

First Principle QCD within fRG approach

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**The QCD Phase Diagram: From Theory to Experimental
Signatures, Oct. 7-11, 2025**

Based on:

Wei-jie Fu, CH, Jan M. Pawłowski, Yang-yang Tan, Li-jun Zhou, ‘*Four-quark scatterings in QCD III*’, Phys. Rev. D 112 (2025), 054047;

Wei-jie Fu, CH, Jan M. Pawłowski, Yang-yang Tan, ‘*Four-quark scatterings in QCD II*’, SciPost Phys. 17(2024), 148;

Wei-jie Fu, CH, Jan M. Pawłowski, Yang-yang Tan, ‘*Four-quark scatterings in QCD I*’, SciPost Phys. 14 (2023) 069,;

fQCD collaboration: <https://fqcd-collaboration.github.io/>



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Outline

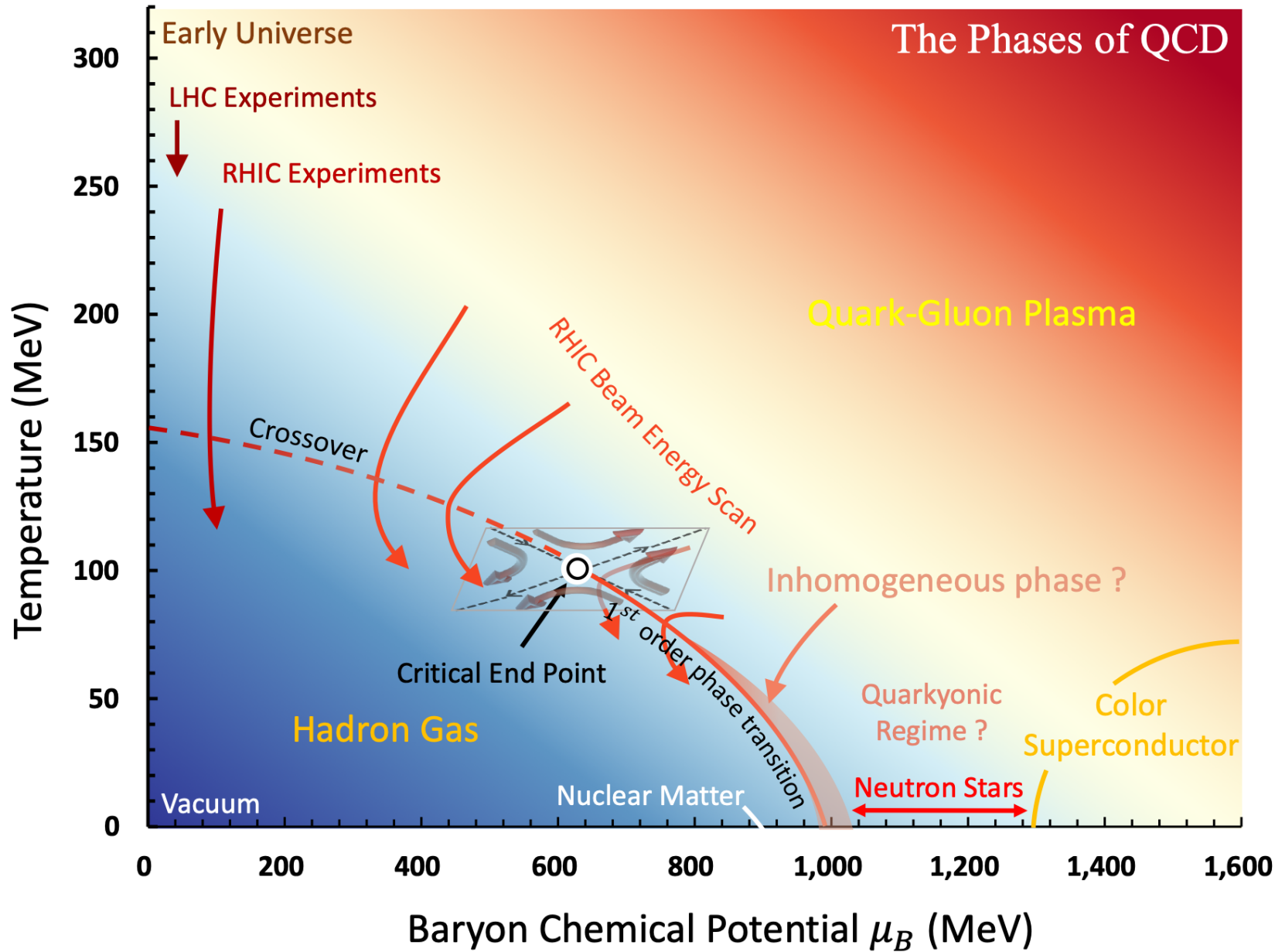
- Introduction
- Computation setting and scale independence
- Chiral symmetry breaking and quark correlation functions
- Matter sector and chiral relation
- Summary and ***outlook***

Outline

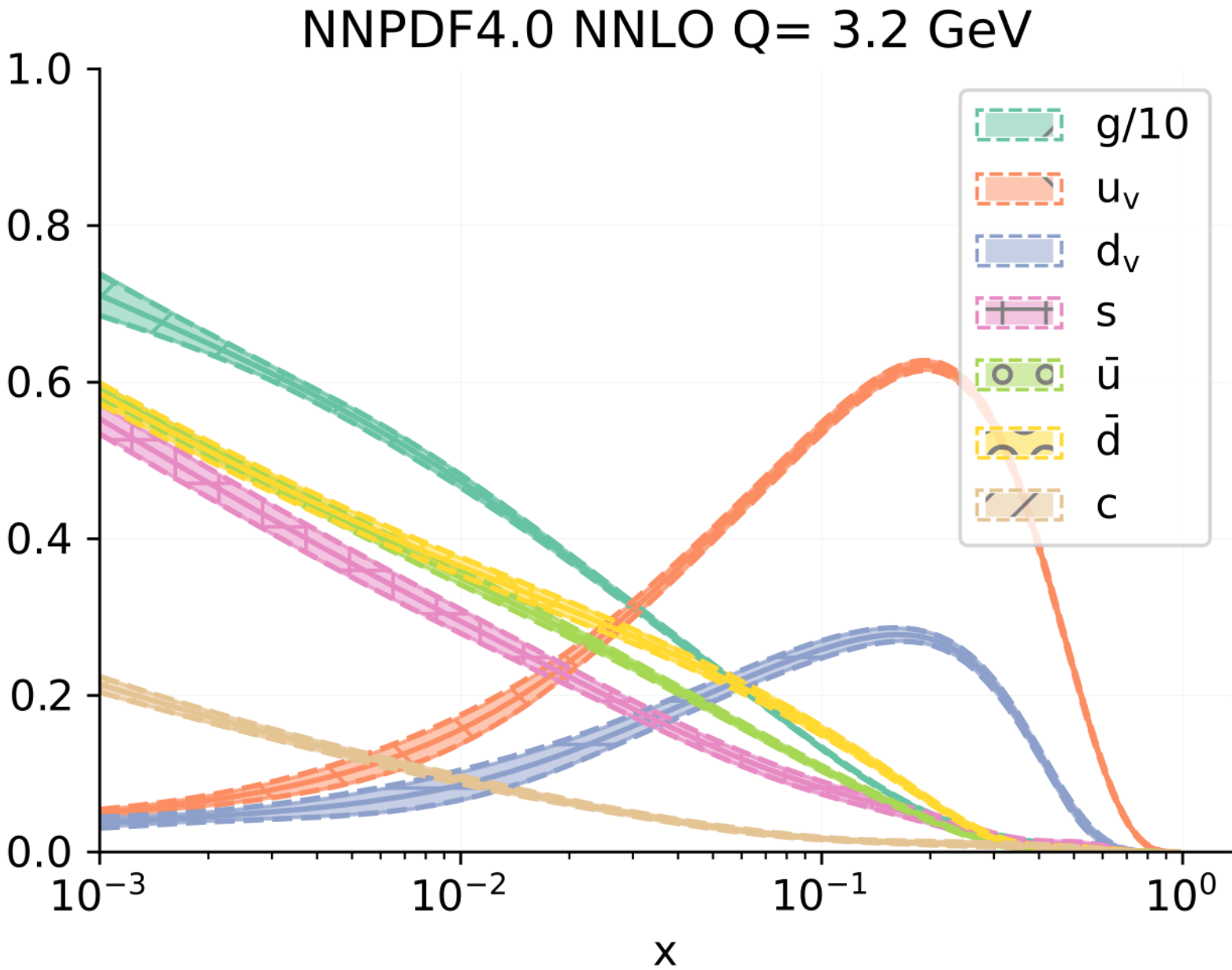
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Topics in QCD Study

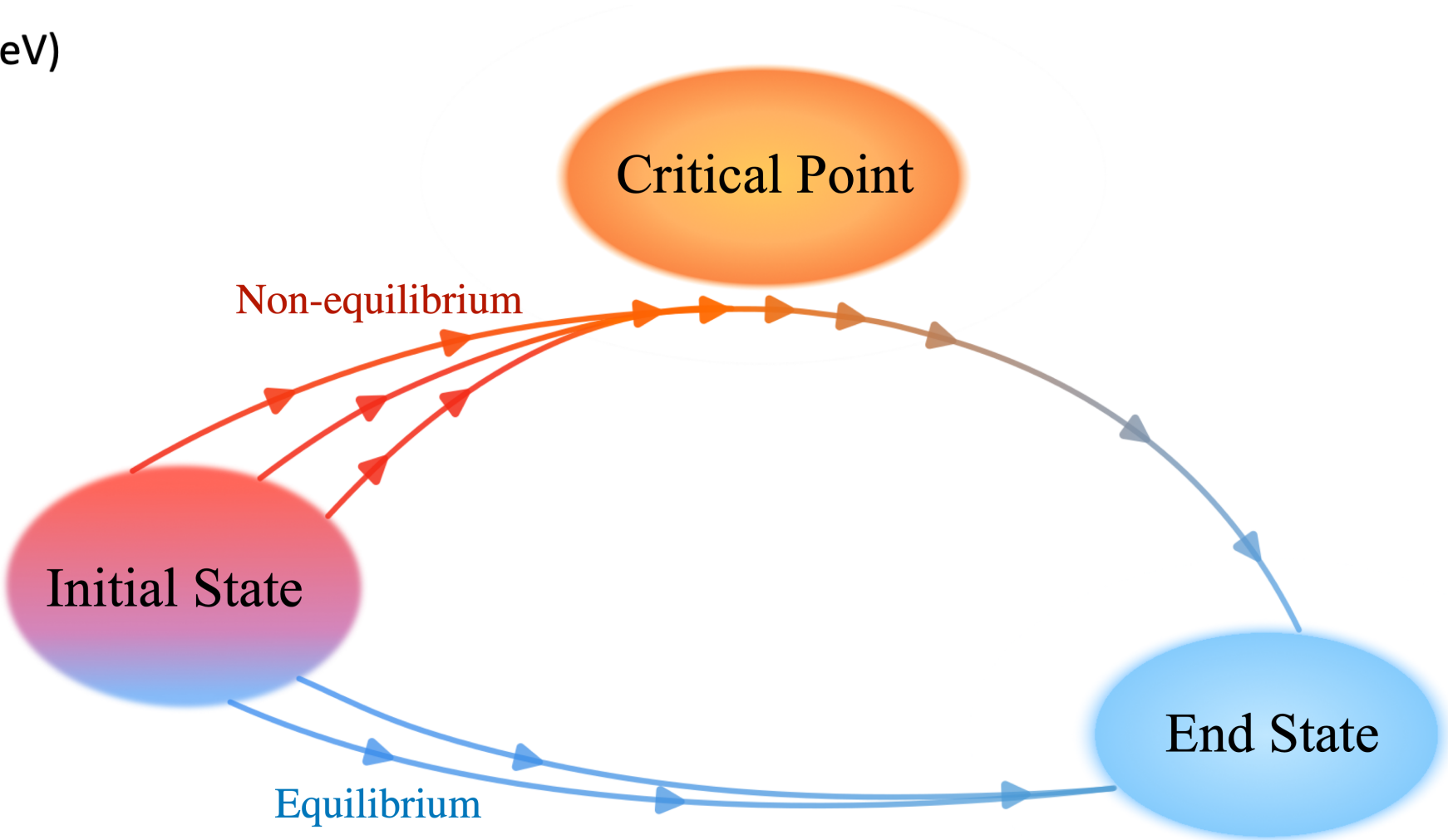
Phase
Structure



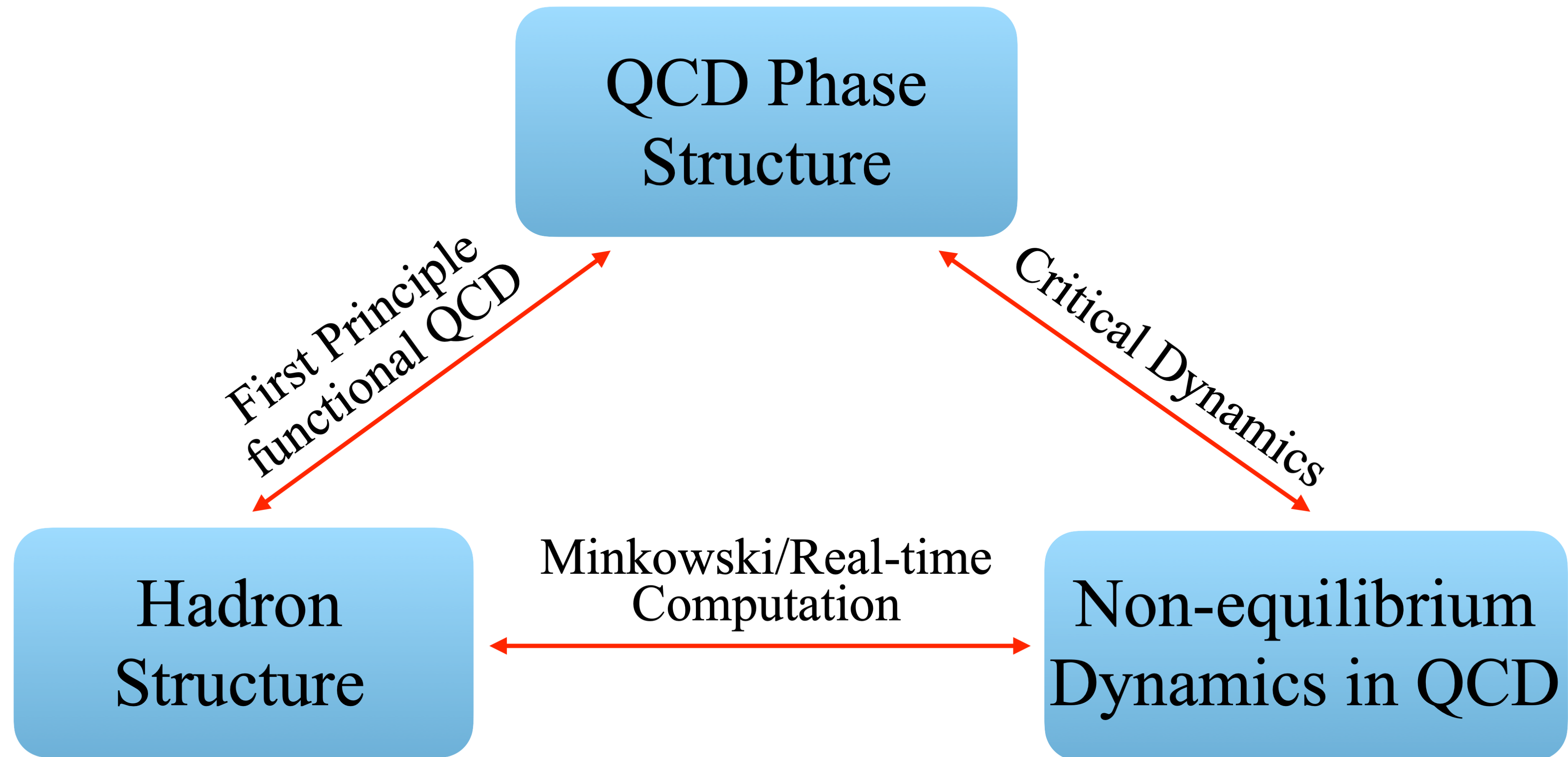
Hadron
Structure



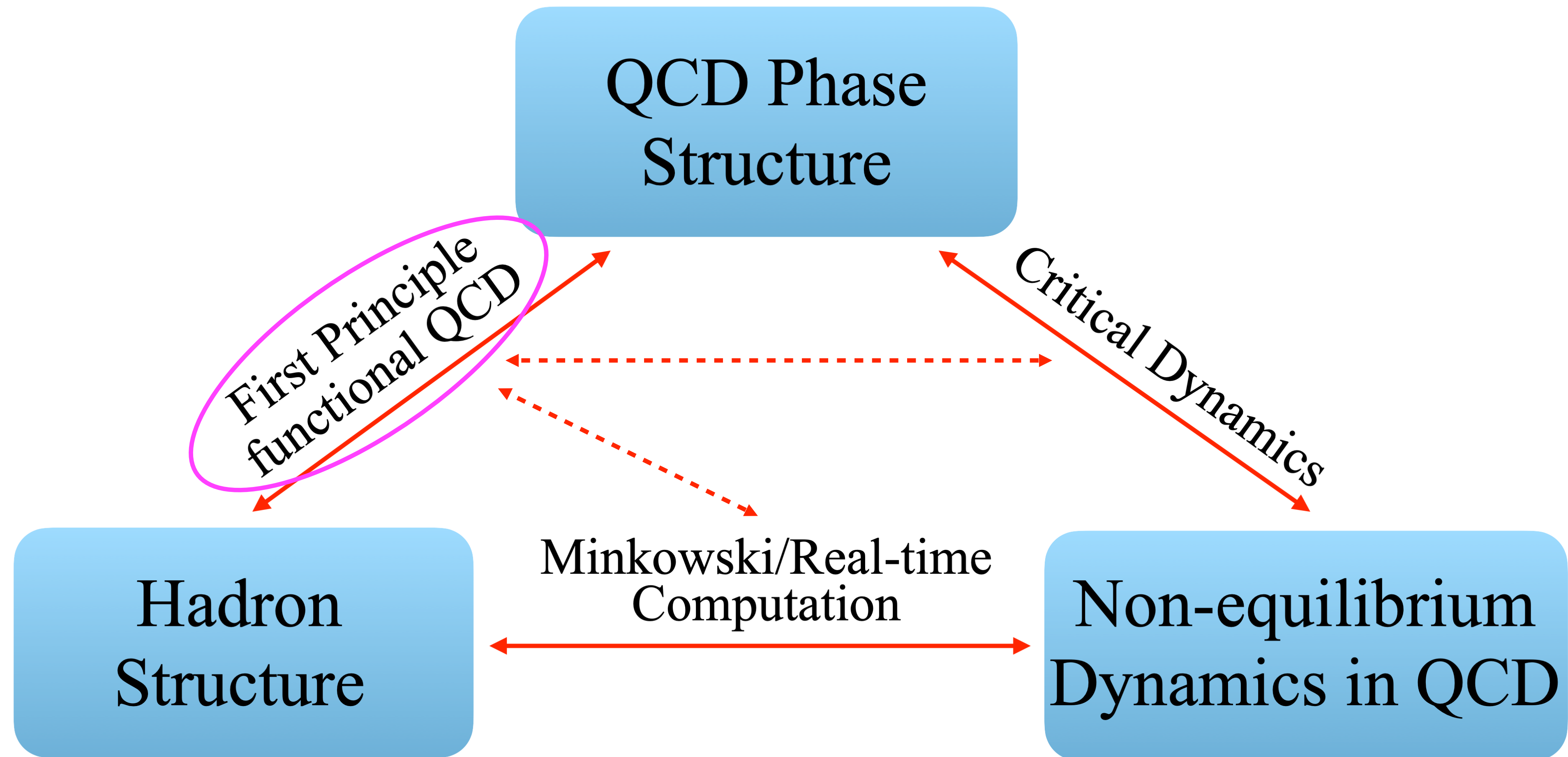
Non-equilibrium
Dynamics



Theoretical Studies of Functional QCD



Theoretical Studies of Functional QCD



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2+1 QCD with fRG

Glue sector

$$\begin{aligned}
 \partial_t \text{ (gluon self-energy) } &= \tilde{\partial}_t \left(\text{gluon loop} + \text{ghost loop} + \text{gluon tadpole} + \frac{1}{2} \text{ (ghost loop)} \right) \\
 \partial_t \text{ (ghost self-energy) } &= \tilde{\partial}_t \left(\text{gluon tadpole} \right) \\
 \partial_t \text{ (3-gluon vertex) } &= \tilde{\partial}_t \left(\text{triangle} + \text{box} \right) \\
 \partial_t \text{ (4-gluon vertex) } &= \tilde{\partial}_t \left(\text{triangle} - \text{box} - \text{triangle} + \frac{1}{2} \text{ (ghost loop)} \right)
 \end{aligned}$$

$$+ \text{ (4-point vertex) } \text{ terms}$$

Matter sector

$$\begin{aligned}
 \partial_t \text{ (quark self-energy) } &= \tilde{\partial}_t \left(\text{triangle} + \text{triangle} - \text{triangle} \right) \\
 \partial_t \text{ (quark-gluon vertex) } &= \tilde{\partial}_t \left(\text{triangle} - \text{triangle} \right) \\
 \partial_t \text{ (quark-quark vertex) } &= \tilde{\partial}_t \left(\text{triangle} - \text{triangle} + \text{triangle} \right)
 \end{aligned}$$

Symmetry point truncation

$$\lambda^{(n)}(p_1, p_2, \dots, p_n) \approx \lambda^{(n)}(\bar{p}) \quad \bar{p} = \sqrt{1/n \sum_i p_i^2}$$

s,t,u-channel truncation

Computation setting

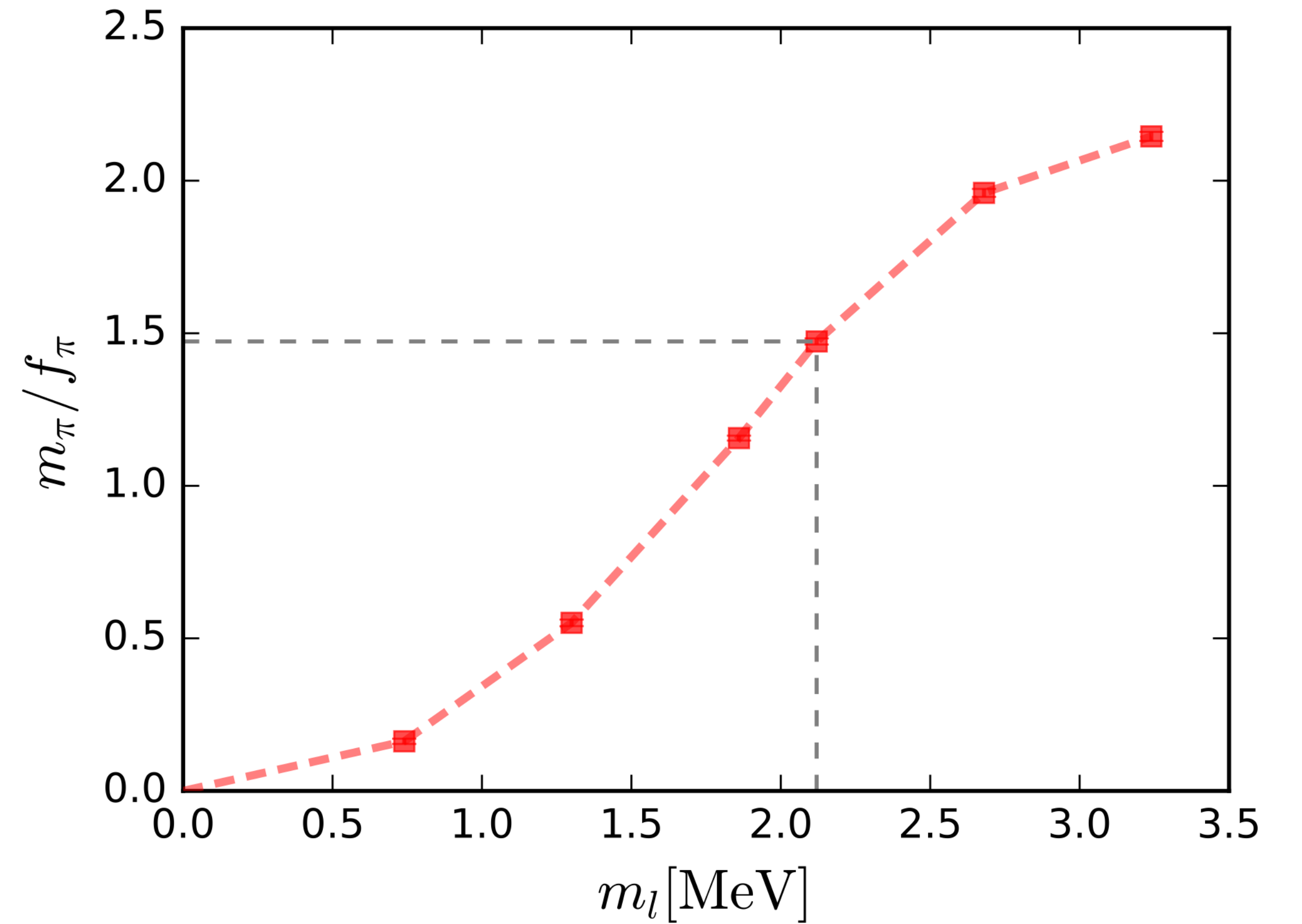
Parameters table

Observables	Value	Parameters	Value
m_π/f_π	137.0/93.0	m_l	2.1 MeV
m_K/f_π	494.0/93.0	m_s	55.9 MeV
M_l	344.5 MeV		
M_s	487.3 MeV		
m_σ	515.2 MeV		
f_K	114.1 MeV		

$$\Lambda = 35.7 \text{ GeV}$$

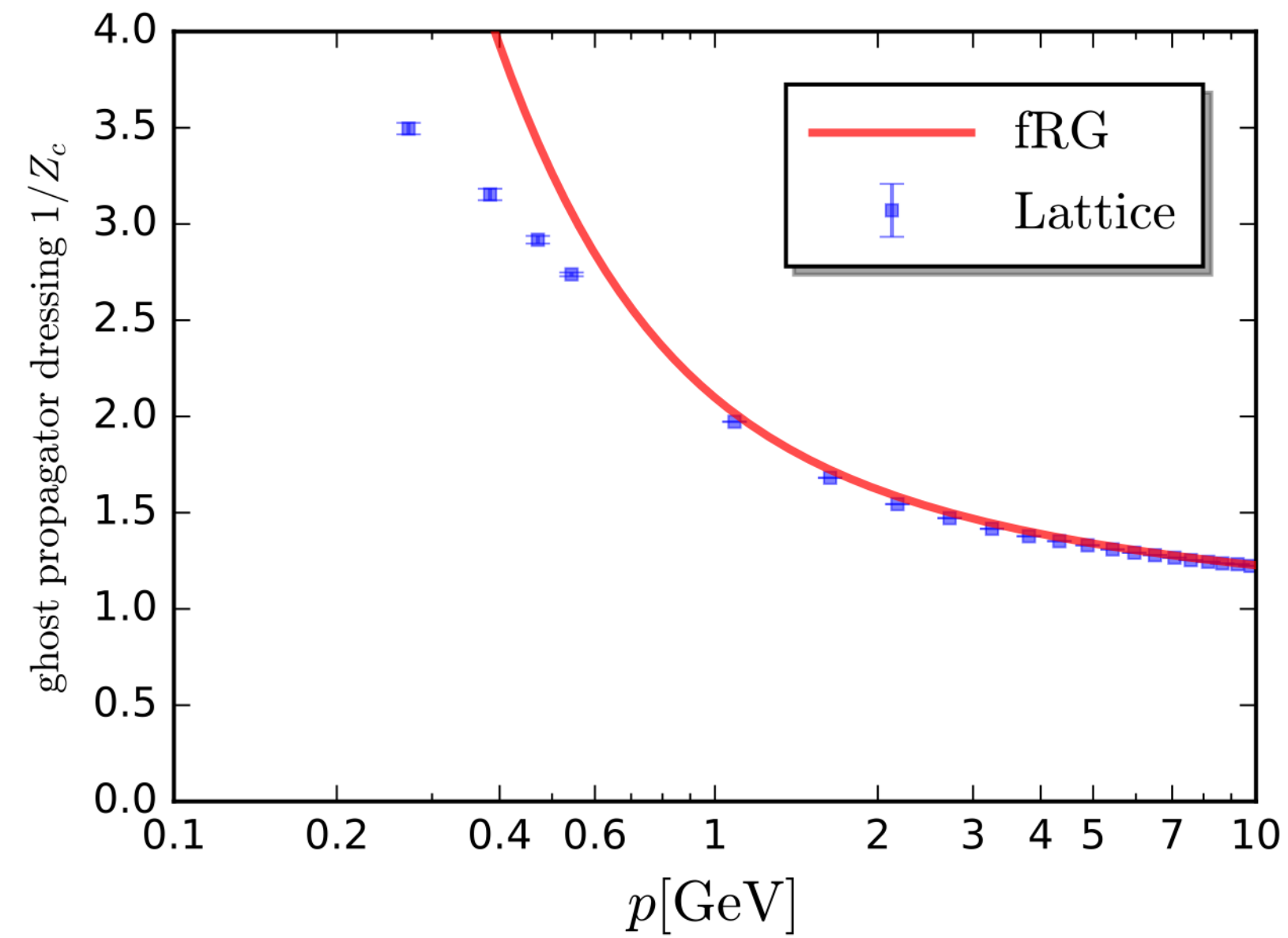
$$\alpha_{s,\Lambda} = 0.179$$

Parameters tuning



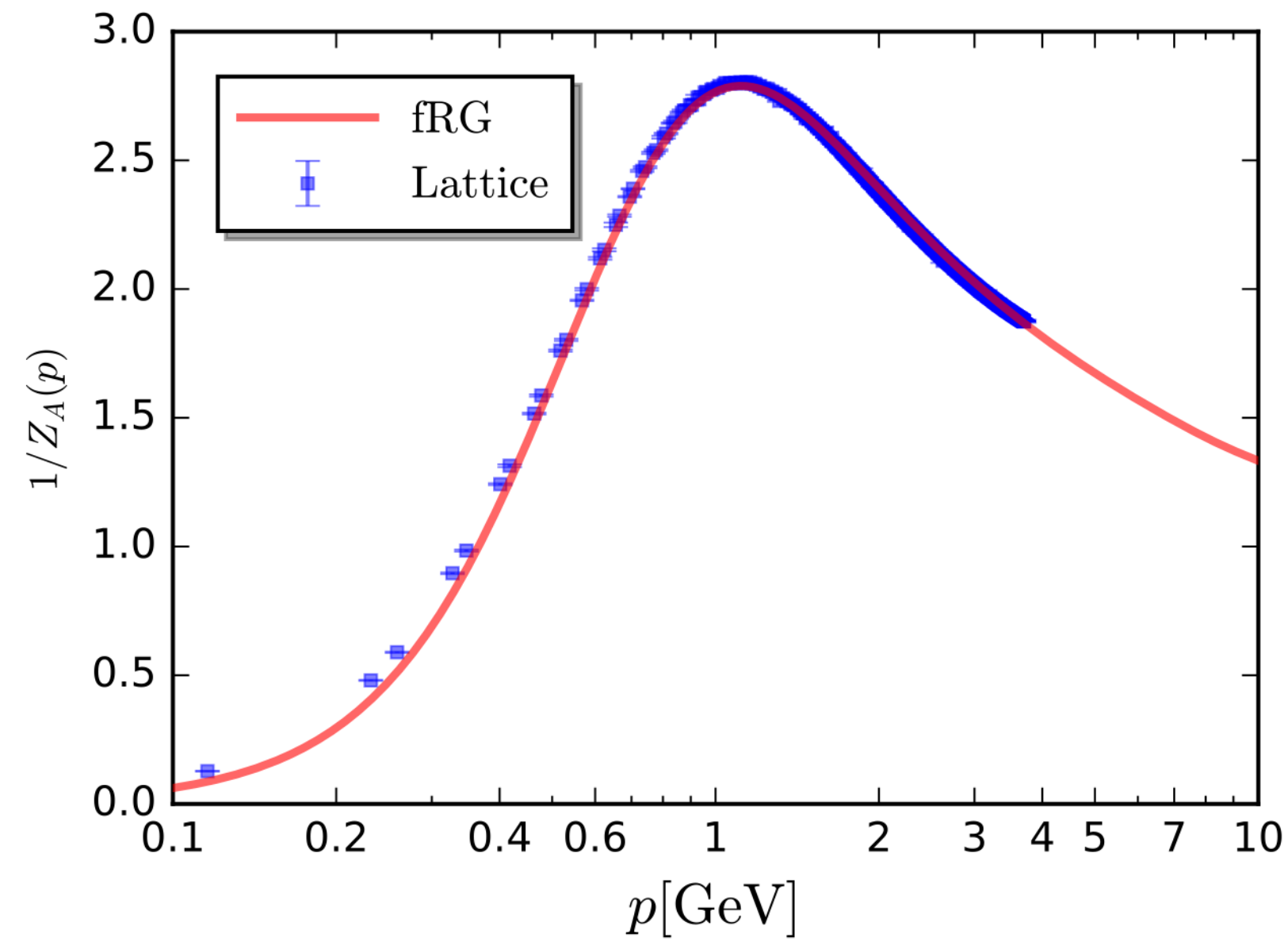
Glue sector in QCD

Ghost dressing



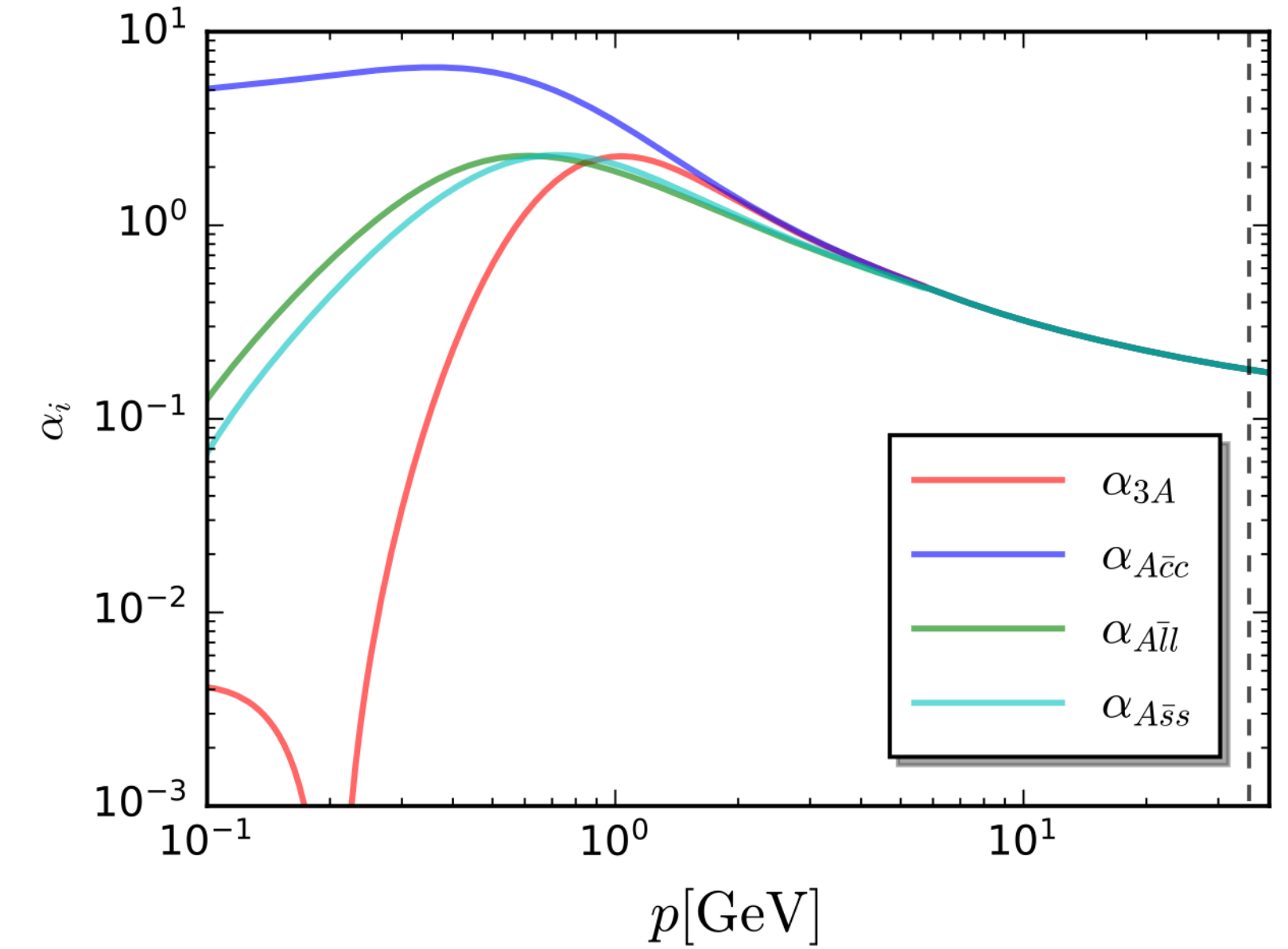
Lattice: Boucaud *et al.*, *PRD* 98 (2018) 114515

Gluon dressing



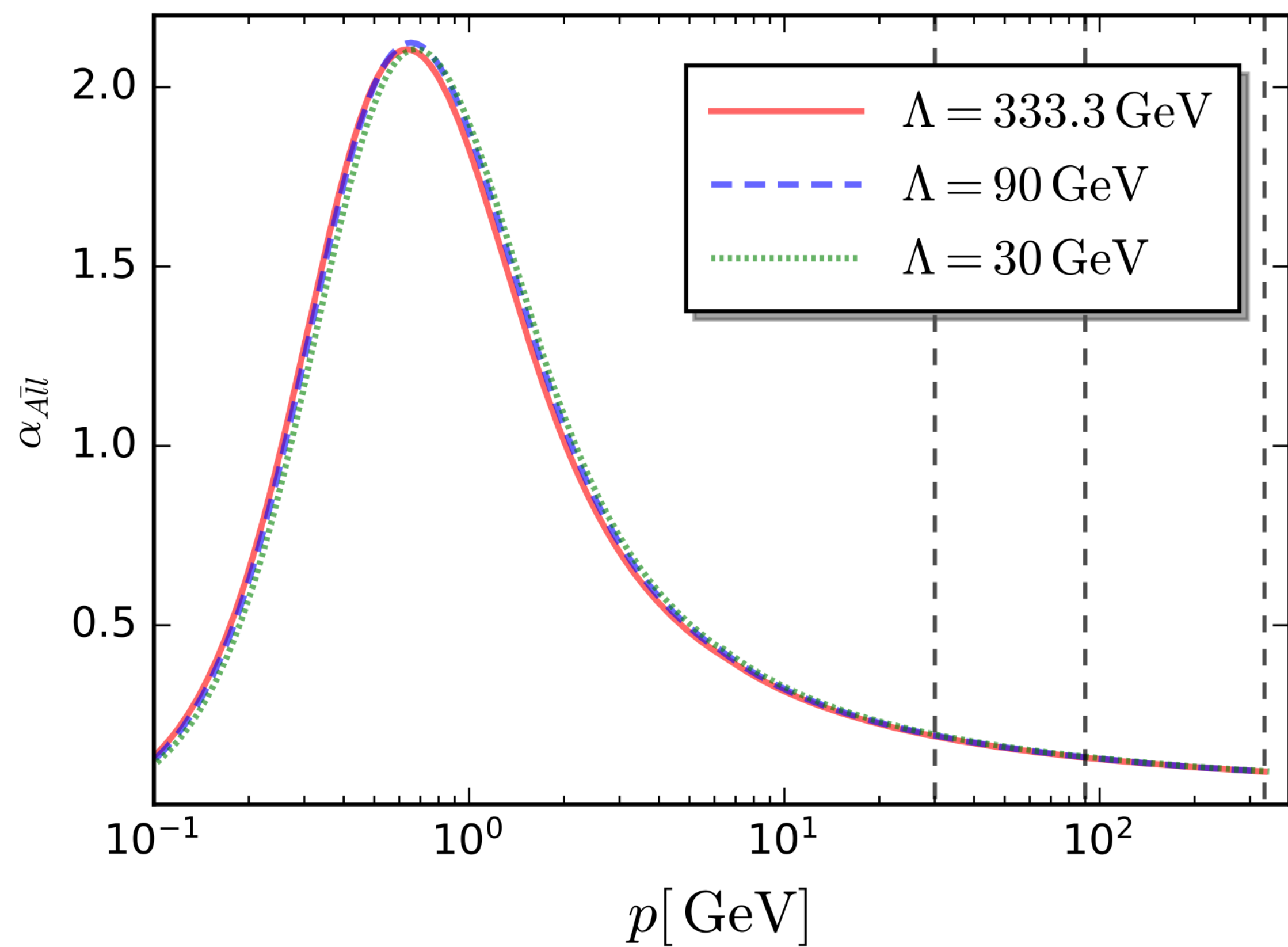
fRG: Fu, CH, Pawłowski, Tan, Zhou, *Phys. Rev. D* 112 (2025), 054047

Couplings

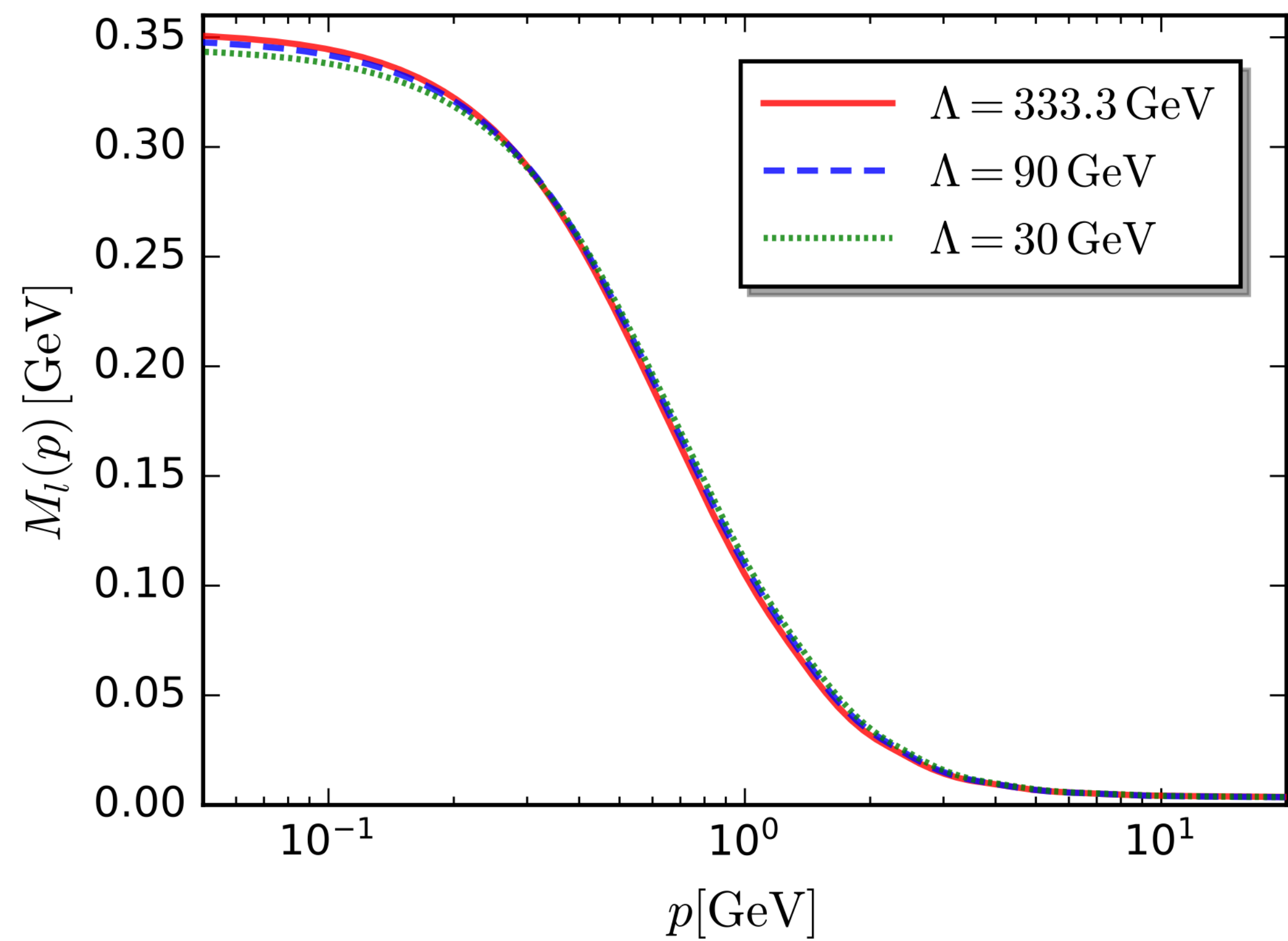


Scale independence

Couplings



Quark mass

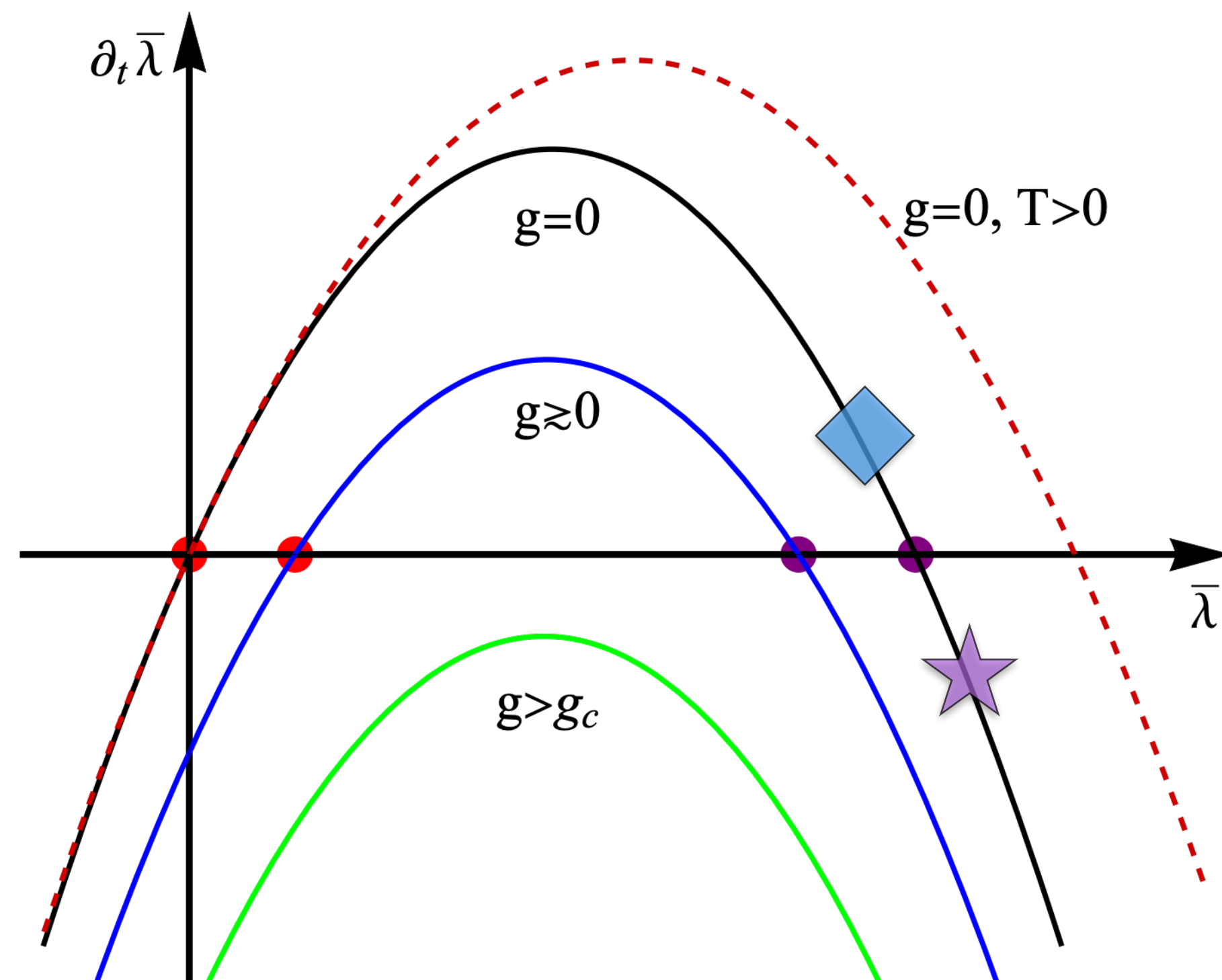


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Chiral symmetry breaking

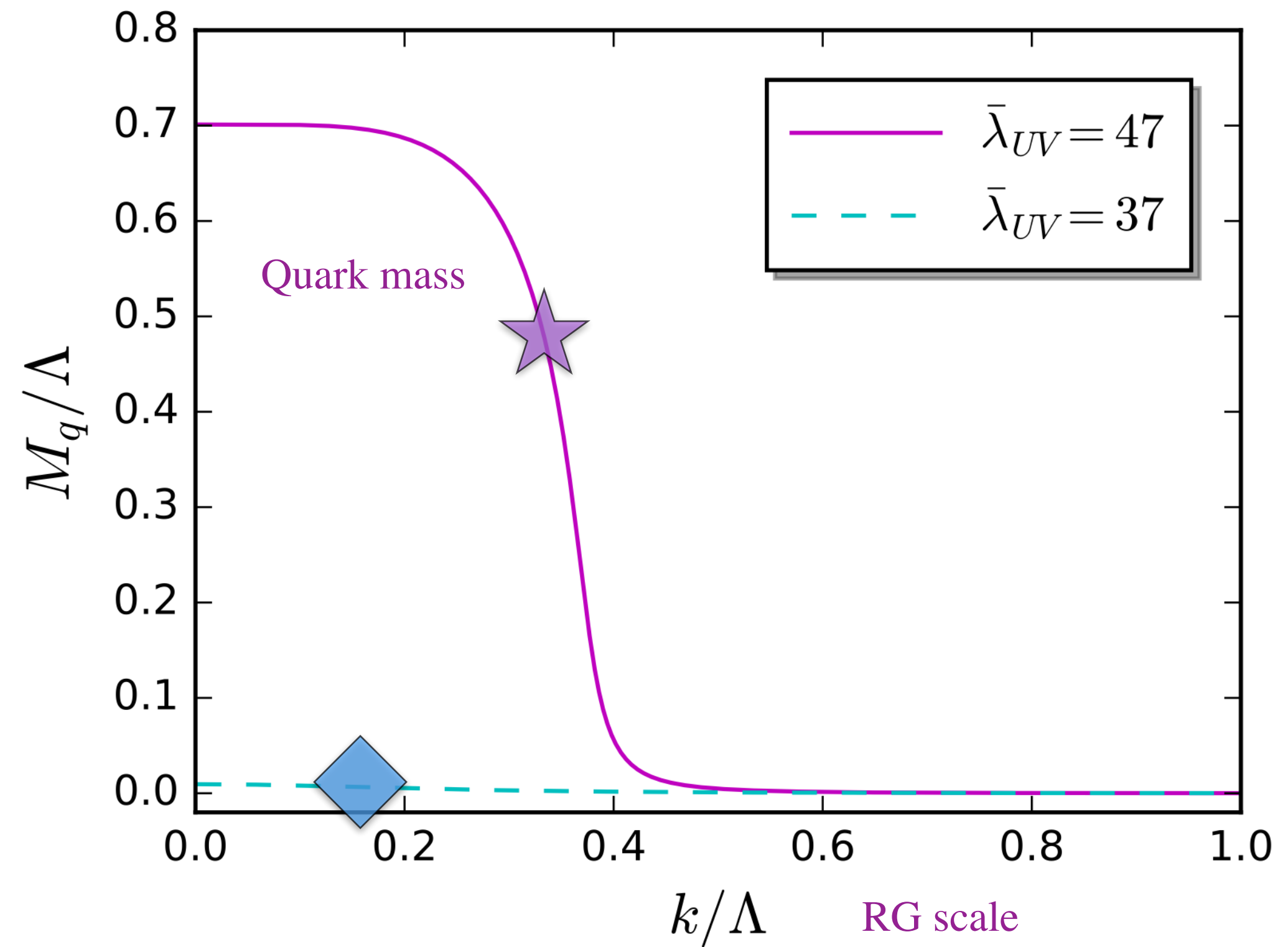
β function of 4-quark coupling



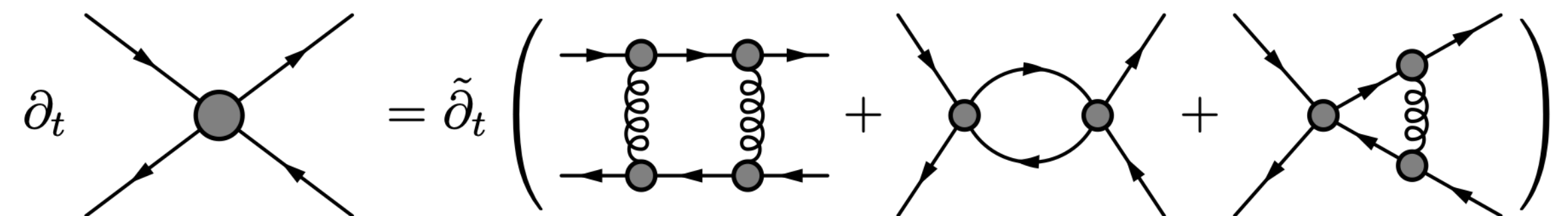
Braun, Gies, *JHEP* 06 (2006) 024.

$$\partial_t \bar{\lambda} = (d-2)\bar{\lambda} - a\bar{\lambda}^2 - b\bar{\lambda}g^2 - cg^4,$$

$$a = C_1, \quad b = c = d = 0$$

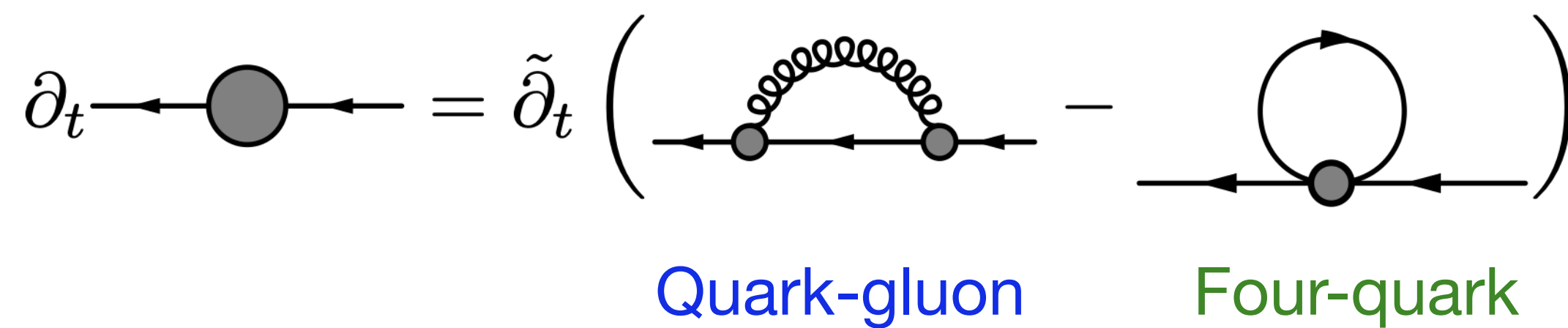
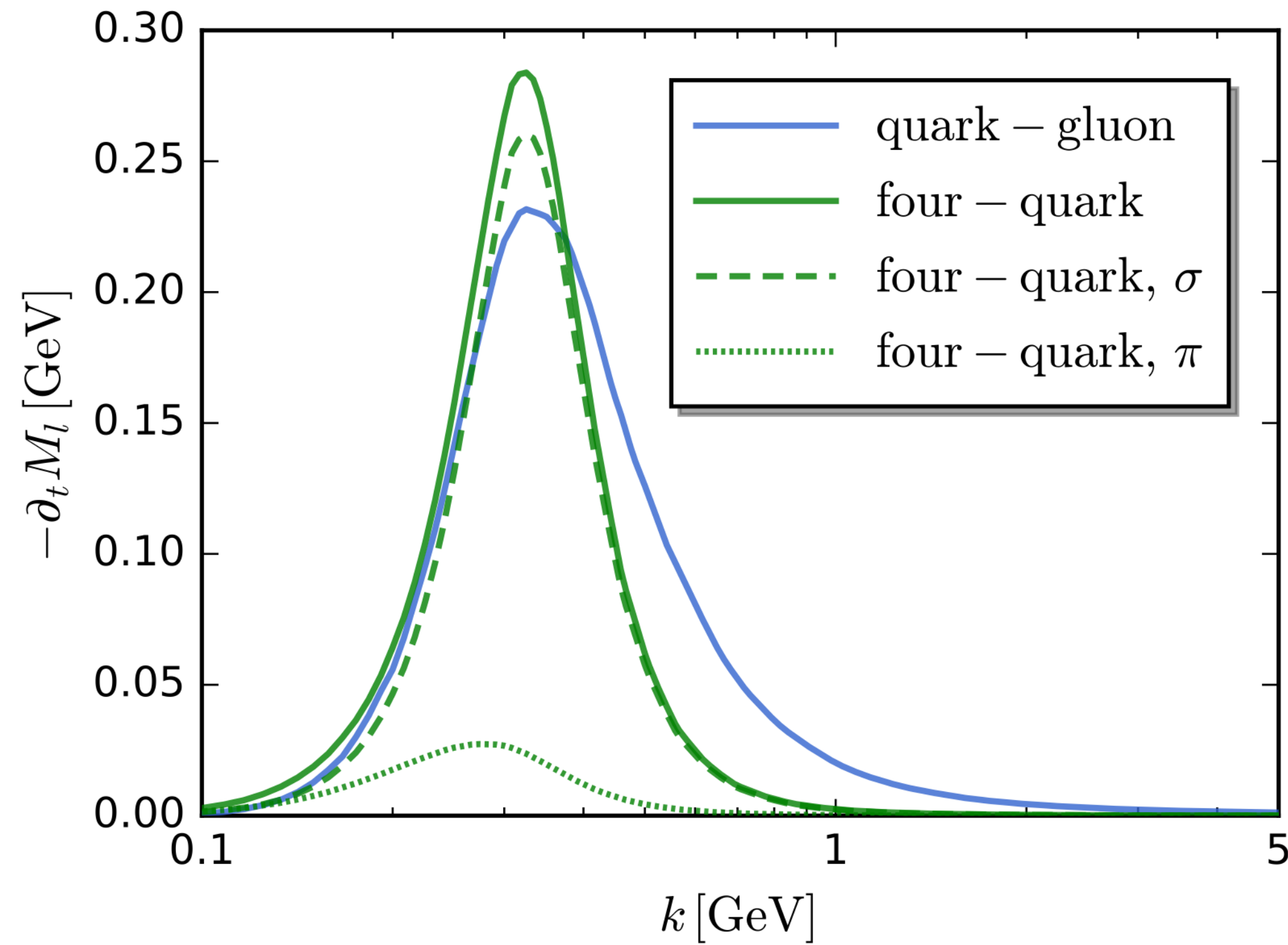


Fu, CH, Pawłowski,
Tan, *SciPost Phys.* 14
(2023) 069

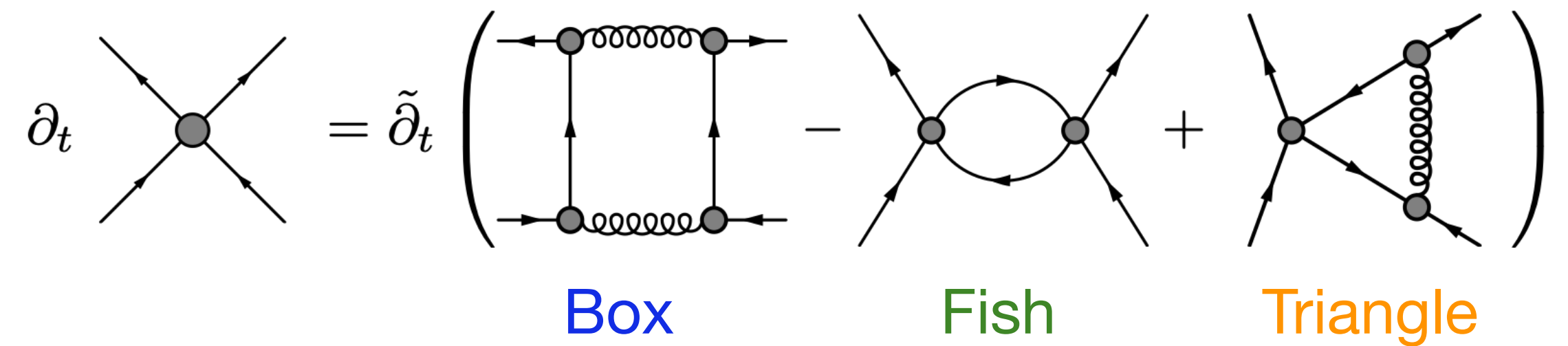
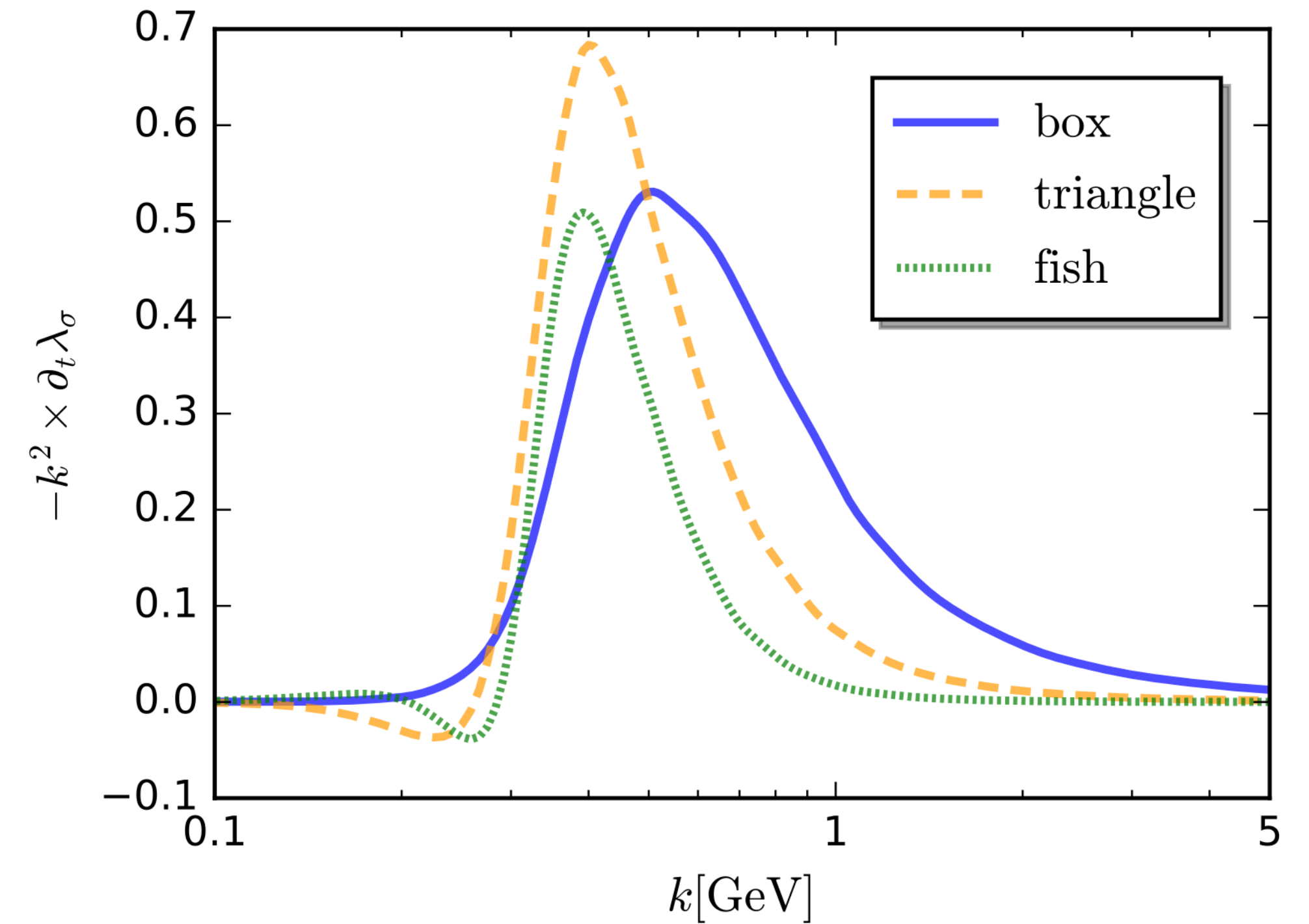


Flow analyze

Flow of the light quark mass

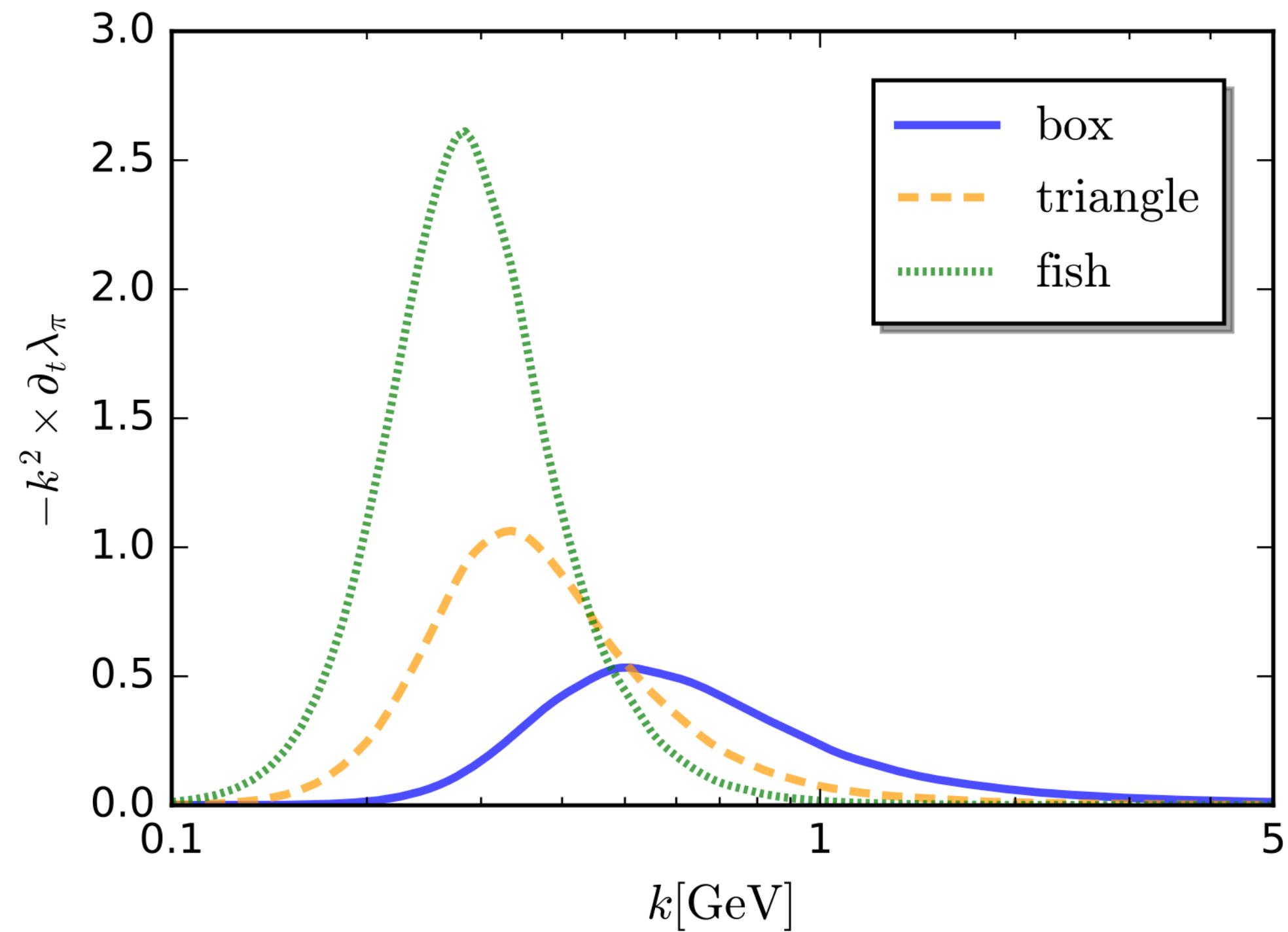


Flow of the λ_σ

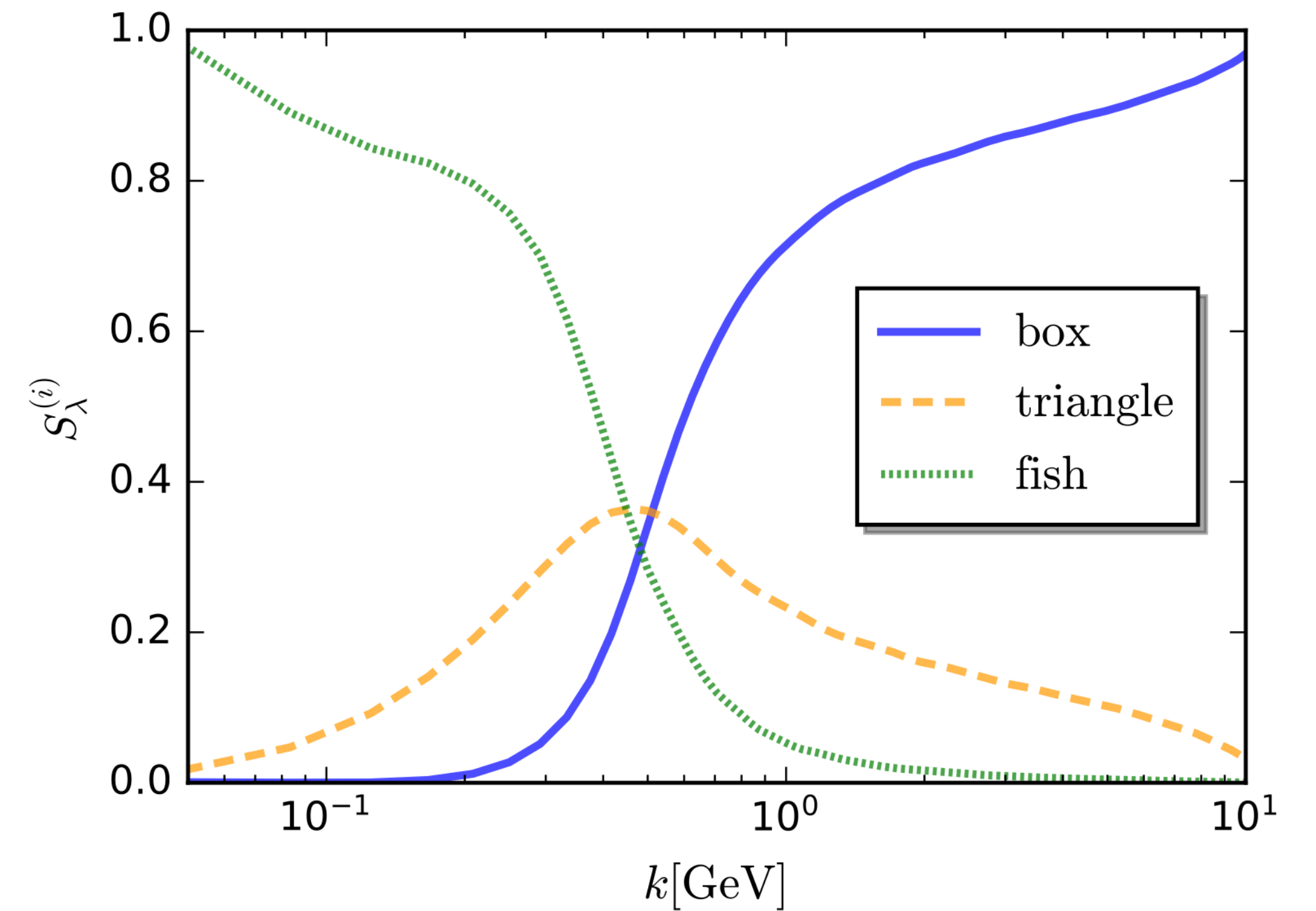


Flow analyze

Flow of the λ_π



Normalised Flow of the λ_π

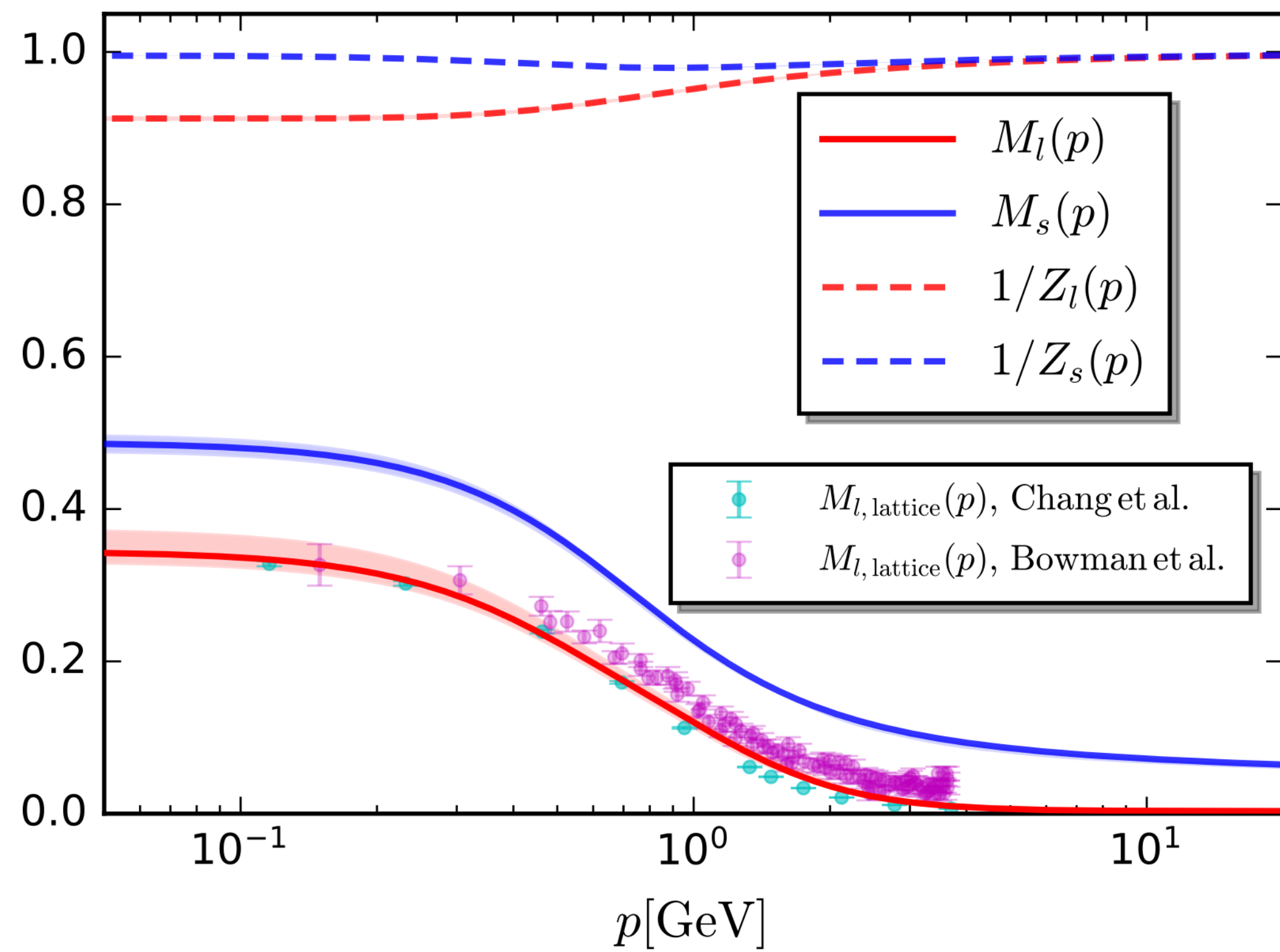


$$\partial_t \text{ (cross)} = \tilde{\partial}_t \left(\text{Box} - \text{Fish} + \text{Triangle} \right)$$

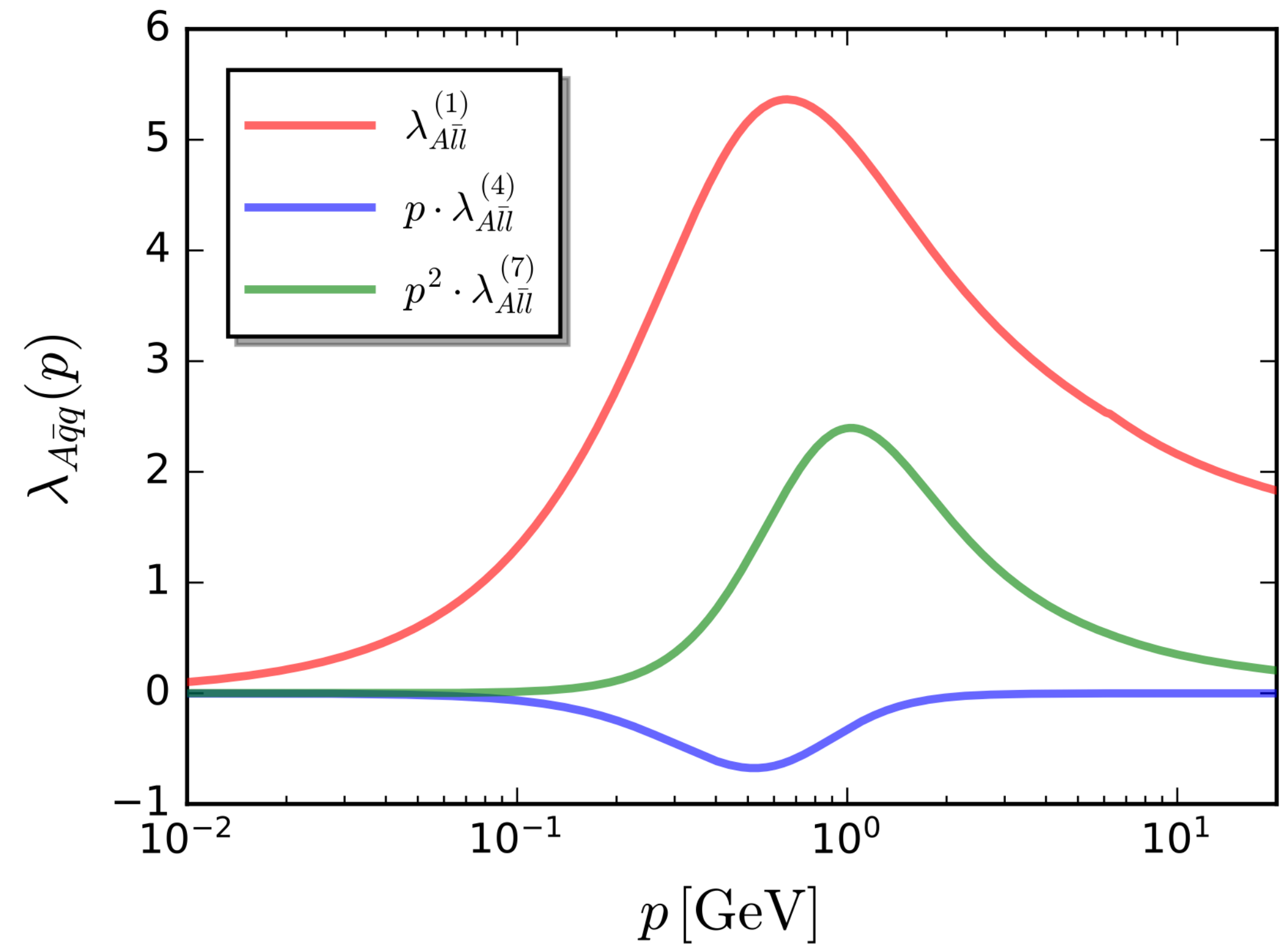
Box
Fish
Triangle

Quark correlation functions

Quark propagators



Quark-gluon vertex



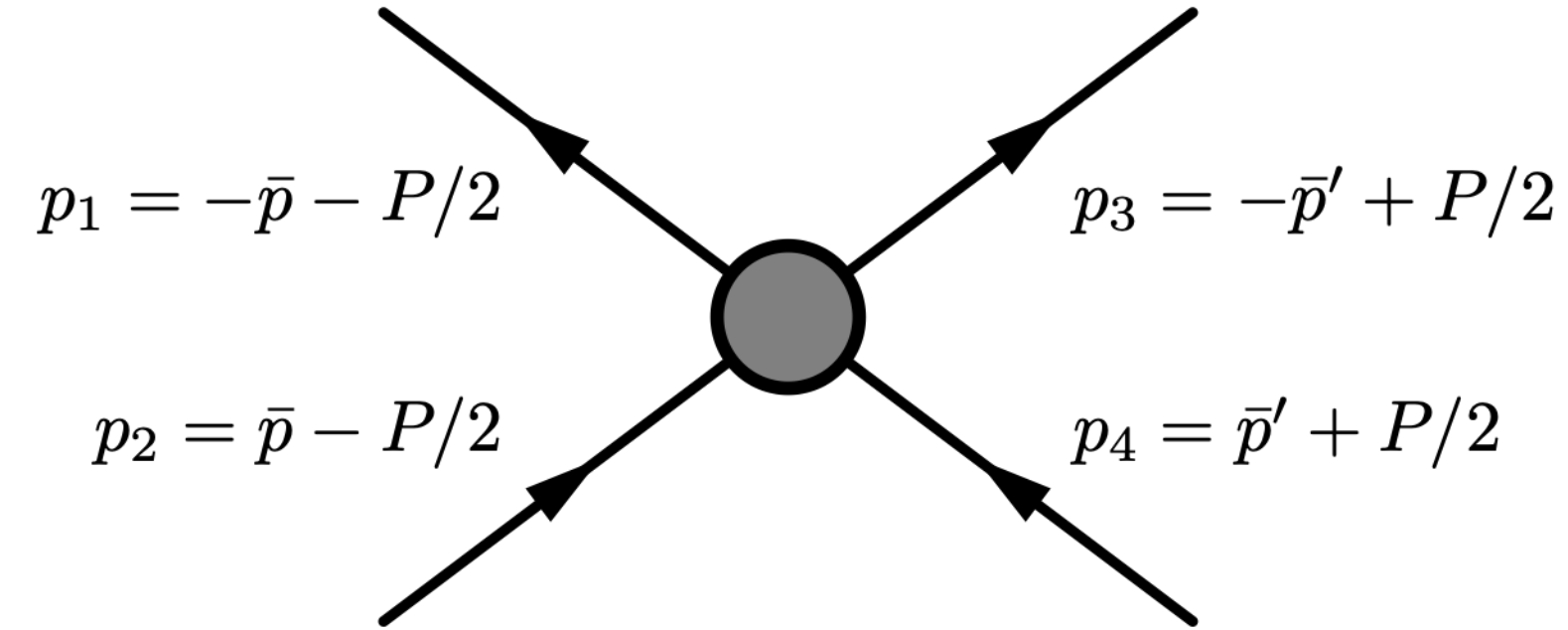
Fu, CH, Pawłowski, Tan, Zhou, Phys. Rev. D 112 (2025), 054047

Outline

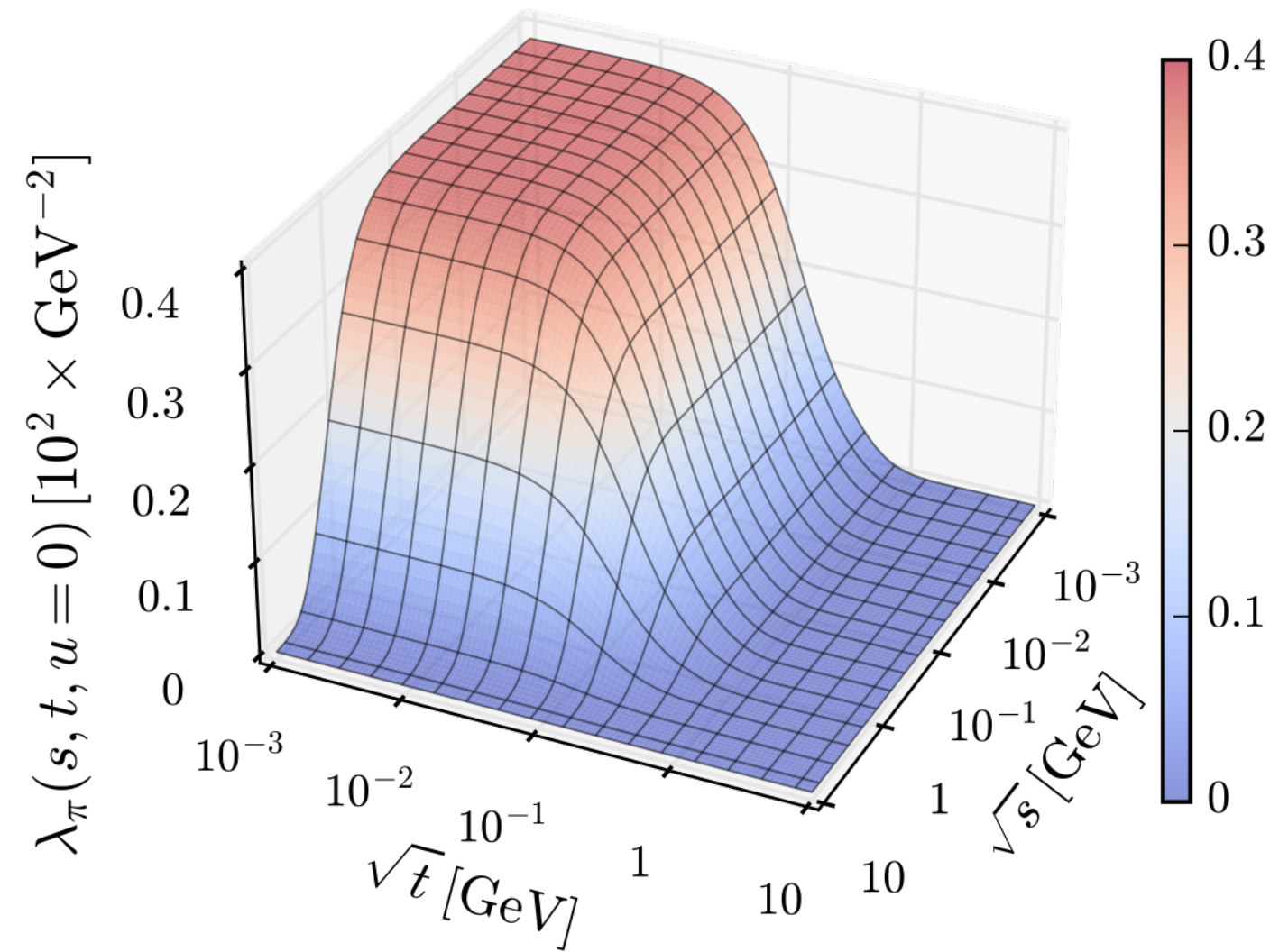
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Four-quark vertex in QCD

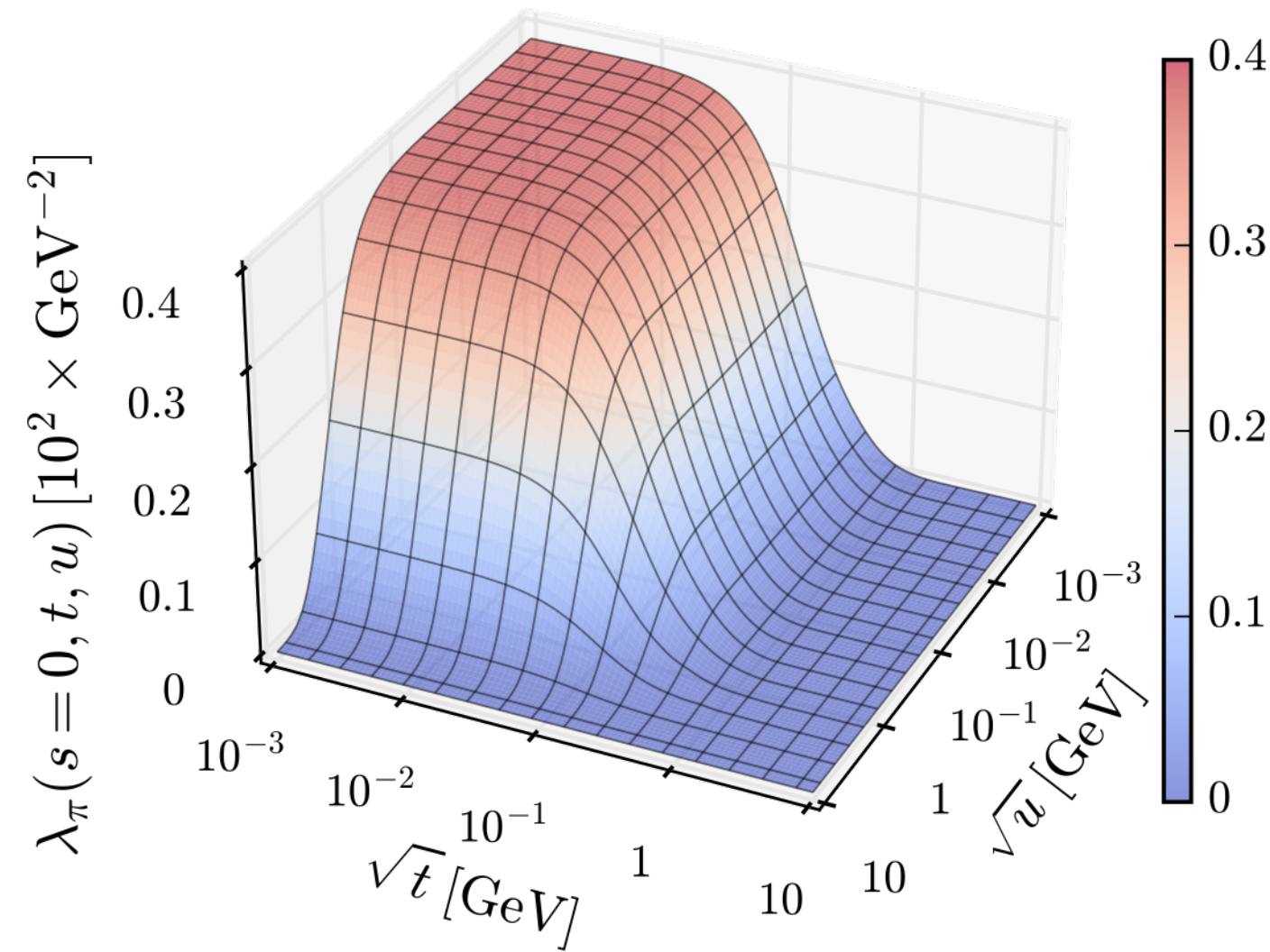
s, t, u-channel truncation



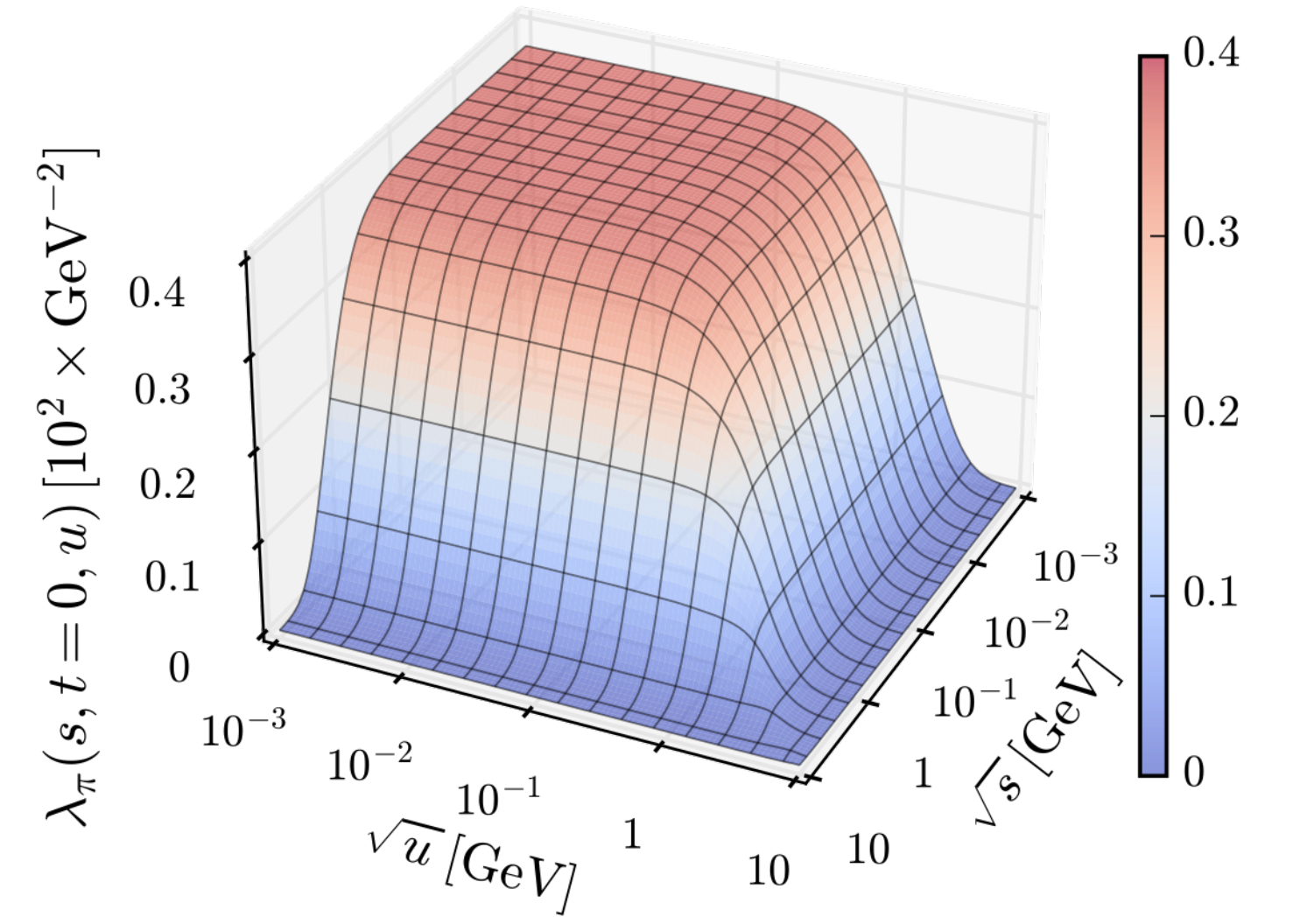
$$\lambda_\alpha(p_1, p_2, p_3, p_4) \approx \lambda_\alpha(s, t, u)$$



s and t channel with u=0



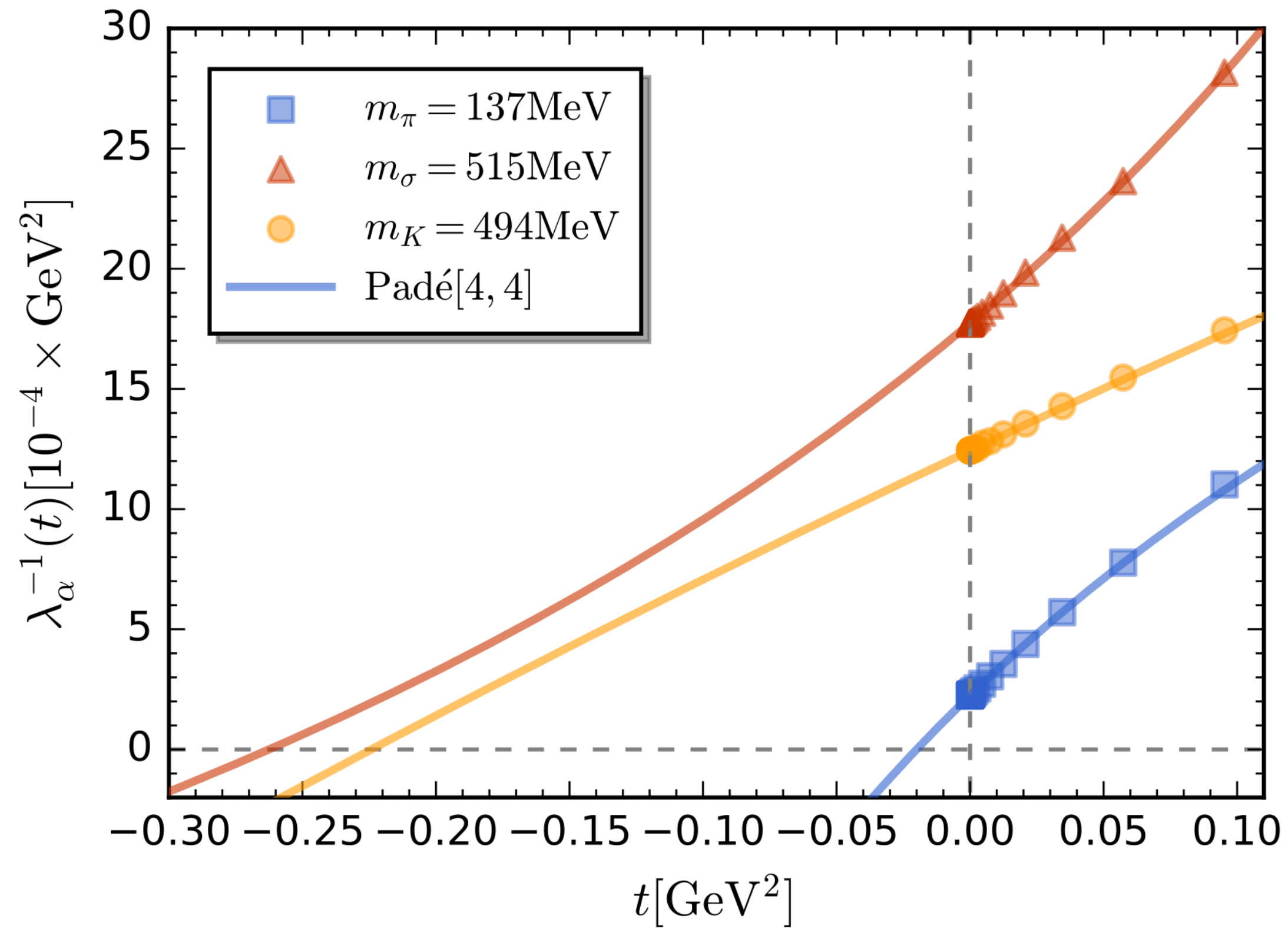
t and u channel with s=0



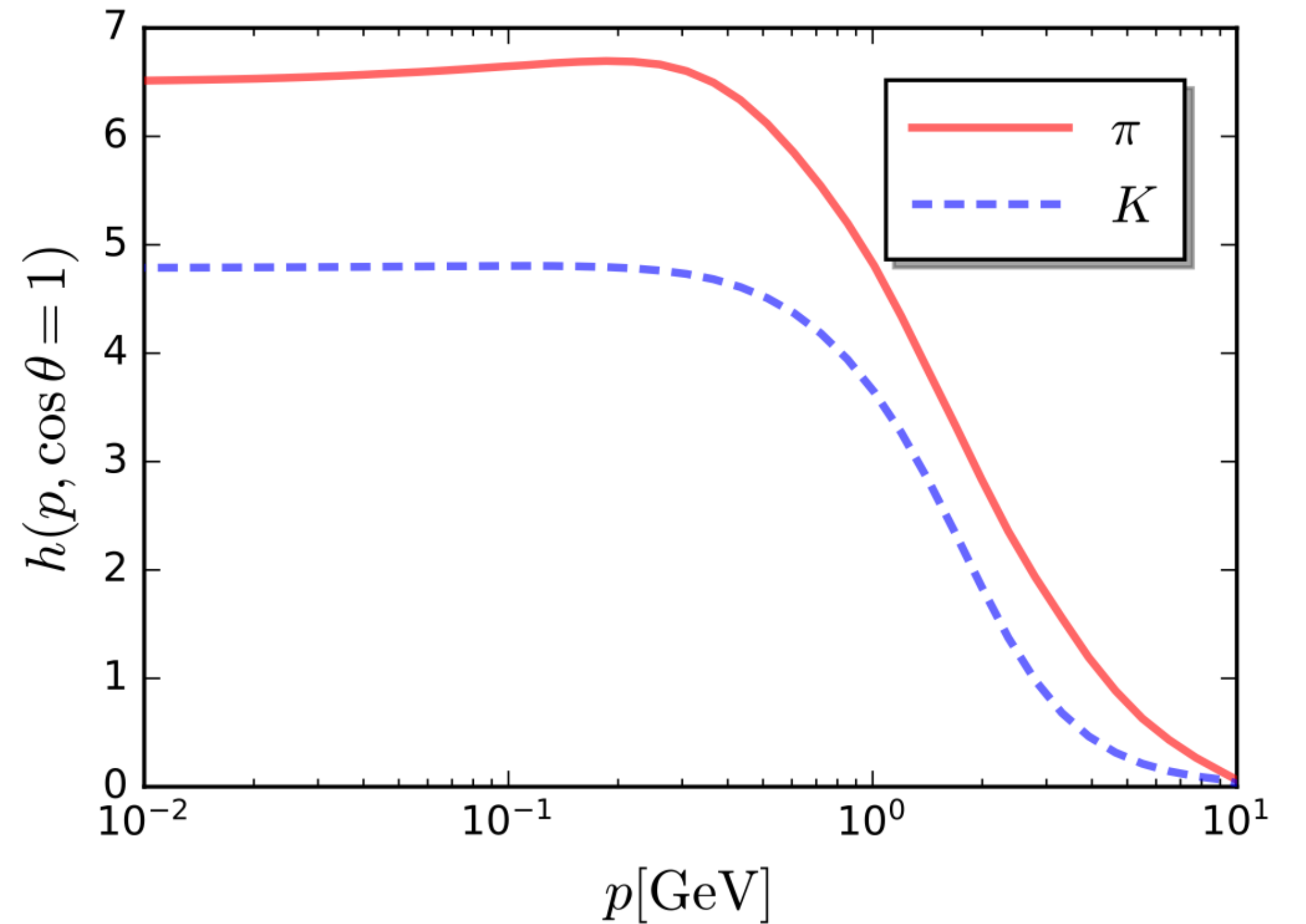
s and u channel with t=0

Extraction of meson mass and BS amplitude

Meson mass



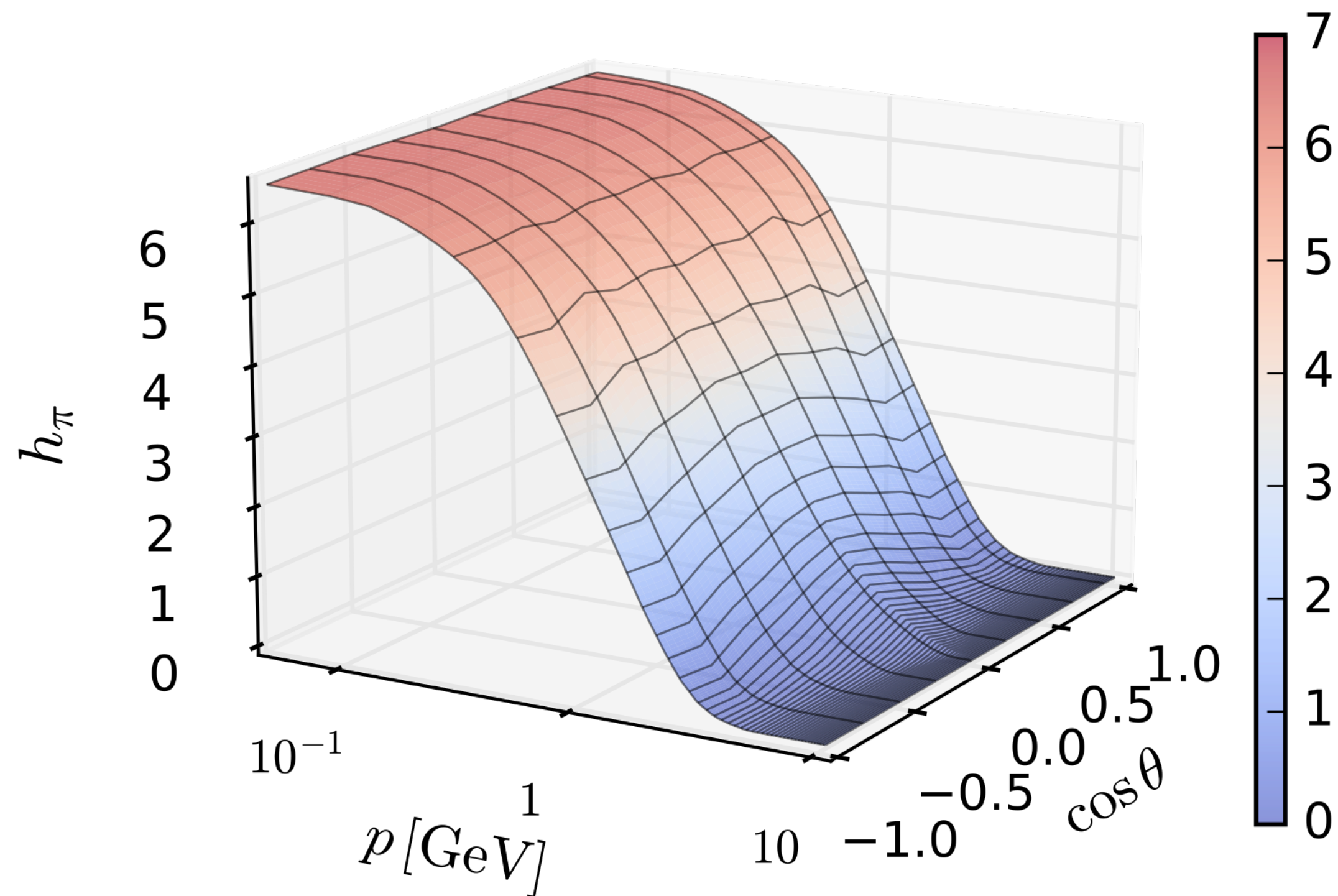
BS amplitude



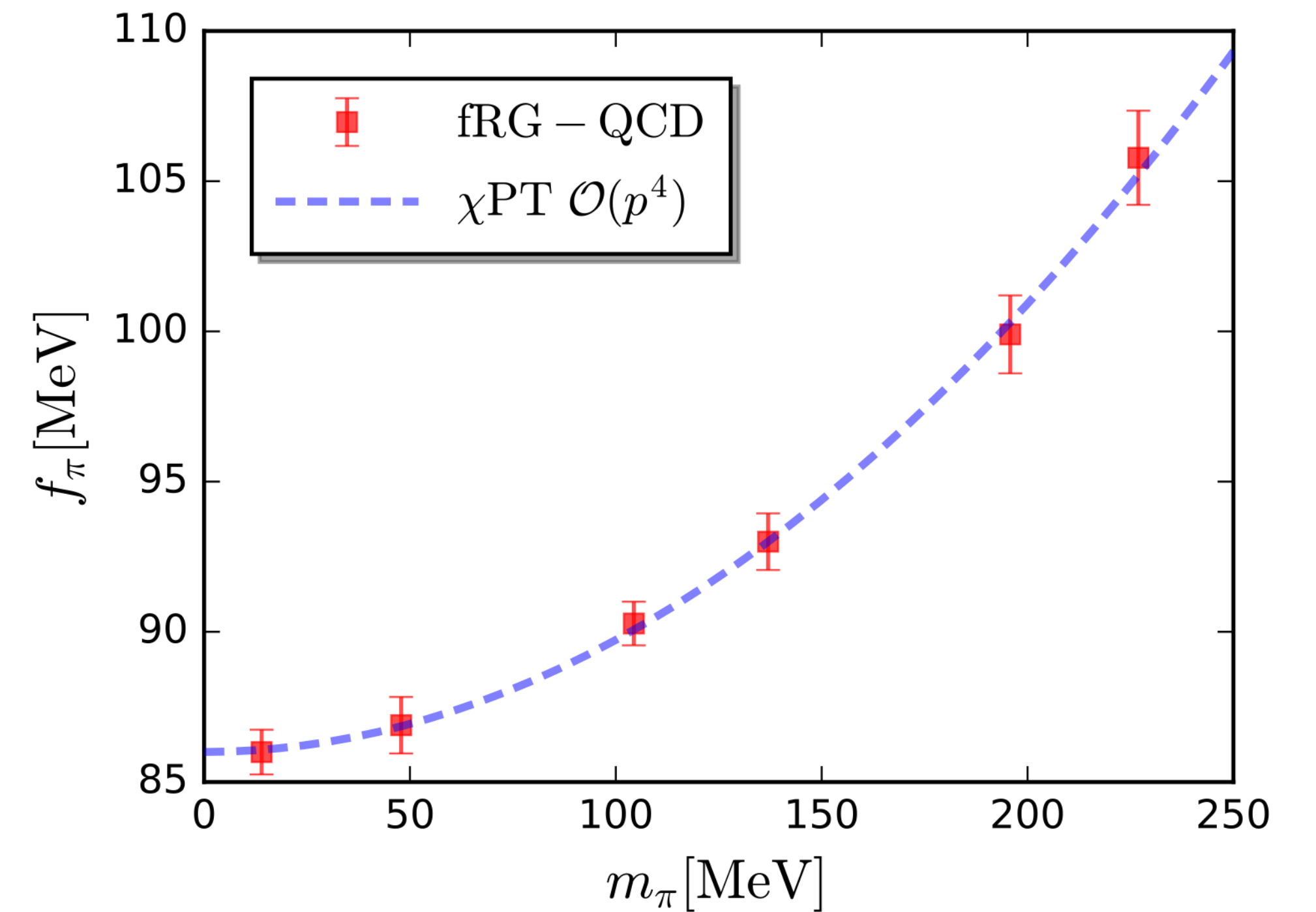
$$\lambda_\pi(P^2, p, \cos \theta) \sim \frac{h_\pi^2(p, \cos \theta)}{P^2 + m_\pi^2}$$

Bethe-Salpeter amplitude and Chiral relation

Pion BS amplitude



Chiral relation



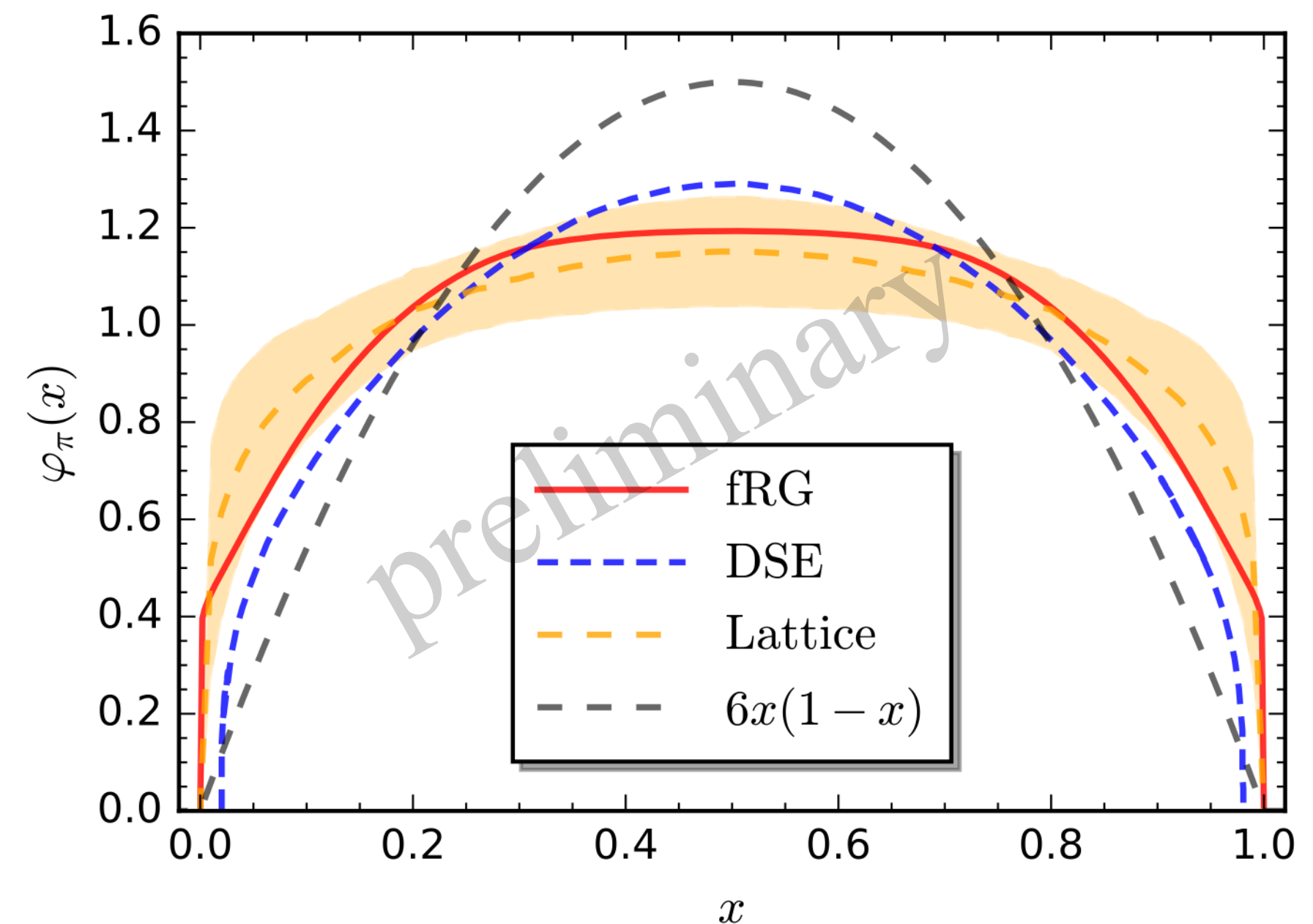
$$\langle 0 | J_{5\mu}^a(x) | \pi^b \rangle = i P_\mu f_\pi \delta^{ab}$$

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Summary and Outlook

- ★ Chiral symmetry breaking under the QCD four-quark scatterings
- ★ Quantitative QCD correlation functions in first-principle functional QCD
- ★ Apply the vacuum first-principle functional QCD calculations to the study of hadron structure
- ★ Quantitative QCD phase diagram: Finite T & μ + momentum dependence + global potential.
(with Franz Sattler, Konrad Kockler and Gan Min...)



Chang, Fu, CH, Pawłowski, Tan, in preparation

Thanks very much for your attentions!