https://fqcd-collaboration.github.io/

Fakultät für Physik

QCD at finite densities: Moat regime and inhomogeneities

Franz R. Sattler

University Bielefeld

The QCD Phase Diagram:

From Theory to Experimental Signatures

Dalian, China

October 8th, 2025

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Moat regime and inhomogeneities

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UNIVERSITÄT BIELEFELD

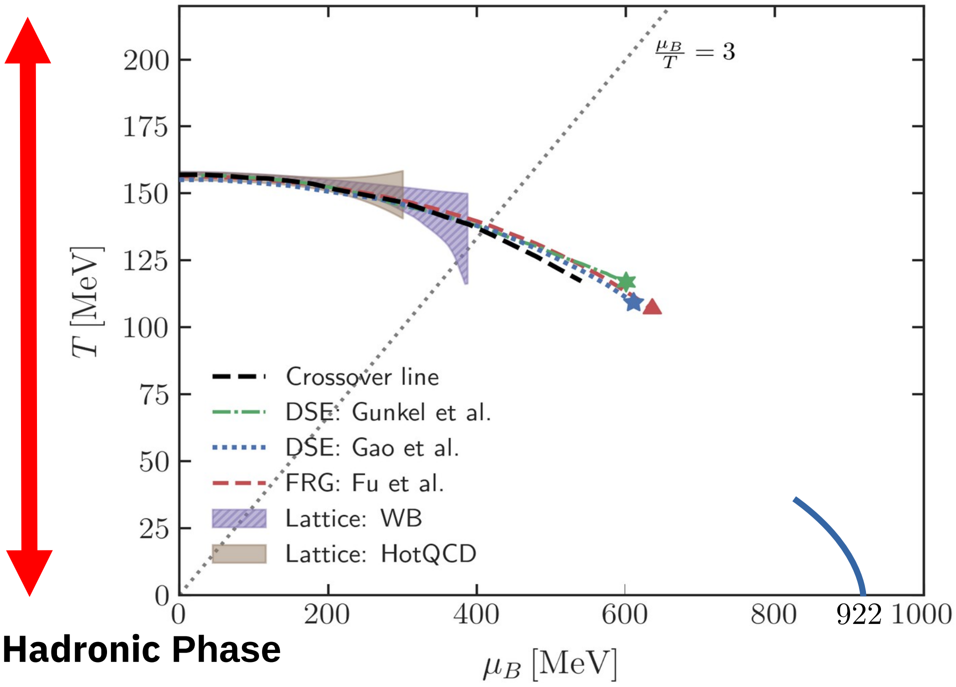
Fakultät für Physik

Outline

- The QCD phase diagram & new phases
- fRG for the QCD phase diagram: recent developments
- Systematic error estimates for fRG computations
- The moat regime & inhomogeneous instabilities

The QCD phase diagram

Quark-Gluon Plasma



Hadronic Phase

High densities: *This work*

Fu, Pawlowski, Rennecke [Phys. Rev. D 101 (2020), 054032]
Gao, Pawlowski [Phys.Lett.B820(2021) 136584]
Gunkel, Fischer [Phys.Rev.D 104 (2021) 5, 054022]

Low densities: *Bellwied et al. (WB) [Phys.Lett.B 751 (2015) 559-564]*
Bazavov et al. (HotQCD) [Phys.Lett.B 795 (2019) 15-21]

Strongly correlated non-perturbative infrared physics:

- **Dynamical Chiral Symmetry Breaking** (dχSB), i.e. condensation of $\langle \bar{q} q \rangle$

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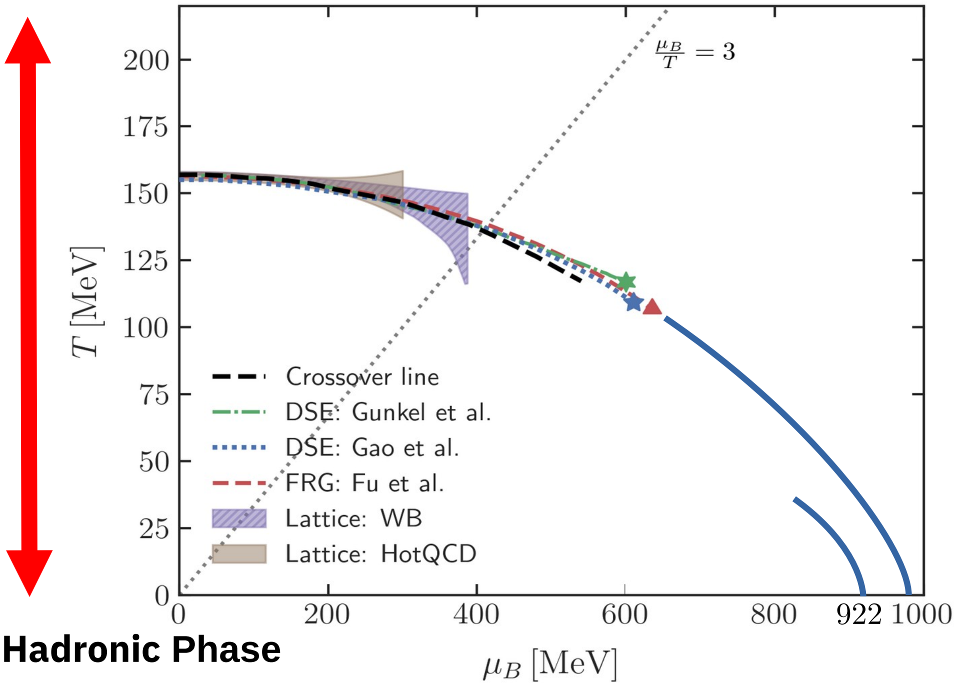

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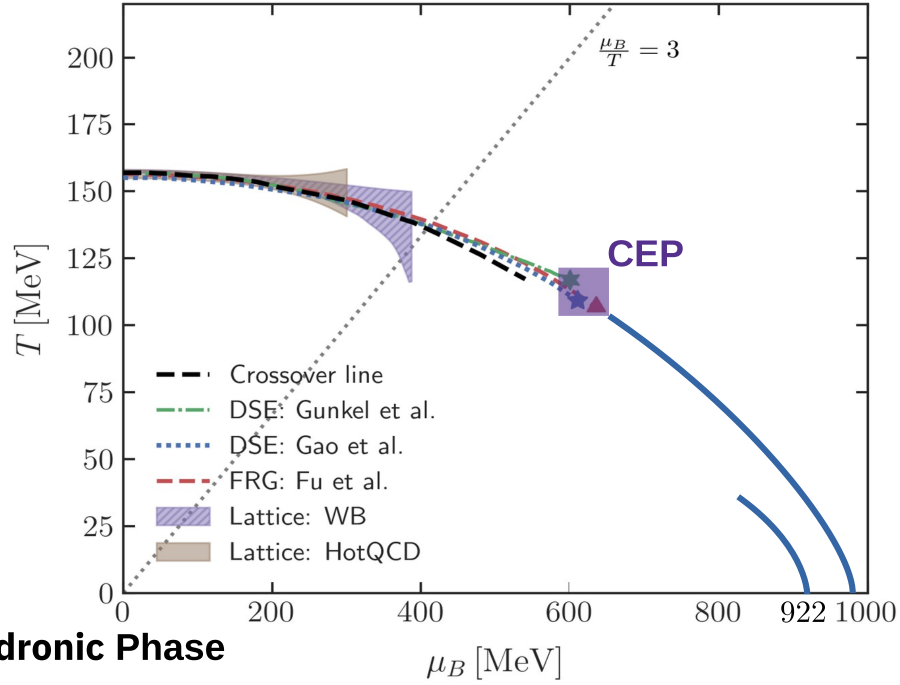

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- **Critical Endpoint** (CEP) at $(T, \mu_B) \approx (110, 630)\mathrm{MeV}$

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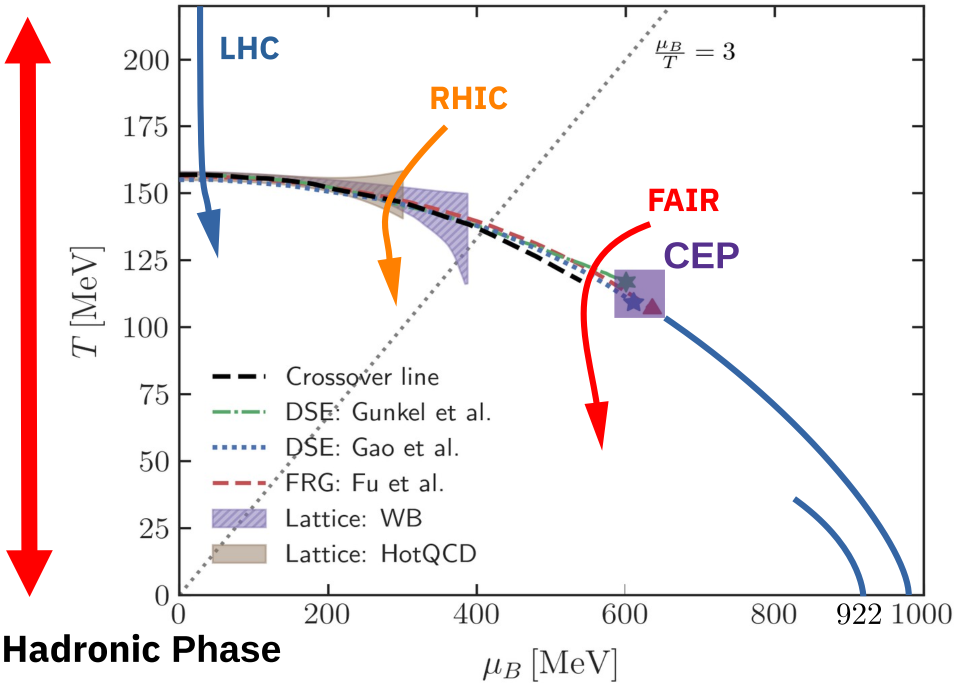

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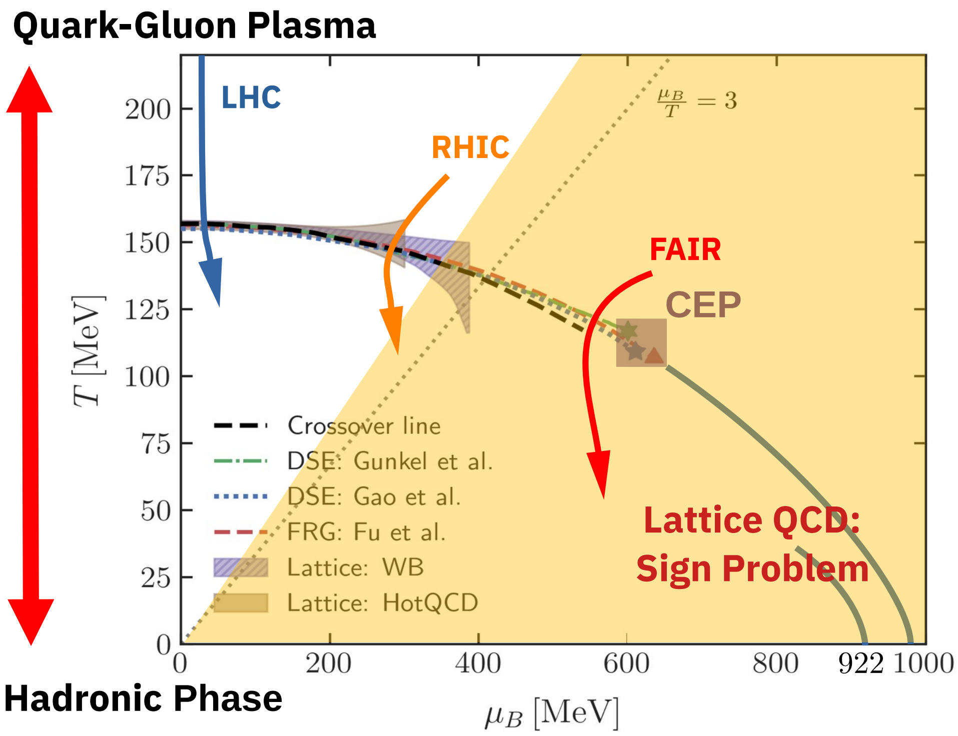

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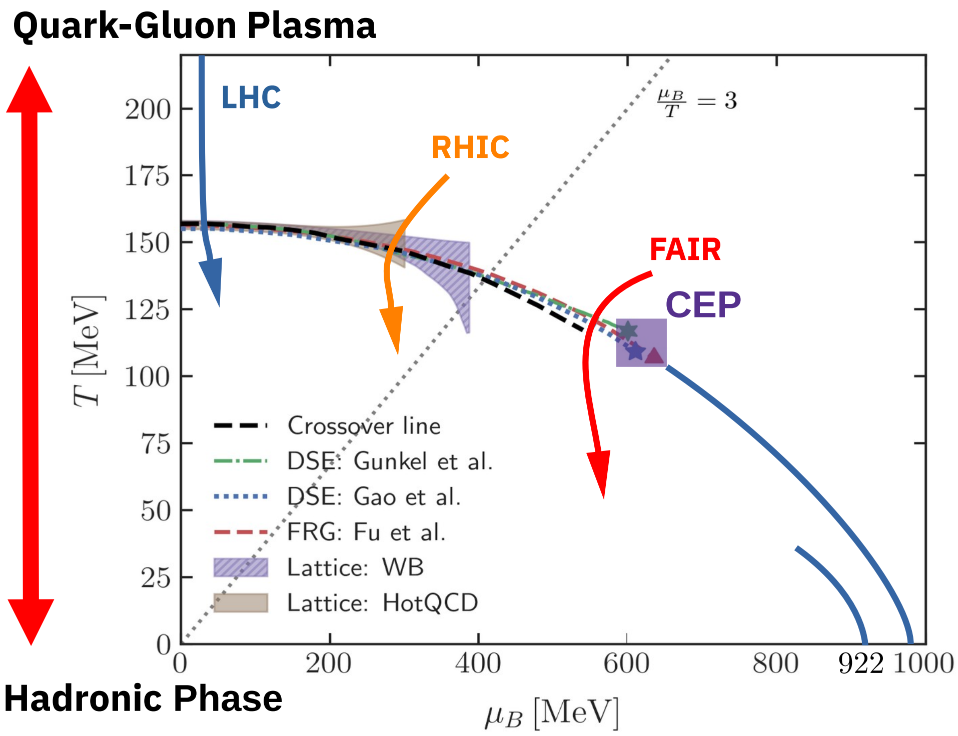

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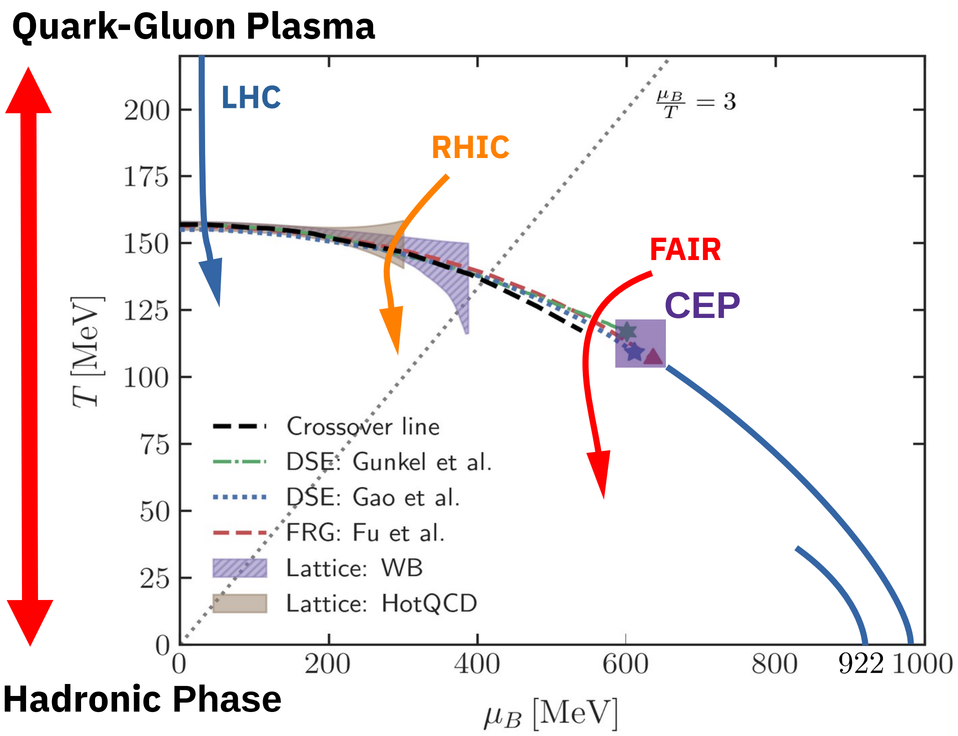
Direct access to phase structure using the

functional Renormalization Group

Non-perturbative, first-principles, **direct access to finite μ**.

New phases in the QCD phase diagram (?)



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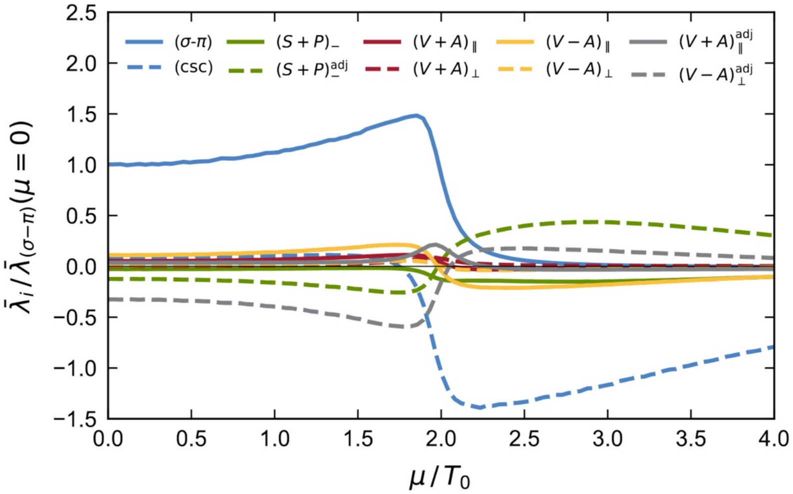
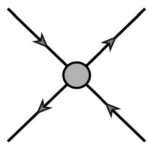
Other four-fermi channels



Braun, Leonhardt, Pospiech
[Phys.Rev.D 101 (2020) 3, 036004]



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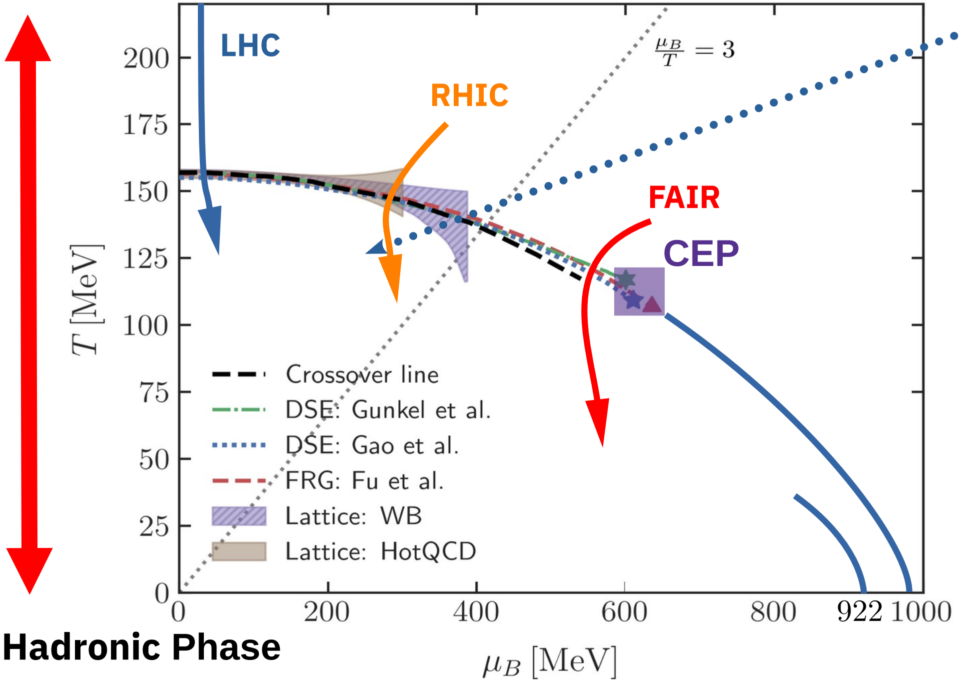

Hadronic Phase

$\langle \bar{q} q \rangle$

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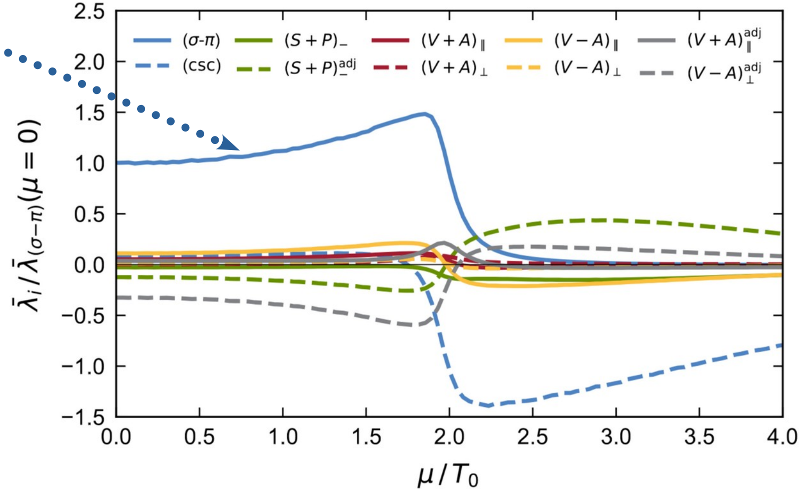

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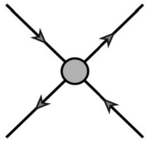

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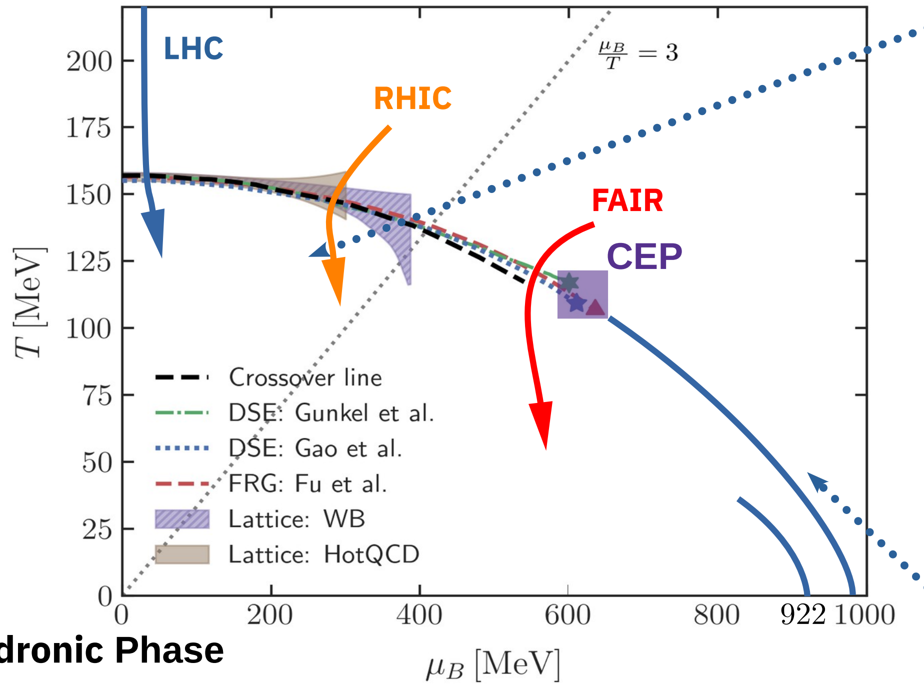

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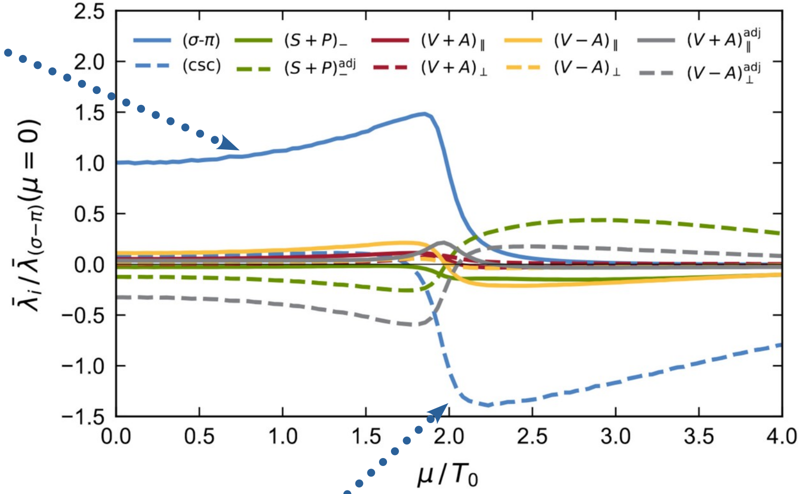

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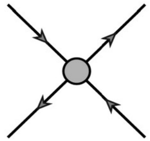

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$\langle d^\star d \rangle$

With Jamaly, Pawlowski
Andreas Geißel's talk!

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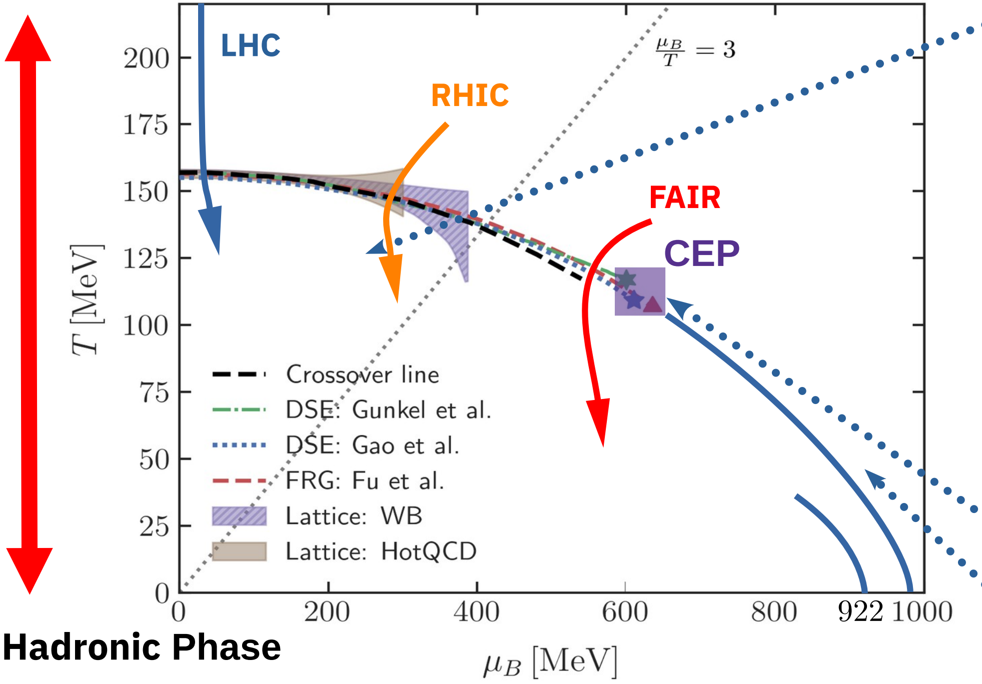

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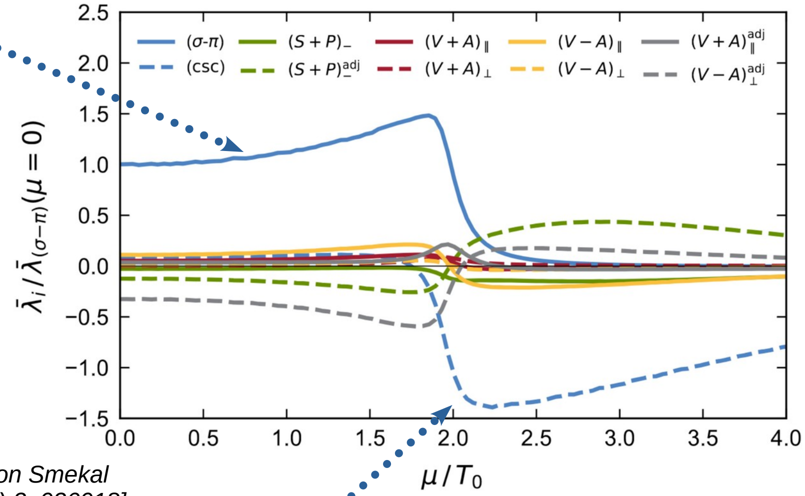

Haensch, Rennecke, von Smekal [Phys.Rev.D 110 (2024) 3, 036018]

$\langle \bar{q} \gamma_0 q \rangle$

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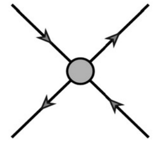

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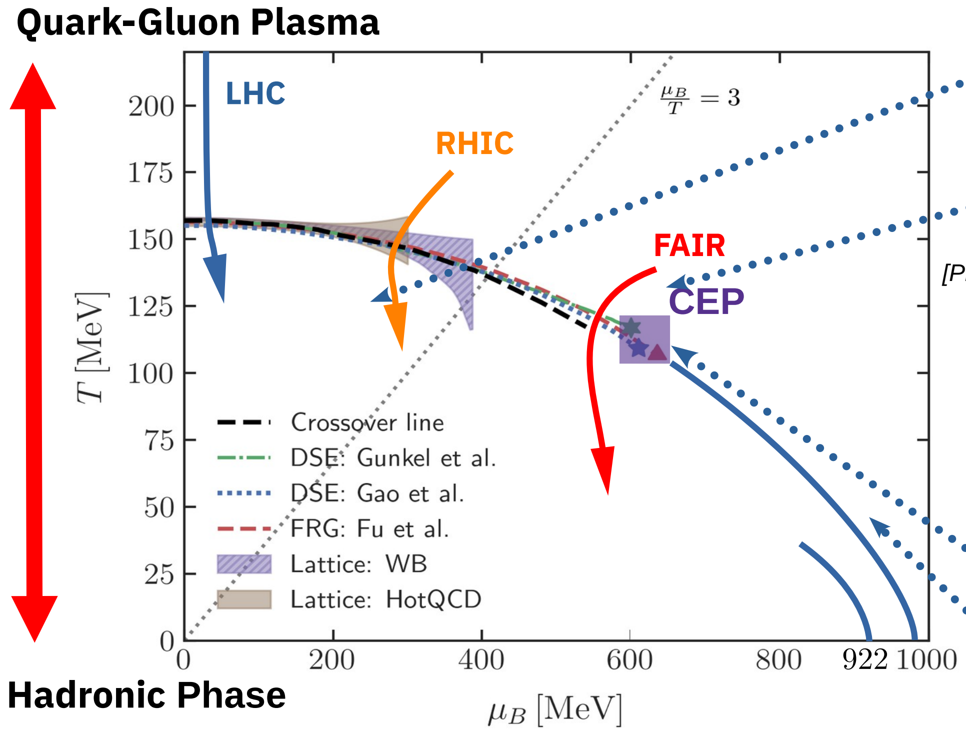

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Inhomogeneous

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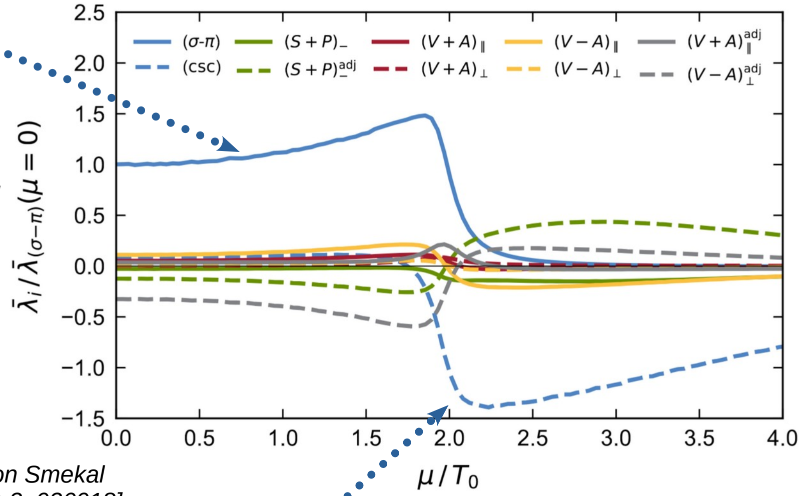

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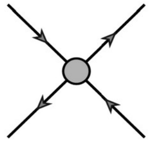

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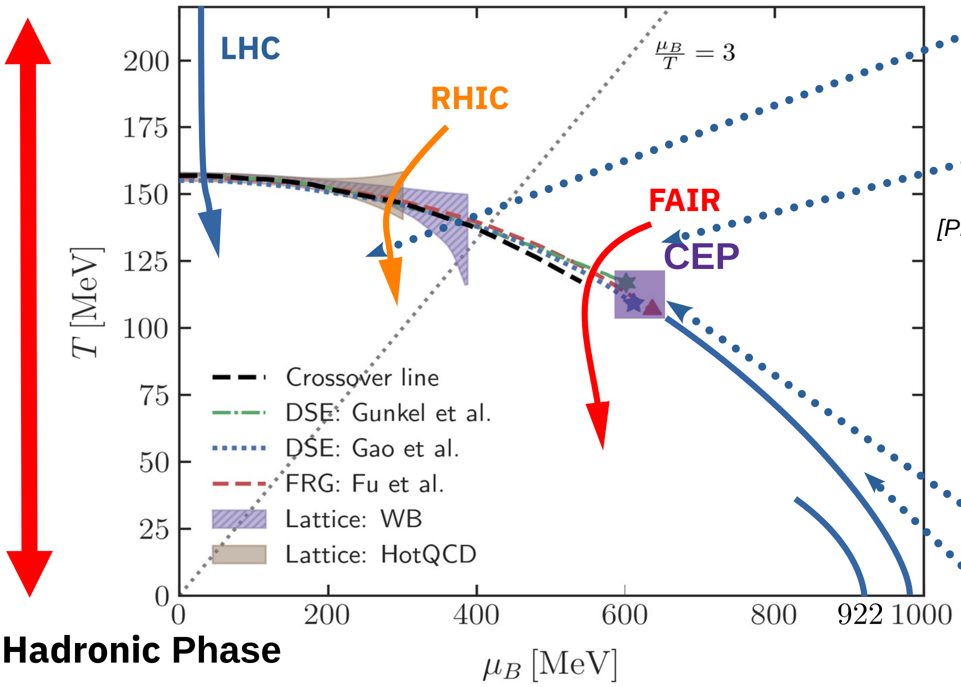

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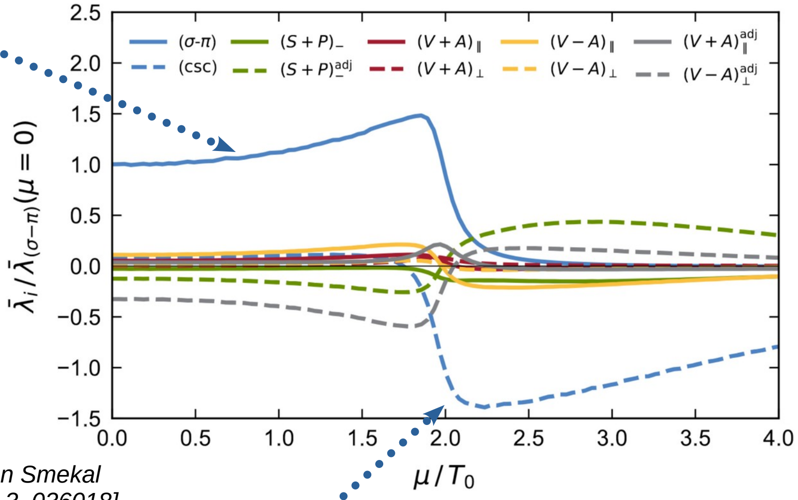

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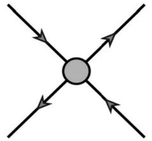

Optimization of bases:

J. Braun, A. Geißel, J. Pawlowski, F.R. Sattler, N. Wink
arXiv:2503.05580 [hep-th]

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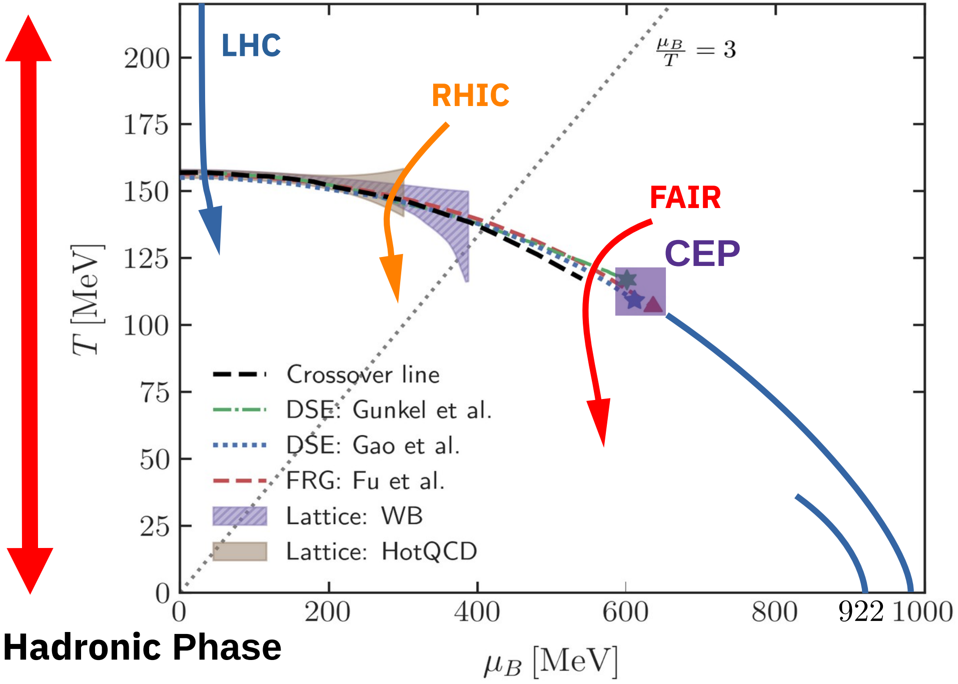

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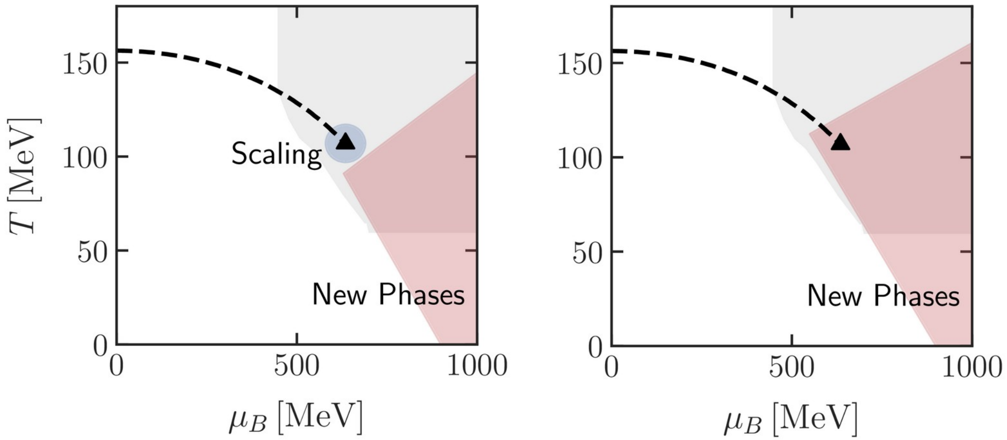
Some Scenarios for the CEP

- Spatial modulations
  (Lifshitz point/regime & Moat)



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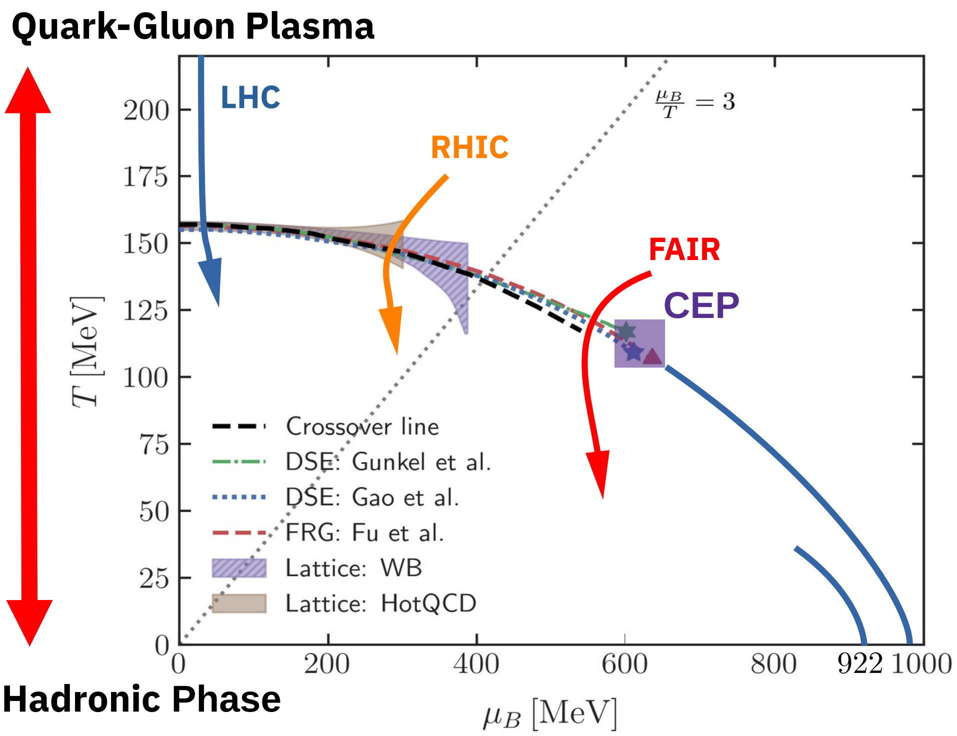

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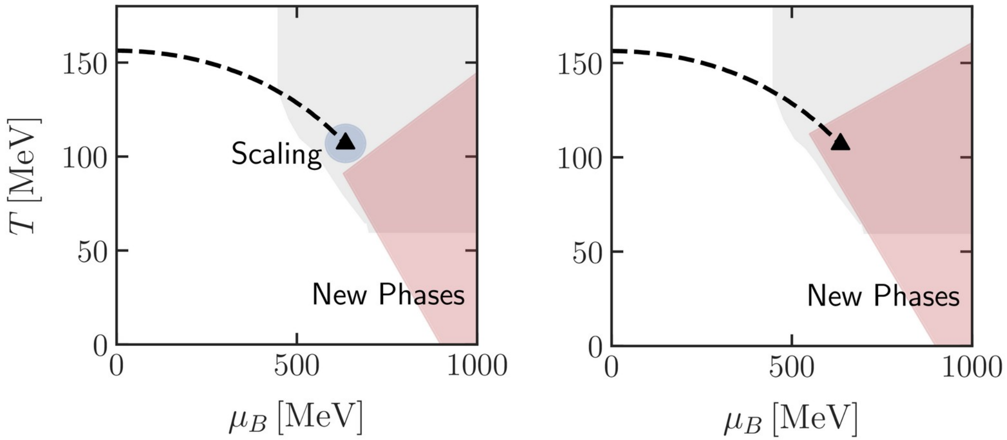
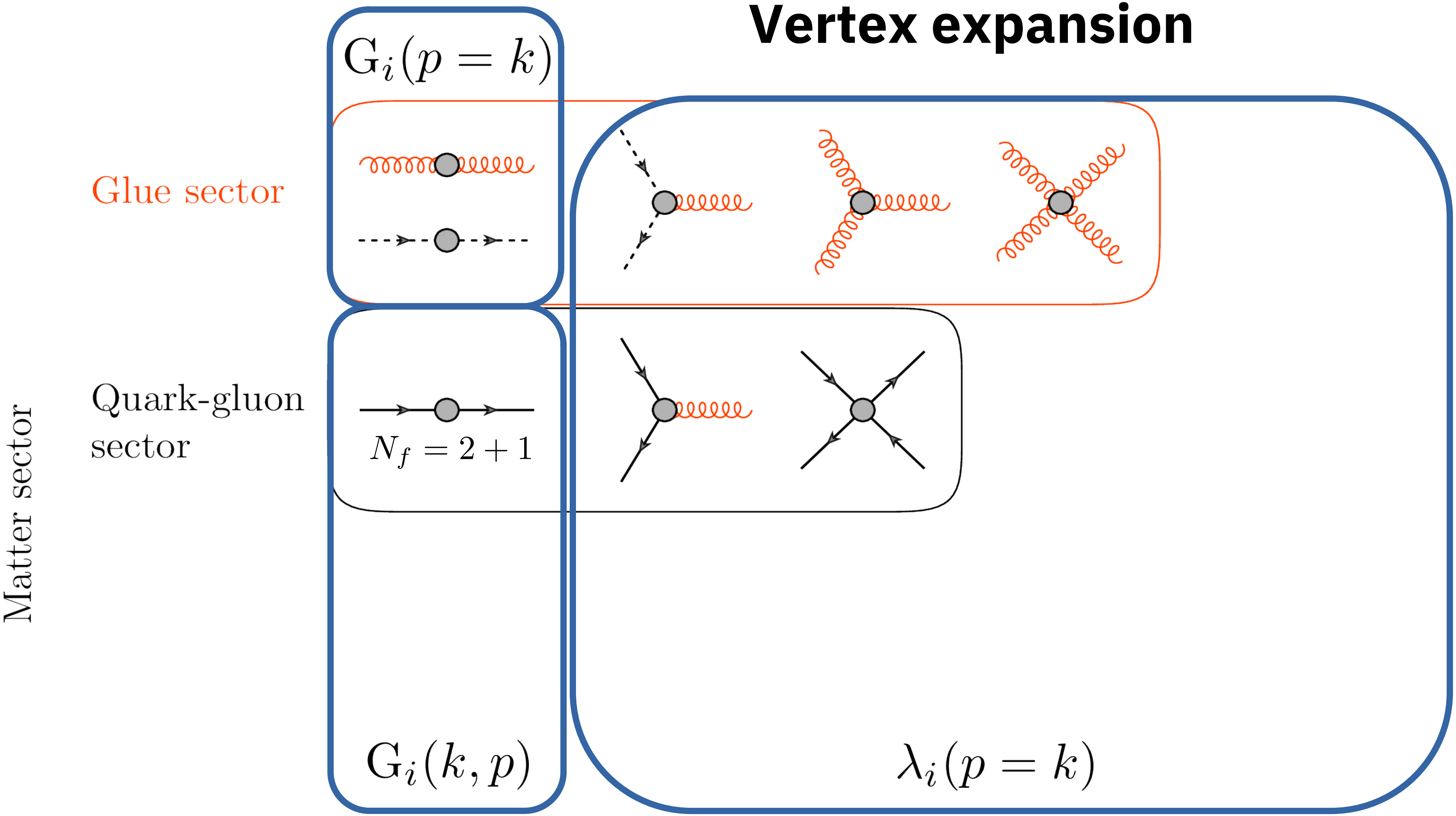
- Diquarks (?) ***Andreas Geißel's talk!***



Vertex expansion



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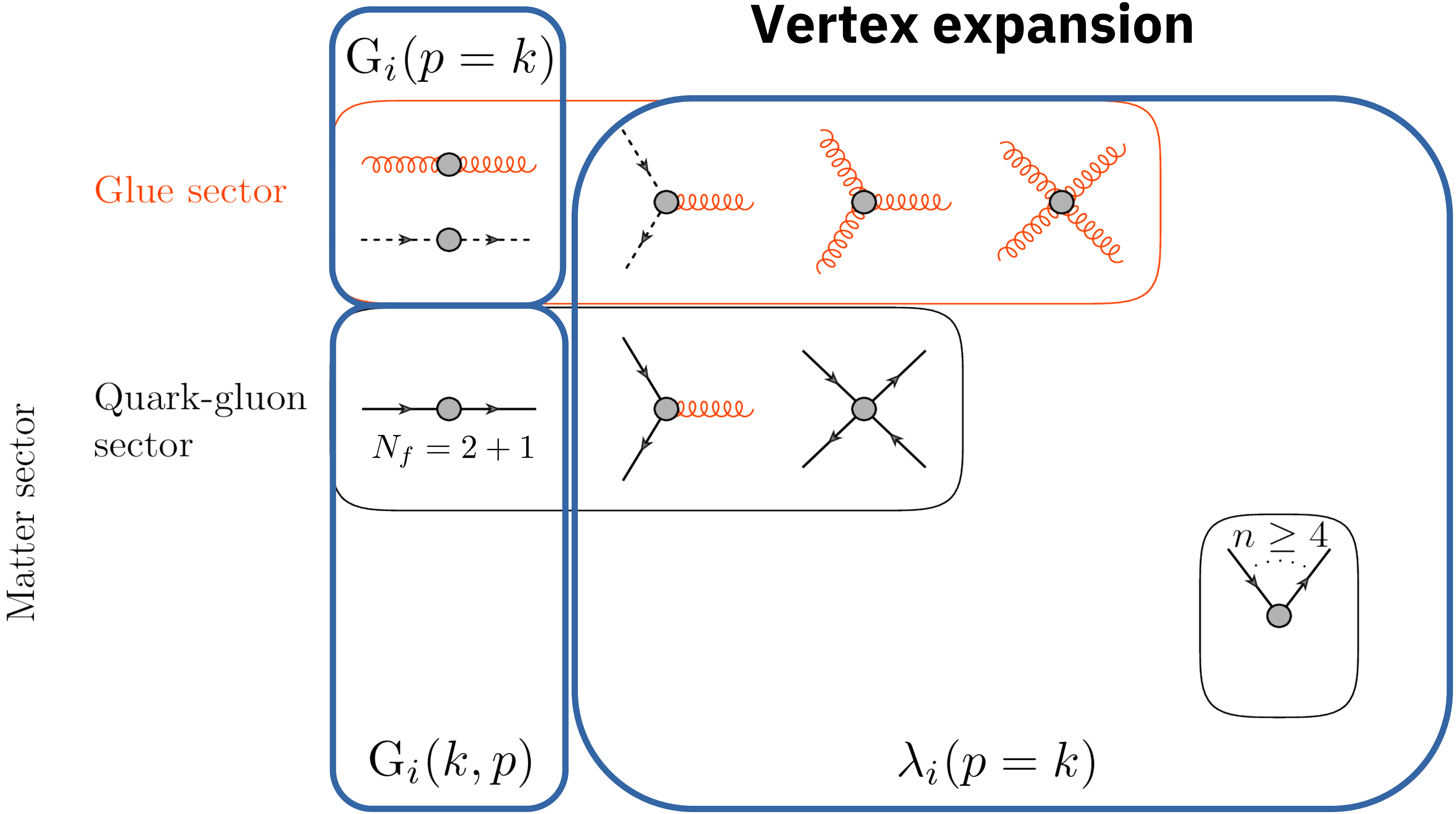

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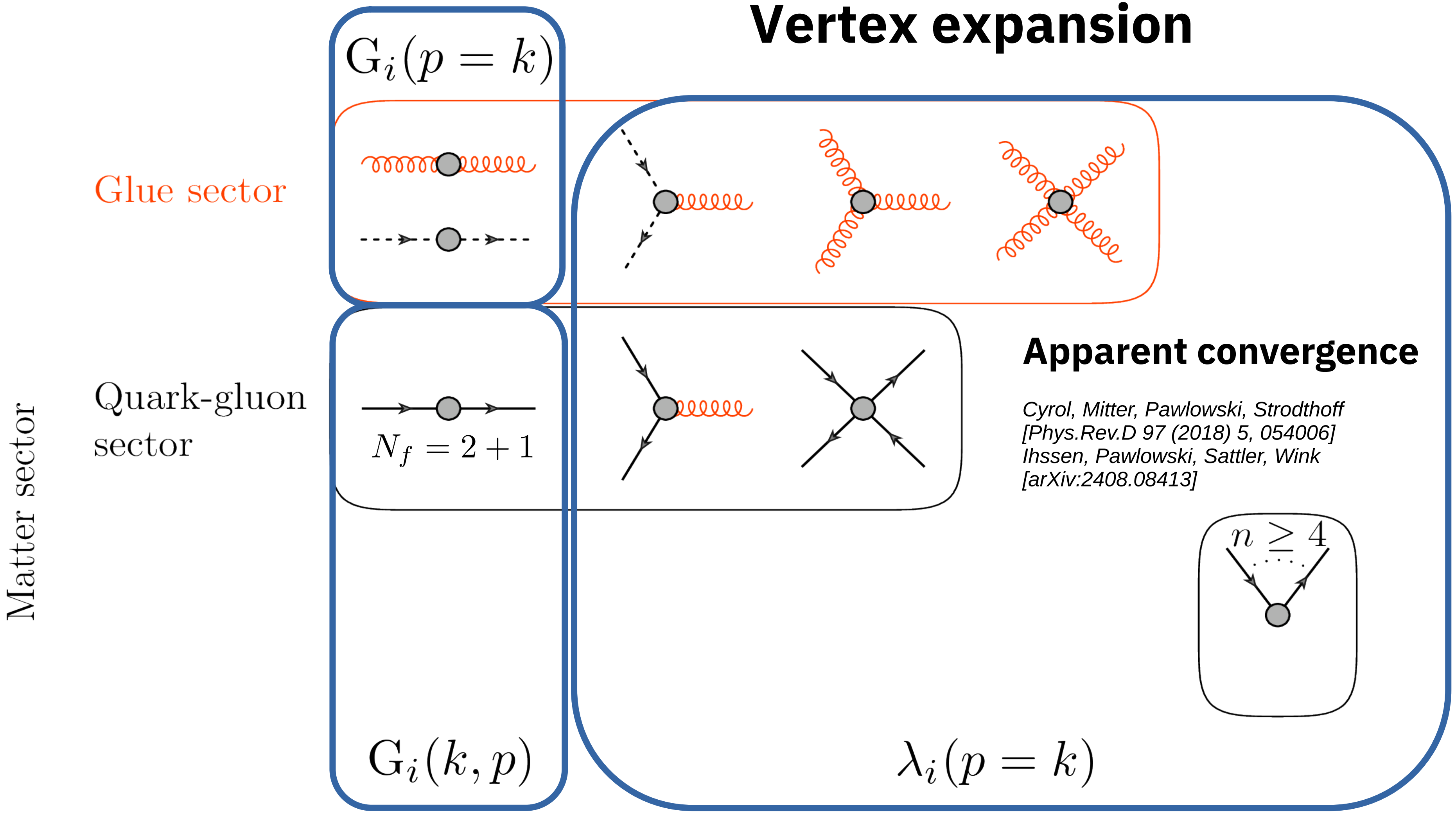

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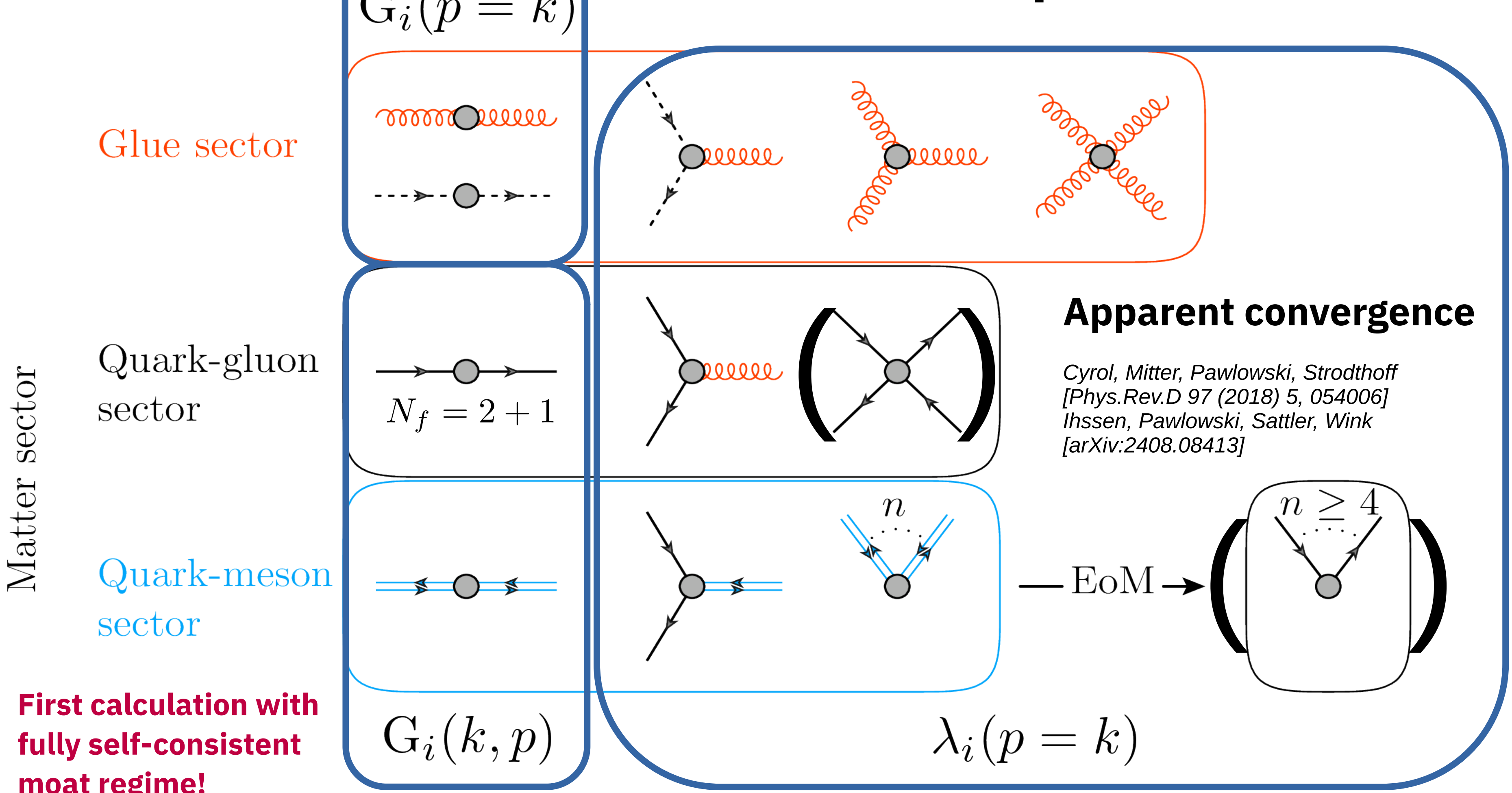
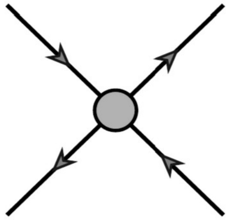


Apparent convergence

Cyrol, Mitter, Pawlowski, Strodthoff [Phys.Rev.D 97 (2018) 5, 054006]
Ihssen, Pawlowski, Sattler, Wink [arXiv:2408.08413]

First calculation with fully self-consistent moat regime!

$\sigma - \pi -$ four-quark flow



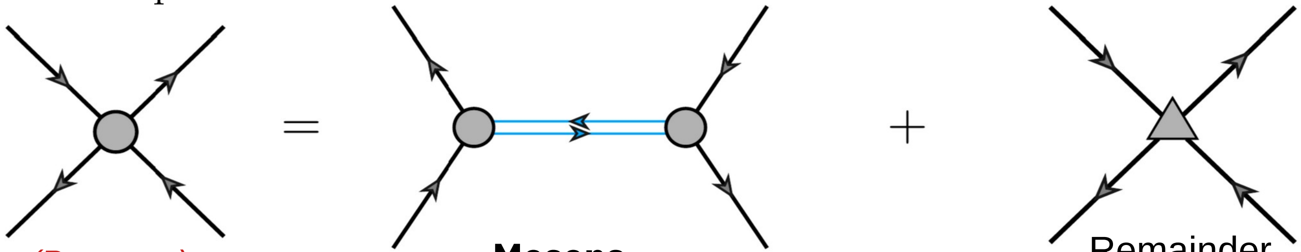
(Resonant)

Fu, Huang, Pawlowski, Tan
[SciPost Phys. 14 (2023) 4, 069]
[arxiv:2401.07638 (2024)]

$\sigma - \pi -$ four-quark flow

$$\pi^i = (u\bar{d},\, d\bar{u},\, \frac{u\bar{u} - d\bar{d}}{\sqrt{2}})$$

$$\sigma = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$$



(Resonant)

Mesons

Remainder

Fu, Huang, Pawlowski, Tan
[SciPost Phys. 14 (2023) 4, 069]
[arxiv:2401.07638 (2024)]

Gies, Wetterich
[Phys.Rev. D 65 (2002), 065001]

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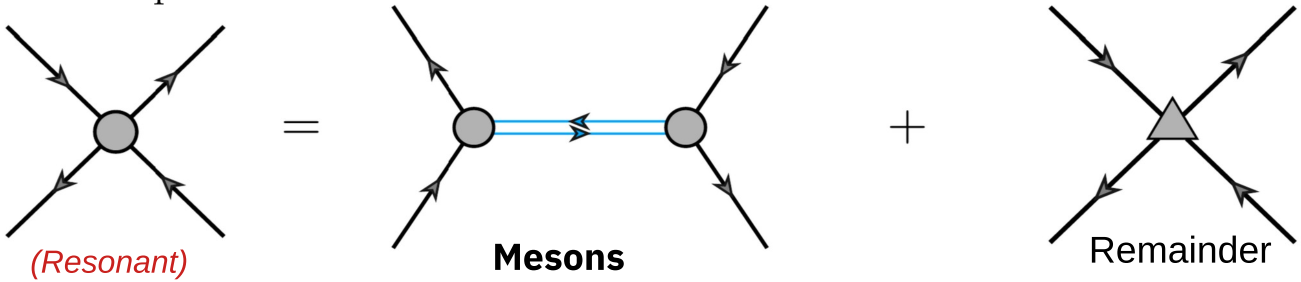

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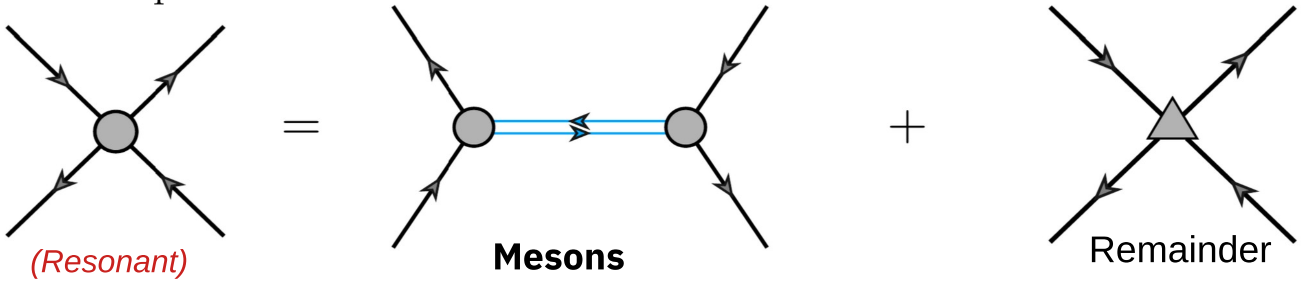
Bethe-Salpeter WF

$$h_\phi(p,q) \cdot \frac{1}{Z_\phi(t^2)(t^2 + m_\phi^2)} \cdot h_\phi(p,q)$$

Meson propagator

$\sigma - \pi -$ four-quark flow



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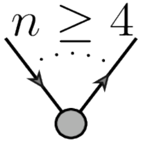
Meson propagator

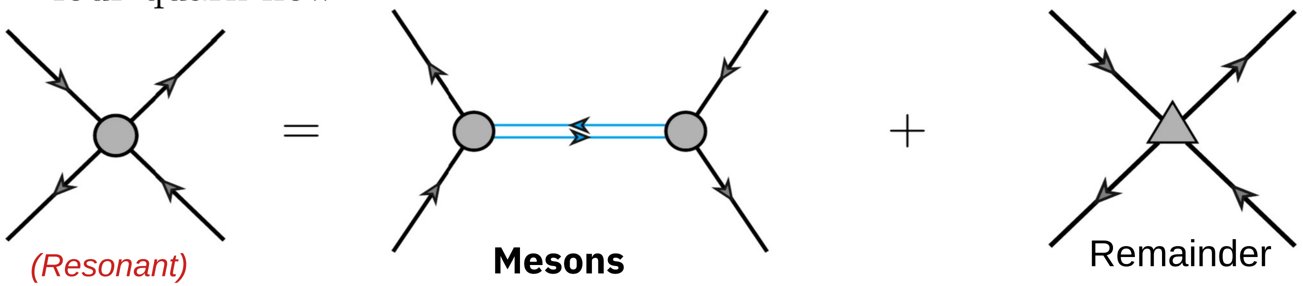
Full scattering potential



$$\longrightarrow \; V\left(\frac{\sigma^2 + \boldsymbol{\pi}^2}{2}\right)$$

All orders of n-meson scatterings

$\sigma - \pi -$ four-quark flow



Fu, Huang, Pawlowski, Tan
[SciPost Phys. 14 (2023) 4, 069]
[arxiv:2401.07638 (2024)]

Gies, Wetterich
[Phys.Rev. D 65 (2002), 065001]

$$\pi^i = \left(u\bar{d},\, d\bar{u},\, \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}\right)$$

$$\sigma = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$$

Bethe-Salpeter WF

$$h_\phi(p,q) \cdot \frac{1}{Z_\phi(t^2)(t^2 + m_\phi^2)} \cdot h_\phi(p,q)$$

Meson propagator

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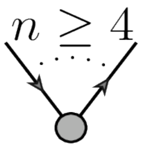

$$\longrightarrow \quad V\left(\frac{\sigma^2 + \boldsymbol{\pi}^2}{2}\right)$$

All orders of n-meson scatterings

Physical DoFs emergent from full QCD

Using flowing field reparametrizations

(Dynamical Hadronisation)

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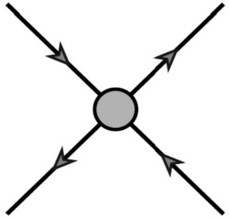
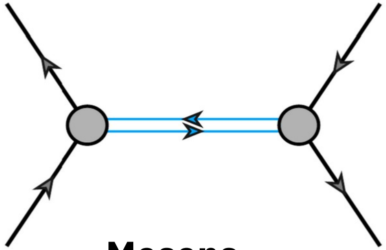


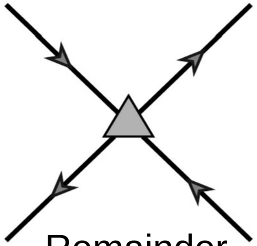
(Resonant)

$=$



Mesons

$+$



Remainder

Fu, Huang, Pawlowski, Tan
[SciPost Phys. 14 (2023) 4, 069]
[arxiv:2401.07638 (2024)]

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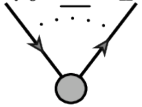
Meson propagator

Full scattering potential

$n \geq 4$



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Physical DoFs emergent from full QCD

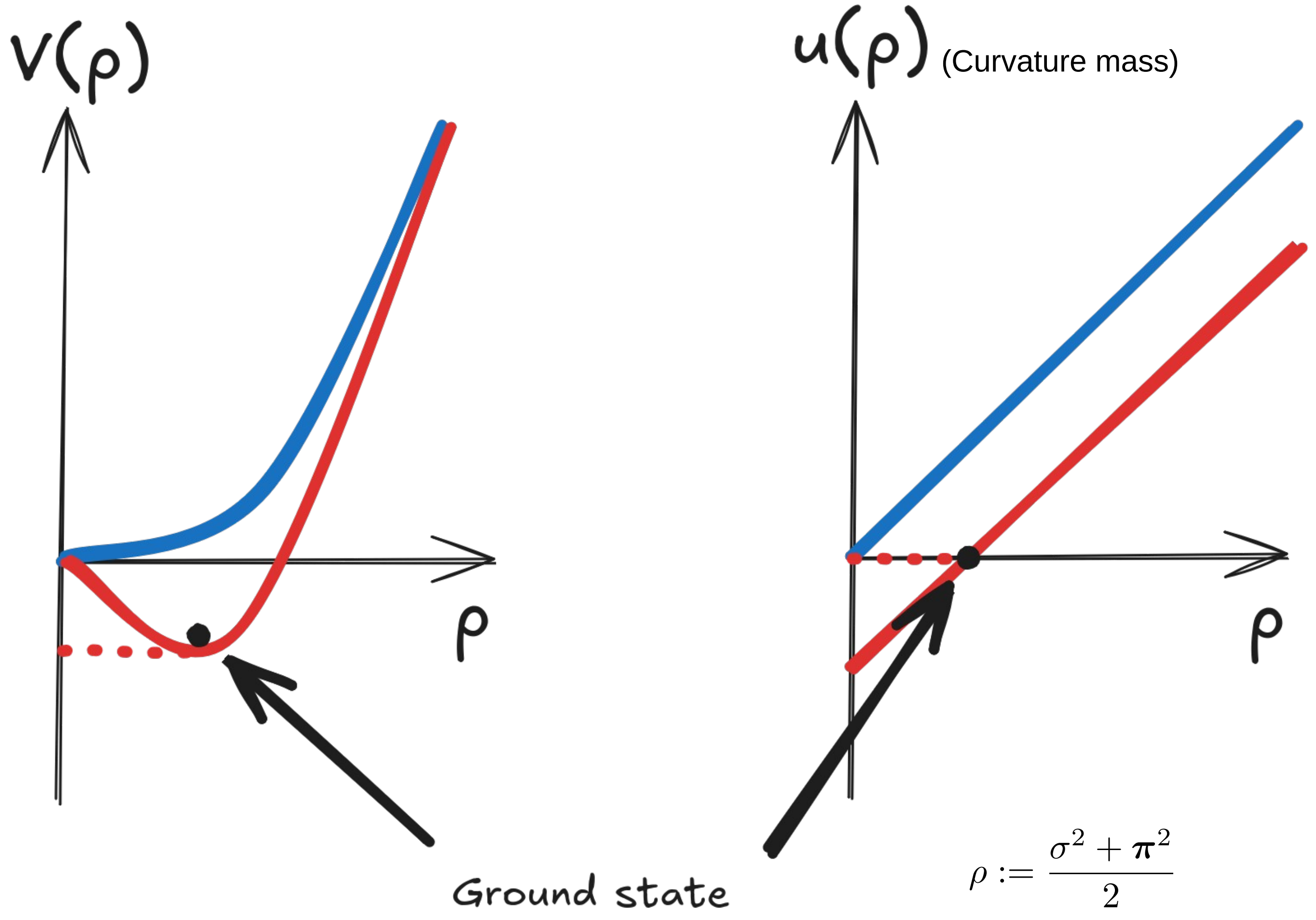
Using flowing field reparametrizations

(Dynamical Hadronisation)

$$\rho := \frac{\sigma^2+\boldsymbol{\pi}^2}{2}$$

$$u(\rho) := \partial_\rho V(\rho) = m^2_{\pi,\mathrm{curv}}(\rho)$$

- Ground state is at minimum of $V(\rho)$ equivalently, at $u(\rho) = 0$.



$$\frac{\delta\Gamma}{\delta\Phi}\Big|_{\mathrm{EoM}} = 0$$

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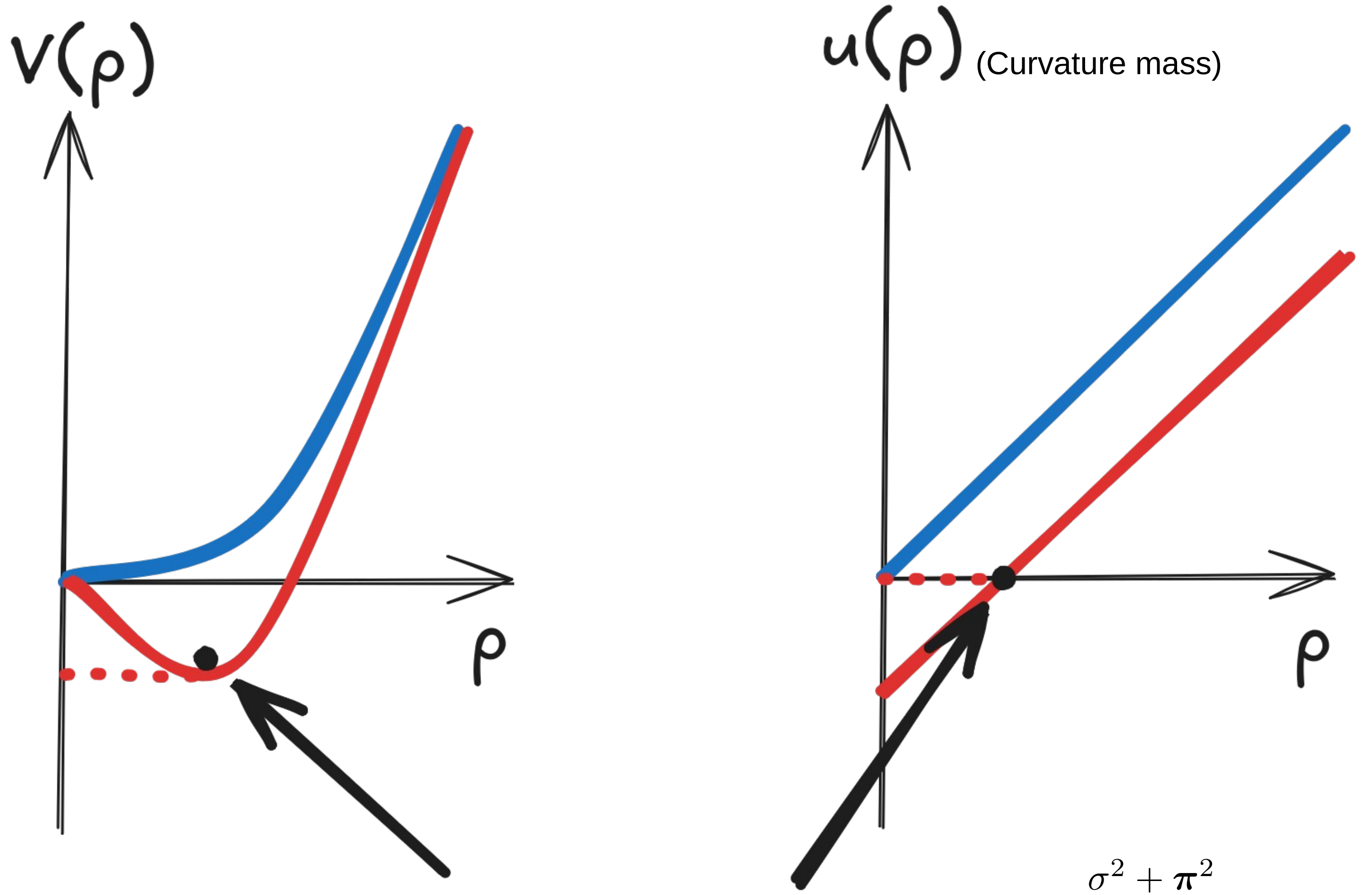
- Quarks act as sinks.



Ground state

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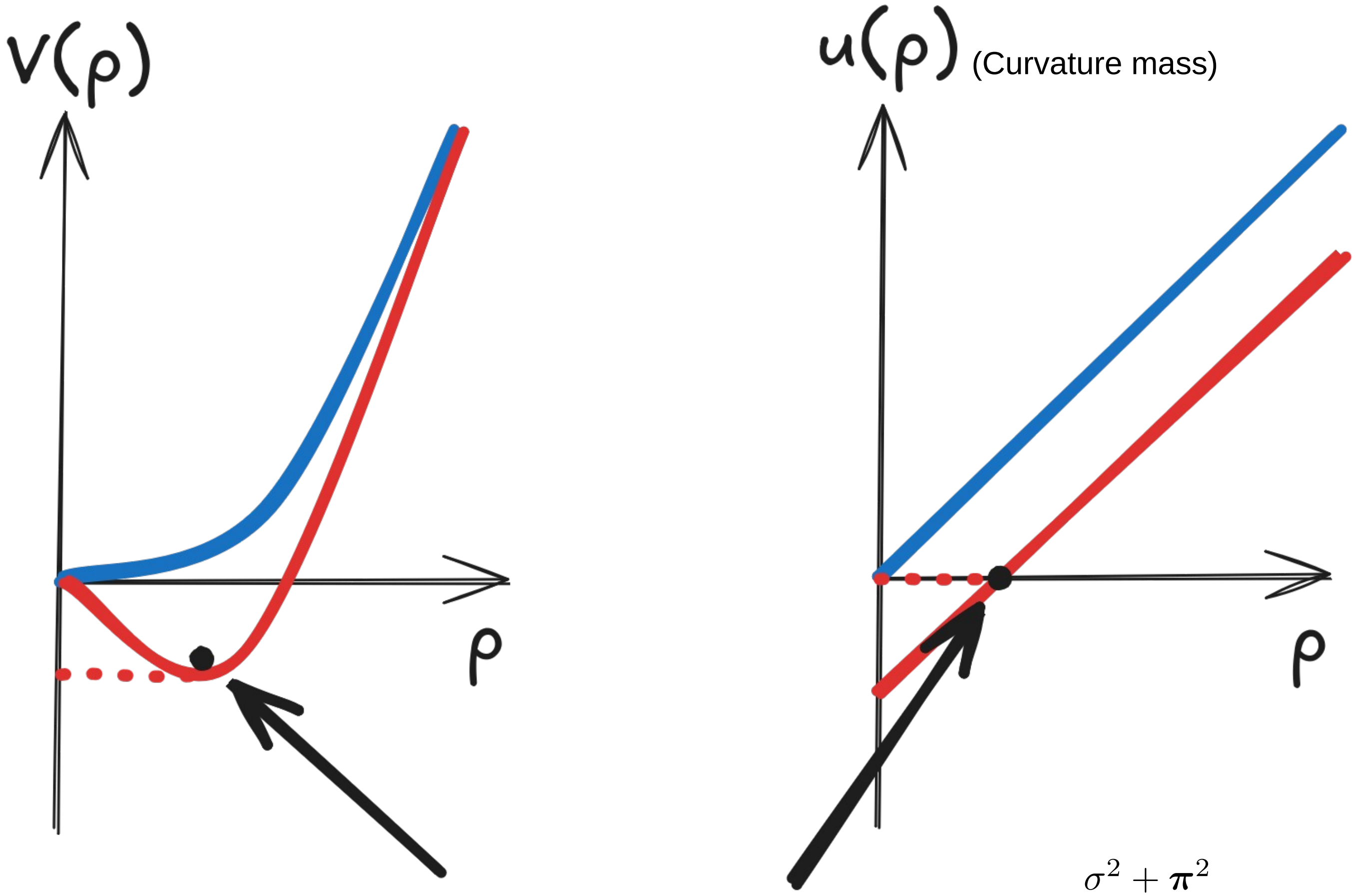
- Ground state is at minimum of $V(\rho)$ equivalently, at $u(\rho) = 0$.
- Quarks act as sinks.
- Mesons introduce advection and diffusion.



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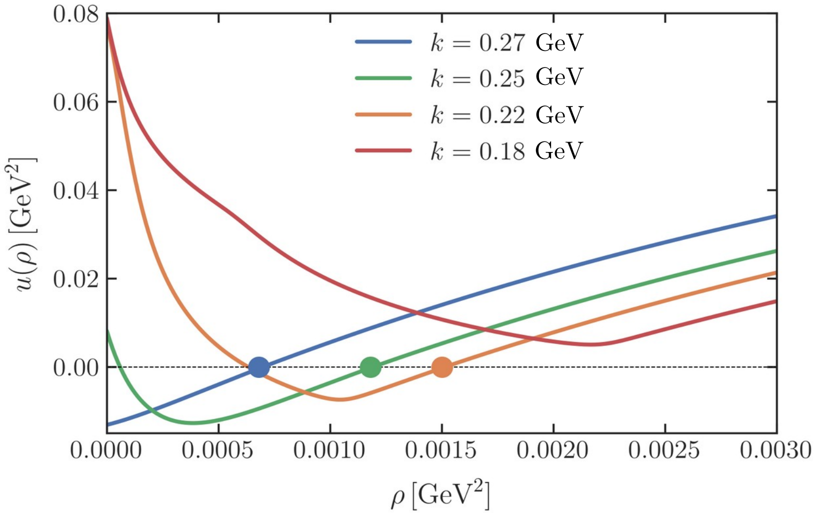
- First-order transitions



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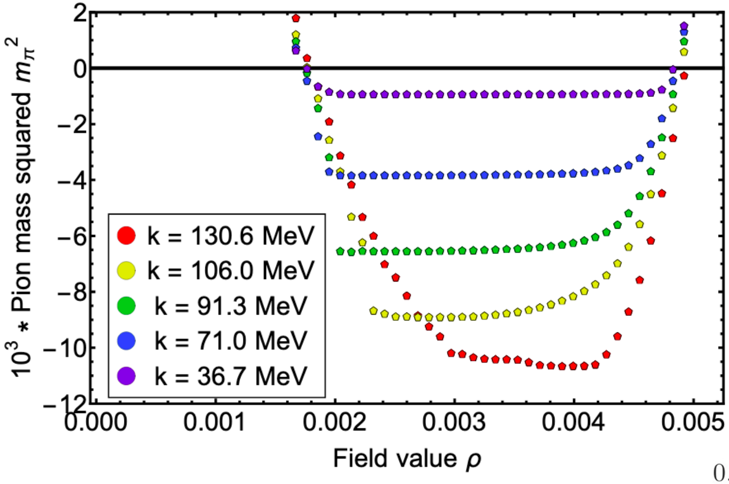
- Possible generation of shocks in $u(\rho)$



Grossi, Ihssen, Pawlowski, Wink
[Phys.Rev.D 104 (2021) 1, 016028]
Koenigstein, Steil, Wink, Grossi, Braun
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Koenigstein, Steil
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[Phys.Rev.D 106 (2022) 6, 065014]

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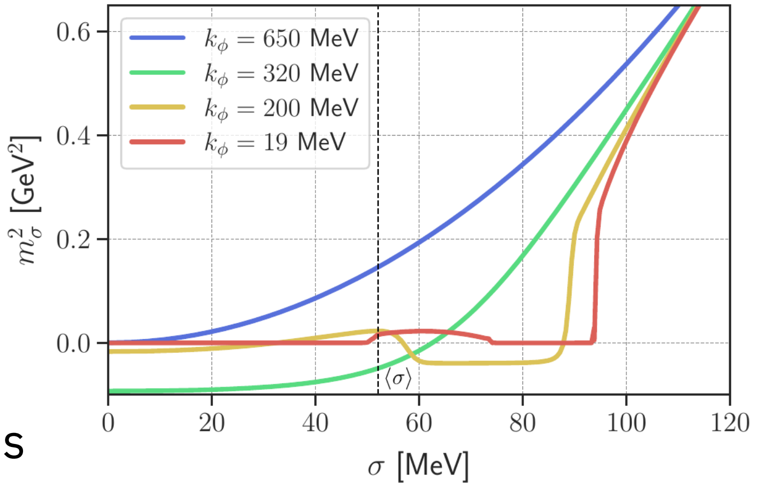

Ihssen, Pawlowski, Sattler, Wink
[Phys.Rev.D 111 (2025) 3, 036030]

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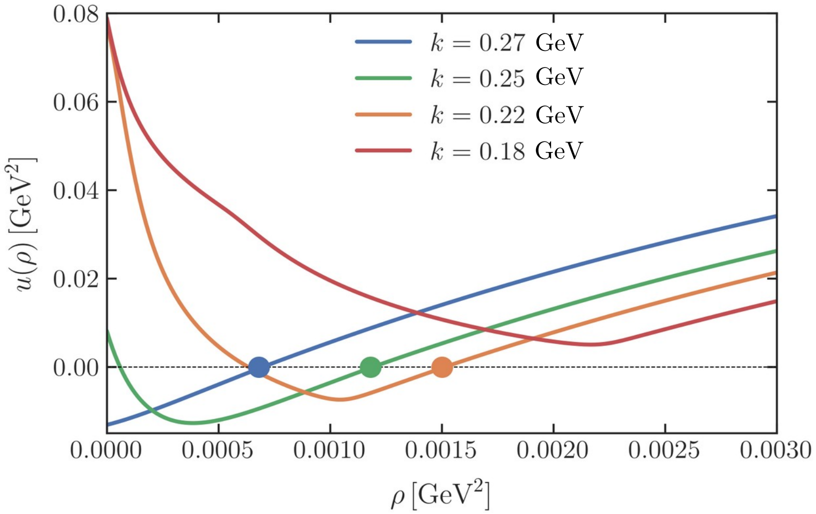

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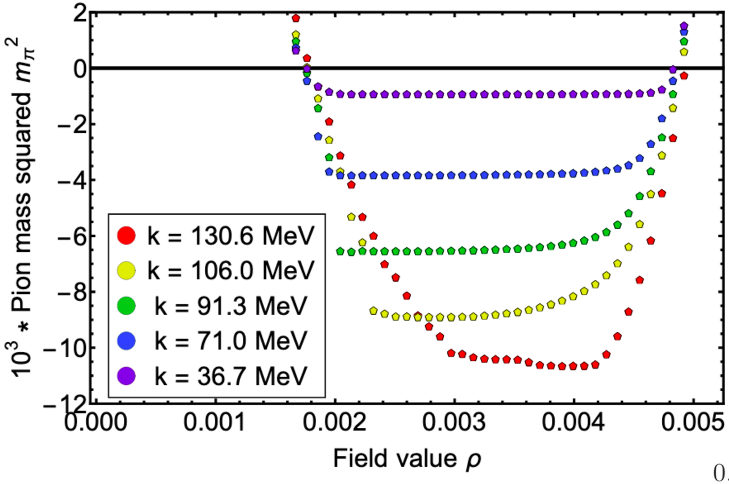

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- Quantitative access to chiral limit!

Ihssen, Pawlowski, Sattler, Wink
[arXiv:2408.08413 (2024)]

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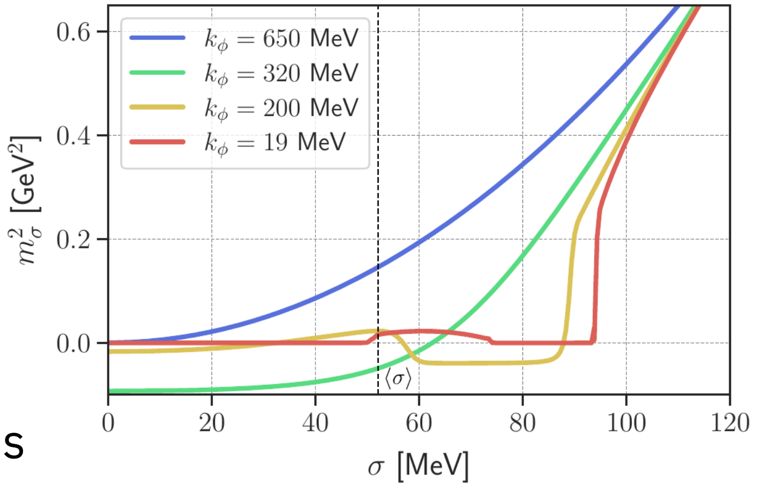

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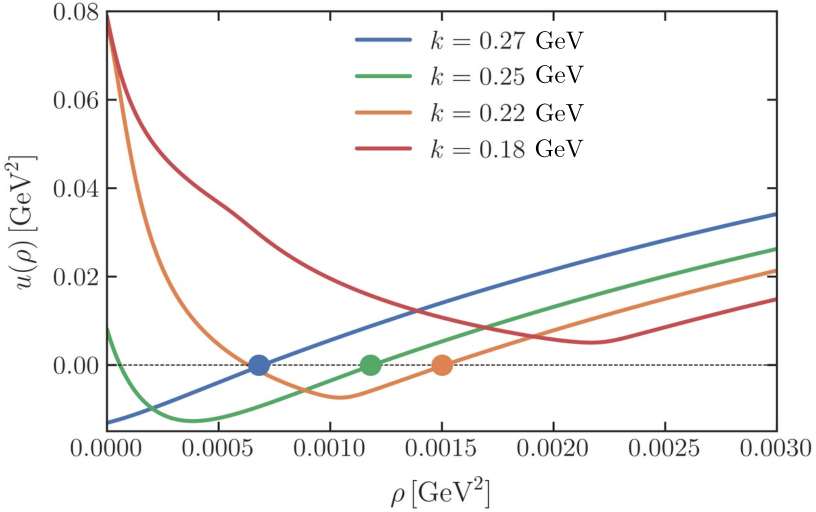

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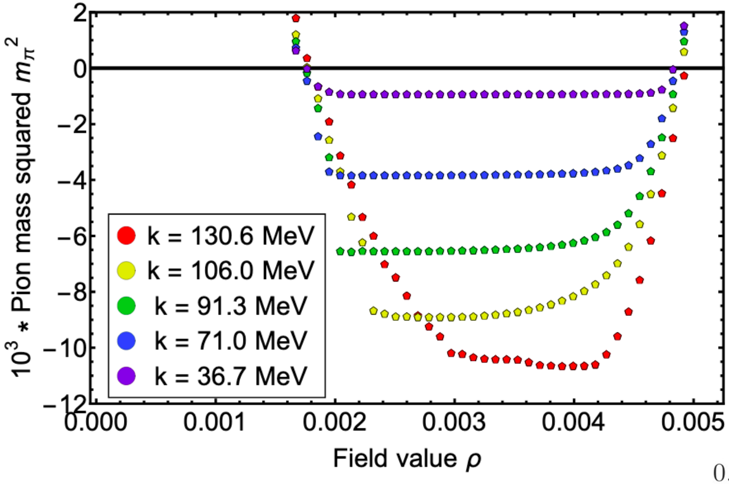

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FEM/FV

Grossi, Wink [SciPost Phys.Core 6 (2023) 071]

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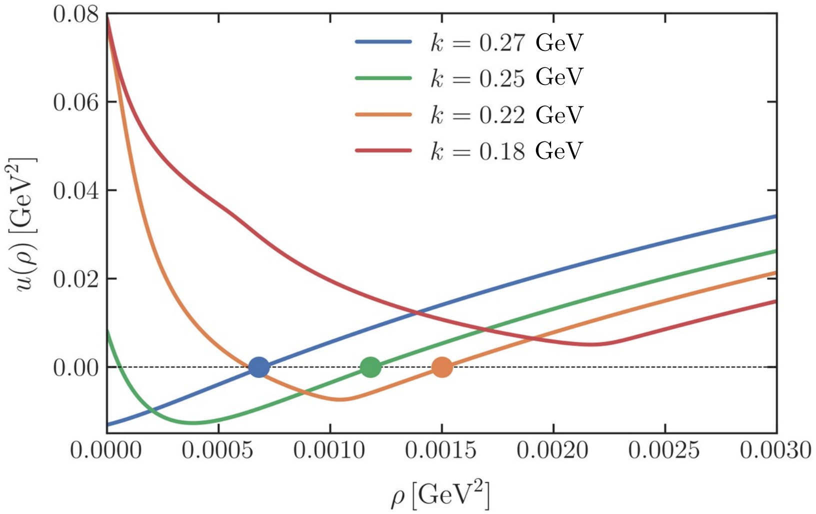

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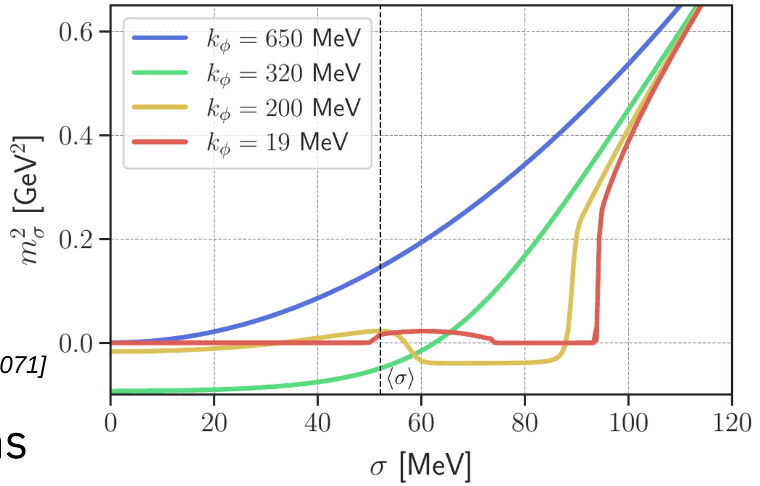

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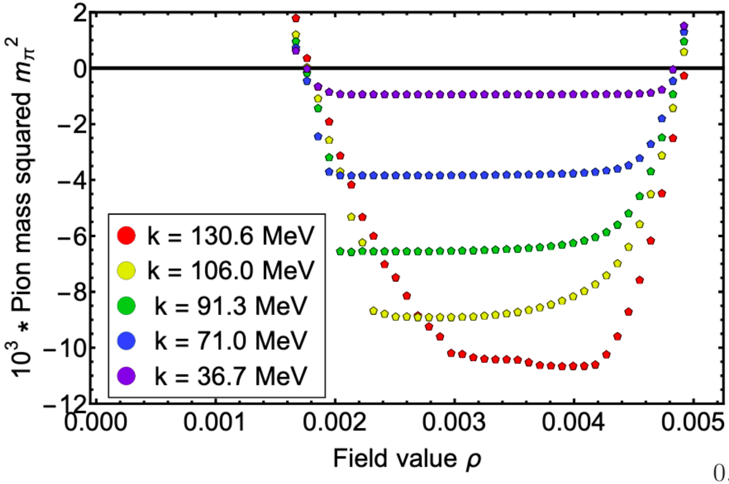

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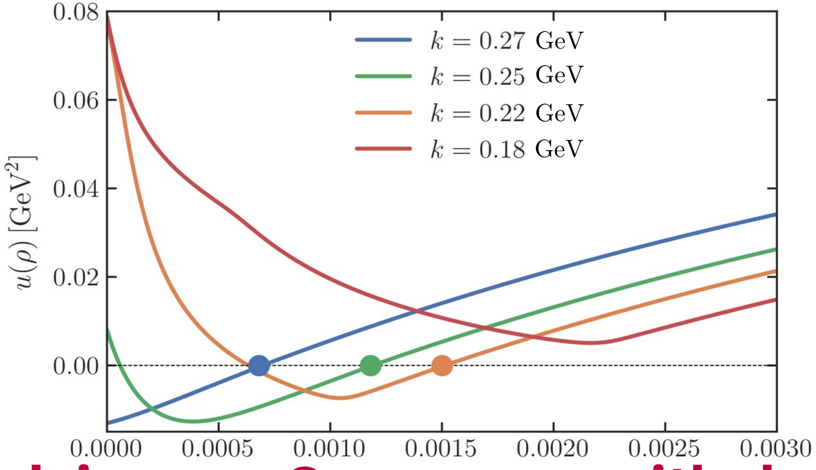

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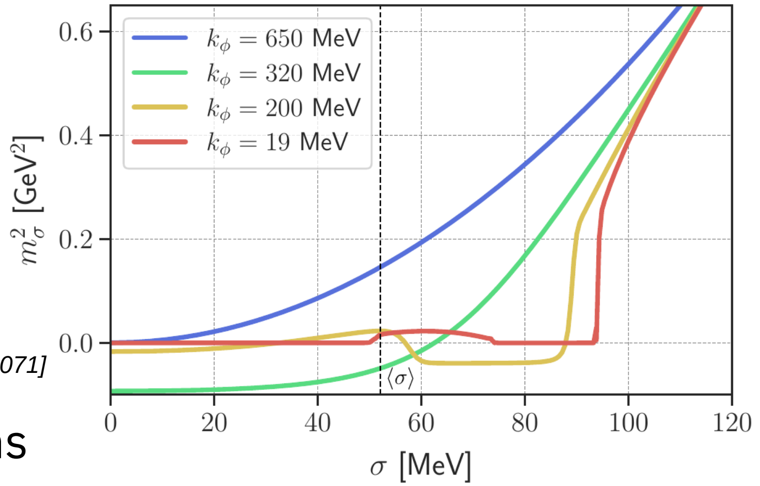

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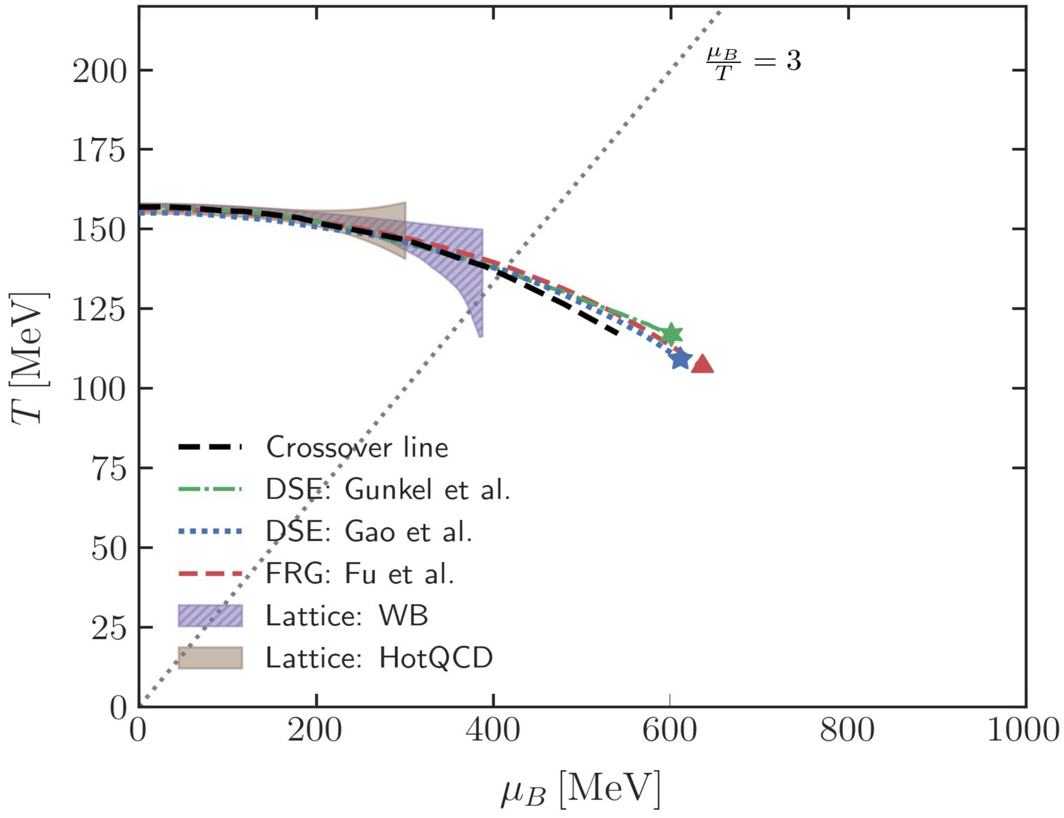
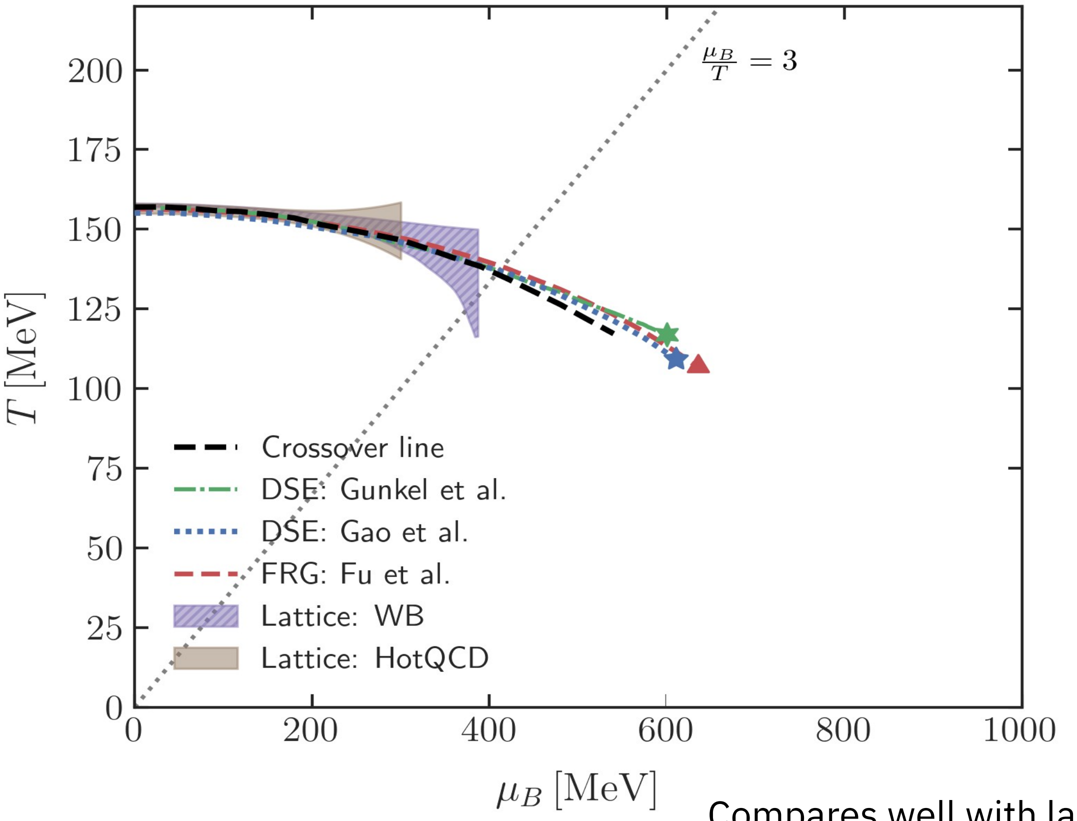
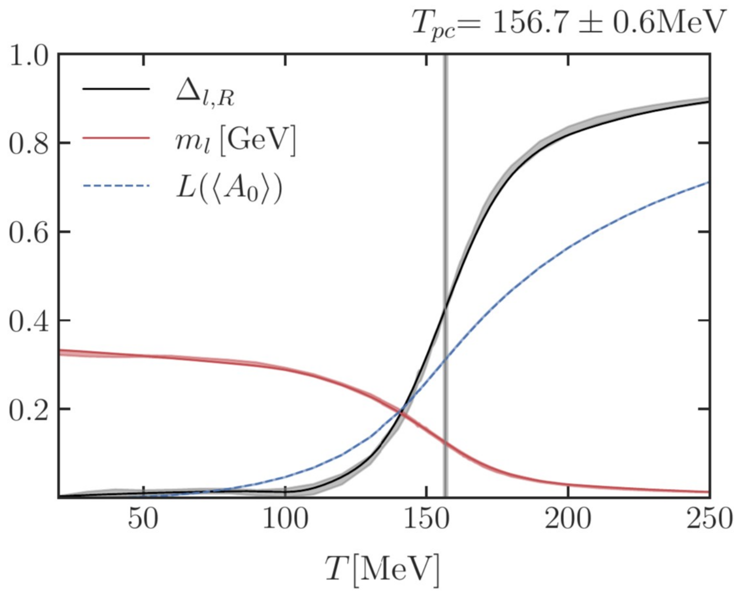
Our framework is open Source, see github.com/satfra



$\frac{\mu_B}{T} = 3$
$T\,[\mathrm{MeV}]$
$\mu_B\,[\mathrm{MeV}]$
0
25
50
75
100
125
150
175
200
0
200
400
600
800
1000
Crossover line
DSE: Gunkel et al.
DSE: Gao et al.
FRG: Fu et al.
Lattice: WB
Lattice: HotQCD





Compares well with lattice results

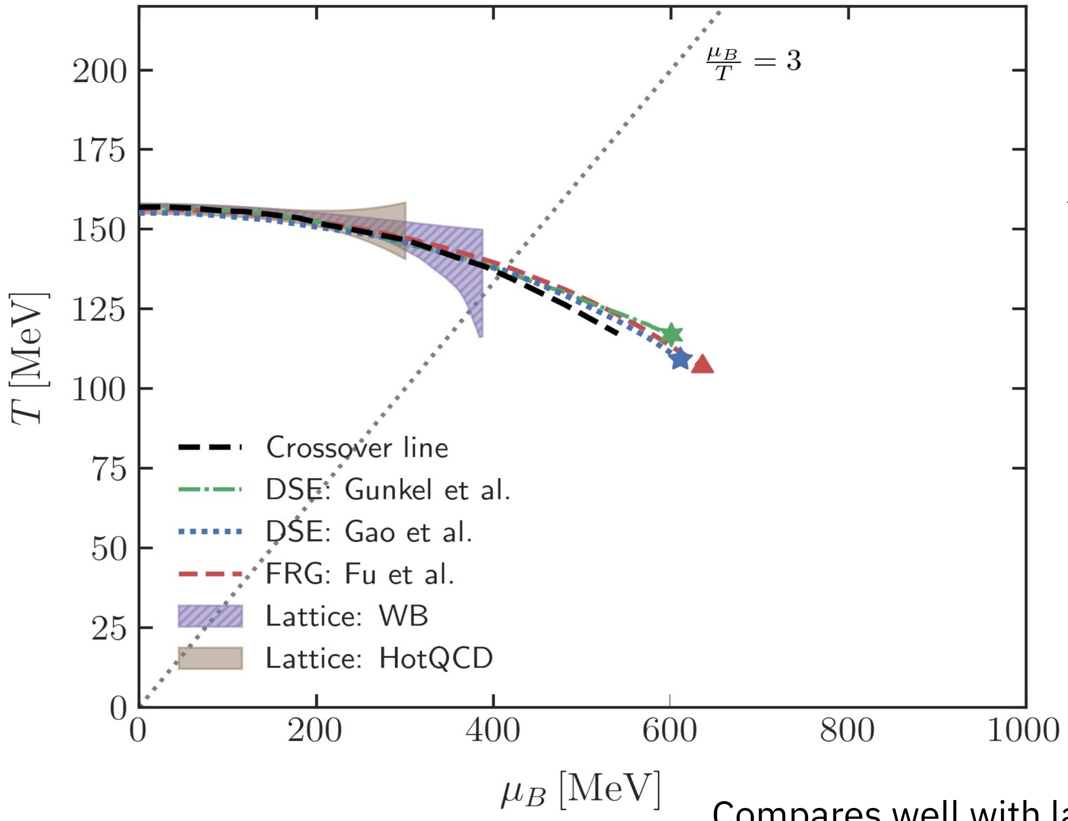
$$T_{pc}^{\mathrm{lat}} = 155 - 158\mathrm{MeV}$$

We have control over

- Gauge consistency
- UV-limit

Partial systematic error estimate from

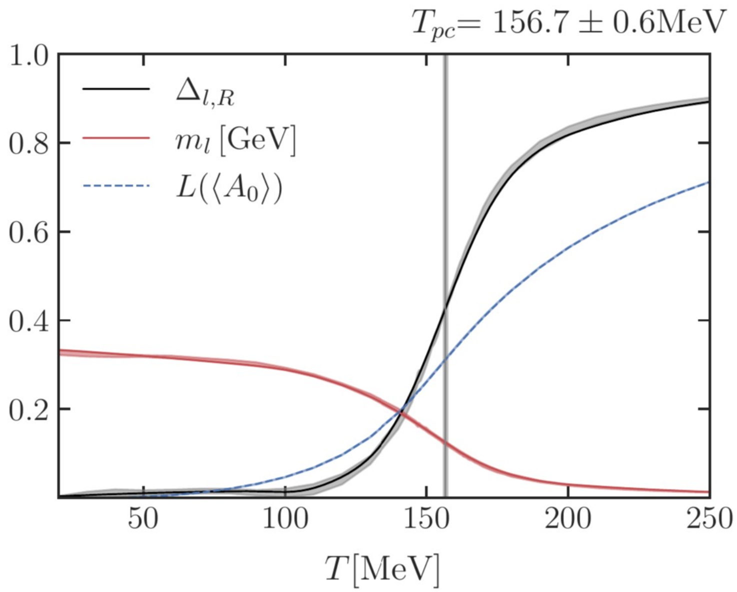
- Regulator variation
- LEGO®



$$T = T_{pc}\left(1 - \kappa\left(\frac{\mu_B}{T_{pc}}\right)^2 + \lambda_2\left(\frac{\mu_B}{T_{pc}}\right)^4 + \lambda_3\left(\frac{\mu_B}{T_{pc}}\right)^6 + \ldots\right)$$

	this work	lattice [52]	lattice [364]	lattice [54]	fRG [42]	DSE [57]
κ	0.0160(21)	0.015(4)	0.0144(26)	0.0153(18)	0.0142(2)	0.0147(5)

[52] A. Bazavov et al, Phys. Lett. B795 (2019)
[364] Bonati et al, Phys. Rev. D98.5 (2018) 054510
[54] Borsanyi, et al, Phys. Rev. Lett. 125.5 (2020) 052001
[42] Fu, Jan M. Pawlowski, and Fabian Rennecke Phys. Rev. D 101.5 (2020), 054032
[57] Gao, Pawlowski Phys. Lett. B 820 (Oct. 2021), 136584



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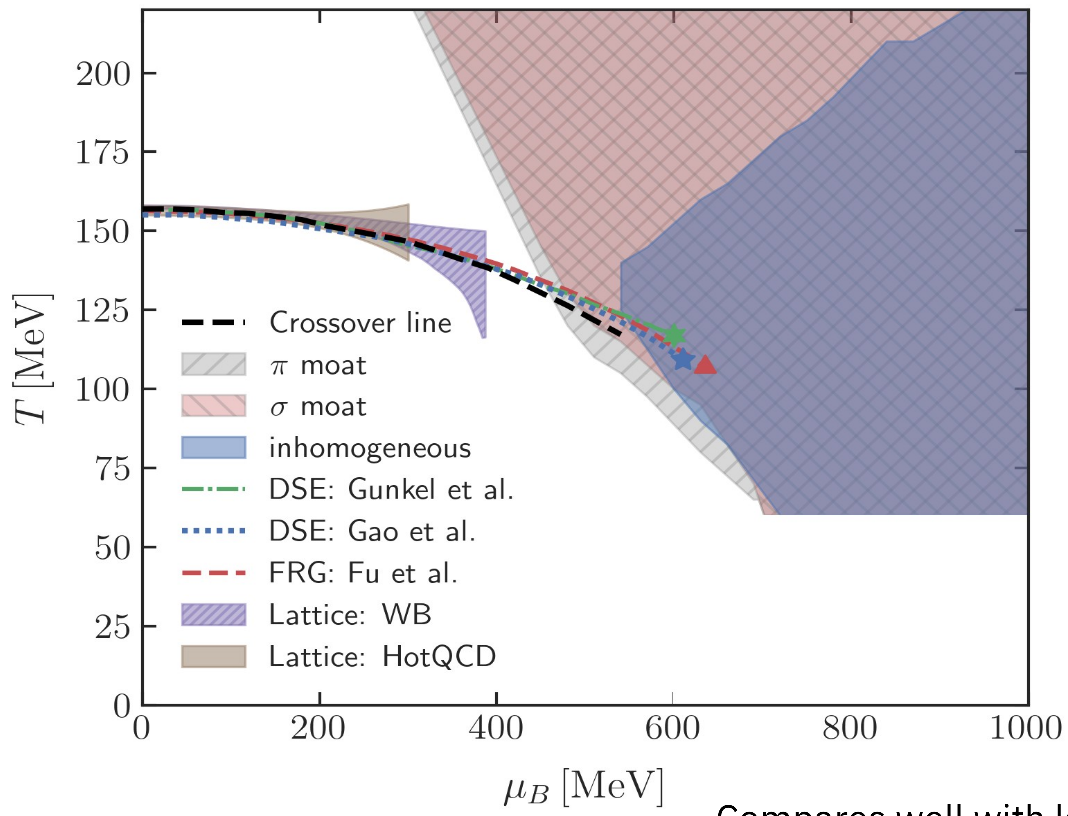

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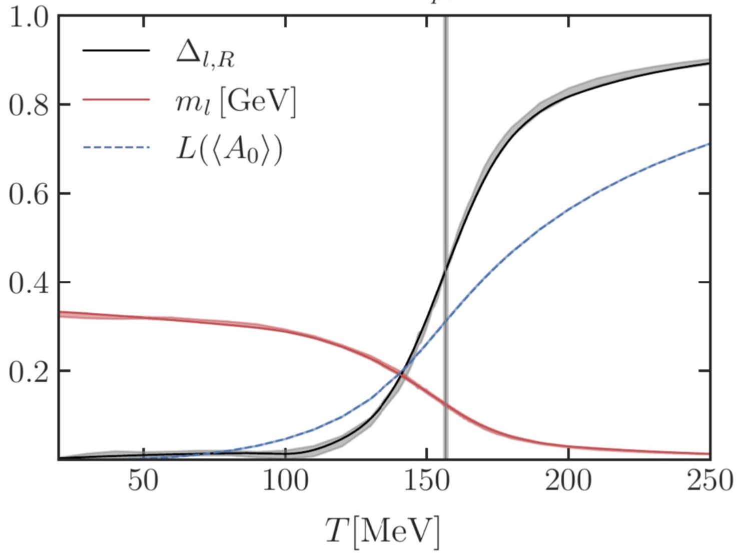
$T_{pc} = 156.7 \pm 0.6\text{MeV}$



Compares well with lattice results

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First calculation with fully self-consistent moat regime!

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- UV-limit

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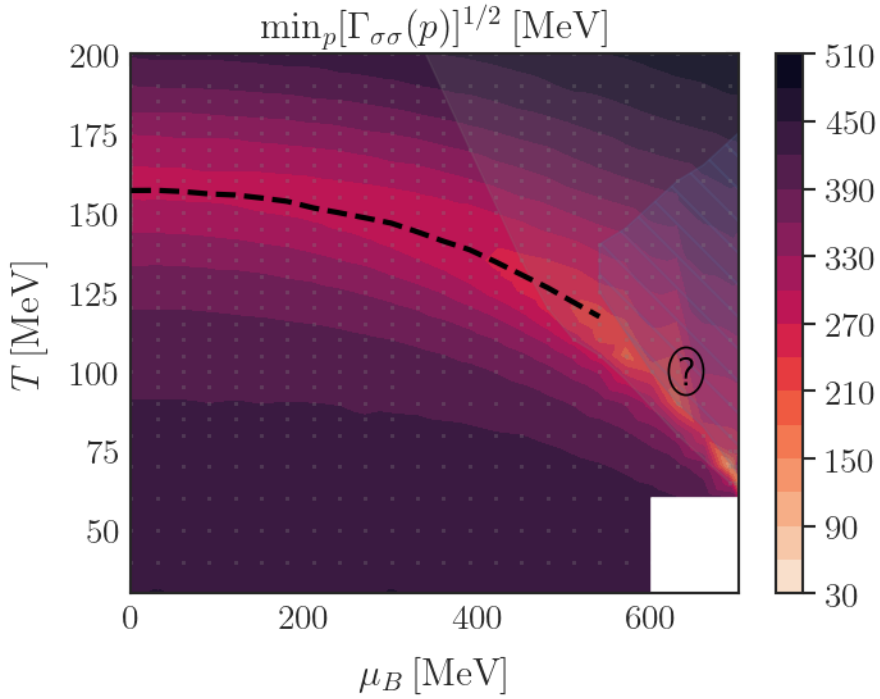
- LEGO®



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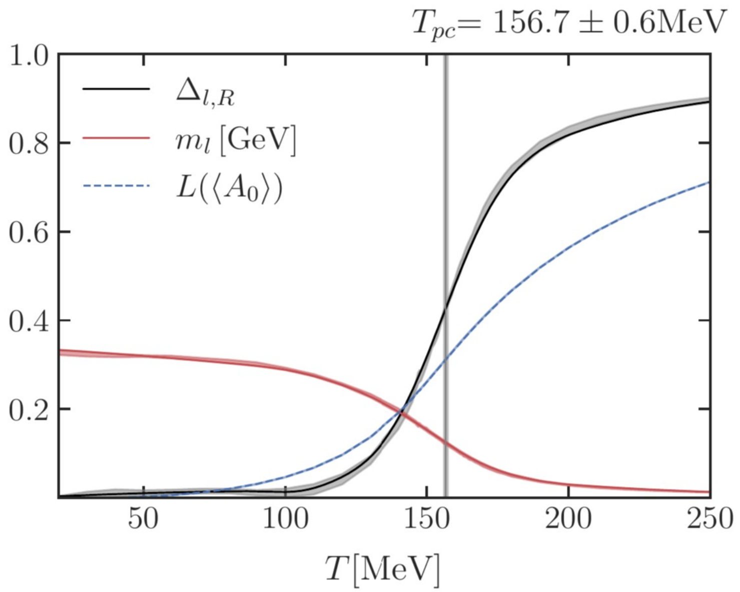

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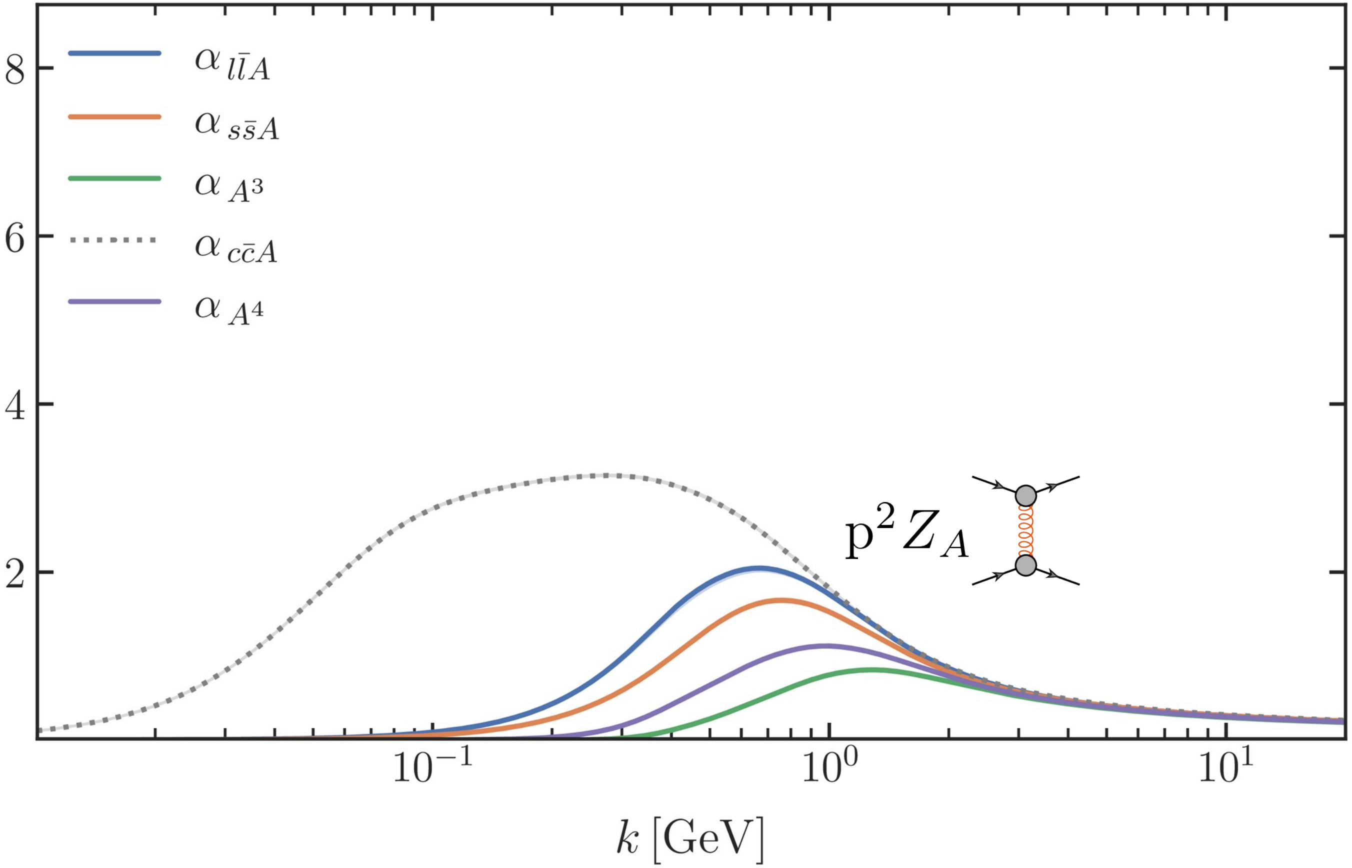
Systematic errors I:
The LEGO® principle



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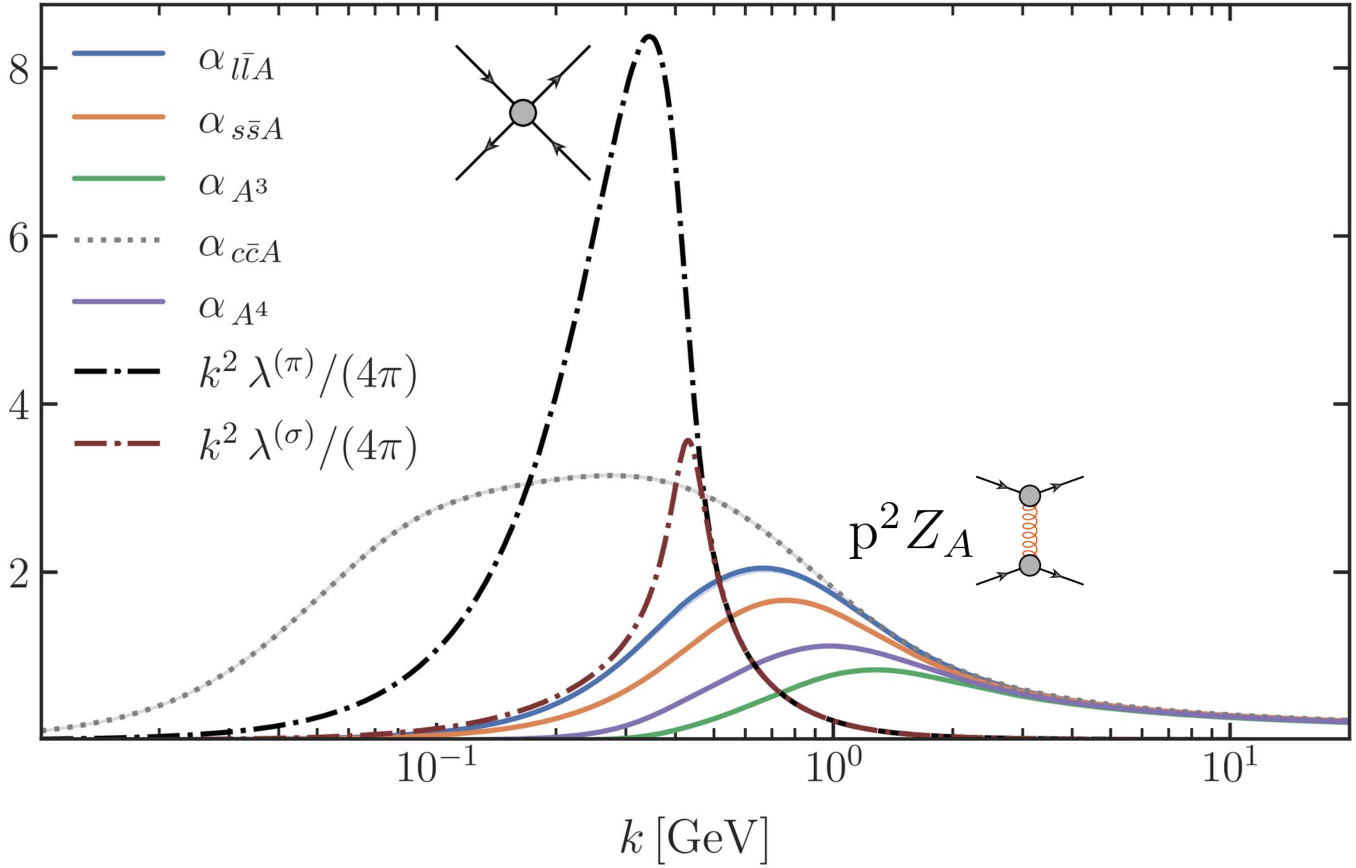

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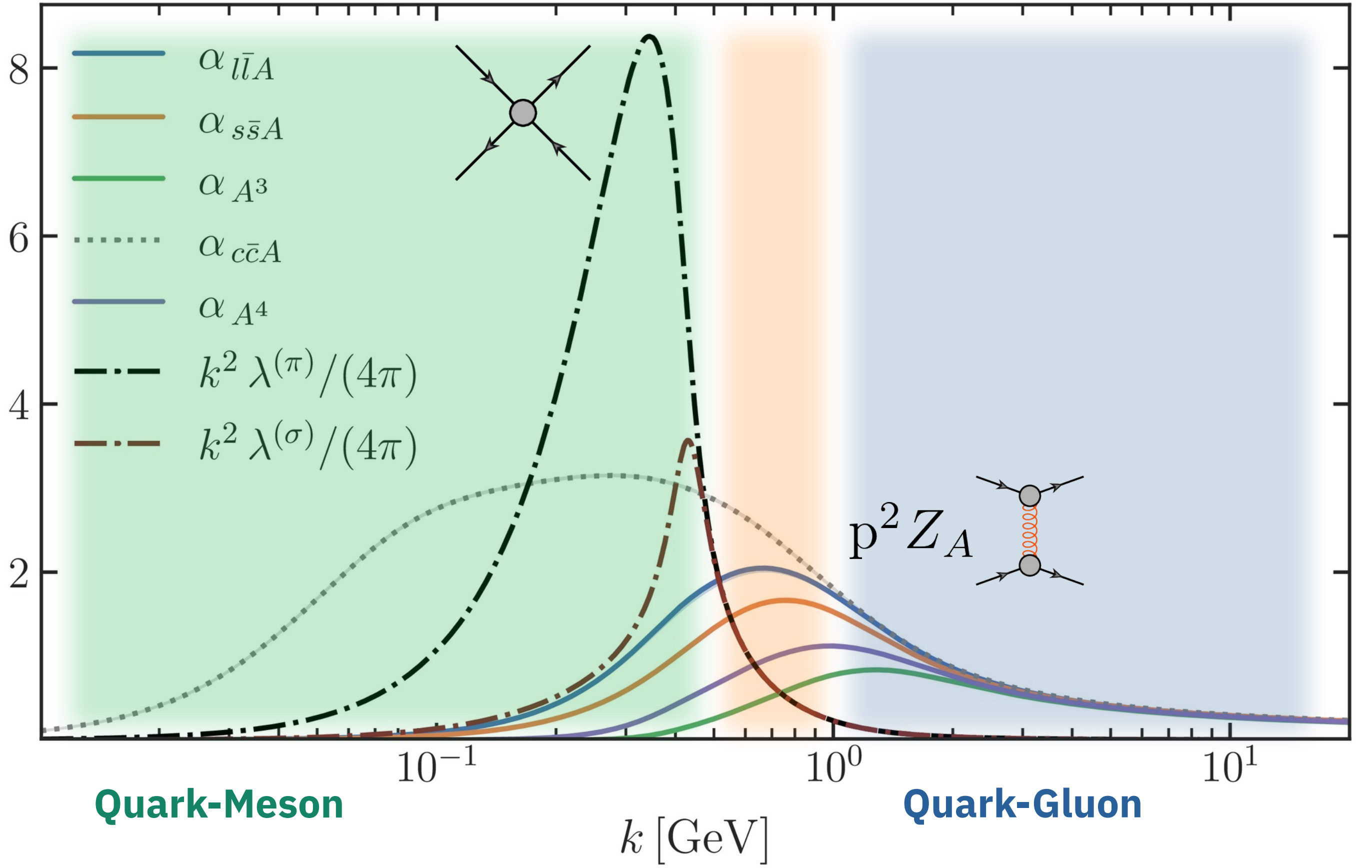

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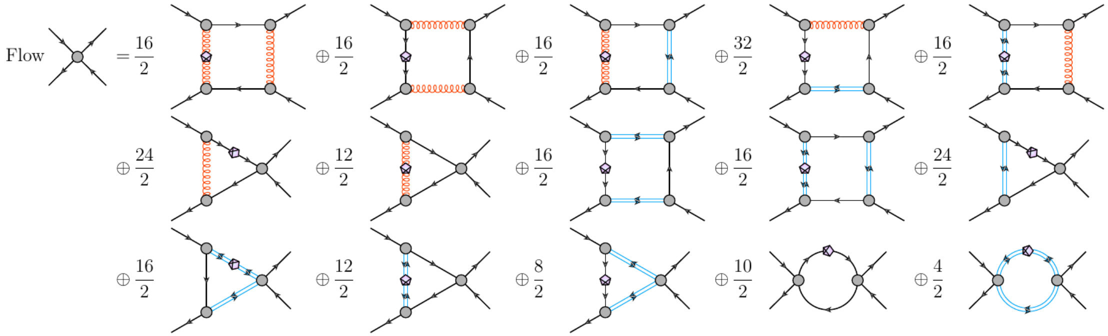
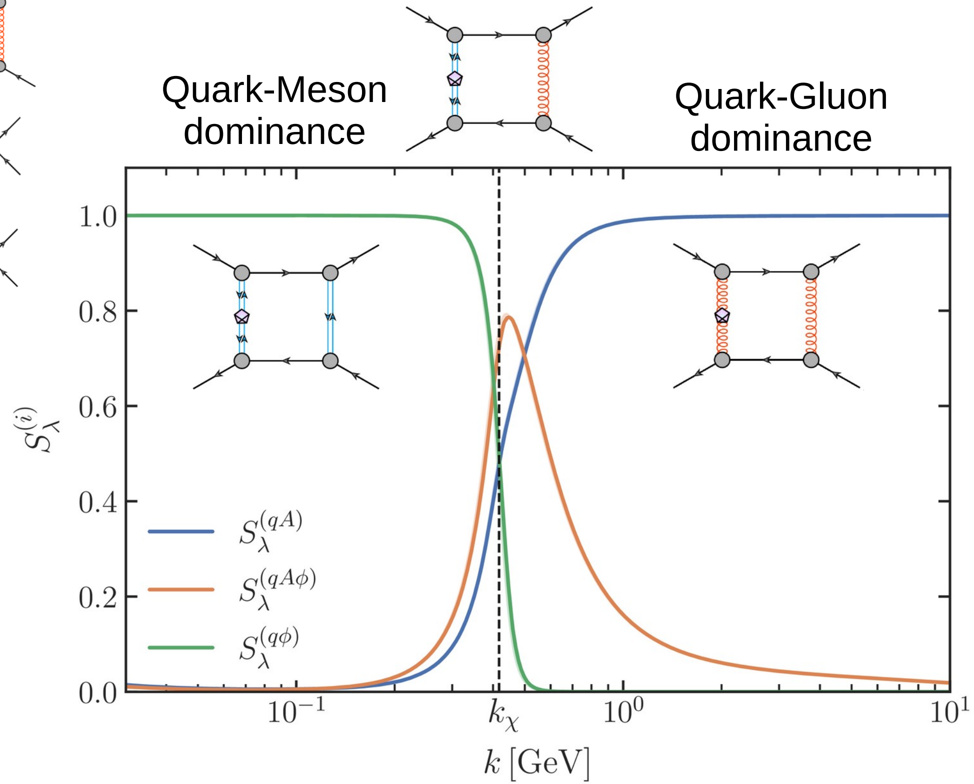
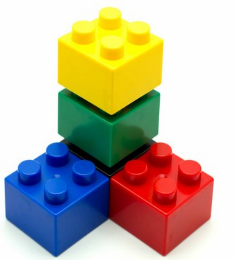




- ➔ Systematic error estimates from subsystems; preliminary estimate 10%.
- ➔ Low-energy effective theories.



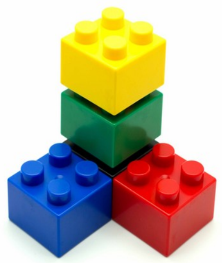
Systematic errors I:
The LEGO® principle

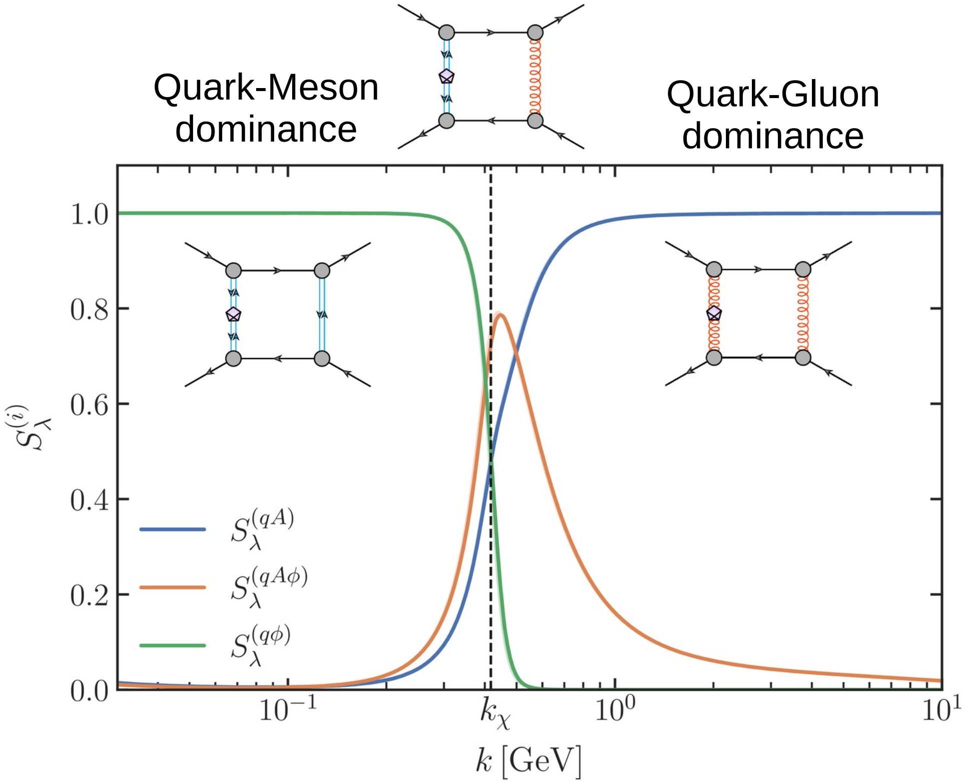
Separate LEGO® blocks:

- Glue subsystem $\{\lambda_{\text{glue}}\} = \{\alpha_{A^3}\,,\,\alpha_{A^4}\,,\,\alpha_{c\bar{c}A}\}$
- Matter subsystem $\{\lambda_{\text{mat}}\} = \{h_\phi(\rho_0)\,,\,\lambda_{\phi,n}(\rho_0)\}$
- Interface blocks $\{\lambda_{\text{inter}}\} = \{\alpha_{l\bar{l}A}\}$

➔ Systematic error estimates from subsystems; preliminary estimate 10%.

➔ Low-energy effective theories.

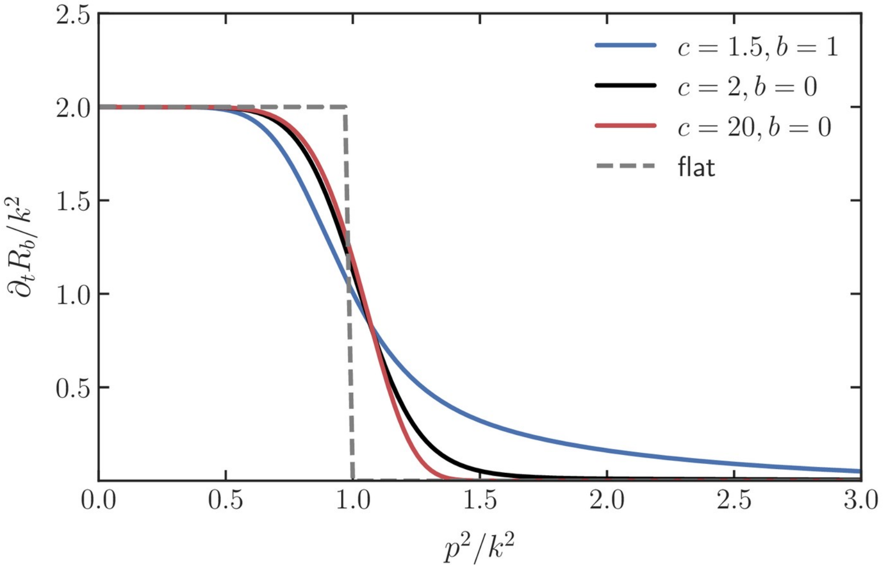




Systematic errors II: Regulator dependences

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \mathrm{Tr}\, G_k \partial_t R_k$$

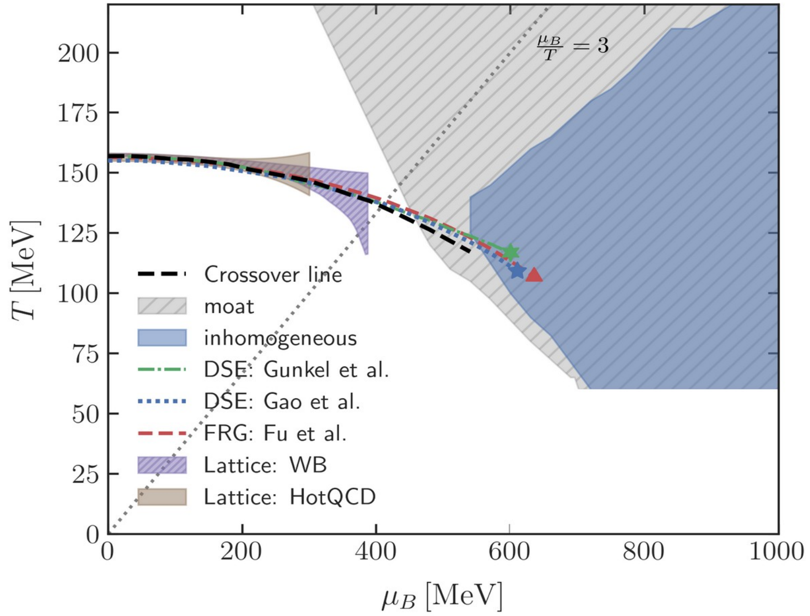
Easy regulator variation thanks to numerical framework:



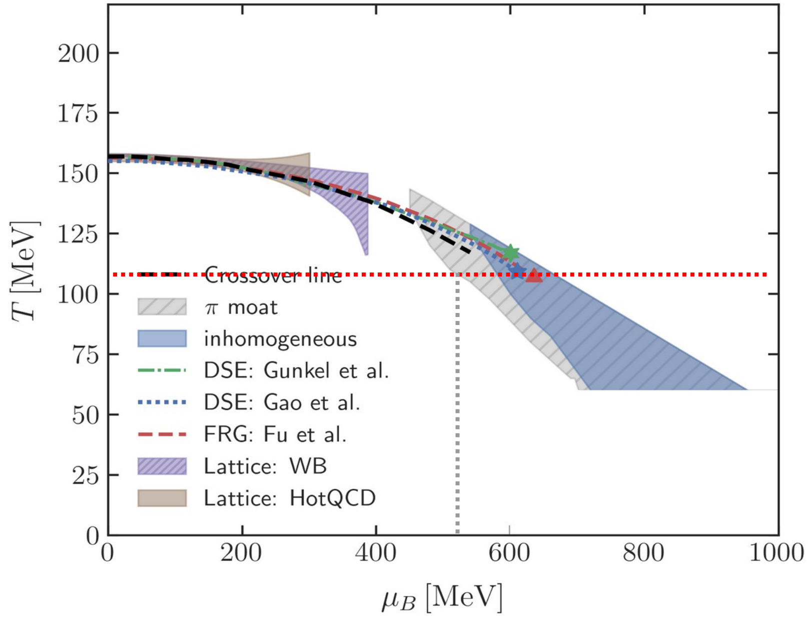
Observable	Value
$(f_K/f_\pi)_\chi$	$1.2168^{+0.0006}_{-0.0007}$
$f_{\pi,\chi}$ [MeV]	$93.2^{+3.5}_{-3.1}$
$m_{l,\chi}$ [MeV]	$311.6^{+0.3}_{-0.1}$
$m_{s,\chi}$ [MeV]	$446.7^{+0.3}_{-0.2}$
$m_{\sigma,\chi}$ [MeV]	$214.7^{+5.4}_{-9.3}$
$\sigma_{l,0,\chi}$ [MeV]	$67.1^{+1.2}_{-0.0}$

Chiral limit observables

Ihssen, Pawlowski, Sattler, Wink [arXiv:2408.08413]



First calculation with fully self-consistent moat regime!



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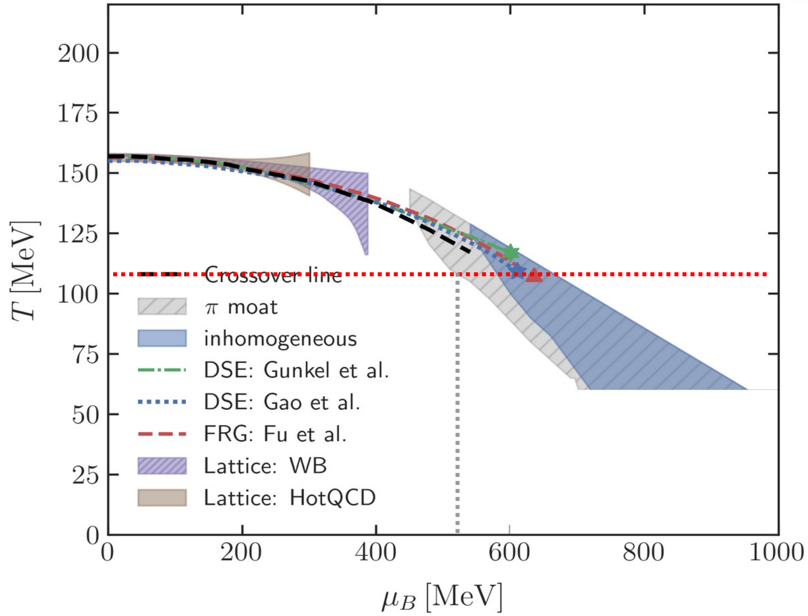
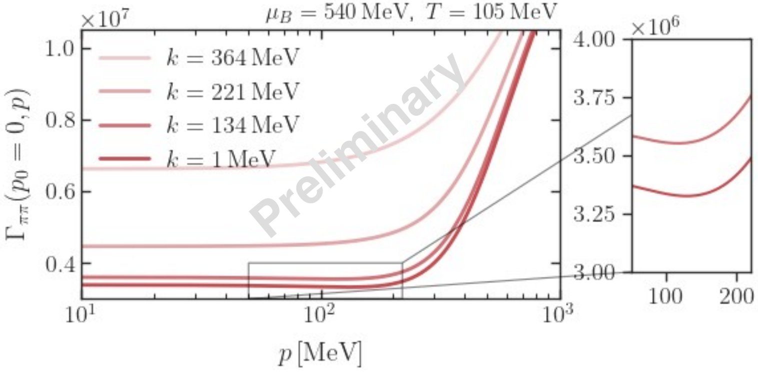
Moat Regime





Shi Yin's talk!

Nunney Castle, England

$$\langle \bar{q}q \rangle(\boldsymbol{x}) \sim \mathrm{const}$$

(homogeneous condensation)

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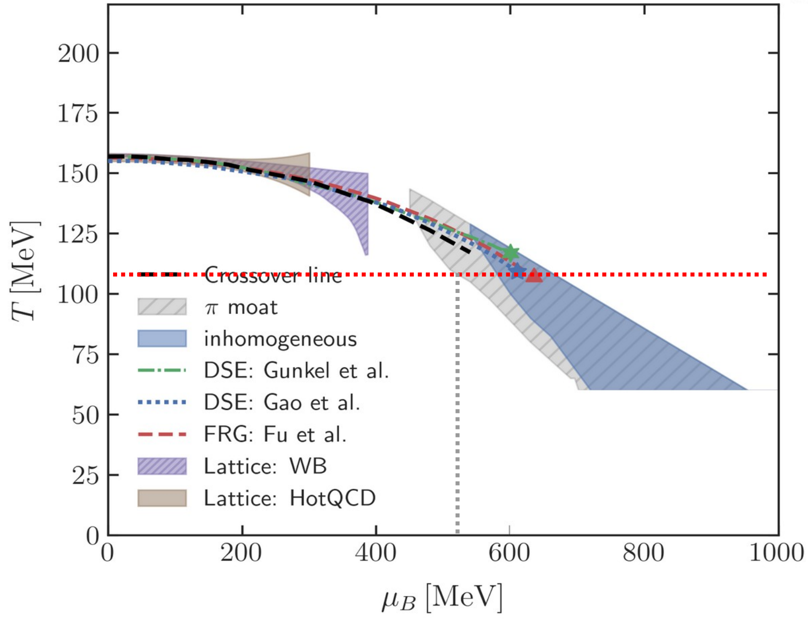

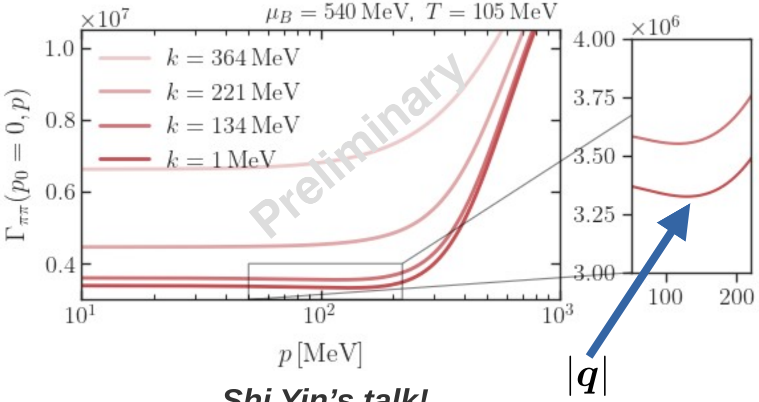

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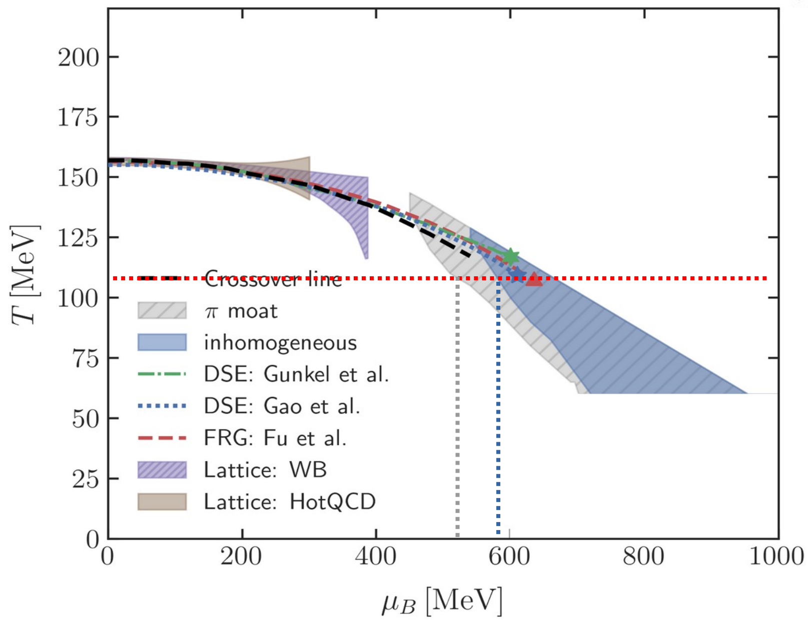

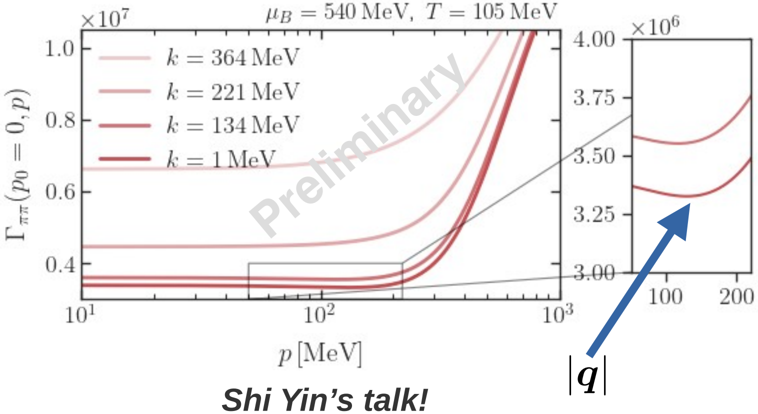

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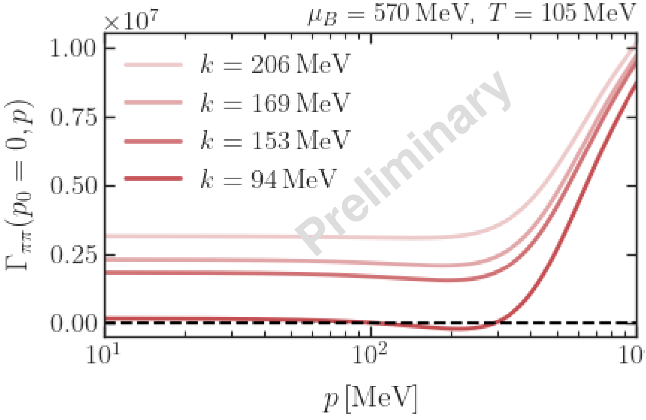

Inhomogeneous Instability

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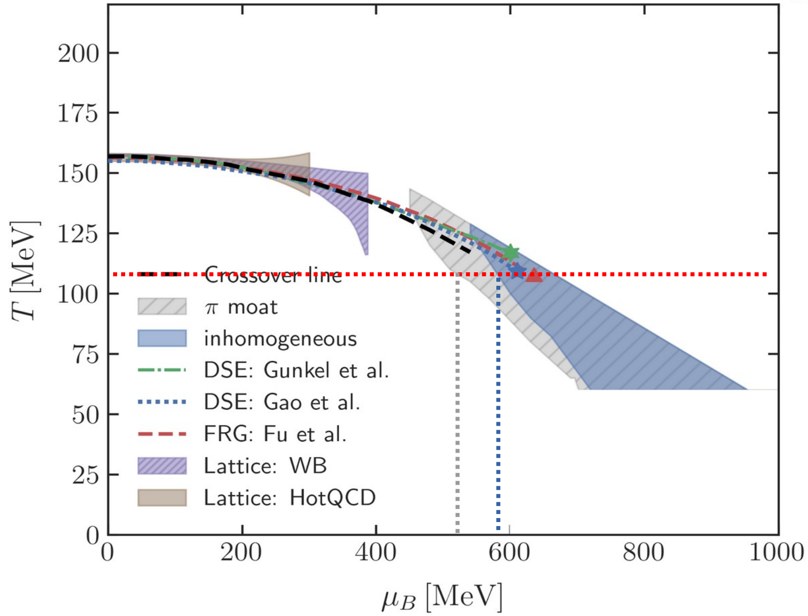

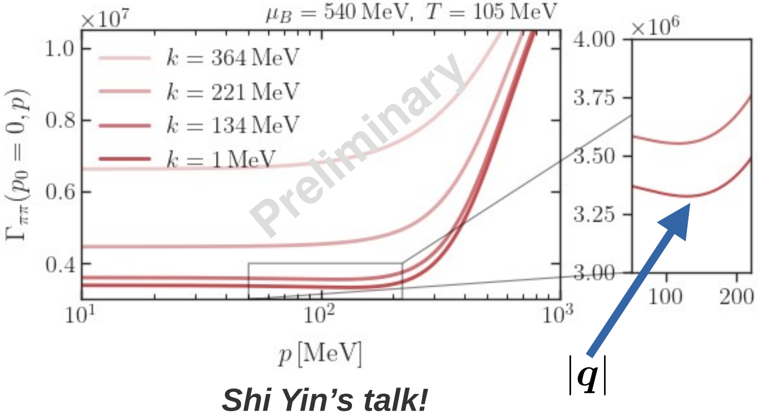

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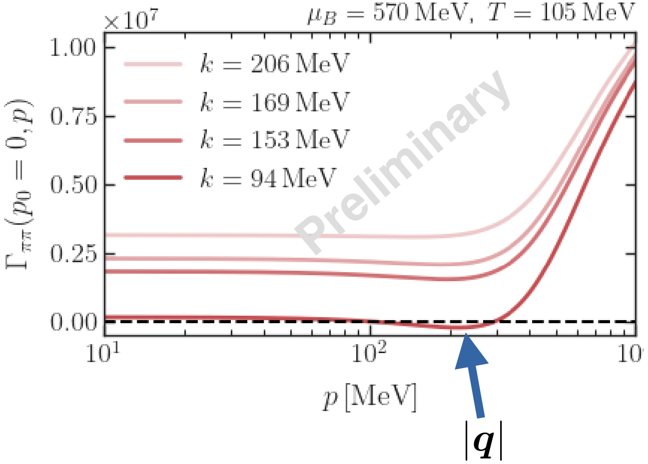

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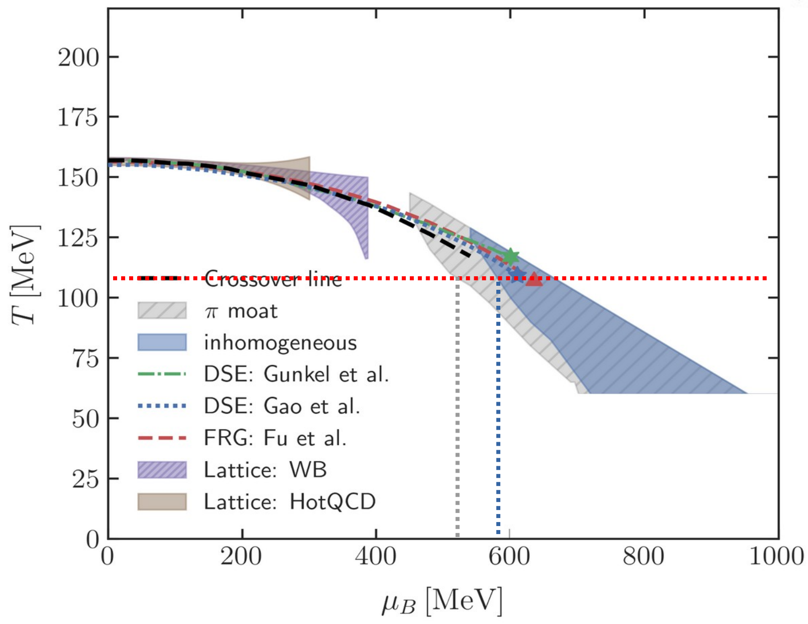

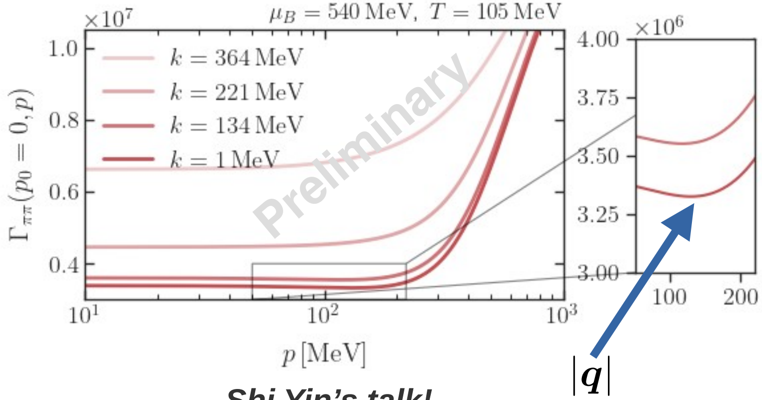

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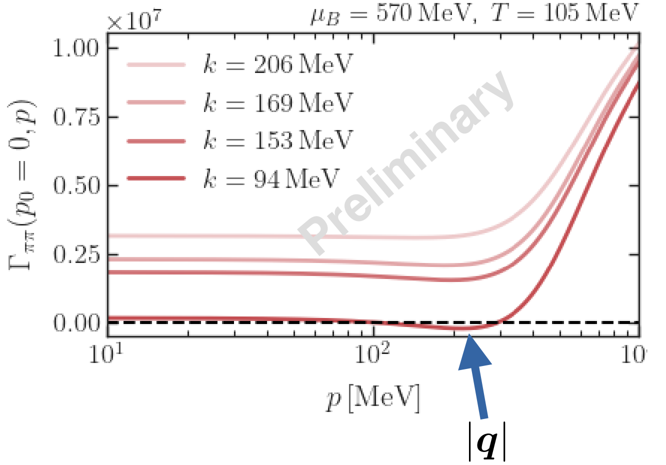

Stable?

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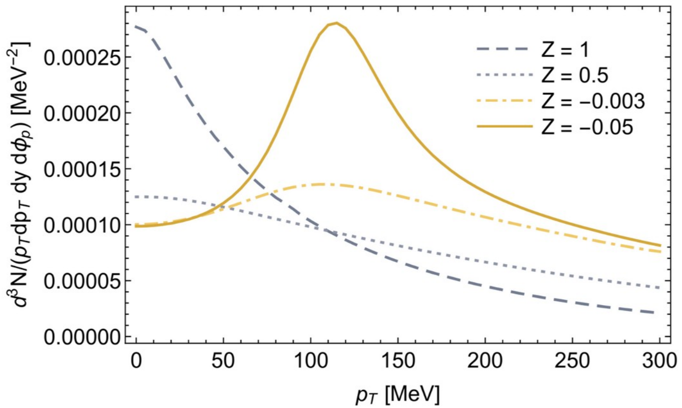
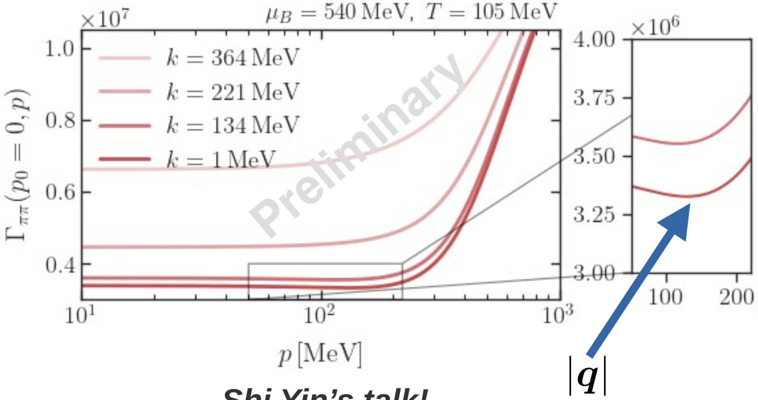
Enhancement in particle production



Rennecke, Pisarski, PoS CPOD2021 (2022)



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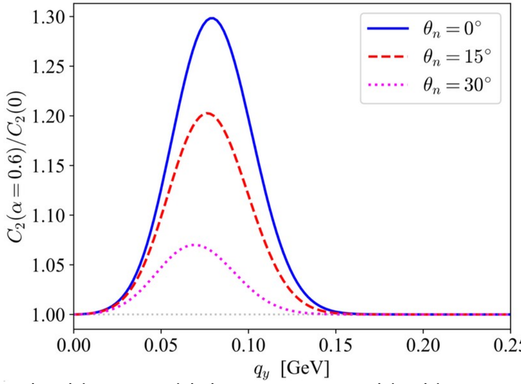
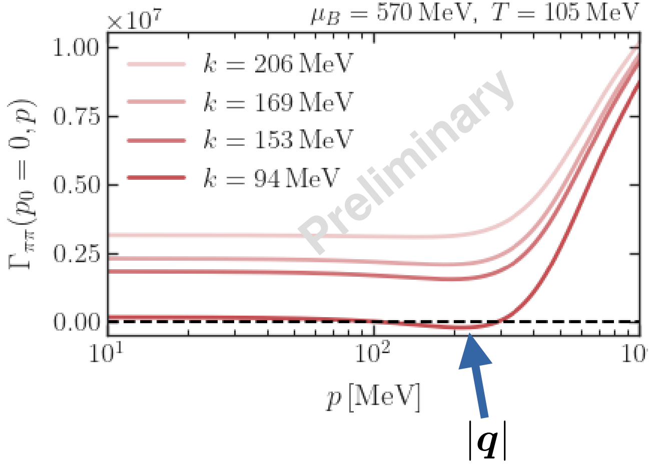
HBT interferometry



K Fukushima, Y. Hidaka, K Inoue, K Shigaki, Y Yamaguchi, Phys. Rev. C 109, L051903 (2024)



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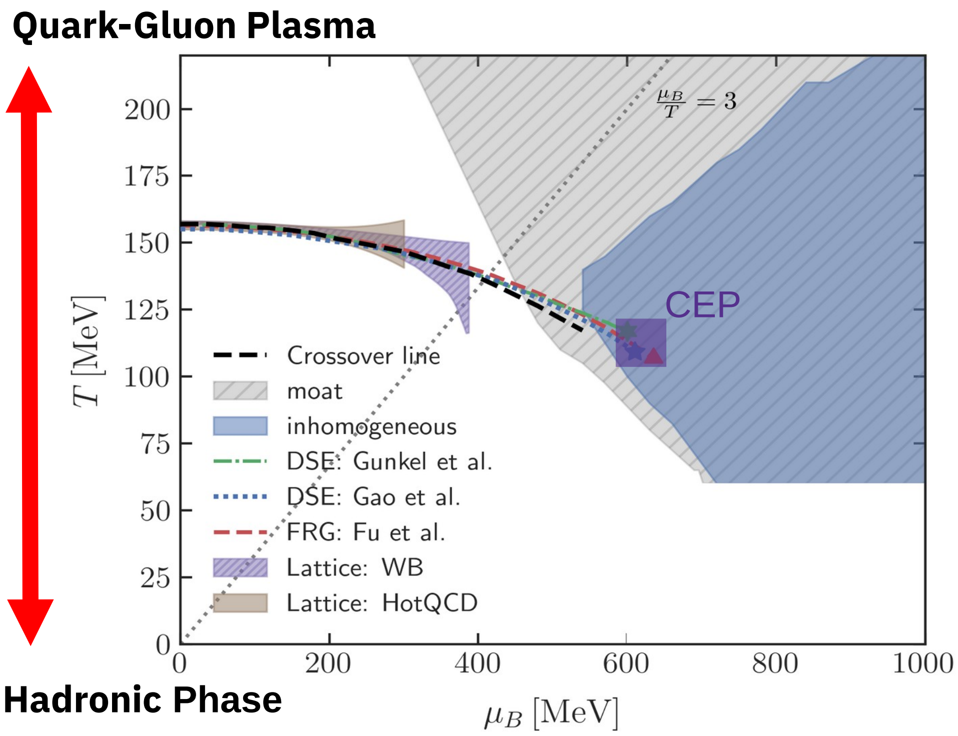
The QCD phase diagram



High densities: *This work*

Fu, Pawlowski, Rennecke [Phys. Rev. D 101 (2020), 054032]

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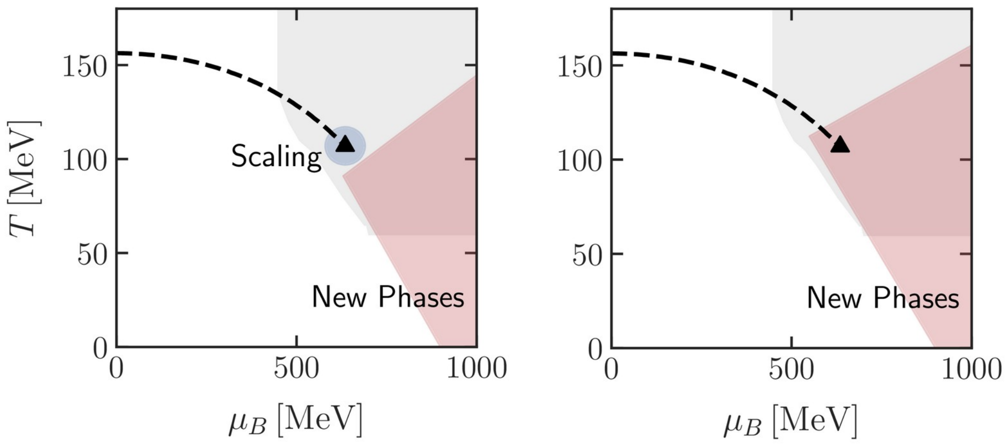
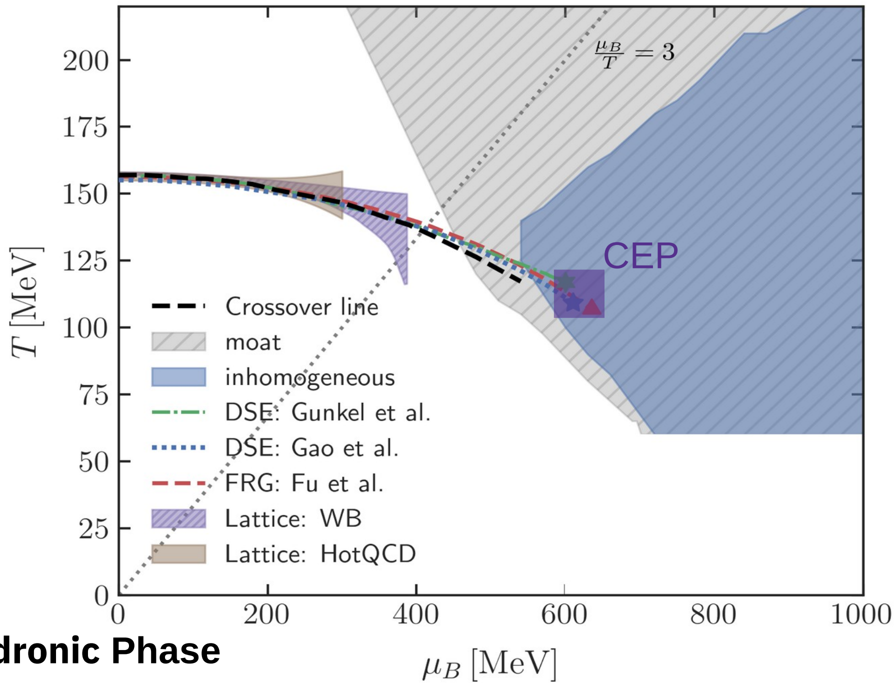


Scenarios for the phase structure

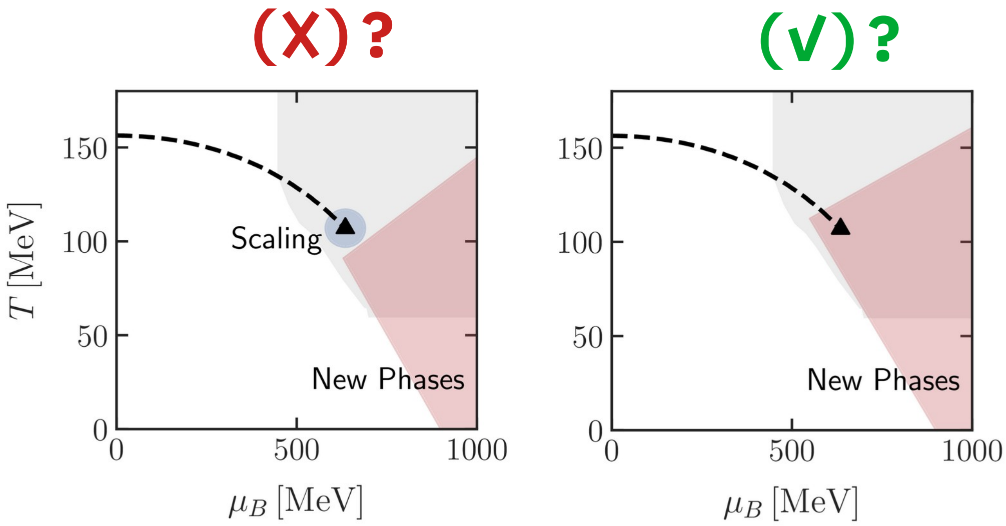
- Inhomogeneous regime (Lifshitz point/regime)
- Moat
- Diquarks

The QCD phase diagram

Quark-Gluon Plasma



Hadronic Phase



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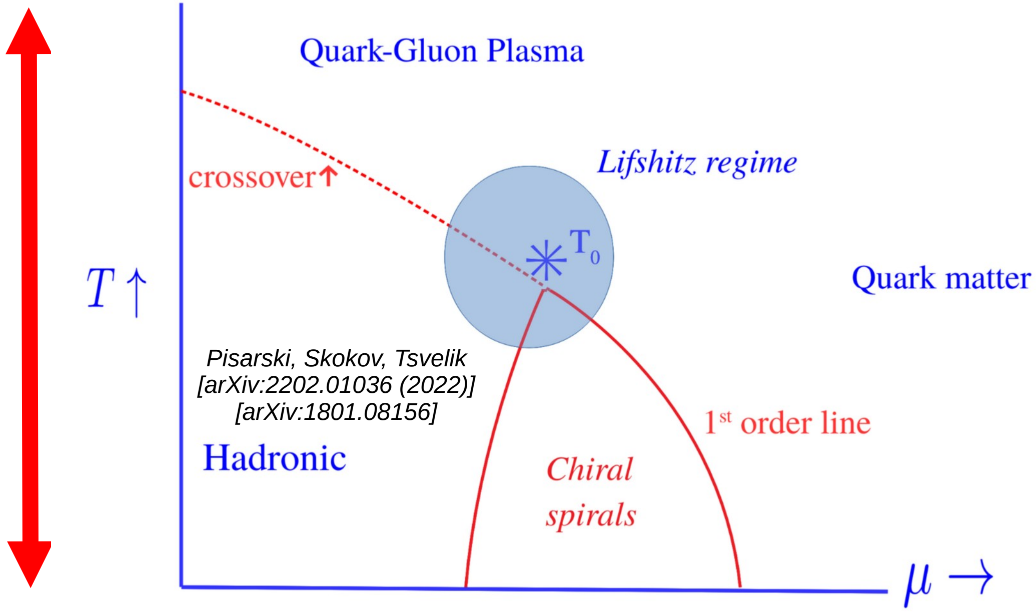

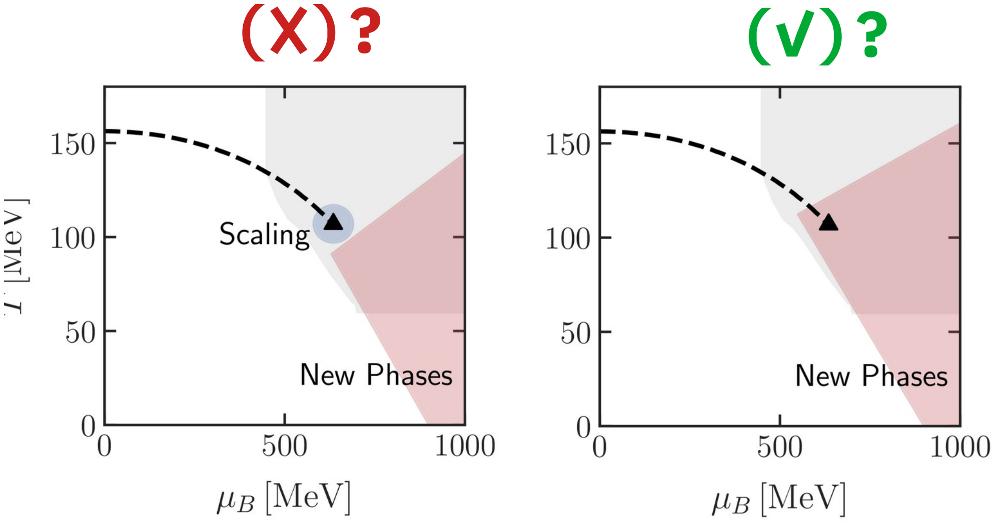

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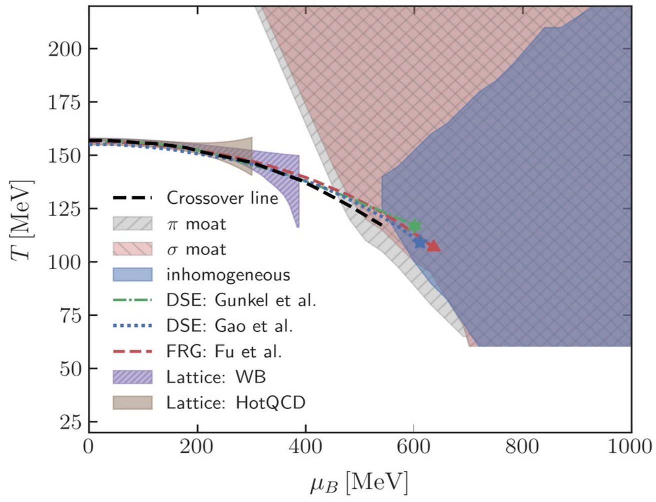

https://fqcd-collaboration.github.io/

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- **Motivation:**
  Direct access to phase structure of QCD with fRG.

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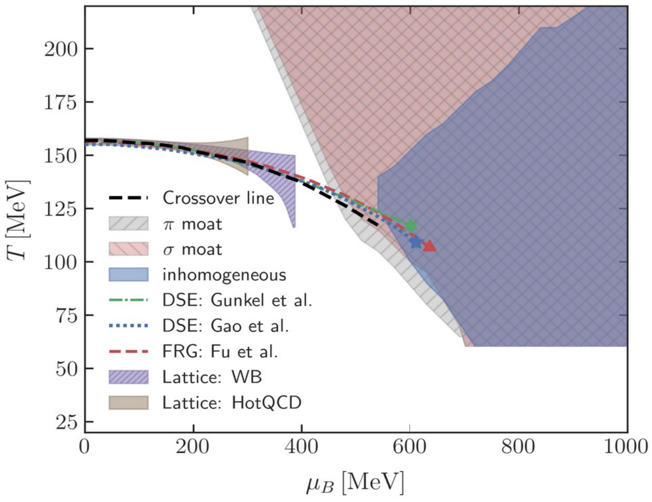

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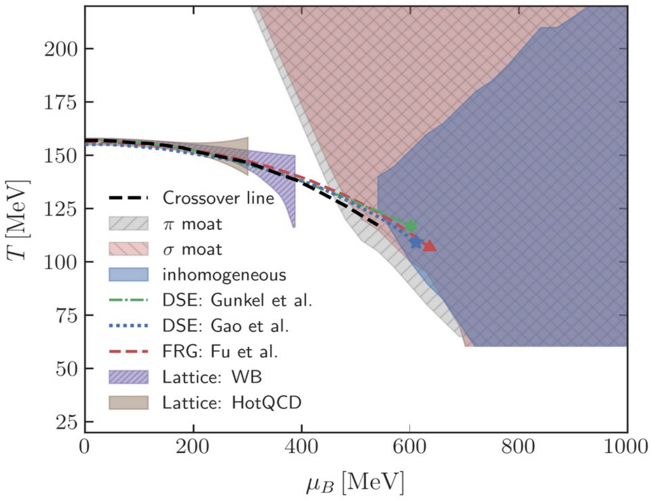

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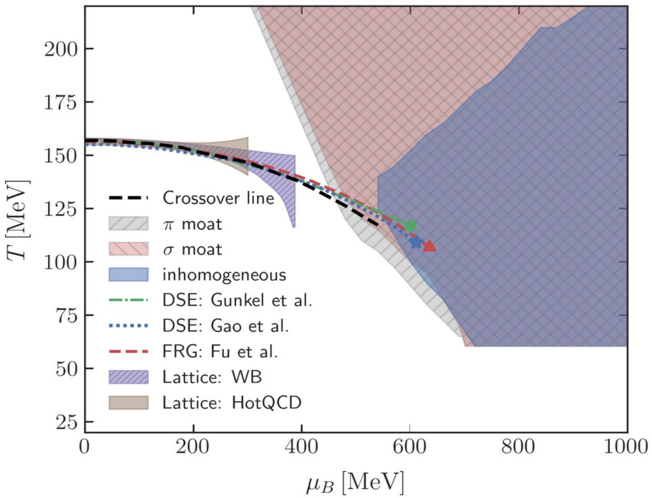

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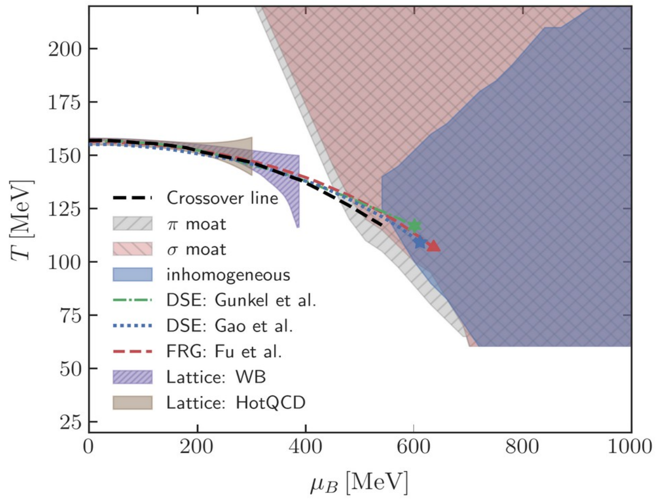

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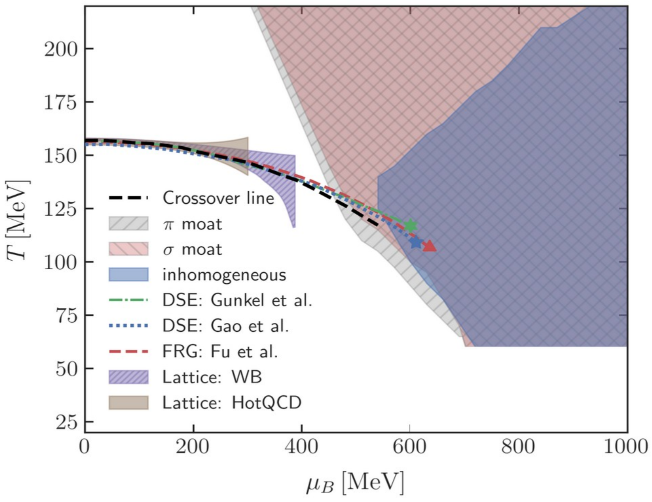
- Improved **Polyakov loop potential** *(with C. Huang & J.Pawlowski)*.



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QCD at finite densitites

Outlook

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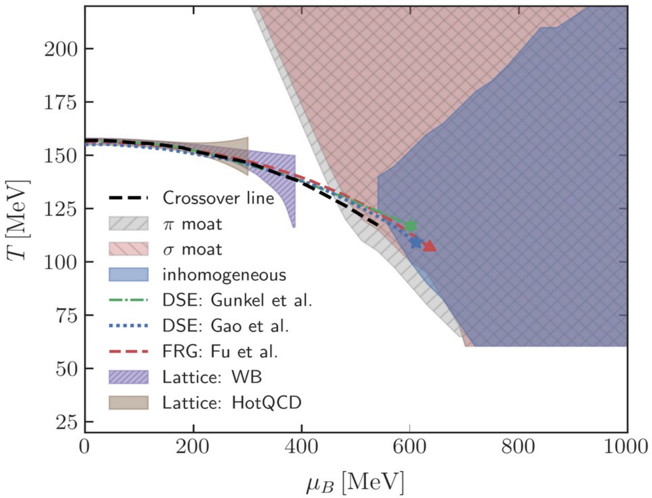
- *Fully self-consistent* treatment of **inhomogeneous condensation.**



➔ **Prediction of correlators, scaling at CEP/Lifshitz point.**

Thank you for your attention!

https://fqcd-collaboration.github.io/

Backup slides

Low-energy QCD

Quark	$\overline{\text{MS}}$ mass at 2GeV	Electric charge
Up	2.16 MeV	2/3
Down	4.67 MeV	-1/3
Strange	93.4 MeV	-1/3
Charm	1.27 GeV	2/3
Bottom	4.18 GeV	-1/3
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$$m_{\text{Baryon}} = 938\,\text{MeV}$$

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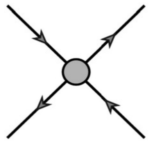
Mass generation through emergence of large four-quark correlations



(Resonant)

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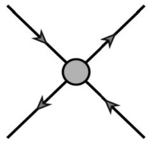
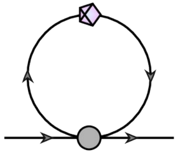


(Resonant)

$$m^{\text{con}}_{u/d} \approx 350\text{MeV} \;\sim$$



Dynamical Chiral Symmetry Breaking

$$\text{SU}(N_f)_A \times \text{SU}(N_f)_V \times \text{U}(1)_V \to \text{SU}(N_f)_V \times \text{U}(1)_V$$

Low-energy QCD

$N_f = 2 + 1$

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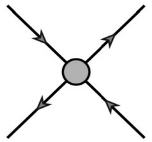

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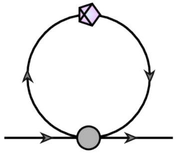

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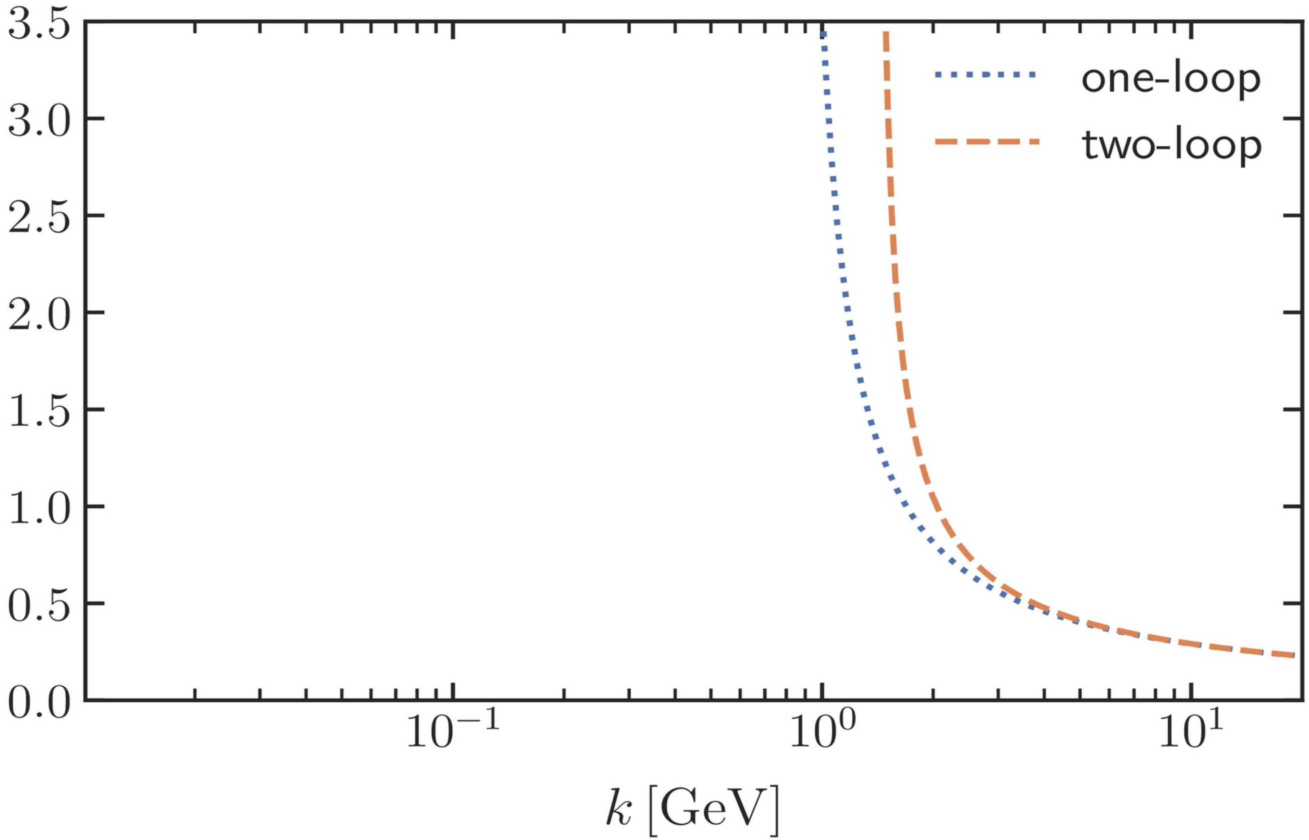
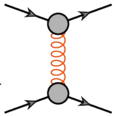
QCD from the fRG perspective



$p^2 Z_A$



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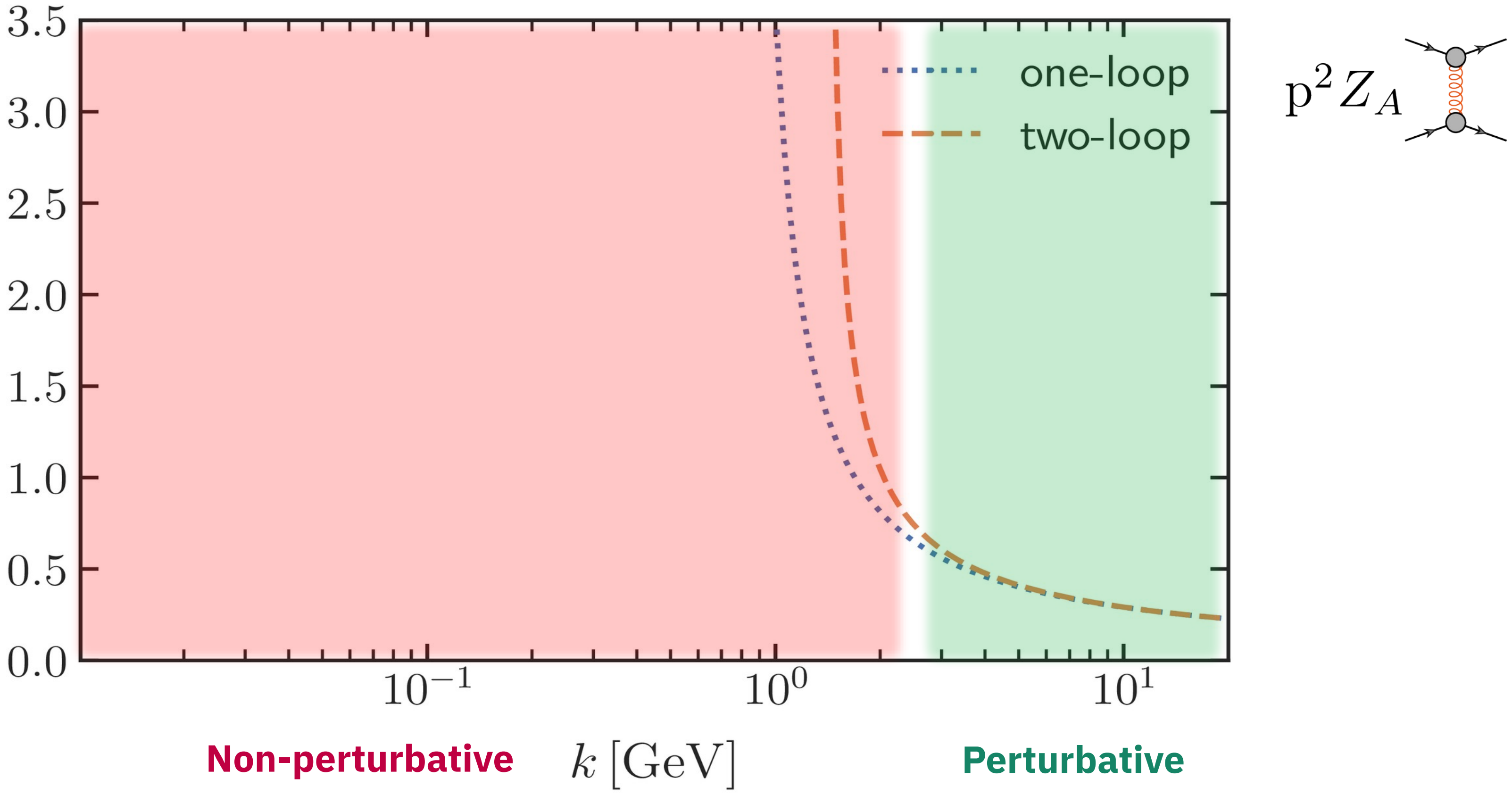

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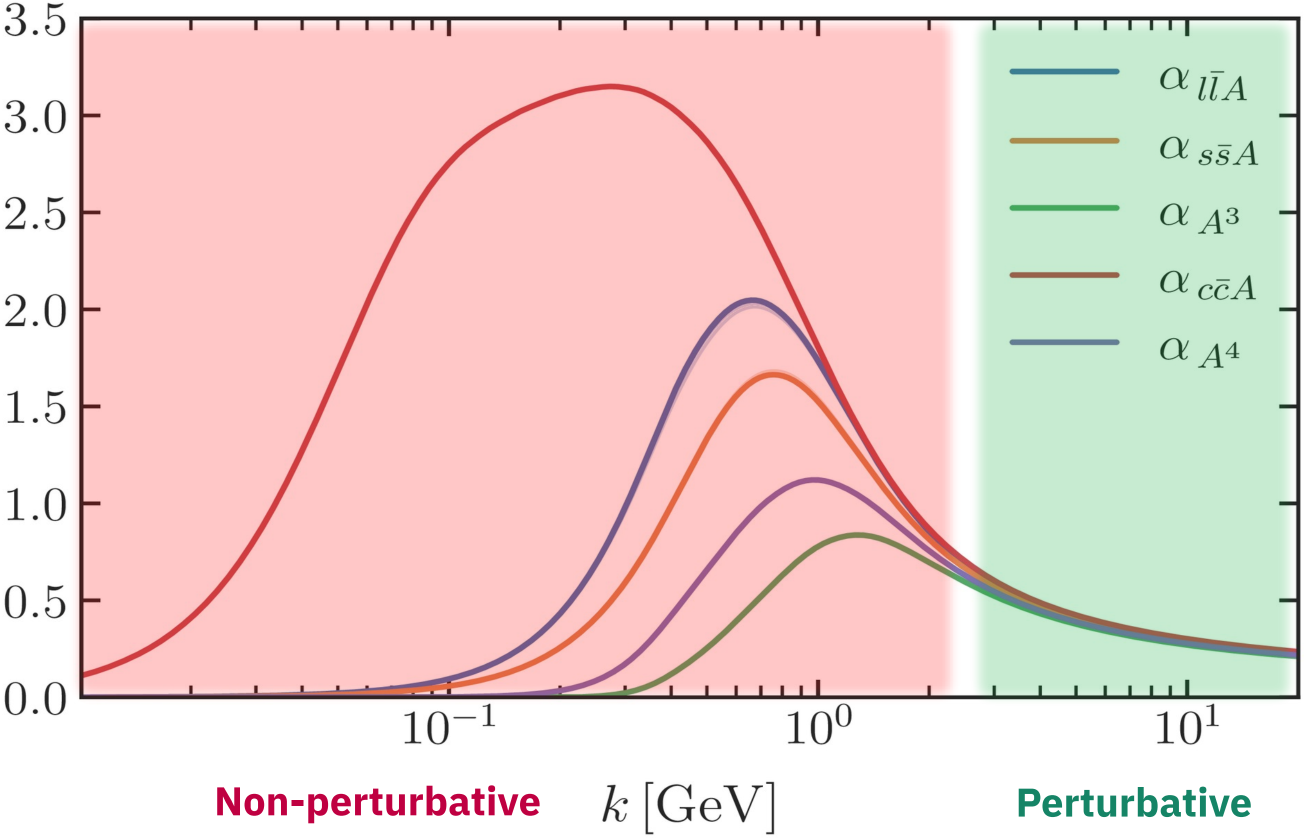

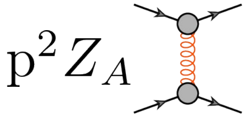

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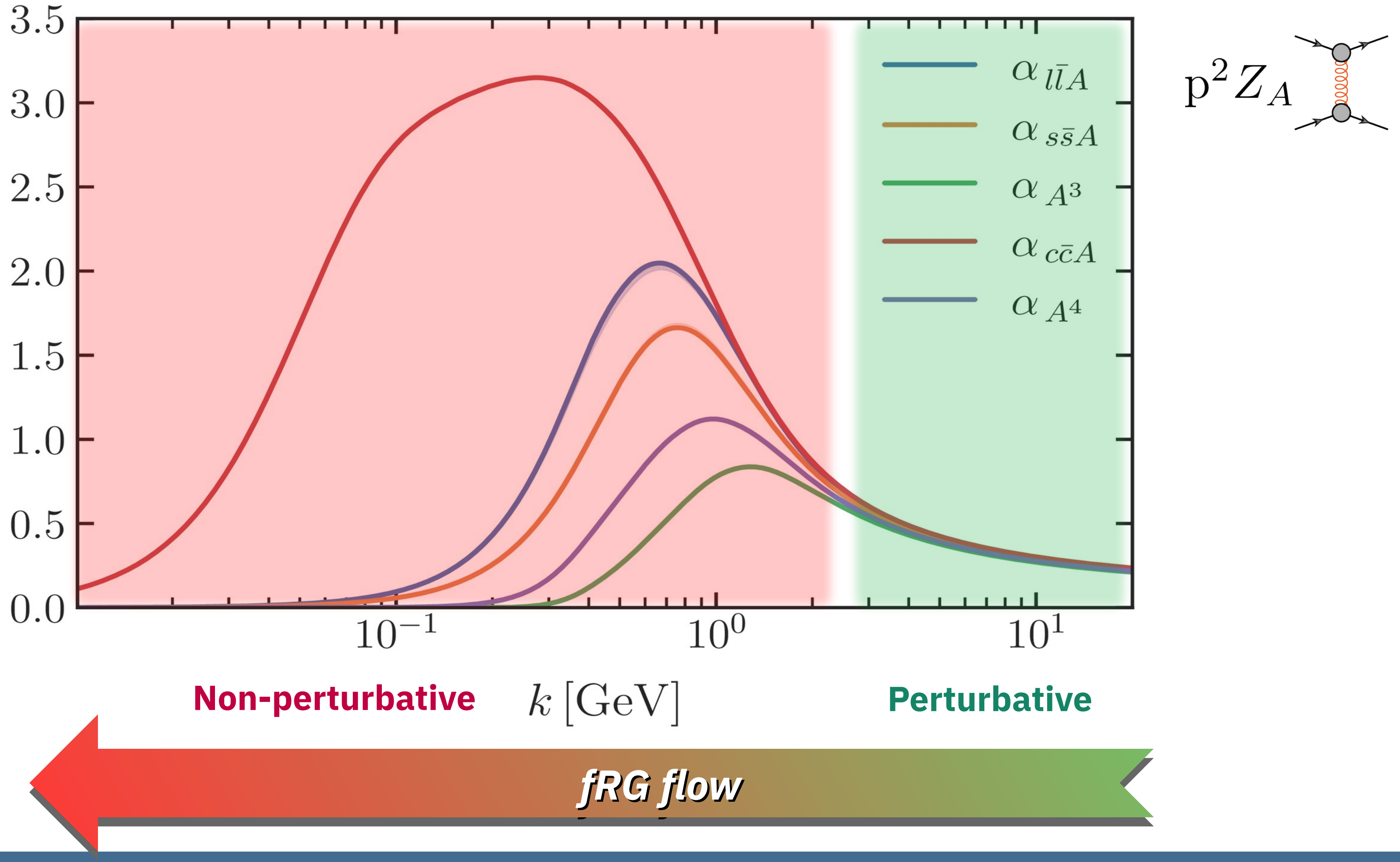
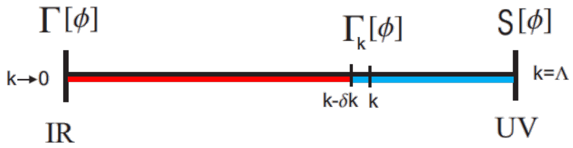


$$Z[J] = \int [D\varphi] e^{-S[\varphi] + \int d^d x J(x) \varphi(x)}$$

Wilsonian approach:

Integrate out momentum shells



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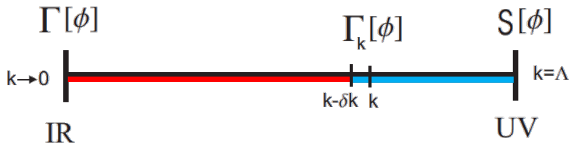

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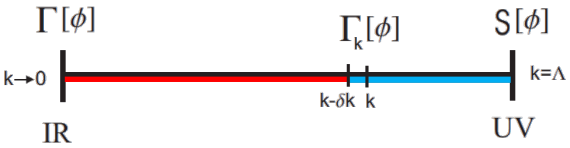

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Introduce mass-like

Regulator

$$[D\varphi]_k = [D\varphi]_{\mathrm{ren}}\, e^{-\frac{1}{2}\int d^dx\, \varphi(x) R_k(x) \varphi(x)}$$

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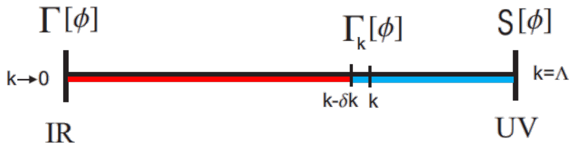
Integrate out momentum shells



$$Z_k[J] = \int [D\varphi]_k e^{-S[\varphi]+\int d^dx J(x)\varphi(x)}$$

Introduce mass-like
Regulator

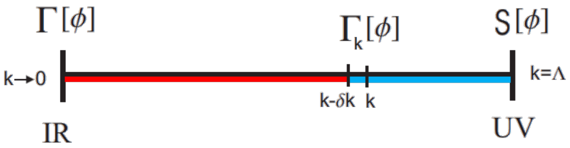
$$[D\varphi]_k = [D\varphi]_{\mathrm{ren}}\, e^{-\frac{1}{2}\int d^dx \varphi(x) R_k(x) \varphi(x)}$$

We can now take derivatives
with respect to k
Obtain differential equation!

$$Z_{\Lambda_{\mathrm{UV}}}[J] \to Z[J]$$

Wilsonian approach:

Integrate out momentum shells



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$$Z_k[J] = \int [D\varphi]_k e^{-S[\varphi] + \int d^dx J(x)\varphi(x)}$$

Simplify the notation a bit: $J_a \varphi^a := \int_x J_i(x)\varphi^i(x)$

$$[D\varphi]_k = [D\varphi]_{\text{ren}}\, e^{-\frac{1}{2}\int d^dx \varphi(x) R_k(x) \varphi(x)}$$

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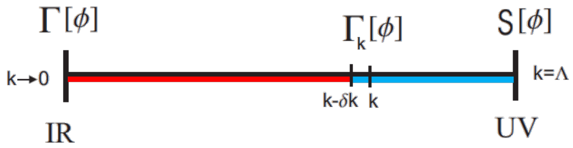

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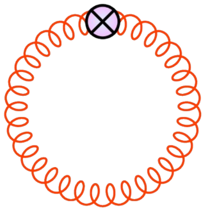
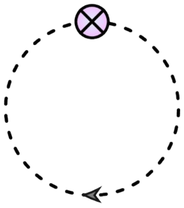
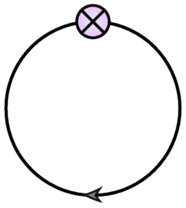
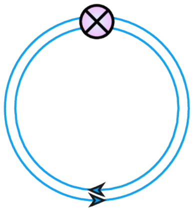
Flow of 1PI effective action (Gibbs free energy):

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} G^{ab} \partial_t R_{k,ab}$$

$= \frac{1}{2}$  $+\frac{1}{2}$  $-$  $+\frac{1}{2}$ 

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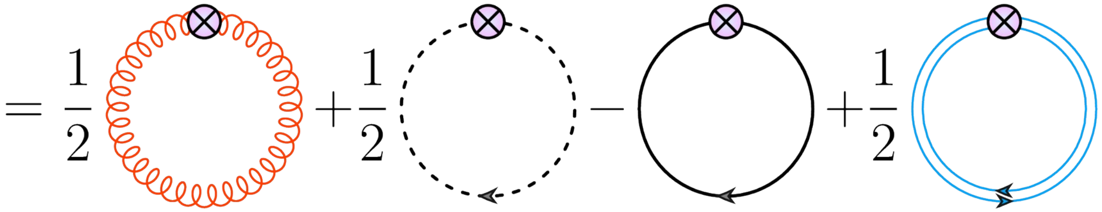

Infinite tower of functional equations

$$\partial_t \Gamma[\bar{\phi}] = \frac{1}{2} \mathrm{Tr}\, G_k\, \partial_t R_k\,,$$

$$\partial_t \Gamma^{(1)}[\bar{\phi}] = -\frac{1}{2} \mathrm{Tr}\, \Gamma_k^{(3)} \left(G_k\, \partial_t R_k\, G_k \right),$$

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$$\partial_t \Gamma^{(3)}[\bar{\phi}] = -\frac{1}{2} \mathrm{Tr}\left[\Gamma_k^{(5)} - 6\,\Gamma_k^{(4)}\, G_k\, \Gamma_k^{(3)} + 6\,\Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)} \right] \left(G_k\, \partial_t R_k\, G_k \right),$$

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$$\vdots \qquad\qquad \vdots$$

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↓

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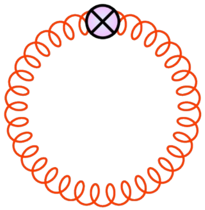
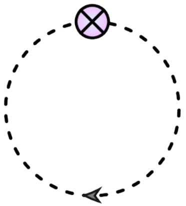
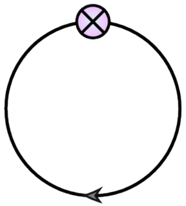
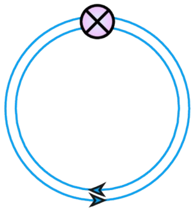
$= \frac{1}{2}$  $+\frac{1}{2}$  $-$  $+\frac{1}{2}$ 

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Quantum Equation of Motion (EoM) $\left.\frac{\delta \Gamma}{\delta \Phi}\right|_{\mathrm{EoM}} = 0$

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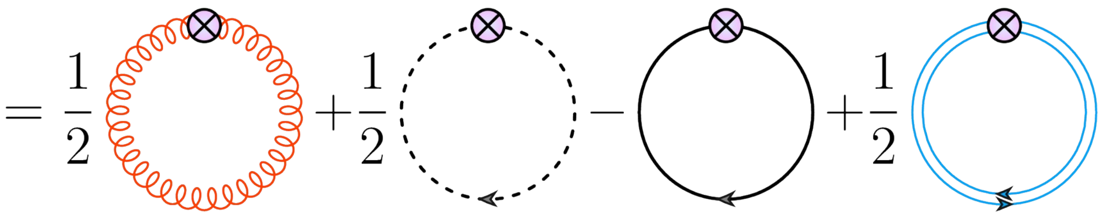

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Quantum Equation of Motion (EoM) $\left.\frac{\delta \Gamma}{\delta \Phi}\right|_{\mathrm{EoM}} = 0$

Choice of truncation

Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2} G^{ab} \partial_t R_{k,ab}$$

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$$\downarrow$$

flowing field reparametrization

$$\dot{\Phi} = \mathcal{O}[\Phi]$$

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flowing field reparametrization

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Generalized Flow equation

Gies, Wetterich [Phys.Rev. D 65 (2002), 065001]
Pawlowski [Annals Phys. 322 (2007) 2831-2915]

$$\left(\partial_t + \dot{\Phi}^a \frac{\delta}{\delta \Phi^a} \right) \Gamma_k = \frac{1}{2} G^{ac} \left(\delta^b_c \partial_t + 2 \frac{\delta \dot{\Phi}^b}{\delta \Phi^c} \right) R_{k,ab}$$

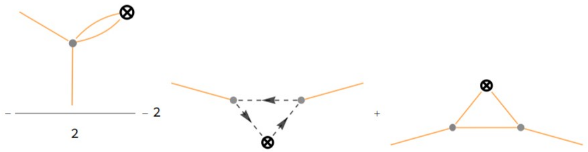
TensorBases

Mathematica package

With J. Braun,
J. Pawlowski, A. Geißel,
N. Wink

- Library of tensor bases, extendable by everyone
- Automatically derived projectors

```
In[24]:= fRGAA = TakeDerivatives[WetterichEquation, {A[i1], A[i2]}] // FTruncate // FSimplify[#, "Symmetries" → symmetryP2] & // FPlot // FRoute // FPrint;
traceExprAA = F[TBGetProjector["AA", 1, {i1, i2} /. fRGAA["1-Loop"]["ExternalIndices"]], (fRGAA["1-Loop"]["Expression"] /. MakeDiagrammaticRules[])]
FlowAA = FormTrace[traceExprAA] // dressingRules // FORMSimplify // PropParam;
MakeCppFunction[FlowAA["1-Loop"], "Name" → "FlowAA", "Parameters" → {"l1", "p", "cos", "gS", "ZA", "Zc", "ZA3", "ZccbA"}]
```



$$\int_{l_1}\left(-2\,G_{\bar{c}^a c^b}(l_1,-l_1)\,\Gamma_{c^b\bar{c}^d A^m}(l_1,p_1-l_1,-p_1)\,G_{\bar{c}^d c^f}(l_1-p_1,p_1-l_1)\,\Gamma_{c^f\bar{c}^h A^n}(l_1-p_1,-l_1,p_1)\,G_{\bar{c}^h c^j}(l_1,-l_1)\,\partial_t R_{c^j\bar{c}^a}(l_1,-l_1)\right)$$
$$+\int_{l_1} G_{A^aA^b}(-l_1,l_1)\,\Gamma_{A^cA^bA^m}(l_1+p_1,-l_1,-p_1)\,G_{A^cA^f}(-l_1-p_1,l_1+p_1)\,\Gamma_{A^gA^fA^n}(l_1,-l_1-p_1,p_1)\,G_{A^gA^j}(-l_1,l_1)\,\partial_t R_{A^jA^a}(-l_1,l_1)$$
$$+\int_{l_1}\left(-\frac{1}{2}\,G_{A^aA^b}(-l_1,l_1)\,\Gamma_{A^cA^bA^iA^j}(l_1,-l_1,-p_1,p_1)\,G_{A^cA^f}(-l_1,l_1)\,\partial_t R_{A^fA^a}(-l_1,l_1)\right)$$

```
Out[27]= auto FlowAA(const auto &l1, const auto &p, const auto &cos, const auto &gS, const auto &ZA, const auto &Zc,
                     const auto &ZA3, const auto &ZccbA)
{
  return -4. *
         (dtZA(pow(1. + powr<6>(k), 0.16666666666666666667)) *
```

FunKit

Mathematica package

DiFfRG framework

C++ library

- Code generation for large fRG systems
- Hydrodynamic methods for full field dependences
- GPU accelerated

Our code is open Source, see github.com/satfra

QCD at finite densitites

Gauge consistency

Slavnov-**T**aylor **I**dentities (STIs) encode the *gauge consistency* in a gauge-fixed setting.

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In terms of BRST transformations $\mathfrak{s}A^a_\mu = D^{ab}_\mu c^b\,, \quad \mathfrak{s}q = -gc^a t^a q\,, \quad \mathfrak{s}c^a = -\frac{g}{2} f^{abc} c^b c^c\,, \quad \mathfrak{s}\bar{c}^a = \frac{1}{\xi}\mathcal{F}^a[A]\,.$

$$\int_x \langle J_A \cdot \mathfrak{s}A - J_c \cdot \mathfrak{s}c - J_{\bar{c}} \cdot \mathfrak{s}\bar{c}\rangle = 0\,.$$

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Adding a regulator leads to modifications ***at k > 0***.

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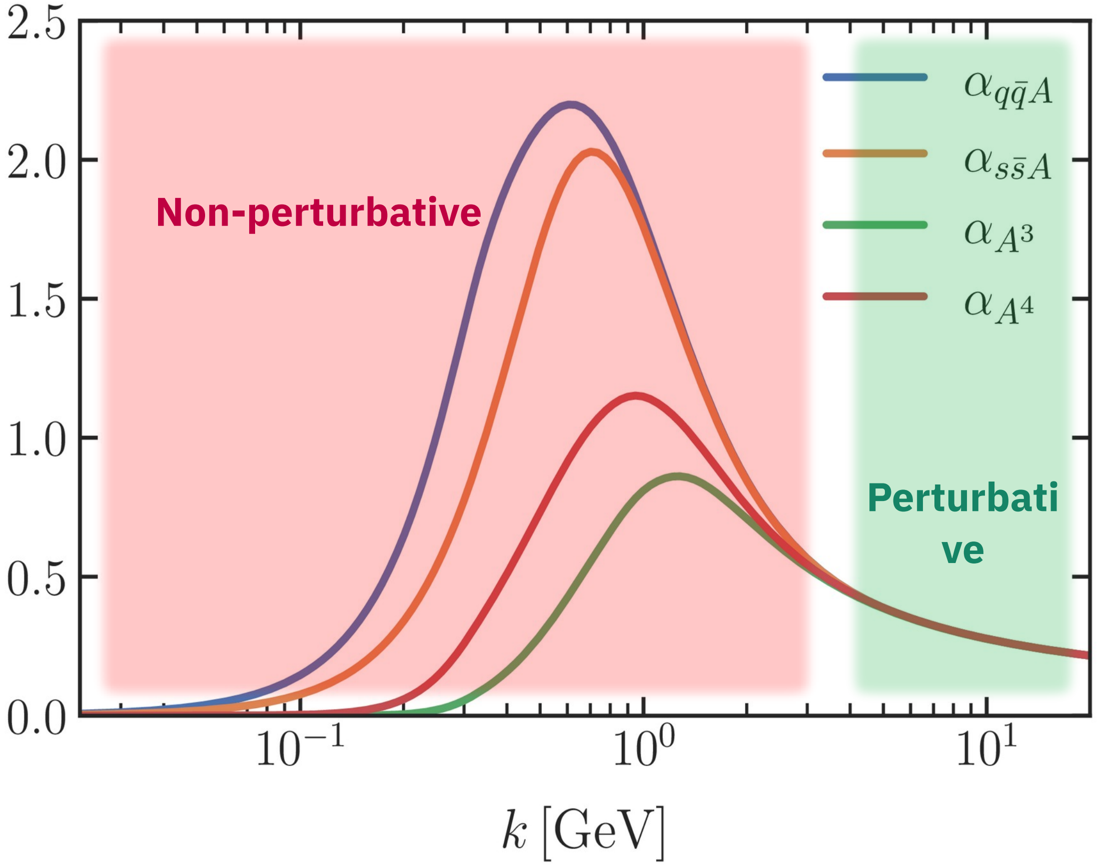
In the perturbative regime, the STIs lead at **k=0** to

$$\alpha_{q\bar{q}A}(\bar{p}) = \alpha_{c\bar{c}A}(\bar{p}) = \alpha_{A^3}(\bar{p}) = \alpha_{A^4}(\bar{p})\,.$$

Gauge consistency



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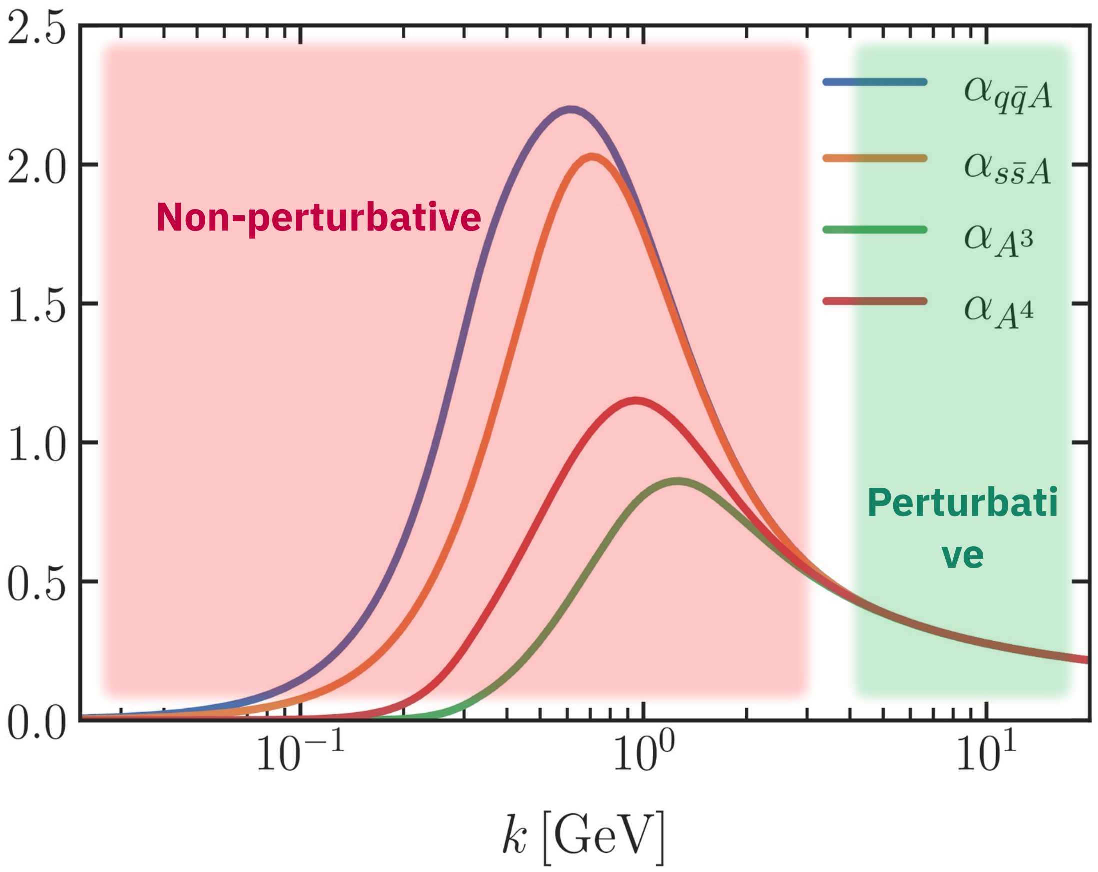
Gauge consistency

We make sure to fulfil perturbative STIs, which already leads to excellent results for the full STIs, see e.g.

Bonini, D'Attanasio, Marchesini, Nucl.Phys.B 437 (1995) 163-186
Bonini, D'Attanasio, Marchesini, Phys.Lett.B 346 (1995) 87-93
Ellwanger, Hirsch, Weber, Z.Phys.C 69 (1996) 687-698,
…….
Fischer, Maas, Pawlowski, Annals Phys. 324 (2009) 2408-2437
Cyrol, Fister, Mitter, Pawlowski, Strodthoff, Phys.Rev.D 94 (2016) 5, 054005
Pawlowski, Schneider, Wink, arXiv:2202.11123 [hep-th]

Particularily important: *Gauge-consistency of the quark-propagator*

With Kockler, Pawlowski



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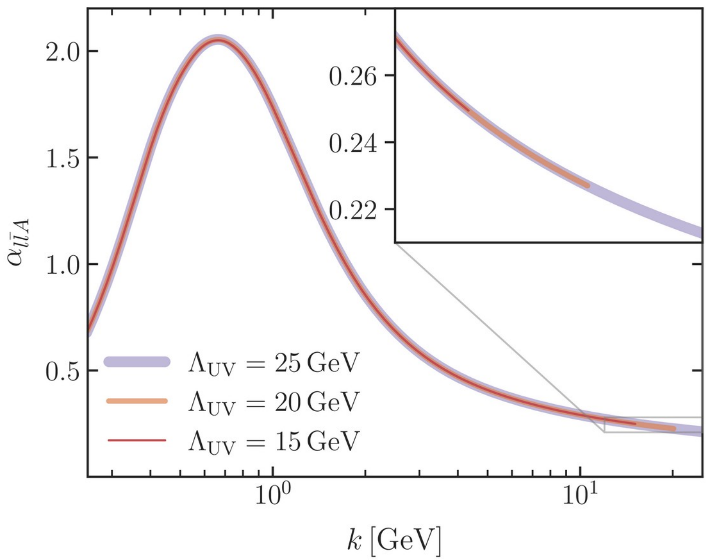
Do we fulfil perturbation theory?



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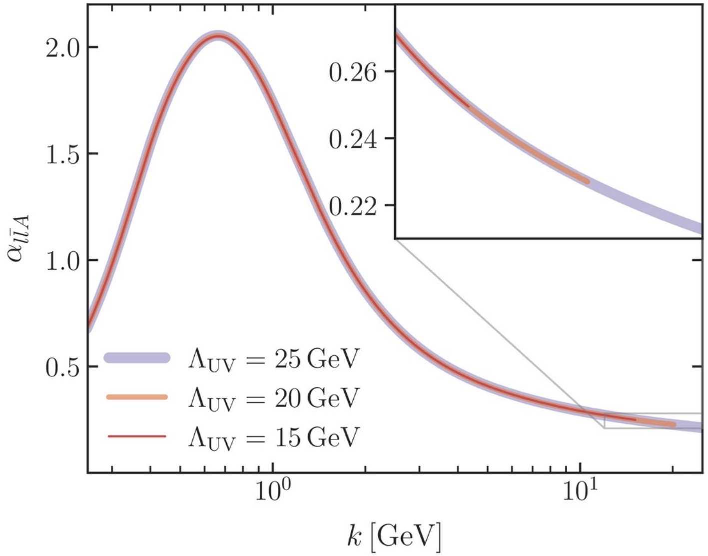

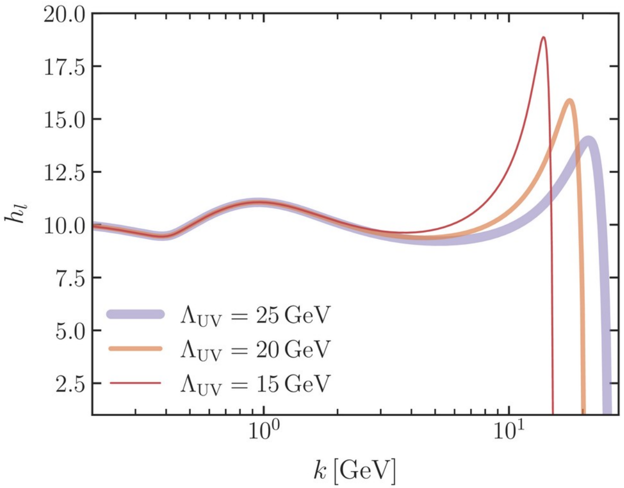

- Start of rebosonisation does not influence matter:
  fixed Point

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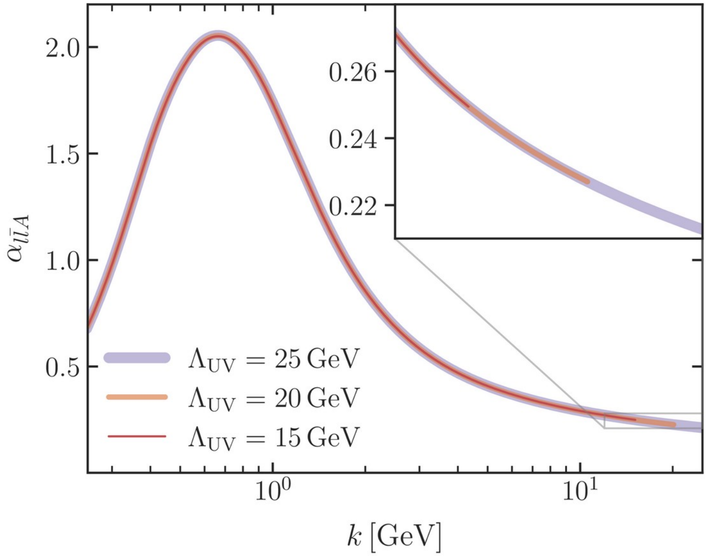

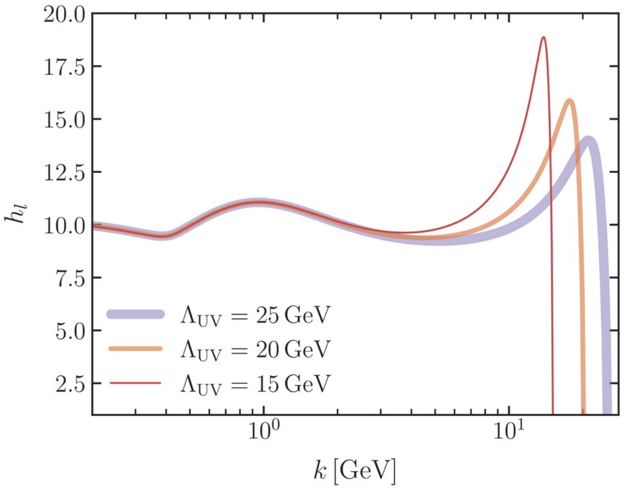

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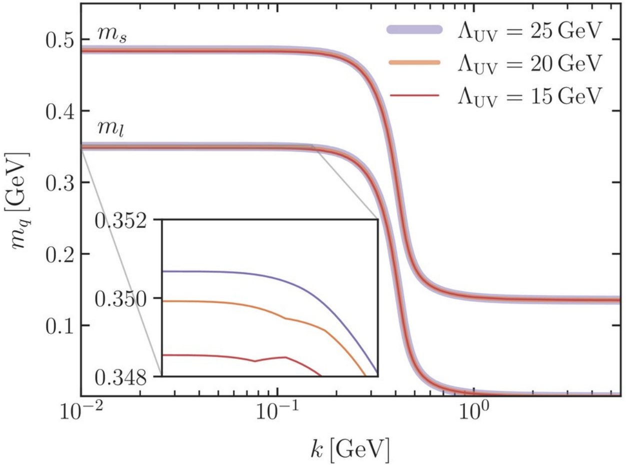
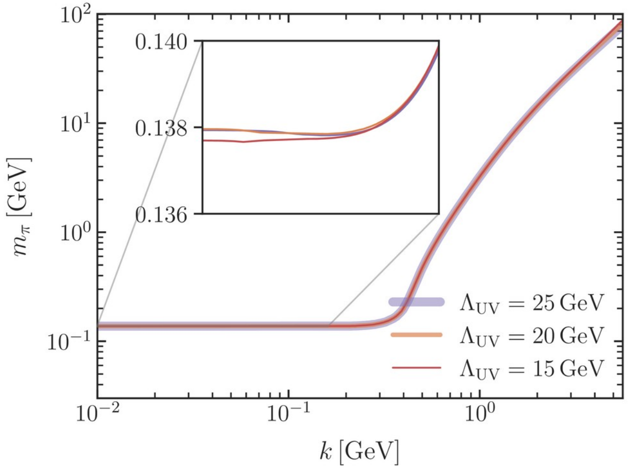
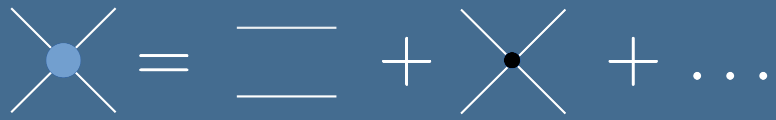


- Low-energy quantities:
  Error due to initial cutoff
  ~ 1MeV



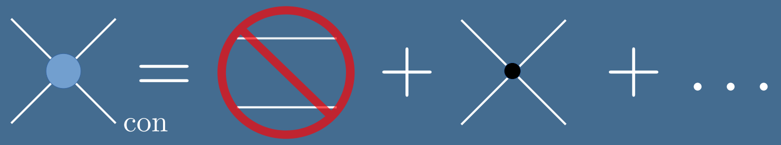
$$Z_k[J]$$



Schwinger functional

Connected diagrams

$$W_k[J] = \log Z_k[J]$$



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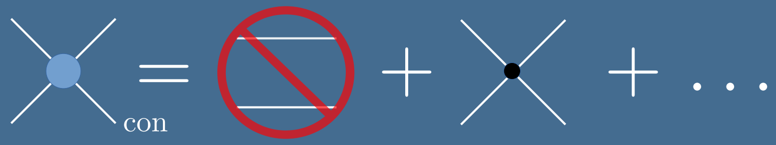

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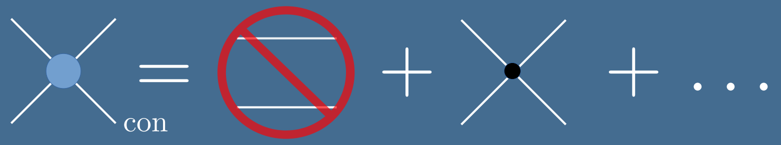

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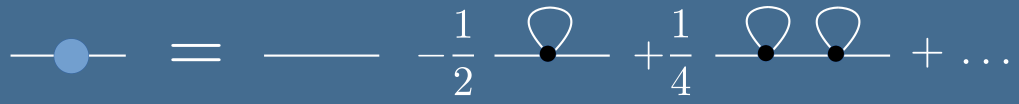
$G =$



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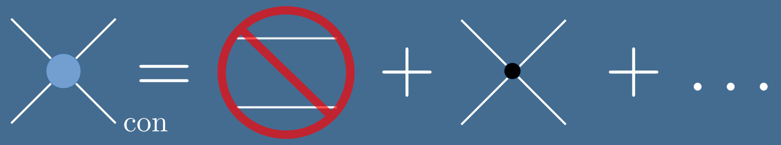
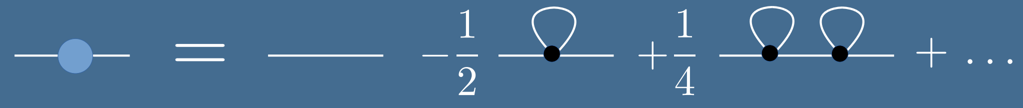


Polchinski equation

$$k\partial_k W_k[\Phi] = \frac{1}{2}\Big(G_k^{ab} + \Phi^a \Phi^b\Big) k\partial_k R_{k,ab}$$

$G =$



Schwinger functional

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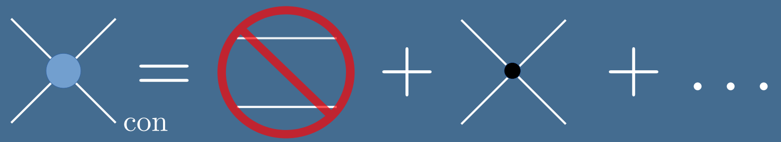

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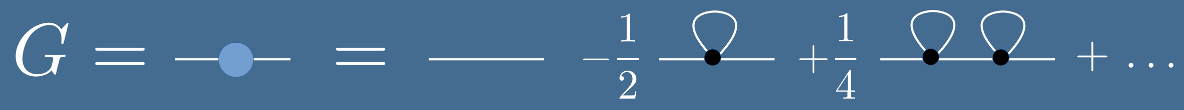

Legendre Transf.

$$\Phi = \langle \varphi \rangle_W$$

1PI effective action (Gibbs Free Energy)

1PI diagrams

$$\Gamma_k[\Phi] = \max_J \left(J_a \Phi^a - W[J]\right) - \frac{1}{2}\Phi^a R_{k,ab} \Phi^b$$

Schwinger functional

Connected diagrams

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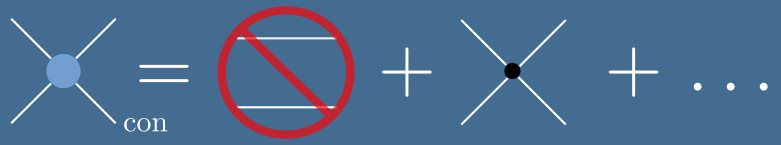

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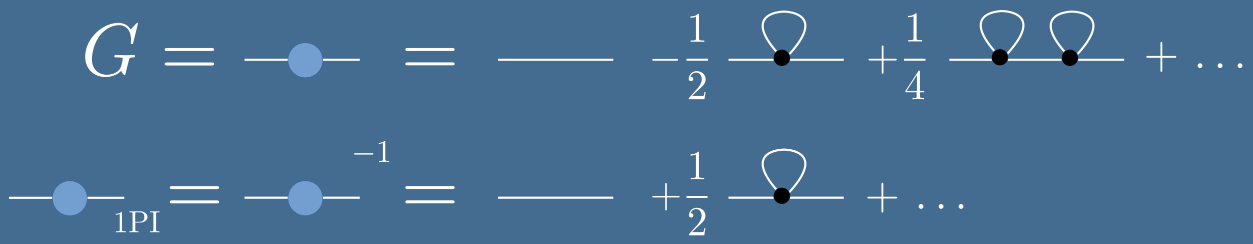

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Wetterich equation

Wetterich [Phys.Lett.B 301 (1993) 90-94]

$$\partial_t\Gamma_k[\Phi] = \frac{1}{2}G^{ab}\partial_t R_{k,ab}$$

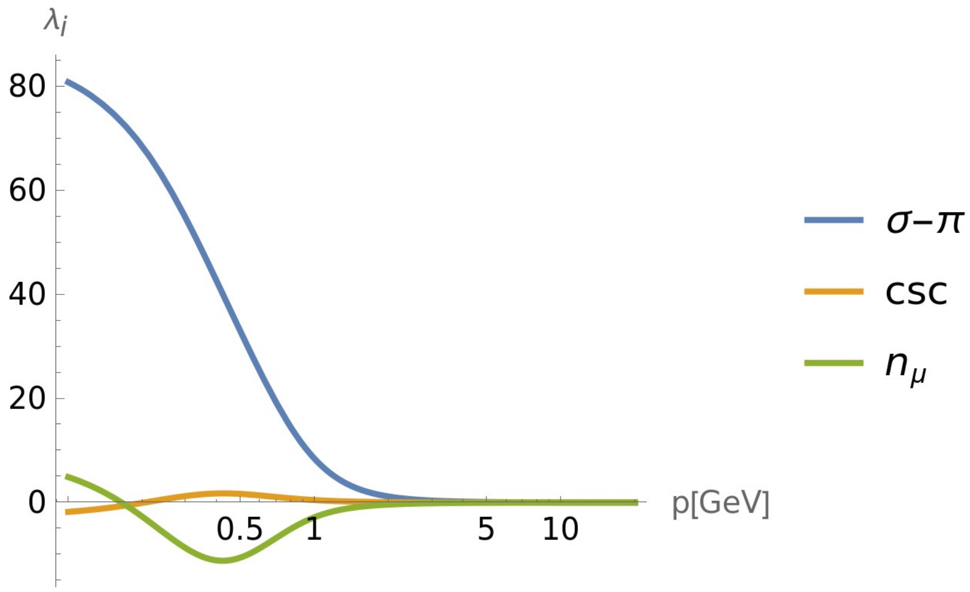
$$t = \ln\left(\frac{k}{\Lambda_{\mathrm{UV}}}\right)$$

What about the other channels?

Clear hierarchy known:

- Scalar-Pseudoscalar (σ-π)

- Diquarks (csc)
- Density channel (n_μ)



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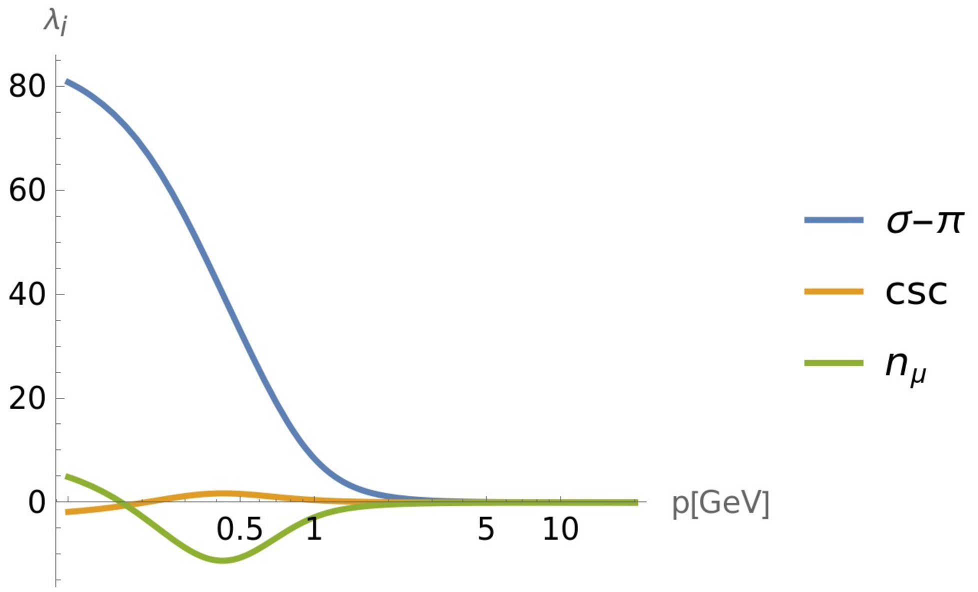

Optimization of bases:

J. Braun, A. Geißel, J. Pawlowski, F.R. Sattler, N. Wink
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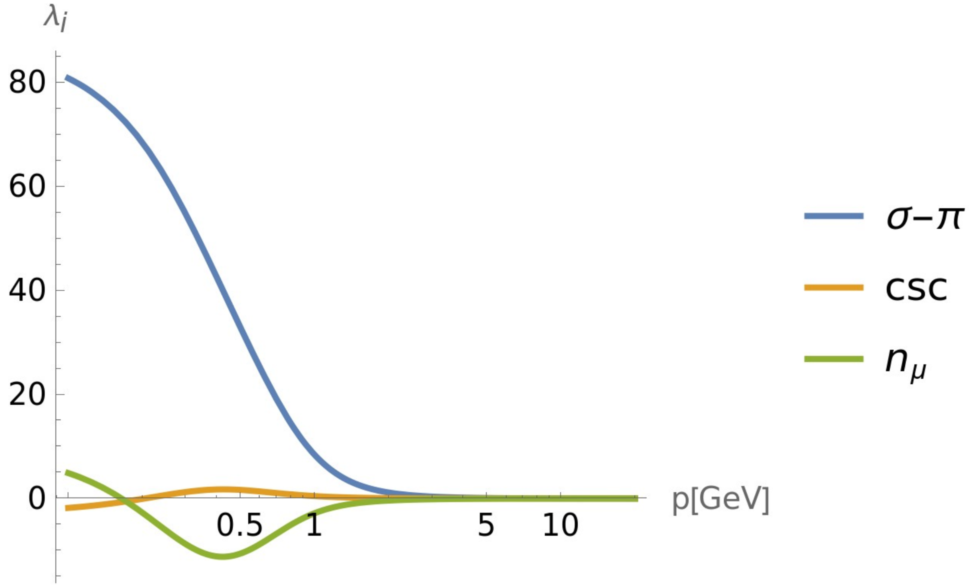

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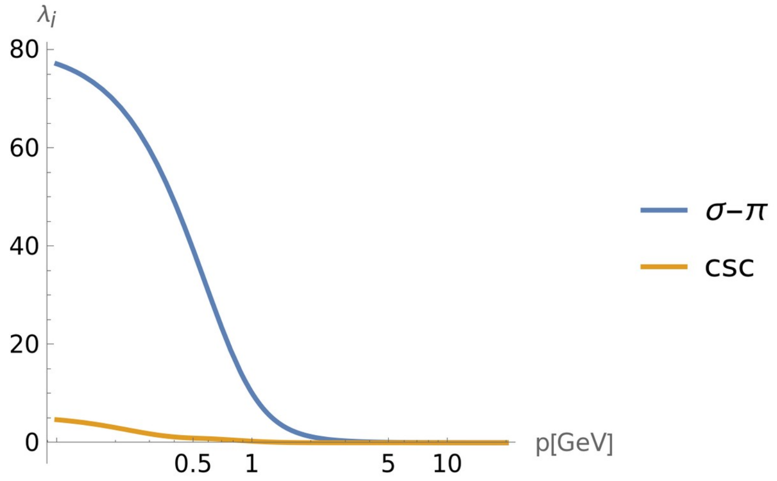

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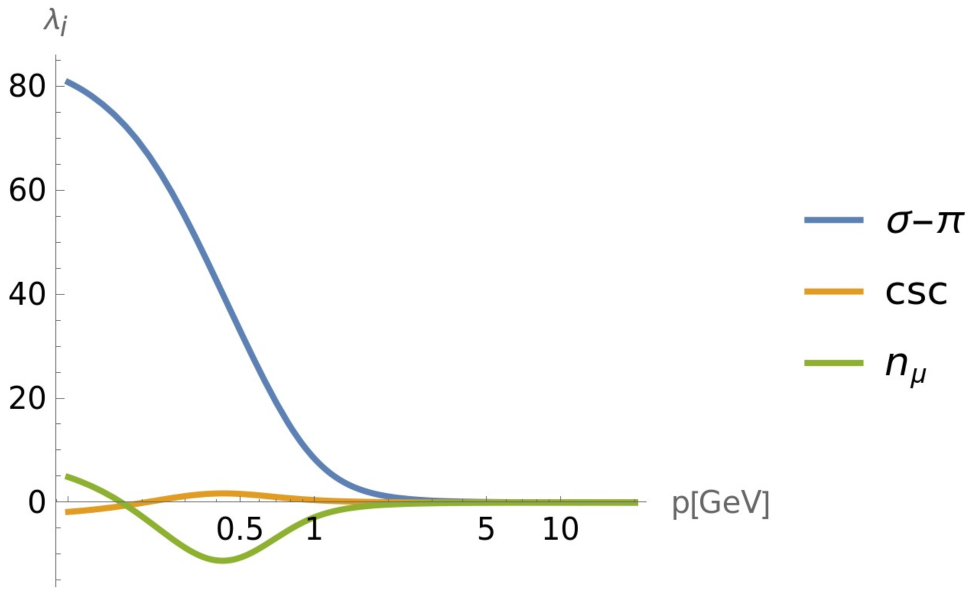

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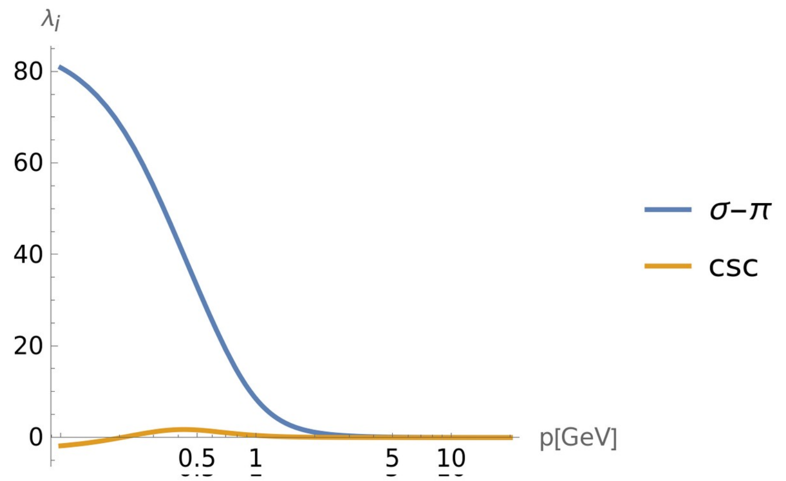

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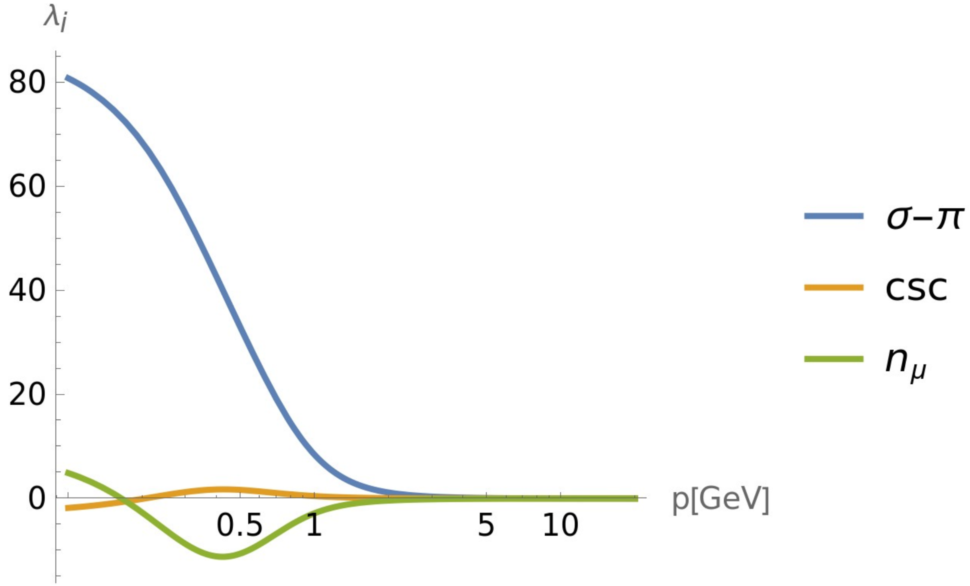

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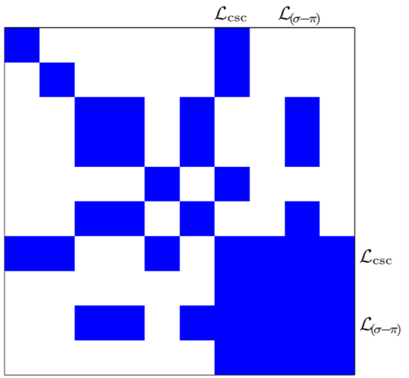
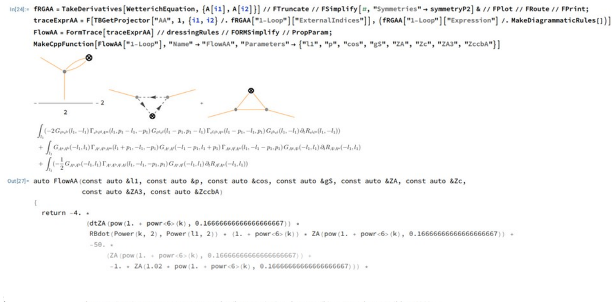


- Diagrammatically based block-diagonalization of bases.
- Removal of projection singularities.
- Mathematica package for typical tasks.



Grossi, Wink [SciPost Phys.Core 6 (2023) 071]
Koenigstein, Steil, Wink, Grossi, Braun
[Rev.D 106 (2022) 6, 065012]
Ihssen, Sattler, Wink
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[Comput.Phys.Commun. 300 (2024) 109182]
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[Phys.Rev.D 111 (2025) 3, 036030]

- Flow equation does not depend on lower n-point functions

Hydrodynamic approach to fRG

$$k\partial_k \Gamma^{(n)} = \mathrm{Flow}[\Gamma^{(2)}, \ldots, \Gamma^{(n+2)}]$$

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- Full information contained in curvature mass of pion

$$\rho = \frac{\pi^2 + \sigma^2}{2}$$

$$u(\rho) = \partial_\rho V_{\mathrm{eff}}(\rho) = m^2_{\pi,\mathrm{curv}}$$

$$\delta^{(4)}(0)\, V_{\mathrm{eff}}(\rho) \sim \Gamma[\Phi]\big|_{p=0}$$

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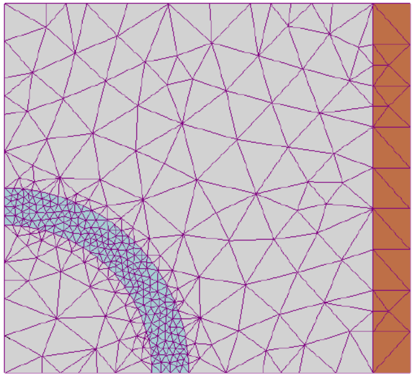
- Convection-Diffusion equation with sources(sinks)

$$k\partial_k u(\rho) = \partial_\rho F_k(u, \partial_\rho u, \rho)$$

Finite element methods for couplings

1. Put it on a Mesh



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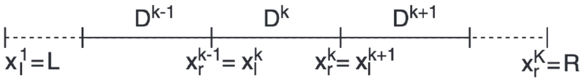

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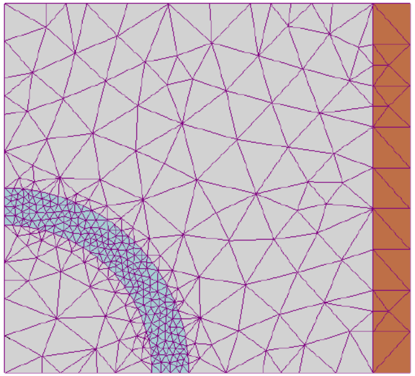

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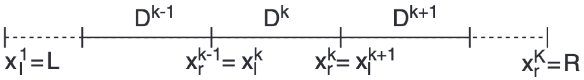


2. Choose basis functions

$$u_h(x,t)\Big|_{D_k} = \sum_{n}^{N} u_n(t)\psi_n(x)$$

+ sensible time integration

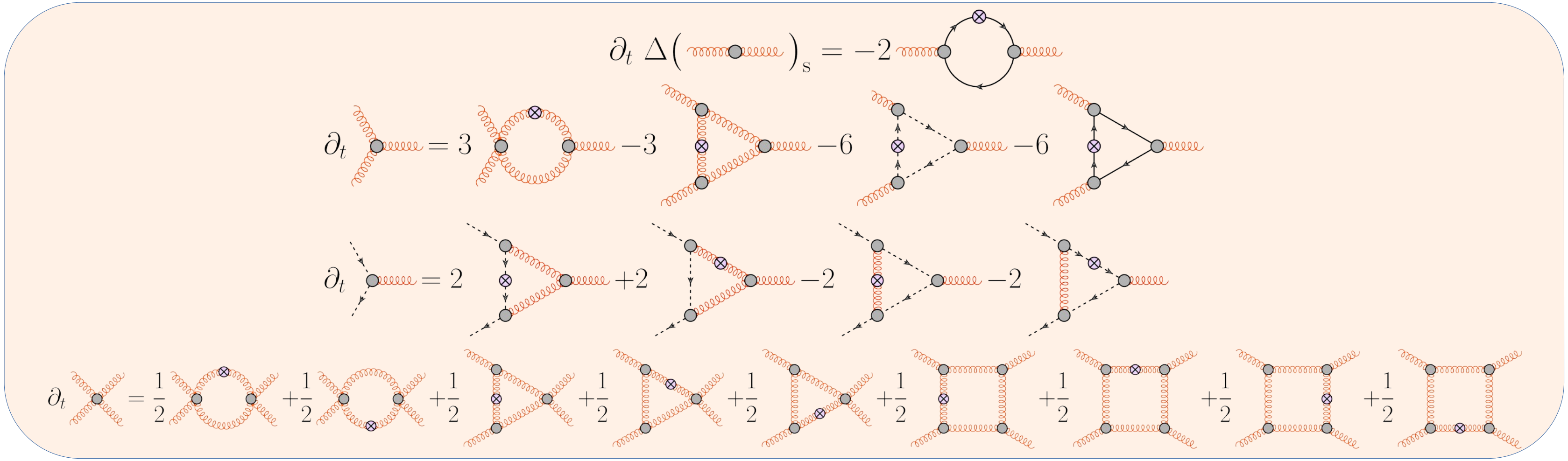
Sattler, Ihssen, Wink, [Phys.Rev.D 107 (2023) 11, 114009]

3. Discretise the PDE

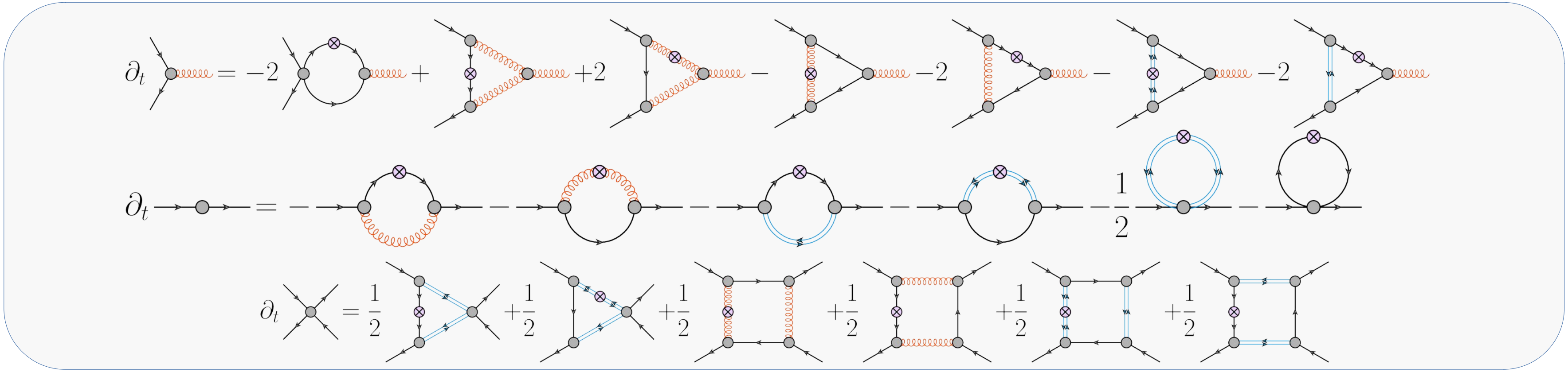
$$\partial_t u + \partial_x F(u) = 0$$

$$\int_{D^k} \left((\partial_t u_h^k)\phi_n - F(u_h^k)\partial_x \phi_n \right) \mathrm{d}x = -\int_{\partial D^k} F^* \cdot \hat{n}\, \phi_n \, \mathrm{d}x$$

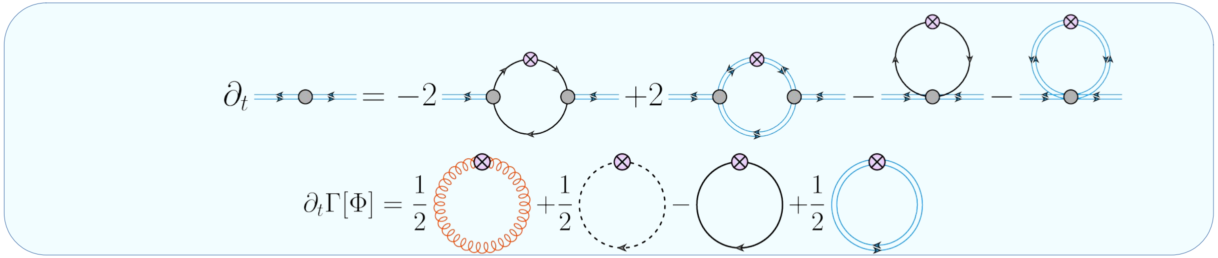
Gluons and Ghosts



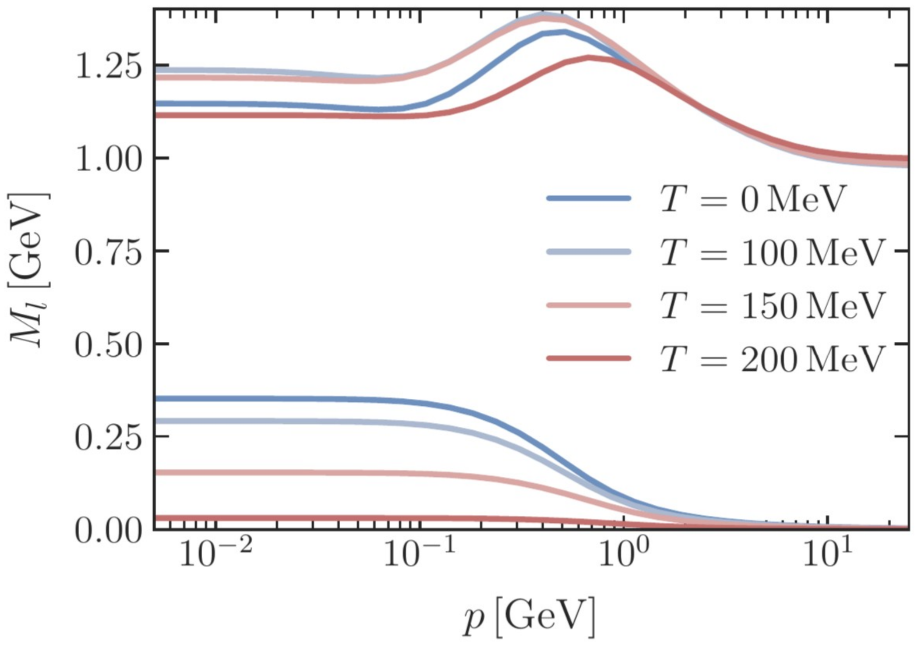
Quarks

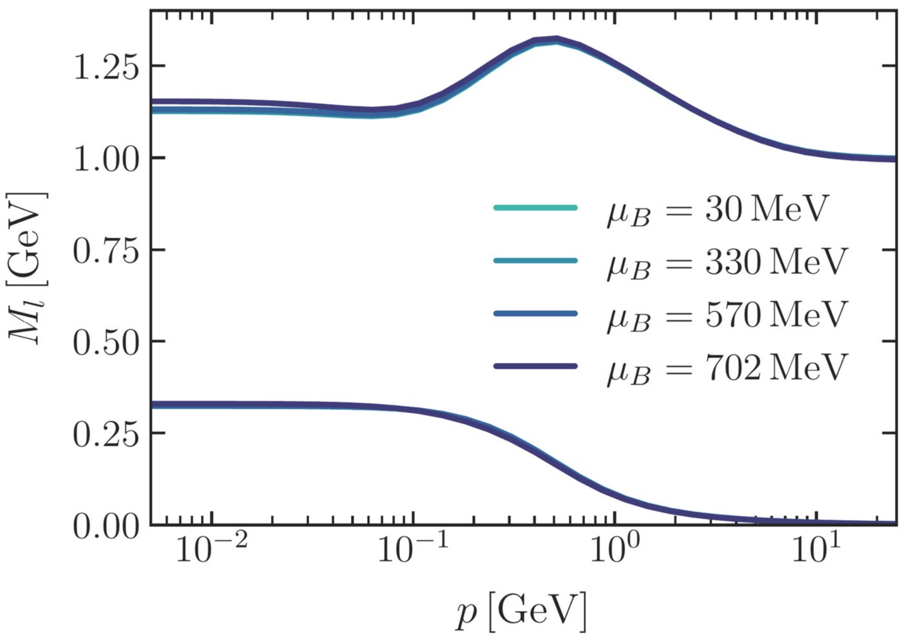


Mesons



QCD at finite densitites

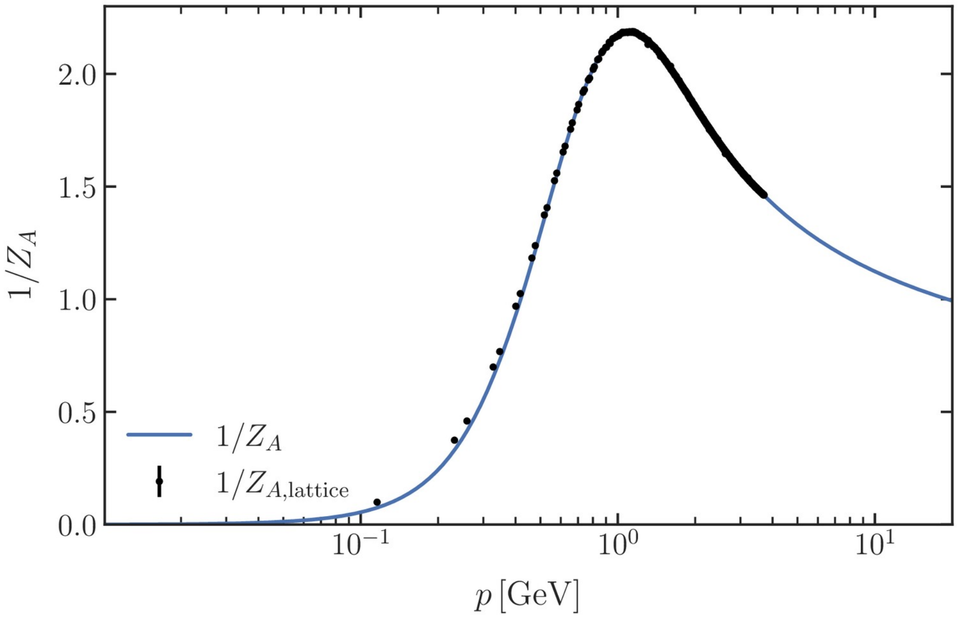




Momentum dependence of quark propagator

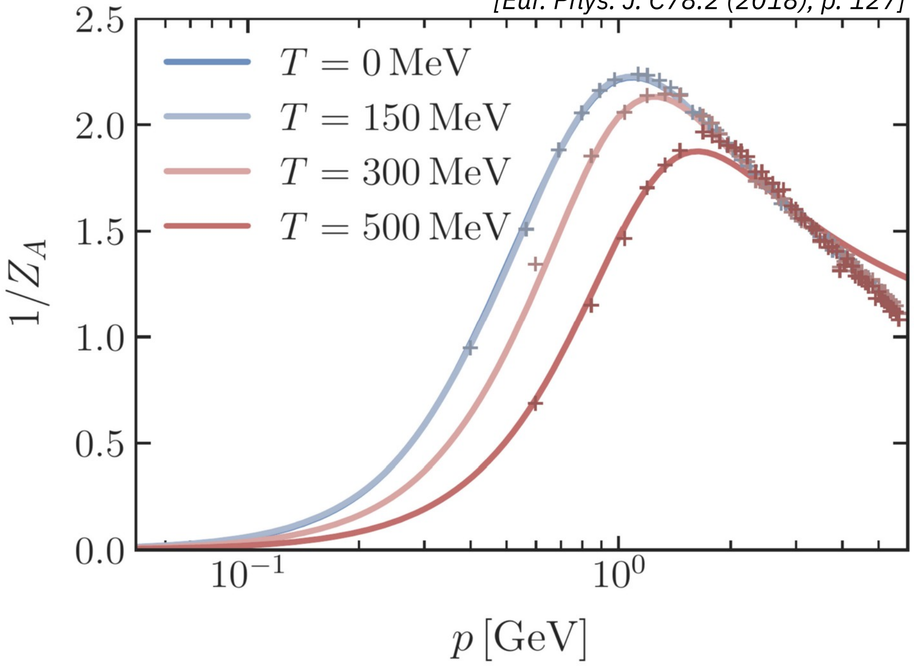
Silver Blaze symmetry

Lattice data from Boucaud et al.
[Phys.Rev.D 98 (2018) 11, 114515]



Vacuum: fRG and lattice results fit except in deep infrared (solution branches)

Lattice data from Ilgenfritz et al
[Eur. Phys. J. C78.2 (2018), p. 127]



Temperature dependence of Gluon propagator fits lattice calculations well.

$$k\partial_k \Gamma_k[\Phi] = \frac{1}{2}\int_p G^{(2)}_{ab}[\Phi](p)\, k\partial_k R_k^{ab}(p)$$

$$k\partial_k \Gamma_k[\Phi] = \frac{1}{2}\int_p \frac{1}{\Gamma^{(2)}_{ab} + R_k^{ab}}(p)\, k\partial_k R_k^{ab}(p)$$

(i) Suppression of IR modes

$$\lim_{p^2 \to 0} R_k(p^2) > 0$$

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(iii) UV limit

$$\lim_{k \to \Lambda \to \infty} R_k(p^2) = \infty\,.$$

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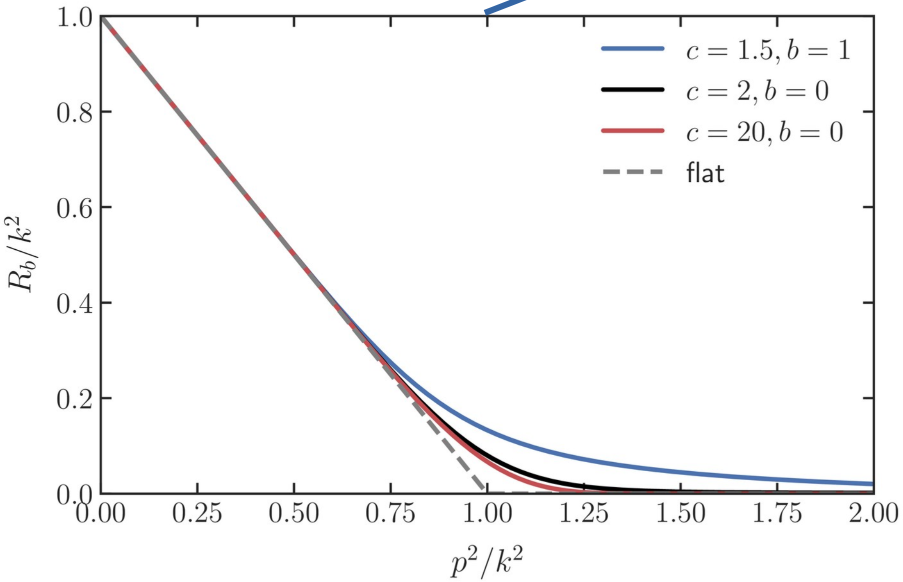

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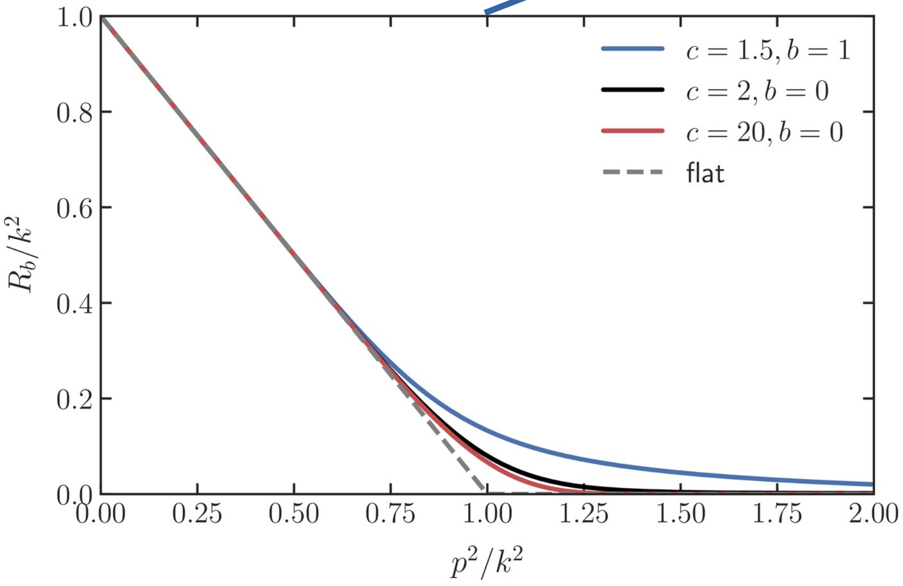

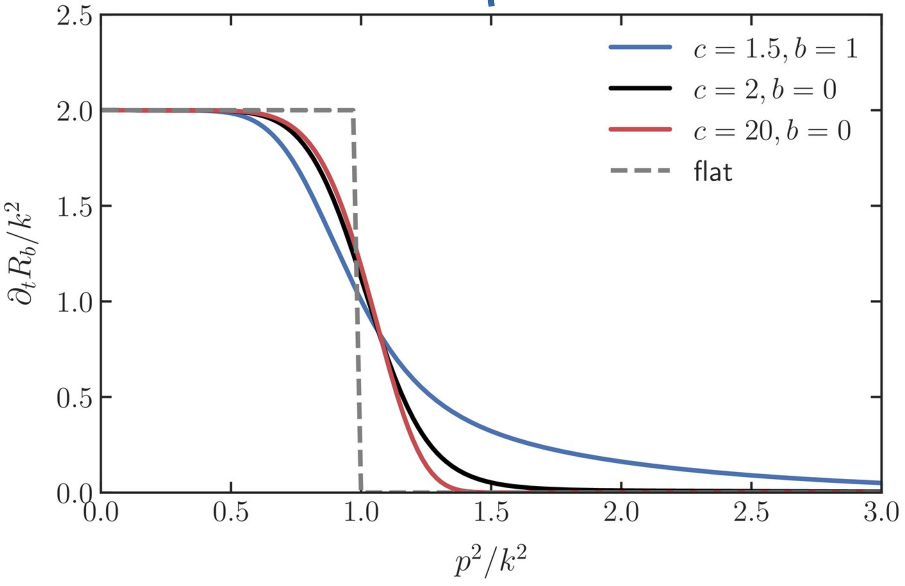

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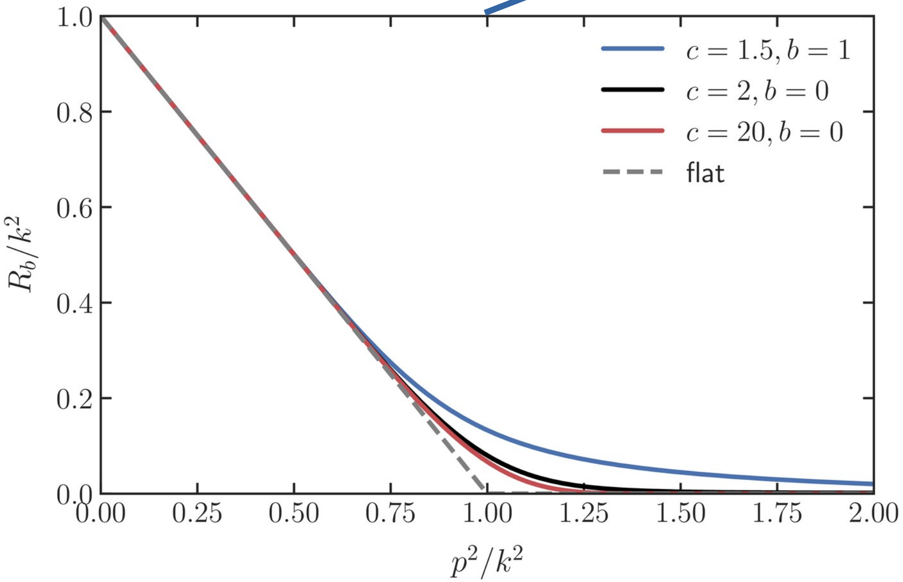
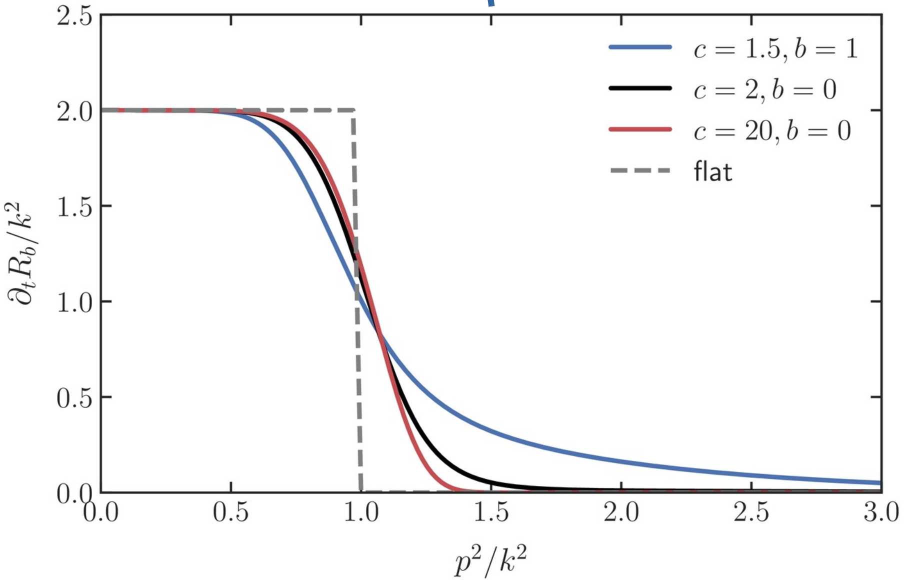
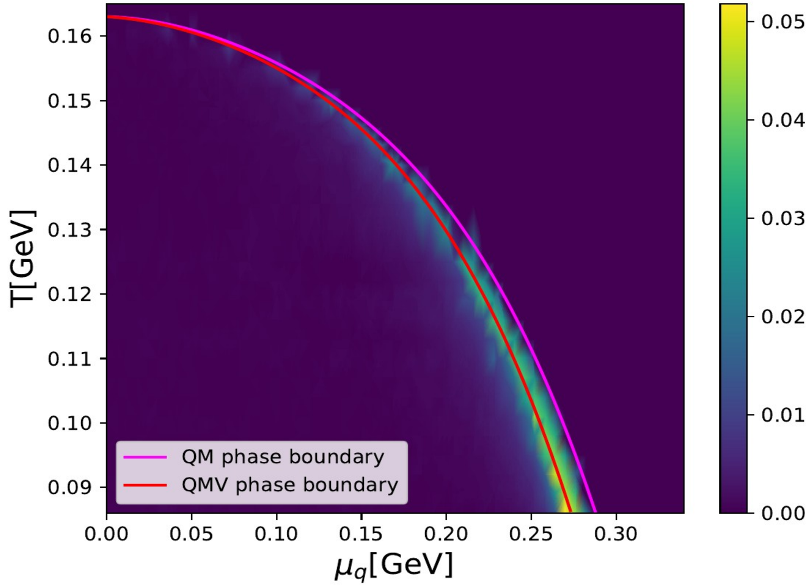
Optimized regulators for symmetry breaking

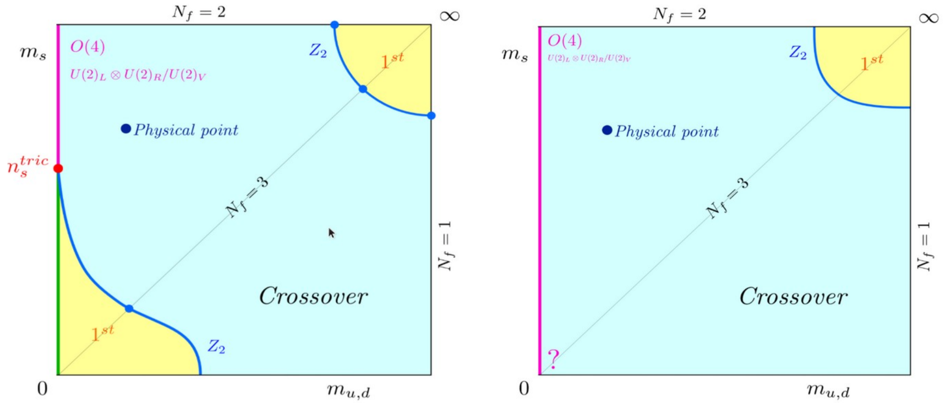




Inclusion of Density mode $\langle \bar{q}\gamma_0 q \rangle$ and Diquarks

Ihssen, Hendricks, Pawlowski, Sattler (in preparation)





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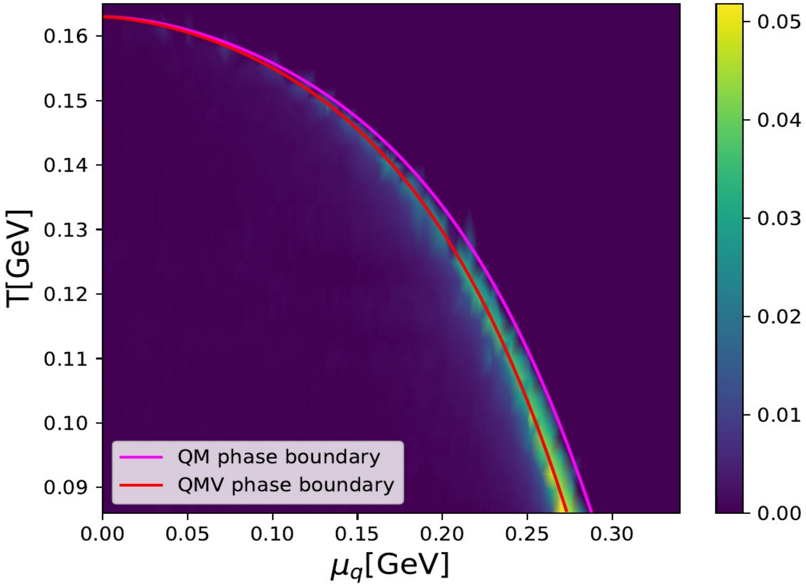

Full Nf = 2+1

Pawlowski, Sattler, Steck (in preparation)

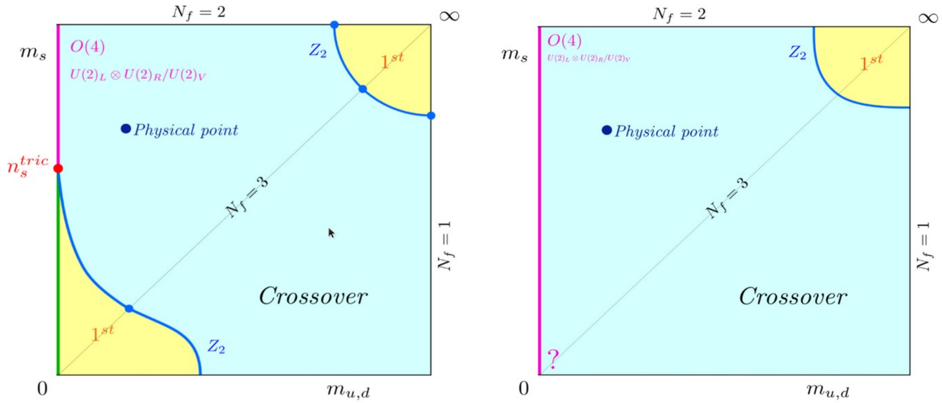

Figure from Owe Philipsen (arXiv:2111.03590)

Observables	Value	Parameter in $\Gamma_{\Lambda_{\mathrm{UV}}}$
$m_{\pi,\mathrm{pol}}$ [MeV]	138(9)	$c_{\sigma_l} = 4.67\,\mathrm{GeV}^3$
f_K/f_π	1.1914	$\Delta m_{sl} = 134.2\,\mathrm{MeV}$
$\alpha_{l\bar{l}A,\Lambda_{\mathrm{UV}}}$		$\alpha_{l\bar{l}A,\Lambda_{\mathrm{UV}}} = 0.227$
m_l [MeV]	350	$a = 0.0251 \quad b = 2\,\mathrm{GeV}$
f_π [MeV]	$97.2^{+4.0}_{-2.2}$	———
m_s [MeV]	$485.0^{+0.0}_{-0.3}$	———
$m_{\pi,\mathrm{cur}}$ [MeV]	138	———
m_σ [MeV]	$388.1^{+0.0}_{-1.1}$	———
$\sigma_{0,l}$ [MeV]	$69.^{+1.2}_{-0.2}$	———

Results on physical point of QCD