

COMPLEX HEAVY QUARK POTENTIAL FROM THE LATTICE WHERE DO WE STAND?

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Department of Physics
Korea University
South Korea

References

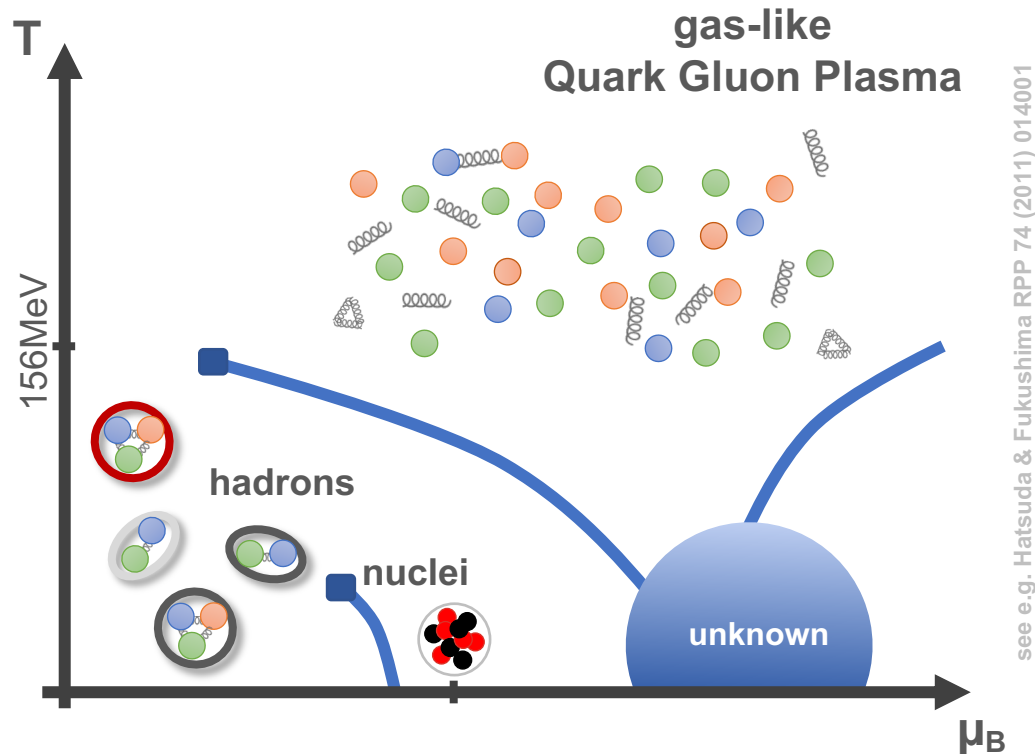
with **J. Skullerud & R. Horohan D'Arcy (FASTSUM)** in progress
with **HotQCD**: PRD 109 (2024) 7, 074504, PRD 105 (2022) 5, 054513
with **R. Larsen & G. Parkar**: PRD 110 (2024) 11, 114501
see also: **D. Leinweber et.al.** PRD 111 (2025) 3, 034508

Outline

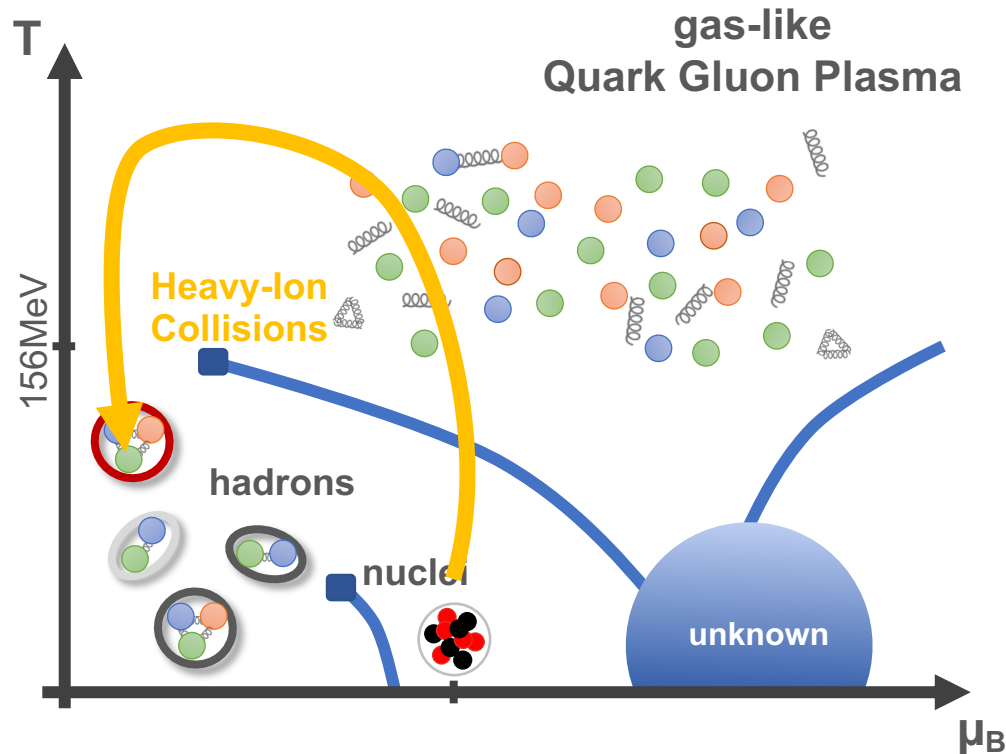


- Motivation: Exploration of phases of QCD
- The complex heavy quark potential from (lattice) QCD
- From past to recent results from lattice QCD
- Summary

The phases of nuclear matter (I)

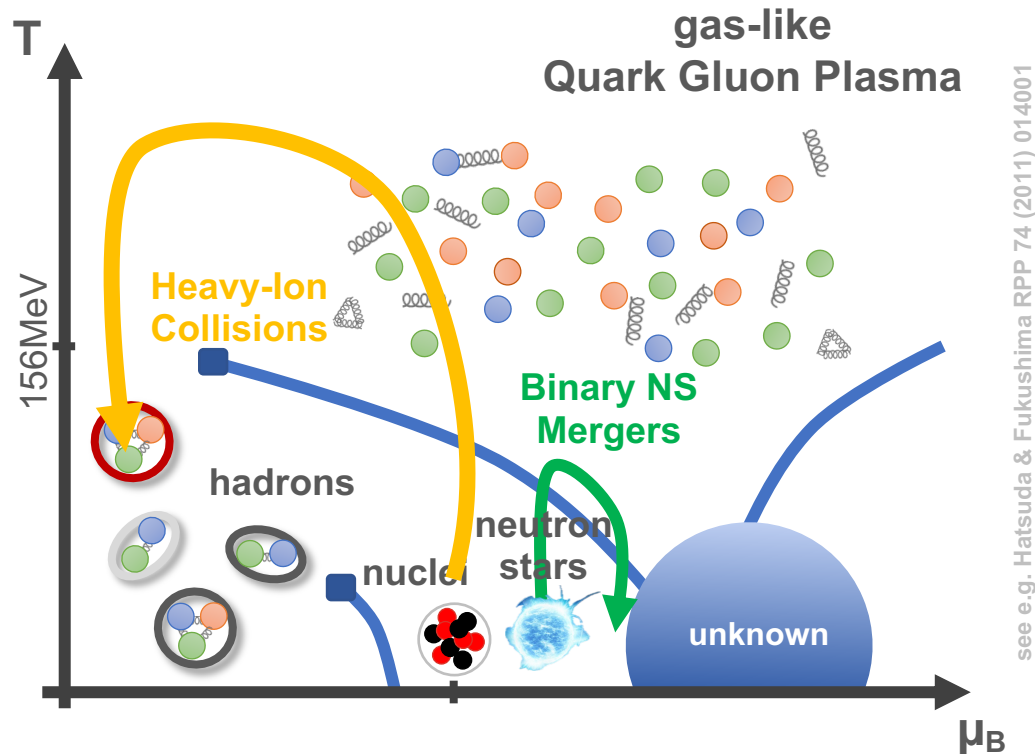


The phases of nuclear matter (I)



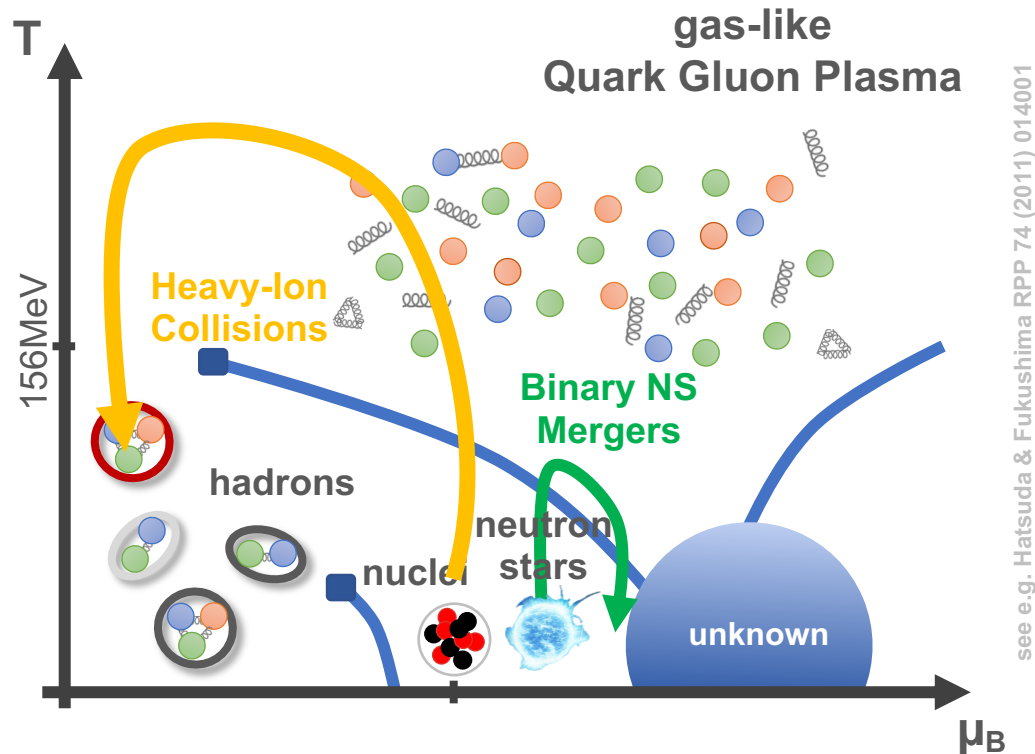
see e.g. Hatsuda & Fukushima RPP 74 (2011) 014001

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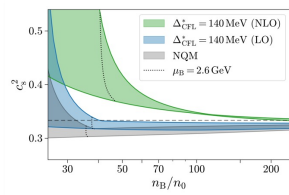
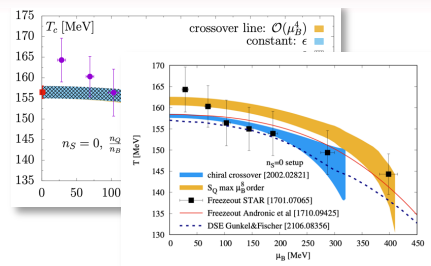
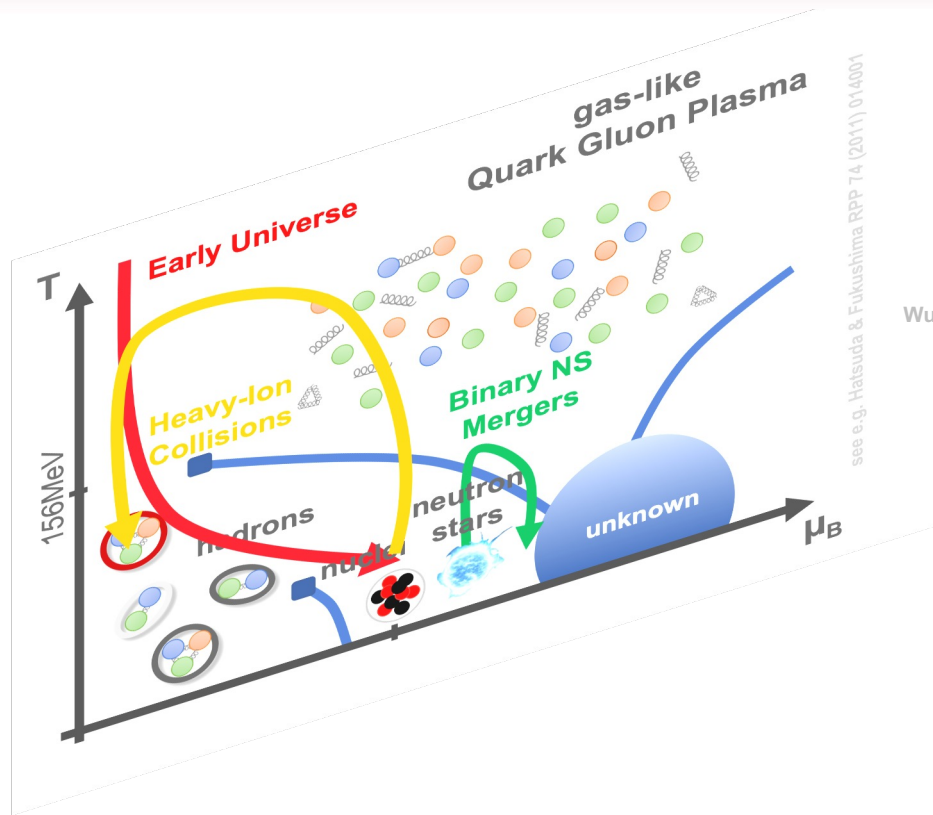
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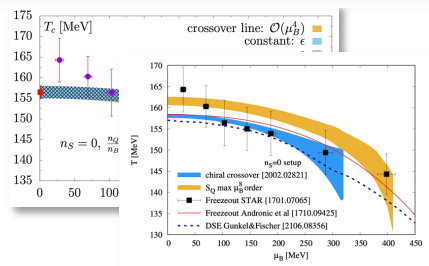
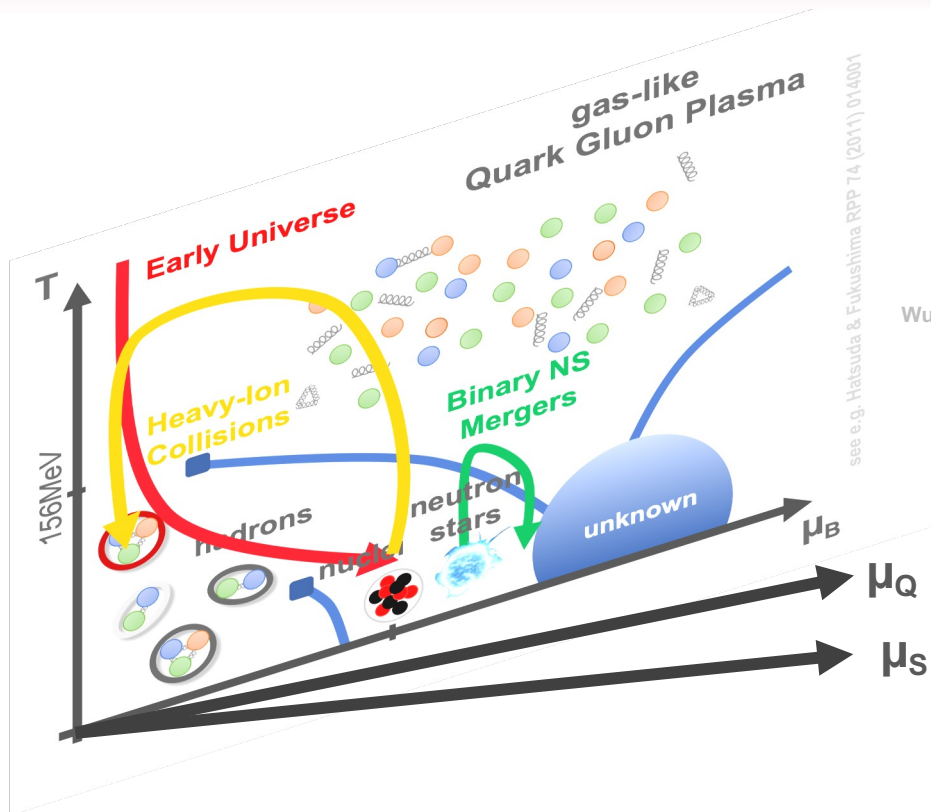
chiral symmetry restoration & deconfinement as key indications of phase change

The phases of nuclear matter (II)

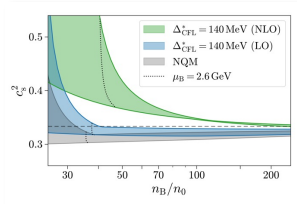


The phases of nuclear matter (II)

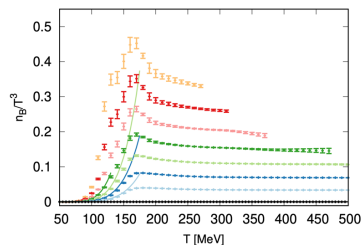
more and more differential analyses



see e.g. HotQCD PLB 795 (2019) 15 or
Wuppertal Budapest PRD 110 (2024) 11, 114507



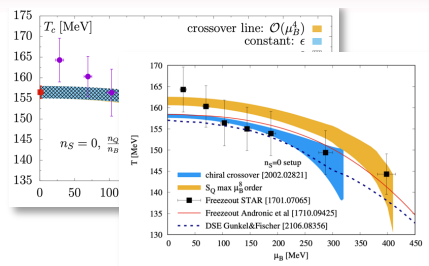
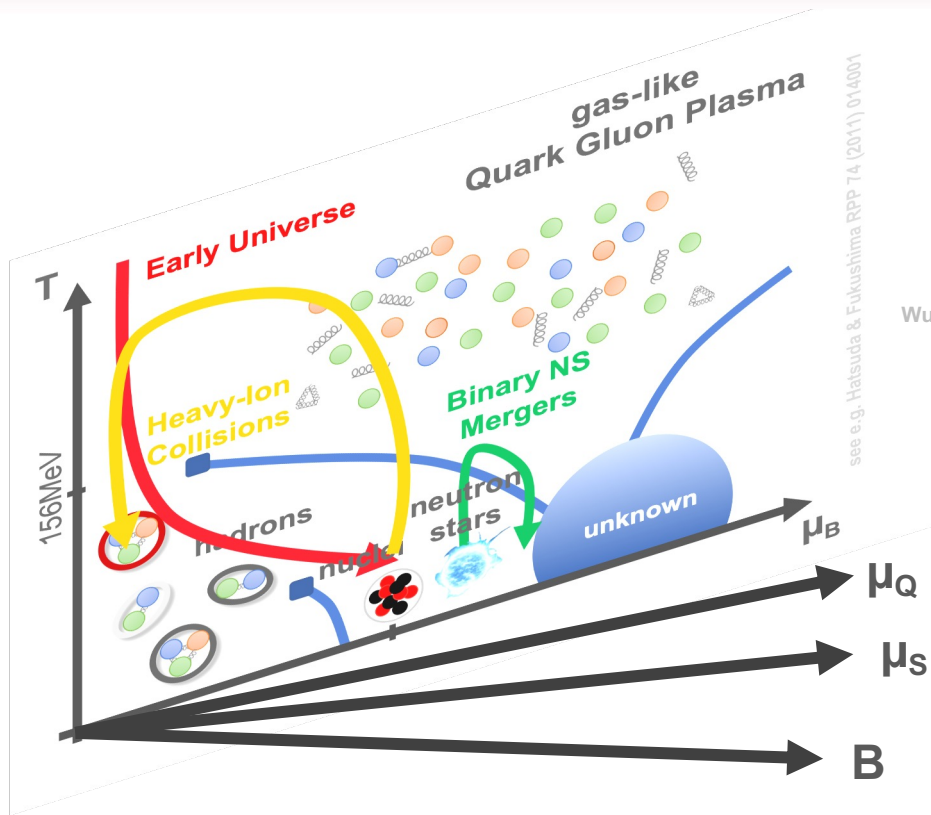
A. Geißel et.al. arXiv:2504.03834



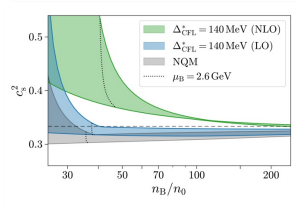
see e.g. Wuppertal
Budapest arXiv:2504.01881

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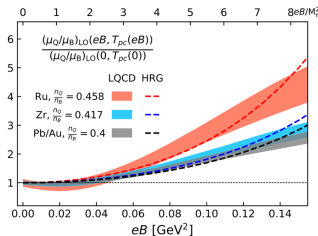
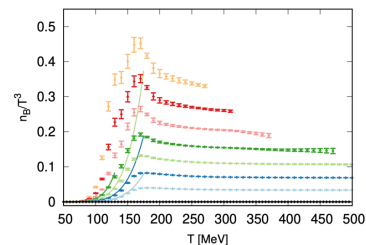
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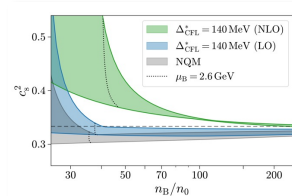
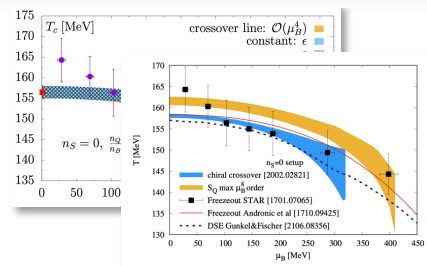
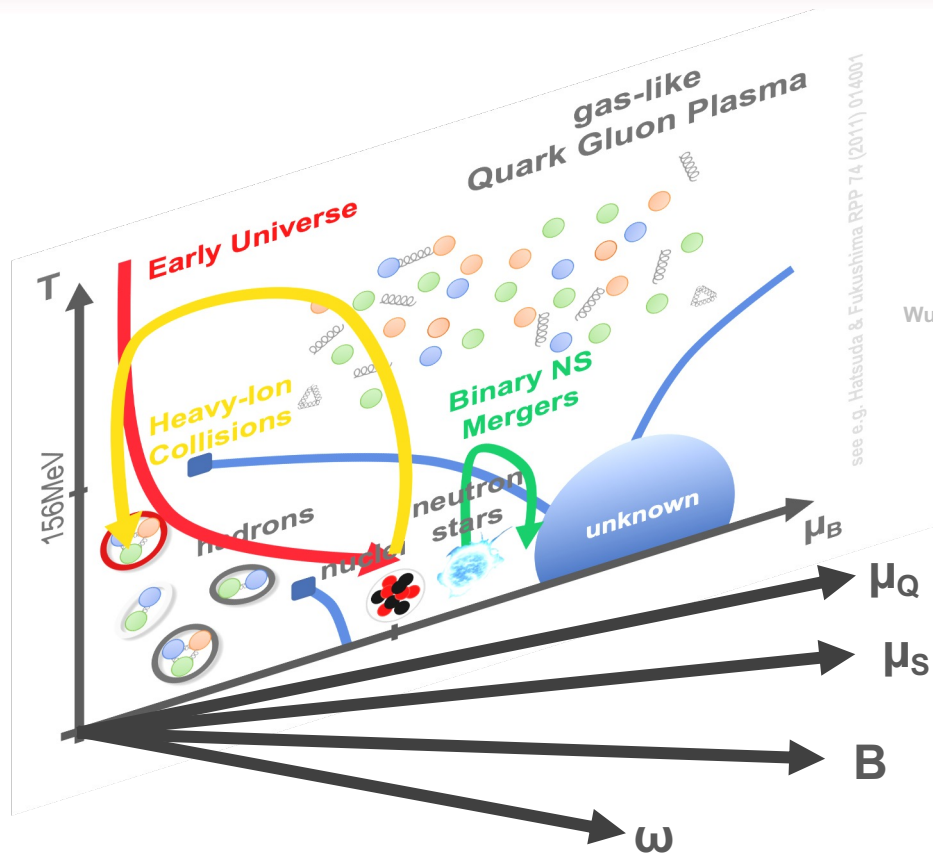
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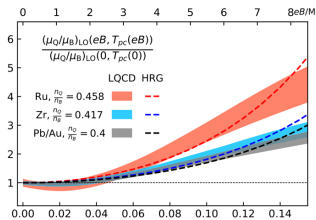
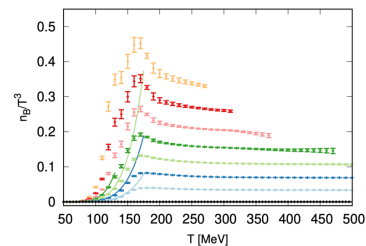
H.T. Ding et.al.
PRL 132, 201903 (2024)

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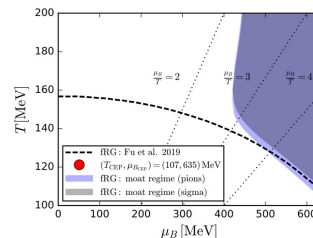
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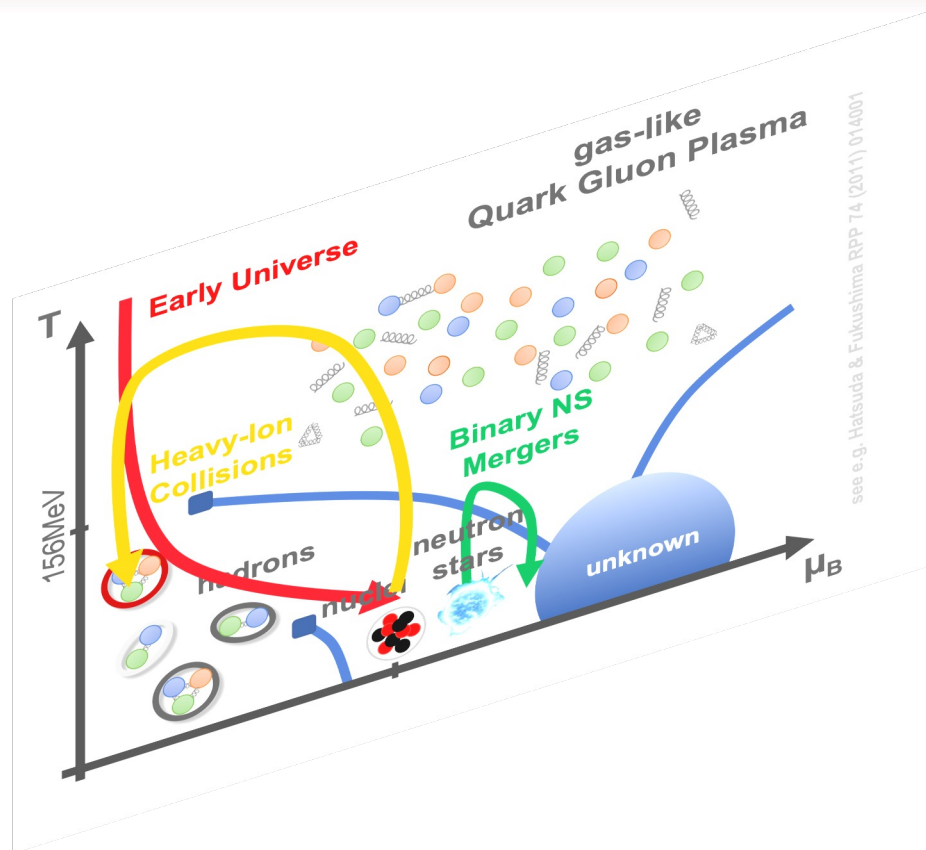


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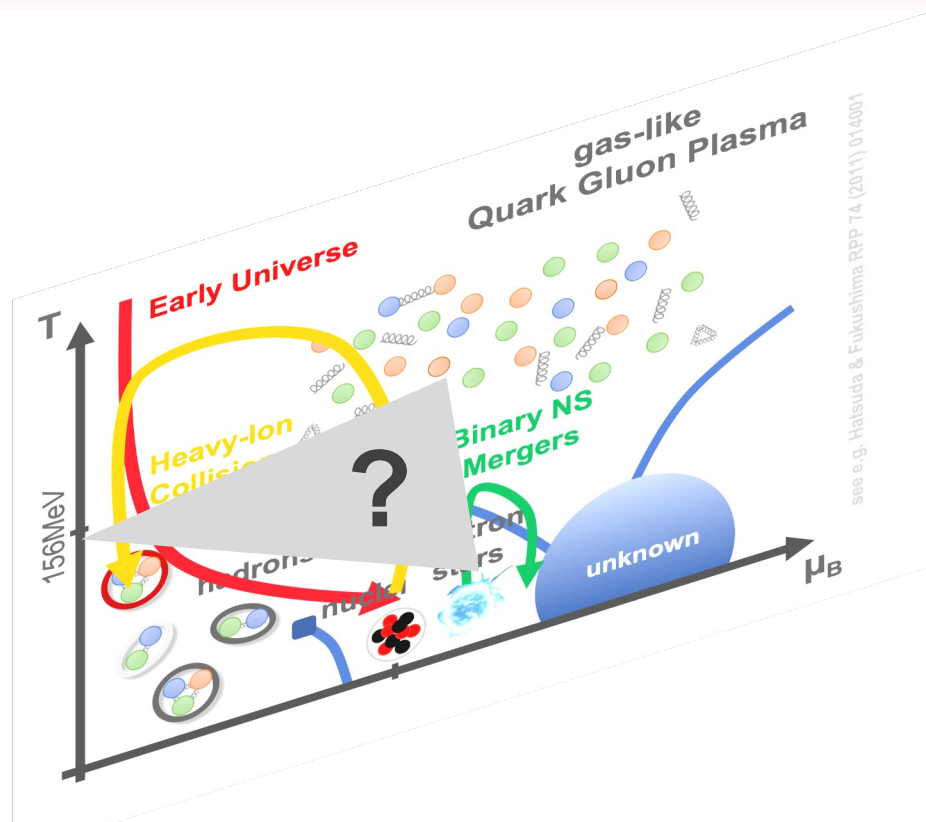


W.J. Fu et.al.
PRD 111, 094026 (2025)

Heavy quark physics and phase structure



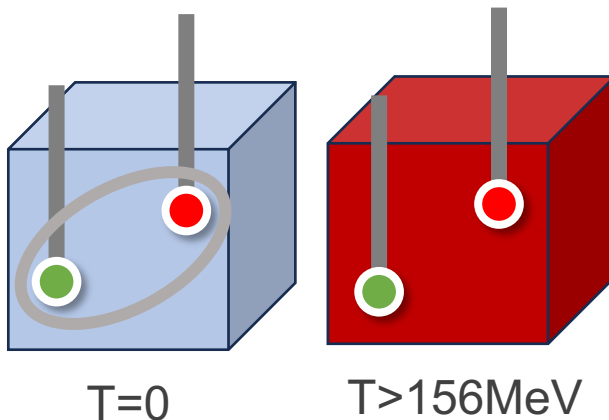
Heavy quark physics and phase structure



Goal: understand crossover transition in more detail – fate of confinement & active d.o.f.

Heavy quark physics and phase structure

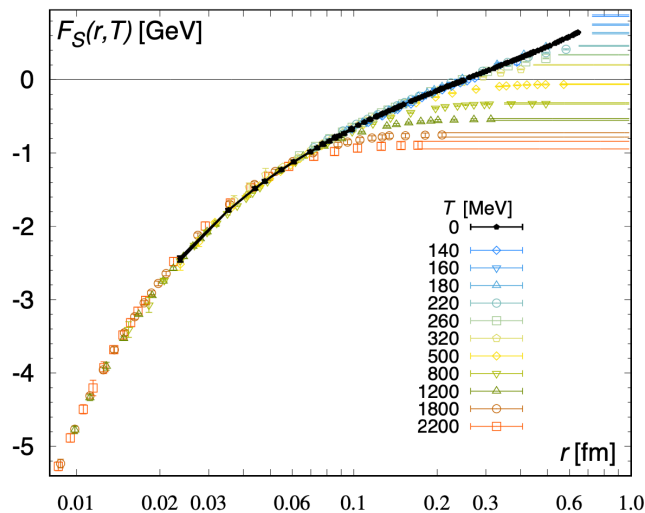
static / thermodynamic



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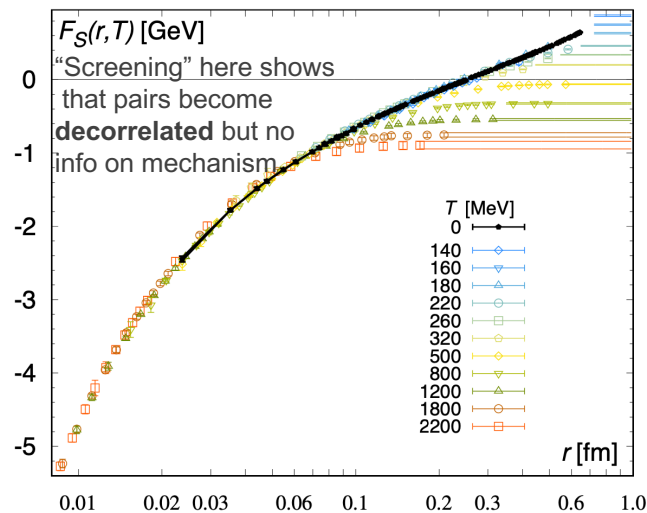


TUMQCD PRD98 (2018) 5, 054511

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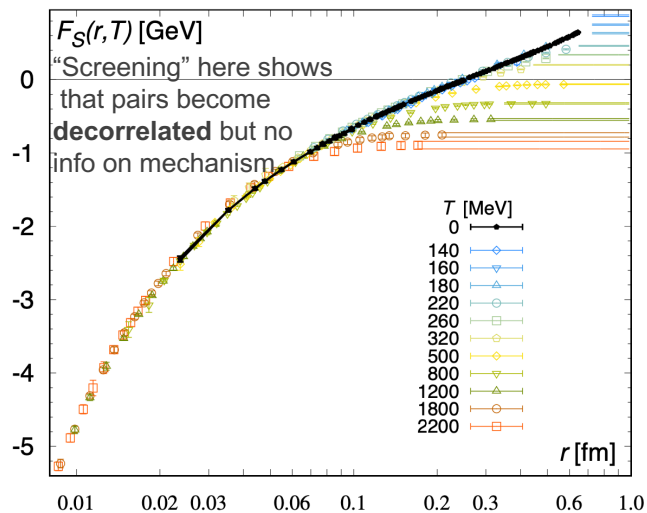


TUMQCD PRD98 (2018) 5, 054511

Goal: understand crossover transition in more detail – fate of confinement & active d.o.f.

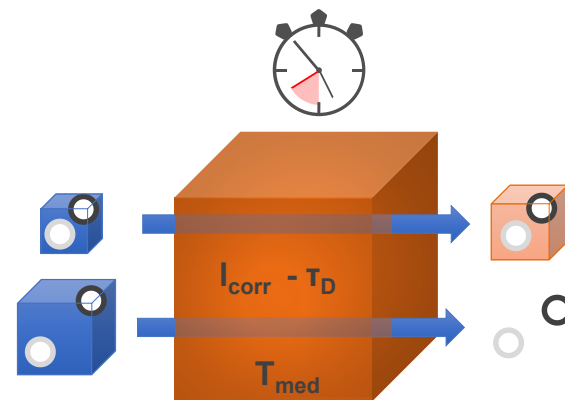
Heavy quark physics and phase structure

static / thermodynamic



TUMQCD PRD98 (2018) 5, 054511

dynamic / real-time

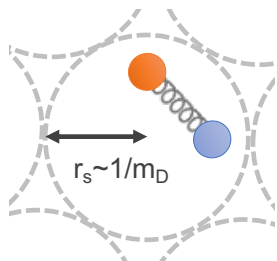


Open-quantum system
described by complex potential

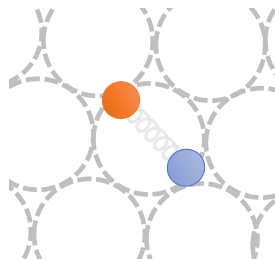
Goal: understand crossover transition in more detail – fate of confinement & active d.o.f.

Heavy quark pair real-time evolution

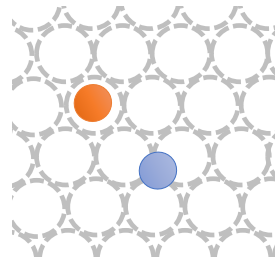
screening



binding force acts efficiently among Q and $\bar{Q}$



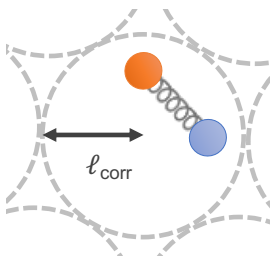
medium impedes gluons mediating force



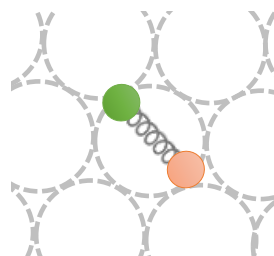
Binding force related to
real-part of static potential
(de-confinement)

decoherence

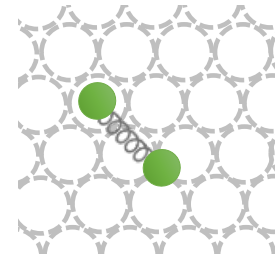
increasing temperature



color rotation acts coherently on Q and $\bar{Q}$



color rotation acts individually on Q and Q



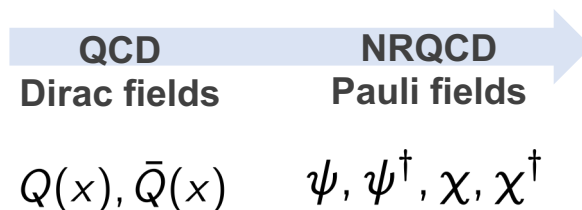
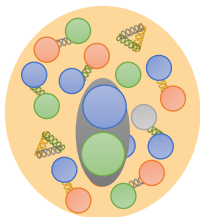
Color decoherence related
to imaginary part
(active color d.o.f.s)

see discussion in S. Kajimoto, Y. Akamatsu, M. Asakawa, A.R., PRD97 (2018), 014003

Defining the potential from QCD

Exploit $\frac{\Lambda_{\text{QCD}}}{m_Q} \ll 1$, $\frac{T}{m_Q} \ll 1$, $\frac{\mathbf{p}}{m_Q} \ll 1$ to treat heavy quarks non-relativistically
 see Brambilla et. al. Rev.Mod.Phys. 77 (2005) 142

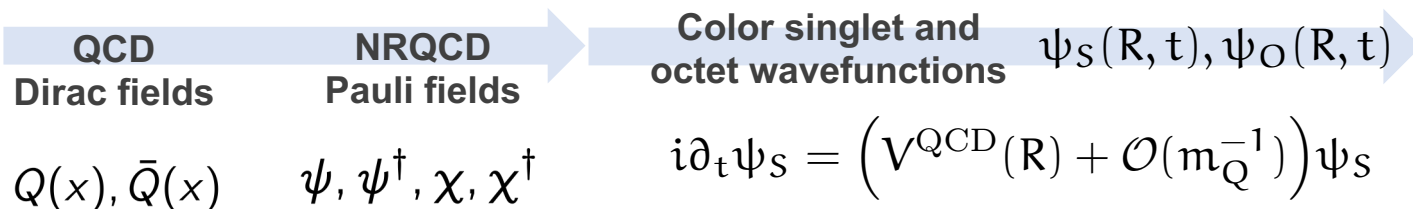
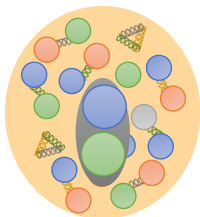
Relativistic $T > 0$
 field theory



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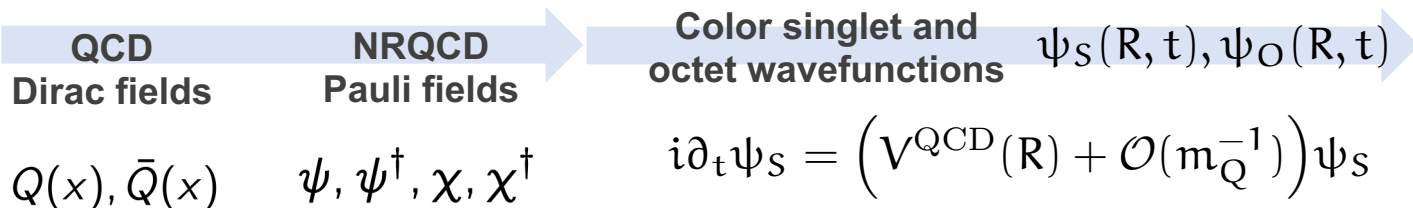
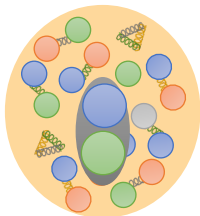
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 field theory



Defining the potential from QCD

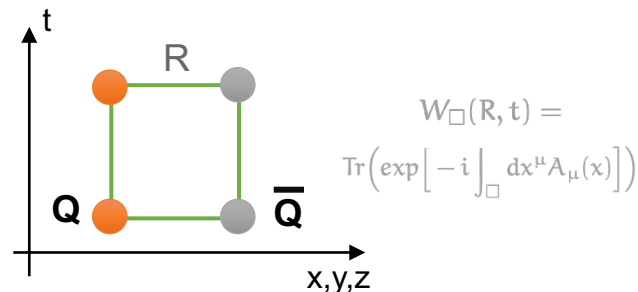
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Relativistic $T > 0$
field theory



Matching through the QCD Wilson loop

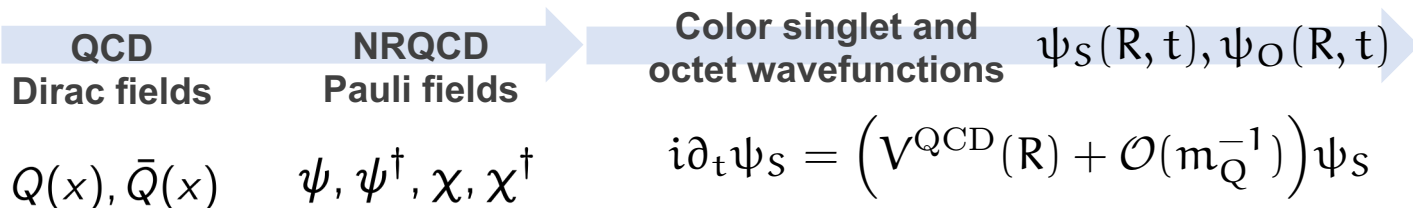
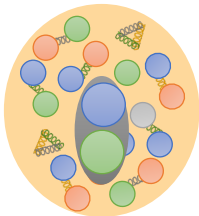
$$V^{\text{QCD}}(R) = \lim_{t \rightarrow \infty} \frac{i\partial_t W_\square(R, t)}{W_\square(R, t)}$$



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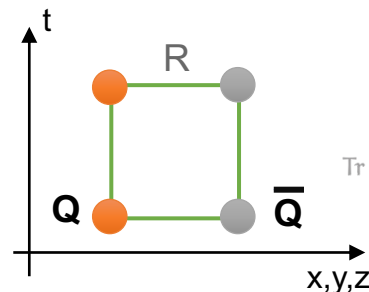
Relativistic $T > 0$
field theory



Matching through the QCD Wilson loop

$$V^{\text{QCD}}(R) = \lim_{t \rightarrow \infty} \frac{i\partial_t W_\square(R, t)}{W_\square(R, t)} \in \mathbb{C}$$

Im[V]: Laine et al. JHEP03 (2007) 054; Beraudo et. al. NPA 806:312,2008



$$W_\square(R, t) = \text{Tr} \left(\exp \left[-i \int_\square dx^\mu A_\mu(x) \right] \right)$$

Perturbation theory predicts: Debye screened Re[V], Im[V] increases with T

Extracting the potential from the lattice

■ Spectral functions as bridge between the Euclidean and real-time Wilson loop

see A.R., T.Hatsuda & S.Sasaki, PRL 108 (2012) 162001, Y.Burnier, A.R. Phys.Rev. D86 (2012) 051503

$$W_{\square}(R, t) = \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \rho_{\square}(R, \omega) \quad \longleftrightarrow \quad W_{\square}(R, \tau) = \int_{-\infty}^{\infty} d\omega e^{-\omega \tau} \rho_{\square}(R, \omega)$$

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Spectral Decomposition

$$V^{\text{QCD}}(R) = \lim_{t \rightarrow \infty} \frac{\int_{-\infty}^{\infty} d\omega \omega e^{-i\omega t} \rho_{\square}(R, \omega)}{\int_{-\infty}^{\infty} d\omega e^{-i\omega t} \rho_{\square}(R, \omega)}$$

A.R., T.Hatsuda & S.Sasaki PoS LAT2009 (2009) 162

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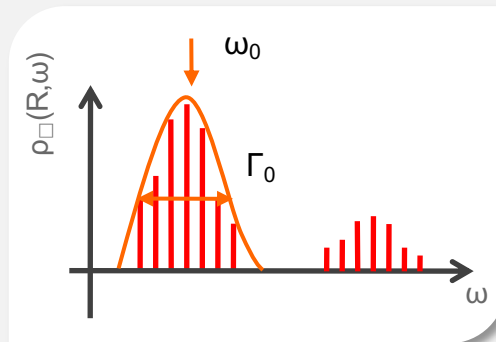
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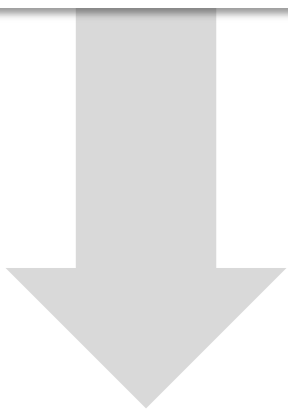
A.R., T.Hatsuda & S.Sasaki PoS LAT2009 (2009) 162



$$V(R) = \omega_0(R) - i\Gamma_0(R)$$

A battery of reconstruction approaches

- Most reconstruction methods can be cast in the language of **Bayesian Inference**: amend ill-posed inverse problem with prior knowledge to select unique answer

$$P[\rho|D, I] \propto P[D|\rho, I]P[\rho|I]$$


for a review: A.R. Front.Phys. 10 (2022) 1028995

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$$P[D|\rho, I] \propto e^{-\chi^2}$$

$$P[\rho|I] \propto e^S$$

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Bayesian Inference

Explicit regulator based on prior information (positivity)

Maximum Entropy Method (MEM)

R. Bryan, Eur. Bio. Phys. J. 18 (1990)

Bayesian Reconstruction (BR)

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$$\left. \frac{\delta}{\delta \rho} P[\rho|D, I] \right|_{\rho=\rho^{\text{Bayes}}} = 0$$

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Linear Methods

Regularize fit by limited choice of basis functions

Backus Gilbert (BG)

Lupo et.al. PoS LATTICE2023 (2024) 004

Gaussian Processes (GP)

see e.g. Horak et.al. PRD 105 (2022) 3, 036014

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Neural Networks

Limited basis functions via NN geometry & regularization in training cost functional

Application to PDFs & spectra

see e.g. Karpie et.al., JHEP 04 (2019) 057, L. Kades et.al. PRD 102 (2020) 9, 096001, L. Wang et.al PRD 106 (2022) 5, L051502

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Pade / Prony

Rational approx. of Euclidean correlator, explicit continuation

Direct Pade

see e.g. R.A. Tripolt et.al. Phys.Lett.B 774 (2017) and A.R. et.al. Nucl.Phys.A 982 (2019) 735

Prony analysis

Zhang et.al. PRB 110 (2024) 23, 235131

$$P[\rho|D, I] \propto P[D|\rho, I]P[\rho|I]$$

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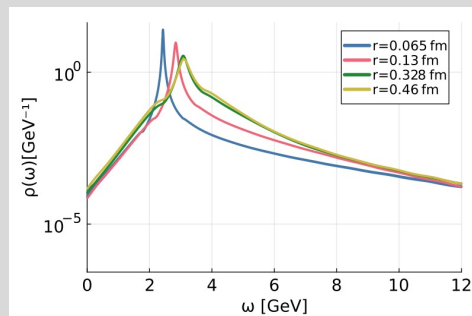
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see e.g. Karpie et.al., JHEP 04 (2019) 057, L. Kades et.al. PRD 102 (2020) 9, 096001, L. Wang et.al PRD 106 (2022) 5, L051502

Mock-data closure tests

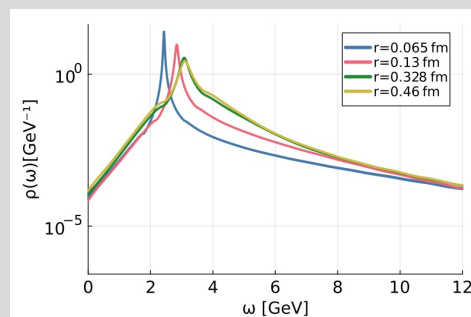
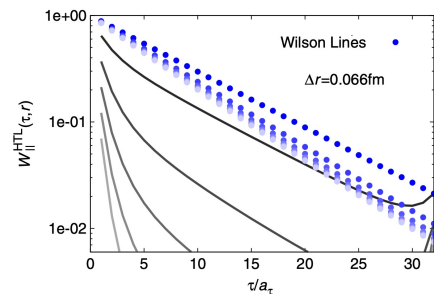


Realistic mock spectral
functions via resummed
perturbation theory (HTL)

HotQCD and A.R., G. Parkar
PRD 105 (2022) 5, 054513

Mock-data closure tests

Bayesian Reconstruction

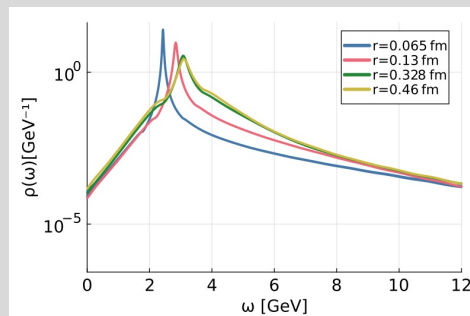
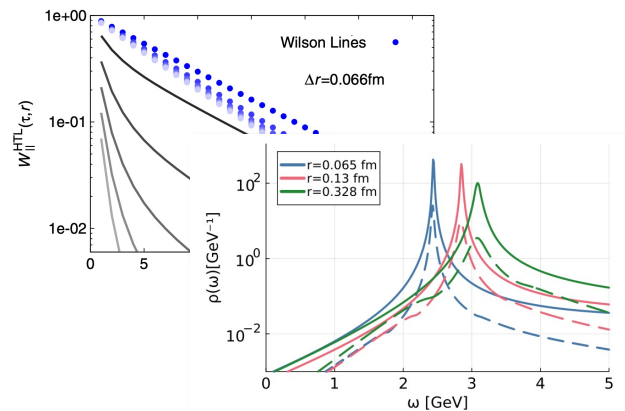


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Mock-data closure tests

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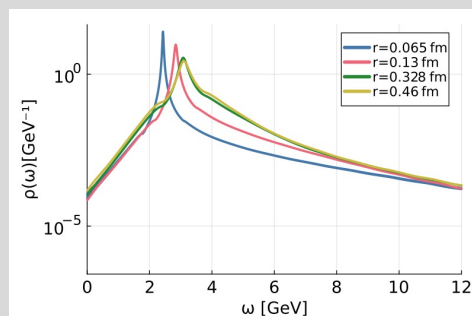
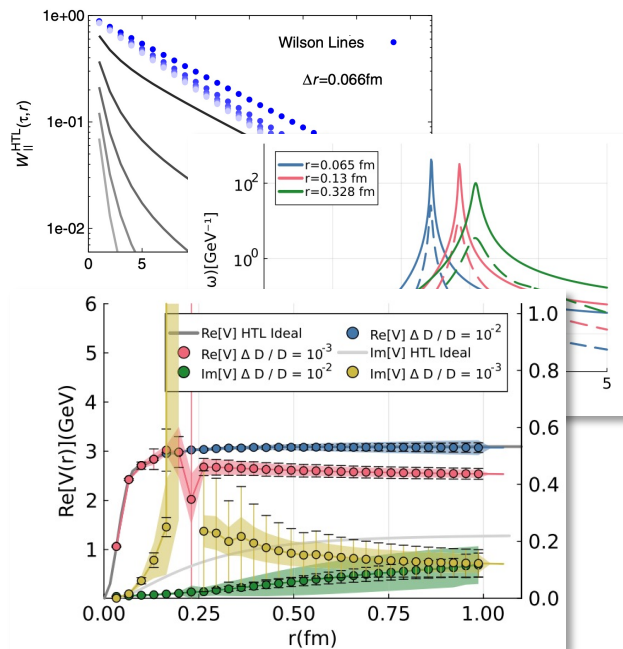


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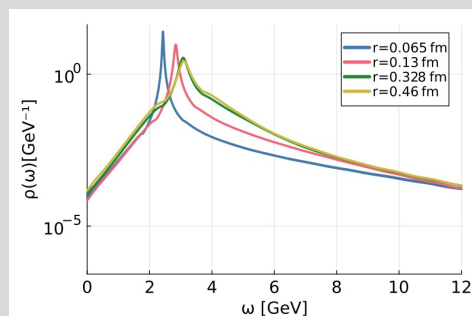
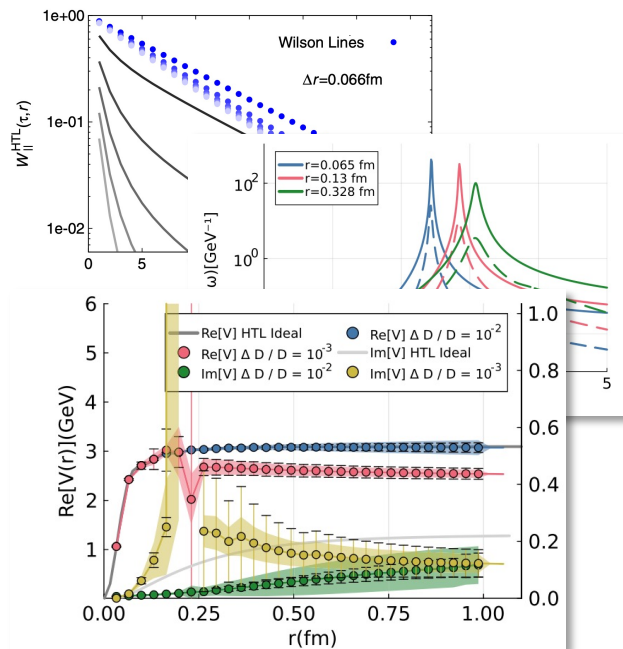


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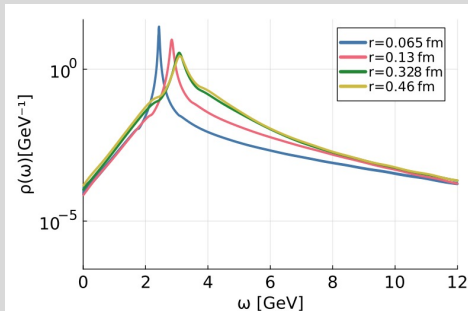
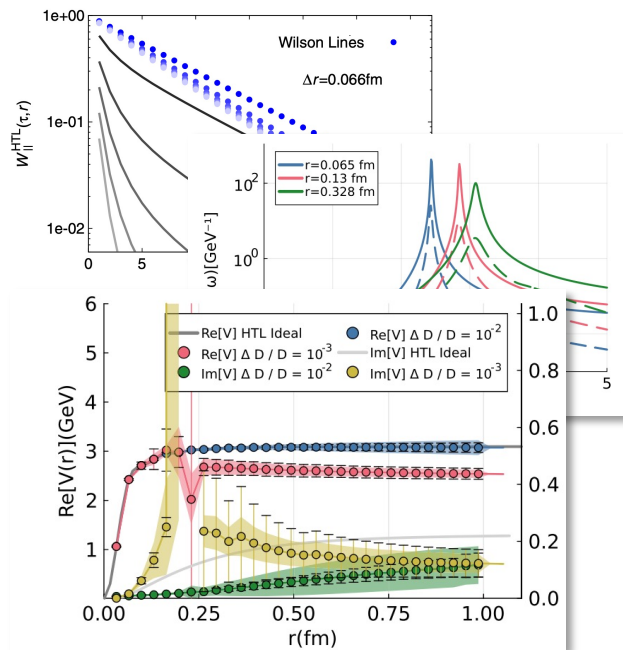
Realistic mock spectral functions via resummed perturbation theory (HTL)

HotQCD and A.R., G. Parkar
PRD 105 (2022) 5, 054513

Failure modes of $\text{Re}[V]$ & $\text{Im}[V]$: reconstruction underestimates true values

Mock-data closure tests

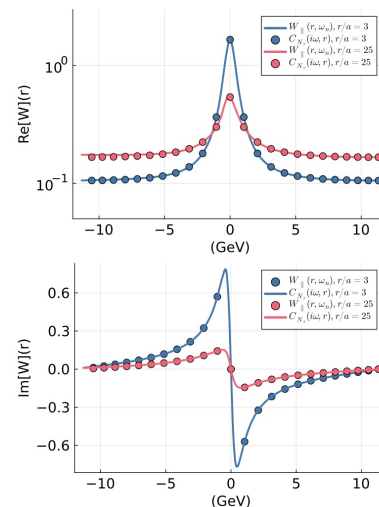
Bayesian Reconstruction



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PRD 105 (2022) 5, 054513

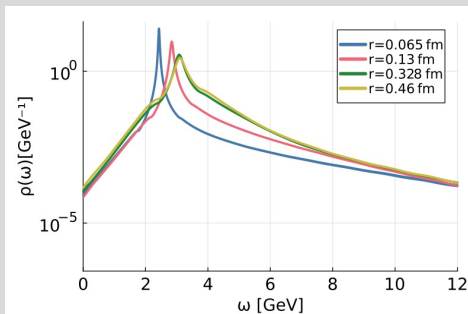
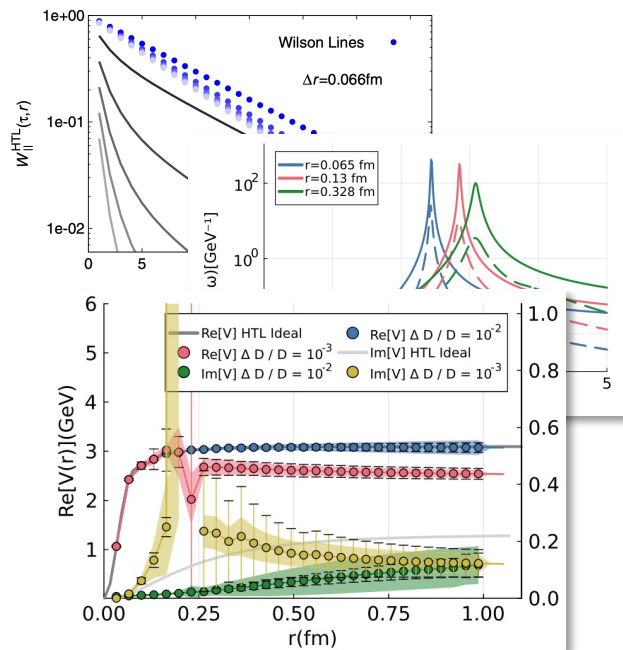
Pade Reconstruction



Failure modes of $\text{Re}[V]$ & $\text{Im}[V]$: reconstruction underestimates true values

Mock-data closure tests

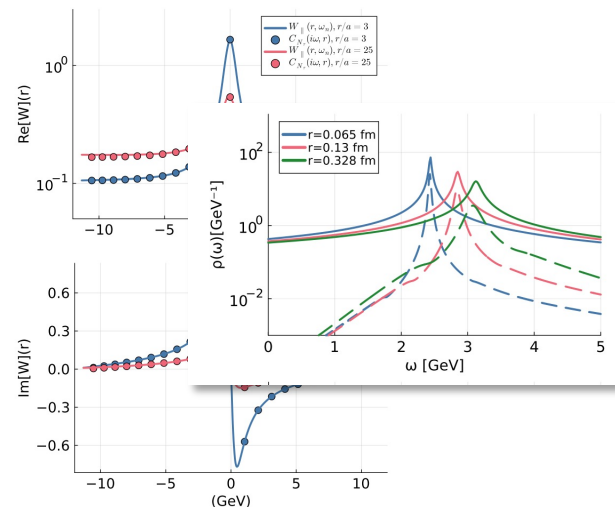
Bayesian Reconstruction



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PRD 105 (2022) 5, 054513

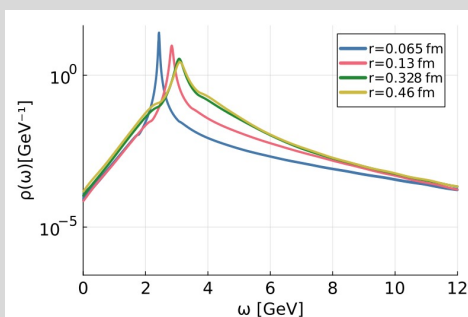
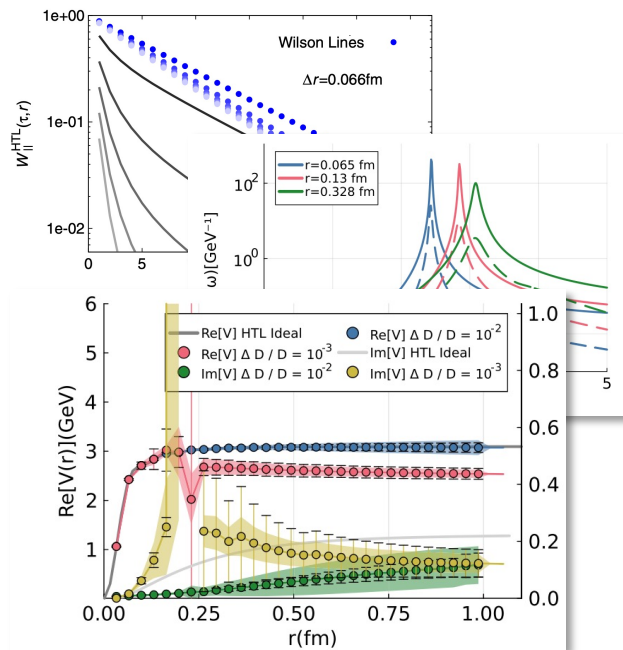
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Mock-data closure tests

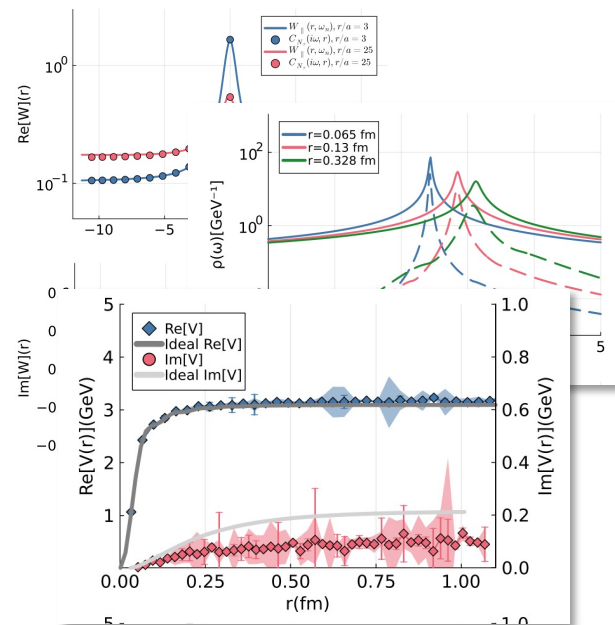
Bayesian Reconstruction



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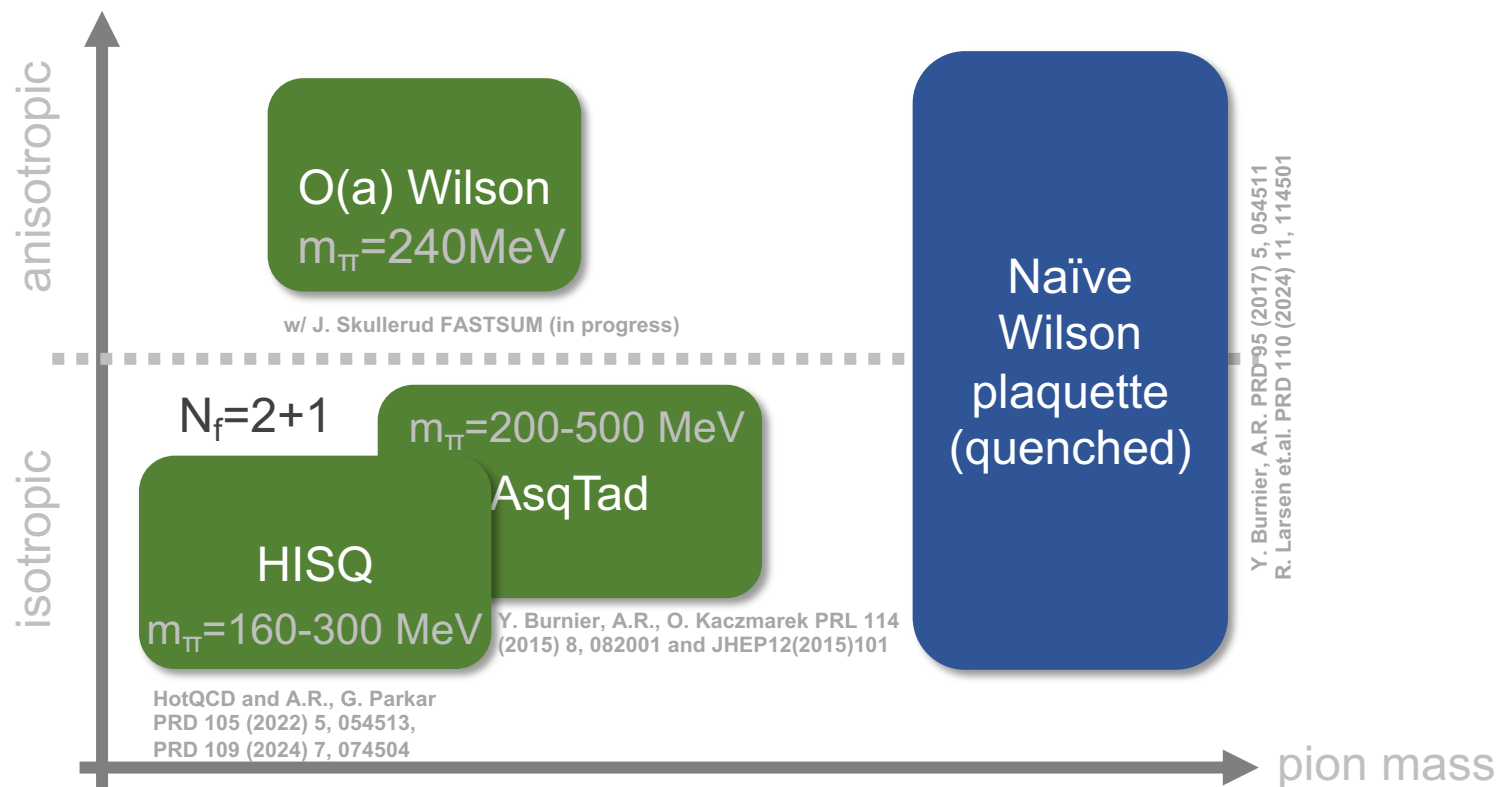
HotQCD and A.R., G. Parkar
PRD 105 (2022) 5, 054513

Pade Reconstruction



Failure modes of $\text{Re}[V]$ & $\text{Im}[V]$: reconstruction underestimates true values

Exploration of the complex static potential



Quenched QCD – $\text{Re}[V]$



- Purely gluonic medium – absence of light dynamical quarks: deconfinement phase transition at $T=271\text{MeV}$

Y. Burnier, A.R. PRD 95 (2017) 5, 054511 & R. Larsen et.al. PRD 110 (2024) 11, 114501

Quenched QCD – $\text{Re}[V]$

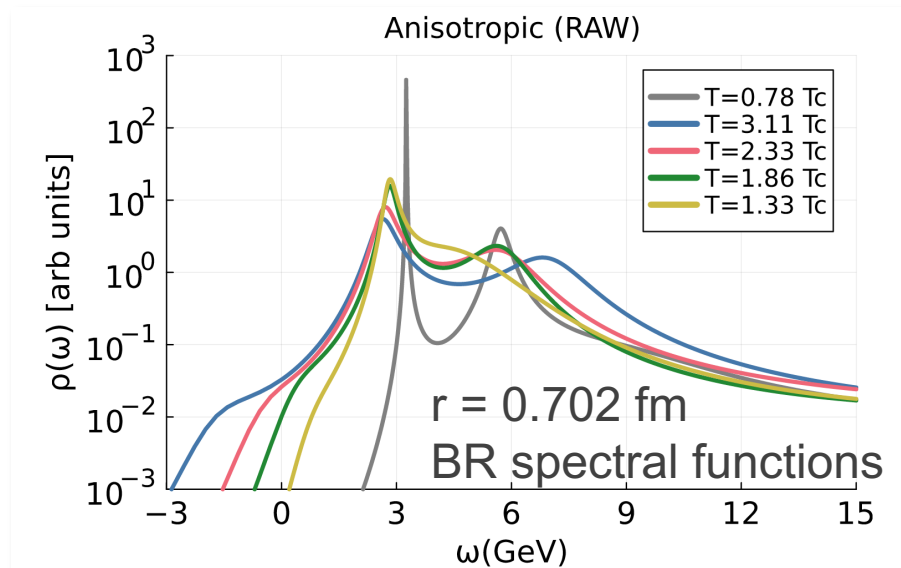
- Purely gluonic medium – absence of light dynamical quarks: deconfinement phase transition at $T=271\text{MeV}$
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- Moderate numerical cost,
fine grids: $N_\tau=20-96$

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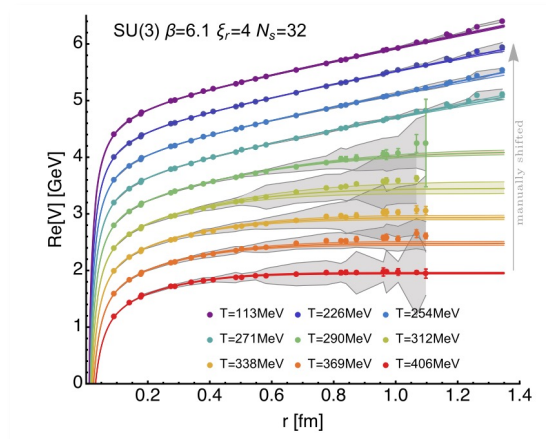


Quenched QCD – Re[V]

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- Moderate numerical cost, fine grids: $N_\tau=20-96$

- Based on Bayesian Reconstruction (BR) (crosschecked with Pade)



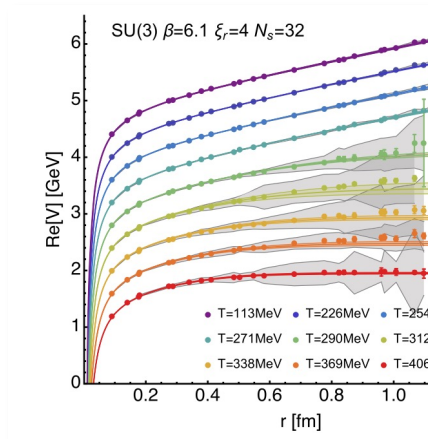
Wilson plaquette action

Quenched QCD – Re[V]

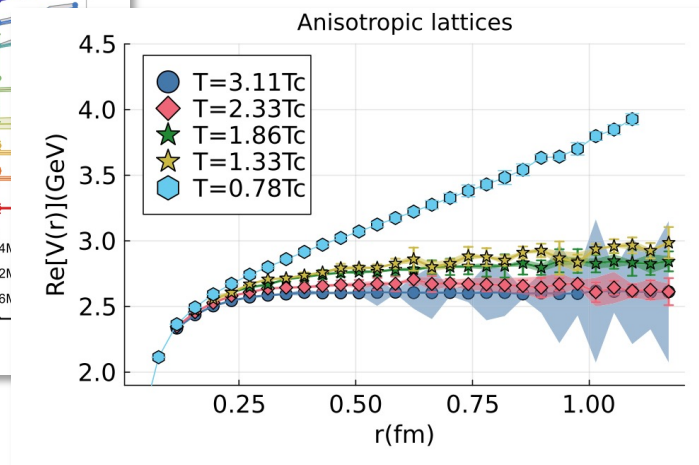
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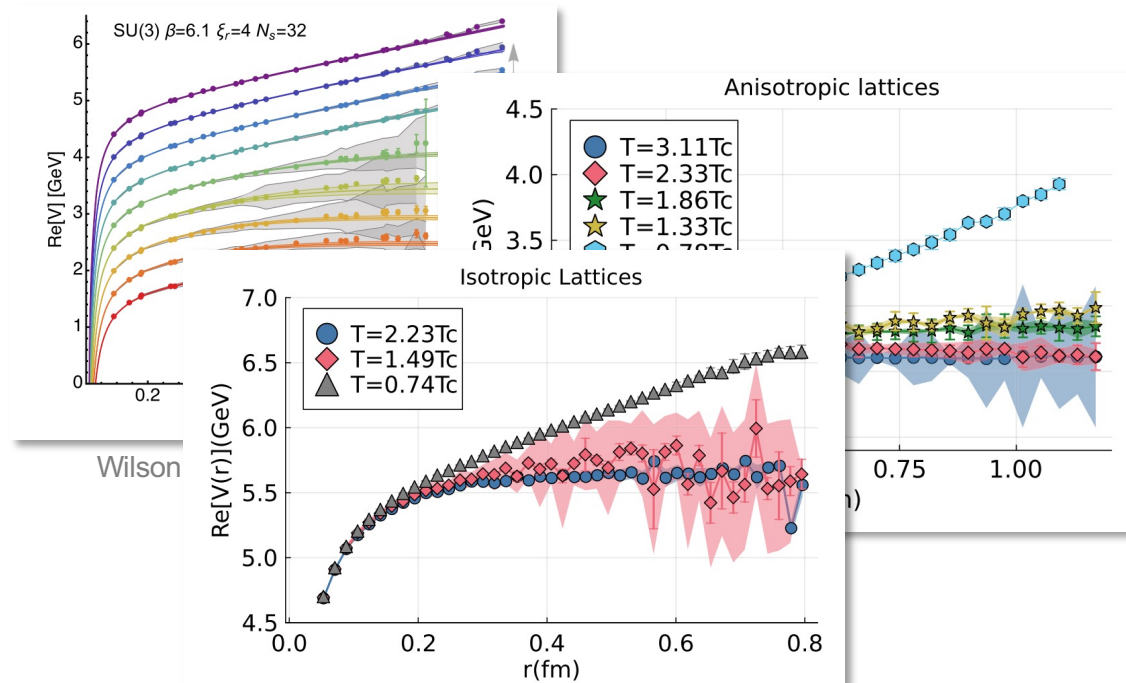


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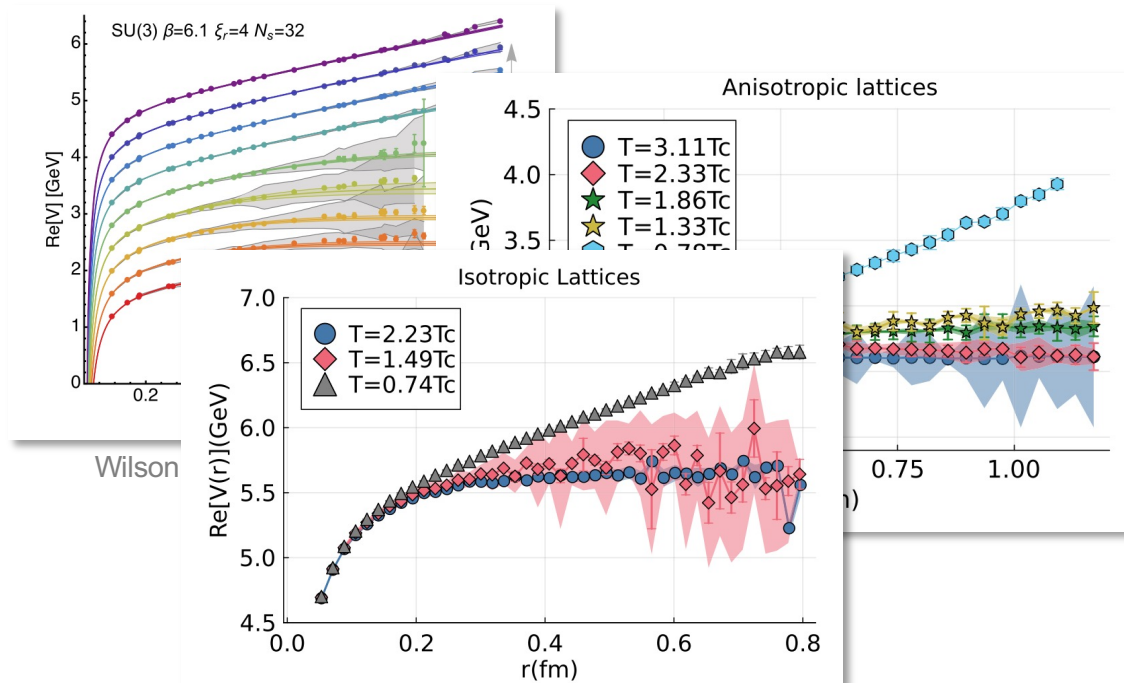
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Quenched QCD – Re[V]

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Y. Burnier, A.R. PRD 95 (2017) 5, 054511 & R. Larsen et.al. PRD 110 (2024) 11, 114501
- Moderate numerical cost, fine grids: $N_\tau=20-96$
- Based on Bayesian Reconstruction (BR) (crosschecked with Pade)
- Consistent picture:
 no modification at $T < T_C$
 Debye screening at $T > T_C$



Quenched QCD – $\text{Im}[V]$

- Purely gluonic medium – absence of light dynamical quarks: deconfinement phase transition at $T=271\text{MeV}$

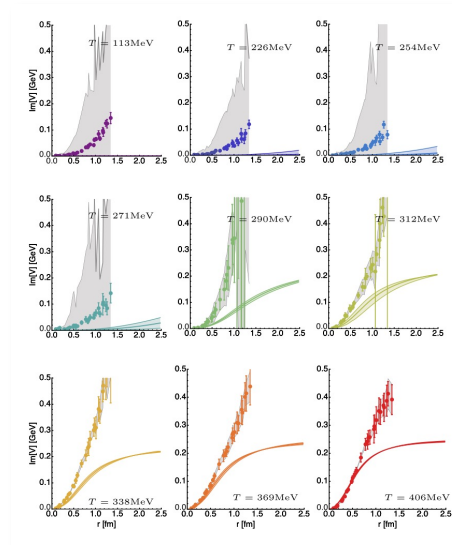
Y. Burnier, A.R. PRD 95 (2017) 5, 054511 & R. Larsen et.al. PRD 110 (2024) 11, 114501
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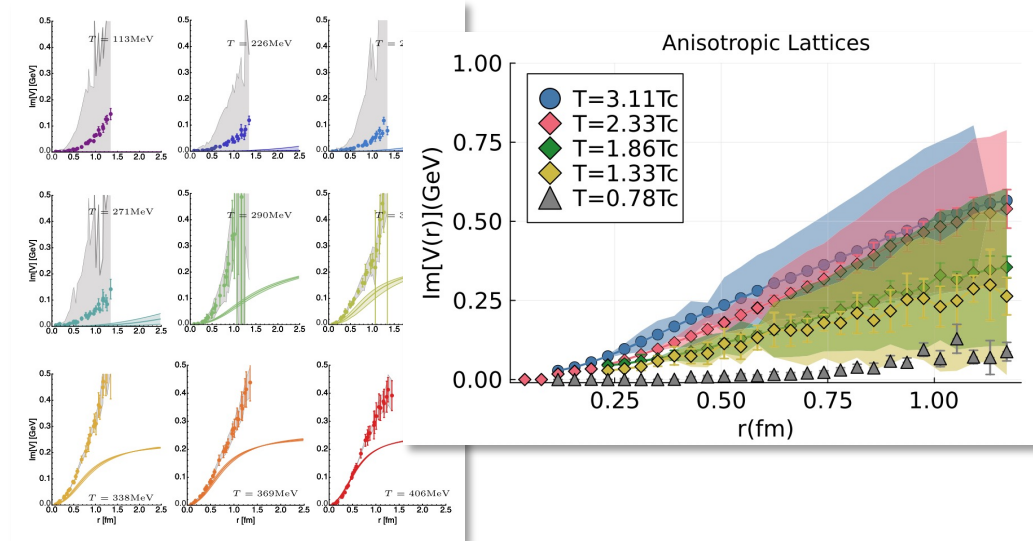


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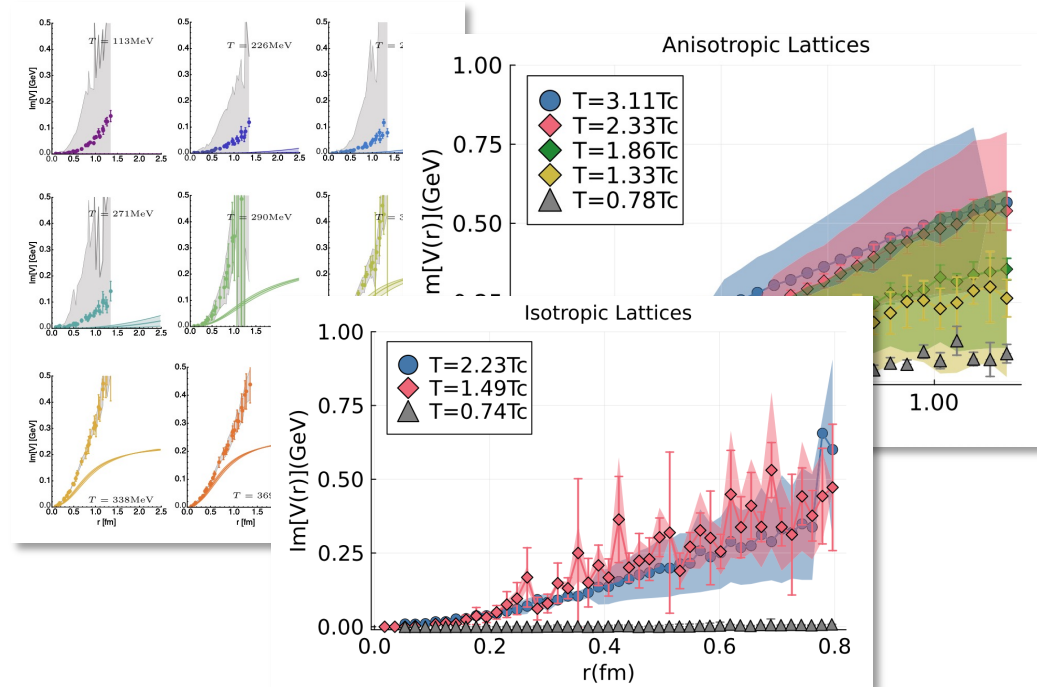


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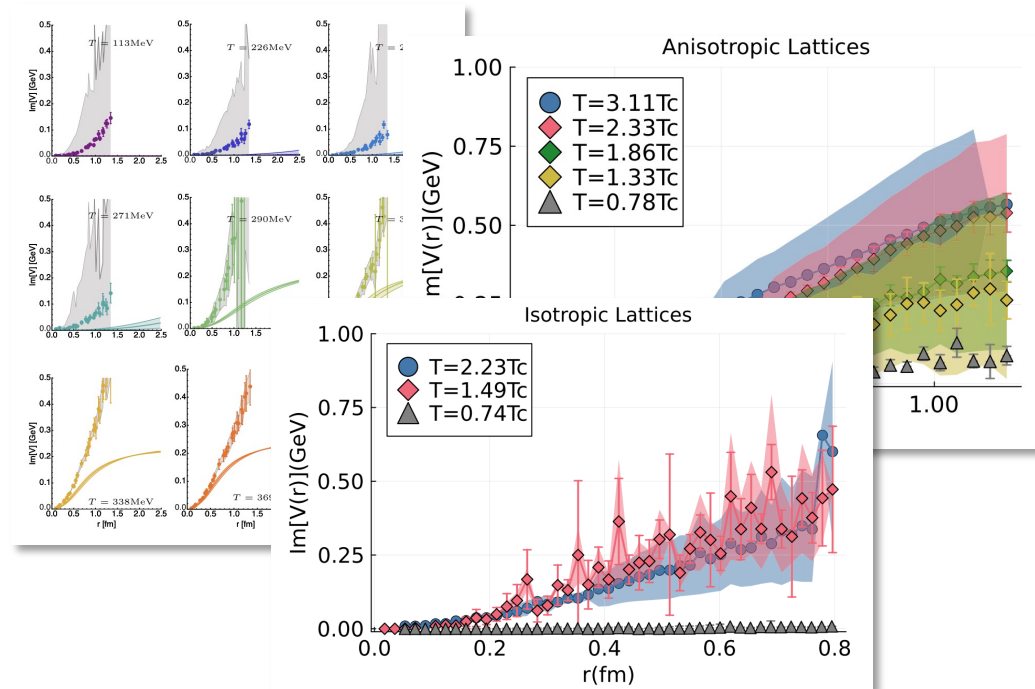
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Quenched QCD – $\text{Im}[V]$

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Y. Burnier, A.R. PRD 95 (2017) 5, 054511 & R. Larsen et.al. PRD 110 (2024) 11, 114501
- Moderate numerical cost, fine grids: $N_\tau=20-96$
- Based on both Bayesian Reconstruction (BR) and Pade Reconstruction
- Much more difficult to reliably extract $\text{Im}[V]$ but statistically significant signal for $\text{Im}[V]>0$ at $T>T_C$



Legacy isotropic full QCD - $\text{Re}[V]$ & $\text{Im}[V]$



■ $N_f=2+1$ AsqTad dynamical quarks w/ $m_\pi=300\text{MeV}$ but still coarse $N_\tau=12$

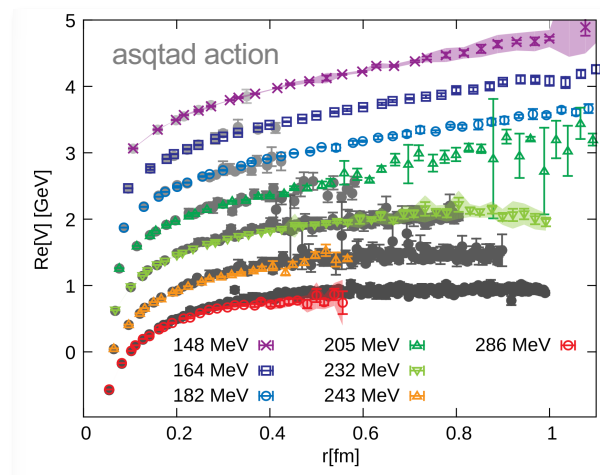
Y. Burnier, A.R., O. Kaczmarek JHEP12(2015)101

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Y. Burnier, A.R., O. Kaczmarek JHEP12(2015)101

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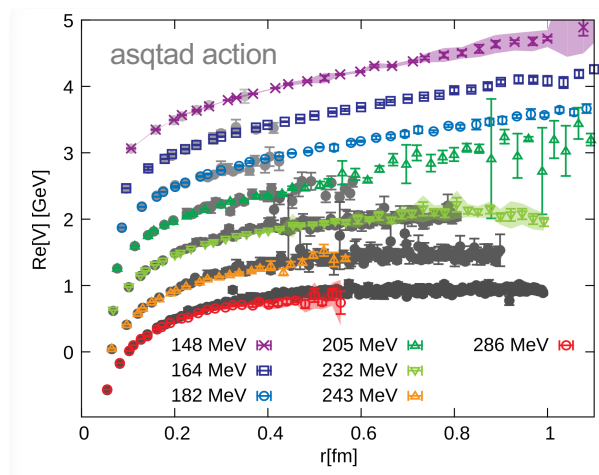


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- Based on Bayesian Reconstruction (BR)
- Smooth crossover manifest as monotonous weakening of $\text{Re}[V]$ as T increases. (residual σ but agreement w/ F^1)

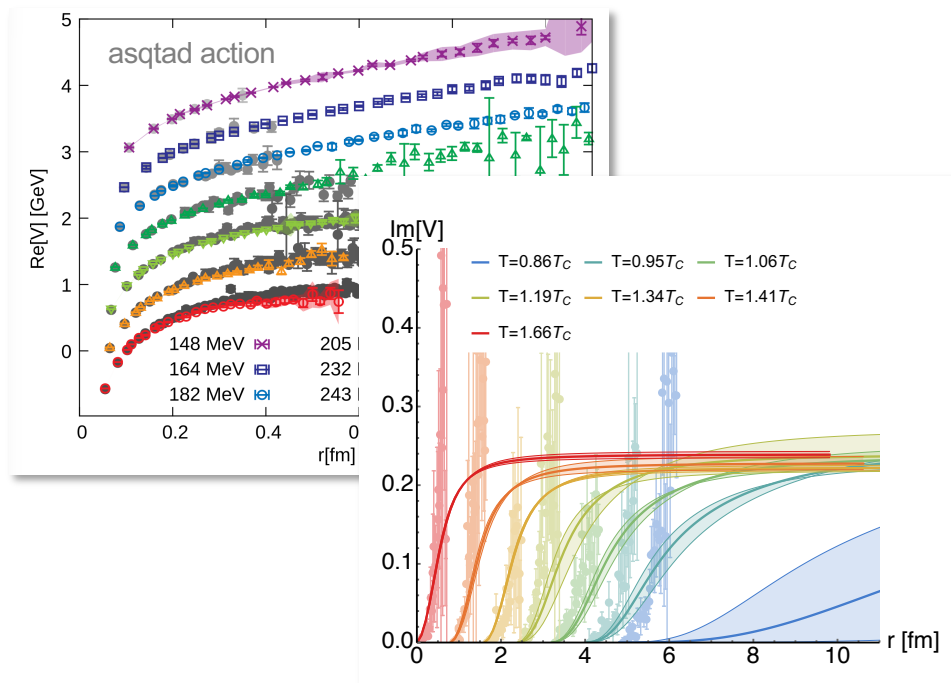


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Y. Burnier, A.R., O. Kaczmarek JHEP12(2015)101

- Based on Bayesian Reconstruction (BR)
- Smooth crossover manifest as monotonous weakening of $\text{Re}[V]$ as T increases. (residual σ but agreement w/ F^1)
- Imaginary part must be taken with grain of salt. Above T_C finite signal significant.



Isotropic full QCD - $\text{Re}[V]$ & $\text{Im}[V]$ (I)

■ $N_f=2+1$ HISQ dynamical quarks w/ mostly $m_\pi=160\text{MeV}$ but still coarse $N_\tau=12$

HotQCD and A.R., G. Parkar PRD 105 (2022) 5, 054513

Isotropic full QCD - $\text{Re}[V]$ & $\text{Im}[V]$ (I)

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HotQCD and A.R., G. Parkar PRD 105 (2022) 5, 054513

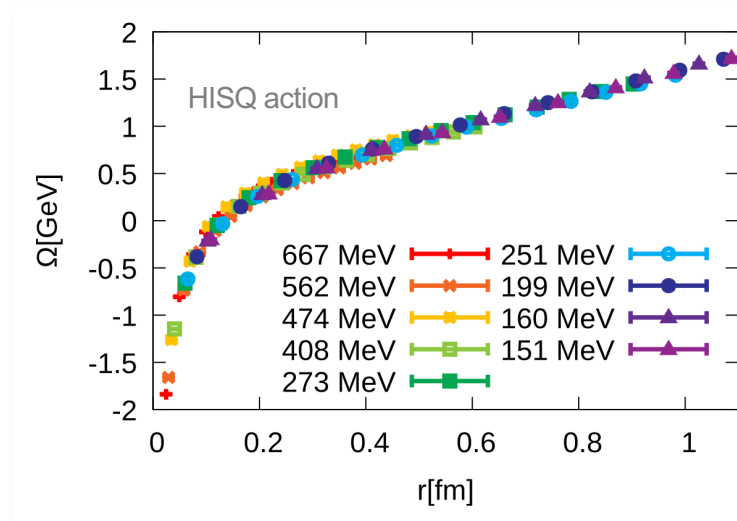
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HotQCD and A.R., G. Parker PRD 105 (2022) 5, 054513

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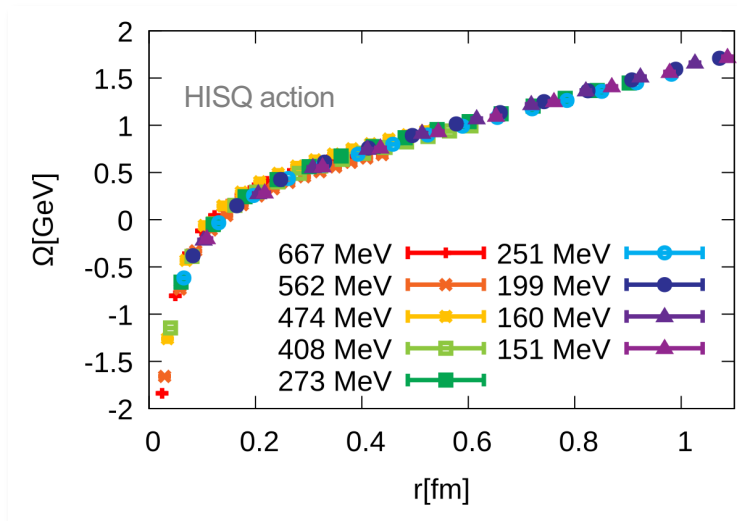
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HotQCD and A.R., G. Parker PRD 105 (2022) 5, 054513

- Based on both Pade reconstruction and model fits

- Surprising outcome: $\text{Re}[V]$ appears independent of T



Isotropic full QCD - $\text{Re}[V]$ & $\text{Im}[V]$ (I)

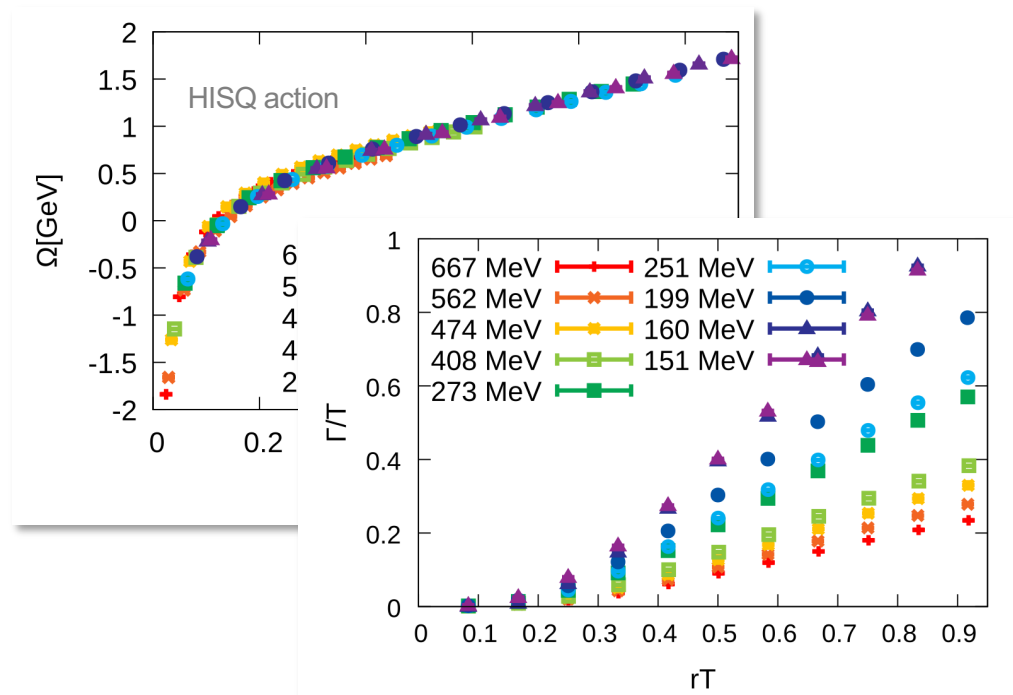
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HotQCD and A.R., G. Parker PRD 105 (2022) 5, 054513

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$\text{Im}[V]$ monotonously increases with temperature.



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HotQCD and A.R., G. Parkar PRD 105 (2022) 5, 054513

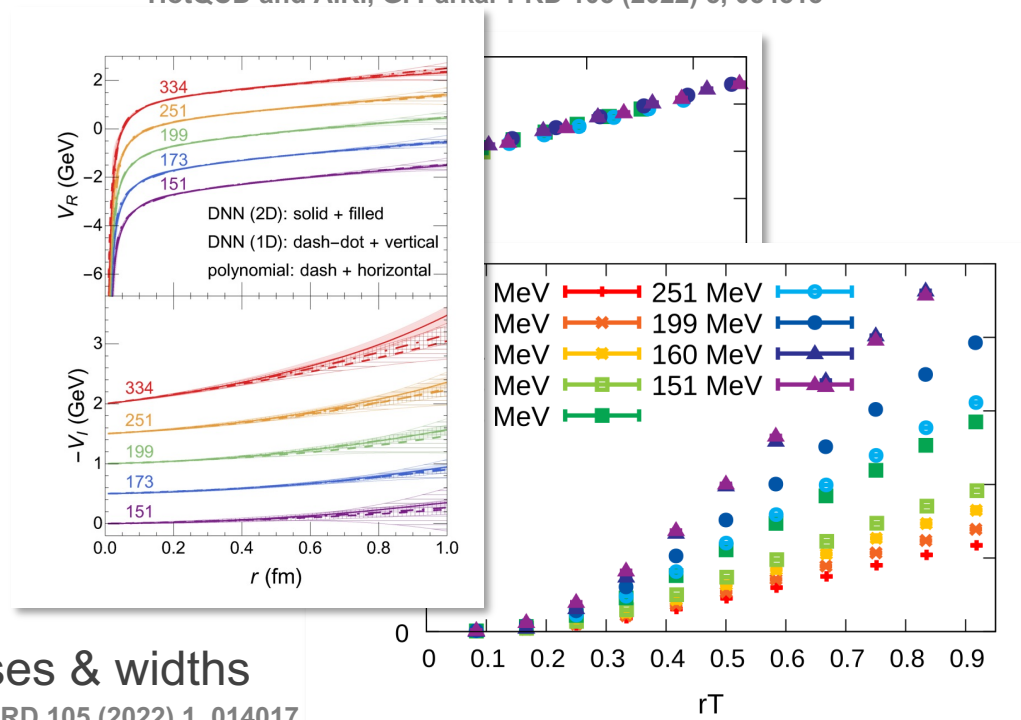
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- Surprising outcome: $\text{Re}[V]$ appears independent of T

$\text{Im}[V]$ monotonously increases with temperature.

- Interesting crosscheck: NN-parameterization of potential trained on quarkonium $T>0$ masses & widths

S. Shi, K. Zhou, J. Zhao, S. Mukherjee and P. Zhuang PRD 105 (2022) 1, 014017



Isotropic full QCD - $\text{Re}[V]$ & $\text{Im}[V]$ (II)

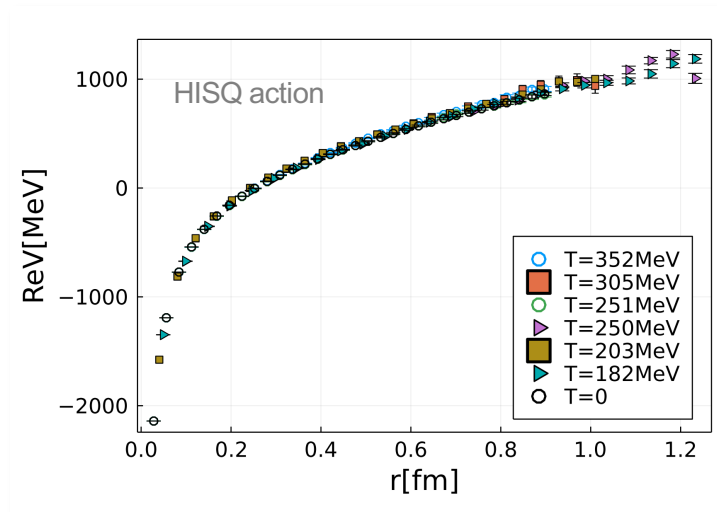
- $N_f=2+1$ HISQ dynamical quarks w/ mostly $m_\pi=160\text{MeV}$ & better resolution $N_\tau=16-36$
HotQCD and A.R. PRD 109 (2024) 7, 074504

Isotropic full QCD - $\text{Re}[V]$ & $\text{Im}[V]$ (II)

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HotQCD and A.R. PRD 109 (2024) 7, 074504

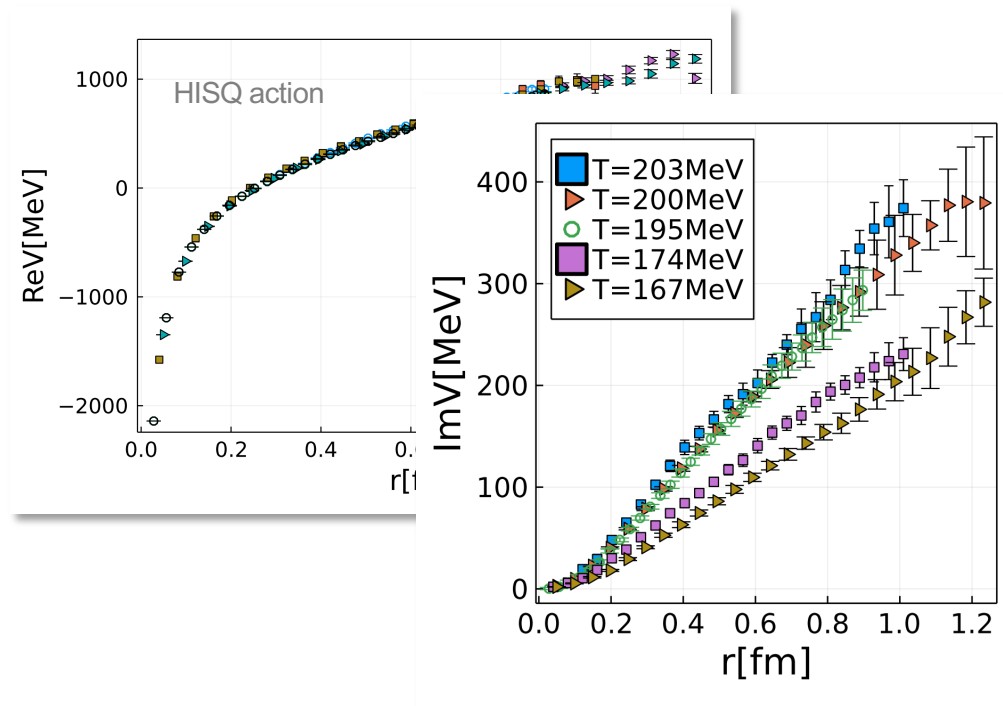
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- Reconfirms earlier result: $\text{Re}[V]$ appears independent of T



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- Reconfirms earlier result: $\text{Re}[V]$ appears independent of T
 $\text{Im}[V]$ monotonously increases with temperature.



Anisotropic full QCD - $\text{Re}[V]$ & $\text{Im}[V]$ (I)

- $O(a)$ Wilson quarks with heavy pion mass ($m_\pi=300\text{MeV}$) w/ $N_\tau=20-64$
(now w/ extra statistics for $N_\tau=24$)

FASTSUM Gen2L ensembles PRD 105, 034504

Anisotropic full QCD - $\text{Re}[V]$ & $\text{Im}[V]$ (I)

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 (now w/ extra statistics for $N_\tau=24$) FASTSUM Gen2L ensembles PRD 105, 034504

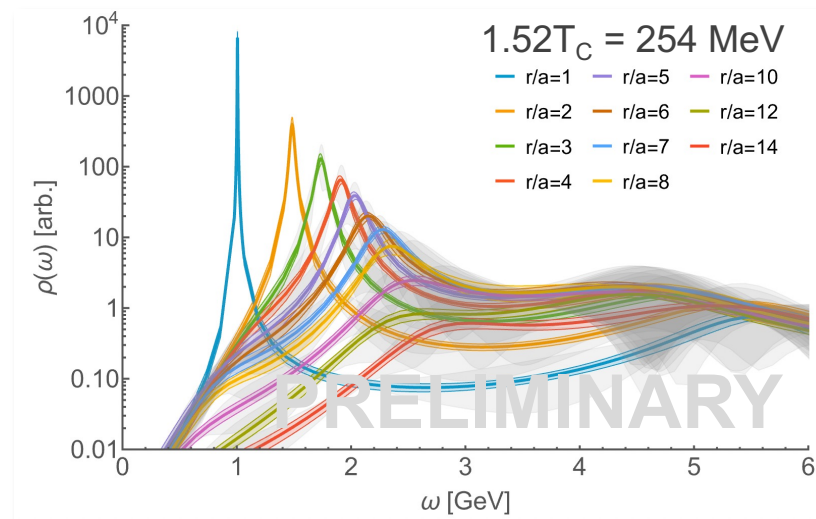
- Based on Bayesian Reconstruction (BR)
 (model fits in progress)

Anisotropic full QCD - $\text{Re}[V]$ & $\text{Im}[V]$ (I)

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FASTSUM Gen2L ensembles PRD 105, 034504

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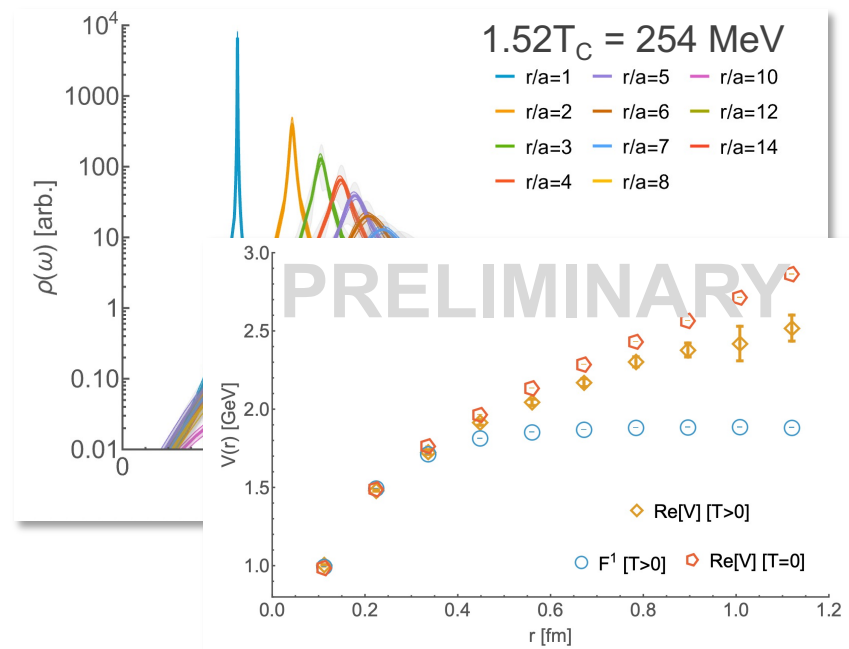


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FASTSUM Gen2L ensembles PRD 105, 034504

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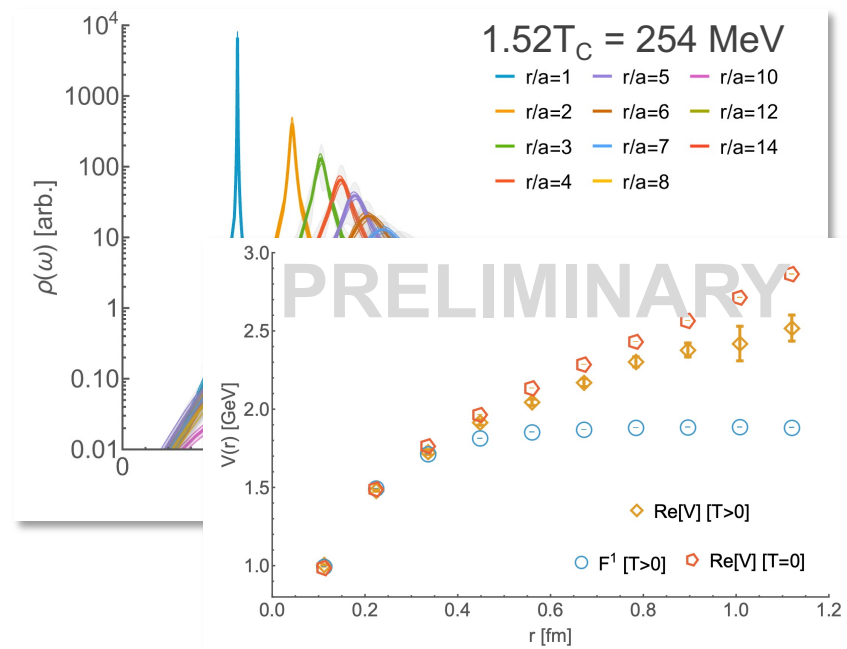


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FASTSUM Gen2L ensembles PRD 105, 034504

- Based on Bayesian Reconstruction (BR)
(model fits in progress)
- Slight departure from $T=0$ results
but significantly different from F^1

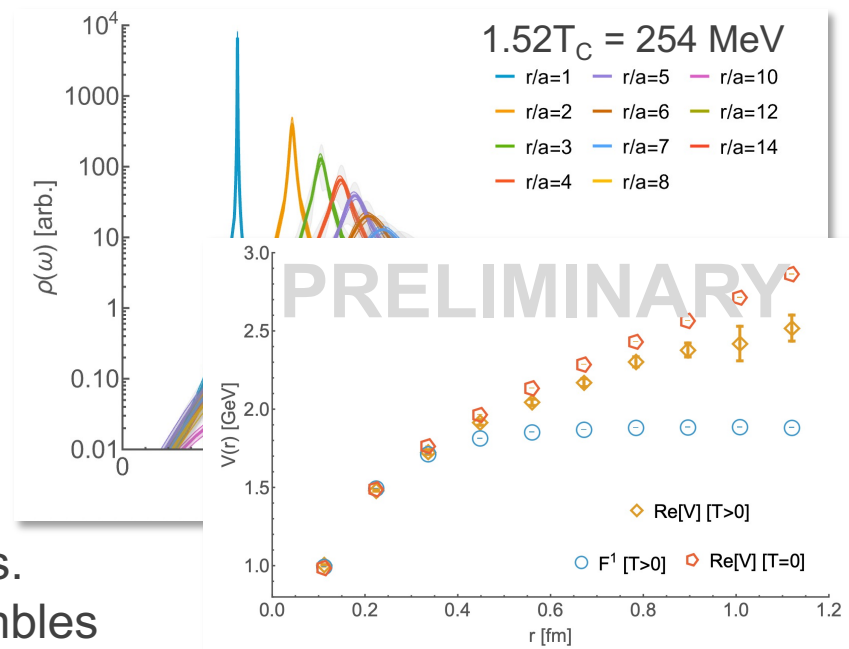


Anisotropic full QCD - $\text{Re}[V]$ & $\text{Im}[V]$ (I)

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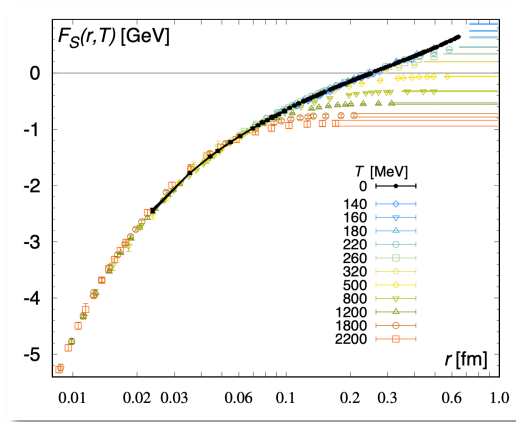
FASTSUM Gen2L ensembles PRD 105, 034504

- Based on Bayesian Reconstruction (BR)
(model fits in progress)
- Slight departure from $T=0$ results but significantly different from F^1
- For $N_\tau < 24$ default model & truncation uncertainty too large for reliable results.
Stay tuned for FASTSUM Gen3 ensembles

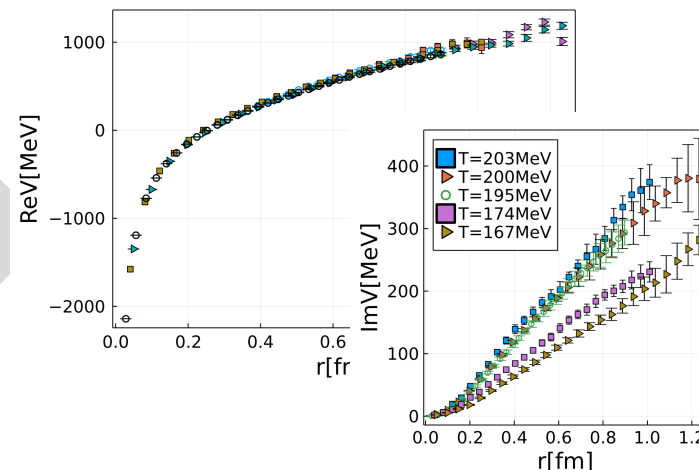


Consistency of recent results

- Presence of significant imaginary part will lead to decoherence - decorrelation



TUMQCD PRD98 (2018) 5, 054511



HotQCD & A.R. PRD 109 (2024) 7, 074504

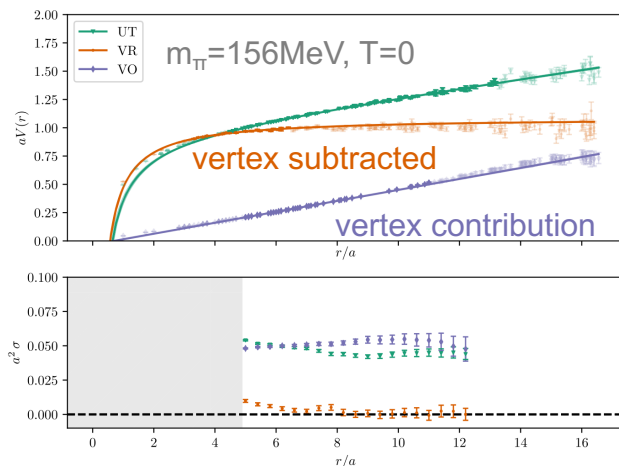
- No apriori inconsistency between potential and thermodynamic behavior

Tantalizing hints for additional structure

- Recent study of topological center vortices at zero & finite temperature

Tantalizing hints for additional structure

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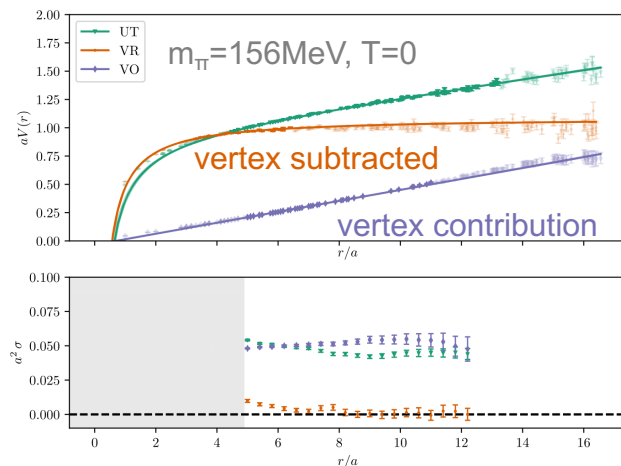


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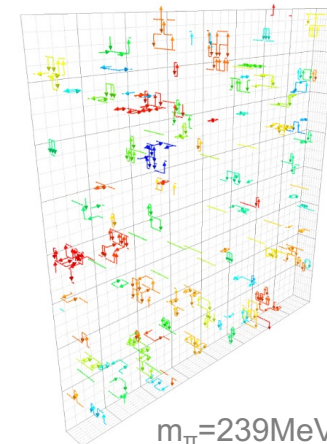
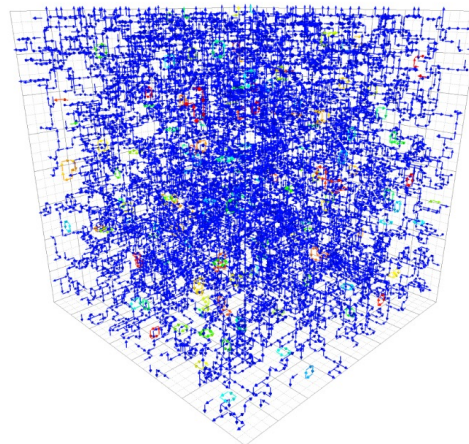
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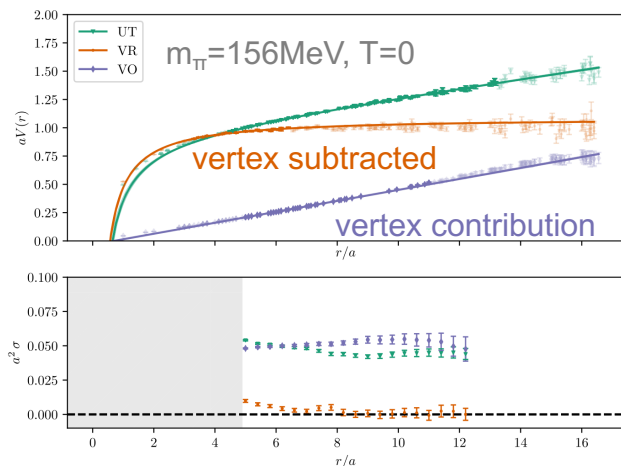
$m_\pi = 239 \text{ MeV}$, $T=0$
O(a) improved Wilson fermions

J. Mickley et.al. PRD 111, 034508

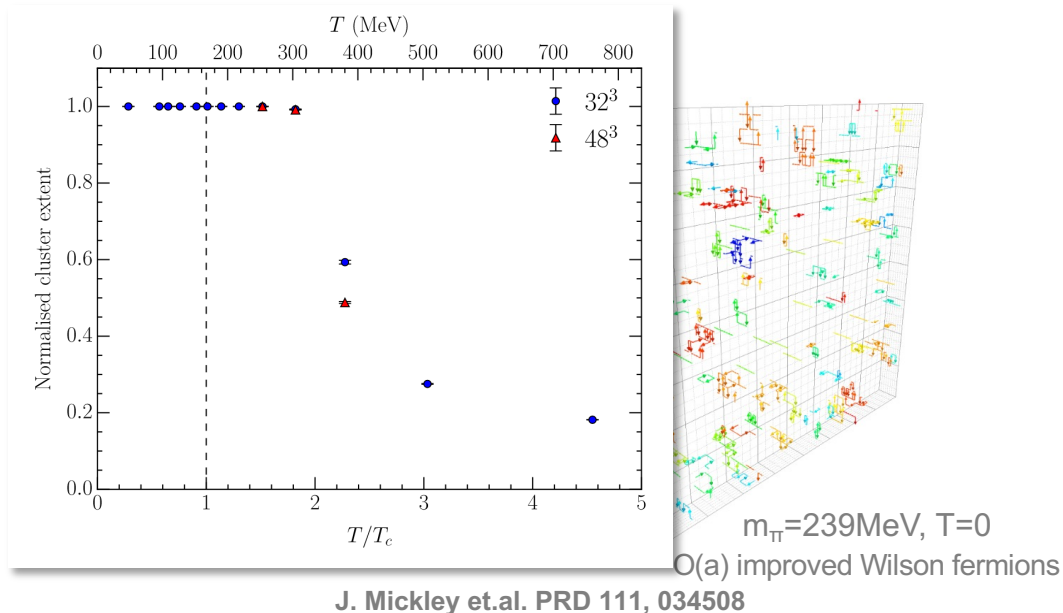
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- At $T > 2T_c$ show percolation transition which may indicate confinement persists

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谢谢

Thank you for your attention