Phase structure and thermodynamics of cold and dense quark matter

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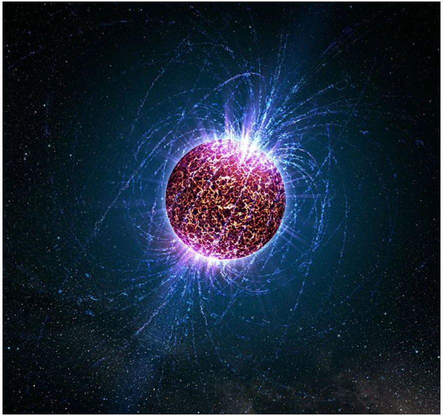
Dalian, October 8, 2025

SFB 1245

TECHNISCHE UNIVERSITÄT DARMSTADT

What do we know about neutron stars?

- stable object that emerges from the gravitational collapse of a massive star
- solution of Einsteins equation within a star and the equation of state of a neutron gas
- except for black holes, smallest and **densest** known stellar objects
- $R \sim 10$ km
- $M_N \sim 1.4 \ldots 2.4\ M_\odot$
- $T \sim 10^6 \ldots 10^{12}$ K



[space.com]

What do we need to describe a neutron star?

- gravity (general relativity)
- electroweak interaction (β-equilibrium)
- isospin imbalance ($\sim$ more neutrons than protons?!)
- strong-interaction matter

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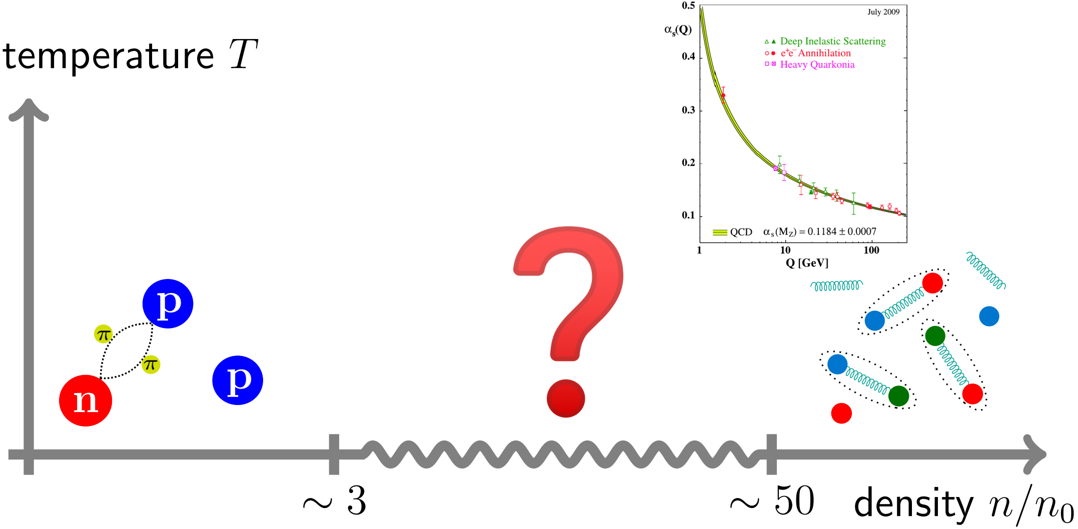
This talk!

Equation of state of cold and dense quark matter

- What is the ground state?
- understanding of neutron stars requires knowledge about the equation of state over a wide density range
- macroscopic properties (pressure, speed of sound, transport coefficients...) over a wide range of length scales
- microscopic degrees of freedom crucially determine the properties of the macroscopic object (neutron star)
- suitable description in terms of effective degrees of freedom

Degrees of freedom



QCD phase diagram of methods

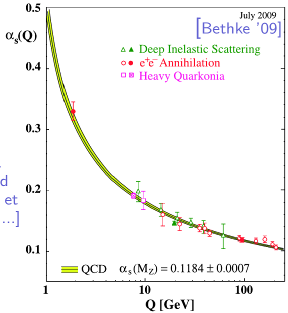
temperature T

lattice

[H. T. Dings et al. (HotQCD) '19; S. Borsanyi et al. '20; F. Cuteri et al. '22; A. Chabane and O. Philipsen '22, O. Philipsen '23; ...]

model studies

[M. Drews and W. Weise '15; R.-A.Tripold, C. Jung, L. v. Smekal, J. Wambach, '21; K. Otto, M. Oertel, and B.-J. Schäfer '19 and '20; Y. Fujimoto et al. '23; L. McLerran and S. Reddy '18;...]



functional QCD studies

[D. T. Son '99; R. Pisarski and D. Rischke '00, T. Schäfer and F. Wilczek '99, J. Berges and K. Rajagopal '99; K.Fukushima, J. M. Pawlowski et al. '21; J. Braun, F. Rennecke et al. '23; W. Fu, F. Rennecke al. '23; F. Ihssen et al. '23 ...]

χEFT

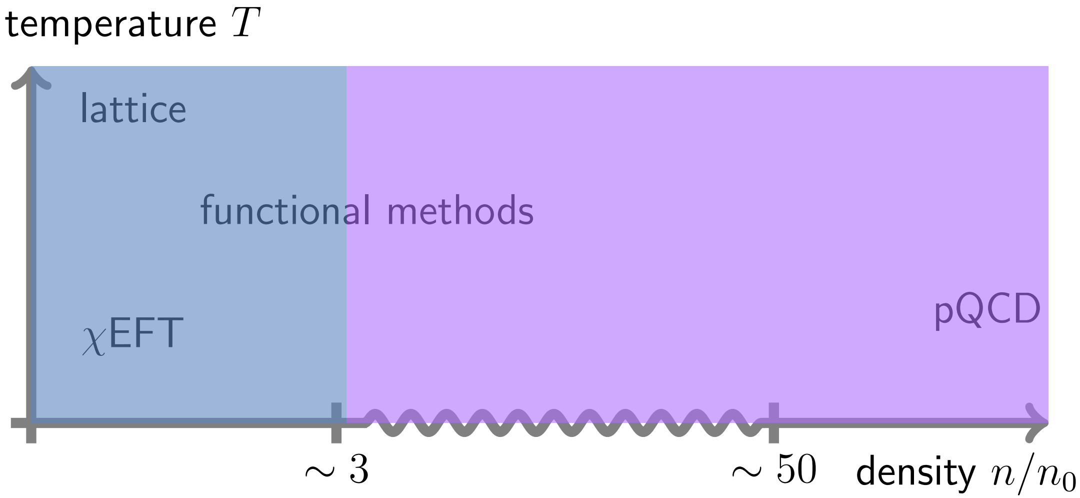
[K. Hebeler '20; E. Epelbaum, H.W. Hammer, and Ulf-G. Meißner '08; Cirigliano et al. '24, Ingo Tews et al. '18; . . .]

pQCD

[A. Kurkela, P. Romatschke, and A. Vuorinen '10, A. Kurkela et al. '14; T. Gorda et al. '18; . . .]

~ 3 ~ 50 density n/n_0

What is this talk about?



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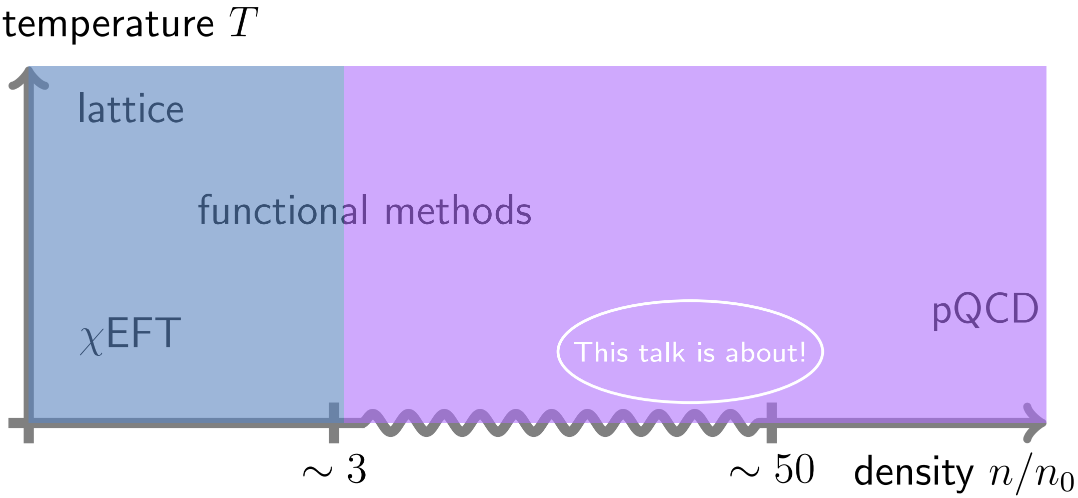
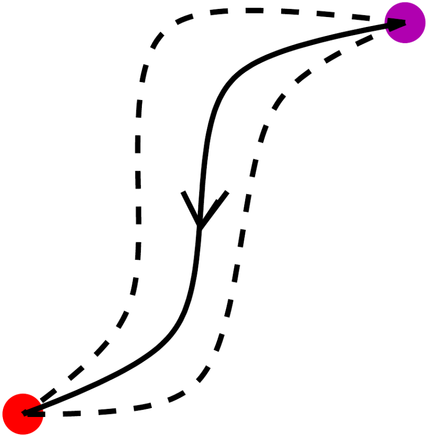


Renormalization group perspective

[Wetterich '93; (figure adapted from) Gies '06]

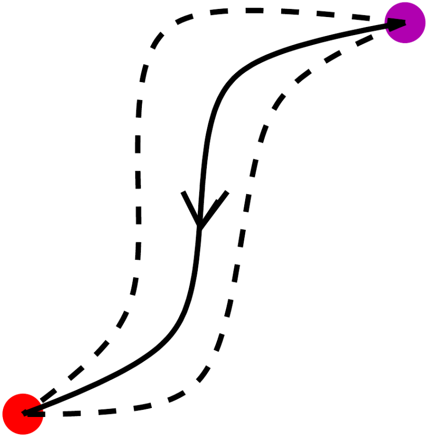
$\Gamma_{k=\Lambda} = S_{\mathrm{QCD}}$



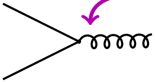
$\Gamma_{k=0} = \Gamma \to$ pressure, expectation values, ...

Renormalization group perspective

[Wetterich '93; (figure adapted from) Gies '06]



$$\Gamma_{k=\Lambda} = S_{\mathrm{QCD}} = \int_x \bar{\psi}\left(i\partial\!\!\!/ - i\gamma_0\mu\right)\psi + g\bar{\psi}A\!\!\!/\psi + \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}$$



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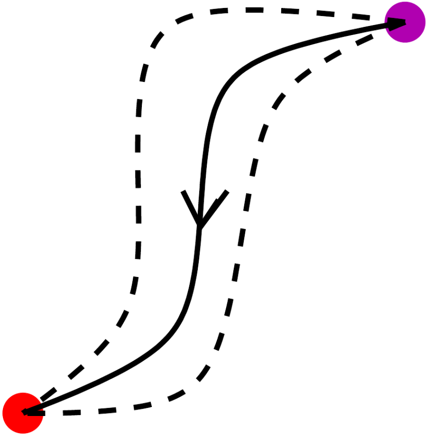

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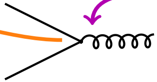

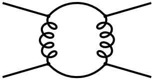

e.g., four-quark interactions λ_i

$\Gamma_{k=0} = \Gamma \to$ pressure, expectation values, ...

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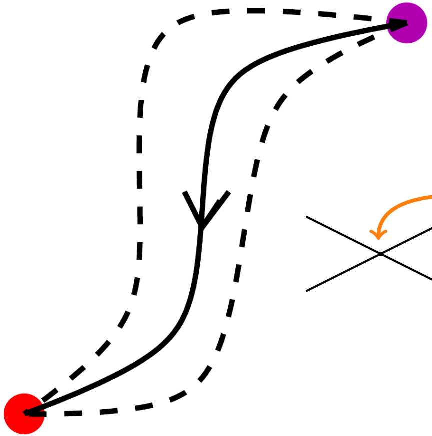

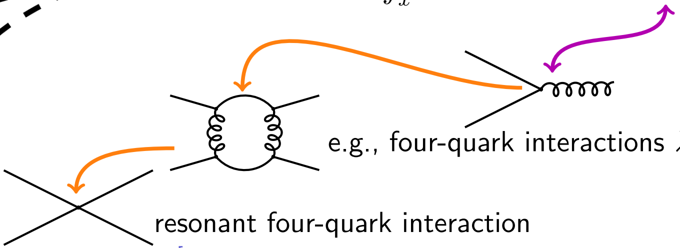

[M. Leonhardt et al. '19; W. Fu et al. '19, J. Braun and B. Schallmo '21, ...]

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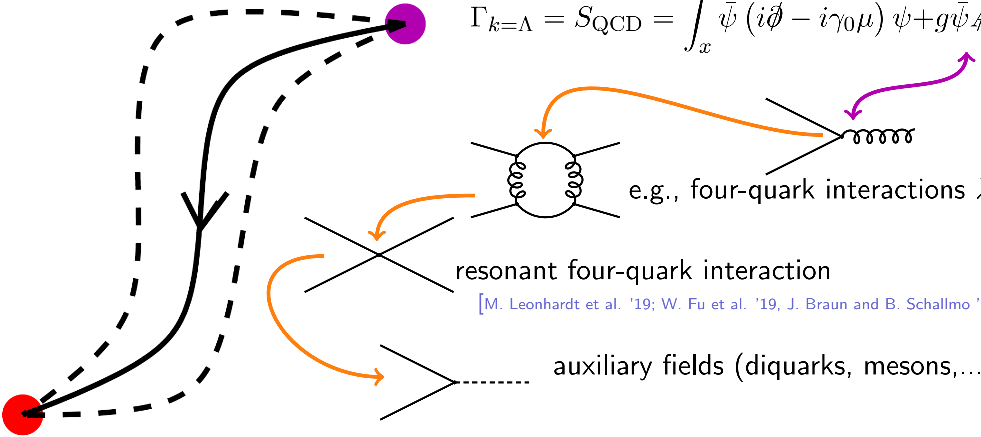

$\Gamma_{k=0} = \Gamma \rightarrow$ pressure, expectation values, ...

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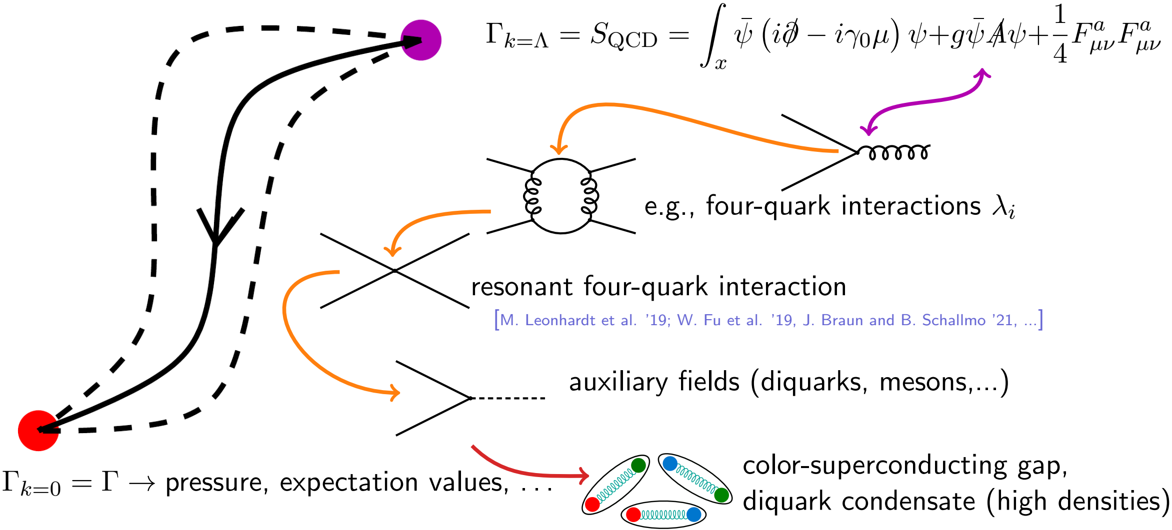
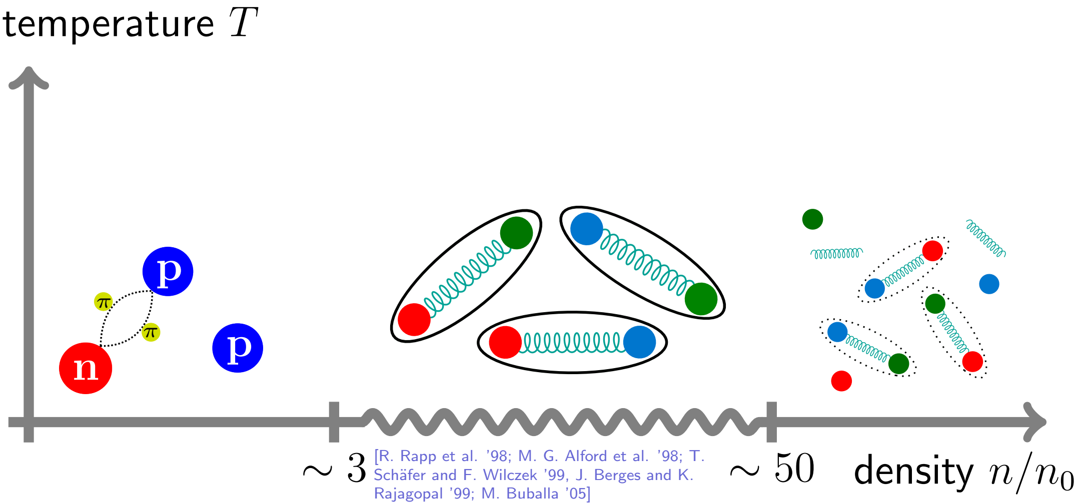


Degrees of Freedom



[R. Rapp et al. '98; M. G. Alford et al. '98; T. Schäfer and F. Wilczek '99, J. Berges and K. Rajagopal '99; M. Buballa '05]

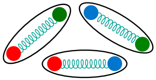
Color superconductivity

$N_{\mathrm{f}} = 2$

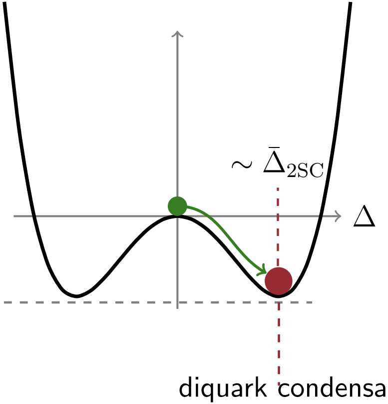
$\mathrm{U}(1)_V$ symmetry breaking

quark cooper pairs

$\Delta_a \sim \langle \psi^T \mathcal{C} \gamma_5 \tau_2 \epsilon_a \psi \rangle$



ground state lowered



two-flavor diquark condensate is chirally symmetric

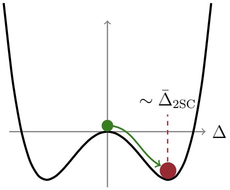
Expansion of the pressure

[J. Braun, AG, and B. Schallmo '22]

- effective action Γ in the presence of a two-flavor diquark condensate:

$\boxed{N_{\mathrm{f}} = 2}$

$$p = -\frac{\Gamma}{V_4}\bigg|_{\mu,\,\Delta=\bar{\Delta}_{2\mathrm{SC}}} + \mathrm{const.}$$



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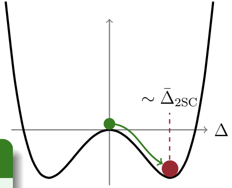


expansion about vanishing gap

$$p = p_{\mathrm{free}}\left(\gamma_0(g) + \gamma_1(g)\left(\frac{|\bar{\Delta}_{2\mathrm{SC}}|}{\mu}\right)^2 + \ldots\right)$$

- expanding the γ_i perturbatively
- gap implicitly depends on strong coupling g

Expansion of the pressure

[AG, T. Gorda, and J. Braun '24]

$\boxed{N_{\mathrm{f}} = 2}$

double expansion

$$p \approx p_{\text{free}} \left(1 - \frac{g^2}{2\pi^2} + \left(2 + 1.09(4)\, g^2\right) \left(\frac{|\bar{\Delta}_{2\mathrm{SC}}|}{\mu}\right)^2\right)$$

$\bar{\Delta}_{2\mathrm{SC}}(g)$

free pressure

pert. contribution

gapped quark loop

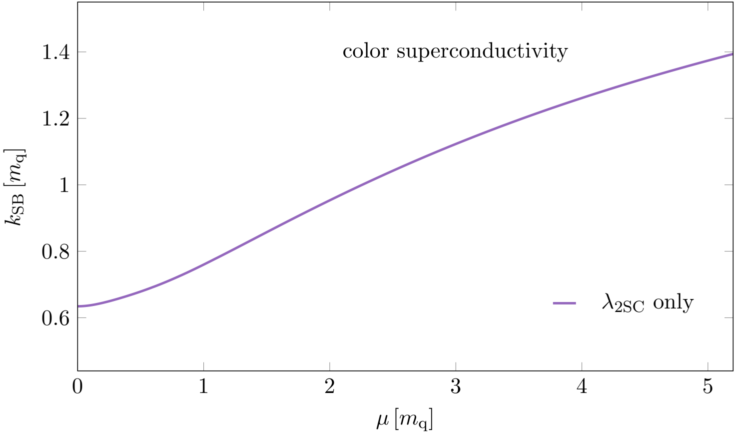
gapped 2-loop contribution

Diquark gap from first principles

[AG and J. Braun (in prep.)]

- functional renormalization group and dynamical bosonization
- spontaneous symmetry breaking indicated by k_{SB}

$N_{\mathrm{f}} = 2$

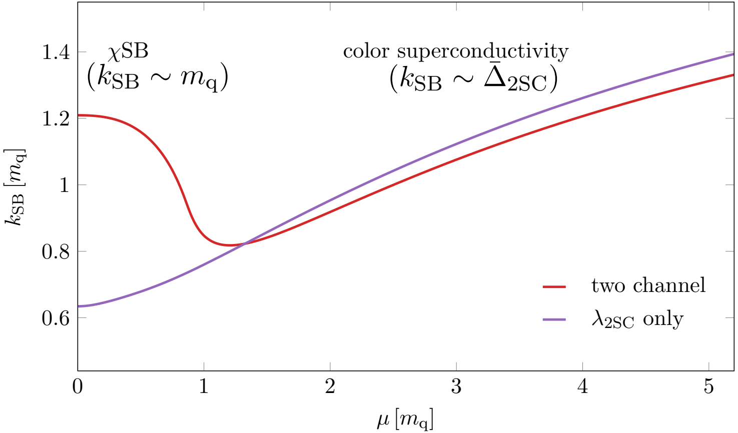


Diquark gap in two-channel approximation

[AG and J. Braun (in prep.)]

- two-channel approximation including 2SC and chiral channel
- strong coupling g at a perturbative scale is the only input

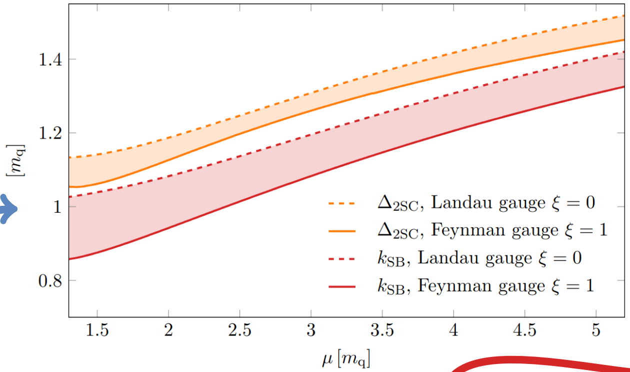
$\boxed{N_{\mathrm{f}} = 2}$



Diquark gap

[AG, and J. Braun (in prep.)]

no Taylor expansion about $g = 0$



ext. truncation

two-channel approximation

very high densities (weak-coupling)

$$\bar{\Delta}_{\text{gap}} \sim \Lambda_{\text{QCD}} \exp\left(-\frac{c}{g^2}\right)\,,\;\; c > 0$$

$$\bar{\Delta}_{\text{gap}} \sim \frac{\mu}{g^5} \exp\left(-\frac{\bar{c}}{g}\right)\,,\;\; \bar{c} > 0$$

[Son '99; Pisarski and Rischke '00, Schäfer and Wilczek '99]

Stiffness of the equation of state

- Speed of sound

$$c_{\mathrm{s}}^2 = \frac{1}{\mu}\left(\frac{\partial p}{\partial \mu}\right) \Bigg/ \left(\frac{\partial^2 p}{\partial \mu \partial \mu}\right)$$

- Causality

$$c_{\mathrm{s}}^2 \leq 1$$

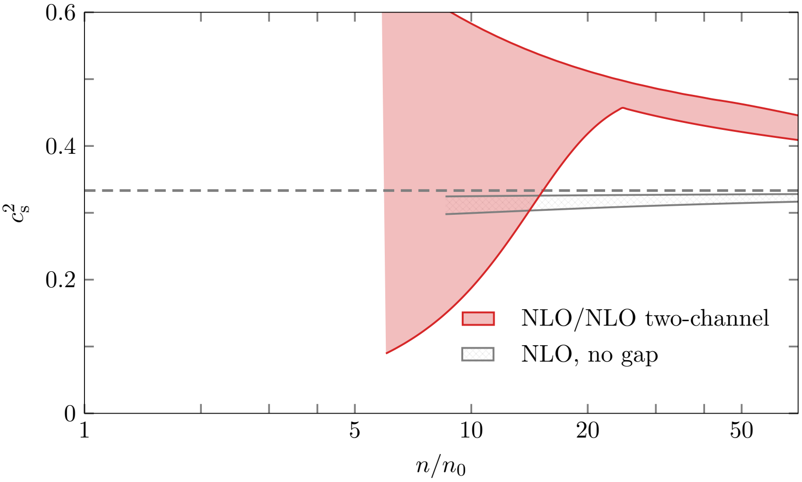
- Thermodynamic stability

$$c_{\mathrm{s}}^2 \geq 0$$

- Speed of sound is a measure for the **stiffness** of the equation of state.
  Stiffness is needed to prevent a neutron star from collapsing to a black hole.

Speed of sound

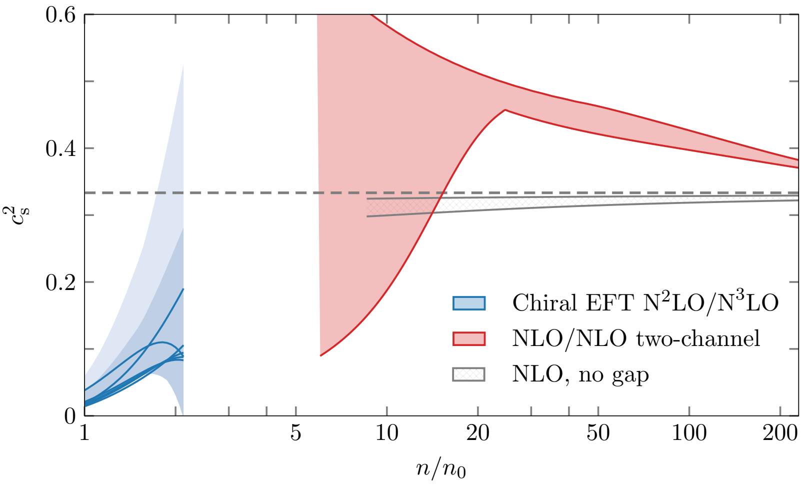
[AG, and J. Braun (in prep.), AG, T. Gorda, and J. Braun '24, B. A. Freedman and L. D. McLerran '77]



$N_{\mathrm{f}} = 2$

Speed of sound from dense to dilute

[AG, and J. Braun (in prep.), M. Leonhardt, et al '19, B. A. Freedman and L. D. McLerran '77]



$N_f = 2$

From two-flavors to color-flavor locking

[AG, T. Gorda, and J. Braun '25]

$\boxed{N_\mathrm{f} = 3}$

double expansion

$$p \approx p_\mathrm{free} \left(1 - \frac{g^2}{2\pi^2} + \left(4 + 3.25(4)\, g^2\right) \left(\frac{|\bar{\Delta}_\mathrm{CFL}|}{\mu} \right)^2 \right)$$

$\bar{\Delta}_\mathrm{CFL}(g)$

free pressure

pert. contribution

gapped quark loop

gapped 2-loop contribution

Gap constraints at high densities

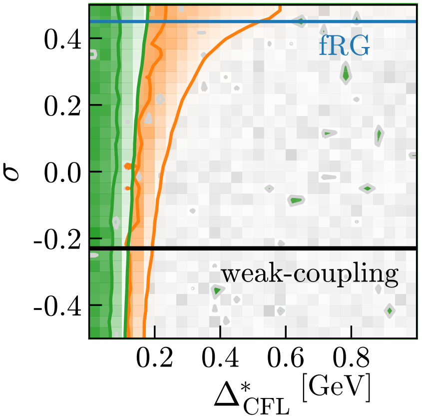
[AG, T. Gorda, and J. Braun '25]

- scaling of the gap

$$\bar{\Delta}_{\mathrm{CFL}} = \Delta^*_{\mathrm{CFL}} \left(\frac{\mu}{\mu^*}\right)^{\sigma},$$

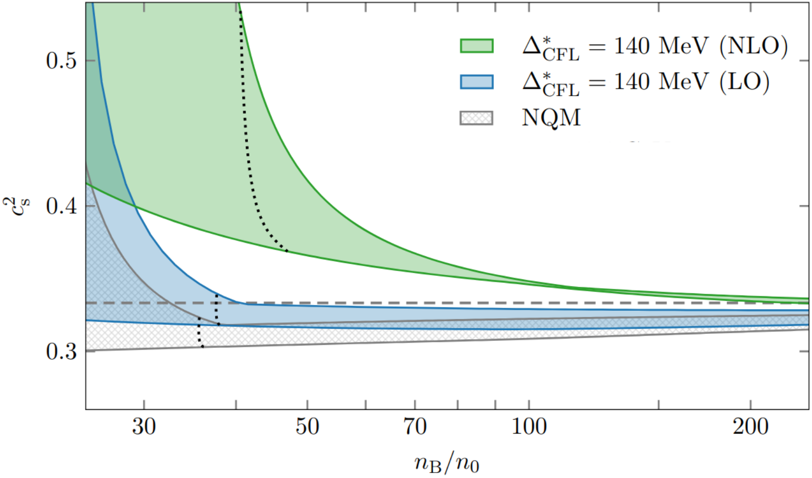
- finite strange quark mass
- state-of-the-art pQCD
- chiral EFT information
- astrophysical observations
- thermodynamic stability
- causality (orange) and $c_s^2 < \frac{2}{3}$ (green)
- under neutron star conditions:

$$\Longrightarrow \Delta^*_{\mathrm{CFL}}(40 n_0) \lesssim 140\,\mathrm{MeV}$$



Speed of sound

[AG, T. Gorda, and J. Braun '25]



$N_f = 3$

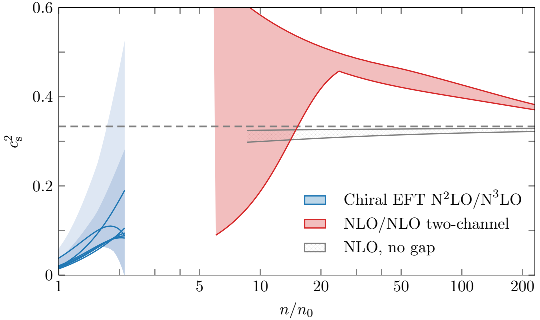
Summary

[M. Thies, AG, J. Braun (in prep.), AG, and J. Braun (in prep.)]

- pairing gap induced correction enhance pressure and speed of sound
- gap calculation from first principles in two-channel approximation
- maximum in the speed of sound at supranuclear densities
- constrains on the CFL gap

$$\Longrightarrow \quad \Delta^*_{\mathrm{CFL}}(40n_0) \lesssim 140\,\mathrm{MeV}$$

- preliminary fRG calculations of CFL gap are in accordance with this constrain



Thank you for your attention!

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