Chiral phase transition under rotation and acceleration

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Content

- Introduction
  Why acceleration and rotation are interesting for quark matter study

- Rotational effect on chiral condensate
  Models versus lattice results

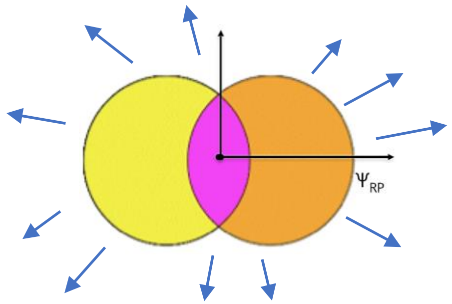
- Acceleration effects on chiral condensate
  Models versus lattice results

- Summary

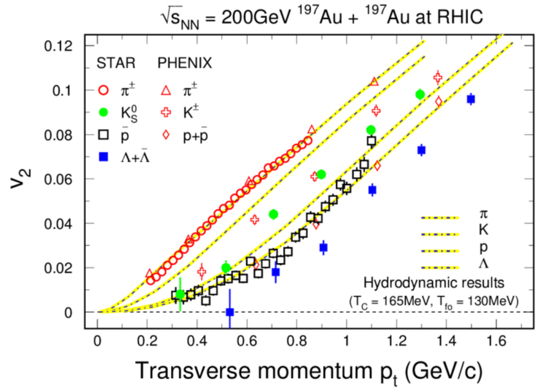
Introduction

Where is accelerating and rotating QCD matter

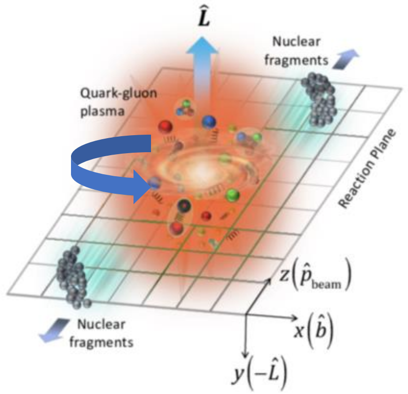
- A typical example is quark gluon matter in heavy-ion collisions



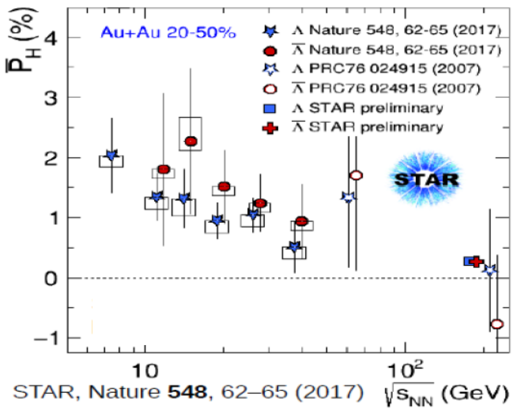
$$a = \frac{dv}{dt} = -\frac{1}{\varepsilon + P}\nabla P$$



Elliptic flow due to fluid acceleration



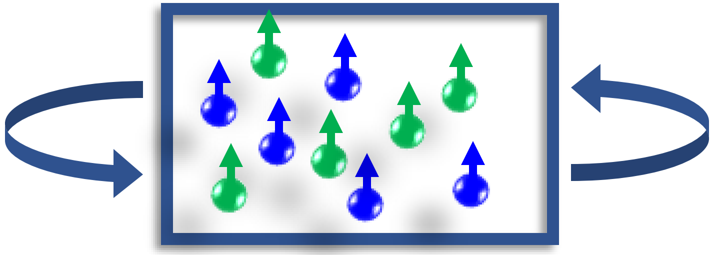
$$\Omega = \frac{1}{2}\nabla \times v$$



Spin polarization due to fluid rotation (vorticity)

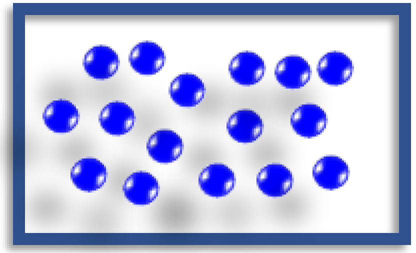
Effect of rotation: Comparison with chemical potential

- Hints for possible rotation effect: comparison with chemical potential



Rotation

$$H = H_0 - \Omega J_z$$



Chemical potential

$$H = H_0 - \mu N$$

For massless Dirac fermions

$$P = \frac{7\pi^2}{180\beta^4} + \frac{(\Omega/2)^2}{6\beta^2} + \frac{(\Omega/2)^4}{12\pi^2}$$

(At rotating axis, for unbounded system)

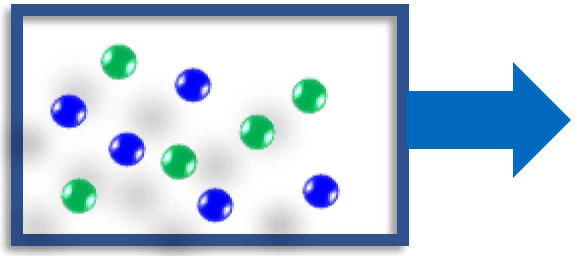
$$P = \frac{7\pi^2}{180\beta^4} + \frac{\mu^2}{6\beta^2} + \frac{\mu^4}{12\pi^2}$$

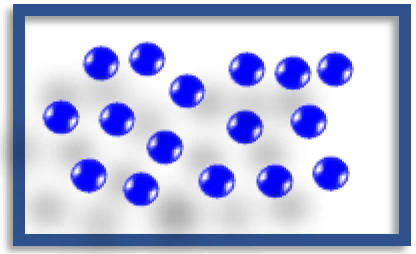
(Ambrus and Winstanley 2019; Palermo etal 2021)

>>> Both have sign problem on lattice

Effect of acceleration: Comparison with chemical potential

- Hints for possible acceleration effect: comparison with chemical potential





Acceleration

Chemical potential

$$H = H_0 - aK_z$$

$$H = H_0 - \mu N$$

For massless Dirac fermions

$$P = \frac{7\pi^2}{180\beta^4} + \frac{(a/\sqrt{12})^2}{6\beta^2} - \frac{17}{20}\frac{(a/\sqrt{12})^4}{12\pi^2}$$

$$P = \frac{7\pi^2}{180\beta^4} + \frac{\mu^2}{6\beta^2} + \frac{\mu^4}{12\pi^2}$$

(Prokhorov etal 2019; Palermo etal 2021)

>>> Sign problem for acceleration can be avoided

Accelerating and rotating thermal equilibrium

- Many-body system can remain equilibrium with acceleration and rotation
- Local equilibrium (LE) density operator

$$\rho_{\mathrm{LE}} = \frac{1}{Z_{\mathrm{LE}}} \exp\left[- \int d\Xi_\mu \left(T^{\mu\nu}\beta_\nu - \frac{1}{2} J^{\mu\rho\sigma}\varpi_{\rho\sigma} \right) \right]$$

- True global equilibrium ρ_{eq} is a time-independent LE

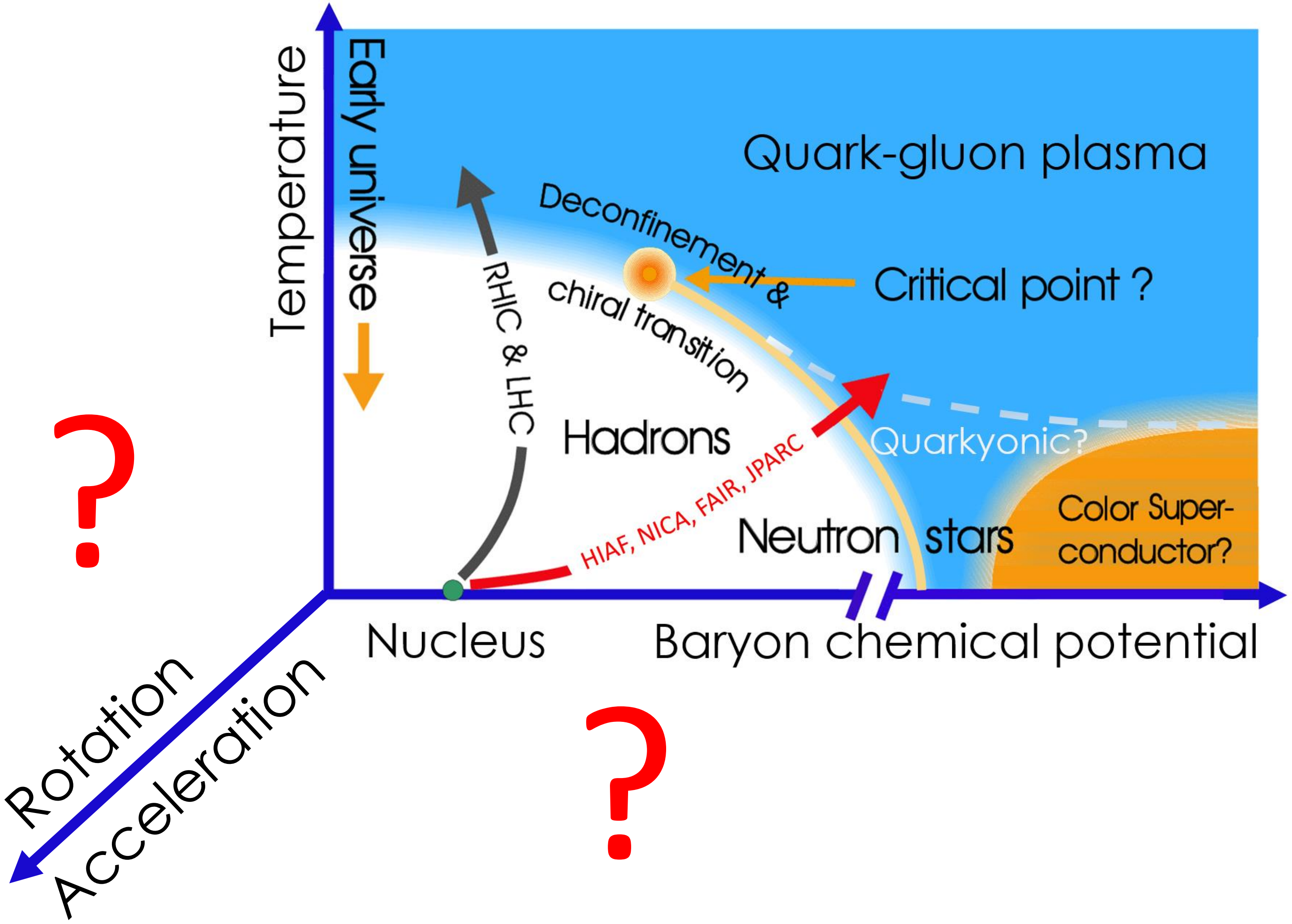
$\Rightarrow$ β^μ and $\varpi^{\mu\nu}$ are constants

$$\begin{cases} \beta^\mu = (\beta, \mathbf{0}) \\ \varpi_{0i} = \beta a^i \\ \varpi_{ij} = \beta \epsilon^{ijk}\Omega^k \end{cases} \Rightarrow \rho_{\mathrm{eq}} = \frac{1}{Z_{\mathrm{eq}}} \exp\left[-\beta \left(H - \boldsymbol{a} \cdot \boldsymbol{K} - \boldsymbol{\Omega} \cdot \boldsymbol{J} \right) \right]$$

Boost operator

Angular momentum

QCD phase diagram



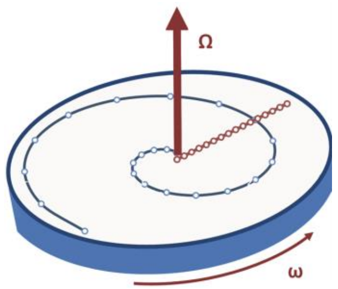
- Chiral condensate and confinement?
- Effects combined with finite B field, densities, … ?
- Possible signatures in HICs?

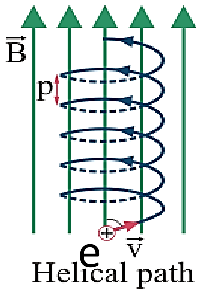
Rotation effects on chiral condensate

Angular momentum polarization

- Consider a scalar (or pseudoscalar) pair of fermions

- Thus in general, one expects that rotation tends to suppress $\sigma, \pi, \ldots\ldots$
- Compare with magnetic catalysis (dimensional reduction)

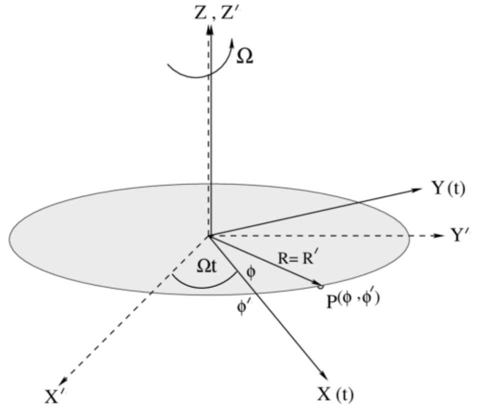




Rotating fermions

- Let us consider fermions; bosons can be similarly discussed.

- Consider a rotating frame



$$\begin{cases} x' = x\cos\Omega t - y\sin\Omega t \\ y' = x\sin\Omega t + y\cos\Omega t \\ z' = z \\ t' = t \end{cases} \Rightarrow \quad g_{\mu\nu} = \begin{pmatrix} 1-\Omega^2 r^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Fermion field

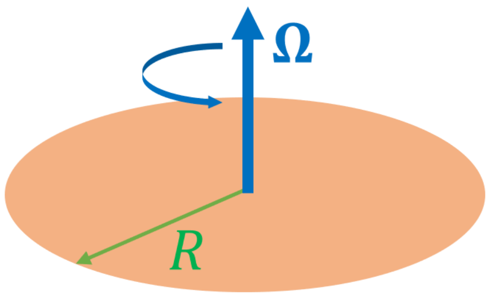
$$S = \int d^4x \sqrt{-g}\,\bar{\psi}\left(i\gamma^\mu \nabla_\mu - m_0\right)\psi \qquad \nabla_\mu = \partial_\mu + i\hat{Q}A_\mu + \Gamma_\mu$$

$$\Downarrow$$

$$H = \hat{Q}A_0 + m_0\beta + \boldsymbol{\alpha}\cdot\boldsymbol{\pi} - \boxed{\boldsymbol{\Omega}\cdot(\mathbf{r}\times\boldsymbol{\pi}+\boldsymbol{\Sigma})}$$

Rotating fermions

- Uniformly rotating system must be finite



$$\boldsymbol{\Omega R \leq 1}$$

- Boundary conditions for Dirac fermions in a cylinder
  - Dirichlet B.C. (No)
  - MIT B.C. (Yes)

$$[i\gamma^{\mu}n_{\mu}(\theta) - 1]\psi\Big|_{r=R} = 0 \quad \Rightarrow \quad j^{\mu}n_{\mu} = 0 \quad \text{at} \quad r = R$$

  - No-flux B.C. (Yes)

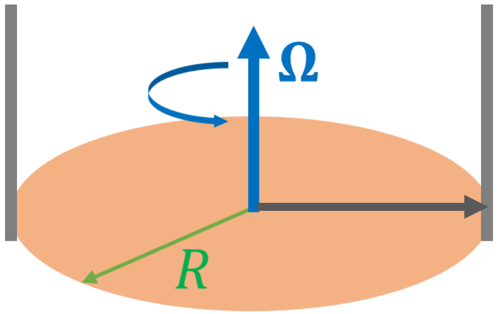
$$\int d\theta\bar{\psi}\gamma^{r}\psi\Big|_{r=R} = 0 \quad \Rightarrow$$

Minimum request for Hermiticity

(Ambrus-Winstanley 2015, Ebihara-Fukushima-Mameda 2016, Chen etal 2017, Chernodub-Gongyo 2016-2017)

Rotating fermions

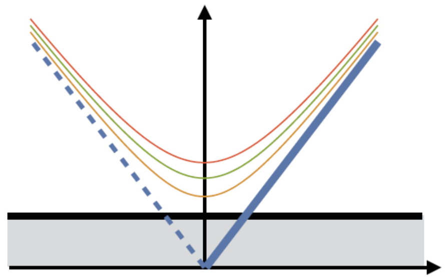
- Consider no-flux B.C.



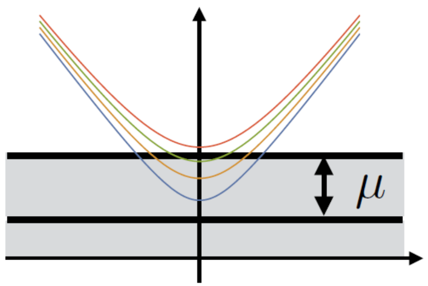
- $p_t = p_{l,k}$ discretized by $J_l(p_{l,k}R) = 0$
- $E = (p_{l,k}^2 + p_z^2 + m^2)^{1/2} > \Omega|l + \frac{1}{2}|$
- Vacuum does not rotate
  (Vilenkin 1979, Ambrus-Winstanley 2015,Ebihara-Fukushima-Mameda 2016)

- To see uniform rotation effect, we need $\boldsymbol{T}$, $\boldsymbol{\mu}$, $\boldsymbol{B}$, ……

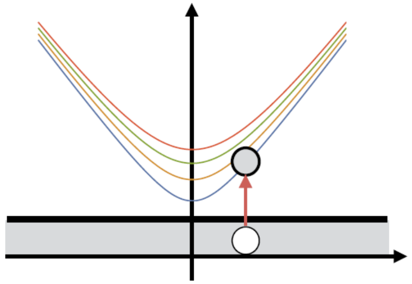
Figures drawn by K.Mameda



B: Chen etal 2015, Liu-Zahed 2017,Chen-Mameda-XGH 2019, Cao-He 2019, Tabatabaee etal 2021…



μ: XGH-Nishimura-Yamamoto 2017, Zhang-Hou-Liao 2018, Huang etal 2018, Nishimura etal 2020,2021…



T: Jiang-Liao 2016, Chernodub-Gongyo 2017, Wang etal 2019, Luo etal 2020, Jiang 2021, …

Rotating Nambu-Jona-Lasinio model

- Take a four-fermion model

$$Z = \int \mathcal{D}[\bar{\psi}, \psi] \exp\left(i \int d^4x \sqrt{-g} \mathcal{L}_{\mathrm{NJL}} \right)$$

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}(i\gamma^\mu \nabla_\mu - m_0)\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma^5 \boldsymbol{\tau} \psi)^2]$$

$$\nabla_\mu = \partial_\mu + i\hat{Q} A_\mu + \Gamma_\mu$$

- Mean-field approximation

$$V_{\mathrm{eff}} = \frac{1}{\beta V} \int d^4 x_E \left\{ \frac{\sigma^2 + \boldsymbol{\pi}^2}{2G} - \sum_{\{\xi\}} \left[\frac{\varepsilon_{\{\xi\}}}{2} + \frac{1}{\beta} \ln(1 + e^{-\beta \varepsilon_{\{\xi\}}}) \right] \Psi^\dagger_{\{\xi\}} \Psi_{\{\xi\}} \right\}$$

$\varepsilon_{\{\xi\}}$ and $\Psi_{\{\xi\}}$: Eigen-energy and eigen-wavefunction with quantum numbers $\{\xi\}$

Rotating Nambu-Jona-Lasinio model

- Consider a simple case: massless, no pion modes, homogeneous

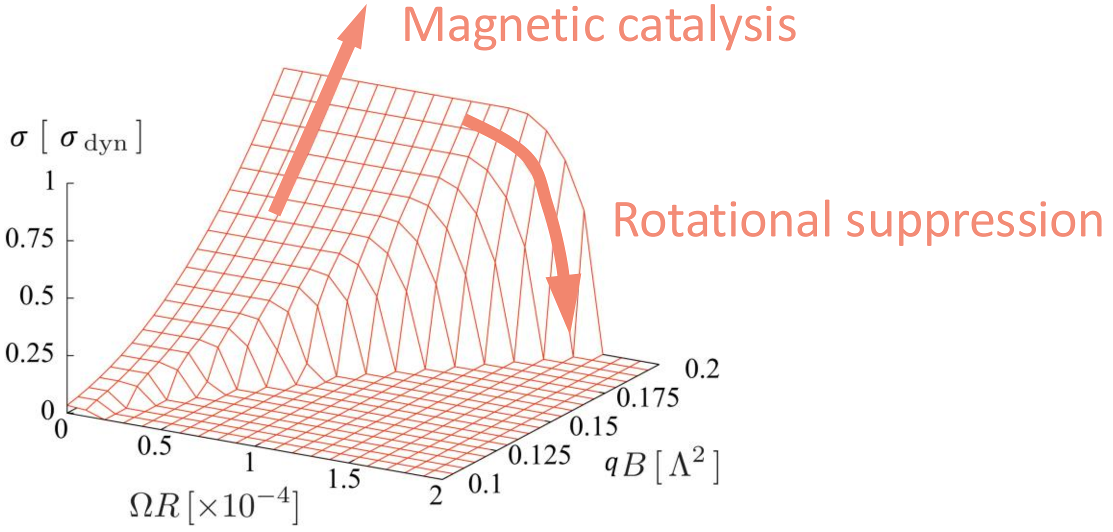
$$\varepsilon_{l,\pm} = \pm\sqrt{p_z^2 + p_t^2 + \sigma^2} - \Omega\left(l + \frac{1}{2}\right)$$

Ration

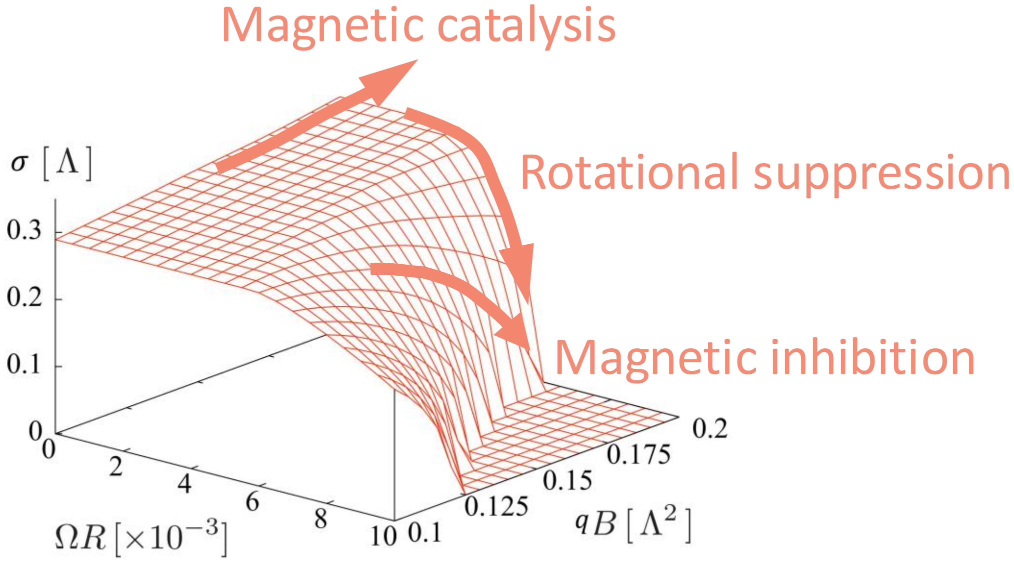
$$\varepsilon_{n,\pm} = \pm\sqrt{p_z^2 + \sigma^2 + 2nqB}$$

Magnetic field

- Chiral condensate vs rotation and/or magnetic field



$G < G_c$



$G > G_c$

(Chen-Fukushima-XGH-Mameda 2015)

Rotating Nambu-Jona-Lasinio model

- Consider a simple case: massless, no pion modes, homogeneous

$$\varepsilon_{l,\pm} = \pm\sqrt{p_z^2 + p_t^2 + \sigma^2} - \Omega\left(l + \frac{1}{2}\right)$$

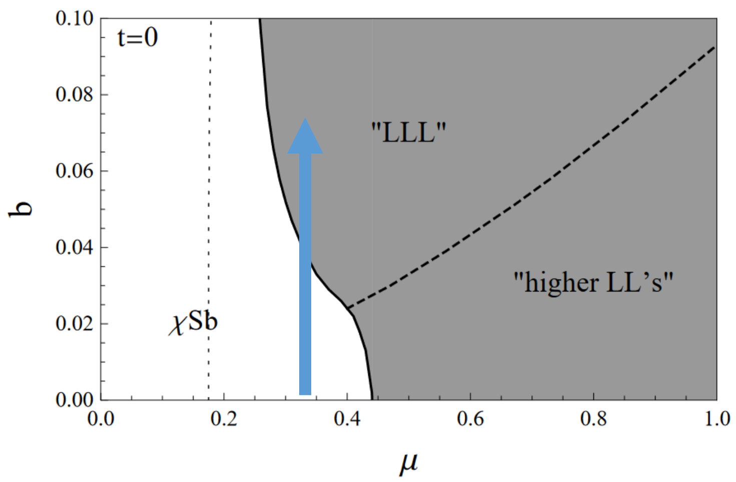
μ

Ration

$$\varepsilon_{n,\pm} = \pm\sqrt{p_z^2 + \sigma^2 + 2nqB}$$

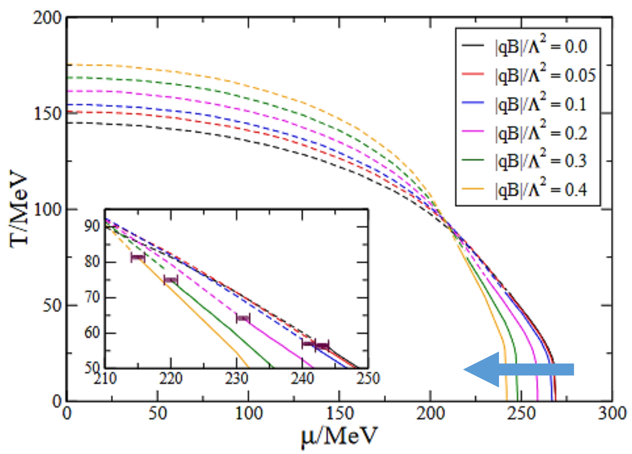
Magnetic field

- Compare with finite-density case:



Sakai-Sugimoto model

(Freis-Rebhan-Schmitt 2010)

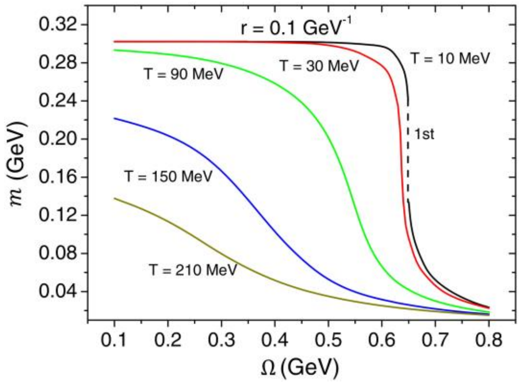


Quark-meson model

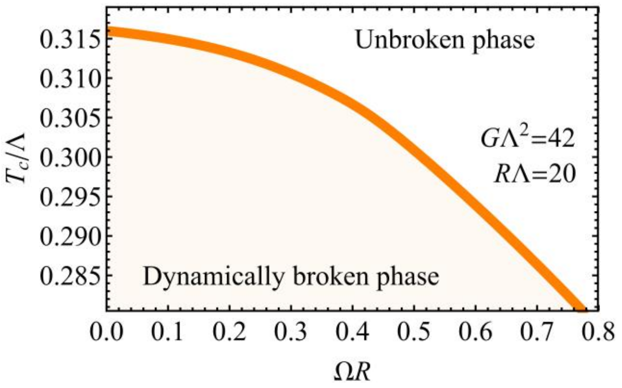
(Andersen-Tranberg 2012)

Rotating Nambu-Jona-Lasinio model

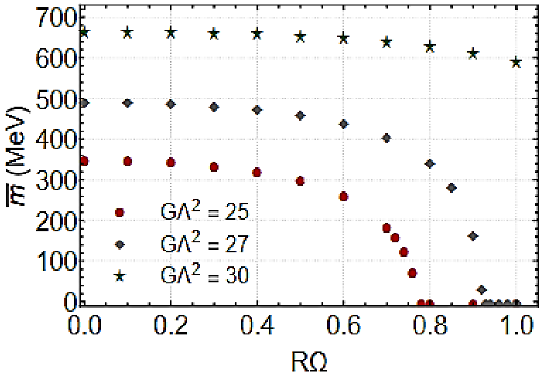
- Many mean-field studies support that rotation suppresses chiral condensate



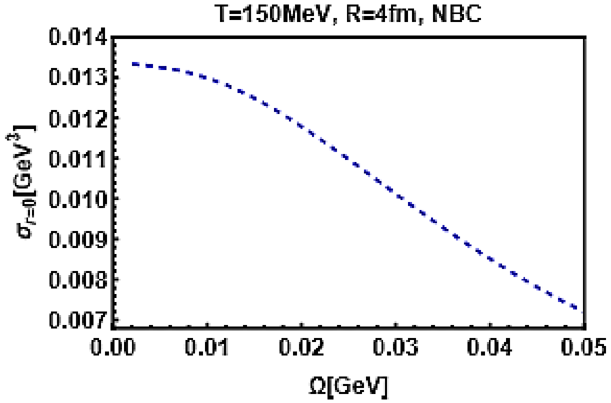
(Jiang-Liao 2016)



(Chernodub-Gongyo 2016)



(Sadooghi-Mehr-Taghinavaz 2022)



(Chen-Li-Huang 2022)

Rotating quark-meson model

- Purpose: beyond mean-field approximation ---- fRG approach
- Quark-meson model is perhaps the simplest model to consider

$$\mathcal{L} = \phi[-(-\partial_\tau + \Omega \hat{L}_z)^2 - \nabla^2]\phi + U(\phi) \;+ \bar{q}[\gamma^0(\partial_\tau - \Omega \hat{J}_z) - i\gamma^i \partial_i + g(\sigma + i\vec{\pi}\cdot\vec{\tau}\gamma^5)]q$$

$$U(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 - c\sigma \qquad \text{with} \qquad \phi = (\sigma, \vec{\pi})$$

- With Dirichlet B.C. for mesons and no-flux B.C. for quarks, solutions for Klein-Gordon eq. and Dirac eq.:

$$\phi = \frac{1}{N_{l,i}^2} e^{-i(\varepsilon - \Omega l)t + il\theta + ip_z z} J_l(p_{l,i} r)$$

$$u_+ = \frac{\mathrm{e}^{-i(\varepsilon - \Omega j) + ip_z z}}{\sqrt{\varepsilon + m}} \begin{pmatrix} (\varepsilon + m)\phi_l \\ 0 \\ p_z \phi_l \\ i\tilde{p}_{l,i}\varphi_l \end{pmatrix}, \qquad \text{with} \qquad \phi_l = \mathrm{e}^{il\theta} J_l(\tilde{p}_{l,i} r)$$

$$u_- = \frac{\mathrm{e}^{-i(\varepsilon - \Omega j) + ip_z z}}{\sqrt{\varepsilon + m}} \begin{pmatrix} 0 \\ (\varepsilon + m)\phi_l \\ -i\tilde{p}_{l,i}\varphi_l \\ -p_z \phi_l \end{pmatrix}, \qquad \varphi_l = \mathrm{e}^{i(l+1)\theta} J_{l+1}(\tilde{p}_{l,i} r)$$

Discretized momenta $p_{l,i}$ and $\tilde{p}_{l,i}$ are determined by B.C.s

Rotating quark-meson model

- The flow equation for effective action

Partition function with an IR regulator

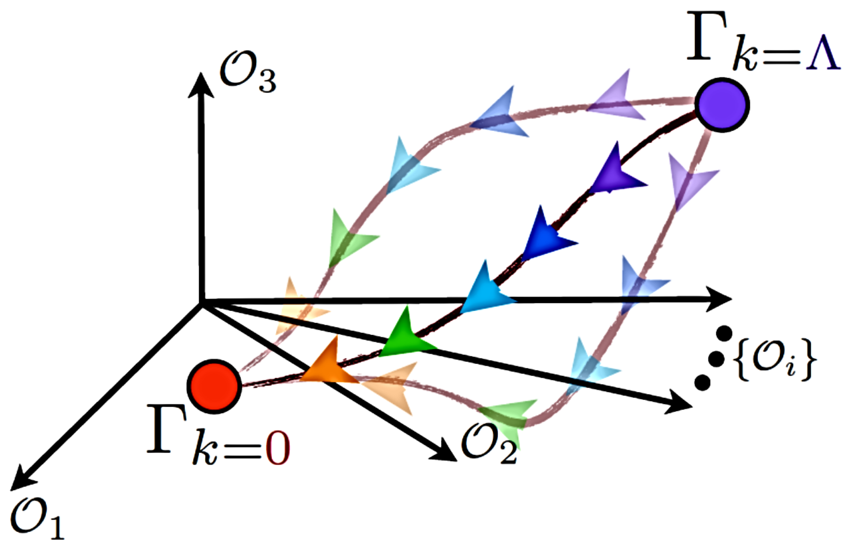
$$Z_k[J] = \int D\chi e^{-S[\chi]+\int_x \chi(x)J(x)-\Delta S_k[\chi]}$$

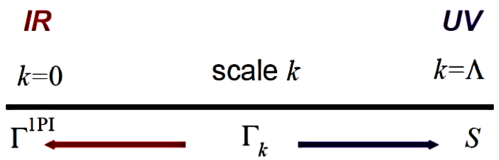
regulator

$$\Delta S_k[\chi] = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}\chi^*(q)R_k(q)\chi(q)$$

Legendre transformation:

$$\Gamma_k[\phi] = -W_k[J] + \int_x \phi(x)J(x) + \Delta S_k[\phi]$$





flow equation (Wetterich 1993)

$$\partial_k\Gamma_k = \frac{1}{2}\mathrm{tr}(G_{\phi,k}\partial_k R_{\phi,k}) - \mathrm{tr}(G_{q,k}\partial_k R_{q,k})$$

with coarse-graining regulators

$$R_{\phi,k} = (k^2 - p^2)\theta(k^2 - p^2)$$

$$\hat{R}_{q,k} = -i\gamma^i\partial_i\left(\frac{k}{\sqrt{-\nabla^2}} - 1\right)\theta(k^2 + \nabla^2)$$

Rotating quark-meson model

- The flow equation for effective action

Partition function with an IR regulator

$$Z_k[J] = \int D\chi e^{-S[\chi]+\int_x \chi(x)J(x)-\Delta S_k[\chi]}$$

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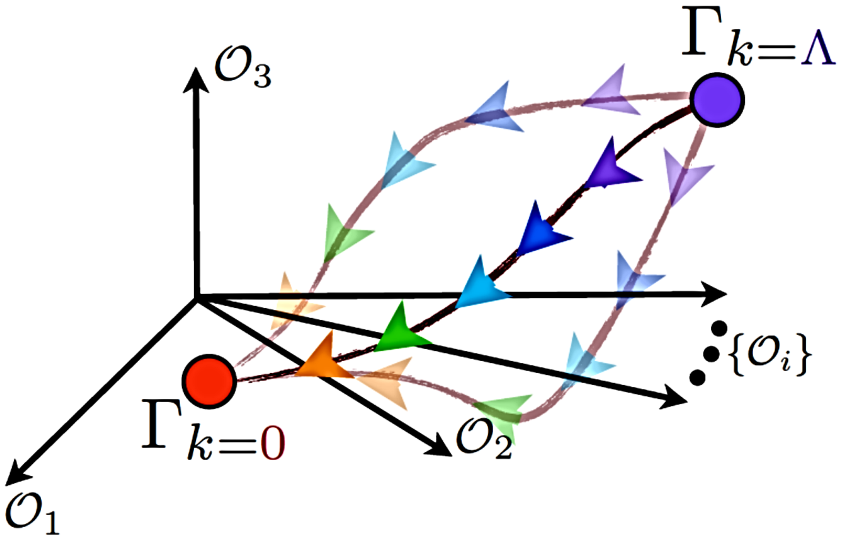
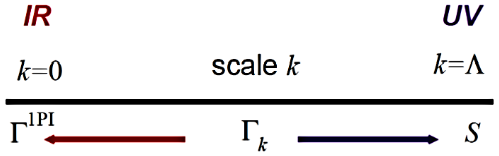
Legendre transformation:

$$\Gamma_k[\phi] = -W_k[J] + \int_x \phi(x)J(x) + \Delta S_k[\phi]$$

flow equation (Wetterich 1993)

$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{tr}(G_{\phi,k}\partial_k R_{\phi,k}) - \mathrm{tr}(G_{q,k}\partial_k R_{q,k})$$ with propagators





$$\hat{G}^{-1}_{\phi,k} = -(-\partial_\tau + \Omega\hat{L}_z)^2 - \nabla^2 + \hat{R}_{\phi,k} + \frac{\partial^2 U}{\partial\phi_i\partial\phi_j}$$

$$\hat{G}^{-1}_{q,k} = \gamma^0(-\partial_\tau + \Omega\hat{J}_z) - \gamma^i\partial_i + \hat{R}_{q,k} + g\phi$$

Rotating quark-meson model

- The flow equation for effective potential: Local potential approximation

$$\partial_k U_k = \frac{1}{\beta V}(\partial_k \Gamma_k^B + \partial_k \Gamma_k^F),$$

$$= \frac{1}{(2\pi)^2}\Bigg\{\sum_{l,i}\frac{1}{N_{l,i}^2}\mathrm{tr}\frac{k\sqrt{k^2 - p_{l,i}^2}}{\varepsilon_\phi}\frac{1}{2}\left[\coth\frac{\beta(\varepsilon_\phi + \Omega l)}{2} + \coth\frac{\beta(\varepsilon_\phi - \Omega l)}{2}\right]J_l(p_{l,i}r)^2\theta(k^2 - p_{l,i}^2)$$

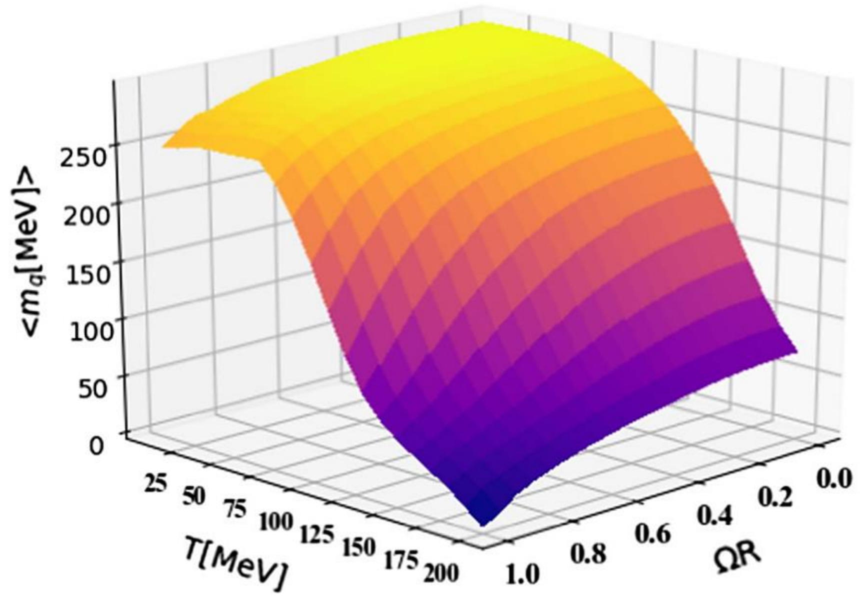
$$-\sum_{l,i}\frac{1}{\tilde{N}_{l,i}^2}2N_cN_f\frac{k\sqrt{k^2 - \tilde{p}_{l,i}^2}}{\varepsilon_q}\frac{1}{2}\left[\tanh\frac{\beta(\varepsilon_q + \Omega j)}{2} + \tanh\frac{\beta(\varepsilon_q - \Omega j)}{2}\right][J_l(\tilde{p}_{l,i}r)^2 + J_{l+1}(\tilde{p}_{l,i}r)^2]\theta(k^2 - \tilde{p}_{l,i}^2)\Bigg\}$$

Depend on U_k

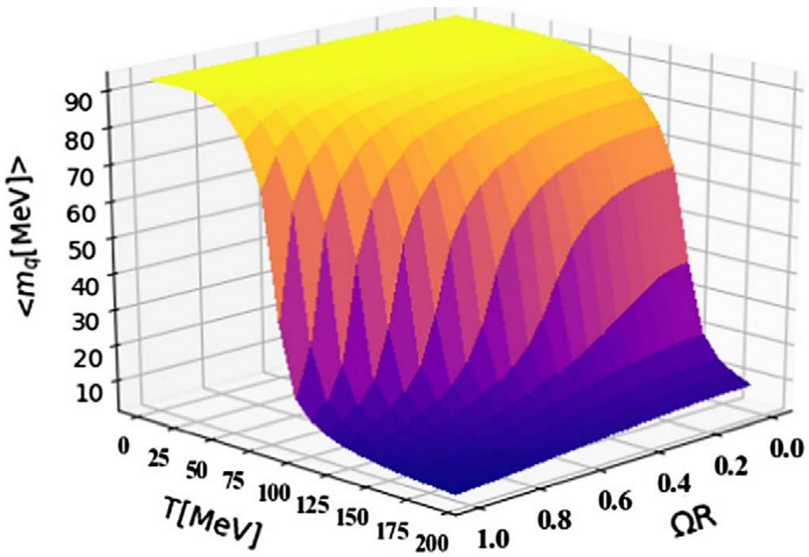
- Solved using grid method with UV cutoff at 1 GeV; System size is 100/GeV, other parameters are fitted to non-rotating results

Rotating quark-meson model

- Chiral condensate on T-Ω plane (Chen-Zhu-XGH 2023)



fRG calculation

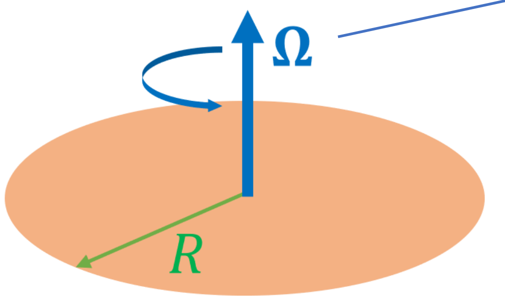


Mean-field calculation

- No surprise: Ω tends to suppress chiral condensate

Formulate rotating lattice

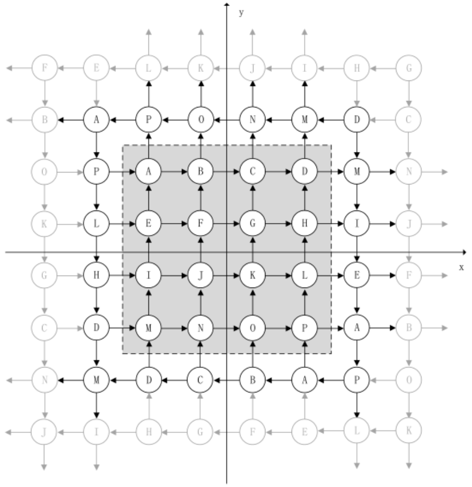
- Gluons and Wilson fermions (Angular momentum) (Yamamoto-Hirono 2013)
- Pure gluons (Polyakov loop) (Braguta etal 2021)
- We consider gluons and 2+1 flavor staggered fermions



Imaginary rotation: $\Omega \to -i\Omega_I$

No sign problem
No causality constraint

Projective-plane B.C for x-y plane
Periodic B.C. for t and z direction



- We measure: (imaginary) angular momentum

Ji decomposition

$$\mathbf{J} = \mathbf{J}_G + \mathbf{s}_q + \mathbf{L}_q$$

$$\begin{cases} \mathbf{J}_G = \sum_a \int d^3x \, \mathbf{r} \times (\mathbf{E}^a \times \mathbf{B}^a) \, , \\ \mathbf{s}_q = \int d^3x \, q^\dagger \frac{\mathbf{\Sigma}}{2} q, \\ \mathbf{L}_q = \frac{1}{i} \int d^3x \, q^\dagger \mathbf{r} \times \mathbf{D} q. \end{cases}$$

Chiral vortical effect

Formulate rotating lattice

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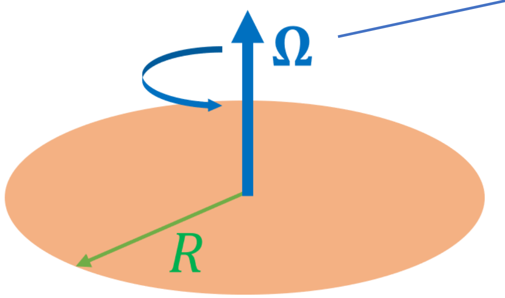
- Pure gluons (Polyakov loop) (Braguta etal 2021)
- We consider gluons and 2+1 flavor staggered fermions



Imaginary rotation: $\boldsymbol{\Omega} \rightarrow -\boldsymbol{i}\boldsymbol{\Omega}_{\boldsymbol{I}}$

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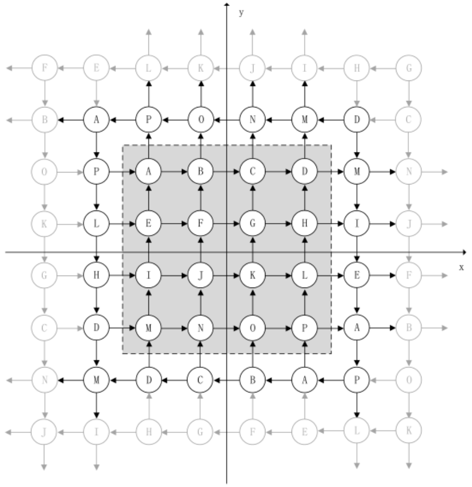
Projective-plane B.C for x-y plane
Periodic B.C. for t and z direction



- We measure: chiral condensate and Polyakov loop

$$\Delta_{l,s}(T,\Omega_I) = \frac{\langle\bar{\psi}_l\psi_l\rangle_{T,\Omega_I} - \frac{m_l}{m_s}\langle\bar{\psi}_s\psi_s\rangle_{T,0}}{\langle\bar{\psi}_l\psi_l\rangle_{0,0} - \frac{m_l}{m_s}\langle\bar{\psi}_s\psi_s\rangle_{0,0}}$$

$$L_{\rm ren} = \exp(-N_\tau c(\beta)a/2)L_{\rm bare}$$

$$L_{\rm bare} = \left|{\rm tr}\left[\sum_{\mathbf{n}}\prod_\tau U_\tau(\mathbf{n},\tau)\right]\right|/3N_x^3$$

Results of angular momentum

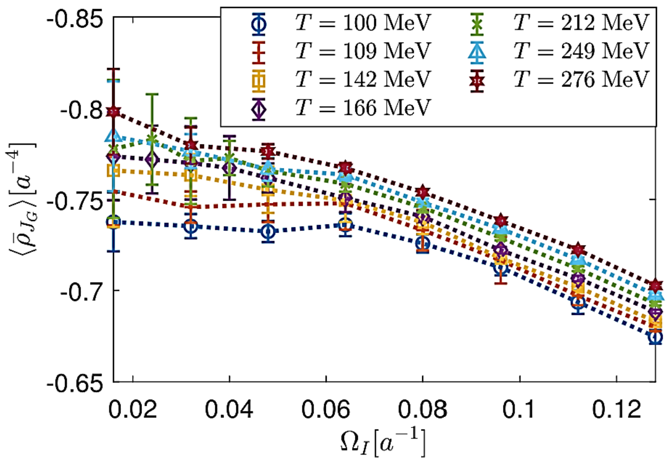
- Angular momentum
- J_G and L_q approximately $\propto r^2$, and s_q approximately independent of r, thus

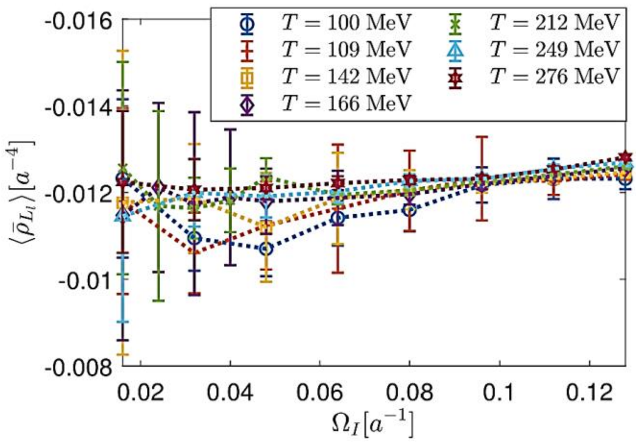
$$\rho_J = \frac{1}{N_{taste} N_{r_{max}}} \sum_{n_x^2+n_y^2<r_{max}^2} \frac{\langle J(n) \rangle}{a\Omega (a^{-1} r)^2}$$

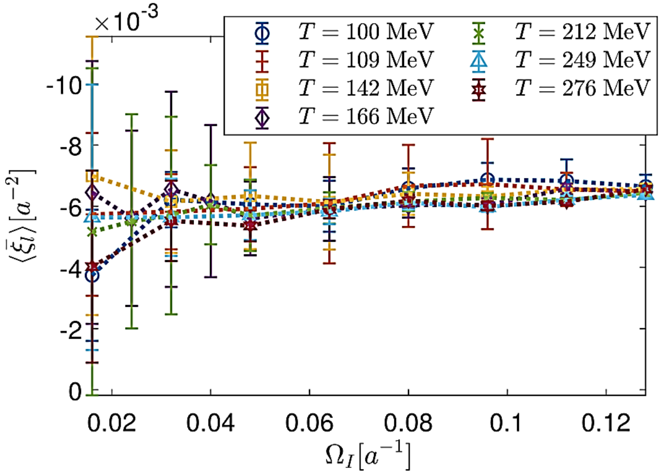
Moment of inertia

$$\xi_q = \frac{1}{4N_{r_{max}}} \sum_{n_x^2+n_y^2<r_{max}^2} \frac{\langle s_q(n) \rangle}{a\Omega}$$

Quark spin susceptibility



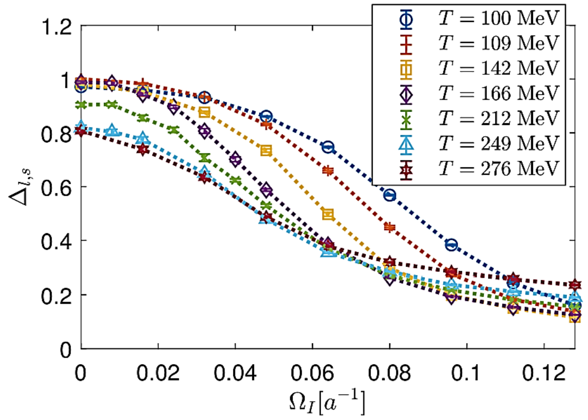


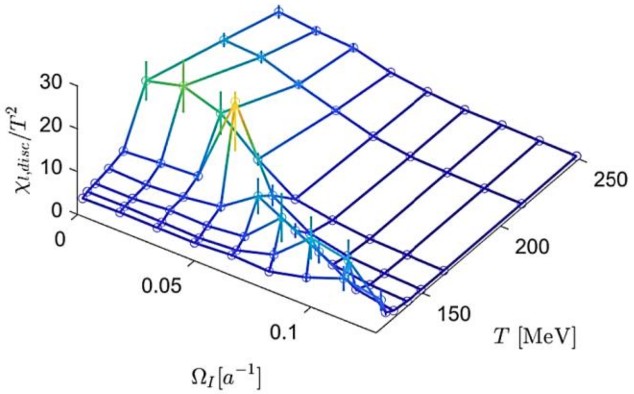


(Yang-XGH 2023)

Results for chiral condensate

- Chiral condensate and chiral susceptibility

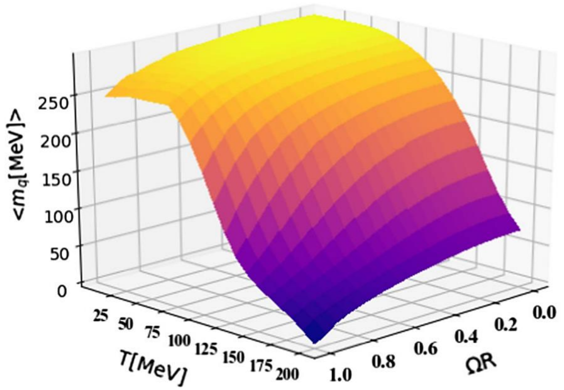




- Analytical continuation to real rotation $\Omega_I \to i\Omega$

Chiral condensate must be even function of Ω

Chiral condensate increase with real Ω!

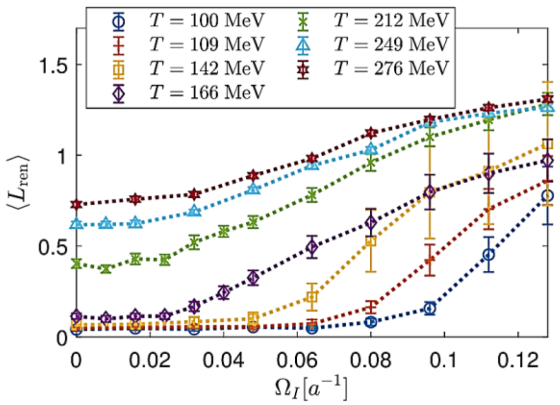


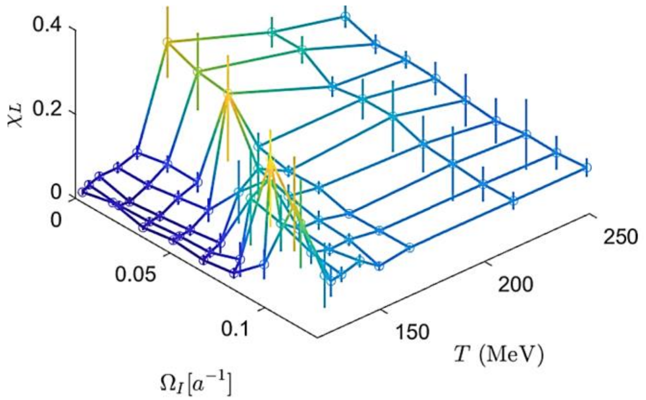
Recall e.g the fRG results for QM model

Sharp conflict between effective models and lattice!

Results for Polyakov loop

- Polyakov loop and its susceptibility: Real rotation catalyze quark confinement



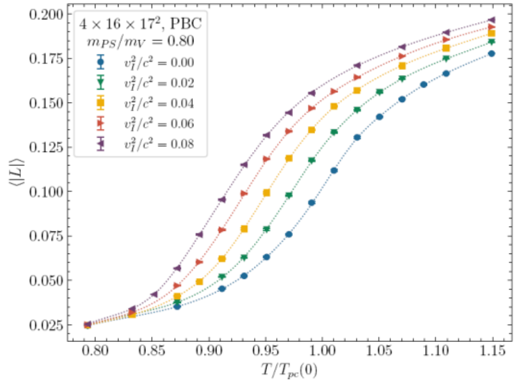


- ~~Pseudo-critical~~ temperature ~~decreases~~ due to ~~imaginary~~ rotation

Critical increases real

- Consistent with previous pure gluon simulation

(Braguta etal 2021)



Contradict with model studies, not understood too

Acceleration effects on chiral condensate

Accelerating frame and Rindler coordinates

- An observer with constant proper acceleration in Minkowski spacetime

$$t_M^2 - \left(z_M - z_M(0) + \frac{1}{a}\right)^2 = -\frac{1}{a^2}, \quad z_M > z_M(0)$$

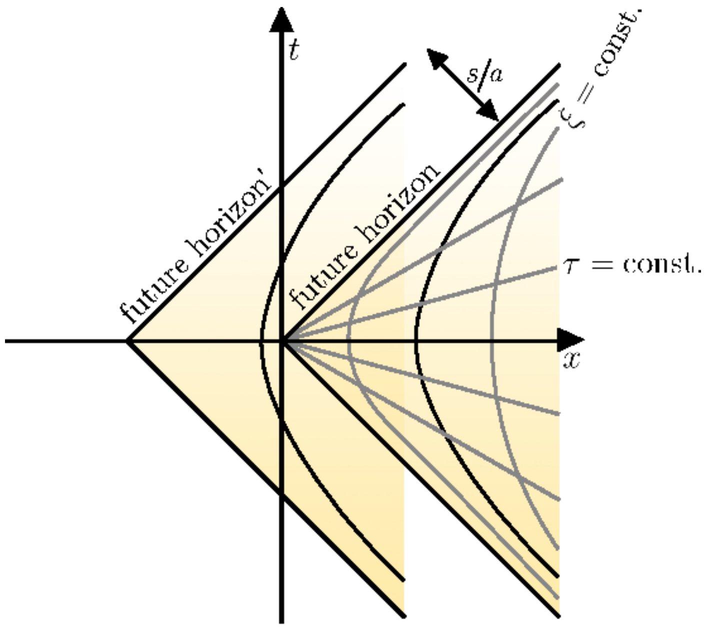
- The coordinates (τ, z) in which the observer is static is the Rindler coordinates

$$t_M = \left(z + \frac{1}{a}\right)\sinh(a\tau),$$
$$z_M = \left(z + \frac{1}{a}\right)\cosh(a\tau) + z_M(0) - \frac{1}{a}.$$

$$ds^2 = dt_M^2 - dz_M^2 = (1 + az)^2 d\tau^2 - dz^2$$

$$t = a\tau,$$
$$\xi = z + 1/a$$

$$ds^2 = \xi^2 dt^2 - d\xi^2$$



Observer at $\xi = 1/a$ has proper acceleration a

Nonlinear sigma model analysis

- The model

$$\mathcal{L}_{NL\sigma M} = -\frac{1}{2}\Phi^T \Box \Phi - M_\pi^2 f_\pi \sigma, \qquad \Phi^T = (\pi^a, \sigma) \qquad \Phi^T\Phi = f_\pi^2.$$

- Effective action

$$\Gamma[\sigma, \lambda] = \int \mathrm{d}^4 x \left(-\frac{1}{2}\sigma \Box \sigma + \frac{\lambda}{2}(\sigma^2 - f_\pi^2) + \frac{N}{2} \ln \frac{-\Box + \lambda}{-\Box} - M_\pi^2 f_\pi \sigma \right)$$

- Gap equations

$$\frac{\delta \Gamma}{\delta \sigma} = -\Box \sigma + \lambda \sigma - M_\pi^2 f_\pi = 0,$$
$$\frac{\delta \Gamma}{\delta \lambda} = \frac{\sigma^2 - f_\pi^2}{2} + \frac{N}{2} G(x, x; \lambda) = 0,$$

- Field operator and Green function (propagator)

$$\hat{\phi} = \int_0^\infty \mathrm{d}\omega \int \frac{\mathrm{d}^2 \mathbf{k}}{2\pi} \sqrt{\frac{\sinh \pi\omega}{\pi^2}} K_{i\omega}(m_\perp \xi)(\hat{a}_{\omega,k} \mathrm{e}^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}} + \hat{a}^\dagger_{\omega,k} \mathrm{e}^{i\omega t - i\mathbf{k}\cdot\mathbf{x}}) \Rightarrow G(x, x') = i\langle 0_R | T\phi(x)\phi(x') | 0_R \rangle,$$

Nonlinear sigma model analysis

- Extend it to Euclidean time

$$G_E(x_E, x'_E) = \int_0^\infty d\omega \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\sinh \pi\omega}{\pi^2} K_{i\omega}(m_\perp \xi) K_{i\omega}(m_\perp \xi') e^{-\omega|t_E - t'_E| + i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')},$$

- Extend it to finite temperature

$$G_\nu(t_E, t'_E) = G_\nu(t_E + \beta_R, t'_E) \text{ where } \beta_R = 1/T_R = a/T \text{ and } \nu = 2\pi T/a$$

$$\begin{aligned} G_\nu(x_E, x'_E) &= \sum_n G_E(t_E - t'_E + \beta_R n) \\ &= \int_0^\infty d\omega \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\cosh \omega(|t_E - t'_E| - \beta_R/2)}{\sinh(\beta_R \omega/2)} \frac{\sinh \pi\omega}{\pi^2} K_{i\omega}(m_\perp \xi) K_{i\omega}(m_\perp \xi') e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \end{aligned}$$

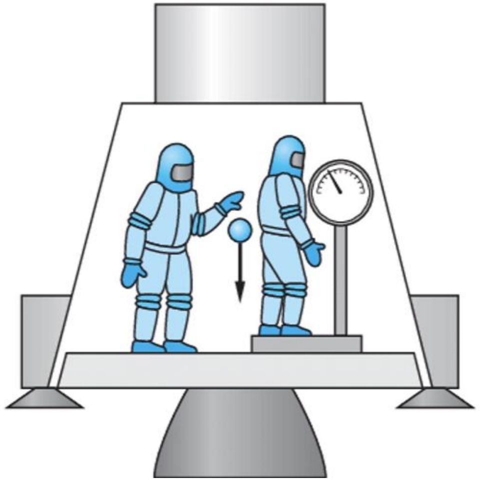
- Gap equation at chiral limit

$$\sigma^2 = f_\pi^2 - N G_\nu(x_E, x_E; 0)$$

- There is divergence in the above one-loop result: vacuum contribution

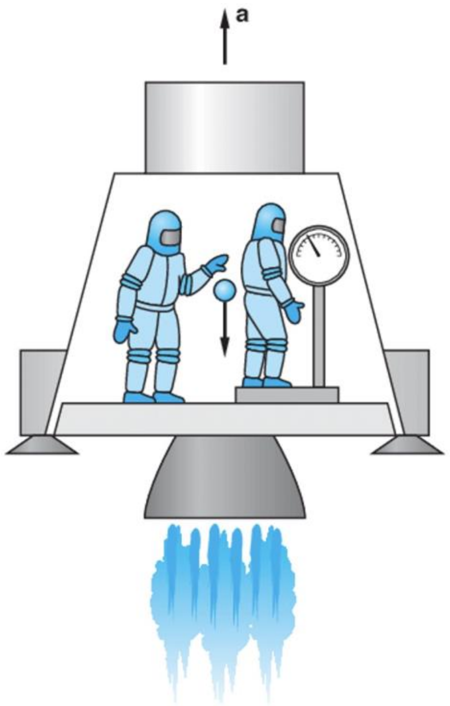
Nonlinear sigma model analysis

- What vacuum contribution to subtract?



Minkowski vacuum

$$a_k^M|0_M\rangle = 0,$$



Rindler vacuum

$$a_k^R|0_R\rangle = 0$$

Nonlinear sigma model analysis

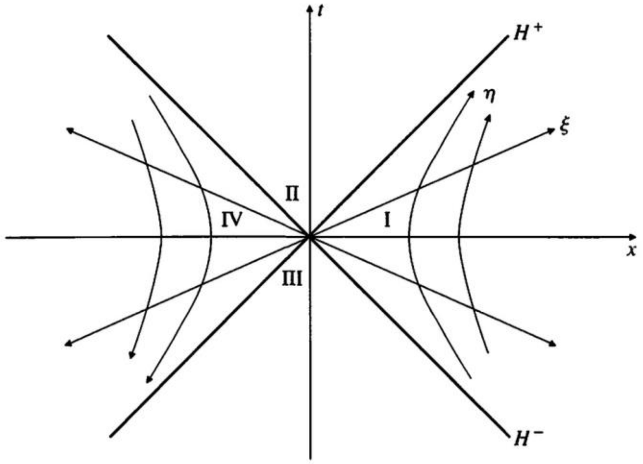
- What vacuum contribution to subtract?

Minkowski vacuum

$$a_k^M|0_M\rangle = 0$$

Rindler vacuum

$$a_k^R|0_R\rangle = 0$$



- Related by a Bogoliubov transformation

$$a_k^R = \left[2\sinh\left(\frac{\pi\omega}{a}\right)\right]^{-1/2}\left(e^{\pi\omega/2a}a_k^{(1)M} + e^{-\pi\omega/2a}a_{-k}^{(2)M\dagger}\right)$$

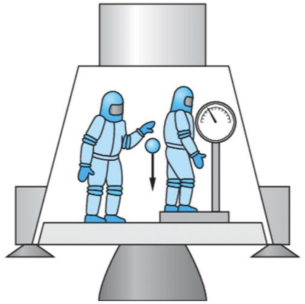
- Perhaps the most profound difference is Unruh effect

$$\langle 0_M|a_k^{R\dagger}a_k^R\rangle 0_M\rangle = \frac{1}{e^{\omega/T_U} - 1} \qquad T_U = \frac{a}{2\pi}$$

Nonlinear sigma model analysis

- Subtraction with respect to the Minkowski vacuum

$$a_k^M|0_M\rangle = 0,$$



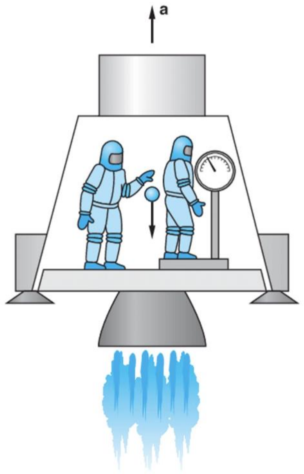
$$\sigma^2 = f_\pi^2 - \frac{1}{(a\xi)^2}\frac{N}{12}\left(T^2 - \frac{a^2}{(2\pi)^2}\right)$$

Local observer with constant acceleration a: $\xi = 1/a$

$$T_c(a) = \sqrt{\frac{12 f_\pi^2}{N} + (\frac{a}{2\pi})^2} = \sqrt{T_{c0}^2 + (\frac{a}{2\pi})^2}.$$

- Subtraction with respect to the Rindler vacuum

$$a_k^R|0_R\rangle = 0$$



$$\sigma^2 = f_\pi^2 - \frac{T^2}{a^2\xi^2}\frac{N}{12}$$

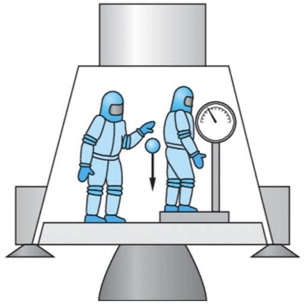
Local observer with constant acceleration a: $\xi = 1/a$

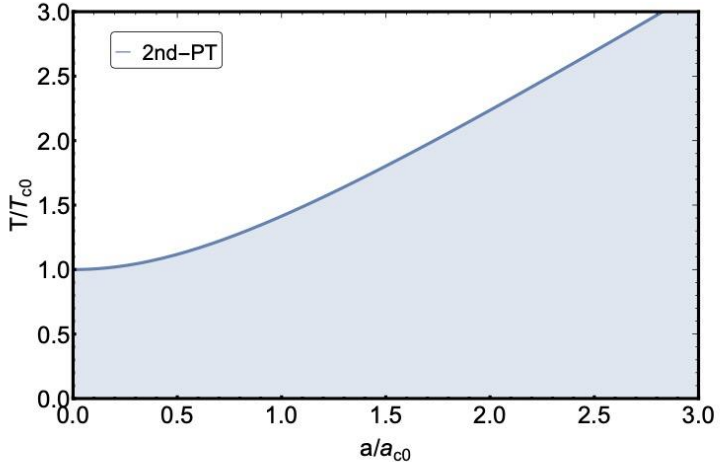
$$T_c(a) = \sqrt{\frac{12 f_\pi^2}{N}} = T_{c0}$$

Nonlinear sigma model analysis: phase diagram

- Subtraction with respect to the Minkowski vacuum

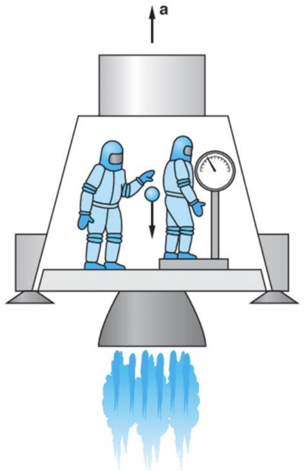
$$a_k^M|0_M\rangle = 0,$$

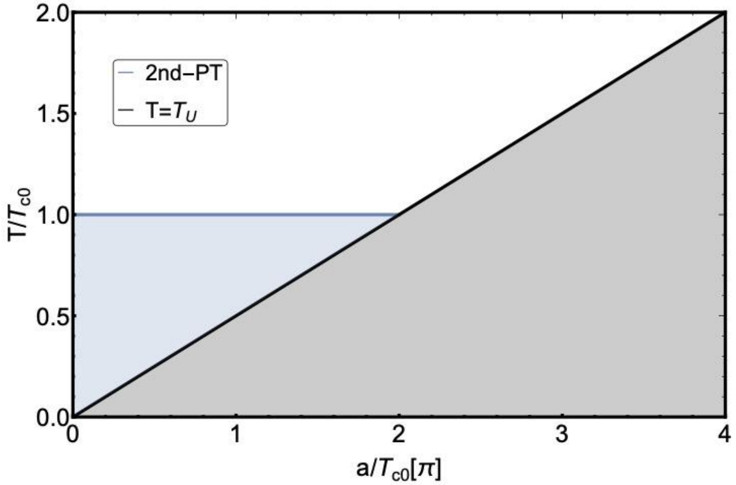




- Subtraction with respect to the Rindler vacuum

$$a_k^R|0_R\rangle = 0$$

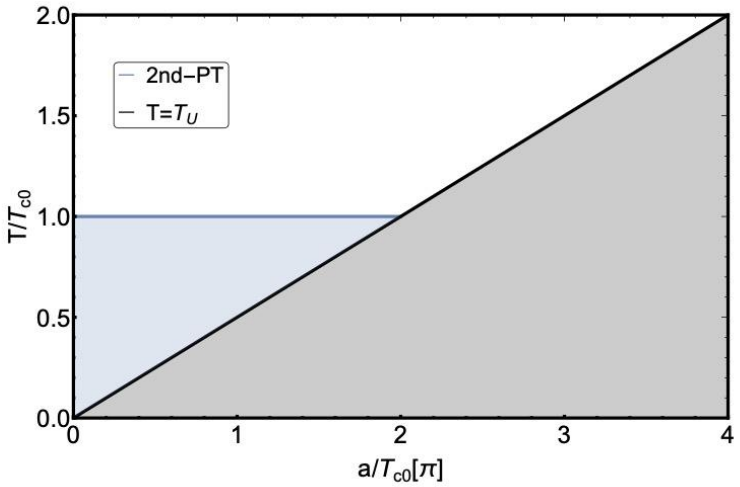


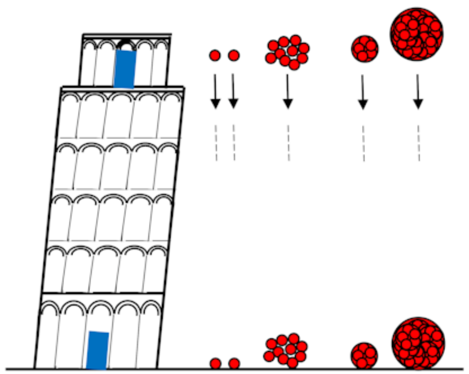


(Zhu-Chen-XGH, to appear)

Nonlinear sigma model analysis: phase diagram

- Which is the good one?
- Perhaps the one with respect to the Rindler vacuum is more reasonable





$$G(x,x') = i\langle 0_R|T\phi(x)\phi(x')|0_R\rangle,$$

Equivalence principle: Local experiments in a free-falling frame give the same results as in inertial frame

NJL model analysis

- Let us go into more micro degree of freedom by considering NJL for quarks

$$\mathcal{L}_{NJL} = \bar{\psi}\left[i\gamma^\mu \nabla_\mu - m_0\right]\psi + \frac{G_\pi}{2}\left[(\bar{\psi}\psi)^2 + \left(\bar{\psi}i\gamma^5\psi\right)^2\right]$$

- Gap equation

$$\frac{m - m_0}{G_\pi} = i\,\mathrm{Tr}(S), \qquad S(x,x') = (\hat{D}+m)G(x,x'),$$

- Euclidean Green function (propagator)

$$G_E(x_E, x'_E) = \int_0^\infty \mathrm{d}\omega \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \frac{\sinh\pi(\omega + i\gamma^{\hat{0}}\gamma^{\hat{3}}/2)}{\pi^2} K_{i\omega-\gamma^{\hat{0}}\gamma^{\hat{3}}/2}(m_\perp\xi) K_{i\omega-\gamma^{\hat{0}}\gamma^{\hat{3}}/2}(m_\perp\xi') e^{-\omega|t_E - t'_E| + i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}$$

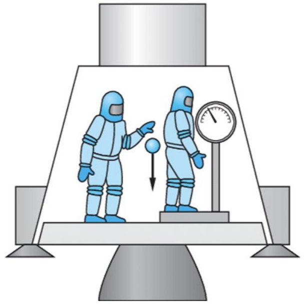
- Extend it to finite temperature

$$\begin{aligned} G_\nu &= \sum_n (-1)^n G_E(t_E - t'_E + \beta_R n) \\ &= \int_0^\infty \mathrm{d}\omega \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \frac{\sinh(\beta_R\omega/2 - \omega|t_E - t'_E|)}{\cosh(\beta_R\omega/2)} \frac{\sinh\pi(\omega + i\gamma^{\hat{0}}\gamma^{\hat{3}}/2)}{\pi^2} K_{i\omega-\gamma^{\hat{0}}\gamma^{\hat{3}}/2}(m_\perp\xi) K_{i\omega-\gamma^{\hat{0}}\gamma^{\hat{3}}/2}(m_\perp\xi') e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}. \end{aligned}$$

NJL model analysis

- Subtraction with respect to the Minkowski vacuum

$$a_k^M|0_M\rangle = 0,$$

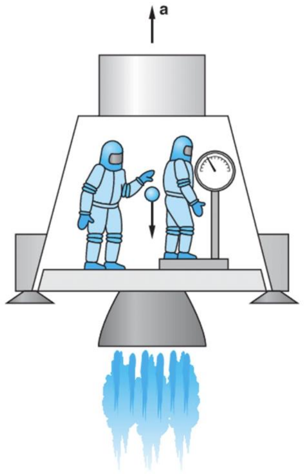


Local observer with constant acceleration a: $\xi = 1/a$

$$T_c(a) = \sqrt{\frac{3\Lambda^2}{\pi^2} - \frac{6}{G} + \frac{a^2}{(2\pi)^2}} = \sqrt{T_{c0}^2 + (\frac{a}{2\pi})^2},$$

- Subtraction with respect to the Rindler vacuum

$$a_k^R|0_R\rangle = 0$$



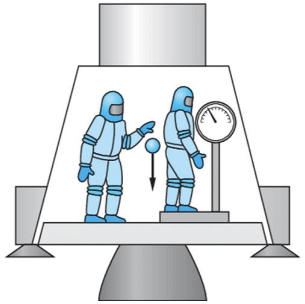
Local observer with constant acceleration a: $\xi = 1/a$

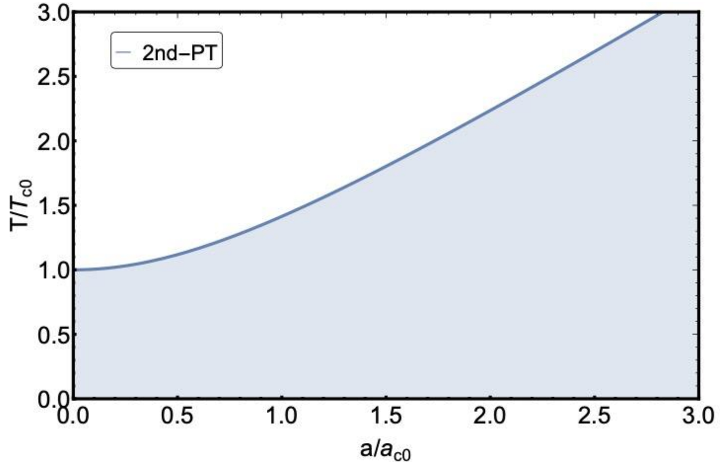
$$T_c(a) = \sqrt{\frac{3\Lambda^2}{\pi^2} - \frac{6}{G_\pi}} = T_{c0}$$

NJL model analysis

- The phase diagram is completely consistent with NLsM analysis

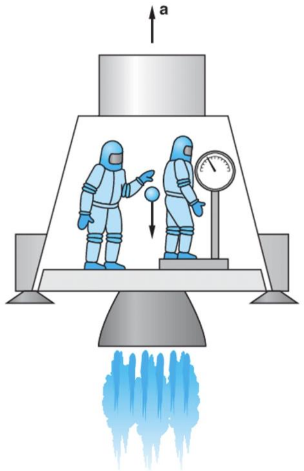
$$a_k^M|0_M\rangle = 0,$$

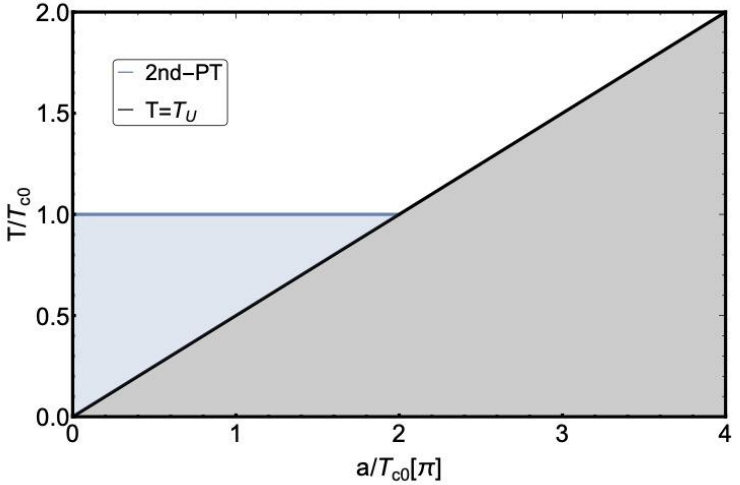




- The phase diagram is completely consistent with NLsM analysis

$$a_k^R|0_R\rangle = 0$$





(Zhu-Chen-XGH, to appear)

Lattice formulation

- Formulate lattice action in the following accelerating metric

$$g_{\mu\nu} = \begin{pmatrix} (1+gz)^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$S_G^{lat} = \frac{\beta}{N_c} \sum_n \left\{ (1+gz) \sum_{i<j<4} \mathrm{Re\,tr}\left[1 - \bar{U}_{ij}^2\right] + \sum_{i=1,2,3} \frac{\mathrm{Re\,tr}\left[1 - \bar{U}_{4i}^2\right]}{1+gz} \right\}$$

$$S_F^{lat} = \sum_{n,n'} \bar{\chi}(n) D \chi(n'),$$

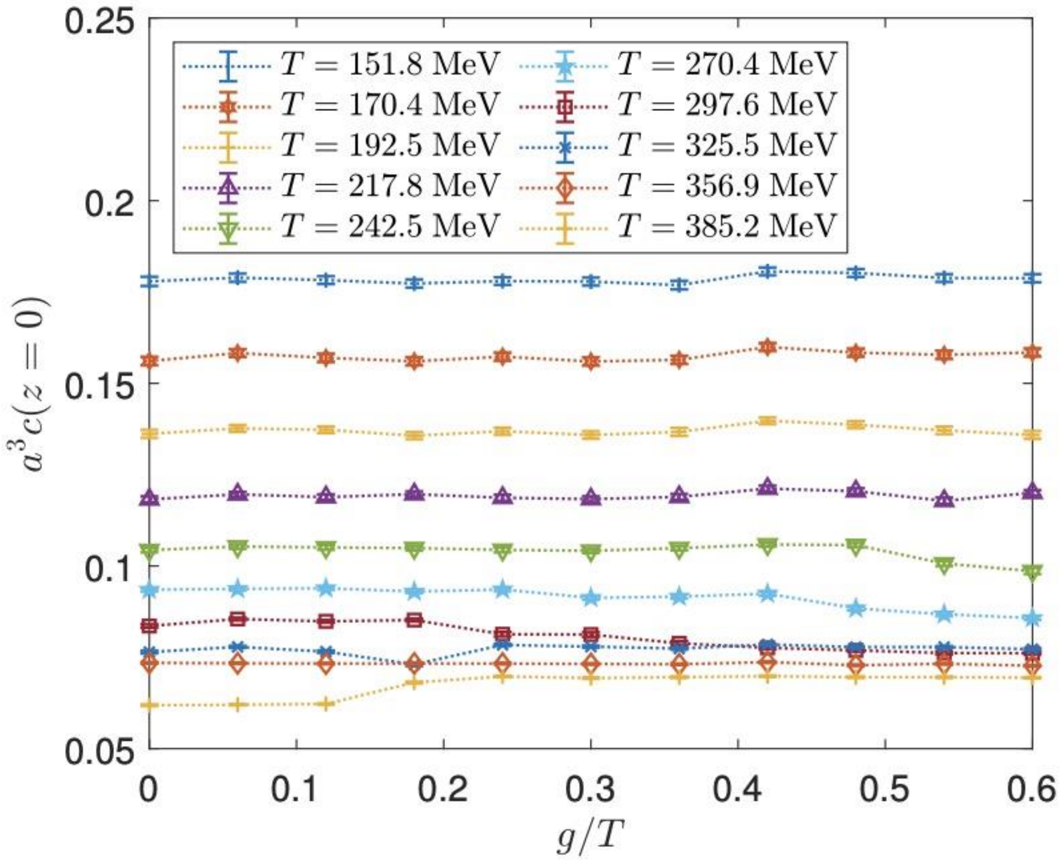
$$D_F = \left\{ \sum_{i=x,y,z} \sum_{\Delta_i = \pm i} (1+g\bar{z}) \eta_{\Delta_i}(n) U_{\Delta_i}(n) \delta_{n,n'-\delta_i} + \eta_\tau(n) \left(U_\tau(n)\delta_{n,n'-\tau} - U_{-\tau}(n)\delta_{n,n'+\tau} \right) \right.$$

$$\left. + \frac{g}{2}\eta_z(n) \left(U_z(n)\delta_{n,n'-z} + U_{-z}(n)\delta_{n,n'+z} \right) + 2(1+gz)am\delta_{n,n'} \right\}$$

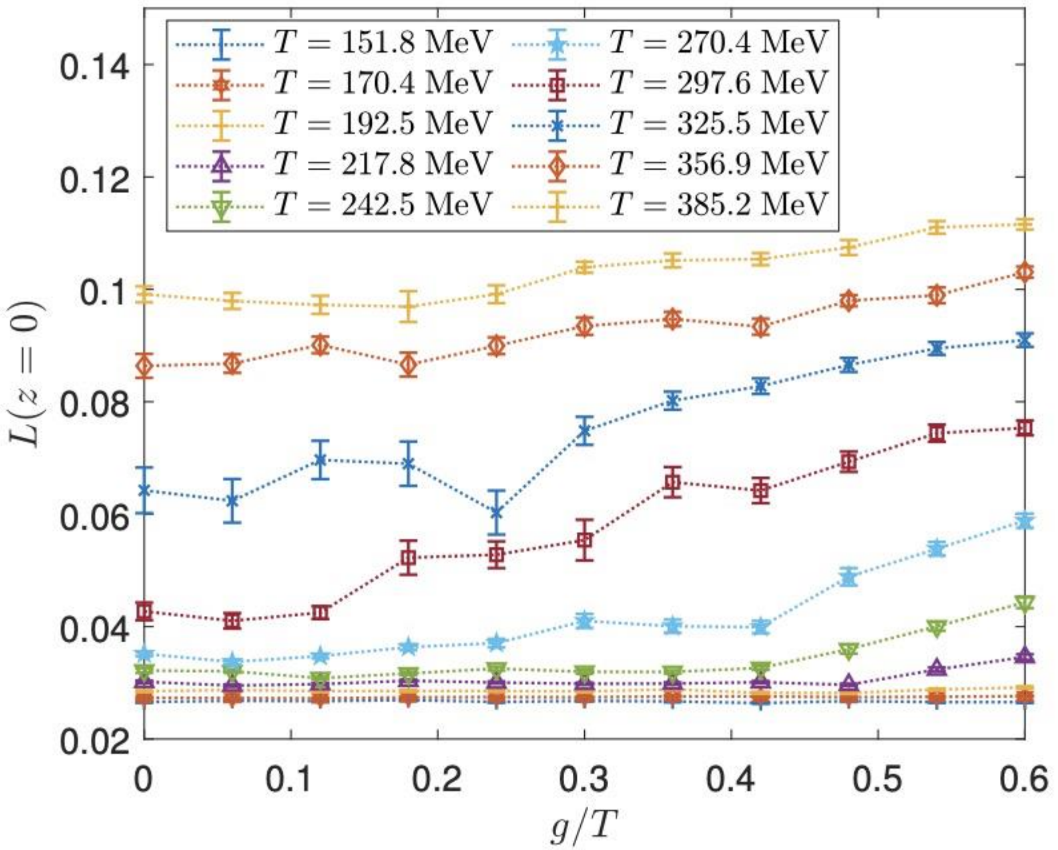
$$= +\frac{1}{2} g \gamma_3^E \qquad \Rightarrow \qquad \text{Sign problem}$$

Lattice results

- Measurements are done at quenched limit



Chiral condensate vs acceleration g



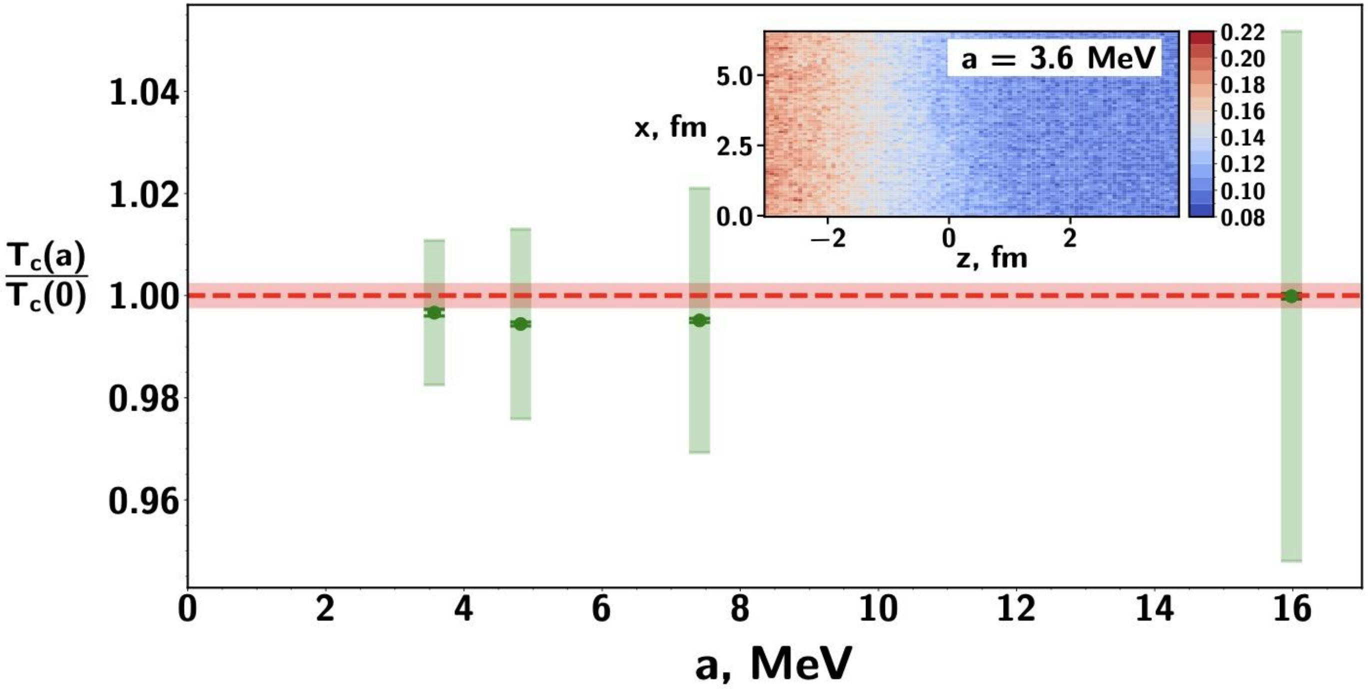
Polyakov loop vs acceleration g

(Yang-XGH, to appear)

No acceleration dependence measured

Lattice results

- A recent simulation for pure Yang-Mills



Deconfinment temperature as determined by Polyakov loop

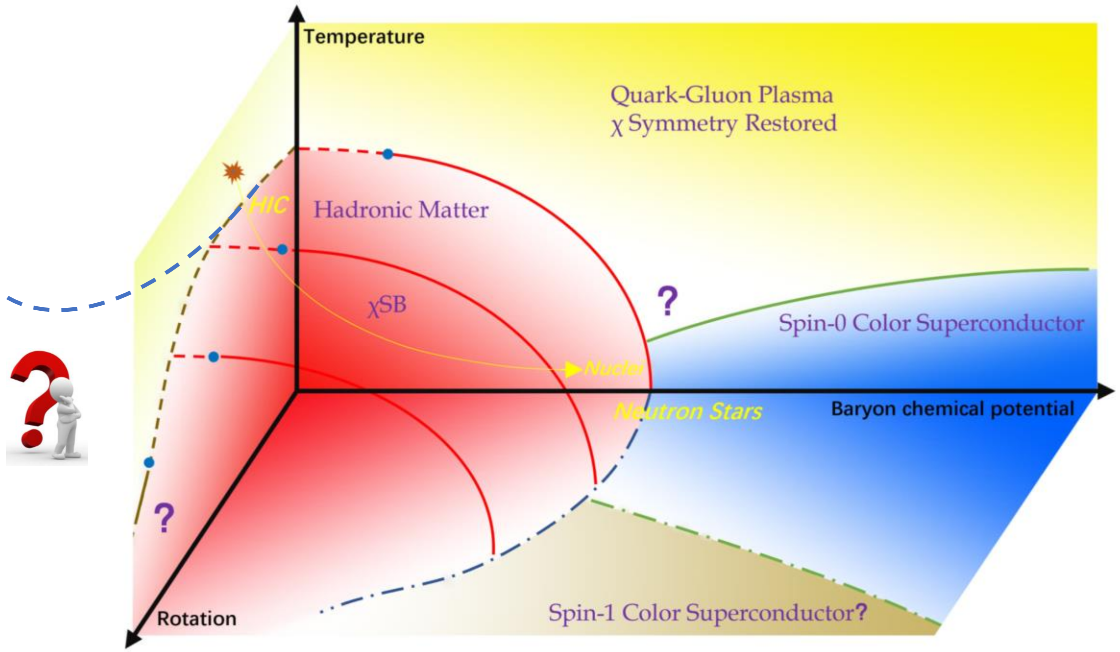
(Braguta etal 2024)

No acceleration dependence measured

Summary and outlooks

Summary and outlooks

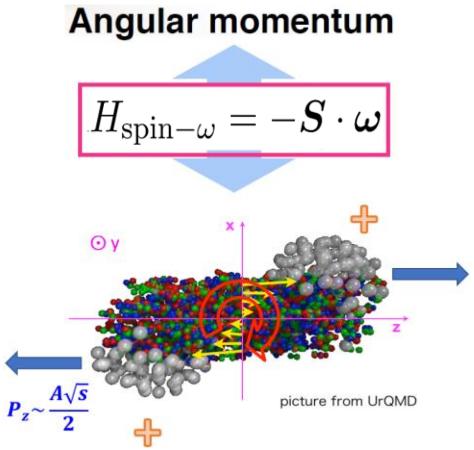
- It is NOT understood how rotation modifies chiral phase transitions.
- Acceleration effects depends crucially on subtraction scheme
- Outlooks:
  - More lattice simulations
  - Cross check torsion effect on chiral condensate and confinement on lattice
    (Yamamoto 2020)
  - Complex Langevin method
    (Azuma-Morita-Yoshida 2023)
  - fRG or DSE for accelerating-rotating QCD
  - More model studies
  - … …



Thank you!

Where rotating quark matter: Quark-gluon plasma

- From global angular momentum to vorticity to hyperon spin polarization

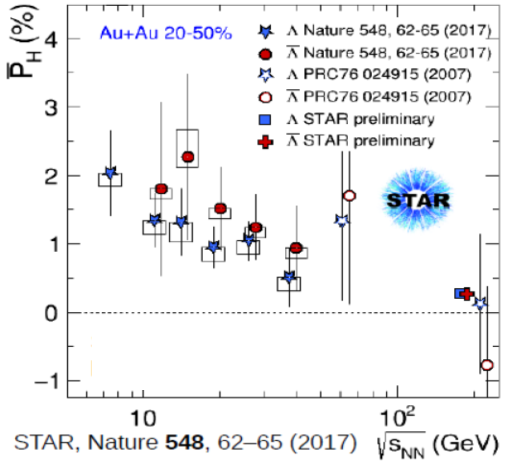


(at thermal equilibrium)

$$\frac{dN_s}{d\boldsymbol{p}} \sim e^{-(H_0 - \boldsymbol{\omega}\cdot\boldsymbol{S})/T}$$

$$\mathrm{P} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \sim \frac{\omega}{2T}$$

- First measurement of Λ polarization by STAR@RHIC *

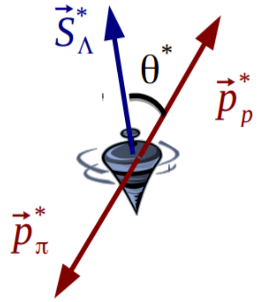


parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P}_\mathbf{\Lambda}\cdot\mathbf{p}_\mathbf{p}^*)$$

α: Λ decay parameter (α_Λ=0.732)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame



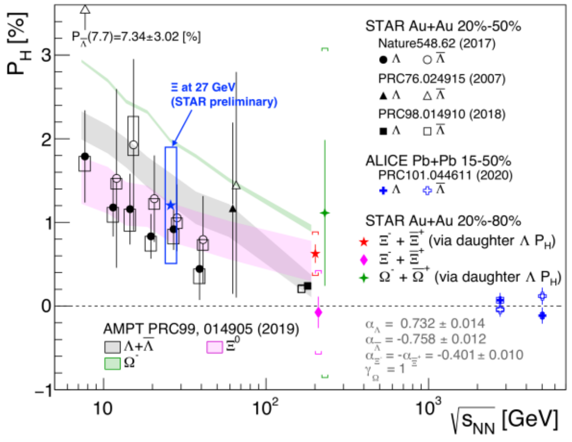
$\Lambda \to p + \pi^-$
(BR: 63.9%, cτ~7.9 cm)

(* First theoretical proposal: Liang and Wang 2004, later by Voloshin 2004)

Where rotating quark matter: Quark-gluon plasma

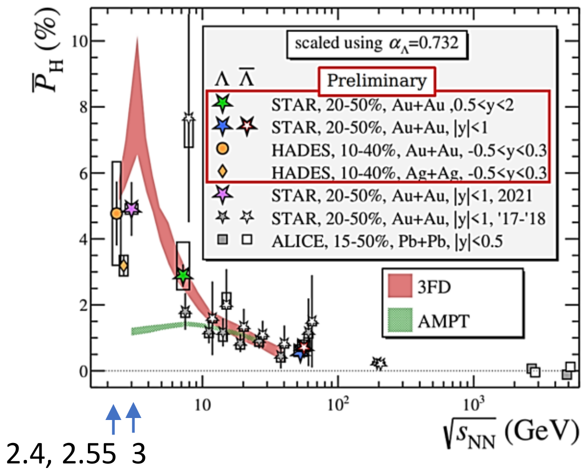
- More recent measurements: Ξ^-, Ω^- by STAR@RHIC, Λ by ALICE@LHC

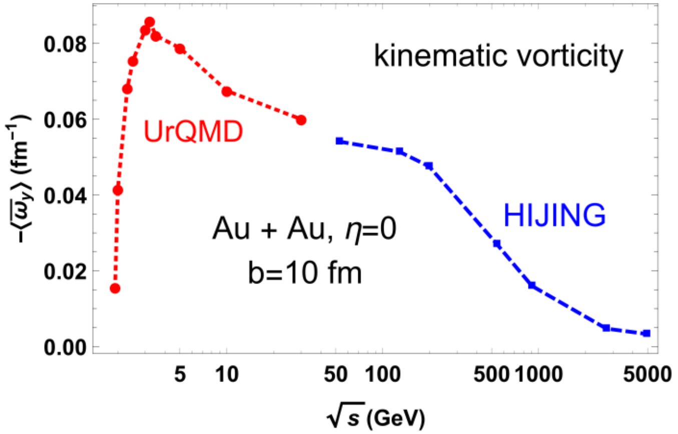
STAR, PRL126, 162301 (2021)



hyperon	decay mode	α_H	magnetic moment μ_H	spin
Λ (uds)	$\Lambda\to p\pi^-$ (BR: 63.9%)	0.732	-0.613	1/2
Ξ^- (dss)	$\Xi^-\to\Lambda\pi^-$ (BR: 99.9%)	-0.401	-0.6507	1/2
Ω^- (sss)	$\Omega^-\to\Lambda K^-$ (BR: 67.8%)	0.0157	-2.02	3/2

- Λ at low energy by STAR@RHIC 2021, HADES@GSI 2021





(Deng-XGH 2016, Deng-XGH-Ma-Zhang 2020)

- "The most vortical fluid": $\omega \sim 10^{20} - 10^{21} s^{-1}$
- Relativistic suppression at high energies

Effect of rotation: Comparison with magnetic field

- Hints for possible rotation effect: comparison with B field

Spin:

Magnetic moment

$$H_{\text{Zeeman}} = -\gamma \boldsymbol{S} \cdot \boldsymbol{B}$$

Magnetic field

Angular momentum

$$\boxed{H_{\text{spin}-\omega} = -\boldsymbol{S} \cdot \boldsymbol{\omega}}$$

Rotation field

Orbital:

In magnetic field, Lorentz force:

$$\boldsymbol{F} = e(\dot{\boldsymbol{x}} \times \boldsymbol{B})$$

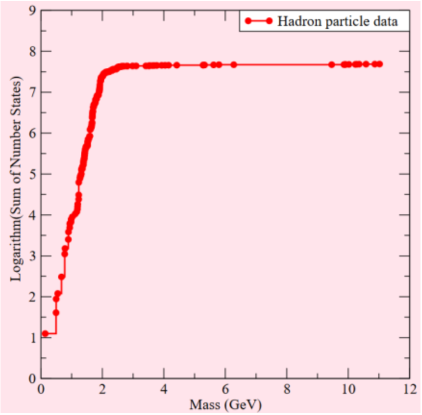
Larmor theorem: $\boldsymbol{eB \sim 2m\omega}$

In rotating frame, Coriolis force:

$$\boldsymbol{F} = \boldsymbol{2m}(\dot{\boldsymbol{x}} \times \boldsymbol{\omega}) + \boldsymbol{O}(\boldsymbol{\omega}^{\boldsymbol{2}})$$

Confinement under rotation

- It is not easy to intuitively imagine the rotational effect on confinement
- Argument based on hadron resonance gas (HRG) model

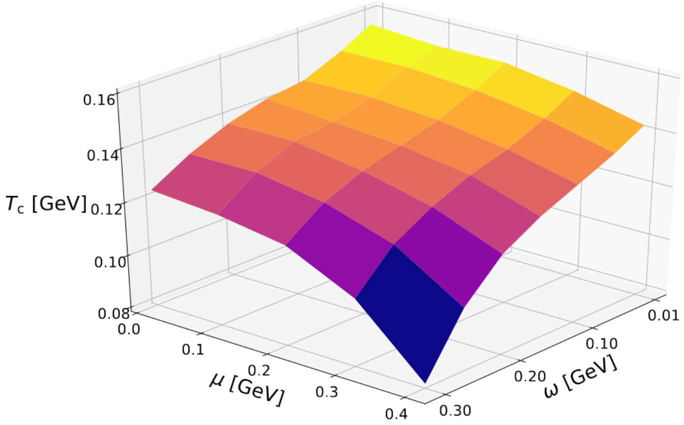


Interpreted as deconf. T

$$\rho(m) = e^{m/T_H} \;\Rightarrow\; Z = \int dm\, \rho(m)\, e^{-m/T} \;\Rightarrow\; \text{diverges for } T > \boxed{T_H}$$

Chosen to be indep. of rotation

$$\left.\begin{aligned} p(T,\mu,\omega;\Lambda) &= \sum_{m;\, M_i \le \Lambda} p_m + \sum_{b;\, M_b \le \Lambda} p_b \\ p_{\mathrm{SB}} &\equiv (N_c^2 - 1)\, p_g + N_c N_f\, (p_q + p_{\bar{q}}) \end{aligned}\right\} \quad \frac{p}{p_{\mathrm{SB}}}(T_c,\, \mu,\, \omega) = \boxed{\gamma} \;\Rightarrow$$



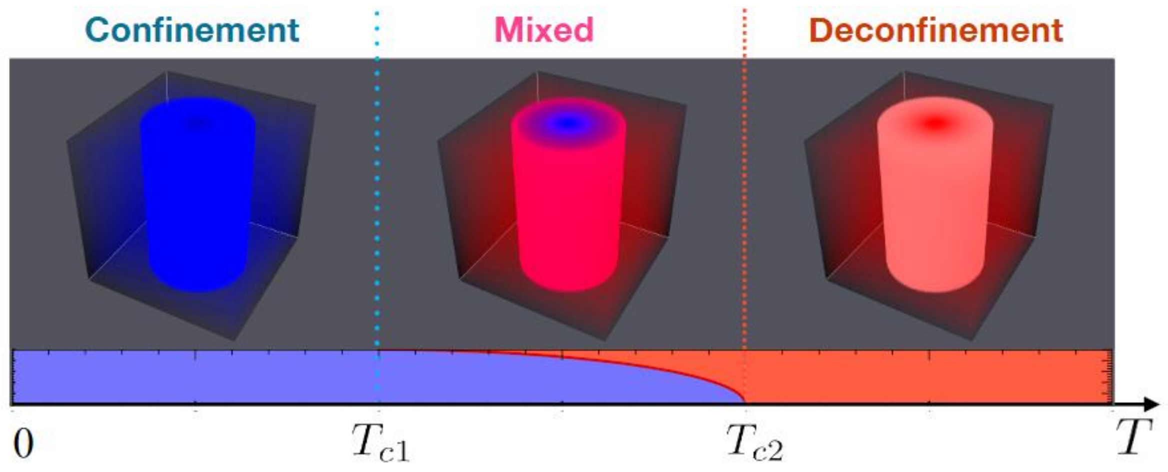
(Fujimoto-Fukushima-Hidaka 2021)

- Rotation favors deconfinement

Confinement under rotation

- It is not easy to intuitively imagine the rotational effect on confinement
- Argument based on Tolman-Ehrenfest temperature

$$\left.\begin{aligned} T(\boldsymbol{x})\sqrt{g_{00}(\boldsymbol{x})} &= T_0 \\ g_{00} &= 1-\rho^2\Omega^2 \end{aligned}\right\} \quad T(\rho) = \frac{T(0)}{\sqrt{1-\rho^2\Omega^2}}$$

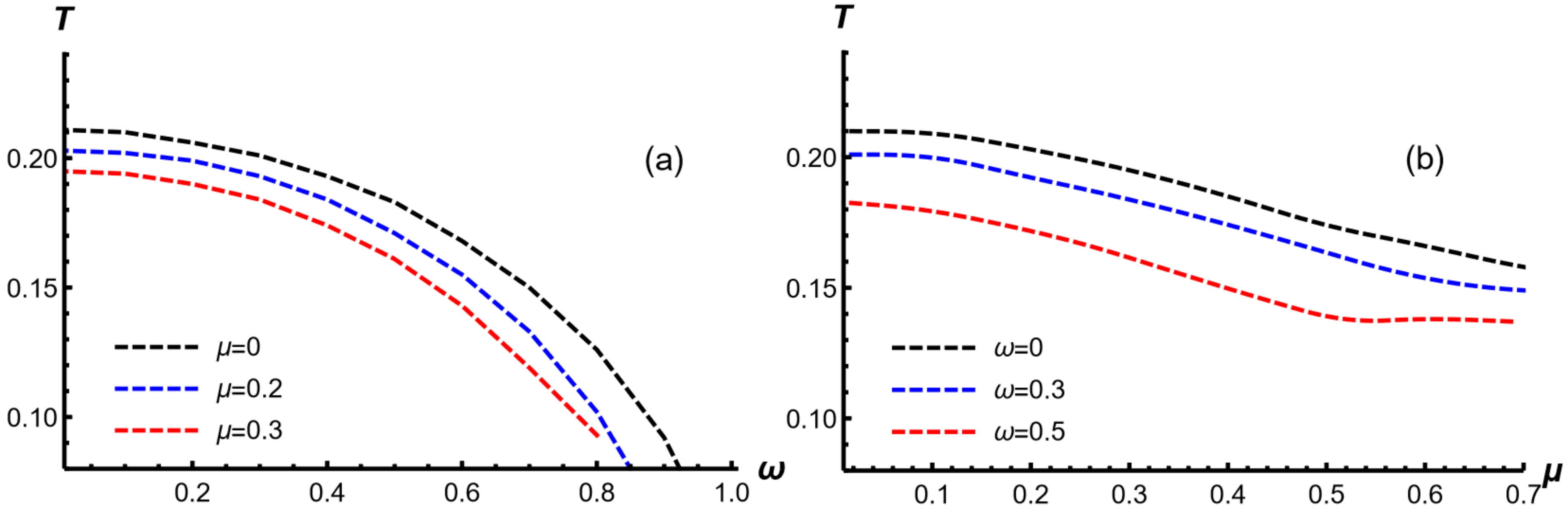


(Chernodub 2020)

- Rotation favors deconfinement

Confinement under rotation

- It is not easy to intuitively imagine the rotational effect on confinement
- Results based on holography

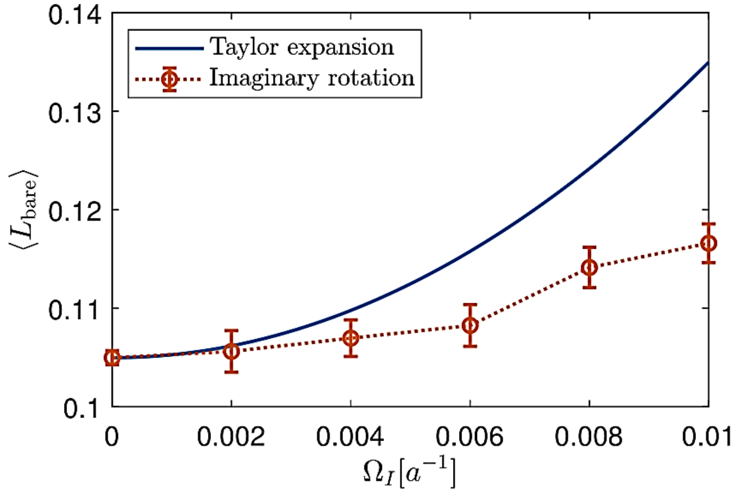


(Chen-Zhang-Li-Hou-Huang 2020)

- Rotation favors deconfinement

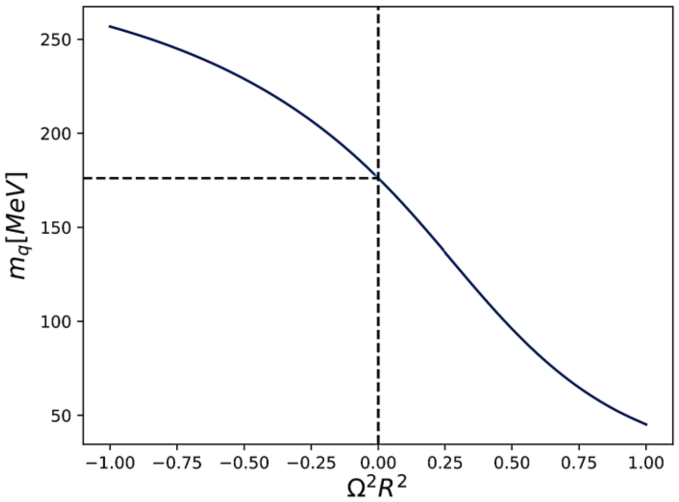
Is analytical continuation sensible?

- For unbounded system, imaginary rotation is always OK, but real rotation is not. So the analytical continuation is problematic.
- For finite system preserving causality, the analytical continuation is OK



Real rotation lattice simulation using Taylor expansion

(Yang-XGH 2023)

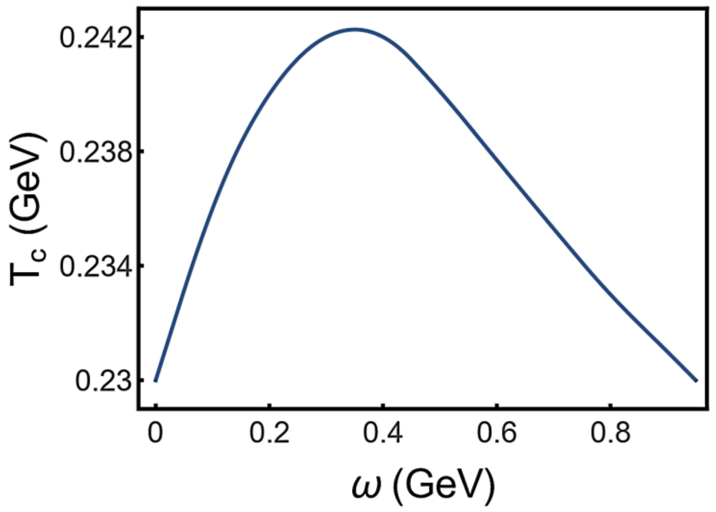


fRG with real and imaginary rotation

(Chen-Zhu-XGH 2023)

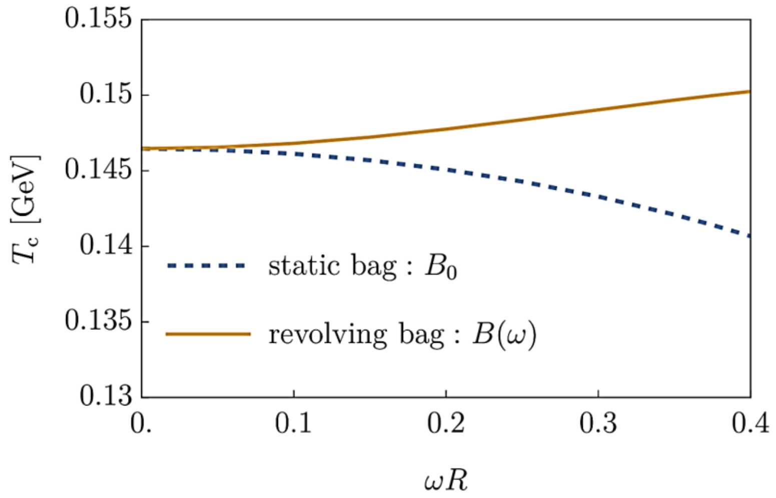
Vacuum does not rotate?

- Natural to expect that the perturbative vacuum does not rotate
- Is it true for QCD vacuum containing nontrivial gluon condensate?



Deconfinement temperature in presence of Caloron background

(Jiang 2023)



Bag constant may response to rotation and enhance the deconfinement temperature

(Mameda 2023)

Important to have other nonperturbative calculations

- fRG for rotating QCD

Gluon propagator

$$G_{\hat{\mu}\hat{\nu}}(x,x') = \sum_n \sum_l \int \frac{p_t \mathrm{d}p_t \mathrm{d}p_z}{(2\pi)^2}\left[\frac{1}{p_l^2}\delta^L_{\hat{\mu}\hat{\nu}} + \left(\frac{1}{p_{l+1}^2} + \frac{1}{p_{l-1}^2}\right)\delta^T_{\hat{\mu}\hat{\nu}} + \left(\frac{1}{p_{l+1}^2} - \frac{1}{p_{l-1}^2}\right)S_{z\hat{\mu}\hat{\nu}}\right]\mathrm{e}^{i\omega_n\Delta\tau + il\Delta\theta + ip_z\Delta z} J_l(p_T r) J_l(p_T r')$$

3 vertex can be handled

$$\int_0^\infty J_{n_1}(k_1\rho)J_{n_2}(k_2\rho)J_{n_3}(k_3\rho)\rho\, d\rho$$

$$= \frac{\Delta}{6\pi A}[\cos(n_1\alpha_2 - n_2\alpha_1) + \cos(n_2\alpha_3 - n_3\alpha_2) + \cos(n_3\alpha_1 - n_1\alpha_3)]$$

4 vertex is difficult

$$\int r\mathrm{d}r J_{l_1}(p_{1t}r)J_{l_2}(p_{2t}r)J_{l_3}(p_{3t}r)J_{l_4}(p_{4t}r)\delta(l_1+l_2+l_3+l_4) \;\; = ?$$