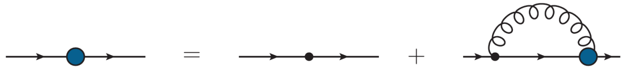
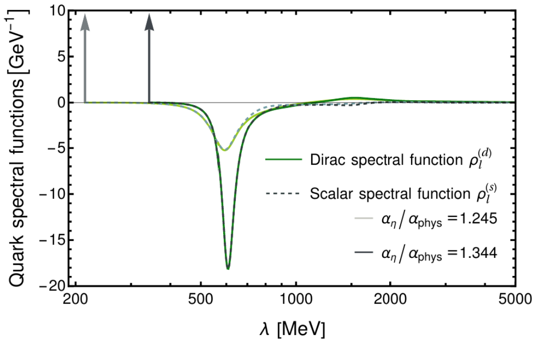
The quark spectral function and (heavy) quark diffusion

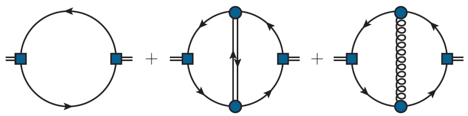


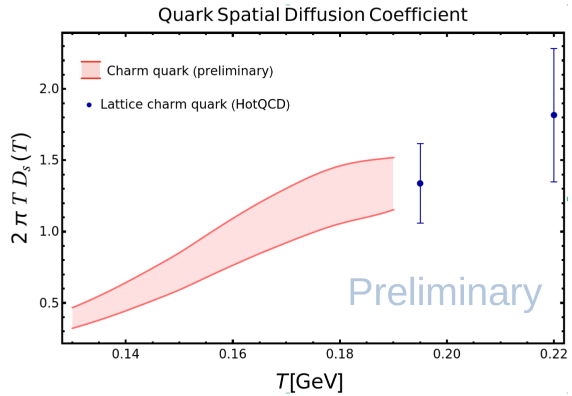


Jonas Wessely

(JLU Giessen)

Dalian 09.10.25





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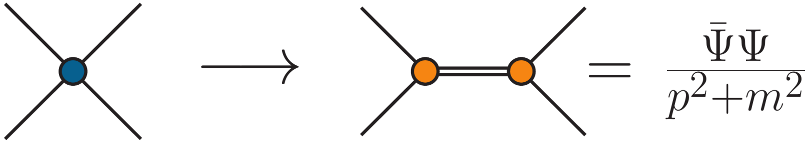
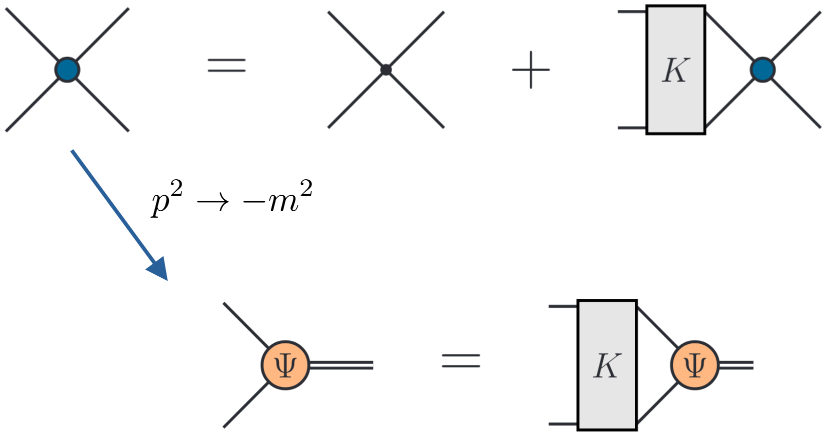
Why real-time?

Hadron Spectroscopy

$\rightsquigarrow$ Boundstate masses and properties





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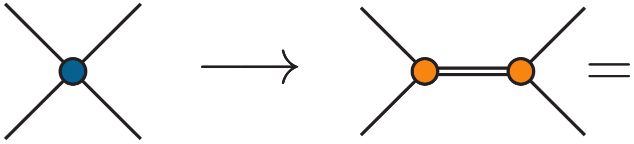
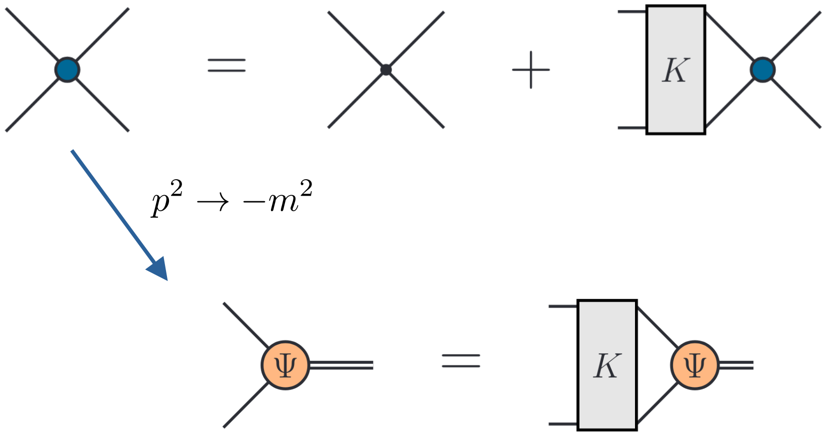
⤳ Boundstate masses and properties

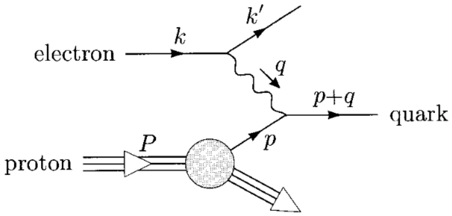


$$= \frac{\bar{\Psi}\Psi}{p^2+m^2}$$



Scattering and cross sections

⤳ PDFs, PDAs, TMDs, ...



Peskin & Schröder

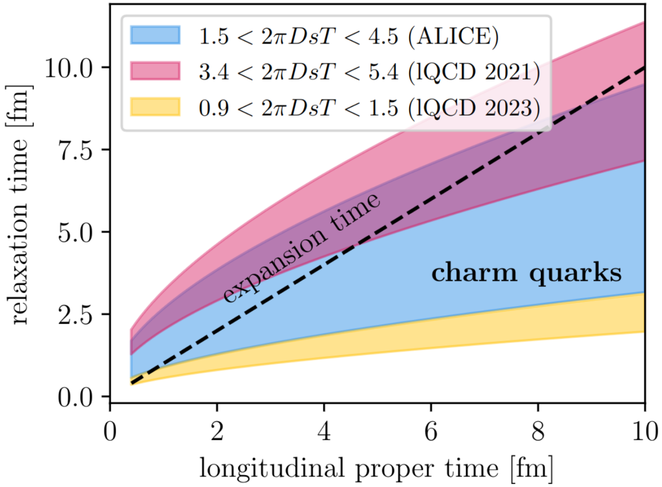
$$\sigma_{eH}(x, Q^2) = \sum_a \int_x^1 \mathrm{d}\xi\, f_{a/H}(\xi)\, \sigma_B(x/\xi, Q^2)$$

Why real-time?

Hydro approach to QGP evolution

$\rightsquigarrow$ Diffusion coefficients, Shear/Bulk viscosity

Capellino er al. EPJ Web Conf. 296 (2024)



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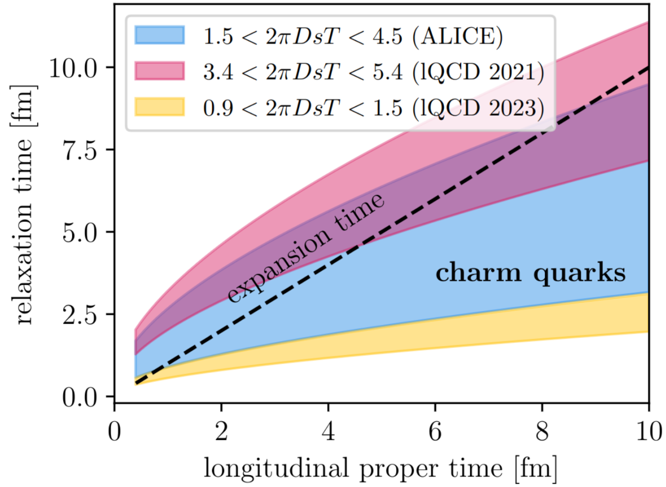


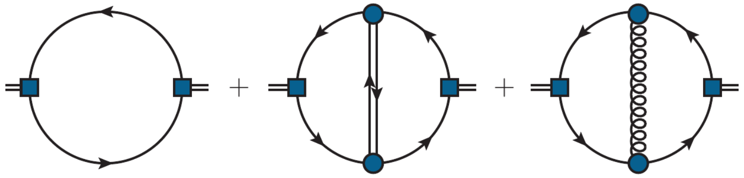
Hydro approach to QGP evolution

⤳ Diffusion coefficients, Shear/Bulk viscosity

$$\mathscr{D}_s = \lim_{\omega \to 0} \frac{\sigma(\omega, \vec{p} = 0)}{\omega \chi_q \pi}$$

$$\sigma(\omega, \mathbf{p}) = \frac{1}{\pi} \int \mathrm{d}t\, e^{i\omega t} \int \mathrm{d}^3 x\, e^{i\mathbf{x}\mathbf{p}} \langle [J_i(t, \mathbf{x}), J_i(0,0)] \rangle$$

$\propto$ Im



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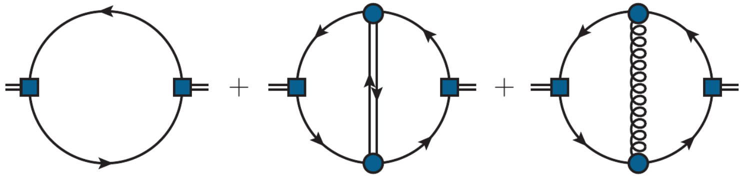
$\propto$ Im 

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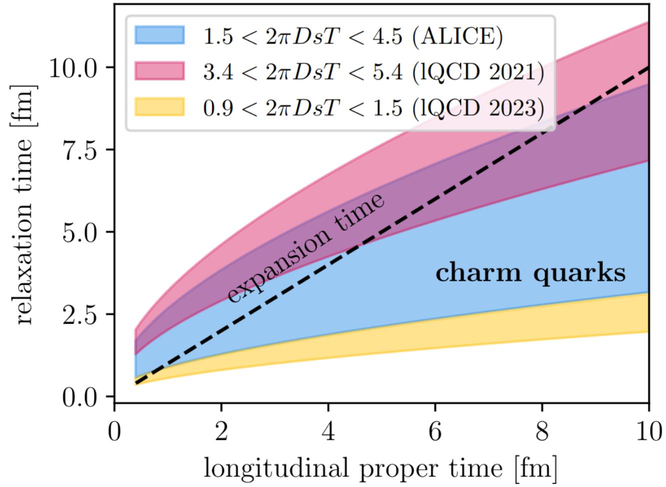
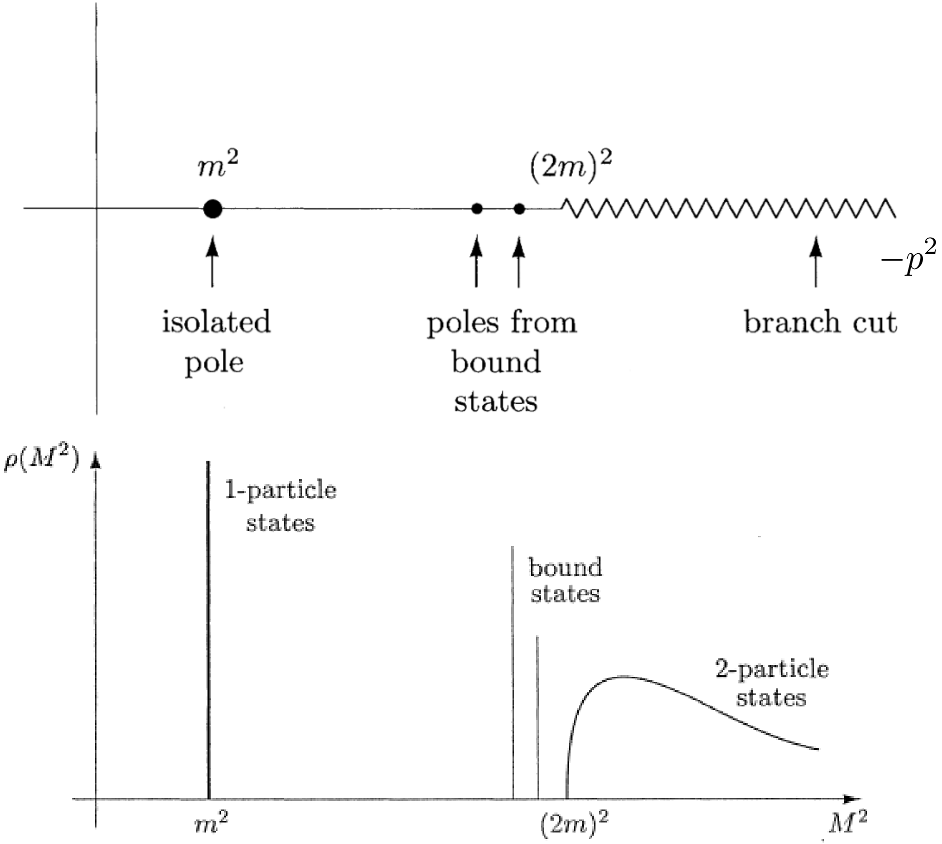


Spectral Functional Methods:

- Access to realtime correlations and finite chemical potentials

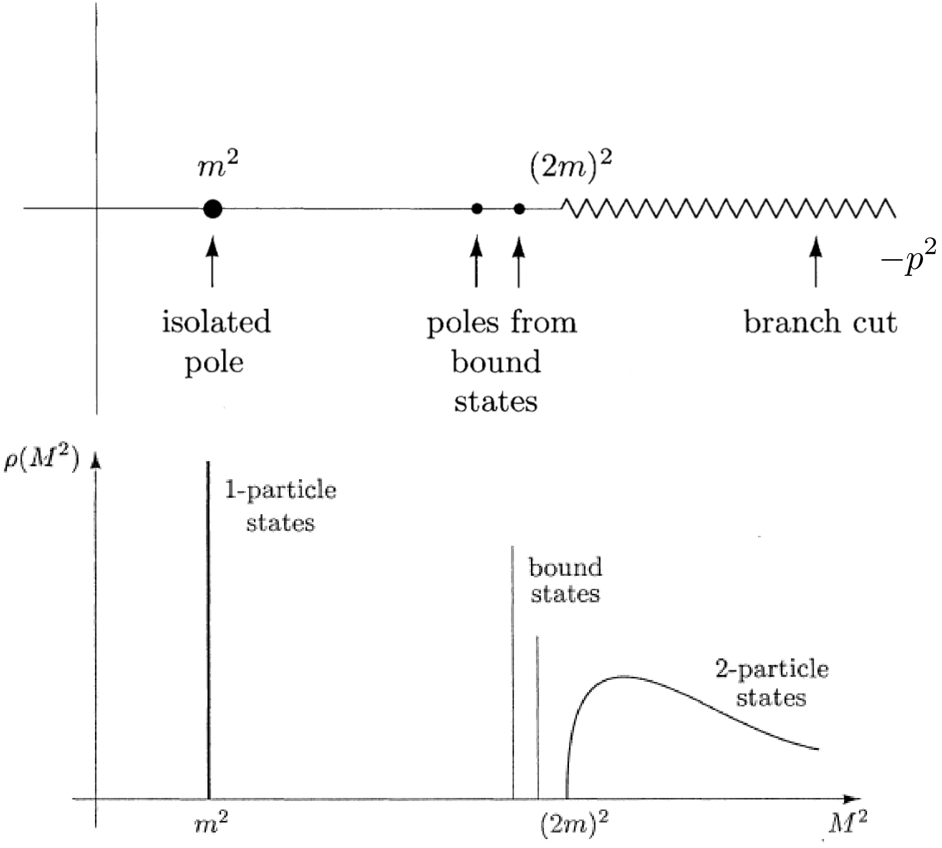
Spectral Functional equations



Peskin & Schröder

$$\boxed{G(p) = \int_\lambda \frac{\rho(\lambda)}{p^2 + \lambda^2}}$$

Spectral Functions



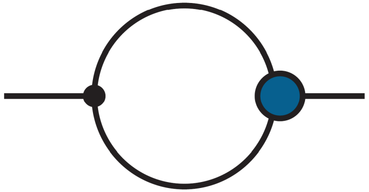
Peskin & Schröder

$$G(p) = \int_\lambda \frac{\rho(\lambda)}{p^2 + \lambda^2}$$

- Spectral functions plays role of (propability) weight for intermediate states of energy λ
- Propagator determined by its discontinuities
- Causality requires analyticity in upper half plane

$$\rho(\omega) = 2\mathrm{Im}\, G(p_0 \to -(\omega + 0^+))$$

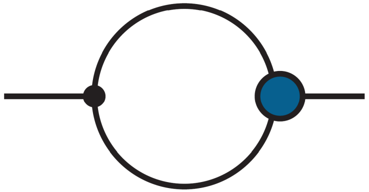
Spectral Functional equations



$$\propto \int_q G(q)G(p+q)$$

$$\boxed{G(p) = \int_\lambda \frac{\rho(\lambda)}{p^2+\lambda^2}}$$

Spectral Functional equations



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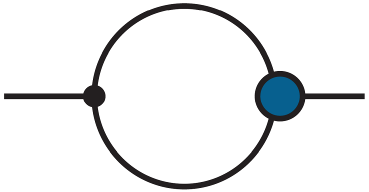
$$= \int_{\lambda_1,\lambda_2} \rho(\lambda_1)\rho(\lambda_2) \int_q \frac{1}{(q^2+\lambda_1^2)((p+q)^2+\lambda_2^2)}$$

Spectral Functional equations



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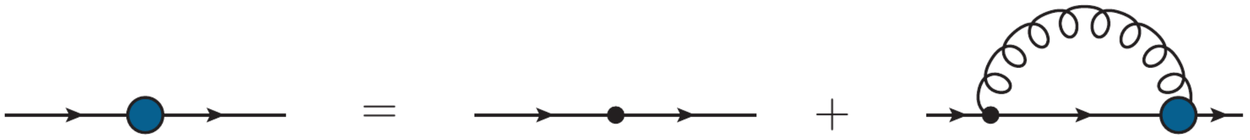
- Loop integrals can be calculated in dimReg
- Access to the full complex plane

- Additional spectral integrals for each correlation function
- Spectral renormalisation for diverging diagrams

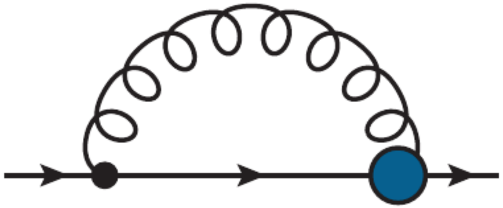
The spectral quark gap equation



J. M. Pawlowski, JW EPJC 85 (2025)

Spectral self-energy

J. M. Pawlowski, JW EPJC 85 (2025)



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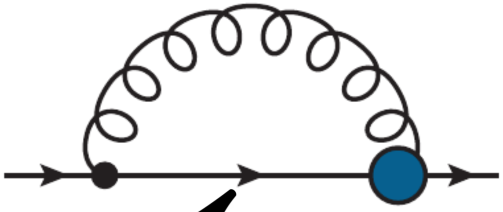
J. M. Pawlowski, JW EPJC 85 (2025)



Compute selfconsistently:

$$G_q(p) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \frac{\rho_q(\lambda)}{\mathrm{i}\not{p} + \lambda}$$

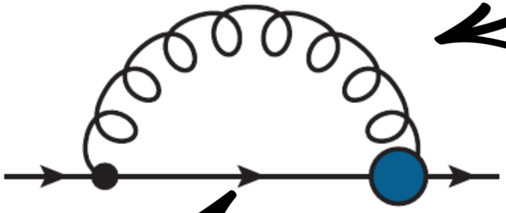
$$= \frac{-\mathrm{i}\not{p} + M_q(p)}{Z_q(p)\left[p^2 + M_q^2(p)\right]}$$

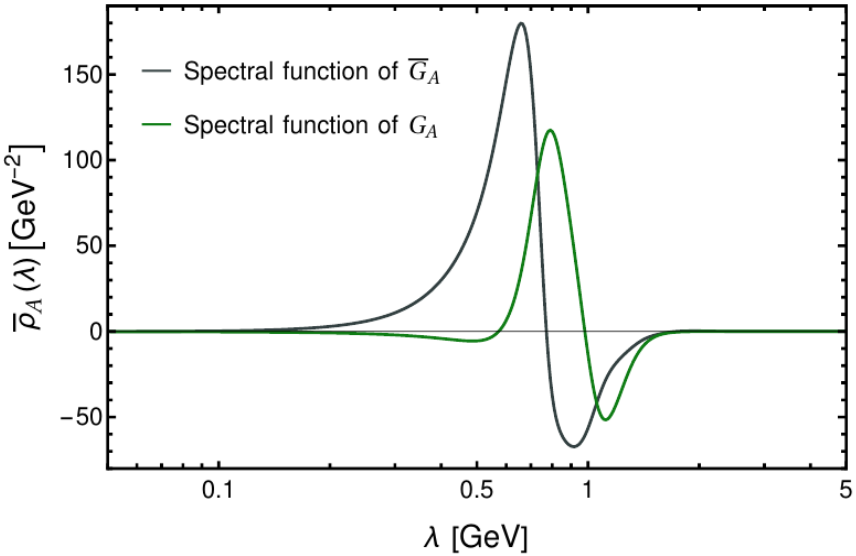
Spectral self-energy

J. M. Pawlowski, JW EPJC 85 (2025)

Gluon from spectral reconstruction

$$G_A(q) = \int_0^\infty \frac{d\lambda}{\pi} \frac{\lambda \rho_A(\lambda)}{q^2+\lambda^2}$$





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Rec. via GPR from:
Horak et. al. PRD 105 (2022)
Horak et. al. PRD 107 (2023)

Spectral self-energy

J. M. Pawlowski, JW EPJC 85 (2025)

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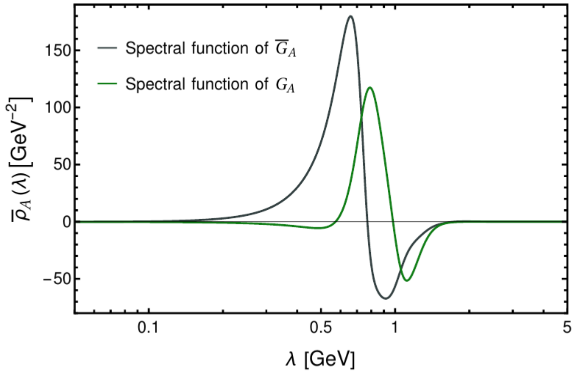

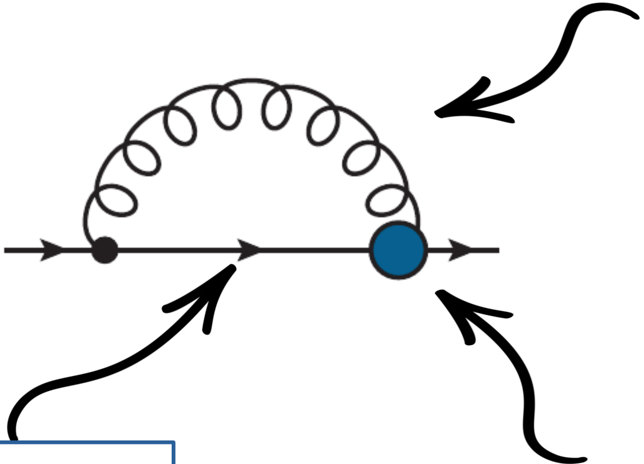

$$\Gamma^\mu \propto \gamma^\mu p^2 G_c(p)$$

Compute selfconsistently:

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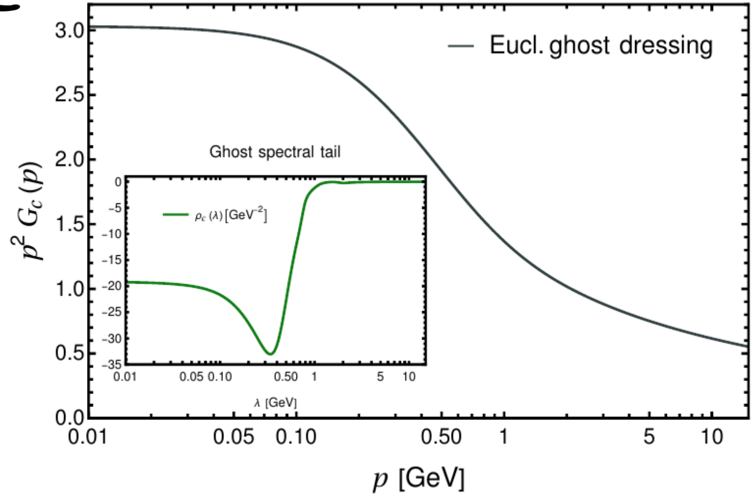
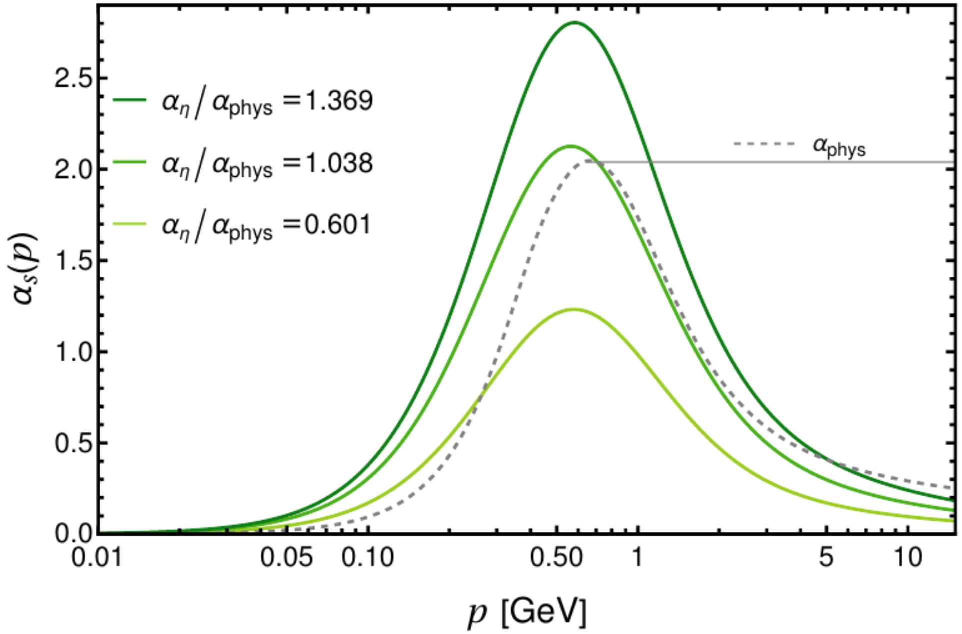
$$= \frac{-\mathrm{i}\not{p} + M_q(p)}{Z_q(p)\left[p^2 + M_q^2(p)\right]}$$



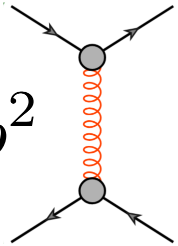
The causal quark gluon coupling



Peak position is well reproduced with given input

Tune height of the peak with global factor η

$$\alpha_s(p) = p^2$$



α_s^{phys} by Ihssen, Pawlowski, Sattler, Wink arXiv: 2408.08413 (2024)

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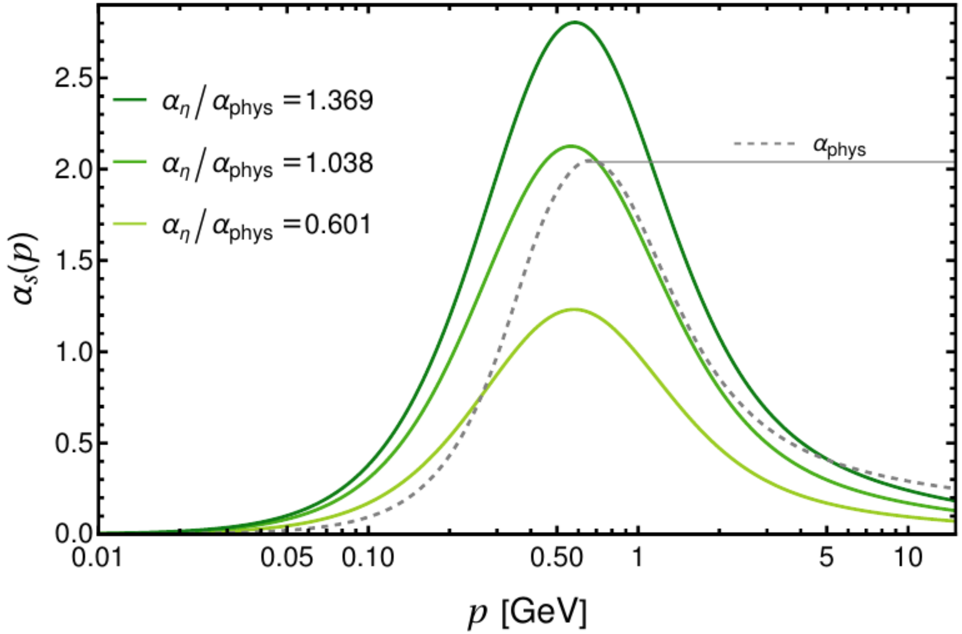

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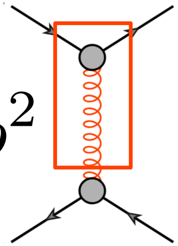
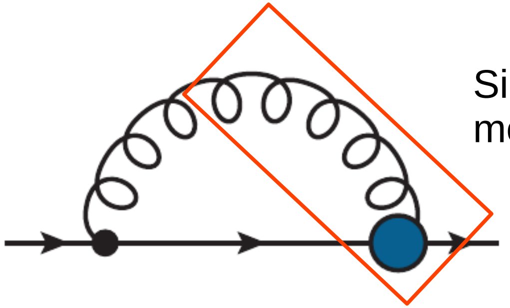
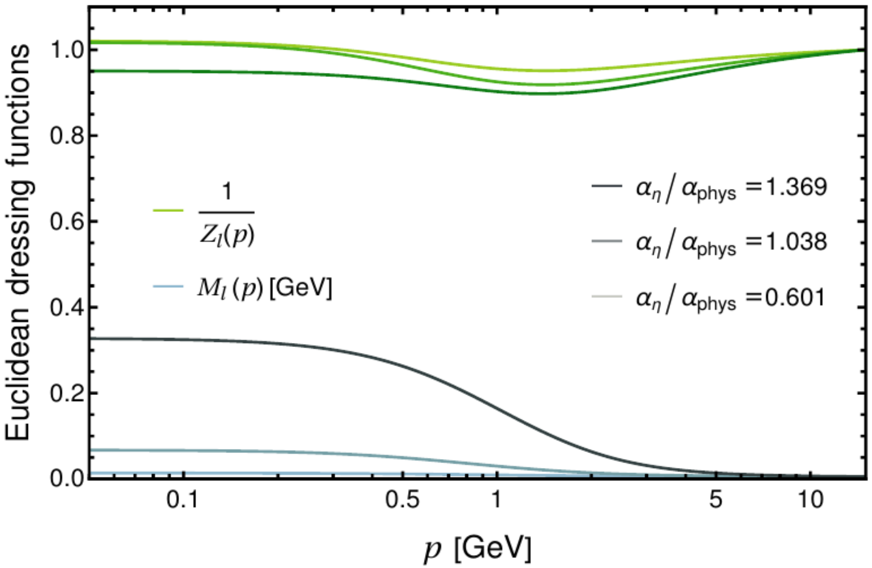




Similar to often used rainbow ladder model ... but causal!

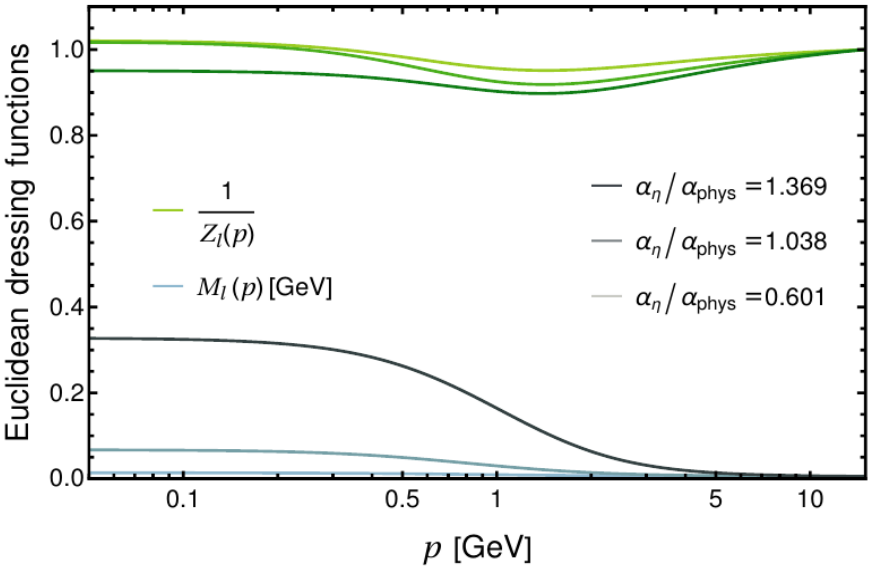
α_s^{phys} by Ihssen, Pawlowski, Sattler, Wink arXiv: 2408.08413 (2024)

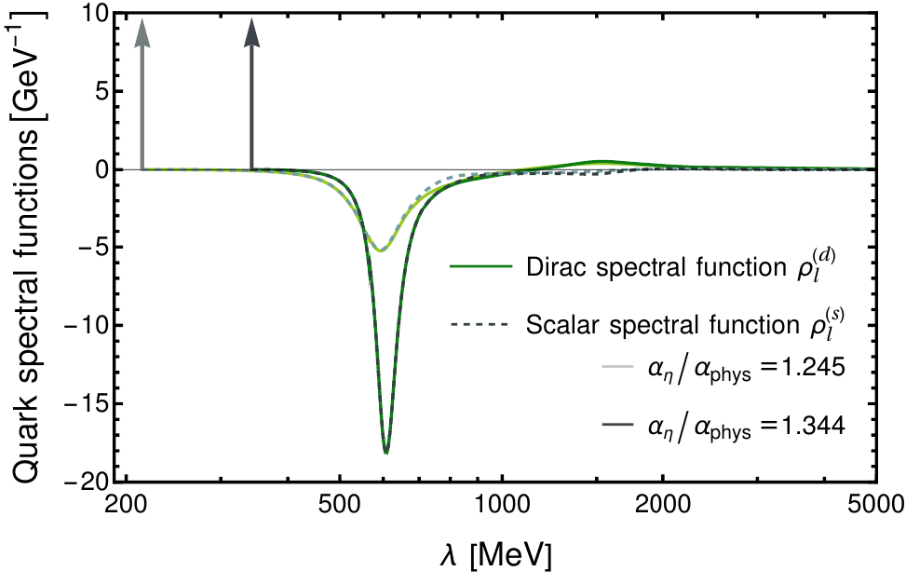
The spectral gap equation II



$\rightsquigarrow$ Causal vertex model reproduces phenomenology of $\mathrm{D}\chi\mathrm{SB}$

The spectral gap equation II



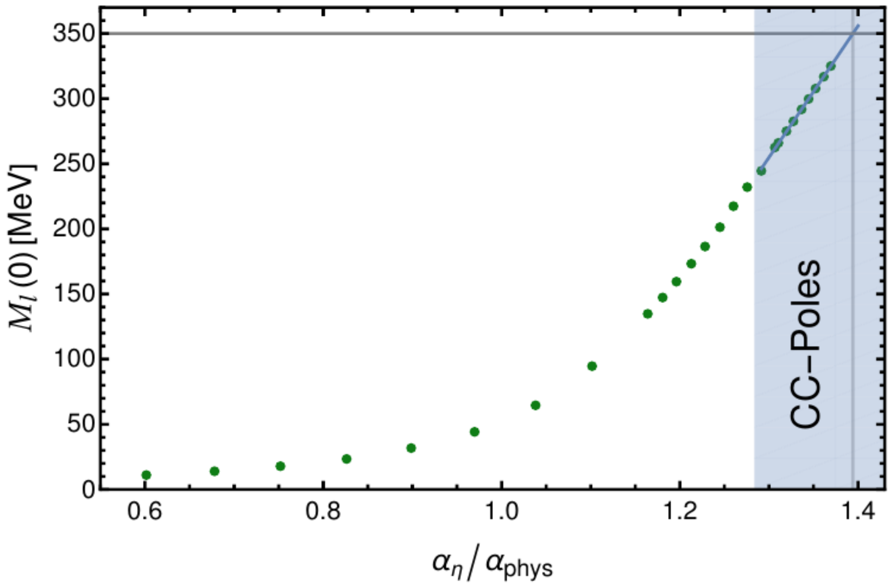


Causal vertex model reproduces phenomenology of $\mathrm{D}\chi\mathrm{SB}$

Real pole at the onset of scattering for converged solution

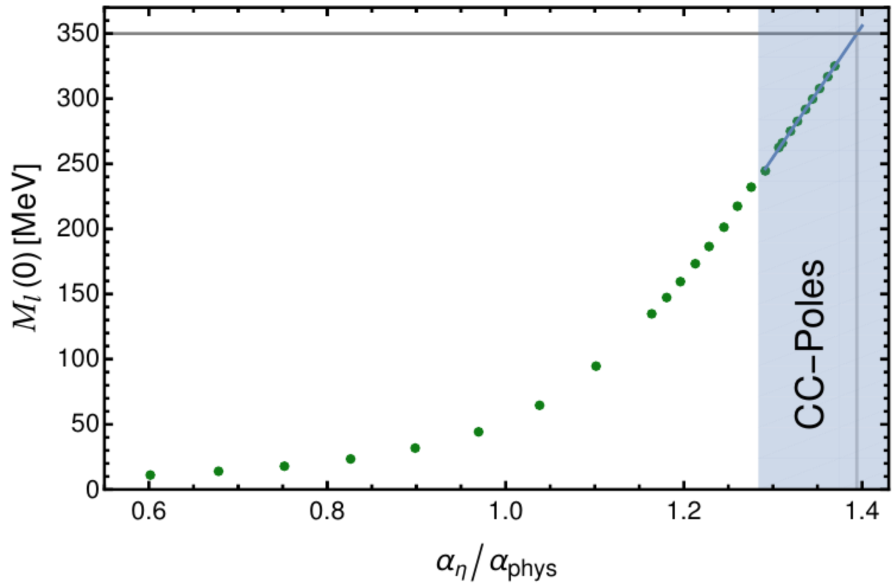
Spectral representation for small Enhancements, including the physical one

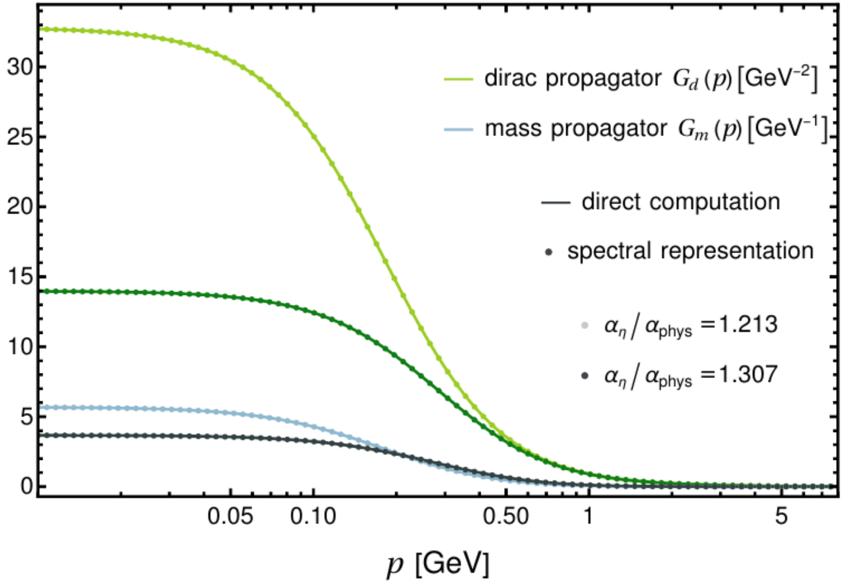
Emergence of acausal structures



- For $\alpha_\eta/\alpha_{\rm phys} \geq 1.283$ additional cc poles emerge in the complex plane.
- Spectral representation is lost for η that reproduce $M_l(0) \approx 350\mathrm{MeV}$

Emergence of acausal structures





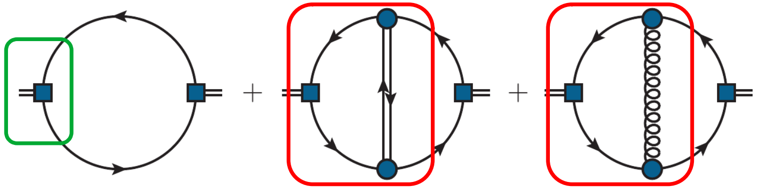
- For $\alpha_\eta / \alpha_{\mathrm{phys}} \geq 1.283$ additional cc poles emerge in the complex plane.
- Spectral representation is lost for η that reproduce $M_l(0) \approx 350\mathrm{MeV}$

- Spectral representation effectively fulfilled up to permille level.
- CC poles likely not effect computation of light hadron properties.

First Shot on Quark Diffusion

$$\mathscr{D}_s = \lim_{\omega \to 0} \frac{\sigma(\omega, \vec{p} = 0)}{\omega \chi_q \pi}$$

$$\propto \ \mathrm{Im}$$



- $\leadsto$ Absorb 2-loop Diagrams in effective vertex
- $\leadsto$ T-dependence on (effective) vertex usually mild
- $\leadsto$ Compensate with global normalisation

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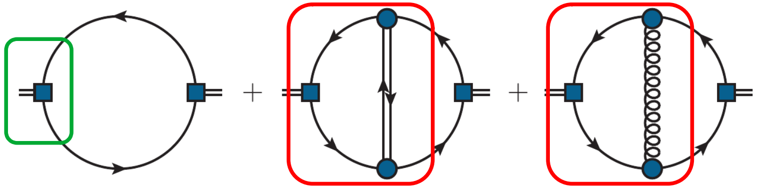
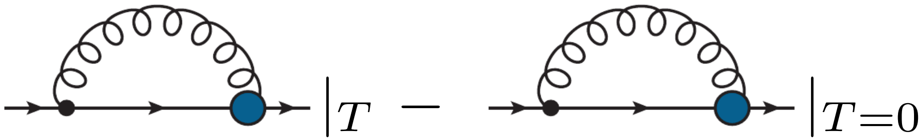
$\propto$ Im 



$\leadsto$ Compute thermal corrections on vacuum input.

$\leadsto$ Gluon propagators from reconstruction of euclidean data at finite T

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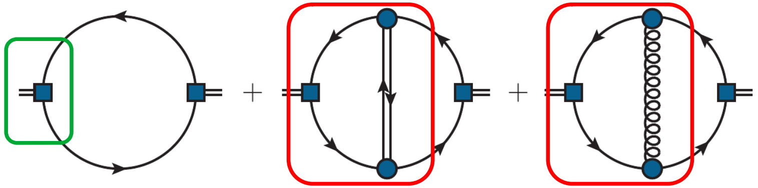
$\propto \ \mathrm{Im}$ 

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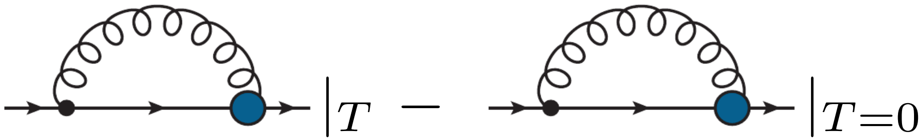
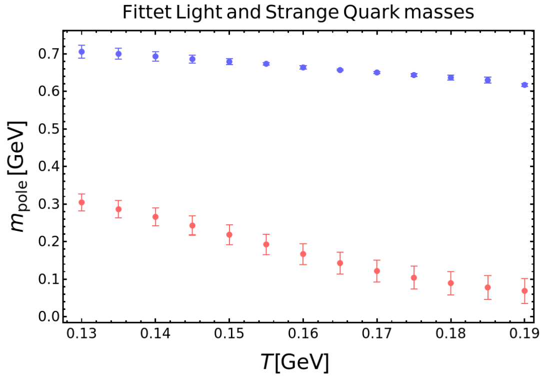


- Compute thermal corrections on vacuum input.
- Gluon propagators from reconstruction of euclidean data at finite T
- Fix open coupling strength to reproduce Euclidean DSE - data at finite T



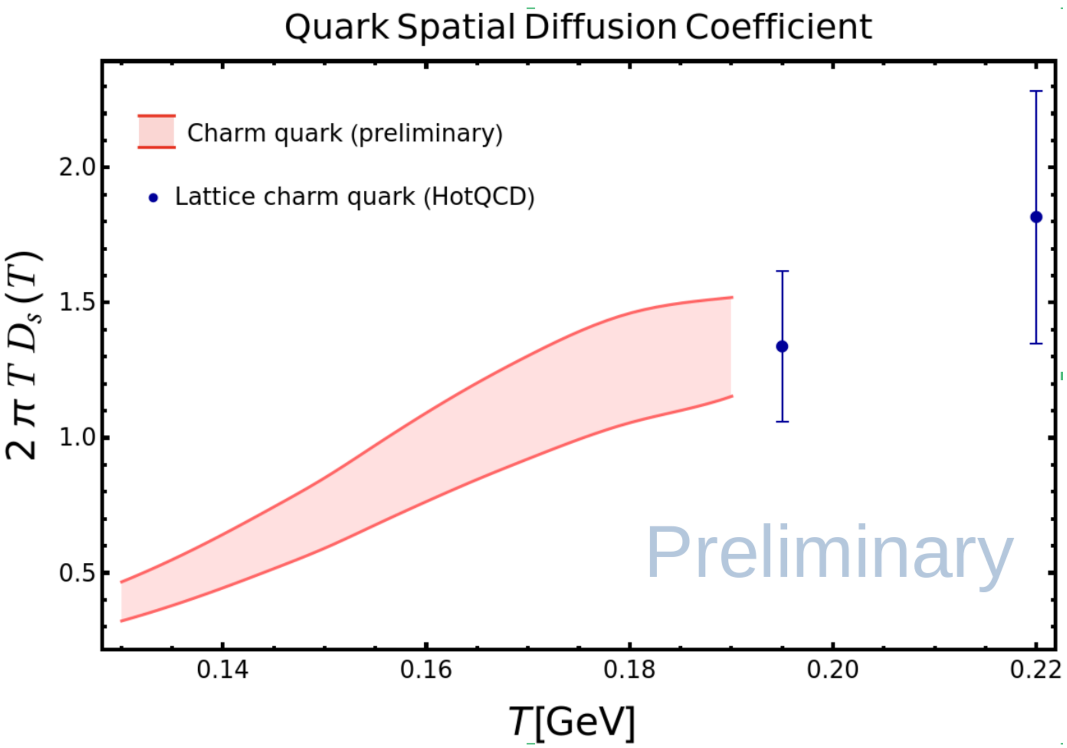
Input masses and effective coupling fittet to euclidean DSE Data:

Lu et al. PRD 110 (2024)

Quark Number suszeptibility from

Bellwied et al. PRD 92 (2015)

First Shot on Quark Diffusion



Normalisation fixed at lattice results:

Altenkord et al PRL 132 (2024)

- Charm Mass estimated from strange quark data
- Error band propagated from error estimate on the masses

Todo:

- Self-consistent solution of finite Temperature gap equation
- Include causal vertex construction
- Go to finite density

Conclusion

- Quark spectral functions with causal vertex construction.
- At physical vertex strengths, quark propagator fulfills spectral representation.
- Enhancement of vertex strength leads to CC-poles for light quarks
- Access to thermal spectral functions and the heavy quark diffusion coefficient.

Conclusion

- Quark spectral functions with causal vertex construction.
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- Enhancement of vertex strength leads to CC-poles for light quarks
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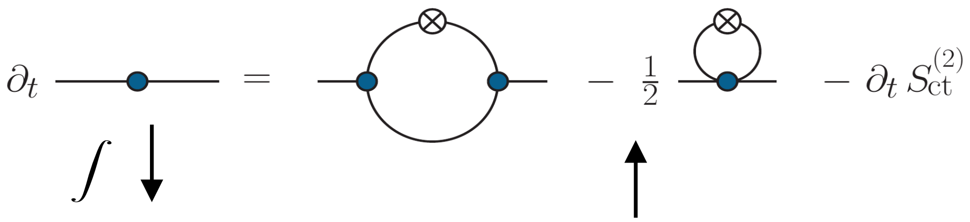
Challenges & Outlook

- Quantitative causal modeling of QGV
  - Include relevant tensor structures
  - Include flavor dependence
- Analytic structure of vertex
- Self-consistency at finite temperature and baryon density

- Hadron (meson) phenomenology
  - Direct bound state calculations and BSWF
  - Pion/Kaon distribution amplitudes and functions
  - Decays of heavy resonances
- Transport coefficients
  - Quark diffusion and electric conductivity
  - Quark contribution to shear viscosity

Backup

Spectral Callan-Symanzik flow

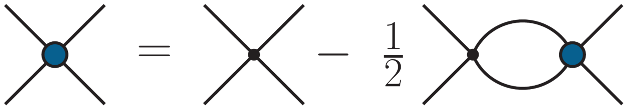


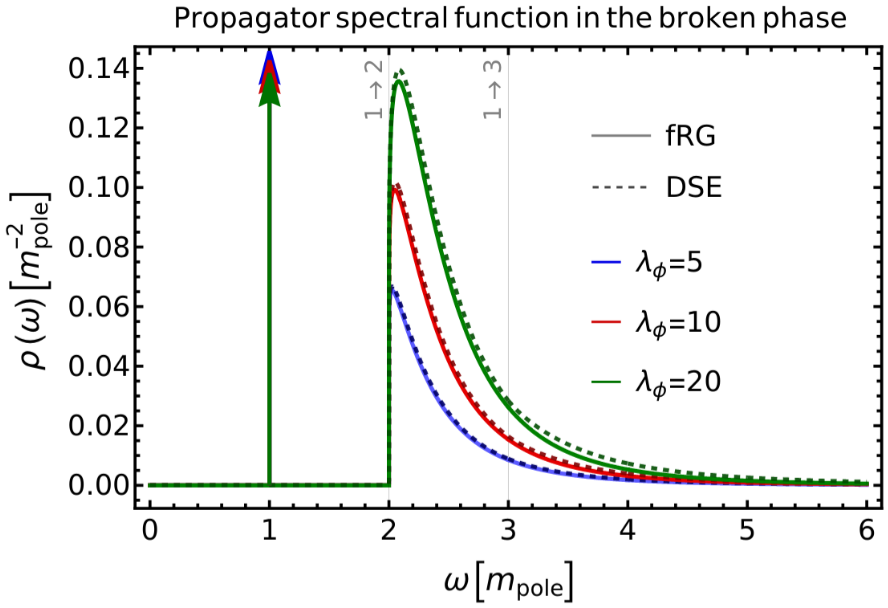
$$\Gamma^{(2)}(p) \longrightarrow \rho(\omega) = 2\,\mathrm{Im}\,G\left(p_0 \to i\omega^+\right)$$

$\rightsquigarrow$ Mass cut-off ensures causality and Lorenz invariance

$\rightsquigarrow$ Renormalise UV-divergency with flowing counter term

$\rightsquigarrow$ Include 4-point function via s-channel resummation





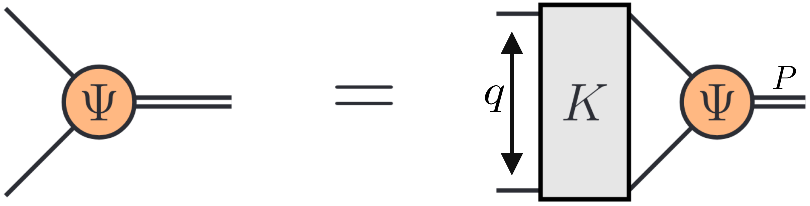
Renormalisation condition can eliminate Fine tuning

$$\rightsquigarrow \Gamma^{(2)}(p^2 = -k^2) = 0$$

Horak, Pawlowski, Wink PRD 102 (2020)
Horak, Ihssen, Pawlowski, JW, Wink, PRD 110 (2024)
Braun et. al. SciPost Phys.Core 6 (2023)

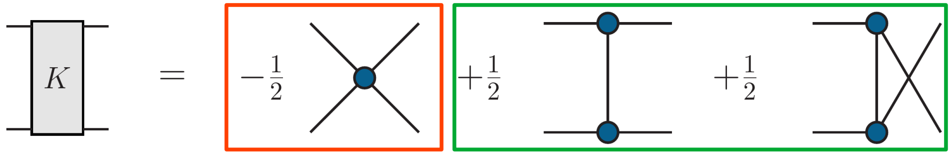
Spectral Bound States

Eichmann, Gomez, Horak, Pawlowski, JW, Wink PRD 109 (2024)



$$\Psi(q,P) = \int_k \underline{K(q,k,P)\, G(k_+)\, G(k_-)}\, \Psi(k,P)$$

- Convert to eigenvalue equation $\mathcal{M}\,\Psi_i = \eta_i \Psi_i$
- Take total momentum onshell $P^2 = -M_i^2$
- Bound-state solution $\longleftrightarrow$ $\eta_i\left(\frac{M_i}{m}, \frac{\lambda_\phi}{m}\right) = 1$



Scaling analysis of massive Wick-Cutkosky model:

- Classical propagator allow to scale out $c = \Gamma_3^2/m^3$
- $c\,\mathcal{M}'\,\Psi_i = c\,\eta_i'\,\Psi_i$