The determination of QCD fluctuations and the freeze out line

Fei Gao

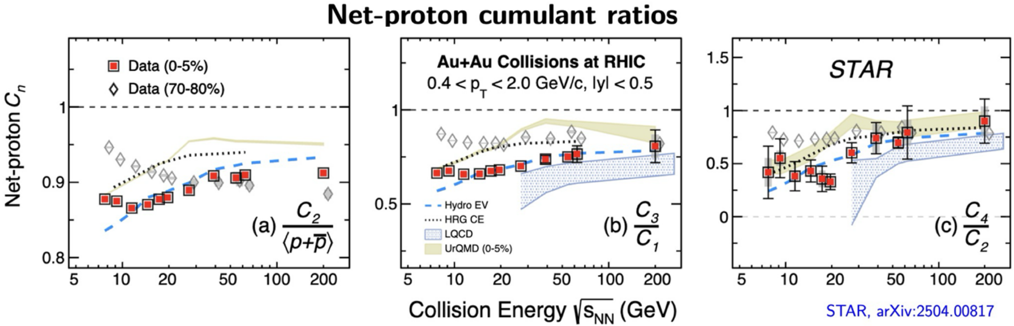
Beijing Institute of Technology, BIT

fQCD collaboration :

Braun, *Chen*, *Fu*, *Gao*, *Ihssen*, *Geissel*, *Huang*, *Lu*, *Pawlowski*, *Rennecke*, *Sattler*, *Schallmo*, *Stoll*, *Tan*, *Toepfel*, *Turnwald*, *Wen*, *Wessely*, *Wink*, *Yin*, *Zorbach*

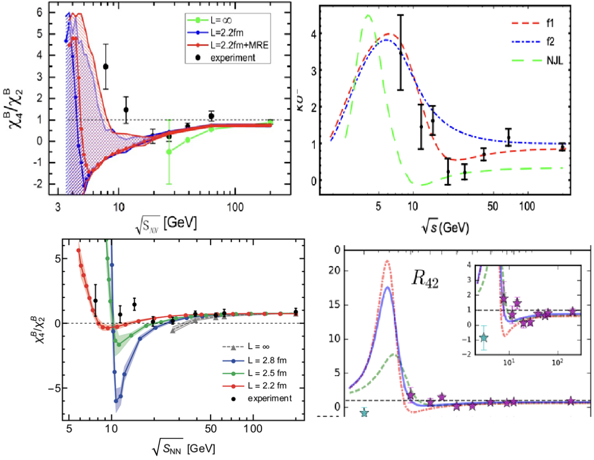
QCD in heavy ion collisions:

QCD phase transitions between hadrons and asymptotic quarks/gluons.



STAR, arXiv:2504.00817

- The measurements are the cumulants/fluctuations at freeze out line.
- The equilibrium calculation gives a base line of the fluctuations, but only if the results are quantitatively correct.
- The non equilibrium effect is important, and should be incorporated based upon the equilibrium results.



DSE with interaction model:
Jing Chen, FG, Yuxin Liu,
arXiv:1510.07543(2015);
Yi Lu et al,
PRD 105, 034012 (2022)
P Isserstedt, M Buballa, CS Fischer, PJ Gunkel
PRD 100, 074011 (2019)
PNJL:
Zhibin Li, Kun Xu, Xinyang Wang, Mei Huang,
EPJC (2019) 79:245;
fRG with LEFT:
Weijie Fu, Xiaofeng Luo, J. Pawlowski, F. Rennecke, S. Yin,
PRD 111, L031502 (2025)

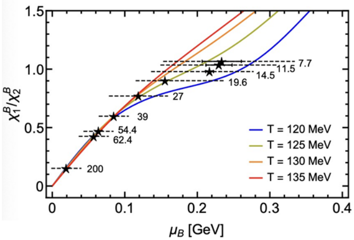
To compare with experiment, one needs to know the freeze out line.

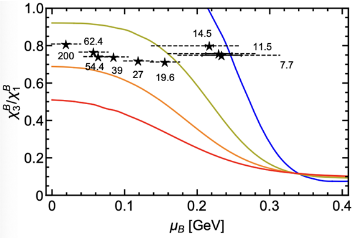
- The freeze out condition can be determined by matching the theoretical calculation of fluctuations with the experimental data as firstly proposed in Lattice simulations.

A. Bazavov et al, PRL 109, 192302 (2012); S.Borsanyi et al, PRL 111, 062005 (2013); S. Borsanyi, et al, PRL 113, 052301 (2014)

- This matching gives an exact 2 to 2 mapping after scanning the T-μ_B plane:

(Jing Chen, FG, Yuxin Liu, *arXiv:1510.07543(2015); Yi Lu, et al, PRD 105, 034012 (2022)*)





$$\frac{\chi_2^B}{\chi_1^B}(T_f, \mu_{B,f}) = \frac{C_2}{C_1}(\sqrt{s_{\mathrm{NN}}})$$

$$\frac{\chi_3^B}{\chi_1^B}(T_f, \mu_{B,f}) = \frac{C_3}{C_1}(\sqrt{s_{\mathrm{NN}}})$$

A self consistent determination of freeze out line and the prediction of kurtosis and also higher order fluctuations at freeze out line.

The "phase transition" from model calculation to first principle QCD

Philosophenweg 16, 69120, Heidelberg

For Dyson-Schwinger equation approach:

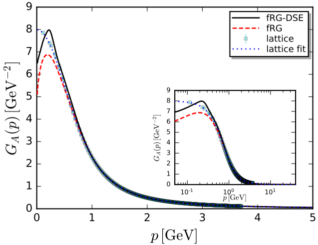
- Step 1: Vacuum input of quark gluon vertex from fRG and STI scaling analysis: (*FG, J. Pawlowski, PRD 102, 034027 (2020)*)
  $T_c^0 = 154$MeV, $\kappa = 0.015$ and CEP at (108,654) MeV.
- Step 2: Calculate the Difference DSE between finite T/μ_B and vacuum self consistently: (*FG, J. Pawlowski, PLB 820 (2021) 136584*)
  $T_c^0 = 152$MeV, $\kappa = 0.015$ and CEP at (109,610) MeV.

Minimal scheme of solving QCD:(Y. Lu, FG, J. Pawlowski, Y. Liu, PRD 110, 014036 (2024))

- Minimal fluctuations: expand the correlation functions on the vacuum data from lattice QCD/YM theory, and calculate the difference DSE.
- Minimal correlation functions: choose the dominant Dirac structures

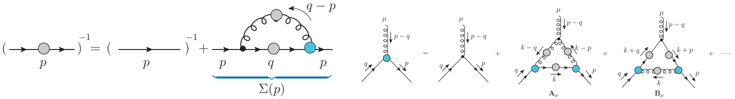
The minimal scheme

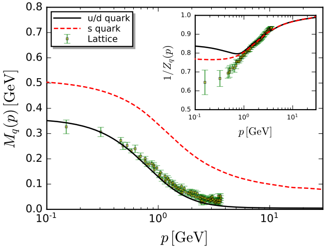
The Yang-Mills sector is relatively separable. One can apply the data in vacuum and compute the difference between finite T/μ and vacuum.



Lattice:
A. G. Duarte et al, PRD 94, 074502 (2016),
P. Boucaud et al, PRD 98, 114515 (2018),
S. Zafeiropoulos et al, PRL122, 162002 (2019)
fRG:
W.-j. Fu et al, PRD 101, 054032 (2020)
Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94 (2016) 5, 054005

Solve the DSEs of quark propagator and quark gluon vertex:





lattice: P. O. Bowman et al, PRD71, 054507 (2005) **fRG**: W.-j. Fu et al, PRD 101, 054032 (2020) **DSE**: FG et al, PRD 103, 094013(2021)

A further simplification on the quark gluon vertex:

Quark gluon vertex In Landau gauge:

$$\Gamma^{\mu}(q,p) = \sum_{i=1}^{8} t_i(q,p) P^{\mu\nu}(q-p) \mathcal{T}_i^{\nu}(q,p) \,,$$

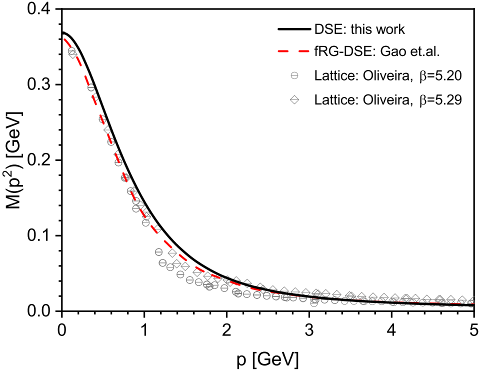
The dominant structures are Dirac and Pauli term:

$$\mathcal{T}_1(p,q) = -i\gamma^{\mu} \,, \mathcal{T}_4^{\mu}(p,q) = \sigma_{\mu\nu}(p-q)^{\nu} \,,$$

$$t_1(p,q) = F(k^2)\frac{A(p^2)+A(q^2)}{2}$$

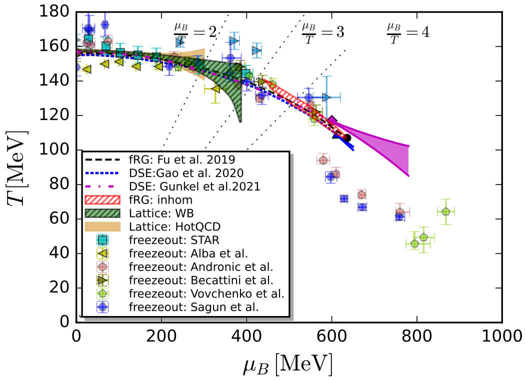
$$t_4(p,q) = \left[Z(k^2)\right]^{-1/2} \frac{B(p^2)-B(q^2)}{p^2-q^2}$$

All quantities are expressed by the running of two point functions.
The Quark Mass function:



FG, J. Papavassiliou, J. Pawlowski, PRD 103.094013 (2021).
Y, Lu, **FG**, YX Liu, J. Pawlowski, PRD 110, 014036 (2024)

Chiral Phase diagram for 2+1 flavour QCD can be directly obtained from quark propagator



The fQCD computations of chiral phase transition are converging:

- $T_c = 155\,\mathrm{MeV}$ and $\kappa \sim 0.015$
- Estimated range of CEP:
  $T \in (100, 110)$ MeV
  $\mu_B \in (600, 700)$ MeV

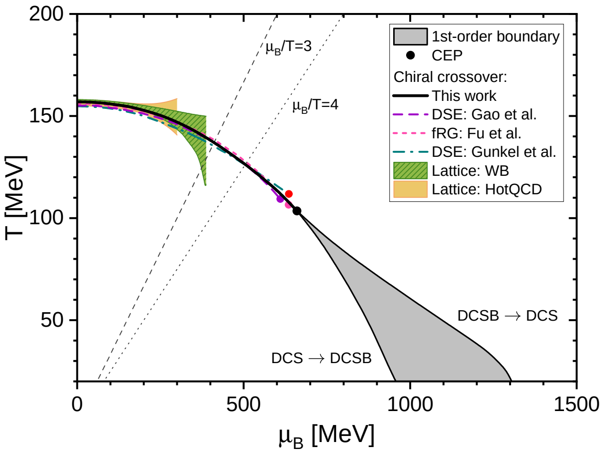
W.-j. Fu et al, PRD 101, 054032 (2020)

FG and J. Pawlowski, PRD 102, 034027 (2020)

FG and J. Pawlowski, PLB 820, 136584(2021)

P.J. Gunkel, C. S. Fischer, PRD 104, 054022 (2021).

Phase diagram in temperature-chemical potential region for 2+1 flavour QCD



- Hadron resonance channel does not have a big impact on QCD phase diagram, as $\delta S/S \sim \delta f = \frac{\text{binding energy}}{\text{nucleon mass}} \approx 0.016$
- Coexistence region slightly above liquid gas transition.
- At zero temperature: Ideal phase transition at $\mu_B \approx 1100$ MeV

[1] Yi Lu, **FG**, Yu-xin Liu. arXiv:2509.02974.

Currently, the functional QCD approaches can only calculate the quark potential directly, while the gluon sector still awaits further investigations.
One may incorporate the lattice QCD simulation at $\mu = 0$ here to combine the advantages of the two methods. One can calculate the quark number densities $\{n_q\}$ at finite chemical potential and obtain the pressure by:

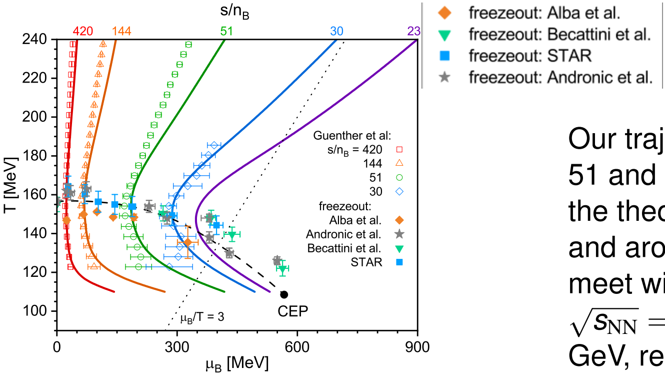
$$n_q^f(T, \mu_B) \simeq -N_c Z_2^f\, T \sum_n \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \mathrm{tr}_D \left[\gamma_4 S^f(p)\right]$$

$$P(T, \boldsymbol{\mu}) = P_{Latt.}(T, \mathbf{0}) + \sum_q \int_0^{\mu_q} n_q(T, \mu)\, d\mu$$

[1] P. Isserstedt, C.S. Fischer and T. Steinert, PRD103 (2021) 054012
[2] **FG**, Yuxin Liu, PRD 94 (2016) 9, 094030
[3] H. Chen, M. Baldo, G. F. Burgio, and H.-J. Schulze, PRD86(2012)045006

QCD thermodynamic properties

isentropic trajectories in the miniDSE scheme:



Our trajectories for $s/n_B = 420$, 144, 51 and 30 which values are chosen in the theoretical studies, and around phase transition line, also meet with the freezeout points at $\sqrt{s_{\rm NN}} = 200$, 62.4, 19.6 and 11.5 GeV, respectively.

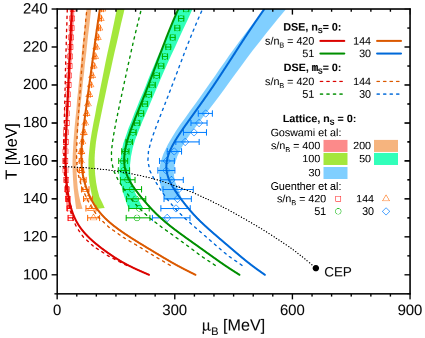
However, the temperature behavior does not match completely. Besides, the low temperature limit of the kurtosis is also wrong.

Y. Lu, FG, J. Pawlowski, Y. X. Liu, PRD 110, 014036 (2024)

"Because you did not include the Polyakov loop/A_0 condensate. "
-J. Pawlowski

A_0 feeds back on the QCD thermodynamic functions via the quark propagator:

$$G_q^{-1}(p) = i(\omega_p + i\mu_q + gA_0)\gamma_4 Z_q^E(p) + i\vec{\gamma}\cdot\vec{p}\, Z_q^M(p) + Z_q^E(p) M_q(p).$$

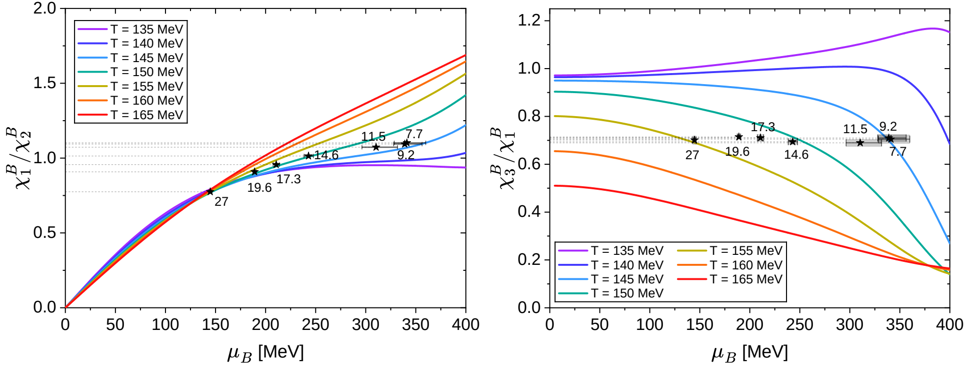


see details in Y. Lu's talk

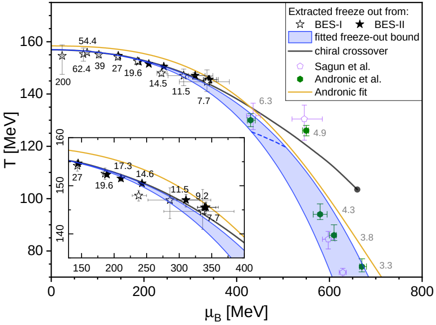
Y. Lu, FG, J. Pawlowski, Y. X. Liu, arXiv:2504.05099

Extraction of the freeze out parameters:

We compare the up to date functional QCD results with the up to date experimental data of BESII from Xiaofeng Luo:



The direct extraction and the fitting procedure of freeze out line



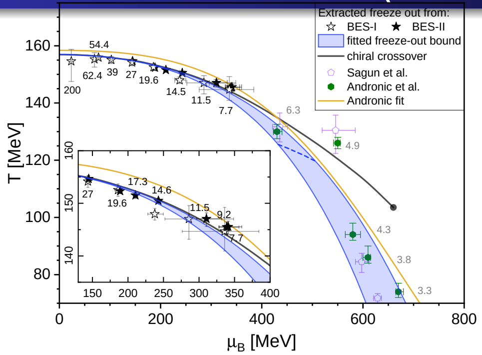
$$T_f = T_0 \left[1 - \kappa_2^f \left(\frac{\mu_{B,f}}{T_0} \right)^2 - \kappa_4^f \left(\frac{\mu_{B,f}}{T_0} \right)^4 + \cdots \right]$$

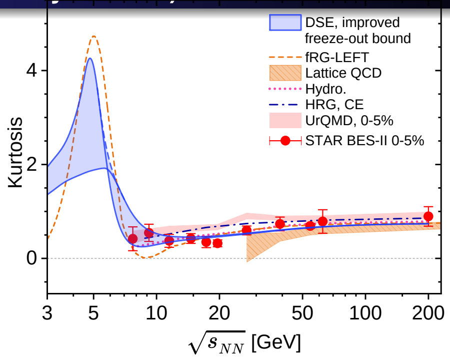
- The direct extraction from data gives: $T_0 = T_c(0) = 157$ MeV; $\kappa_2^f \leq \kappa_2 = 0.0153$.
- Assumption: freeze out after phase transition: $\kappa_2^f \geq \kappa_2 = 0.0153$.

$T_0 = T_c(0) = 157$ MeV; $\kappa_2^f = \kappa_2 = 0.0153$, with only κ_4^f that is determined by one point from the estimate of statistic model at 6.3 GeV from (A. Andronic, et al, Nature 561, 321 (2018).)

- Up to 7.7 GeV, the freeze out line and the phase transition line coincide, and the deviation caused possibly by non equilibrium effects appears from 10 GeV.

The freeze out line and the kurtosis(Preliminary results)





- The peak of kurtosis is located at 4-5 GeV, which can be observed in experiments, as the smoking gun for CEP
- The non equilibrium effects tend to wash out the critical information, and in kurtosis, it means a smaller maximum of the peak, but the location of the peak may not change.
- For instance, the GCE/CE argument gives a suppression factor $1/3$, which gives the observed peak height around 1.

Combination of fQCD computations and BES measurements

The conclusions and discussions:

- A minimal scheme for solving QCD.
- Polyakov loop is essential for thermodynamic quantities.
- The chiral PT and deconfinement is with a CEP at $\mu_B \approx 600 - 700$ MeV.
- The computed fluctuations are consistent with the current measurements in BESII.
- Freeze out line is consistent with phase transition line up to 7.7 GeV.
- In the obtained kurtosis, a peak is located at around 5 GeV, which is not a direct signal of CEP, but if the experiment finds it there too then the CEP would be located at where it should be ($\mu_B \approx 600 - 700$ MeV).

Thank you!