Probing QCD phase structure and thermodynamics via finite-density observables

Yi Lu

Peking University

Based on: YL, Gao, Liu, Pawlowski: 2504.05099

fqcd-collaboration.github.io

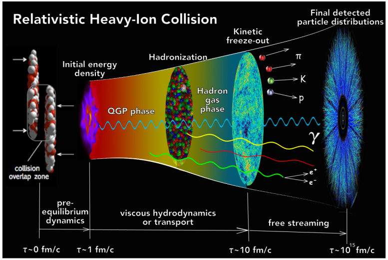
Dalian, 8th 10, 2025

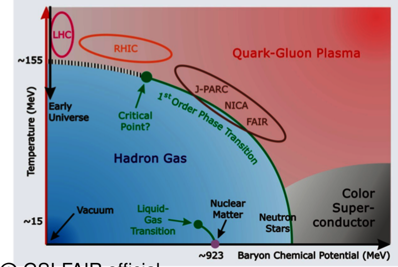
Introduction

Experimental research on QCD phase diagram at high baryon density is entering the precision physics era: CBM (FAIR), CEE+ (HIAF), ...

Great demand on theoretical side:

- Phase structure at high μ_B in parallel of experimental landscape - beyond the μ_B range of current best lattice QCD extrapolation.
- Emergent phenomena - "more is different".
- (Equilibrium) QCD baselines?
  - macroscopically & microscopically.
- How to connect theory with EXP.: thermodynamics & related observables?
- . . .





@ GSI-FAIR official

Theoretical landscape

(Plot courtesy: F. Rennecke, Quark Matter 2025)

• QCD chiral condensate:

Lattice (extrapolation from $\mu_B^2 \leq 0$)

Functional (DSE and fRG, direct computation)

○ Lee-Yang edge singularities:

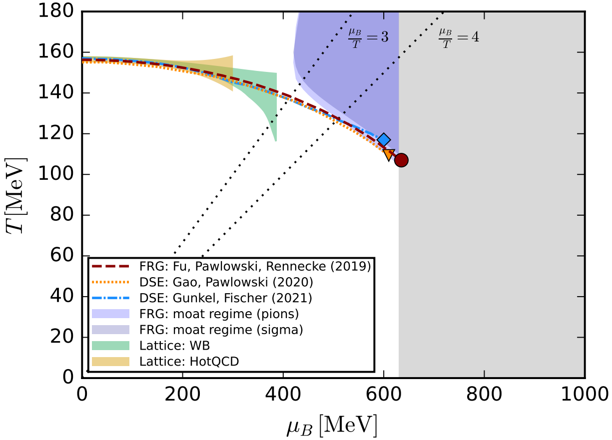
Basar, PRC 110: 015203 (2024),
Clarke et al., 2405.10196,
Schmidt, 2504.00629,

Borsanyi et al., 2507.13254, ...

○ Thermodynamics extrapolations:

Hippert et al., PRD 110: 094006 (2024),
Shah et al. 2410.16206, Borsanyi et al. 2502.10267

(**talk this morning**)

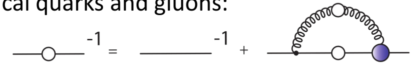


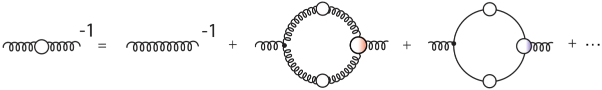
Continuum functional QCD

Dyson Schwinger equations [DSE]

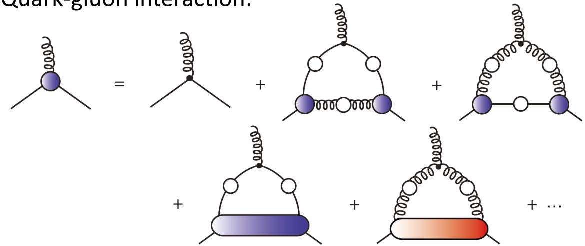
(Plots: Eichmann, 0909.0703)

Dynamical quarks and gluons:





Quark-gluon interaction:

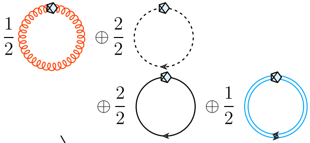


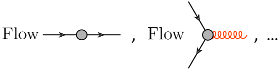
Alkofer & von Smekal, 0007355 Roberts & Schmidt, 0005064
Eichmann at al, 1606.09602 Fischer, 1810.12938 …

functional renormalization group [fRG]

Flow $\Gamma[\Phi] =$

(Wetterich eq.)





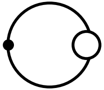
"LEGO principles" 2408.08413

Pawlowski, 0512261 Gies, 0611146
Dupuis at al., 2006.04853 Fu, 2205.00468 …

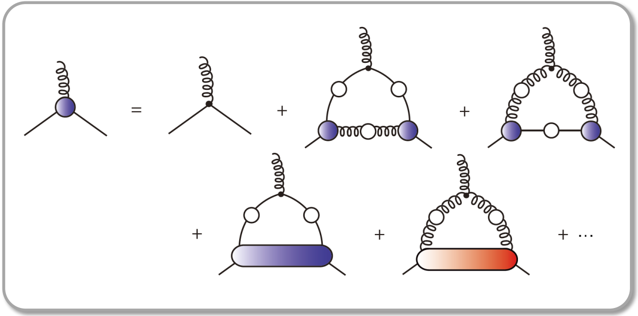
$$\langle \mathbf{vac.}\,|\bar{q}q|\,\mathbf{vac.}\rangle \text{ vs } \sum \langle \mathrm{vac.}\,|\psi\rangle\langle\psi|\,\mathrm{vac.}\rangle$$

$\psi \in$ asymptotic states: $\pi, K, p, n, \ldots$

$$\langle \bar{q}(x)\Gamma q(x)\rangle = -\mathrm{Tr}\,\langle \Gamma q\bar{q}\rangle = -\Gamma$$



- Direct access on QCD correlation functions → off-shell representation;
- All required dynamics and analytical properties of QCD have (to be) captured;
- Free of extrapolations towards high μ_B:
→ necessary condition for probing any "new" physics phenomena.



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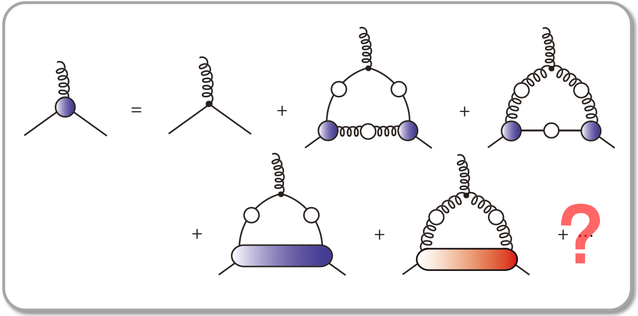

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Price to pay: infinite hierarchy of correlations …

- Inevitably, truncations - judging on convergence and error estimates? ✓

YL, Gao, Liu, Pawlowski, PRD 2024, Ihssen, Pawlowski, Sattler, Wink, 2408.08413, and many others ...

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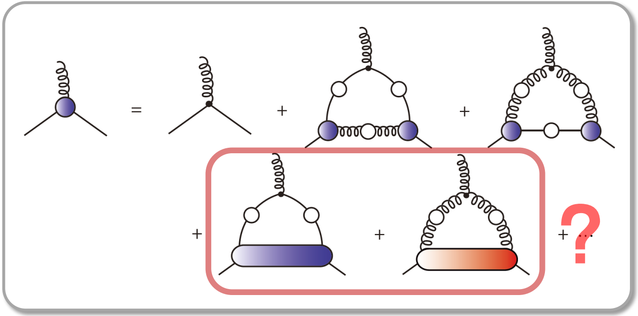

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- Extremely-high density region still less settled: emergent DOF may even dominates...

Theoretical landscape

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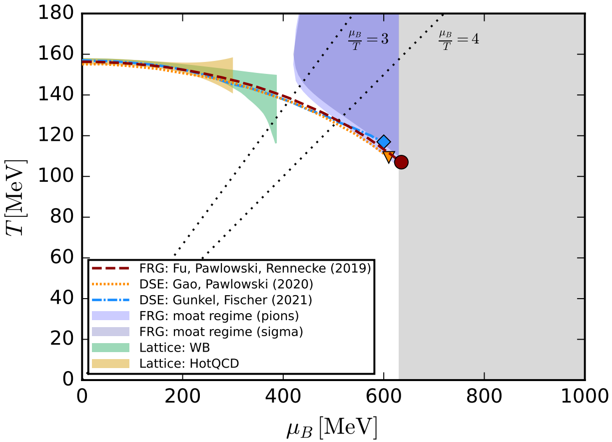


- This talk:

new insights on **(de-)confinement** and **thermodynamics** in fQCD
- underlying chiral phase structure is within the state-of-the-art estimates.

“Going medieval” on QCD (de-)confinement

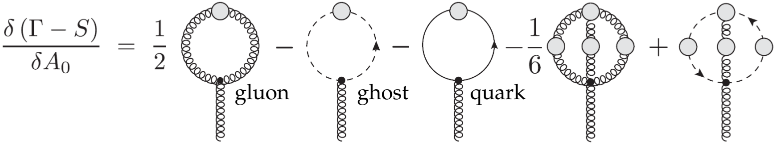
Status before: gluonic background field $\langle A_0 \rangle$, a gauge invariant QCD correlation function, gives a direct access on center symmetry breaking.

Yang-Mills: fRG & DSE: Fister, Pawlowski, PRD 88: 045010 (2013).

2+1-flavor: fRG: Braun, Gies, Pawlowski, PLB 684: 262 (2010); DSE: Fischer, Fister, Luecker, Pawlowski, PLB 732: 273 (2014).

perturbative: Curci-Ferrari model: Reinosa, Serreau, Tissier, PRD 92: 025021 (2015).

Polyakov loop potential; relatively simple truncation in fQCD, e.g. DSE:



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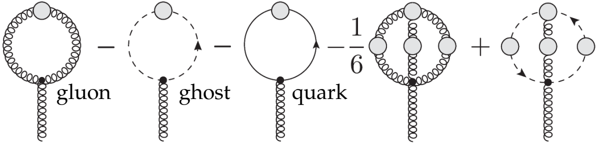
$$\frac{\delta(\Gamma - S)}{\delta A_0} = \frac{1}{2}$$



Meanwhile... the confining dynamics is expected to play a crucial role on QCD thermodynamics; e.g. Fu, Pawlowski, PRD 92: 116006 (2015)

so far, few investigations on thermodynamics in the perspective of A_0 (correlation).

Now: New insights on QCD thermodynamic functions in the view of A_0:

- Complete color SU(3) structure (Cartan sub-algebra) of A_0 at finite μ_B:

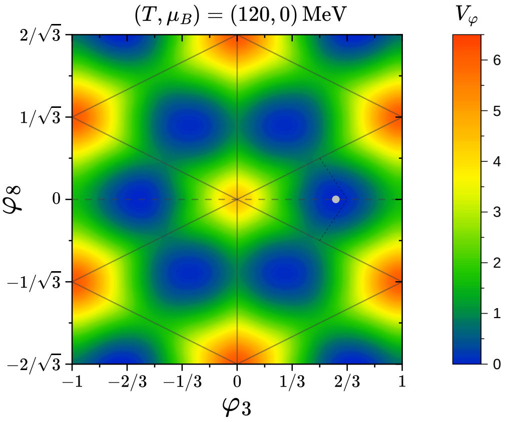
$$A_0 = \frac{2\pi T}{g}\varphi^c; \qquad \text{eigenvalues}: \quad \varphi^c = \begin{cases} \varphi_3/2 \pm \varphi_8/2\sqrt{3},\ \varphi_8/\sqrt{3}, & \text{fund.} \\ 0,\ \pm\varphi_3,\ \pm(\varphi_3 \pm \varphi_8\sqrt{3})/2, & \text{adj.} \end{cases}$$

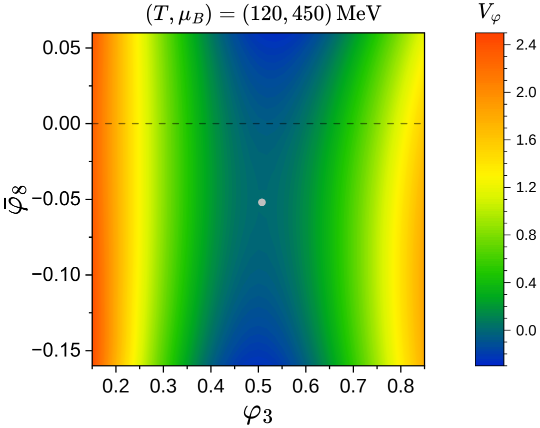
pheno.: difference between the static free energies of quarks and anti-quarks.

$$L(\langle A_0\rangle) = \frac{1}{3}\left[e^{2\pi\bar{\varphi}_8/\sqrt{3}} + 2\, e^{-\pi\bar{\varphi}_8/\sqrt{3}}\cos\pi\bar{\varphi}_3\right], \qquad \bar{L}(\langle A_0\rangle) = \frac{1}{3}\left[e^{-2\pi\bar{\varphi}_8/\sqrt{3}} + 2\, e^{\pi\bar{\varphi}_8/\sqrt{3}}\cos\pi\bar{\varphi}_3\right].$$

Polyakov loop potential in 2+1-flavor QCD, within the field space relevant to physical observables: Reinosa, Serreau, Tissier, PRD (2015), Ihssen and Pawlowski, SciPost Phys. (2023)

$\boxed{\varphi_8 \in \mathbb{R} \quad \text{at} \quad \mu_B = 0}$ $\boxed{\bar{\varphi}_8 = -\mathrm{i}\varphi_8 \in \mathbb{R} \quad \text{at} \quad \mu_B \in \mathbb{R}}$





Polyakov loops L and $\bar{L}$ and their T-susceptibilities:

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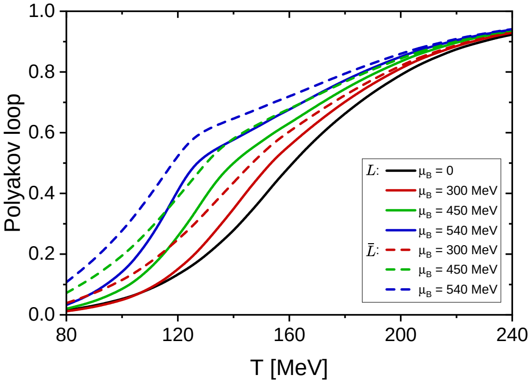

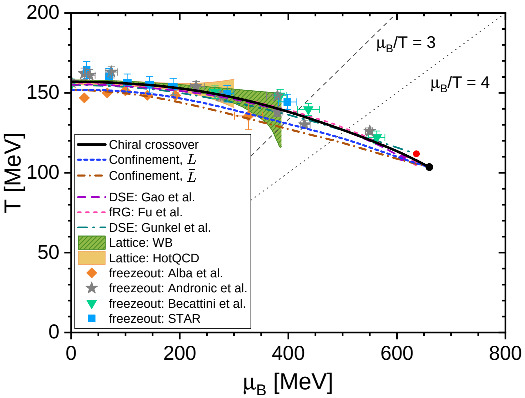

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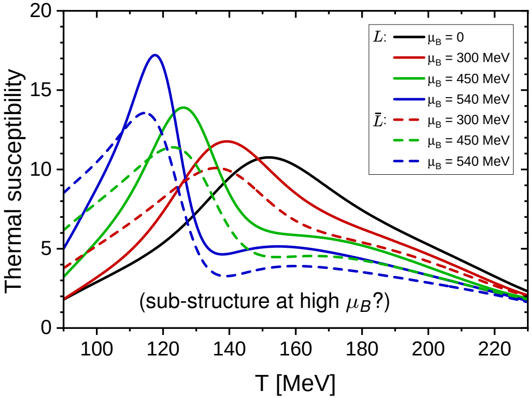

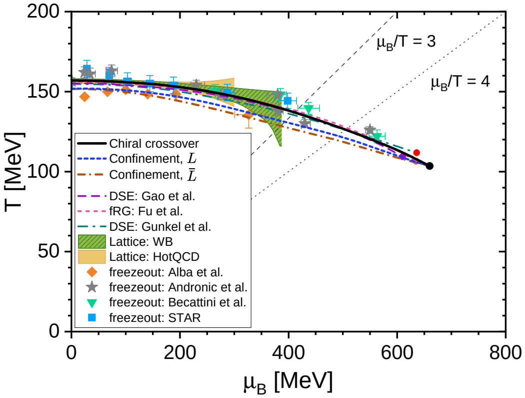

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pheno.: difference between the static free energies of quarks and anti-quarks.

- Matter sector - an off-shell representation is complete: e.g. net density

$n_q = \langle \bar{q}\gamma_0 q \rangle = -\mathrm{Tr}\,[\gamma_0 G_q]$; YL et al., PRD (2024), Isserstadt et al., PRD (2019), ...

both confining and chiral dynamics are included self-consistently:

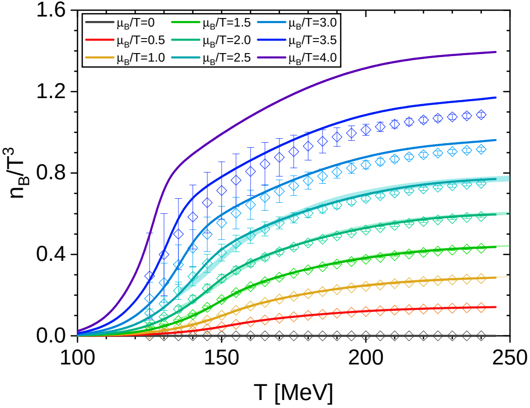
$$G_q^{-1}(p_q) = \mathrm{i}\gamma_0 p_{q,0}\, Z_q^E(p_q) + \mathrm{i}\gamma\cdot\boldsymbol{p}\, Z_q^M(p_q) + Z_q^E(p_q)\boxed{M_q(p_q)}\,,$$

$$p_{q,\mu} = (\omega_{q,n} + \boxed{2\pi T\varphi^c} - \mathrm{i}\mu_q\,,\ \boldsymbol{p}).$$

QCD equation of state: “apples to apples” in continuum & discrete approaches:

fQCD: $n_q(T,\mu_B) = \langle \bar{q}\gamma_0 q\rangle = -\mathrm{Tr}\,[\gamma_0 G_q]$;

lQCD: $n_B(T,\mu_B) = \sum_{n=0}^{N} \frac{P_{2n+2}(T^{(\prime)},0)}{(2n+1)!}\mu_B^{2n+1}$;



- Conserved charge relation: $n_B = \frac{1}{3}(n_u + n_d + n_s)$.

- More than just a success / ‘reproduction’ on e.g. chiral crossover line...

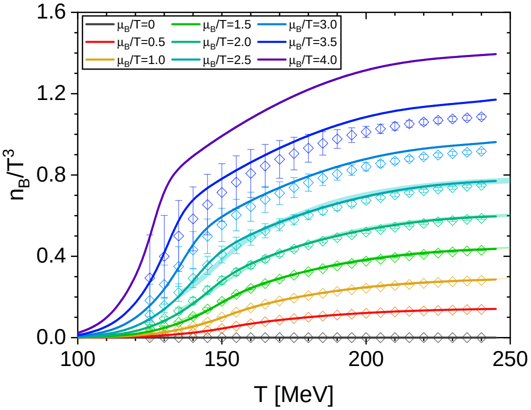
- Testing ground for the convergence of lattice extrapolations: Wen, Yin, Fu, PRD 110: 016008 (2024).

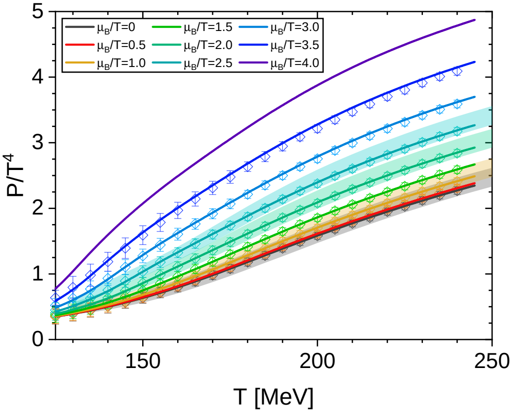
- Going beyond $\mu_B/T \gtrsim 3$: merits on phenomenological studies in parallel of the future experimental landscape.

QCD equation of state: “apples to apples” in continuum & discrete approaches:

fQCD: $n_q(T,\mu_B) = \langle \bar{q}\gamma_0 q\rangle = -\mathrm{Tr}\,[\gamma_0 G_q];$ $\quad P(T,\mu_B) = P(T,0) + \int_0^{\mu_B} n_B(T,\mu)\,d\mu.$

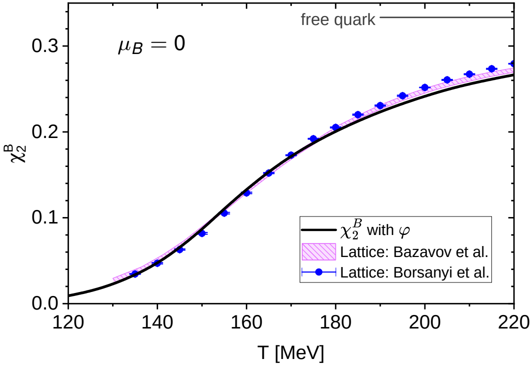
lQCD: $n_B(T,\mu_B) = \sum_{n=0}^{N} \frac{P_{2n+2}(T^{(\prime)},0)}{(2n+1)!}\mu_B^{2n+1};$ $\quad P(T,\mu_B) = \sum_{n=0}^{N} \frac{P_{2n}(T^{(\prime)},0)}{(2n)!}\mu_B^{2n}.$

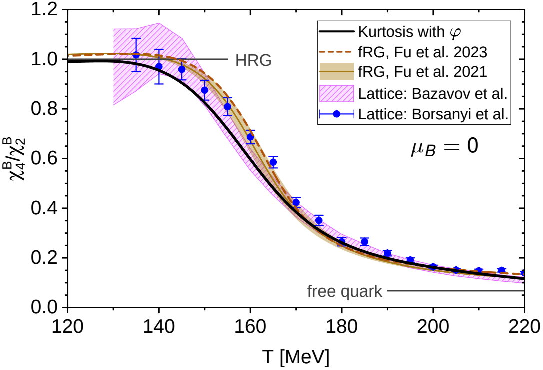




Hints on the finite-μ_B signatures of (de-)confinement phase transition.

$$\chi_k^B = \frac{\partial^k (P/T^4)}{\partial(\mu_B/T)^k} \propto \partial_{\mu_B}^{(k-1)} n_B = \begin{cases} -\frac{1}{3}\sum\limits_{q=u,d,s} \mathrm{Tr}\left[\gamma_0(\partial_{\mu_B}^{(k-1)} G_q)\right], & \text{fQCD;} \\ \sum_{n=0}^{N-k} \frac{P_{2n+k}(T^{(\prime)},0)}{(2n)!}\mu_B^{2n}, & \text{lQCD.} \end{cases}$$





Baryon number fluctuations

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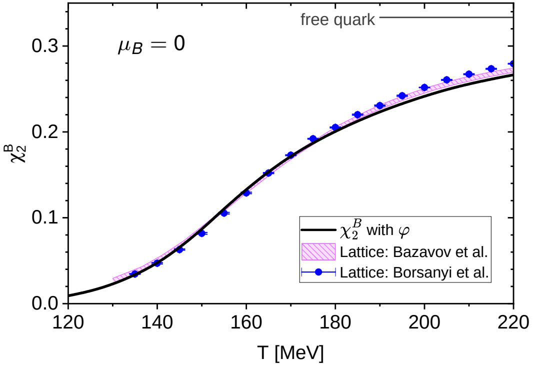
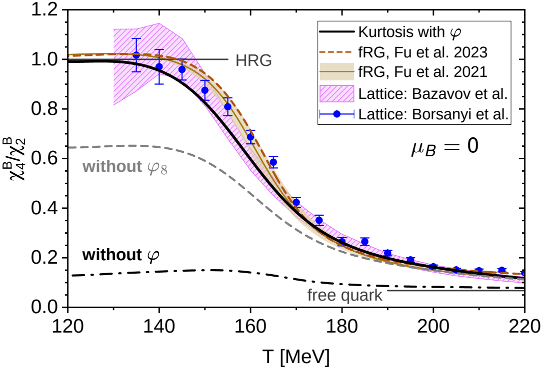




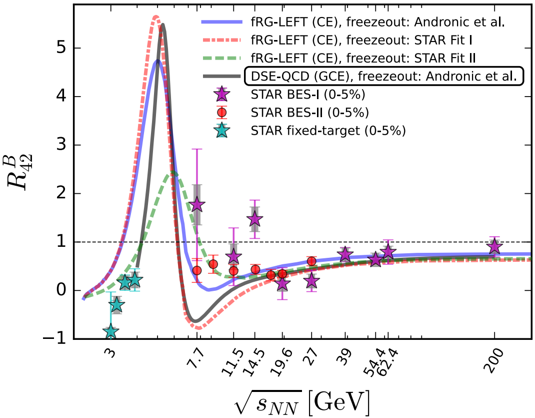
Conserved charge cumulants in the beam energy scan.

Kurtosis in equilibrium (grand canonical):

$$R_{42} = \chi_4^B / \chi_2^B \sim C_4 / C_2 = \kappa\sigma^2.$$

- Agreements on a smooth, monotonic behaviour for $\sqrt{s_{\rm NN}} \gtrsim 20\,\mathrm{GeV}$ ...
- Non-monotonicity can be seen far away from CEP with a **narrowed crossover**.
- EXP signature for CEP could be indirect
  - Fu et al., PRD 111: L031502 (2024)
- Additional effects in realistic collisions? non-equilibrium, inhomongeneity, etc.?

(Plot courtesy: S. Yin and F. Rennecke)



Baryon number fluctuations as an observable

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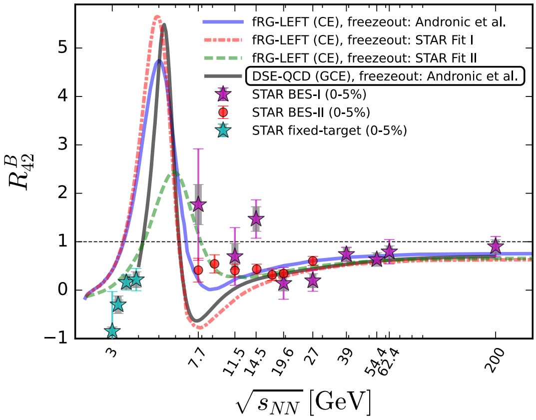
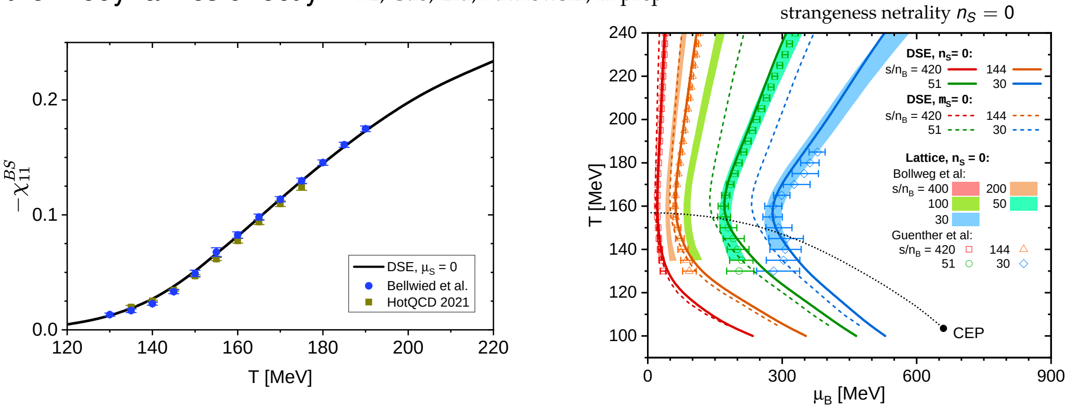


Freeze out condition (matching experiment) can be improved. **F. Gao's Talk, 9th**

Access wide open for extended studies

Strangeness and baryon-strange correlations: $(\mu_u, \mu_d, \mu_s) = (\frac{\mu_B}{3}, \frac{\mu_B}{3}, \frac{\mu_B}{3} - \mu_S)$.

Resolution on $A_0(T, \mu_B, \mu_S, \ldots)$ may serve as a unified approach for probing QCD thermodynamics directly: YL, Gao, Liu, Pawlowski, in prep.



Strange hadron production in heavy-ion collision: EoS(T, μ_B, μ_S) + pheno.

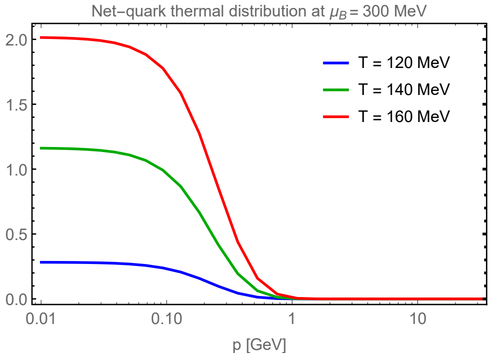
Access wide open for extended studies

Useful by-products at hand: net-quark thermal distribution given by a partial sum:

$$\langle \bar{q}\gamma_0 q \rangle = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} f_{\text{net-}q}(\boldsymbol{p}),$$

$$f_{\text{net-}q}(\boldsymbol{p}) = -T \sum_{\omega_p} \mathrm{tr}[\gamma_0 G_q(\omega_p, \boldsymbol{p})].$$

Pheno. matching: quasi-particles that reflect the QCD equilibrium baseline.



ψ: asymptotic states (experiment)

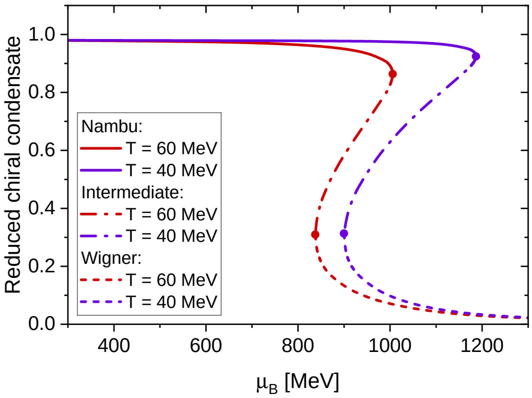
$$\boxed{\langle \psi_{\mathrm{i}} | q(x_1) \cdots \bar{q}(x_n) A_\mu(y_1) \cdots A_\nu(y_m) | \psi_{\mathrm{o}} \rangle \to f_{\psi_{\mathrm{i}}} * f_q * \cdots f_{\bar{q}} * \cdots f_{\text{gluon}} * \cdots * f_{\psi_{\mathrm{o}}}}$$

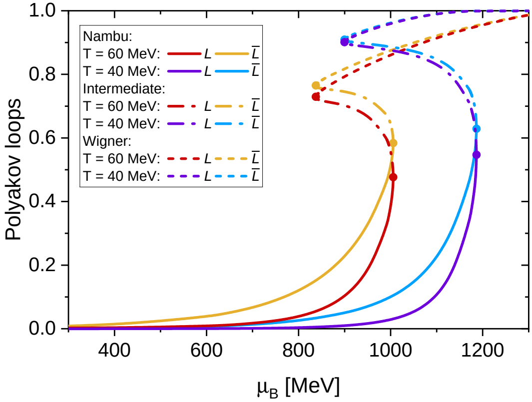
$q, \bar{q}, A_\mu$: first-principles QCD

And further $\delta f_{\text{non-eq.}}$: transport model

Possible spinodal decomposition in chiral and confinement phase transition.

- Case study with only dynamical quarks and gluons (emergent DOFs off at present);
- Stationary phases in quark gap equation - spinodal decomposition (beyond mean-field).



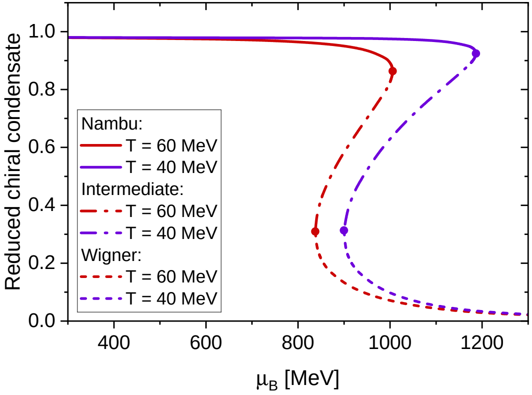


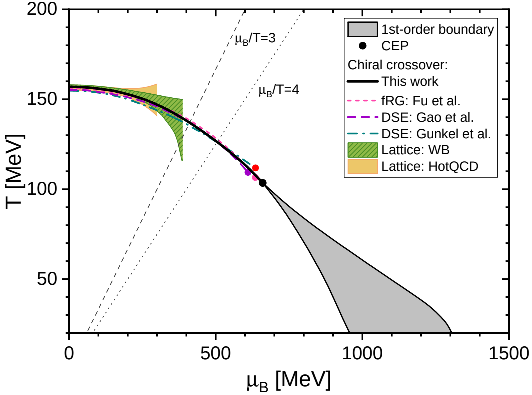
YL, Gao and Liu, 2509.02974

New insight on phase structure beyond CEP in DSE-QCD

Possible spinodal decomposition in chiral and confinement phase transition.

- Case study with only dynamical quarks and gluons (emergent DOFs off at present);
- Stationary phases in quark gap equation - spinodal decomposition (beyond mean-field).





Spinodal-induced inhomogeneity? Overlaps with the moat regime (**F. Sattler's talk**)?

Summary

New insights on several aspects in QCD thermodynamics:

- QCD (de-)confinement and center symmetry at finite T and μ_B.
- Confinement and chiral dynamics in the sub-structure of Polyakov loop.
- A_0 as a unified approach for understanding QCD thermodynamics.
- Baryon number fluc. & confining signatures, and connections to BES.

Access wide open on:

- Strangeness: local/global neutrality, baryon-strange correlations etc.
- Improving non-equilibrium dynamics with reliable (equilibrium) QCD baseline.
- QCD spinodal decomposition and inhomogeneity; observational effects?
- $\cdots$

Thanks for your attention!

Back-up

Representation of the Polyakov loop potential $V(A_0)$ and its DSE

- A_0 eigenvalues (ν) and the mode potential representation:

$$V(A_0) = \sum_{\nu} V_{\rm mode}(\nu) = \sum_{\nu_{\rm adj.}} \left[V^{\rm gluon}_{\rm mode}(\nu_{\rm adj.}) + V^{\rm ghost}_{\rm mode}(\nu_{\rm adj.}) \right] + \sum_{\nu_{\rm fund.}} V^{\rm quark}_{\rm mode}(\nu_{\rm fund.}) .$$

Each mode potential is given by a standard functional loop diagram calculation (w.o. A_0), with a shift on the Matsubara frequencies $\omega_n \to \omega_n + 2\pi T\nu$. Convenience in DSE:

$$\frac{\delta V(A_0)}{\delta A_0} = 0 \quad \to \quad \frac{\partial V(\varphi)}{\partial \varphi_i} = \sum_{\nu} \frac{\partial V_{\rm mode}(\nu)}{\partial \nu} \frac{\partial \nu}{\partial \varphi_i} = 0, \quad i = 3, 8.$$

- Specifically, the full ghost 1-loop diagram:

$$\frac{\partial V^{\rm ghost}}{\partial \nu} = -\frac{1}{T^3 \mathcal{V}_3} \operatorname{Tr} \frac{\partial S^{(2)}_{cc}}{\partial \nu} G_{cc} = -\frac{1}{T} \sum_{n\in\mathbb{Z}} \int \frac{d^3\boldsymbol{p}}{(2\pi)} \frac{n+\nu}{Z_c(x^{(\nu)})\, x^{(\nu)}} , \qquad x^{(\nu)} = (2\pi T)^2 (n+\nu)^2 + \boldsymbol{p}^2 .$$

with the ghost dressing function Z_c, and similar for gluon and quark.

A_0 back-coupling in the chiral dynamics?

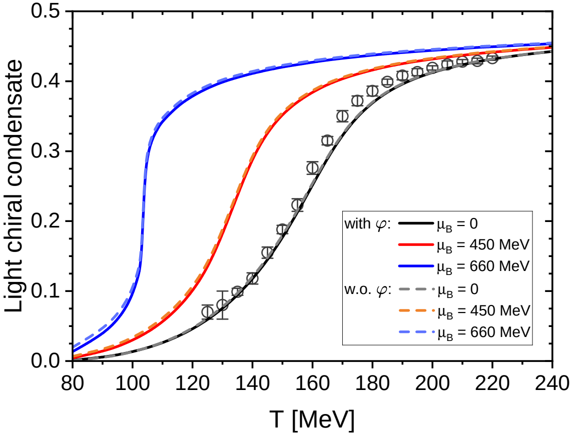
• A_0 back-coupling is seemingly small in the chiral observable:

comparsion of chiral condensate with/without A_0 background indicates that

$$M_q(A_0) \approx M_q(A_0 = 0).$$

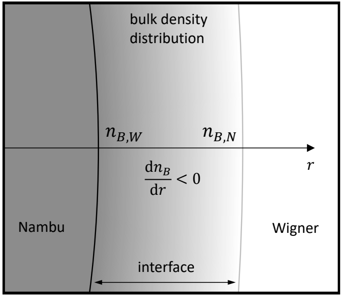
for the quark dynamical mass (function).

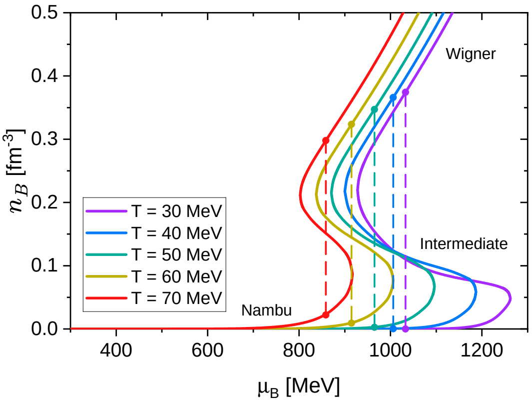
• Further evidence by solving the A_0-dependent quark gap equation in combination of A_0 DSE: Wan et al. 2504.12964.



Spinodal-induced inhomogeneity at high density?

Interface between Nambu & Wigner phases:





Baryon density distribution resolved by the stationary condition of total free energy $\int \mathrm{d}^3\boldsymbol{r}\,\delta F(\boldsymbol{r}) = 0$:

Pheno. model proposed in: Randrup, PRC 79: 054911 (2009)

$$F(\boldsymbol{r}) = \frac{1}{2}C(\nabla_{\boldsymbol{r}} n_B)^2 + F_{\text{bulk}}(\boldsymbol{r}), \qquad \delta F_{\text{bulk}}(\boldsymbol{r}) = \mu_B(n_B)\,\delta n_B(\boldsymbol{r}).$$

Spinodal-induced inhomogeneity at high density?

- Total free energy deficit per unit area of the interface: T-dependent interface tension

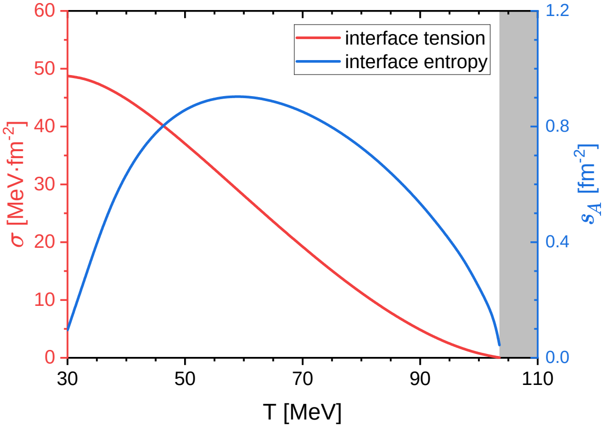
$$\sigma(T) = \int \sqrt{2C\Delta f(n_B)}\,\mathrm{d}n_B,$$

$$\Delta f(n_B) = \int^{n_B} [\mu_B(n) - \mu_B^{\mathrm{MC}}]\,\mathrm{d}n.$$

- Also indicates interface entropy

$$s_A = \frac{\partial S_A}{\partial A} = -\frac{\partial \sigma}{\partial T}.$$

Randrup, PRC 79: 054911 (2009),
Gao and Liu, PRD 94: 094030 (2016)



Observational effects of inhomogeneous patterns in dense strong interaction matter is possible: compact stars, cosmological aspects (relic phase transition) and so on.