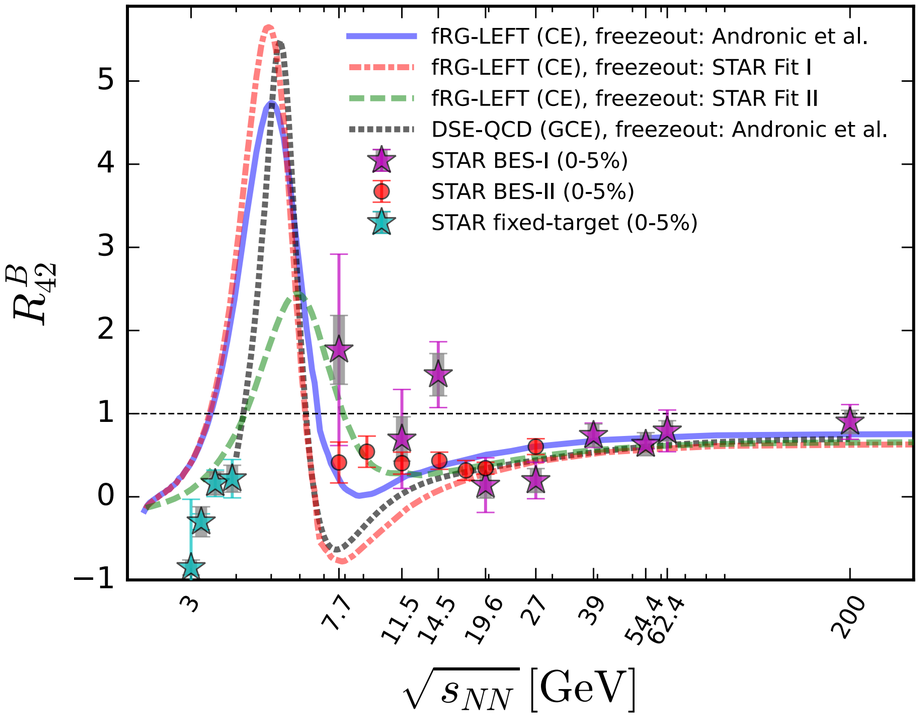
Small scaling regime around potential CEP

!!Great News!!

Location of CEP/New phase accessible via combination of precision measurements & computations

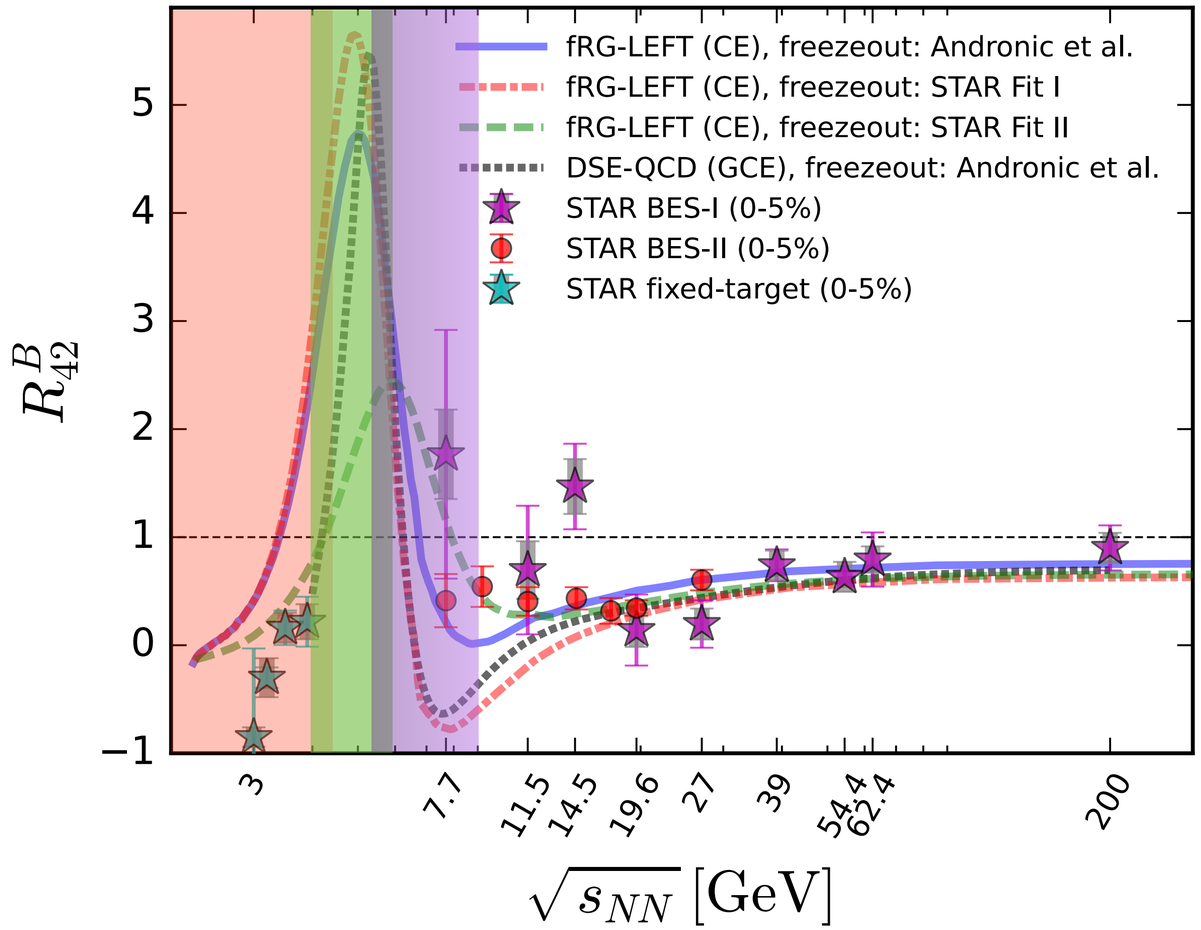


Ripples of the critical and point and new phases



Ripples of the critical point

net-baryon fluctuations in QCD vs net-proton fluctuations at STAR



Dominated by non-critical soft modes

Results:
1st principles functional QCD computations
& low energy effective theories/extrapolations

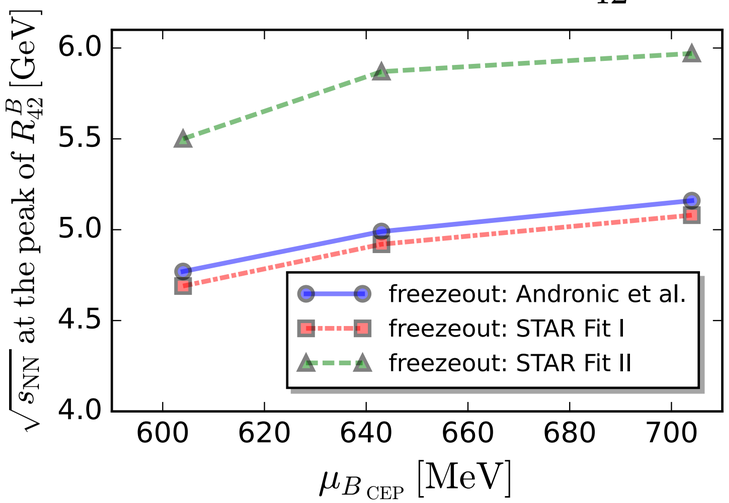
Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9

Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

Lu, Gao, Liu, JMP, arXiv:2504.05099

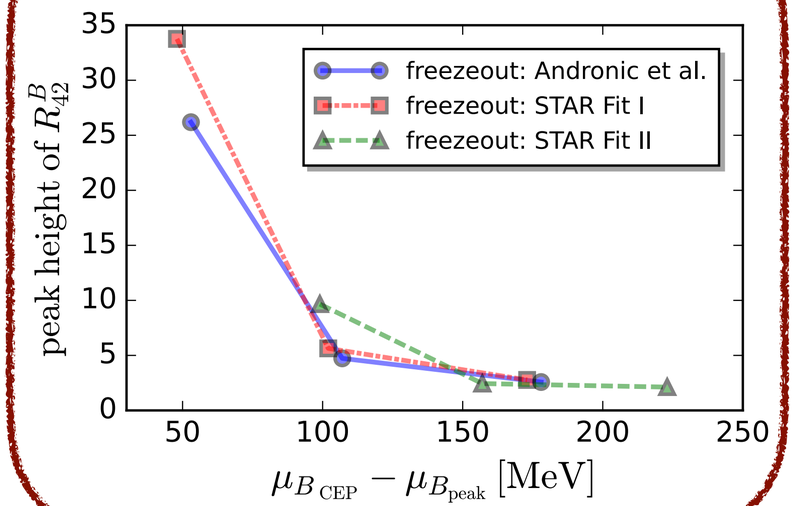
Ripples of the critical point

Position of the peak of R^B_{42}



Reconstructing the CEP

Height of the peak of R^B_{42}



Dominated by non-critical soft modes

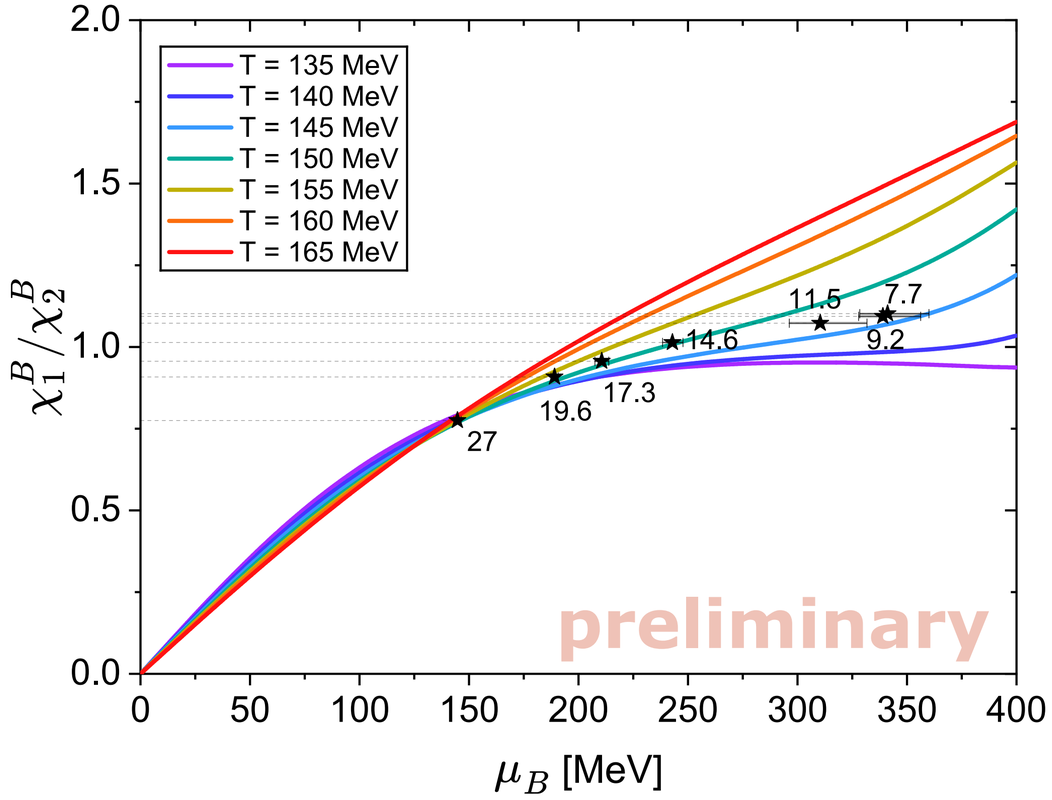
Demand: Precise freeze-out line at high density

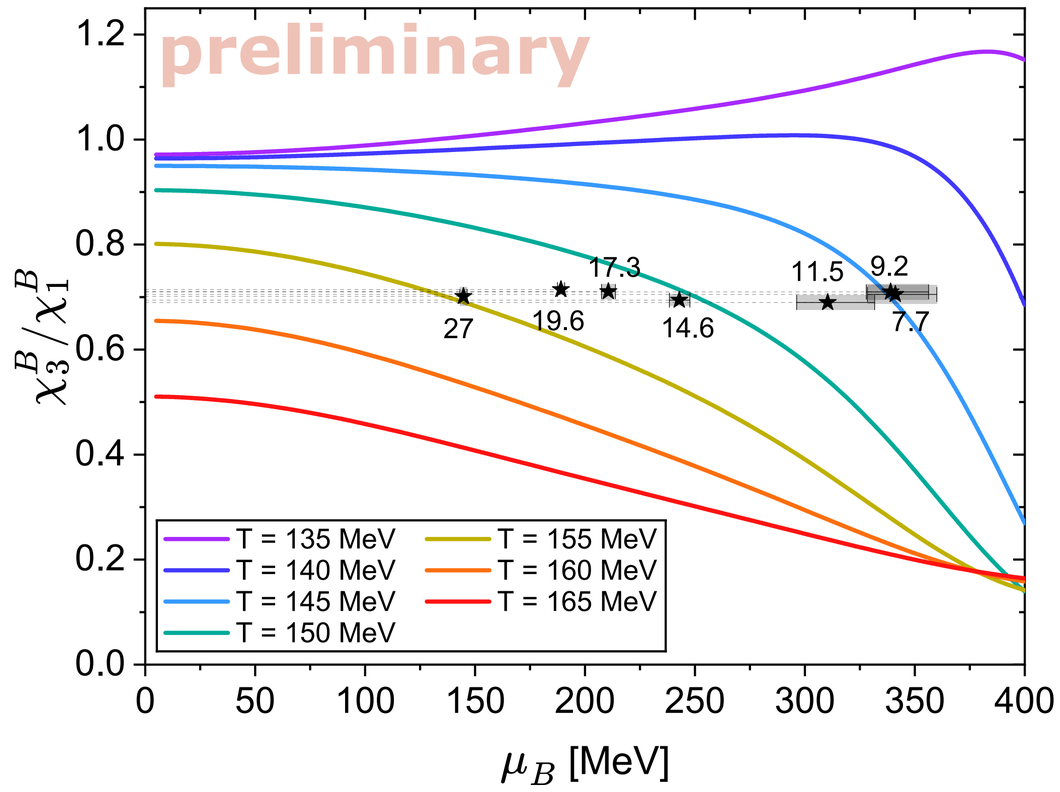
Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 111, L031502

See talk of Fei Gao

Functional Freeze-out

$$(T_f\ ,\ \mu_{B\,f}) \longleftrightarrow \sqrt{s}$$

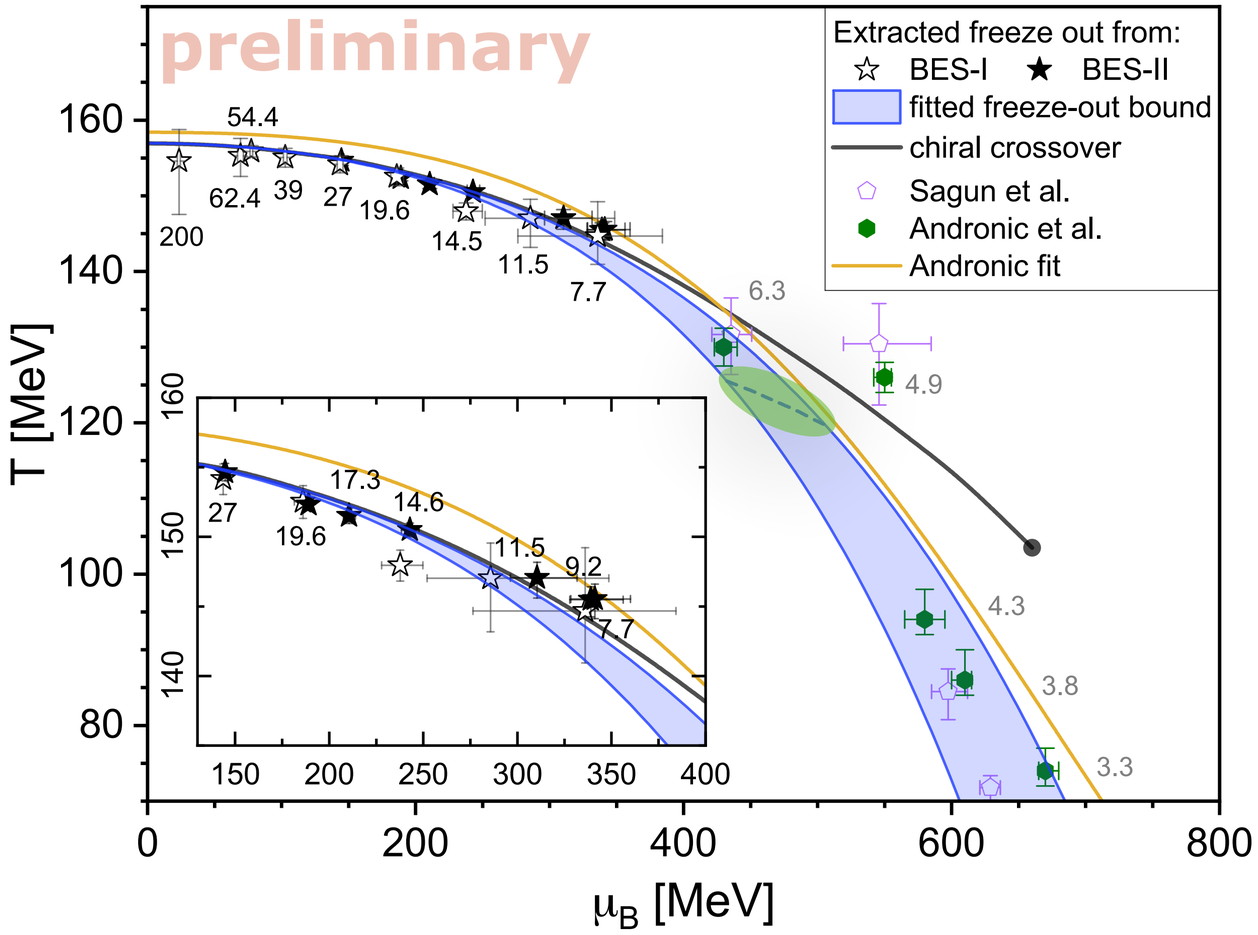


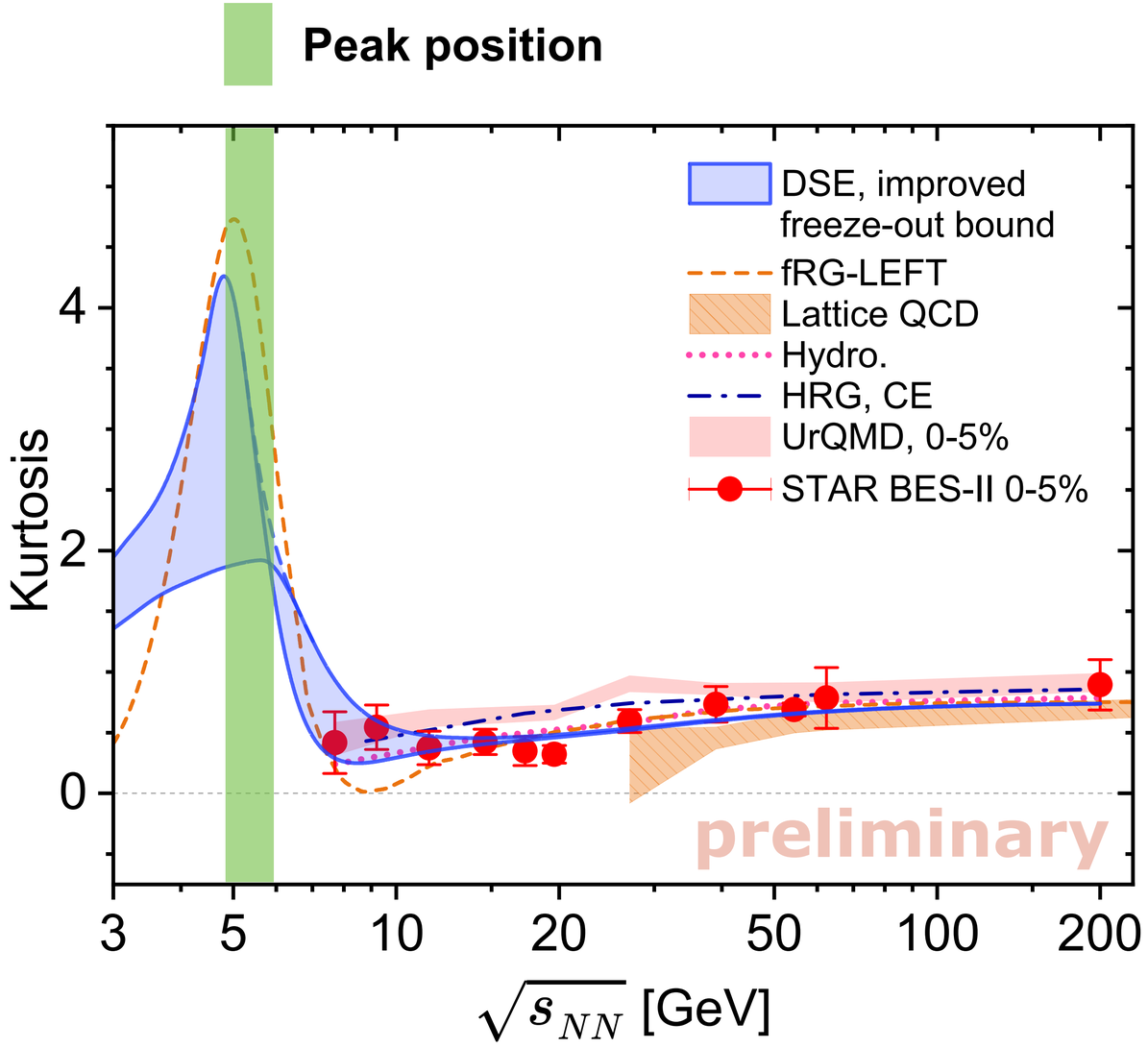


Fischer, Lu, Gao, Liu, JMP, in preparation

See talk of Fei Gao

Functional Freeze-out





Demand: build in dynamics

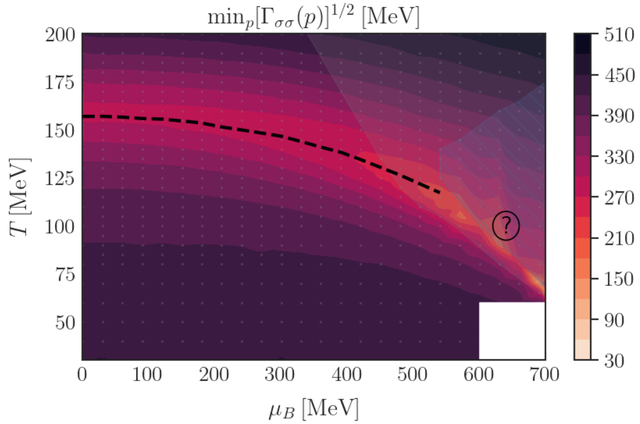
See talks of Yang-Yang Tan and Jonas Wessely

Fischer, Lu, Gao, Liu, JMP, in preparation

QCD moat and inhomogeneous phases

aka

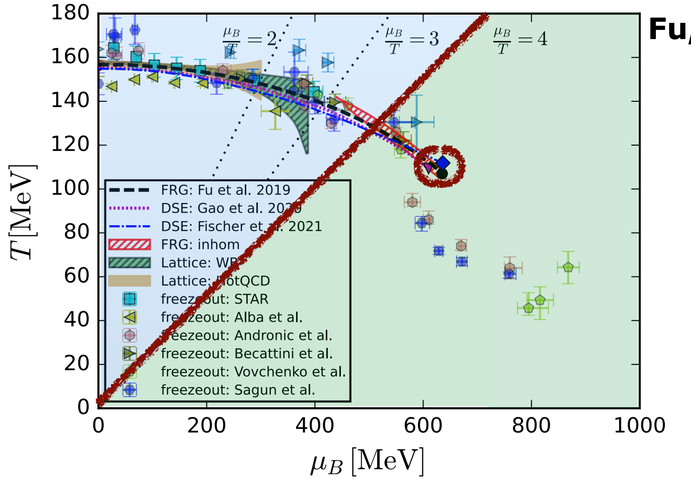
Critical end point and new phases: where do we stand reloaded



See talk of Shi Yin

The QCD Moat

Prediction



Fu, JMP, Rennecke, PRD 101 (2020) 054032

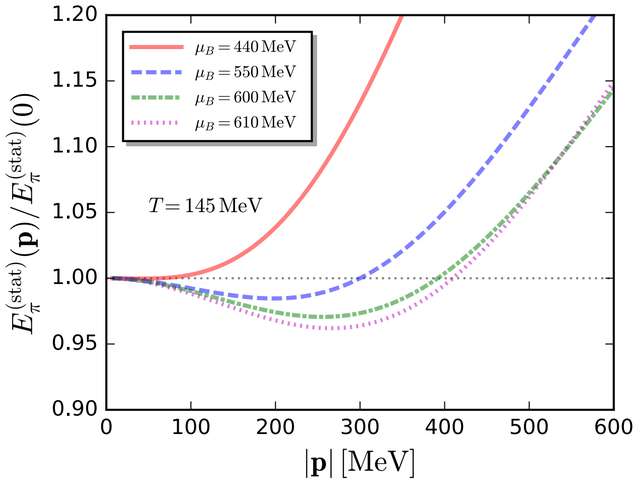
Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

Regime of quantitative reliability of current best truncation

Estimate

Moat regime is not captured quantitatively



Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

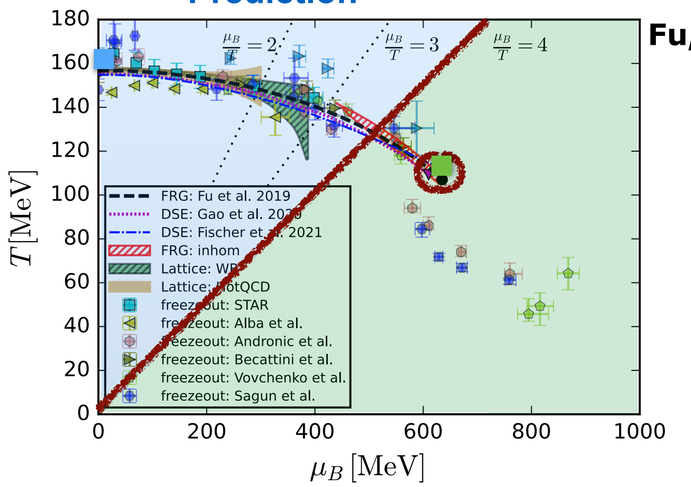
See talk of Shi Yin

The QCD Moat

Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

Prediction



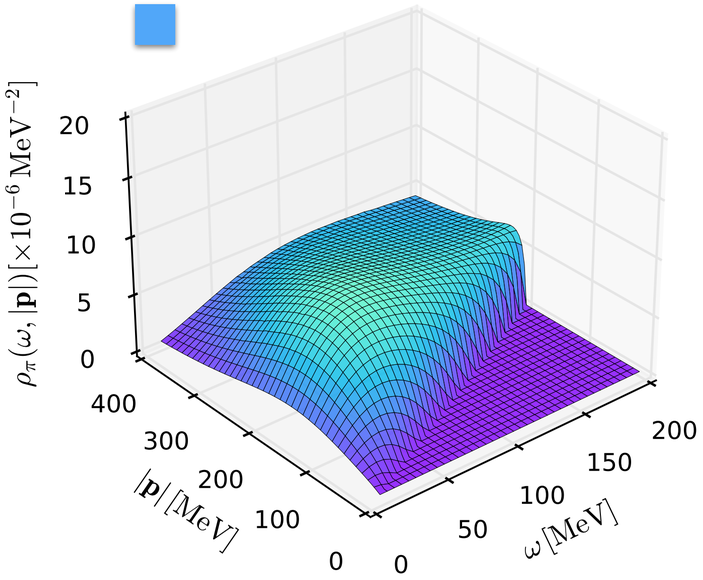
Fu, JMP, Rennecke, PRD 101 (2020) 054032

Regime of quantitative reliability
of
current best truncation

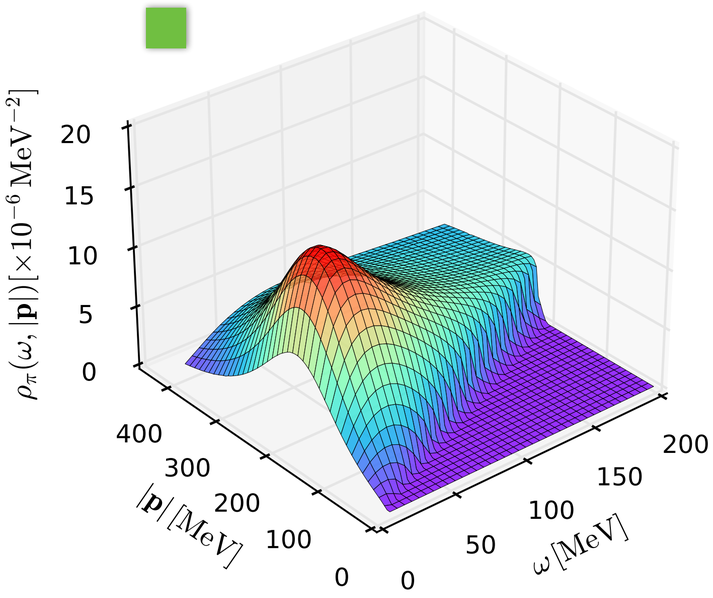
Estimate

Moat regime is not captured quantitatively

T=160 MeV & μ_B =0 MeV



T=114 MeV & μ_B =630 MeV



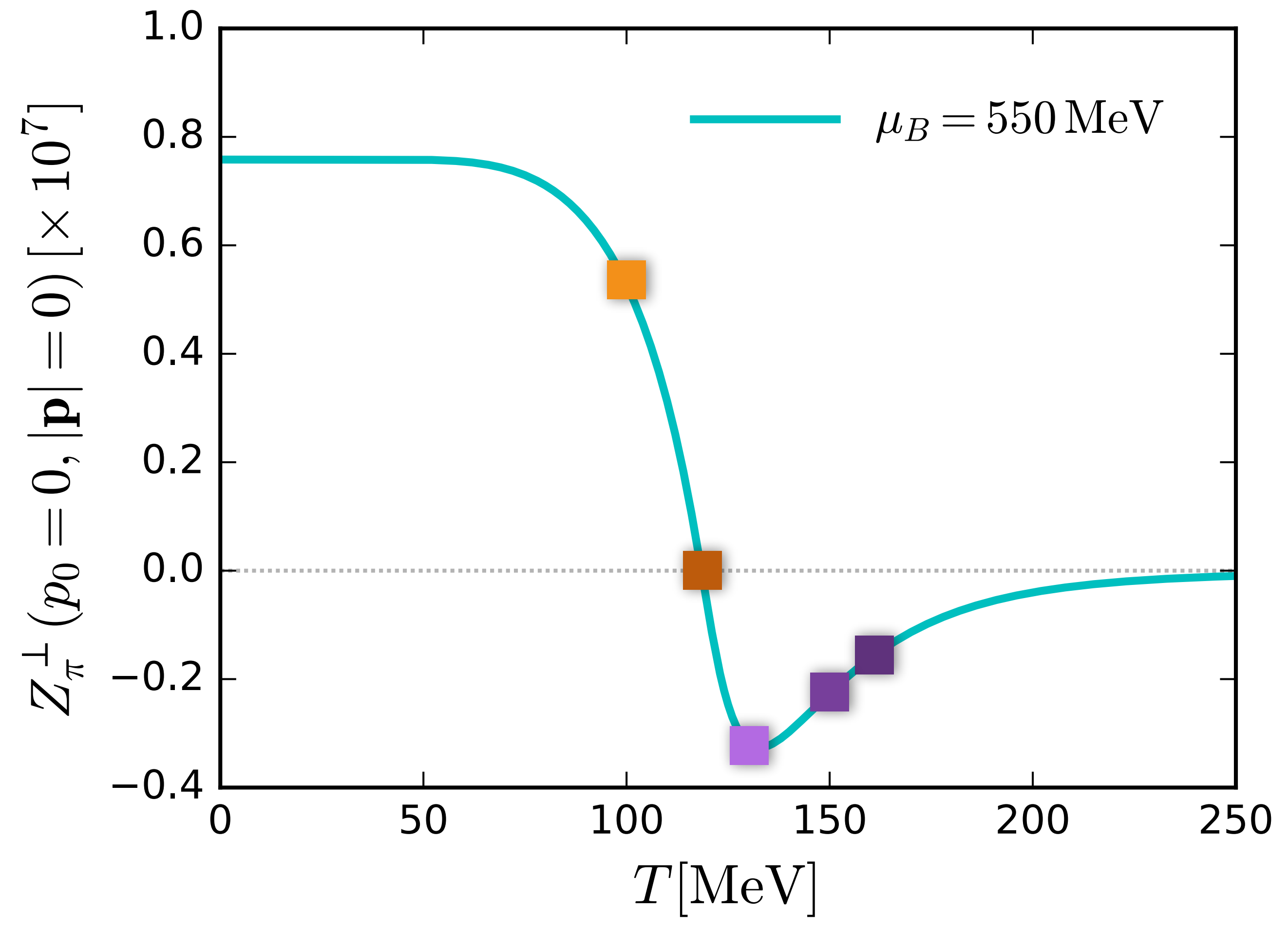
Pion spectral functions

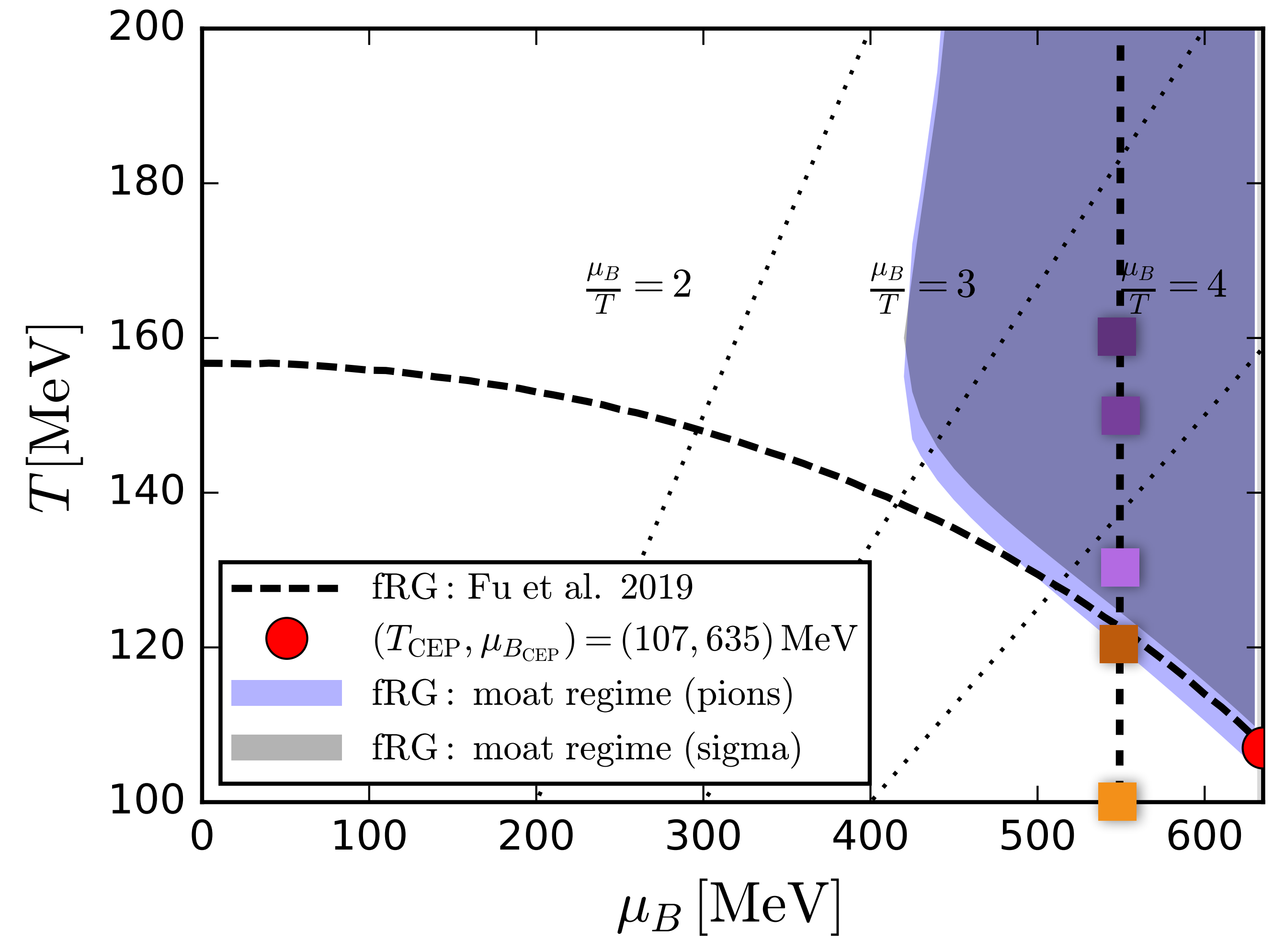
Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

See talk of Shi Yin

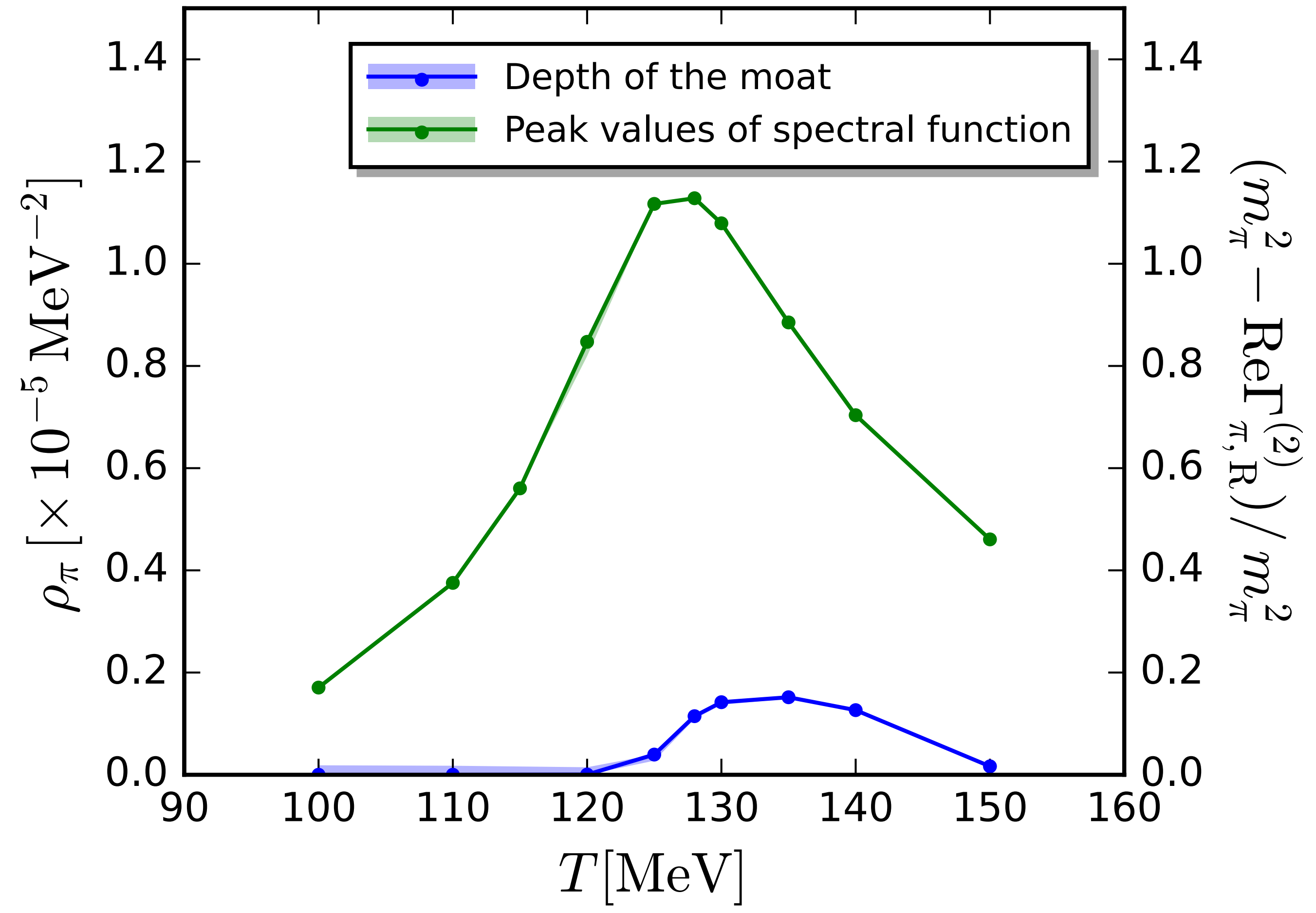
The QCD Moat

Spatial wave function of the pion at p=0





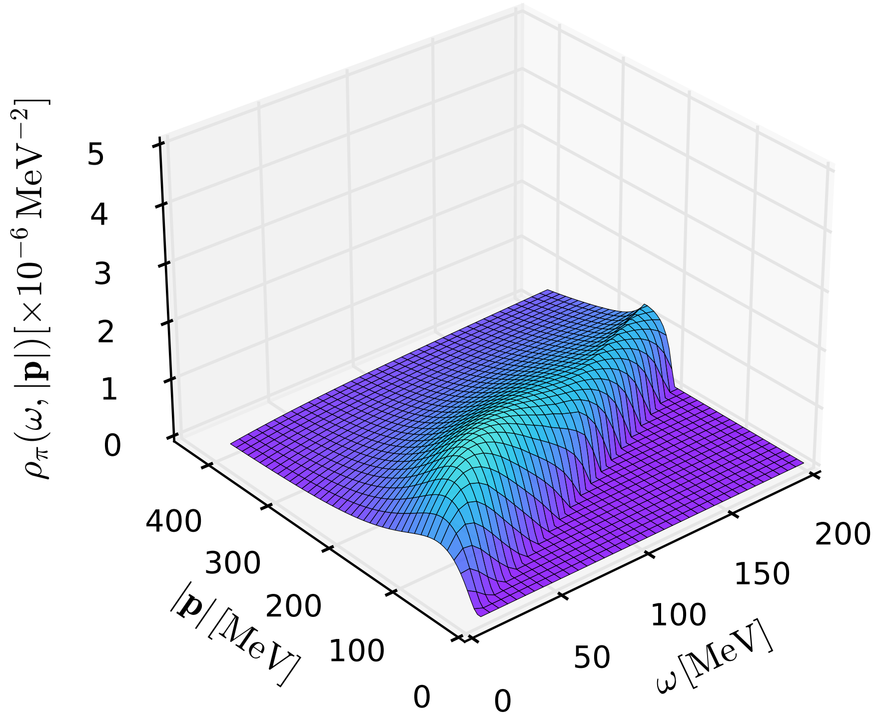
Depth of the moat & spectral peak of the pion

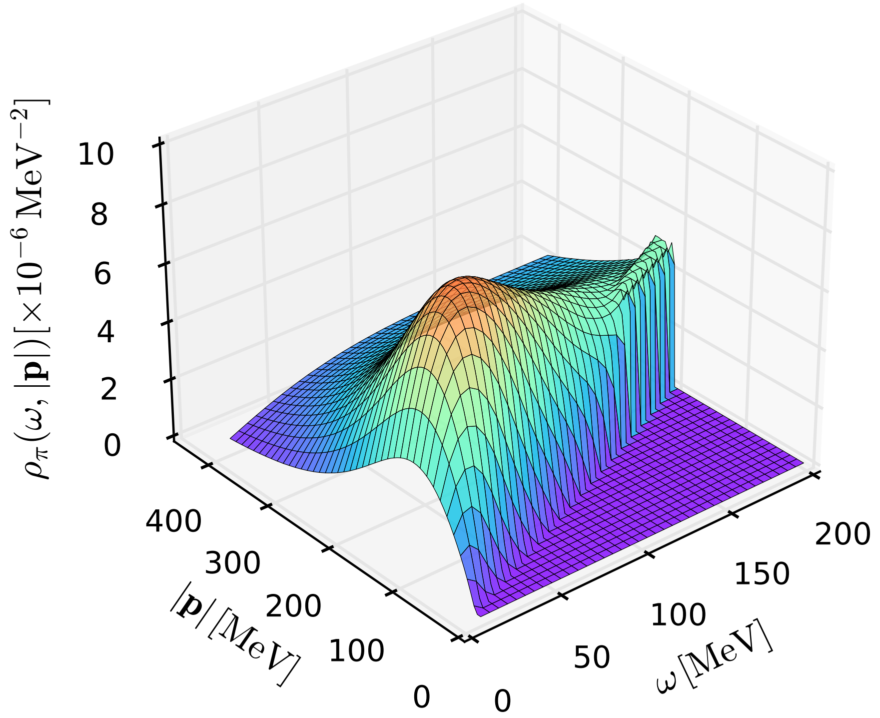


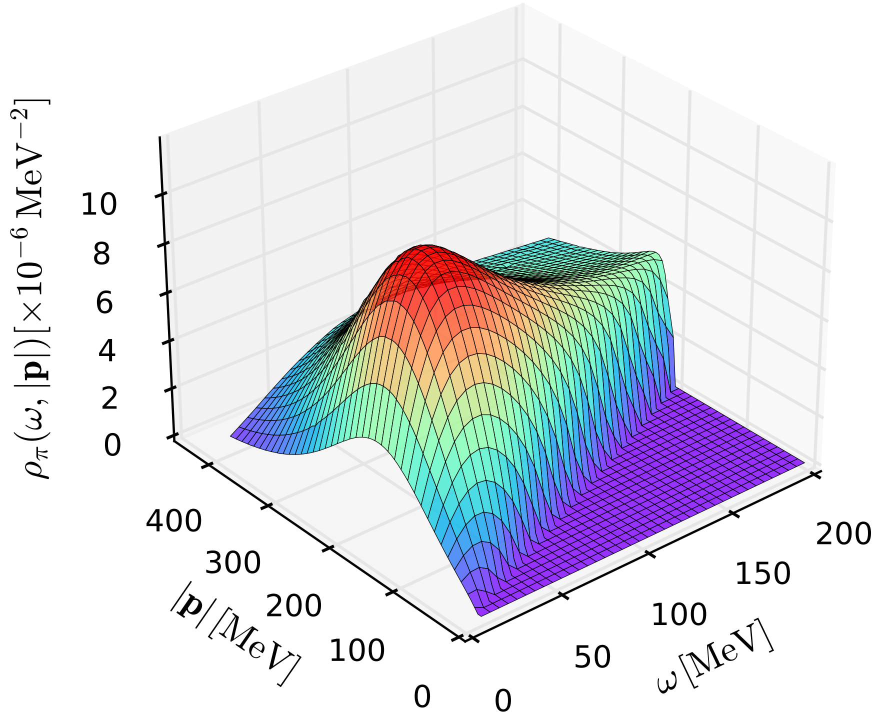
Dissecting the moat & the moaton

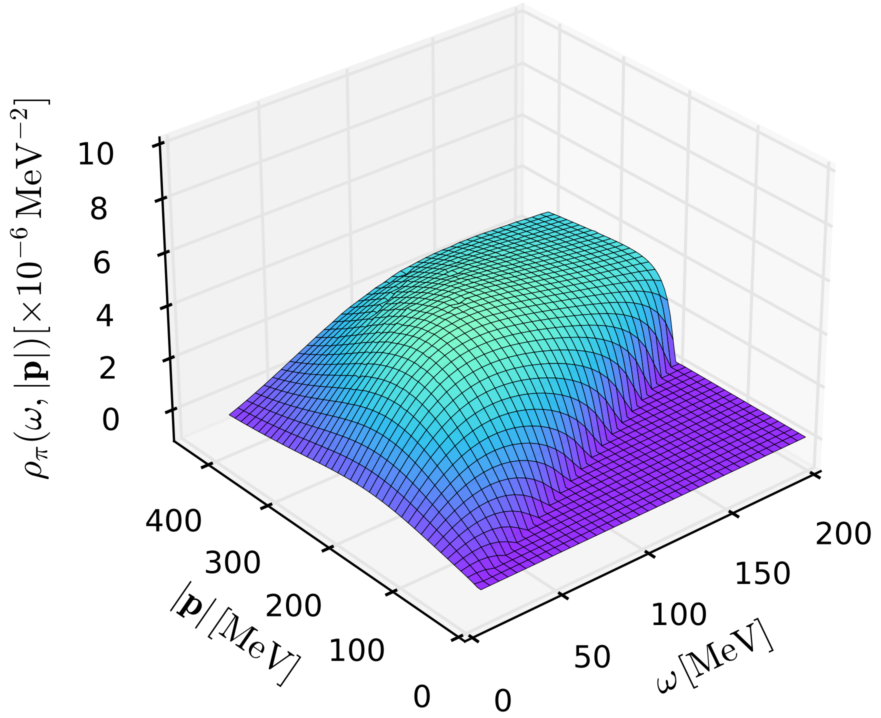
μ_B = 650 MeV

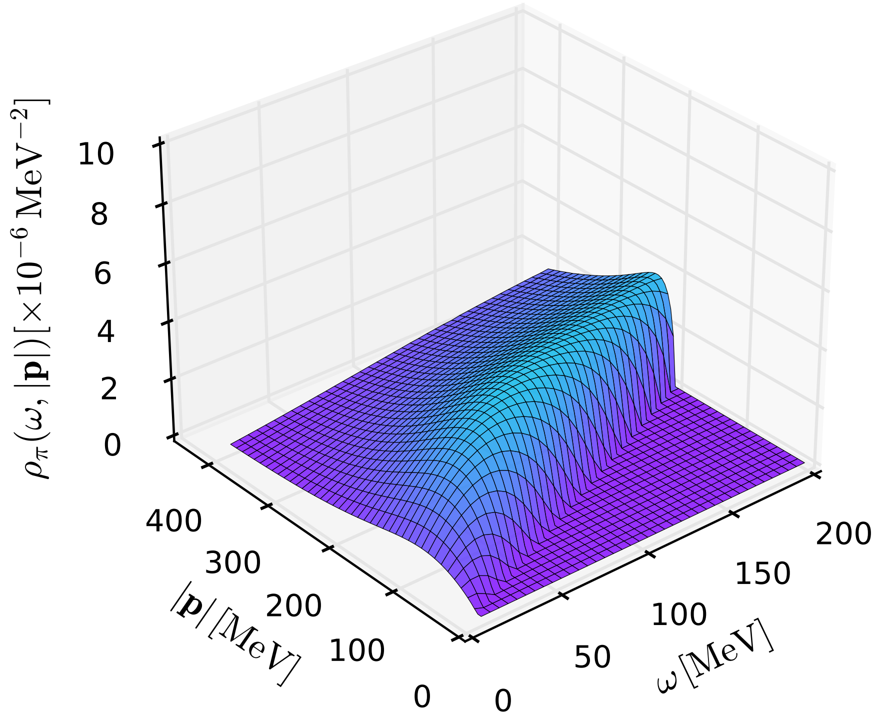
T=100 MeV **T=120 MeV** **T=130 MeV** **T=150 MeV** **T=160 MeV**











Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

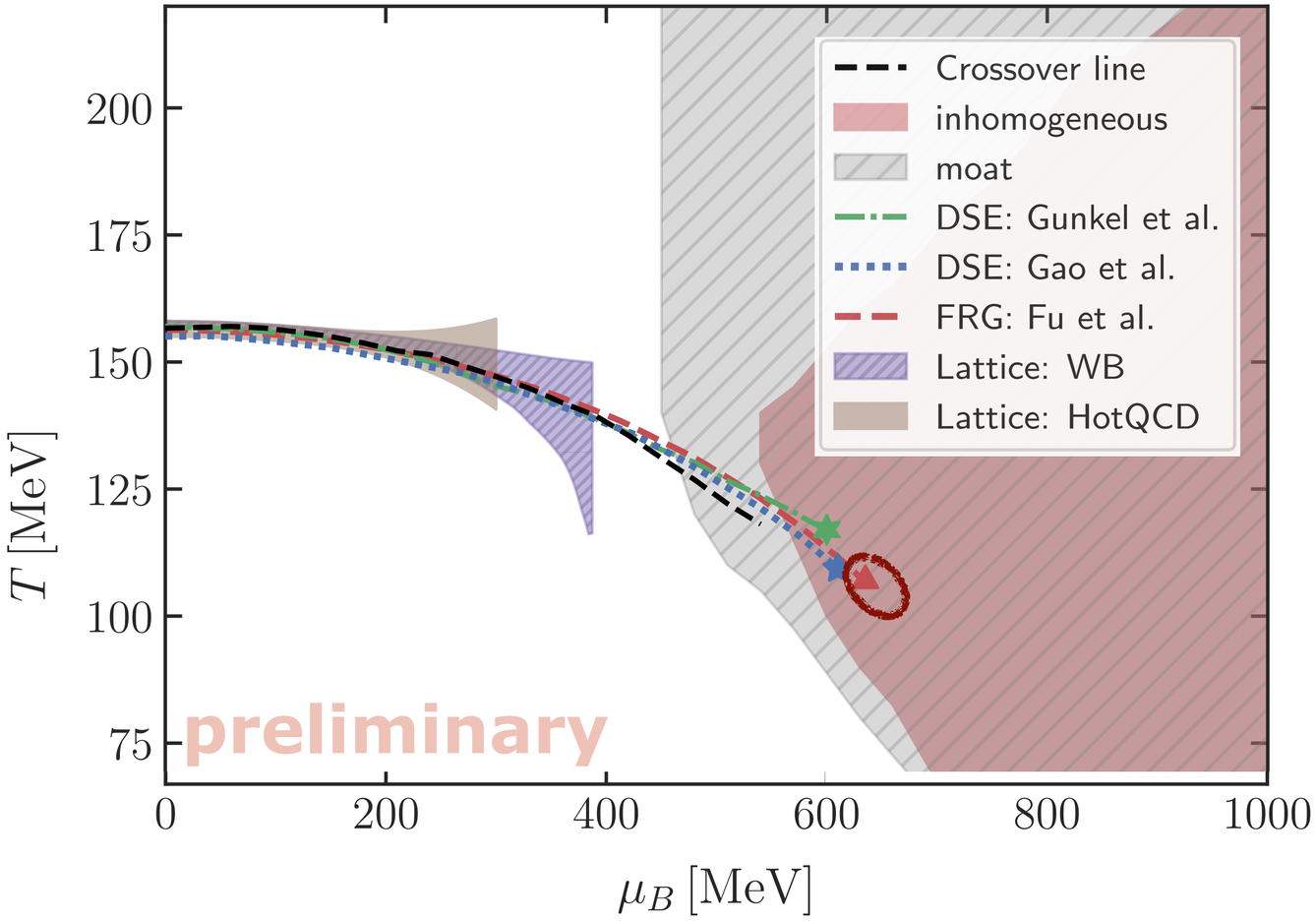
See talk of Franz Sattler

The QCD Moat

resolution of the moat regime

No computation inside the instability regime

Estimated location of CEP

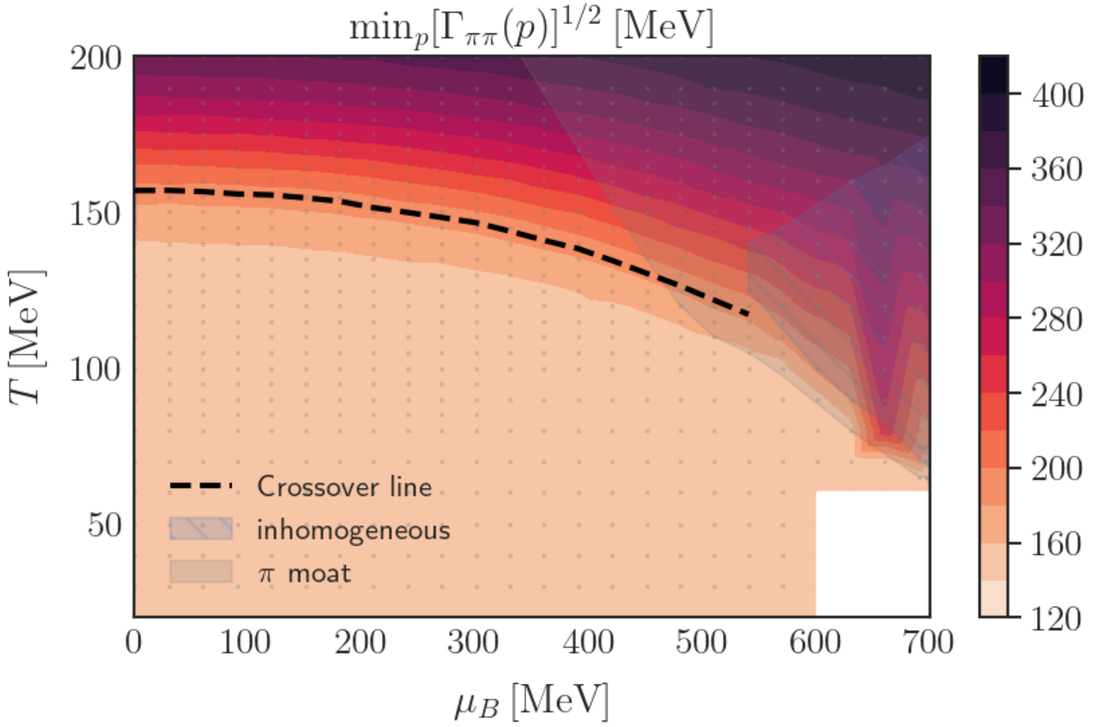


JMP, Rennecke, Sattler, in preparation

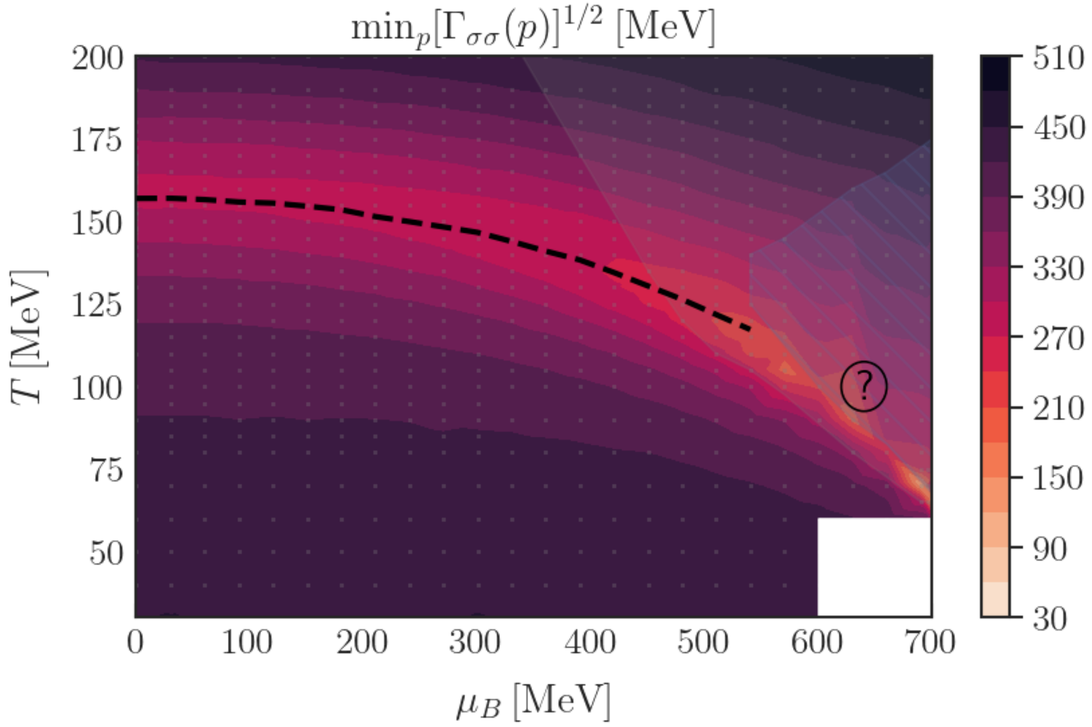
See talk of Franz Sattler

The QCD Moat

Pion ‘minimum’ mass



Scalar mode ‘minimum’ mass



preliminary

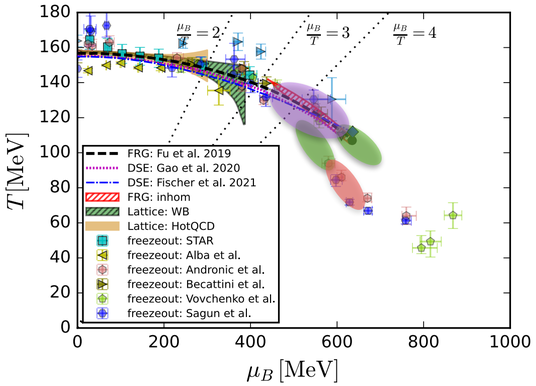
JMP, Rennecke, Sattler, in preparation

Summary

- **Critical end point: where do we stand?**
  - **Predictions:** $\frac{\mu_B}{T} \lesssim 4$
  - **Estimates:** $4 \lesssim \frac{\mu_B}{T} \lesssim 8$

Onset of new phases

$$(\mu_B\,,\,T)_{\rm CEP} \sim (600-650\,,\,105-115)\ {\rm MeV}$$

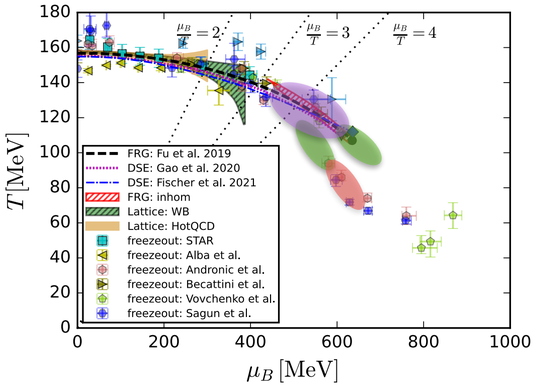


Summary

- **Critical end point: where do we stand?**
  - **Predictions:** $\frac{\mu_B}{T} \lesssim 4$
  - **Estimates:** $4 \lesssim \frac{\mu_B}{T} \lesssim 8$

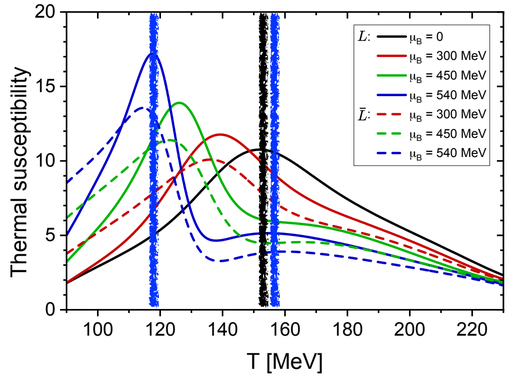
Onset of new phases

$$(\mu_B\,,\,T)_{\rm CEP} \sim (600-650\,,\,105-115)\ {\rm MeV}$$



- **Strong correlations above the chiral crossover**

What are their experimental signatures?

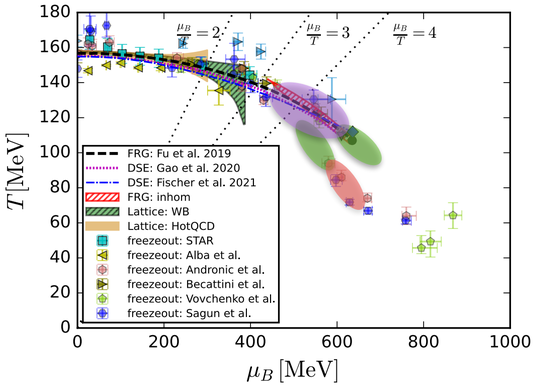


Summary

- **Critical end point: where do we stand?**
  - **Predictions:** $\frac{\mu_B}{T} \lesssim 4$
  - **Estimates:** $4 \lesssim \frac{\mu_B}{T} \lesssim 8$

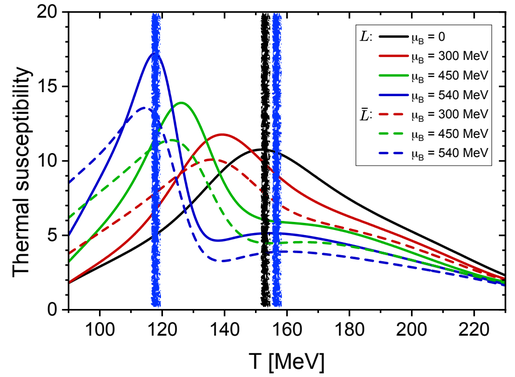
Onset of new phases

$$(\mu_B\,,\,T)_{\rm CEP} \sim (600-650\,,\,105-115)\ {\rm MeV}$$



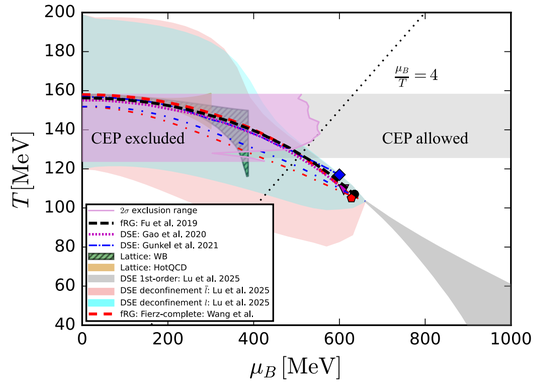
- **Strong correlations above the chiral crossover**

What are their experimental signatures?



- **The physics of soft modes**

Small scaling regime around potential CEP



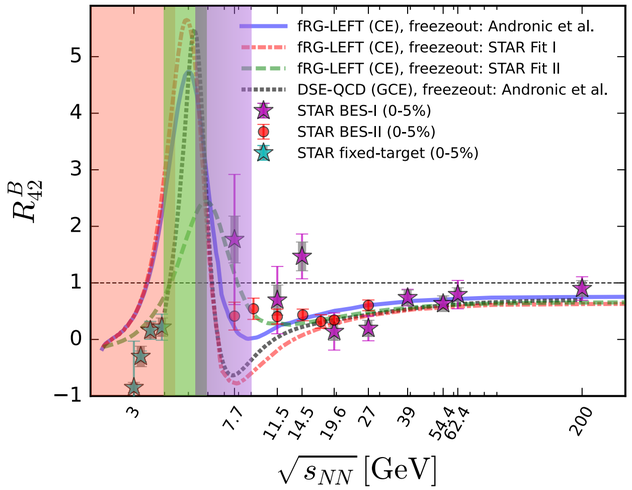
!!Great News!!

Location of CEP/New phase accessible via combination of precision measurements & computations

Summary

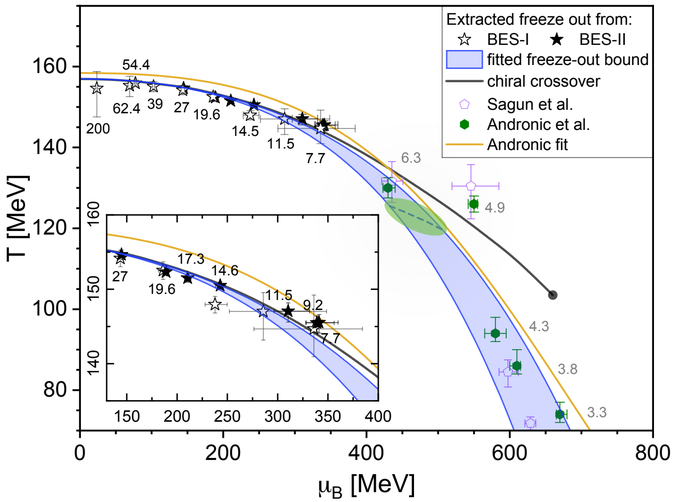
- **Ripples of the critical and point and new phases**

Fluctuations of conserved charges



Dominated by non-critical soft modes

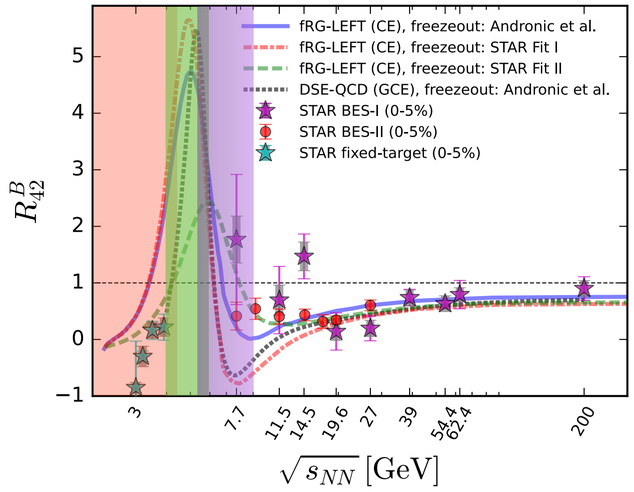
Functional freeze out



Summary

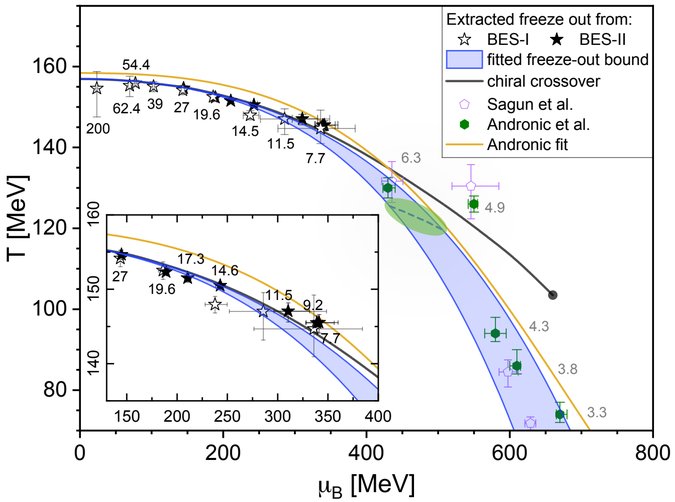
- **Ripples of the critical and point and new phases**

Fluctuations of conserved charges



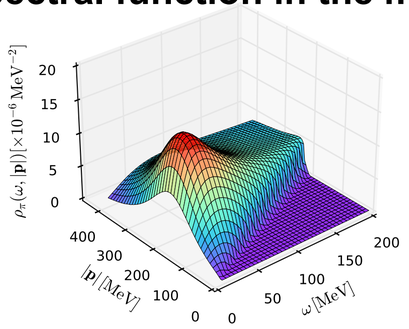
Dominated by non-critical soft modes

Functional freeze out



- **The QCD moat and inhomogeneous phases**

Spectral function in the moat



Signatures?

resolution of the moat regime in QCD