CRITICAL FLUCTUATIONS AND CORRELATIONS OF QUARK SPIN NEAR CEP

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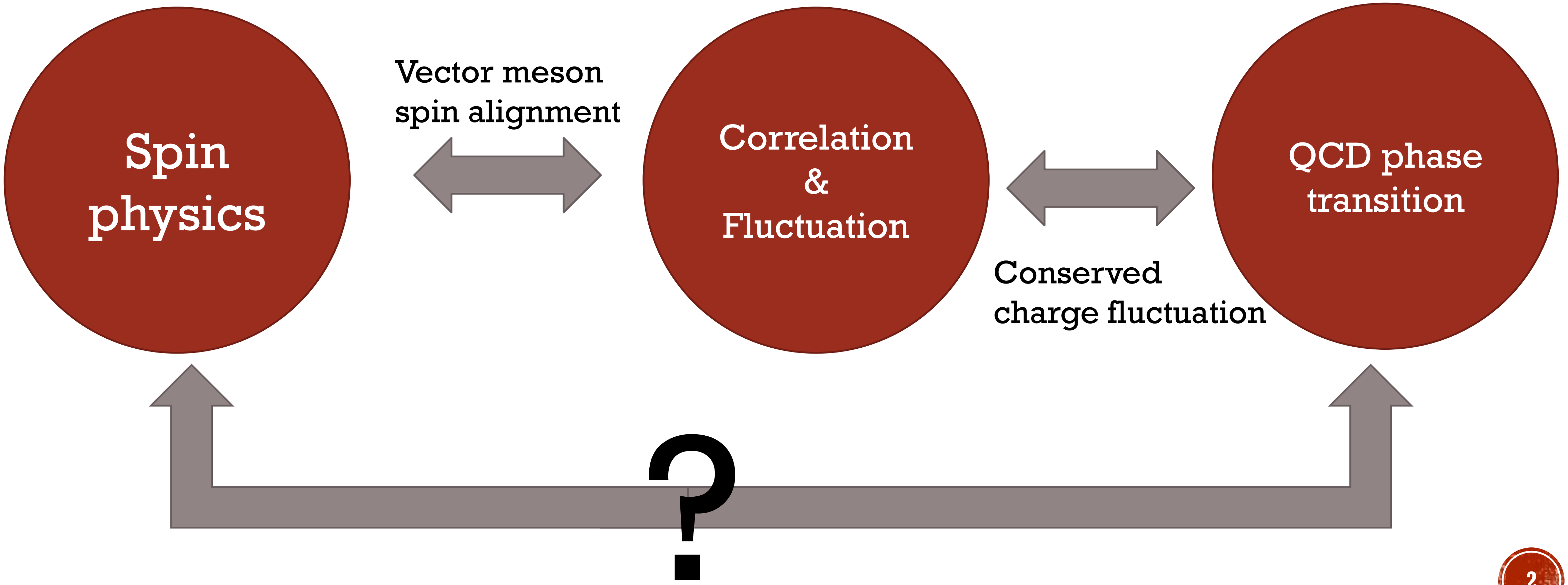
Phys. Rev. Lett. 135， 032302 (2025)

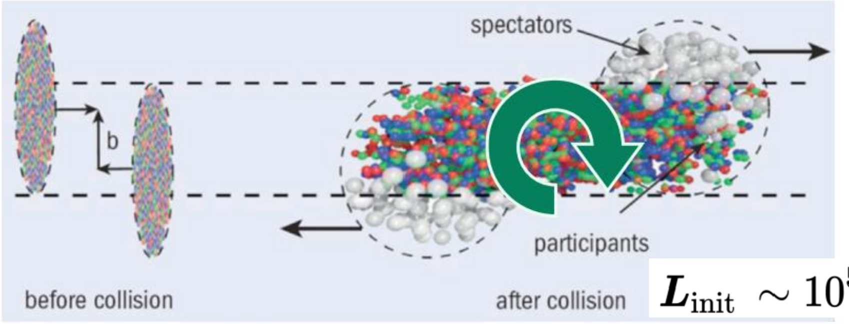
The QCD Phase Diagram:
From Theory to Experimental Signature
@ Dalian, 9th October 2025

OUTLINE


Spin physics
Vector meson spin alignment
Correlation & Fluctuation
Conserved charge fluctuation
QCD phase transition
?

GLOBAL SPIN POLARIZATION

F. Becattini, F. Piccinini, J. Rizzo, PRC 77 (2008) 024906



$L_{\mathrm{init}} \sim 10^5 \hbar$

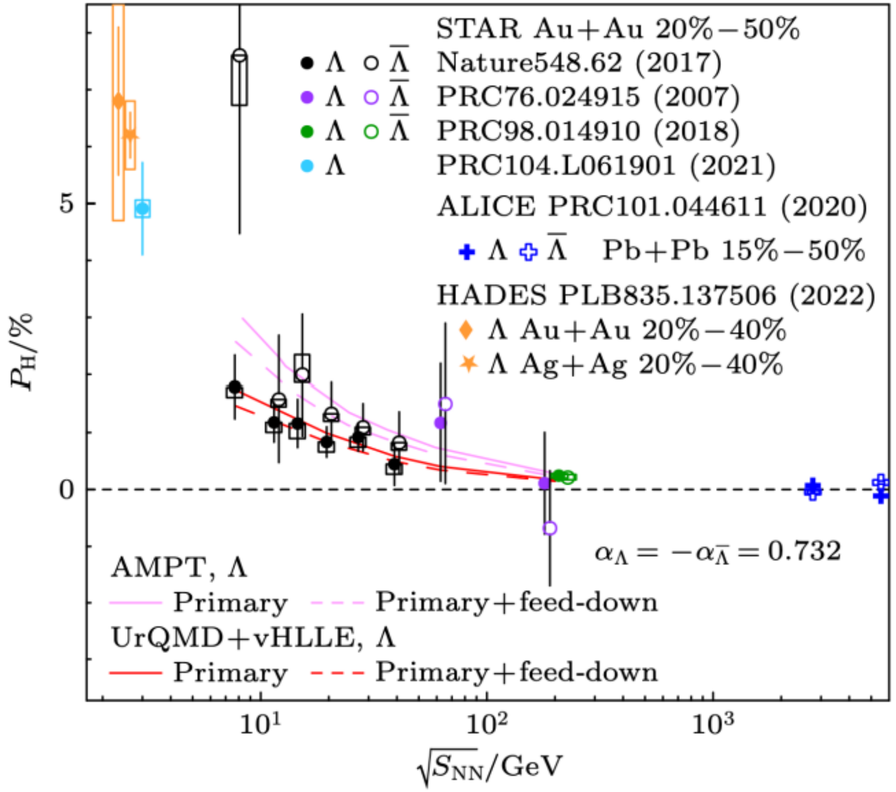
- Quark can be polarized under such a large L
- Coalescence picture : $P_\Lambda = P_s$ Liang, Wang, PRL (2005)
- Estimation of polarization

  Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)

$$\mathbf{P}_\Lambda \simeq \frac{\boldsymbol{\omega}}{2T} + \frac{\mu_\Lambda \mathbf{B}}{T}$$

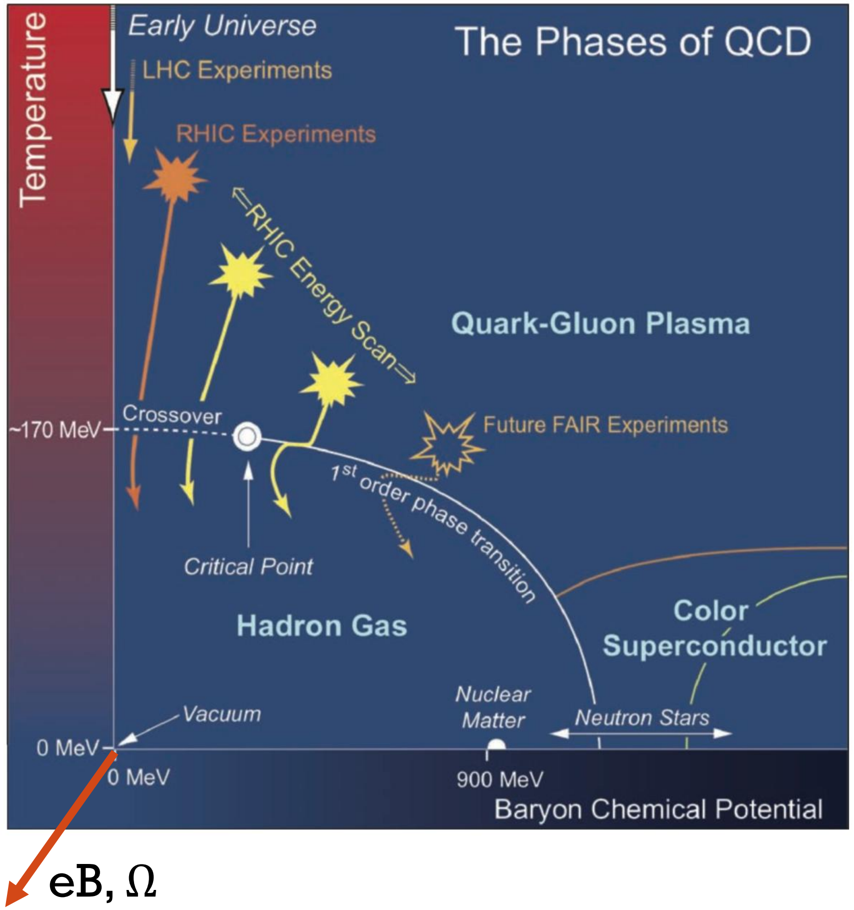
$$\mathbf{P}_{\bar{\Lambda}} \simeq \frac{\boldsymbol{\omega}}{2T} - \frac{\mu_\Lambda \mathbf{B}}{T}$$

- ω can be very large $\sim 10^{21}/s$
- Rotation related effects attract many attentions

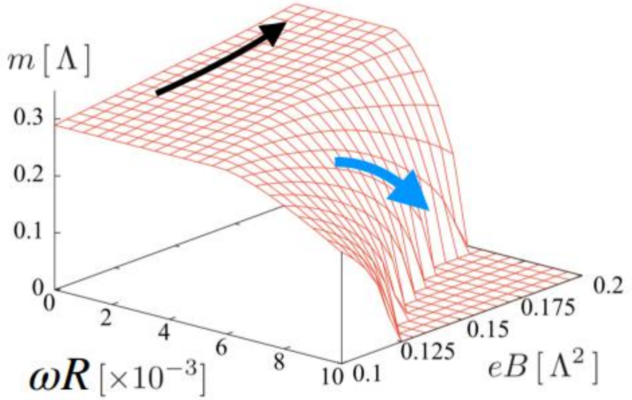


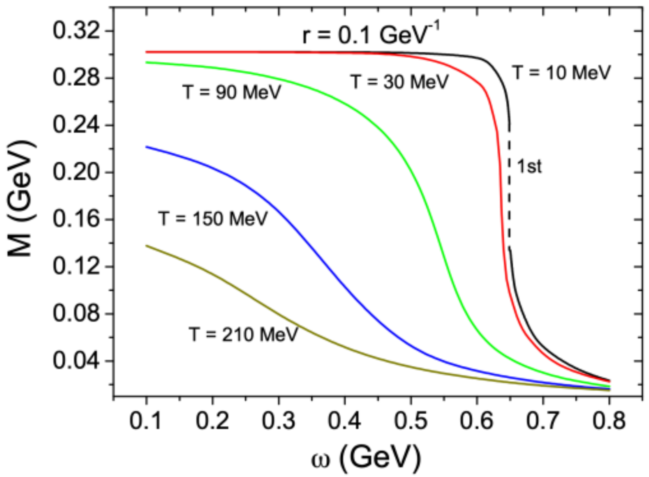
Sun Xu et al., Acta Phys. Sin. 72(7), 072401 (2023)

QCD PHASE DIAGRAM UNDER ROTATION



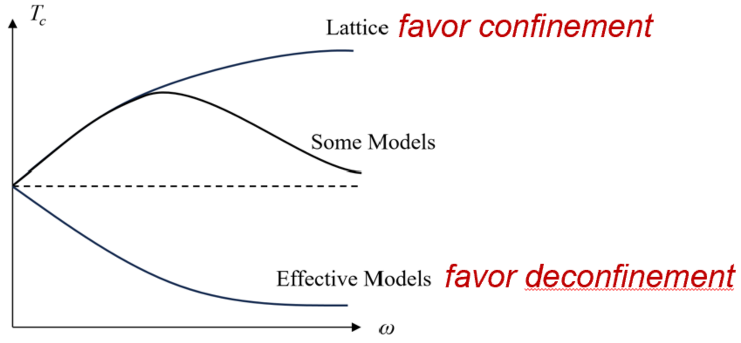
HLC, K. Fukushima, X-G. Huang, K. Mameda, Phys. Rev. D 93, 104052 (2016)





Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016)

- First studied by NJL model (order parameter: quark mass)
- Rotation behaves like chemical potential
- However, lattice studies give opposite result!



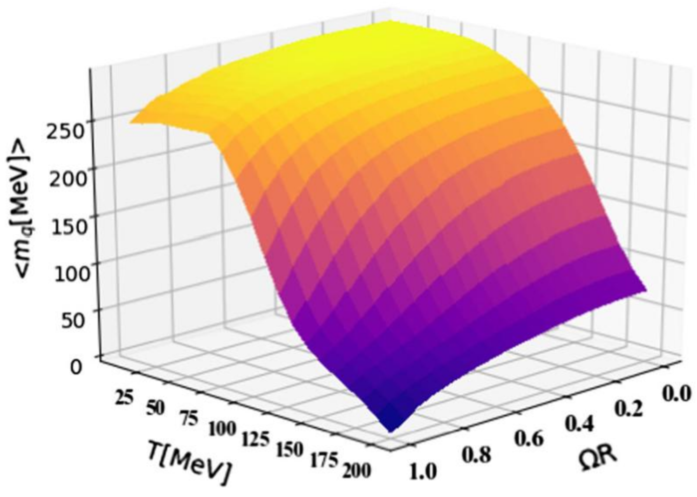
P. Zhuang's talk @PHD2024

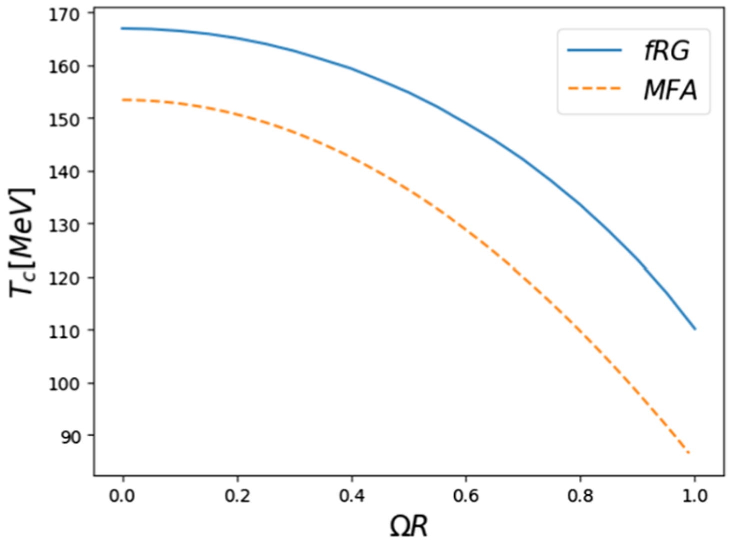
FRG STUDY OF QUARK MESON MODEL UNDER ROTATION

HLC, Z-B. Zhu, X-G. Huang, Phys.Rev.D 108 (2023) 5, 054006

$$\partial_k U_k(r) = \frac{1}{(2\pi)^2}\left\{\sum_{l,i}\frac{1}{N_{l,i}^2}\mathrm{tr}\frac{k\sqrt{k^2-p_{l,i}^2}}{\varepsilon_\phi}\frac{1}{2}\left[\coth\frac{\beta(\varepsilon_\phi+\Omega l)}{2}+\coth\frac{\beta(\varepsilon_\phi-\Omega l)}{2}\right]J_l(p_{l,i}r)^2\theta(k^2-p_{l,i}^2)\right.$$

$$\left.-\sum_{l,i}\frac{1}{\tilde{N}_{l,i}^2}2N_cN_f\frac{k\sqrt{k^2-\tilde{p}_{l,i}^2}}{\varepsilon_q}\frac{1}{2}\left[\tanh\frac{\beta(\varepsilon_q+\Omega j)}{2}+\tanh\frac{\beta(\varepsilon_q-\Omega j)}{2}\right][J_l(\tilde{p}_{l,i}r)^2+J_{l+1}(\tilde{p}_{l,i}r)^2]\theta(k^2-\tilde{p}_{l,i}^2)\right\}.$$





The extension to real QCD under rotation appears to be very challenging.

SPIN ALIGNMENT

- Polarization only relates to single quark
- Vector meson spin alignment relates to quark pair: correlation between quarks will be important
- Coalescence picture : Liang, Wang, PRL (2005)

Spin density matrix element $\rho_{00}^{V} = \frac{1-\langle P_q P_{\bar{q}}\rangle}{3+\langle P_q P_{\bar{q}}\rangle} \approx \frac{1}{3} - \frac{4}{9}\langle P_q P_{\bar{q}}\rangle$

$\rho_{00} \neq 1/3$ indicates spin alignment

- Naïve estimation gives

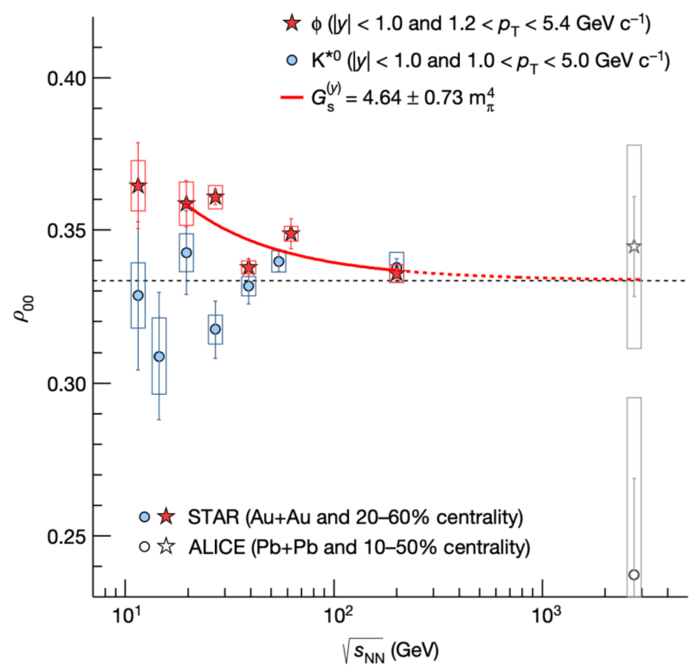
$$\rho_{00} - \frac{1}{3} = -\frac{1}{9}\left(\frac{\omega}{T}\right)^2 \quad \sim 10^{-4}$$

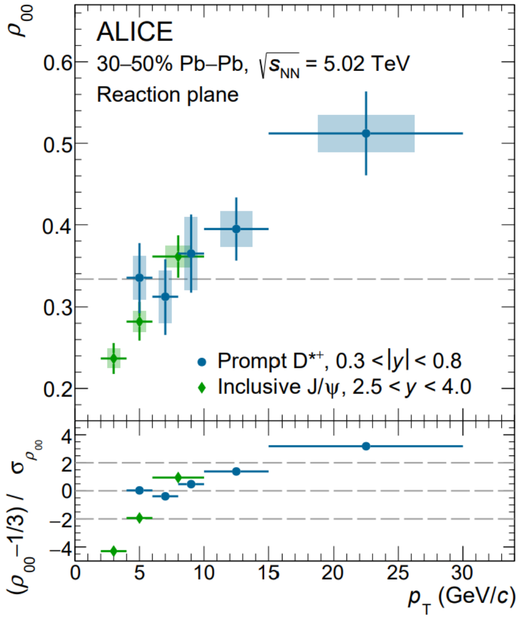
too small

- Further theoretical studies are needed

Correlation and fluctuation are indeed crucial!

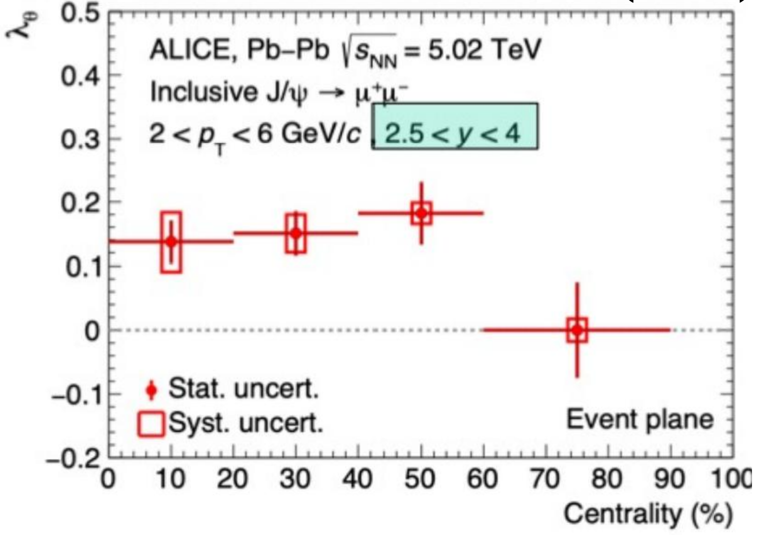
STAR, Nature 614 244 (2023)





ALICE, arXiv: 2504.00714

ALICE, PRL 131 042303 (2023)



$$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}} \qquad \lambda_\theta > 0, \rightarrow \boxed{\rho_{00}^{J/\Psi} < 1/3}$$

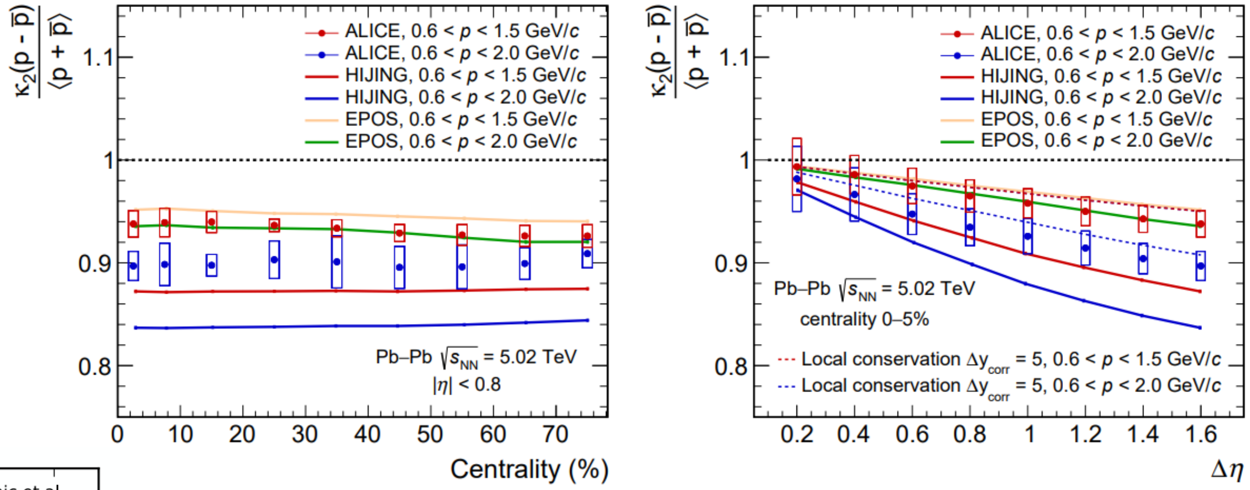
CORRELATION AND FLUCTUATION

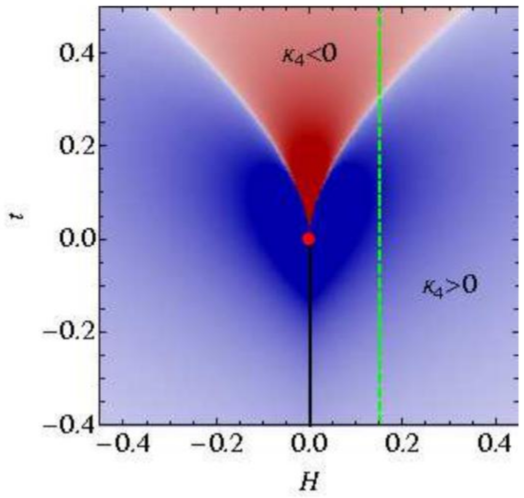
- Baryon number fluctuation

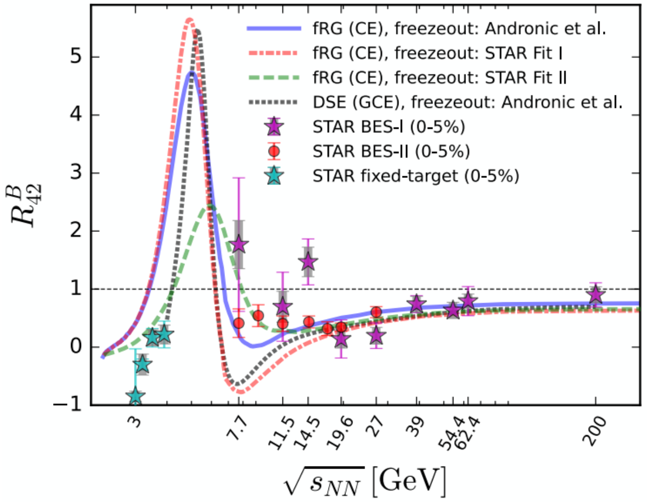
$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4}$$

- Quantifying the nature of the phase transition
- At large density: critical endpoint (CEP)

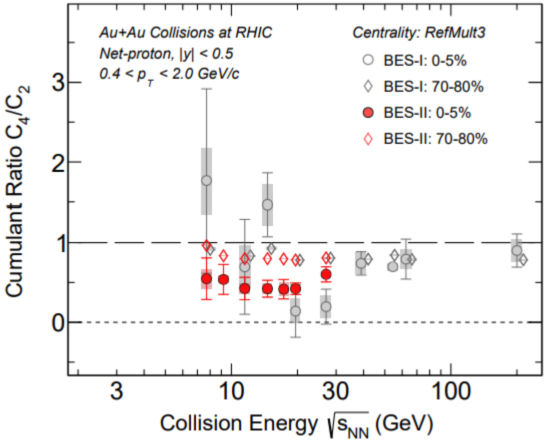
ALICE, PLB 844 (2023) 137545







STAR Collaboration, arXiv:2504.00817



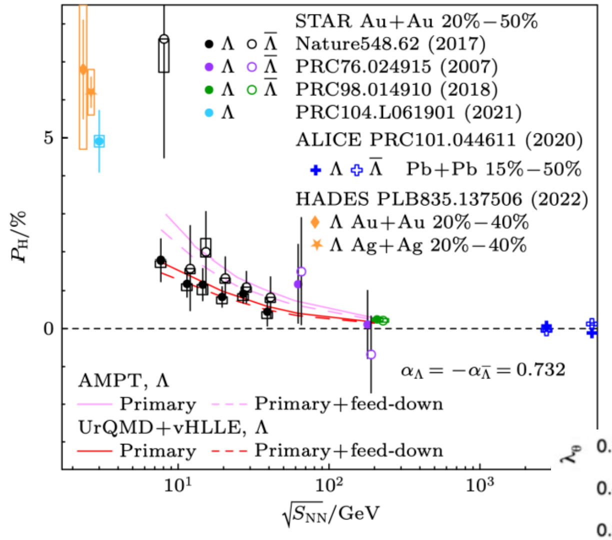
M. Stephanov, PRL 107 (2011) 052301

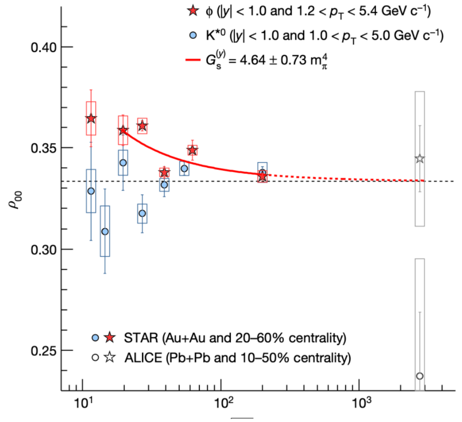
From W. Fu's HENPIC seminar

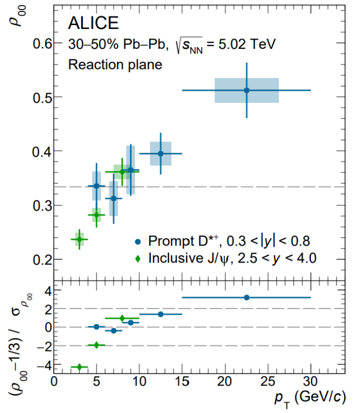
MOTIVATION

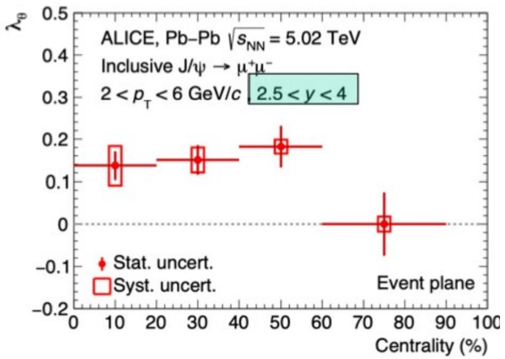
- Almost studied separately

Spin polarization/alignment

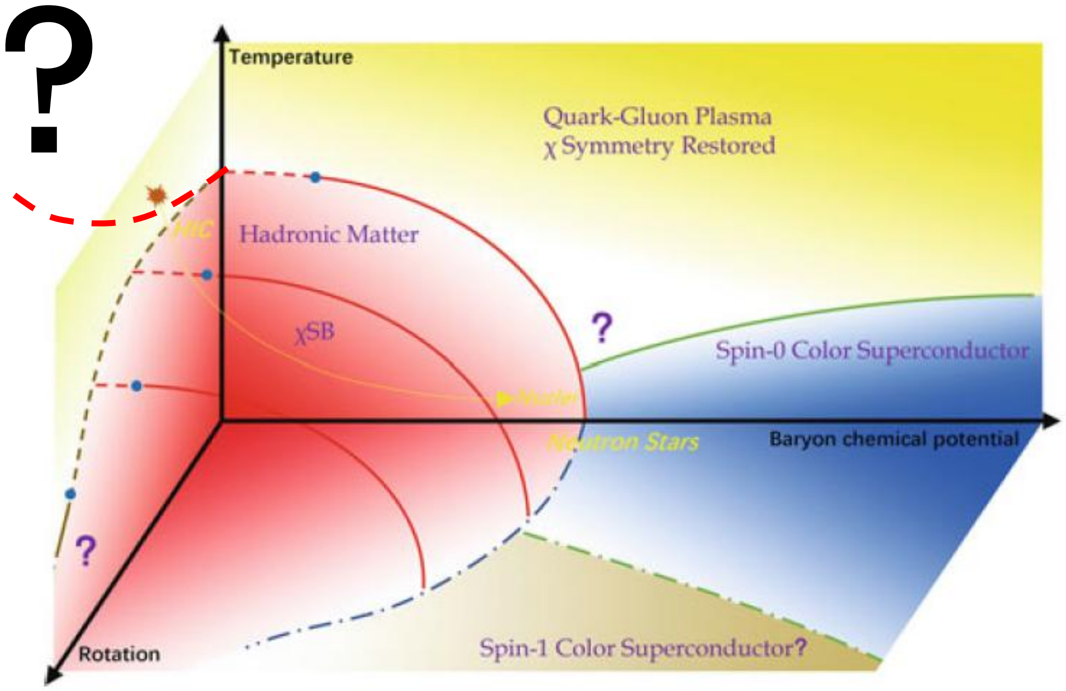








Phase transition

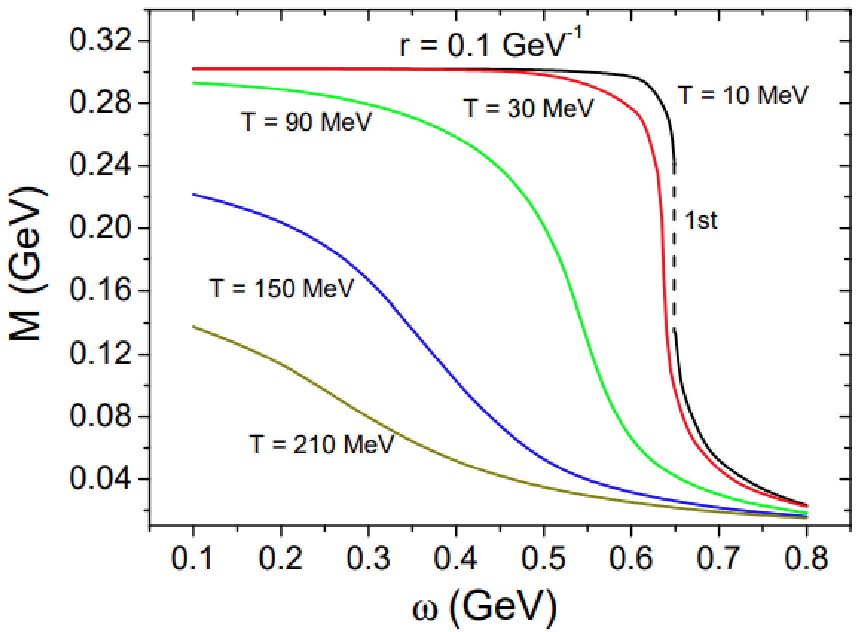


Q: Are these two aspects related?

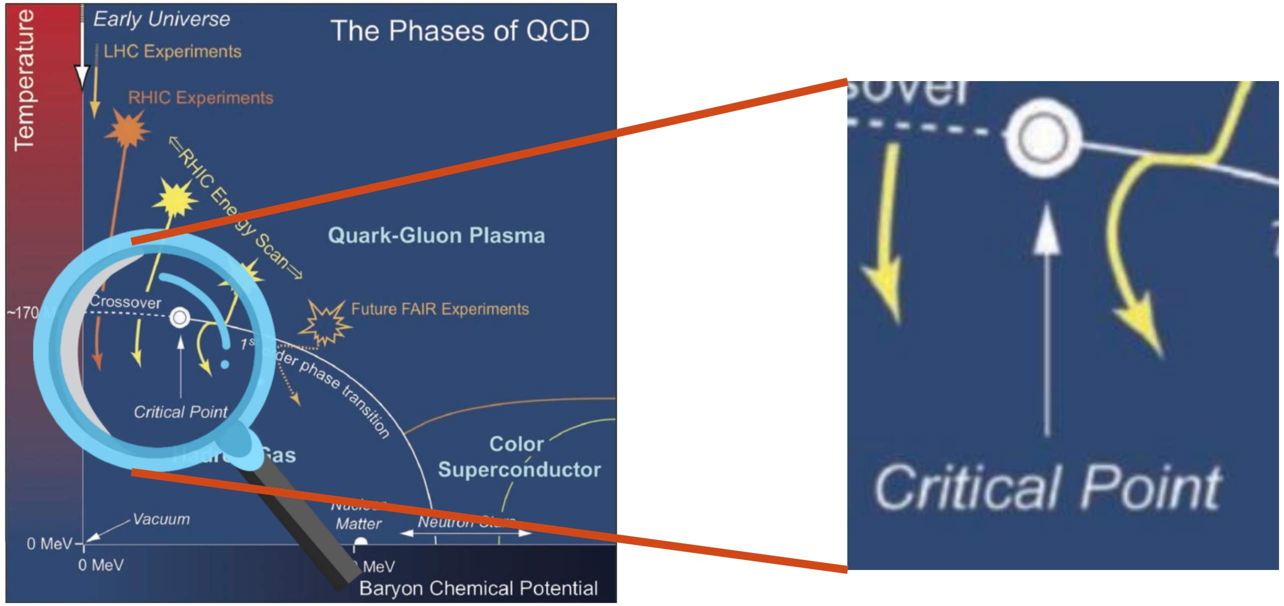
Minghua Wei , Mei Huang, Chin.Phys.C 47 (2023) 10, 104105
Fei Sun, Jingdong Shao, Rui Wen, Kun Xu, Mei Huang, PhysRevD.109.116017
Sushant K. Singh, Jan-e Alam, Eur. Phys. J. C 83, 585 (2023)

NAÏVE ESTIMATIONS

Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016),



- Spin polarization $\sim\Omega$
- Spin alignment $\sim\Omega^2$
- Chiral condensate does not change much at small Ω
- The effect seems not large enough for measurement
- However



!!

Spin fluctuation? Just like baryon number?

QUALITATIVE STUDY: NJL MODEL

Rotating spacetime metric

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- NJL Lagrangian in rotating frame

$$\mathcal{L}_{NJL} = \bar{\psi} i\gamma^\mu \nabla_\mu \psi - m_0 \bar{\psi}\psi + \mu_B \bar{\psi}\gamma^0\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}\gamma^5\vec{\tau}\psi)^2]$$

- QFT in curved spacetime: vierbein formulism

$$e^t_0 = e^x_1 = e^y_2 = e^z_3 = 1, \qquad e^x_0 = y\Omega, \qquad e^y_0 = -x\Omega,$$

- Hamiltonian for fermion

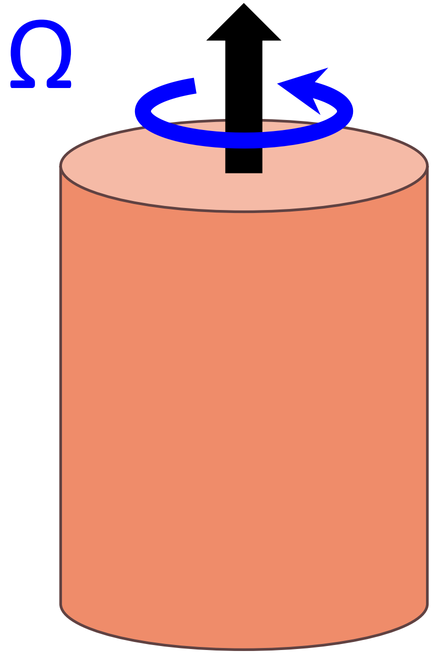
$$\hat{H} = \gamma^0(\vec{\gamma}\cdot\vec{p} + m) - \vec{\omega}\cdot(\vec{x}\times\vec{p} + \vec{S}_{4\times 4}) = \hat{H}_0 - \vec{\omega}\cdot\hat{\vec{J}}.$$

L. Landau and E. Lifshitz, Statistical Physics, Part 1

- Similar as finite density

$$E \to E - \mu$$

Rotation behaves as an effective chemical potential ➡ Restore chiral symmetry



QUALITATIVE STUDY: NJL MODEL

- General local thermodynamic potential under rotation (∂m can be neglected)

$$V_{eff}(r) = \frac{(m-m_0)^2}{4G} - N_c N_f \sum_l \int_0^\Lambda \frac{p_t \mathrm{d}p_t \mathrm{d}p_z}{(2\pi)^2} [\varepsilon_p + T\ln(1+\mathrm{e}^{-\beta(\varepsilon_p-\mu-\Omega j)}) + T\ln(1+\mathrm{e}^{-\beta(\varepsilon_p+\mu+\Omega j)})](J_l^2(p_t r) + J_{l+1}^2(p_t r)).$$

- Away from the center, contribution from orbital angular momentum is dominant
- Since we are interested in spin, we first focus on the physics near the center (r=0)

$$\begin{aligned} V_{eff}^0(\Omega,\mu) = & \frac{(m-m_0)^2}{4G} - N_c N_f \int_0^\Lambda \frac{\mathrm{d}^3 p}{(2\pi)^2} 2\varepsilon_p \\ & + N_c N_f \int_0^\infty \frac{\mathrm{d}^3 p}{(2\pi)^2} [\overset{q,\uparrow}{T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu-\Omega/2)/T})} + \overset{q,\downarrow}{T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu+\Omega/2)/T})} \\ & + \underset{\bar{q},\uparrow}{T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu-\Omega/2)/T})} + \underset{\bar{q},\downarrow}{T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu+\Omega/2)/T})}]. \end{aligned}$$

- We can get information about average spin from this expression

QUALITATIVE STUDY: NJL MODEL

- Without critical fluctuation

$$V^0_{eff}(\Omega,\mu)=\frac{(m-m_0)^2}{4G}-N_cN_f\int_0^\Lambda\frac{\mathrm{d}^3p}{(2\pi)^2}2\varepsilon_p$$

$q,\uparrow$ $q,\downarrow$

$$+N_cN_f\int_0^\infty\frac{\mathrm{d}^3p}{(2\pi)^2}[T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu-\Omega/2)/T})+T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu+\Omega/2)/T})$$

$$+T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu-\Omega/2)/T})+T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu+\Omega/2)/T})].$$

$\bar{q},\uparrow$ $\bar{q},\downarrow$

Since rotation couple to total angular momentum

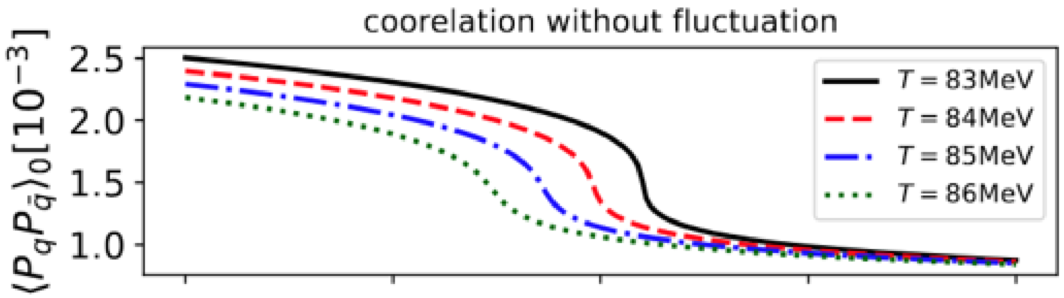
$\Omega(\cancel{l}+s)$

0 (Gap Eq)

$$\langle S\rangle=-\frac{V}{T}\frac{\mathrm{d}V^0_{eff}}{\mathrm{d}(\frac{\Omega}{T})}=-\frac{V}{T}\frac{\partial V^0_{eff}}{\partial(\frac{\Omega}{T})}-\frac{V}{T}\boxed{\frac{\partial V^0_{eff}}{\partial m}}\frac{\partial m}{\partial(\frac{\Omega}{T})}.$$

$$\langle P_qP_{\bar{q}}\rangle_0=\frac{\int\mathrm{d}^3p(f_q^\uparrow-f_q^\downarrow)(f_{\bar{q}}^\uparrow-f_{\bar{q}}^\downarrow)}{\int\mathrm{d}^3p(f_q^\uparrow+f_q^\downarrow)(f_{\bar{q}}^\uparrow+f_{\bar{q}}^\downarrow)}.$$

$$f_q^{\uparrow/\downarrow}=\frac{1}{\mathrm{e}^{\beta(\epsilon_p-\mu\mp\frac{\Omega}{2})}+1},\quad f_{\bar{q}}^{\uparrow/\downarrow}=\frac{1}{\mathrm{e}^{\beta(\epsilon_p+\mu\mp\frac{\Omega}{2})}+1}.$$



However, what we want to see is the fluctuation related to phase transition

QUALITATIVE STUDY: NJL MODEL

- To get correlation, further techniques are needed
- Introducing rotation and chemical potential only act on quark or antiquark

$$
\begin{aligned}
&V_{\text{eff}}(\Omega_q^s, \Omega_{\bar{q}}^s, \Omega, \mu_q, \mu_{\bar{q}}, \mu; r) \\
=&\frac{[m(r)-m_0]^2}{4G} - N_c N_f \int_0^\Lambda \frac{\mathrm{d}^3 p}{(2\pi)^3} 2\varepsilon_p - \sum_{l=-\infty}^{\infty} N_c N_f \int_0^\infty \frac{\mathrm{d}^3 p}{(2\pi)^3} J_l^2(p_t r) \Big[T \ln(1+\mathrm{e}^{-(\varepsilon_p-\mu-\Omega_q^s/2-\Omega l-\mu_q)/T}) \\
&+ T \ln(1+\mathrm{e}^{-(\varepsilon_p-\mu+\Omega_q^s/2-\Omega l-\mu_q)/T}) + T \ln(1+\mathrm{e}^{-(\varepsilon_p+\mu-\Omega_{\bar{q}}^s/2+\Omega l-\mu_{\bar{q}})/T}) + T \ln(1+\mathrm{e}^{-(\varepsilon_p+\mu+\Omega_{\bar{q}}^s/2+\Omega l-\mu_{\bar{q}})/T}) \Big]
\end{aligned}
$$

  - Then by taking derivative, we can get correlation of quark/antiquark spin and particle number

$$
\langle S_q S_{\bar{q}} \rangle - \langle S_q \rangle \langle S_{\bar{q}} \rangle = \frac{\partial^2 V_{eff}}{\partial \Omega_q^s \partial \Omega_{\bar{q}}^s} \Big|_{\Omega_q^s = \Omega_{\bar{q}}^s = \Omega} \qquad \langle N_q N_{\bar{q}} \rangle - \langle N_q \rangle \langle N_{\bar{q}} \rangle = \frac{\partial^2 V_{eff}}{\partial \mu_q \partial \mu_{\bar{q}}} \Big|_{\mu_q = \mu_{\bar{q}} = 0}
$$

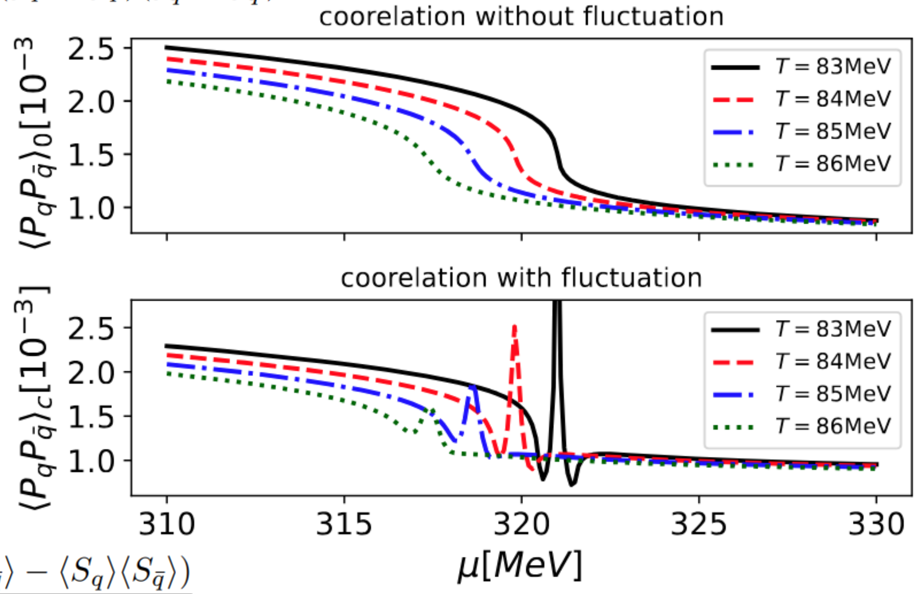
  - Then we can define the spin correlation of quark-antiquark as

$$
\langle P_q P_{\bar{q}} \rangle_c = \frac{4(\langle S_q S_{\bar{q}} \rangle - \langle S_q \rangle \langle S_{\bar{q}} \rangle)}{\langle N_q N_{\bar{q}} \rangle - \langle N_q \rangle \langle N_{\bar{q}} \rangle}
$$

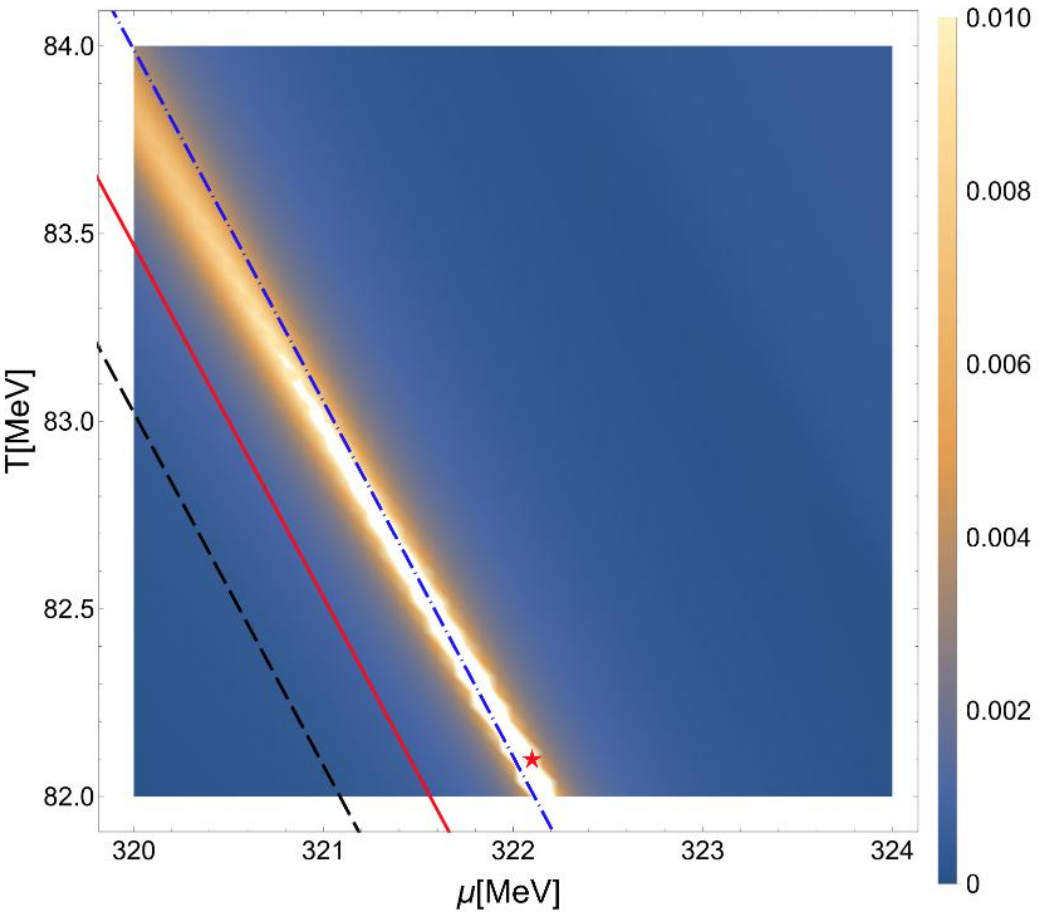
SPIN CORRELATION ENHANCED BY CEP!

- First we consider r=0
- Comparison with the case w/o fluctuation

$$\langle P_q P_{\bar{q}}\rangle_0 = \frac{\int \mathrm{d}^3p(f_q^\uparrow - f_q^\downarrow)(f_{\bar{q}}^\uparrow - f_{\bar{q}}^\downarrow)}{\int \mathrm{d}^3p(f_q^\uparrow + f_q^\downarrow)(f_{\bar{q}}^\uparrow + f_{\bar{q}}^\downarrow)}.$$



$$\langle P_q P_{\bar{q}}\rangle_c = \frac{4(\langle S_q S_{\bar{q}}\rangle - \langle S_q\rangle\langle S_{\bar{q}}\rangle)}{\langle N_q N_{\bar{q}}\rangle - \langle N_q\rangle\langle N_{\bar{q}}\rangle}$$



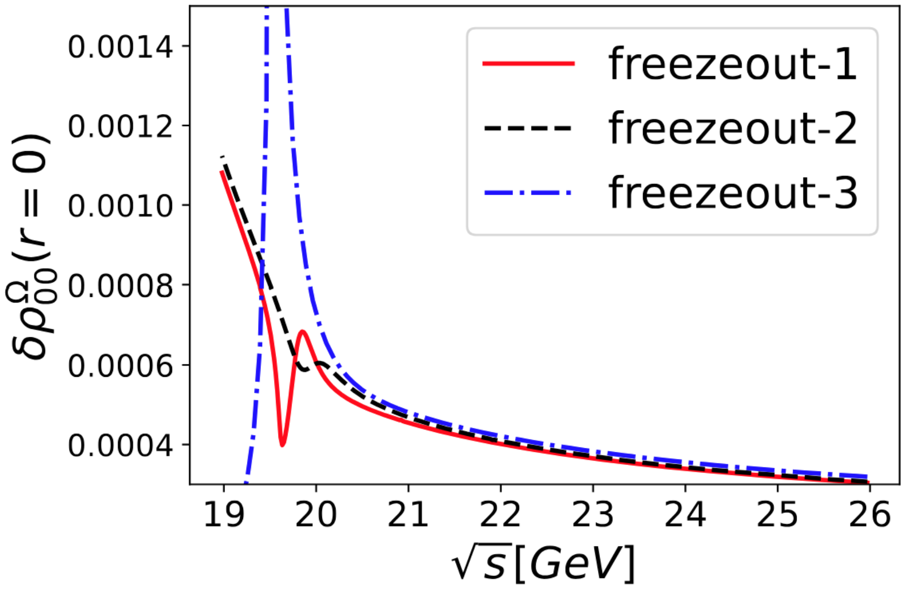
VECTOR MESON SPIN ALIGNMENT

- Along imaginary freezeout lines

$$\rho_{00} = \frac{1-\langle P_q P_{\bar{q}}\rangle}{3+\langle P_q P_{\bar{q}}\rangle} \approx \bar{\rho}_{00} \boxed{-\,\delta\rho^{\Omega}_{00}}.$$

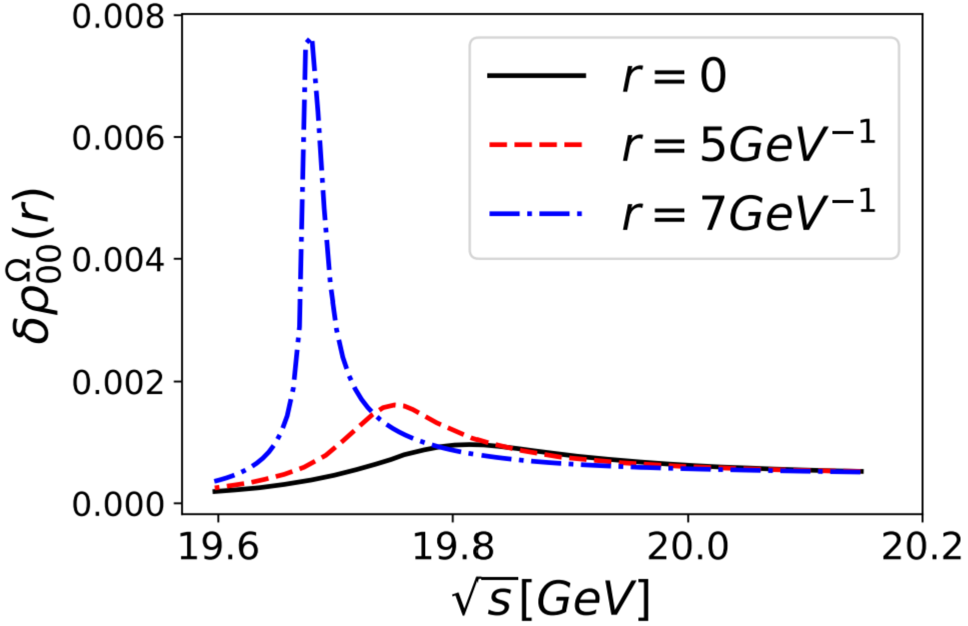
Contribution from critical fluctuation

At r=0



$$V_{\rm eff}(\Omega^s_q,\Omega^s_{\bar{q}},\Omega,\mu_q,\mu_{\bar{q}},\mu;r)$$
$$=\frac{[m(r)-m_0]^2}{4G}-N_cN_f\int_0^{\Lambda}\frac{\mathrm{d}^3p}{(2\pi)^3}2\varepsilon_p-\sum_{l=-\infty}^{\infty}N_cN_f\int_0^{\infty}\frac{\mathrm{d}^3p}{(2\pi)^3}J_l^2(p_tr)\Big[T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu-\Omega^s_q/2-\Omega l-\mu_q)/T})$$
$$+T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu+\Omega^s_q/2-\Omega l-\mu_q)/T})+T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu-\Omega^s_{\bar{q}}/2+\Omega l-\mu_{\bar{q}})/T})+T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu+\Omega^s_{\bar{q}}/2+\Omega l-\mu_{\bar{q}})/T})\Big]$$

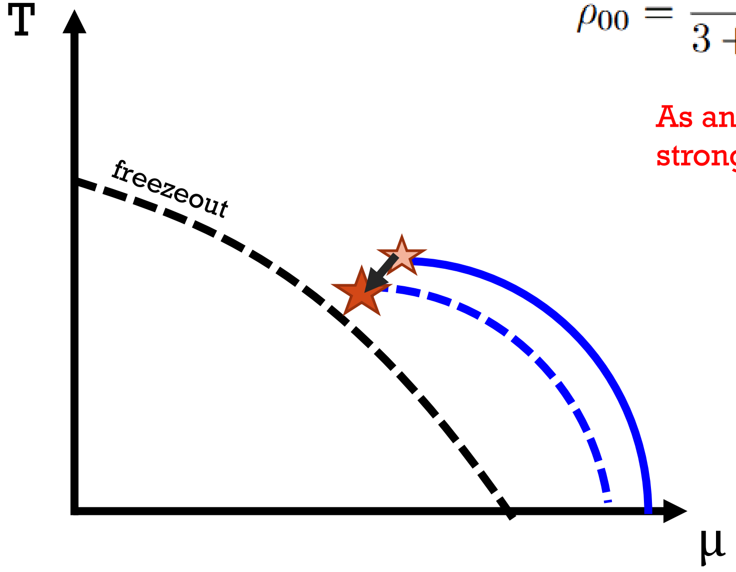
Orbit angular momentum contribution to Freezeout – 2



PHI MESON SPIN ALIGNMENT

- The CEP is shifted closer to freezeout line by orbit angular momentum contribution
- The peak structure is enhanced

A SCHEMATIC FIGURE

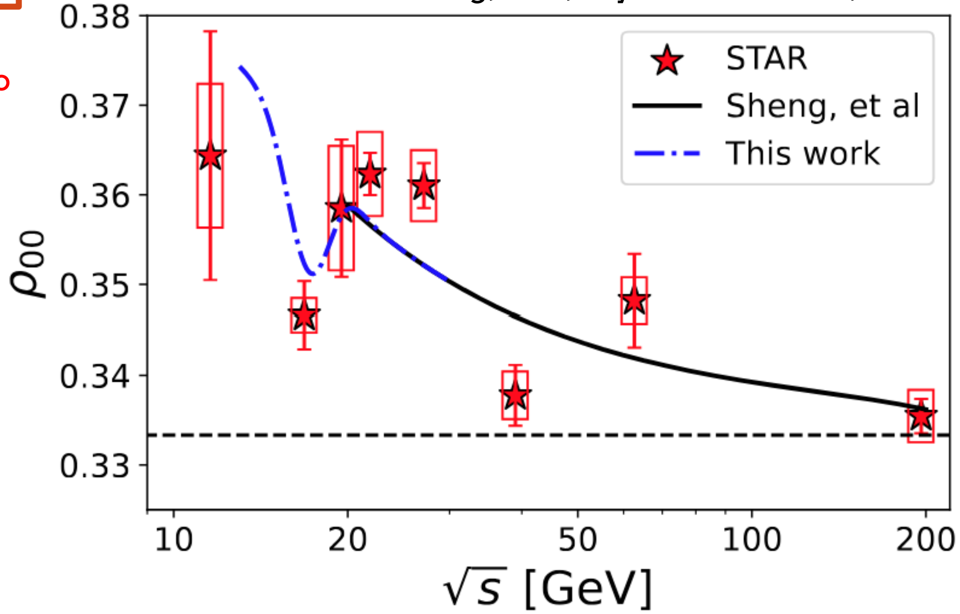


$$\rho_{00} = \frac{1 - \langle P_q P_{\bar{q}} \rangle}{3 + \langle P_q P_{\bar{q}} \rangle} \approx \bar{\rho}_{00} - \boxed{\delta \rho_{00}^{\Omega}}.$$

As an additional contribution (to strong force field contribution)

G. Wilks' talk@SQM2024

X.-L. Sheng, et al, Phys. Rev. Lett. 131, 042304 (2023)



OTHER CORRELATIONS

Kun Xu, Mei Huang, Phys.Rev.D 110 (2024) 9, 094034

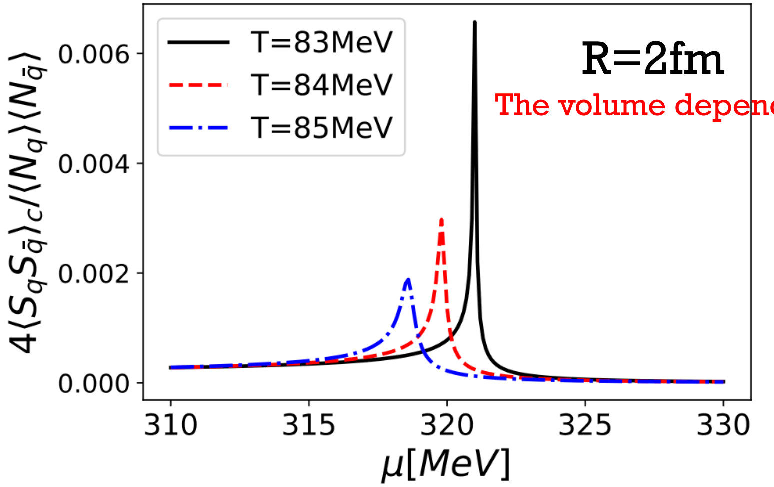
$$\delta\rho_{00}(\phi) \approx -\frac{4}{9}\frac{\langle\delta N_s \delta N_{\bar{s}}\rangle}{N_s N_{\bar{s}}} = -\frac{32}{9c^2}\frac{T}{V}\frac{N_c^2 G_A}{\rho_s^2}L^2$$

- Analogy to baryon number case

angular velocity $\omega \Longleftrightarrow \mu$ chemical potential

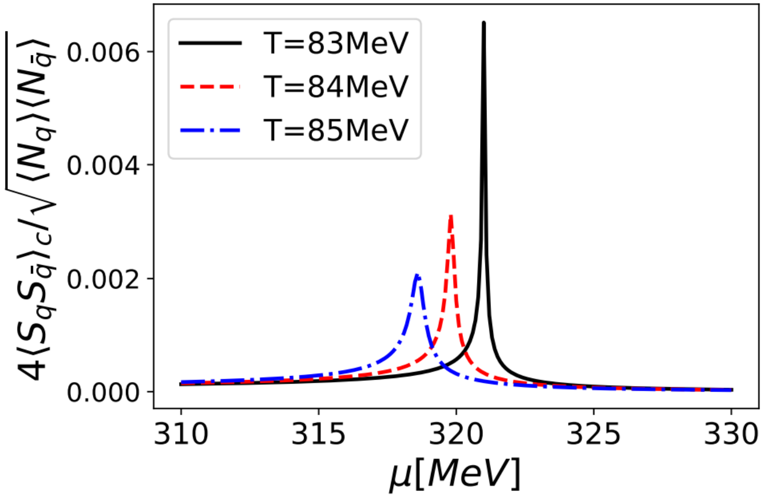
conserved charge : spin $S \Longleftrightarrow N$ quark number

$$\frac{4(\langle S_q S_{\bar{q}}\rangle - \langle S_q\rangle\langle S_{\bar{q}}\rangle)}{\langle N_q\rangle\langle N_{\bar{q}}\rangle}$$

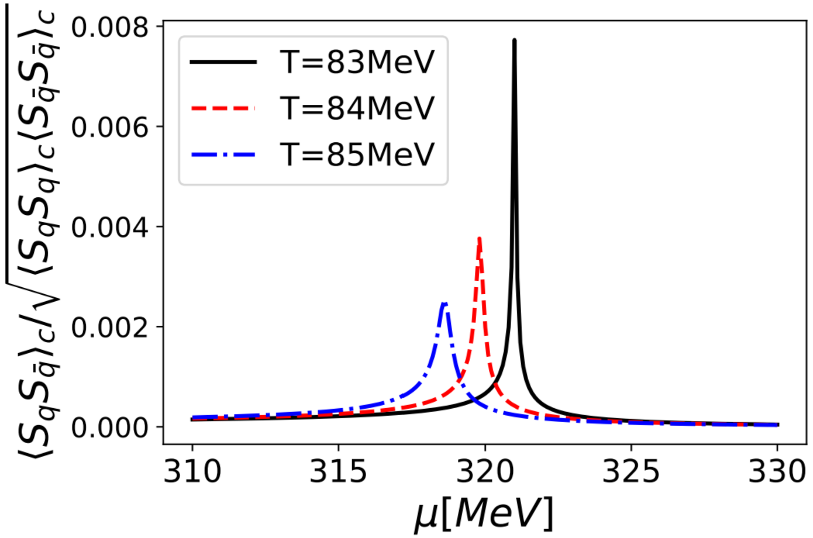


The volume dependence is annoying

$$\frac{4(\langle S_q S_{\bar{q}}\rangle - \langle S_q\rangle\langle S_{\bar{q}}\rangle)}{\sqrt{\langle N_q\rangle\langle N_{\bar{q}}\rangle}}$$



$$\frac{\langle S_q S_{\bar{q}}\rangle_c}{\sqrt{\langle S_q S_q\rangle_c \langle S_{\bar{q}} S_{\bar{q}}\rangle_c}}$$ Pearson correlation coefficient

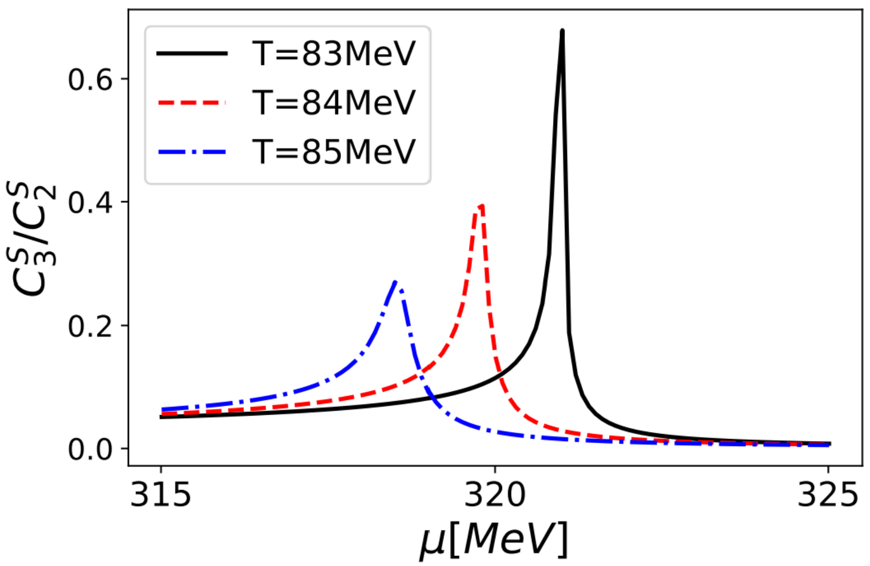


Second order quark-antiquark spin correlations

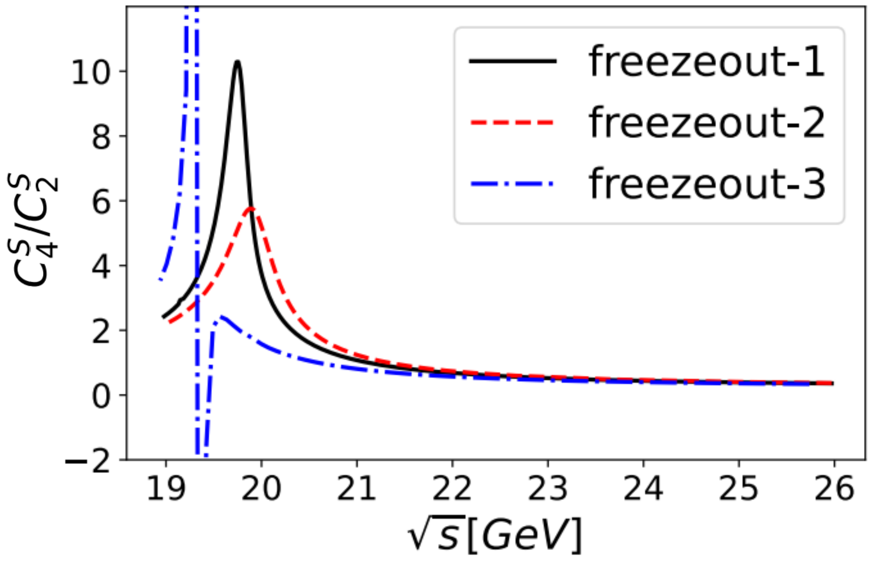
OTHER CORRELATIONS

- Cumulants $C_n^S = VT^{n-1}\frac{\partial^n p}{\partial \omega^n}$
- Higher orders are more sensitive to CEP

Skewness of spin fluctuation:



Kurtosis of spin fluctuation:



SUMMARY

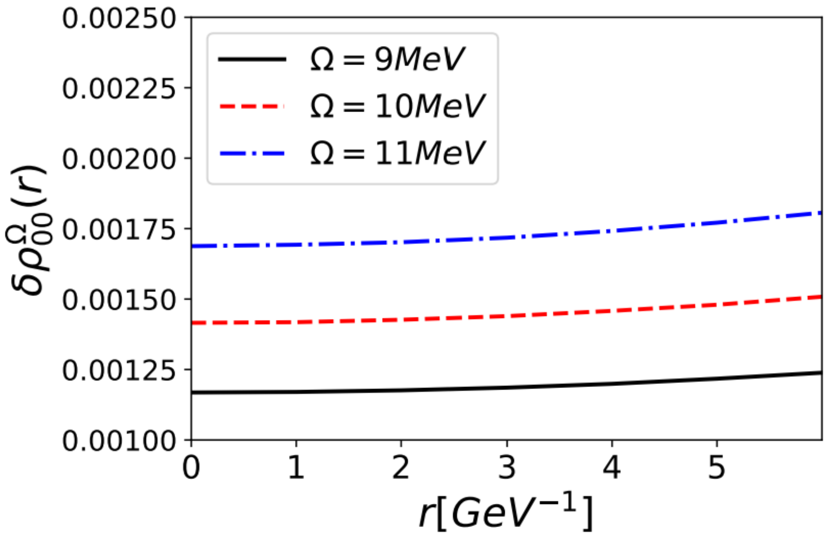
- Interaction might be important to understand quark spin correlation
- Critical fluctuation near CEP can lead to non-monotonic behavior of spin alignment & Hyperon-anti-Hyperon correlation
- Spin alignment & Hyperon-anti-Hyperon correlation can serve as signatures for CEP
- Connection between spin and phase transition is an interesting direction
- Spin alignment of other vector mesons
- More realistic and detailed studies in future

THANK YOU FOR ATTENTION！

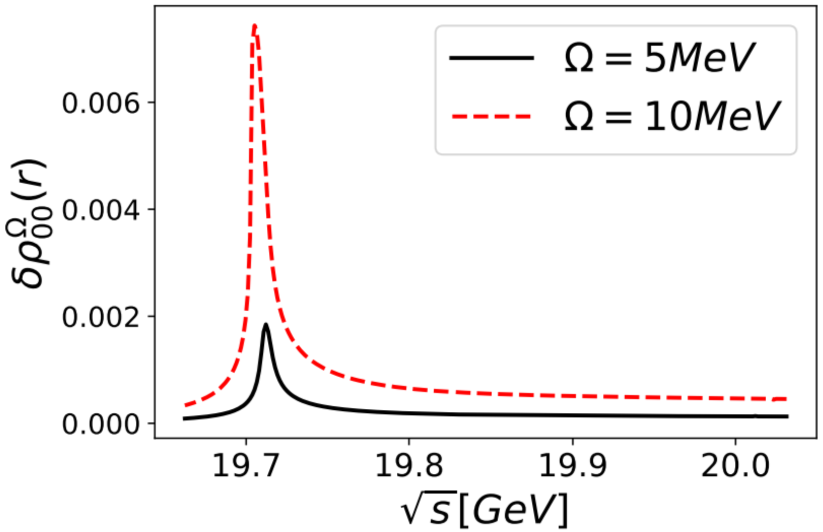
BACK UP

ROTATION DEPENDENCE

Far from CEP



Freezeout-2 at different rotation



PNJL MODEL

Λ [MeV]	m_0 [MeV]	$G_{\mathrm{PNJL}}\Lambda^2$	N_f	a_0	a_1	a_3	b_3	T_0[MeV]
651	5.5	2.135	2	3.51	-2.47	15.2	-1.75	210

$$
\begin{aligned}
&V_{\mathrm{PNJL}}(\Omega_q^s, \Omega_{\bar{q}}^s, \mu_q, \mu_{\bar{q}}; r=0) \\
=&\frac{[m-m_0]^2}{4G_{\mathrm{PNJL}}} - 2N_f \int_0^\Lambda \frac{\mathrm{d}^3 p}{(2\pi)^3} 3\varepsilon_p \\
&- N_f \int_0^\infty \frac{\mathrm{d}^3 p}{(2\pi)^3} \Big[T \ln(1 + 3\Phi \mathrm{e}^{-(\varepsilon_p - \mu - \Omega_q^s/2 - \mu_q)/T} + 3\bar{\Phi} \mathrm{e}^{-2(\varepsilon_p - \mu - \Omega_q^s/2 - \mu_q)/T} + \mathrm{e}^{-3(\varepsilon_p - \mu - \Omega_q^s/2 - \mu_q)/T}) \\
&+ T \ln(1 + 3\Phi \mathrm{e}^{-(\varepsilon_p - \mu + \Omega_q^s/2 - \mu_q)/T} + 3\bar{\Phi} \mathrm{e}^{-2(\varepsilon_p - \mu + \Omega_q^s/2 - \mu_q)/T} + \mathrm{e}^{-3(\varepsilon_p - \mu + \Omega_q^s/2 - \mu_q)/T}) \\
&+ T \ln(1 + 3\Phi \mathrm{e}^{-(\varepsilon_p + \mu - \Omega_{\bar{q}}^s/2 - \mu_{\bar{q}})/T} + 3\bar{\Phi} \mathrm{e}^{-2(\varepsilon_p + \mu - \Omega_{\bar{q}}^s/2 - \mu_{\bar{q}})/T} + \mathrm{e}^{-3(\varepsilon_p + \mu - \Omega_{\bar{q}}^s/2 - \mu_{\bar{q}})/T}) \\
&+ T \ln(1 + 3\Phi \mathrm{e}^{-(\varepsilon_p + \mu + \Omega_{\bar{q}}^s/2 - \mu_{\bar{q}})/T} + 3\bar{\Phi} \mathrm{e}^{-2(\varepsilon_p + \mu + \Omega_{\bar{q}}^s/2 - \mu_{\bar{q}})/T} + \mathrm{e}^{-3(\varepsilon_p + \mu + \Omega_{\bar{q}}^s/2 - \mu_{\bar{q}})/T}) \Big] \\
&+ T^4 \left\{ -\frac{1}{2} \left[a_0 + a_1 (\frac{T_0}{T}) + a_2 (\frac{T_0}{T})^2 \right] \bar{\Phi}\Phi + b_3 (\frac{T_0}{T})^3 \ln \left[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2 \right] \right\},
\end{aligned}
$$

- Quantitatively agree with NJL model results