QCD phase structure under external electromagnetic fields

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The QCD Phase Diagram: From Theory to Experimental Signatures, Dalian, 2025.10.8-2025.10.11

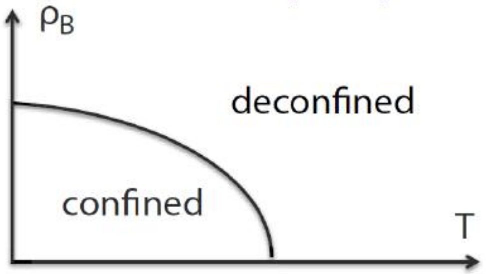
Outline

1. background
2. inverse magnetic catalysis (IMC)
3. electric catalysis (EC)
4. summary and outlook

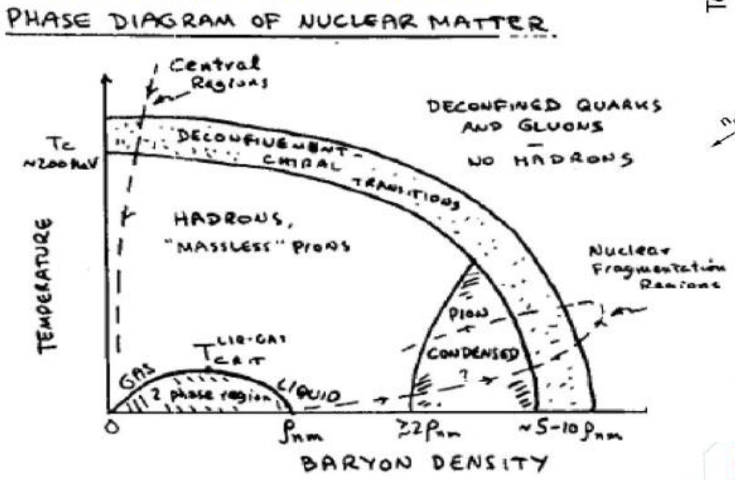
1 Background

QCD phase structure

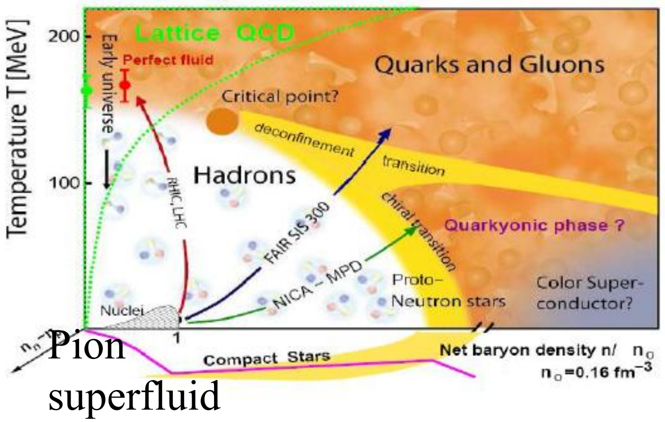
Deconfinement , N.Cabbibo and G.Parisi, PLB59, 67(1975)



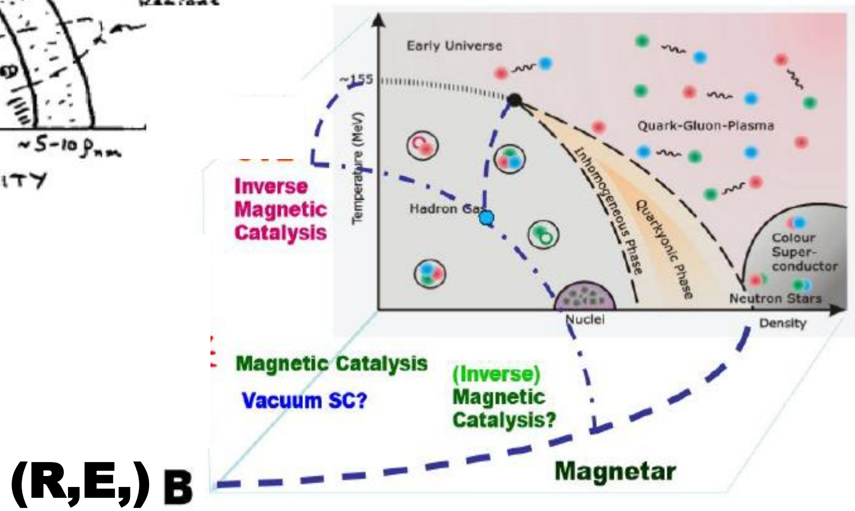
+ Chiral restoration G.Baym, NSAC Long Rang Plan, 1983



+ CSC, quarkyonic phase and critical point, ... (1998)



+ eB: IMC,Vacuum SC (2010)

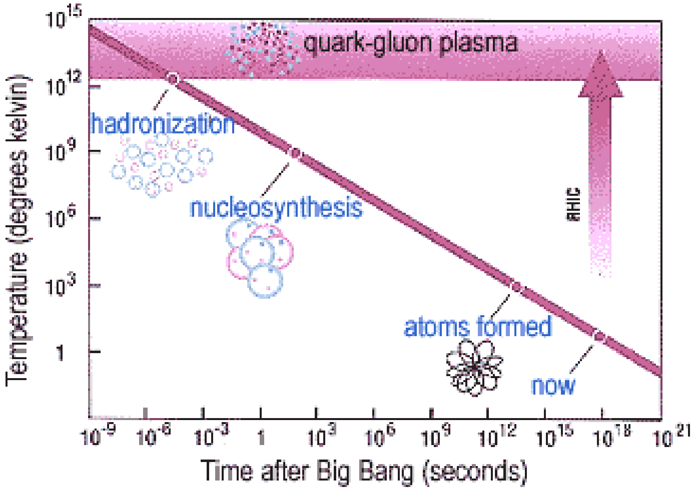


Theoretical Methods:

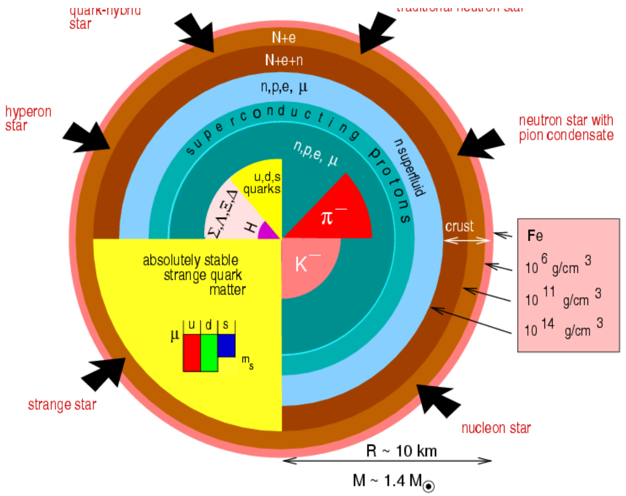
Lattice QCD
Functional RG
Resumed pQCD
Dyson-Schwinger equation
AdS/CFT
Effective models
Machine learning

1 Background

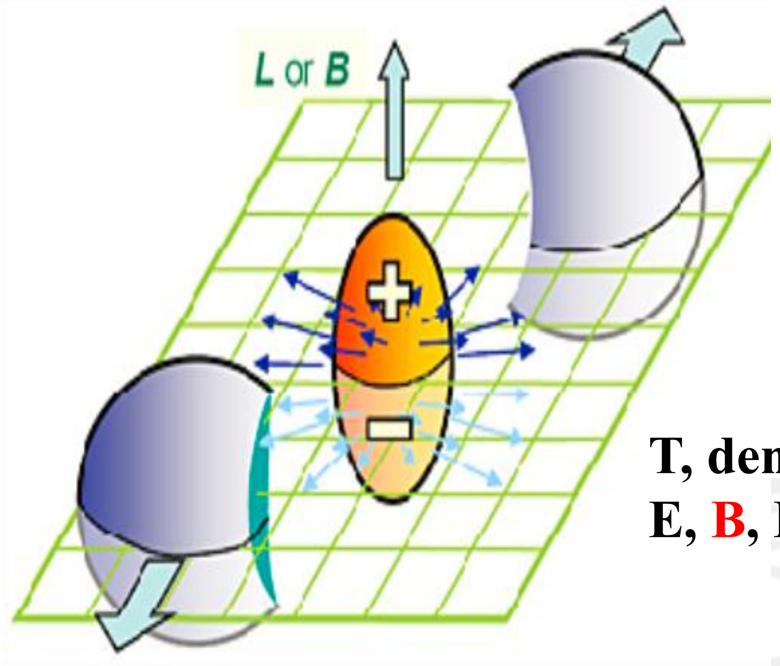
Early Universe (T, B)



Compact stars (Density, B, R)



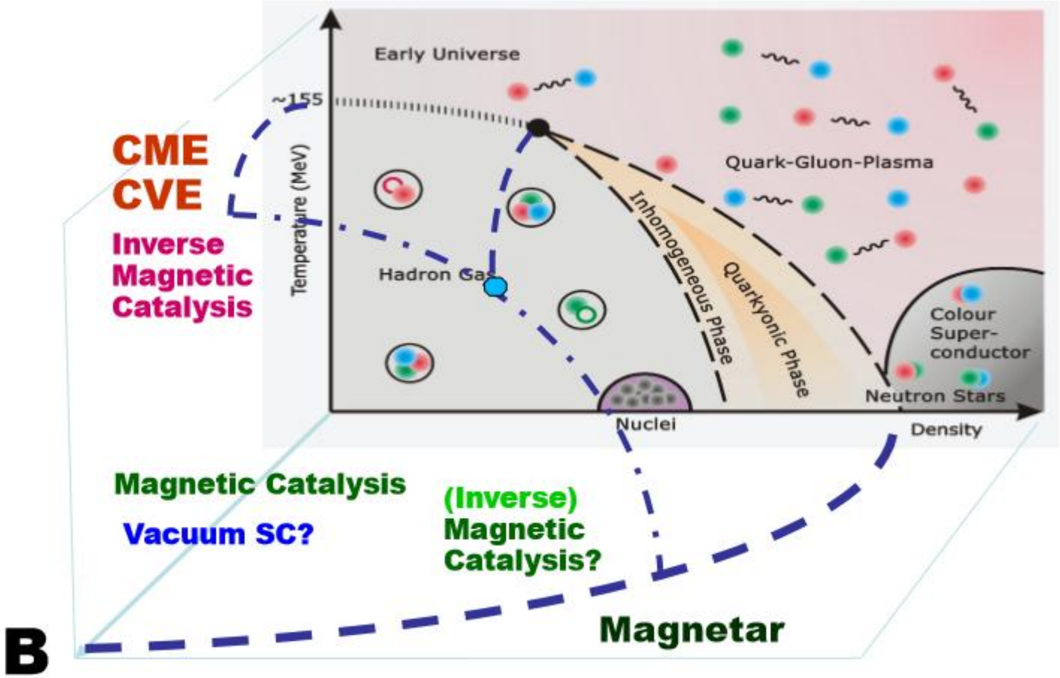
HIC: LHC, RHIC, FAIR,NICA, …



T, density, E, B, R

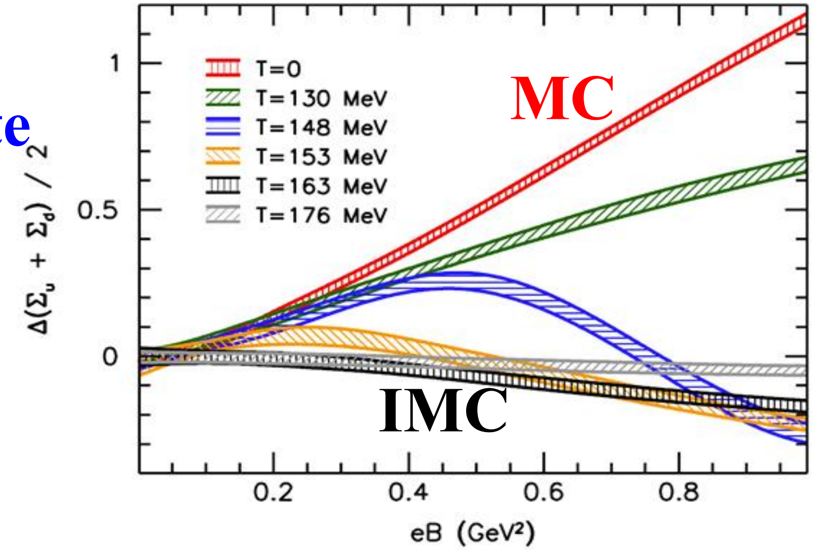
$$B, E \sim \gamma \frac{Z\alpha_{EM}}{R_A^2} \sim 10^{18\sim 20}\,\text{Gauss}$$

2 Inverse Magnetic Catalysis (IMC)

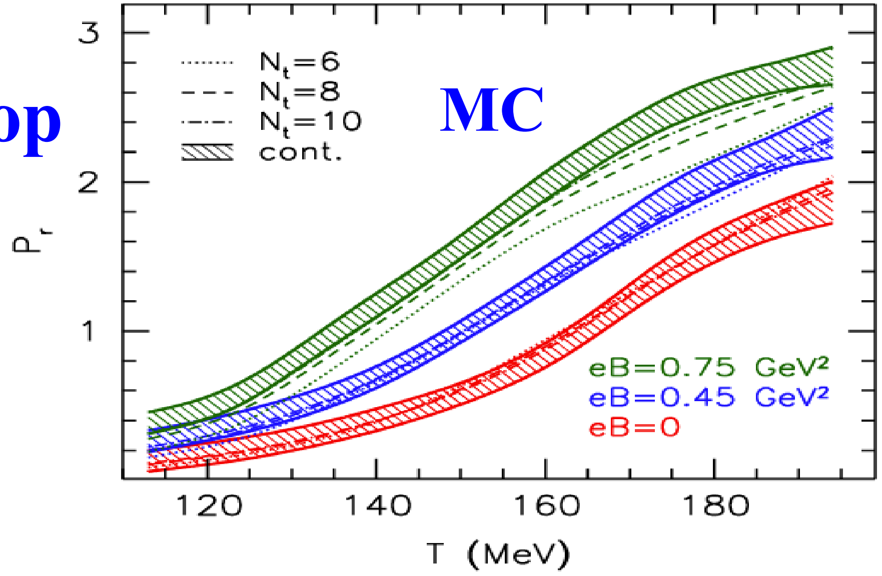


2 Inverse Magnetic Catalysis (IMC)

chiral condensate

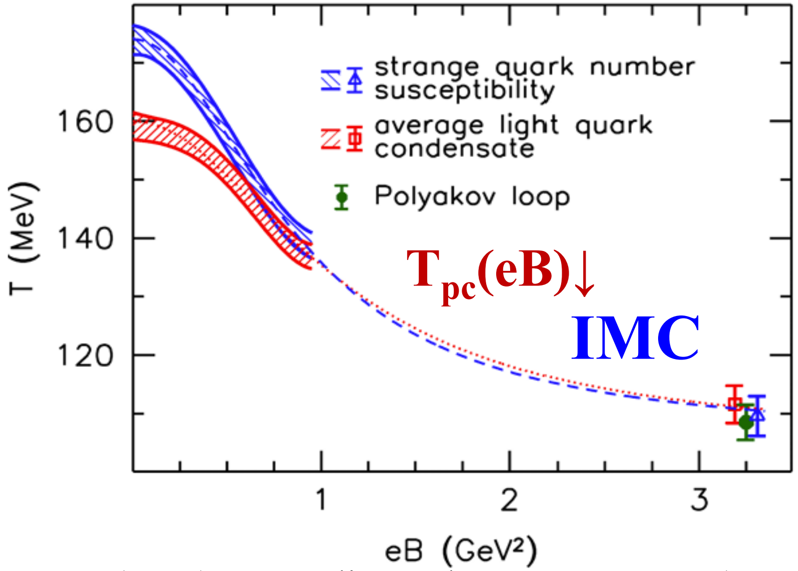


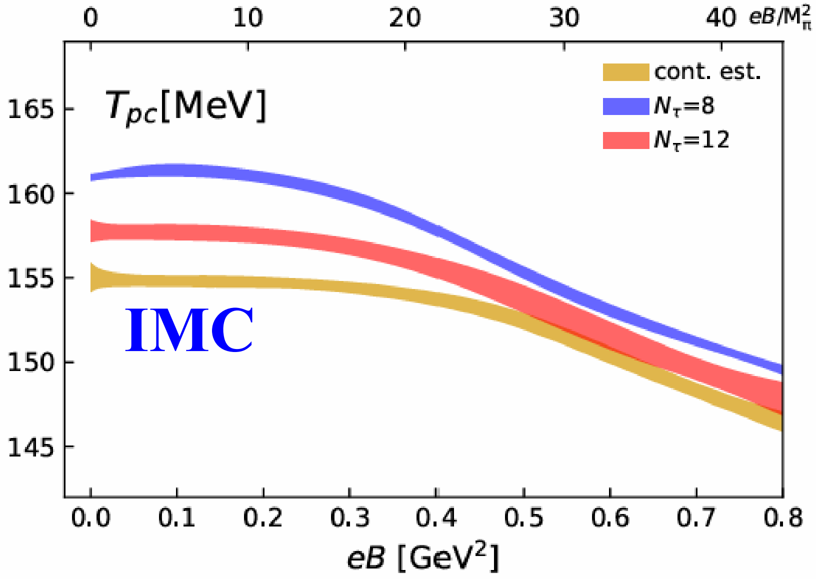
Polyakov loop



critical temperature

$T_{pc}(eB) \downarrow$



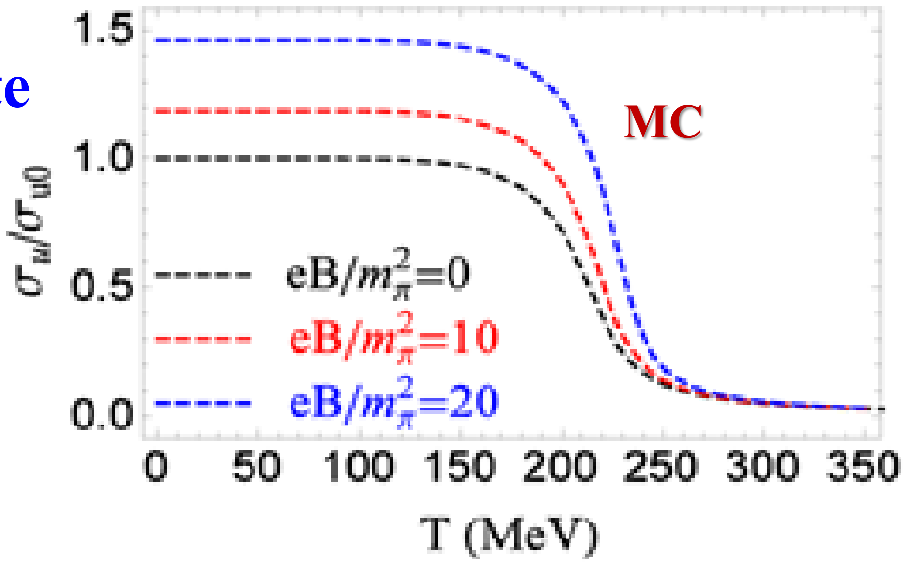


G. Bali, et.al, PRD86, 071502(2012);JHEP02,044(2012); M.D'Elia, et.al, PRD98, 054509(2018);H.T. Ding, et.al, PRD105, 034514(2022); PRD111, 114522 (2025).

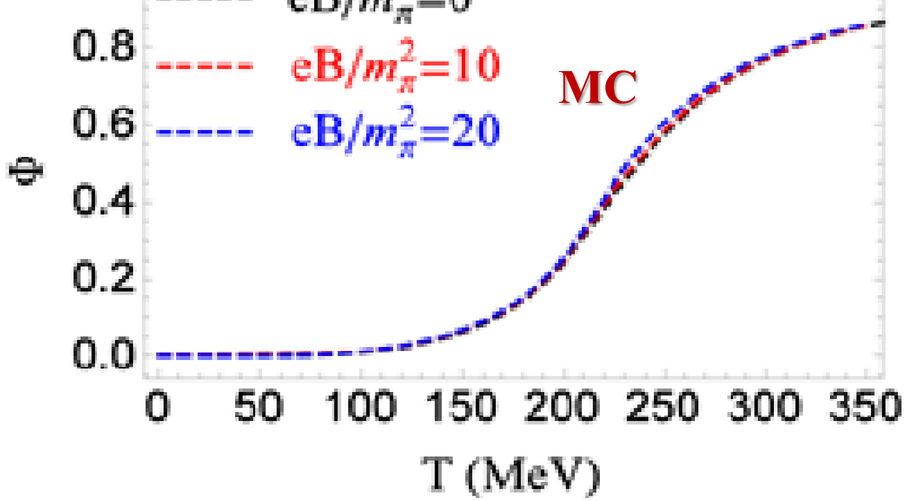
2.1 Analytical efforts ----- Mean field results: MC

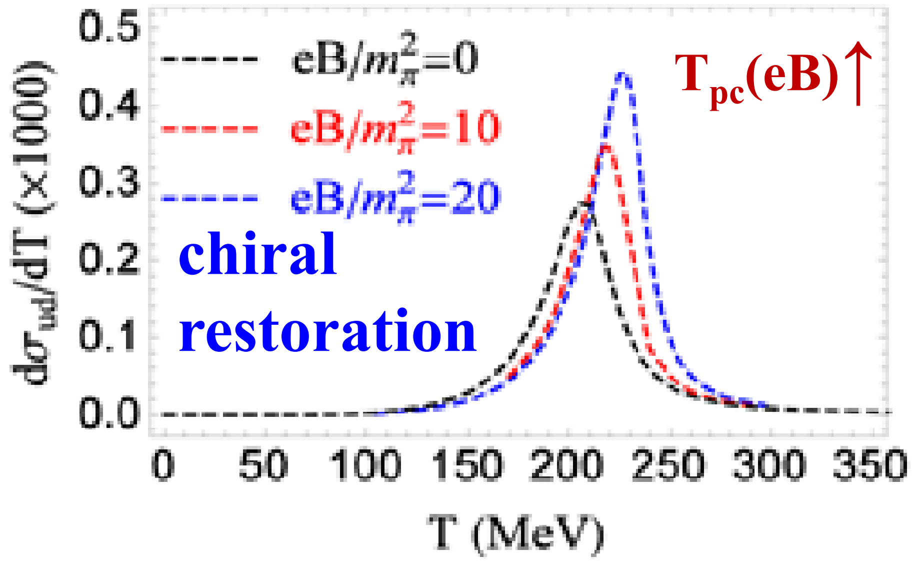
$$E_f = \sqrt{p_z^2 + 2l|Q_f B| + m_f^2}$$

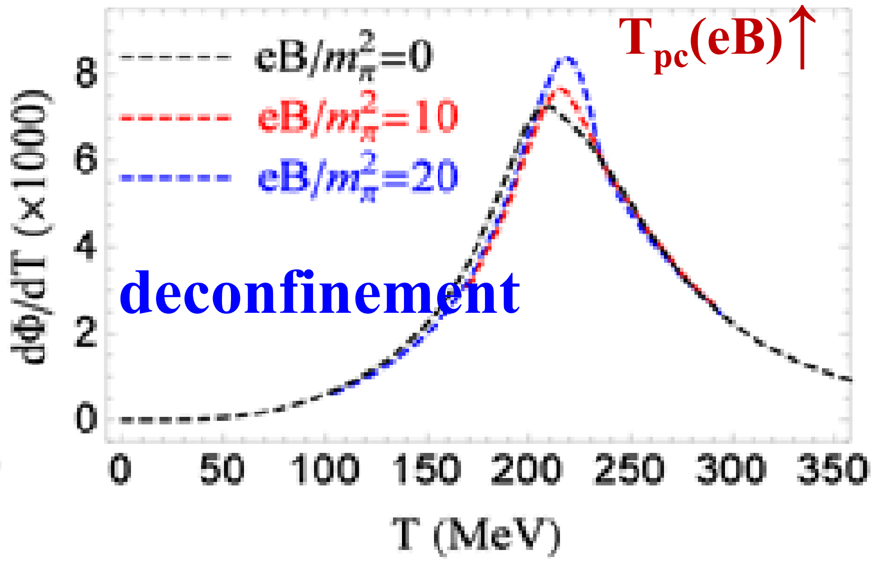
chiral condensate



Polyakov loop







2.1 Analytical efforts for IMC ---- Beyond mean field

Quarks

(1) magnetic inhibition: feedback from neutral pion to quarks

Fukushima et al., PRL110, 031601 (2013); Mao, PLB758,195(2016); PRD94,036007(2016); PRD97,011501(2018); Mei, Mao, Huang et al., PRD110, 034024(2024).

(2) thermo-magnetic effect: fluctuations @(P)QM

Kamikado et al. , JHEP 03,009(2014); Ayala et al.PRD90,036001(2014); PRD89,116017(2014); PRD92,096011(2015); EPJA 57, 234(2021).

(3) weakening (eB dependence) of coupling between quarks

Farias et al. , PRC 90, 025203 (2014); Ferreira et al. , PRD89,116011(2014); PRD89, 036006(2014); PRD89,016002(2014); PRD89, 056013(2014); Mueller et al. , PRD91,116010(2015); Braun et al. , PLB755, 265(2016); Mao,arXiv:2406.13531.

(4) quark anomalous magnetic moment:

Chaudhuri et al. , PRD99, 116025 (2019); EPJA56, 213 (2020); PRD103, 096021 (2021); Mao et al. , PRD102, 114035 (2020); PRD106, 034018 (2022); Ghosh, PRD103, 116008 (2021); Xu, Chao, Huang, PRD103,076015(2021); Feng, PRD**107**, 076004(2023); Tavares, PRD109, 016011 (2024) .

Gluons

(5) eB-dependent interaction between Polyakov loop and quarks

Fraga, PLB731, 154(2014); Ferreira, PRD89, 016002(2014); Mao, PRD110, 054002(2024).

(6) sphalerons & instantons :chirality imbalance, $U_A(1)$ anomaly

Chao, Huang et al. , PRD88, 054009 (2013); Fukushima PRD81, 114031 (2010); Mao et.al JPS Conf. Proc. 20, 011009 (2017);Zhou, Mao, arXiv:2025.XXXXX.

2.1.1 B influence the quark-quark interaction

(1) magnetic inhibition: feedback from neutral pion to quarks

Fukushima et al., PRL110, 031601 (2013); Mao, PLB758,195(2016); PRD94,036007(2016); PRD97,011501(2018); Mei, Mao, Huang et al., PRD110, 034024(2024).

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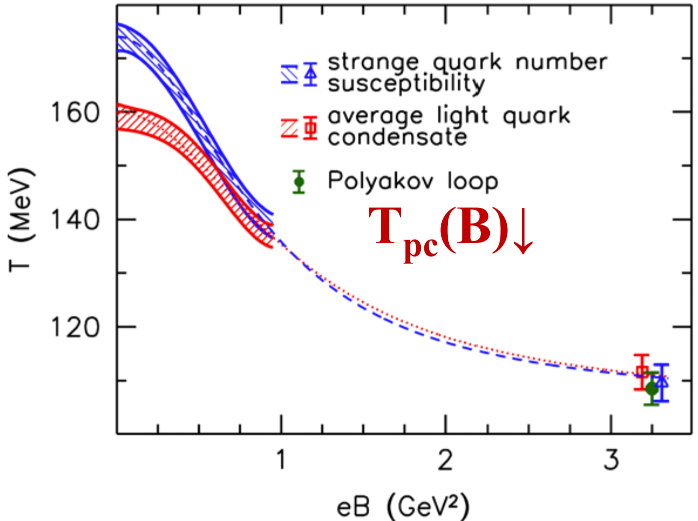
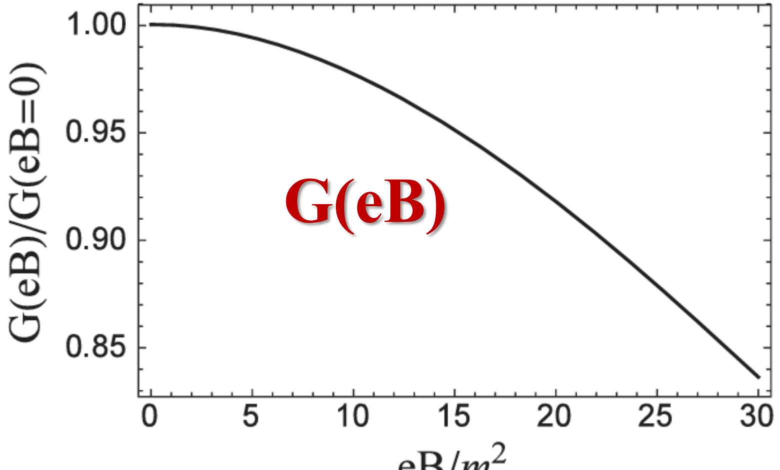


$$\mathcal{L} = \bar{\psi}(i\gamma_\mu D^\mu - \hat{m}_0)\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\tau\psi\right)^2\right]$$



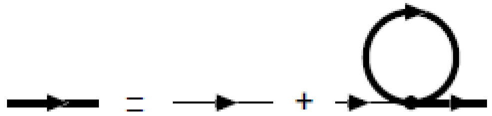
feeddown from pion G(eB)

NJL model beyond mean field

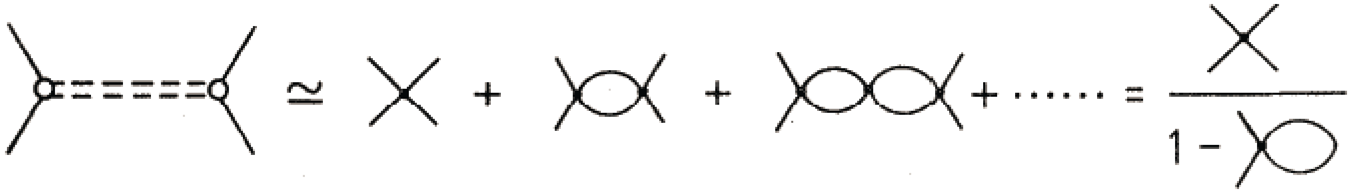
$$\mathcal{L} = \bar{\psi}\left(i\gamma_\nu D^\nu - m_0\right)\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right] \qquad \mathbf{B} = (0, 0, B)$$

idea: strong interaction & strong correlation

(1)Quarks： mean field



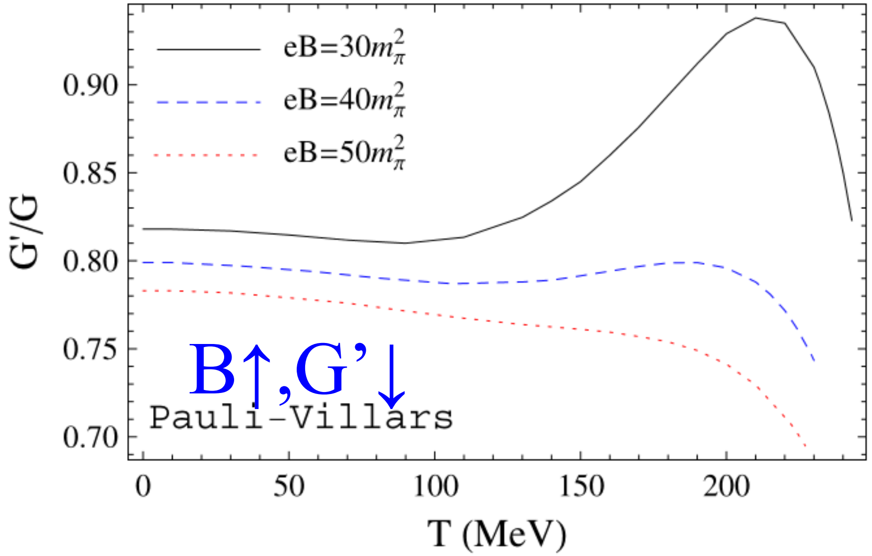
(2)Mesons: RPA resummation (quantum fluctuation)



(3)Q-M system： $\Omega = \Omega_q + \Omega_M \rightarrow \Omega(G'), \boxed{G'(B,T)}$

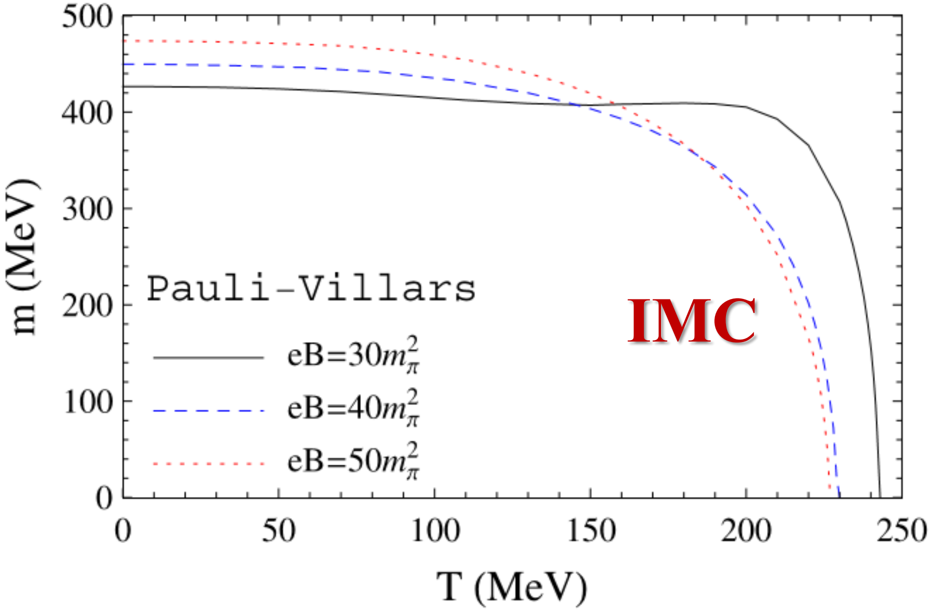
feed-down from mesons to quarks

2.1.1 Chiral limit eB VS G



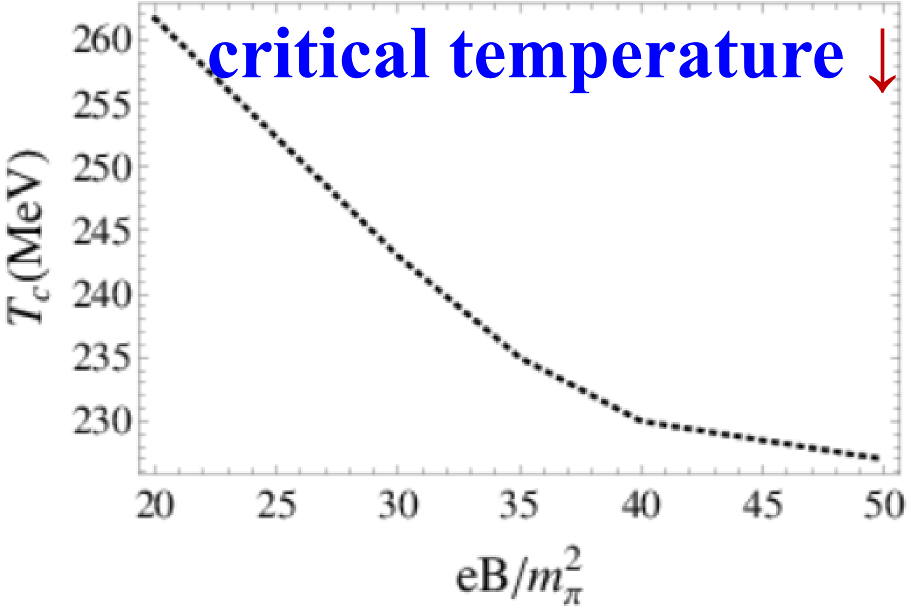
B↑,G'↓

order parameters



IMC

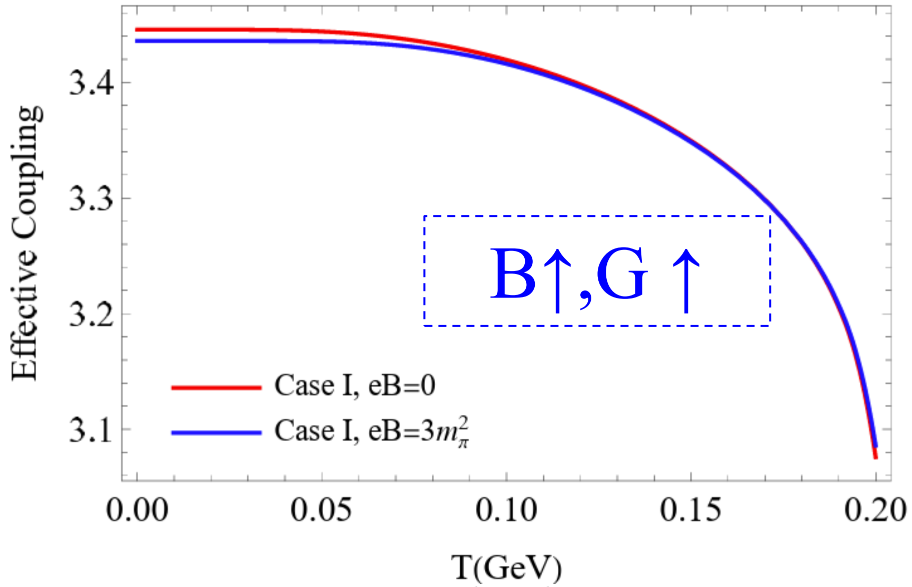
critical temperature ↓

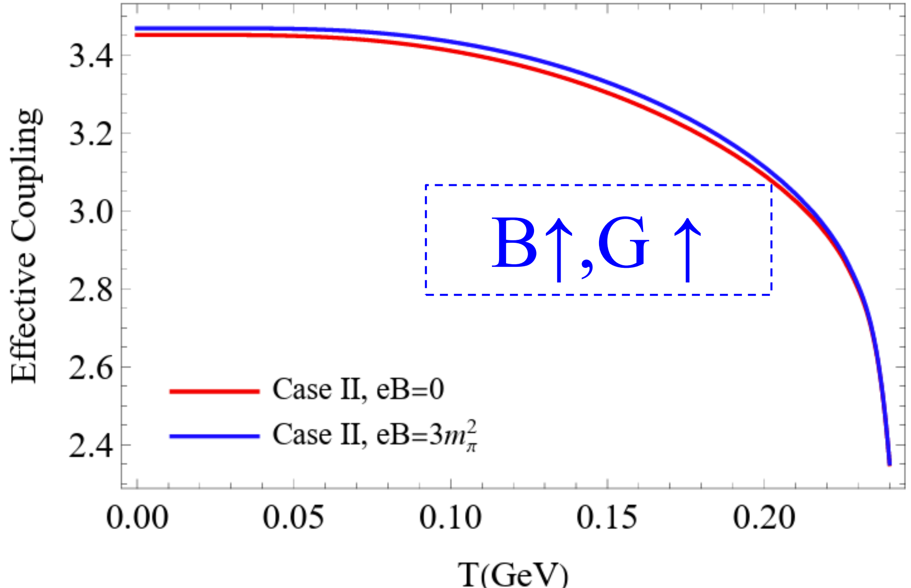


Phys. Lett. B758, 195-199 (2016);
Phys. Rev. D94, 036007 (2016);
Phys. Rev. D97, 011501(R) (2018);

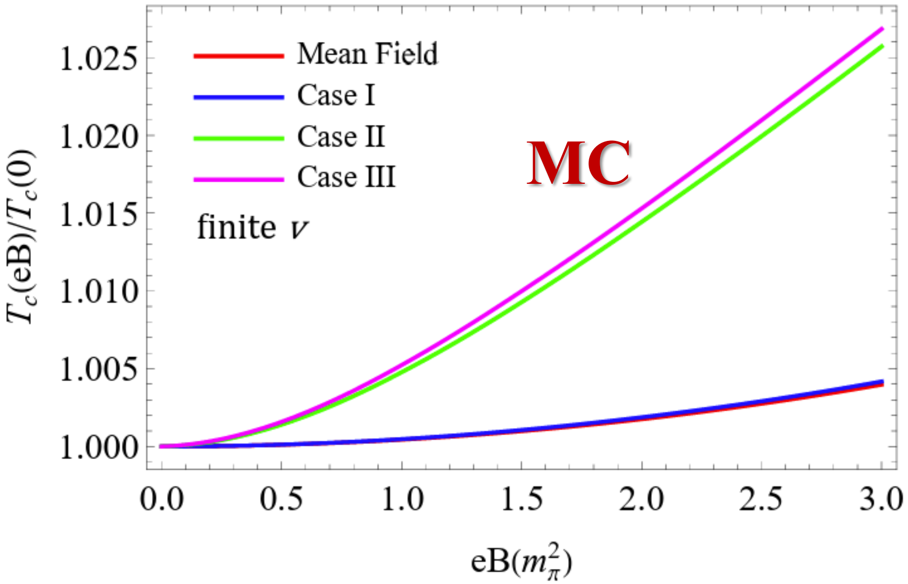
2.1.1 Physical case eB VS G

J.Mei, et.al, PRD110, 034024(2024)



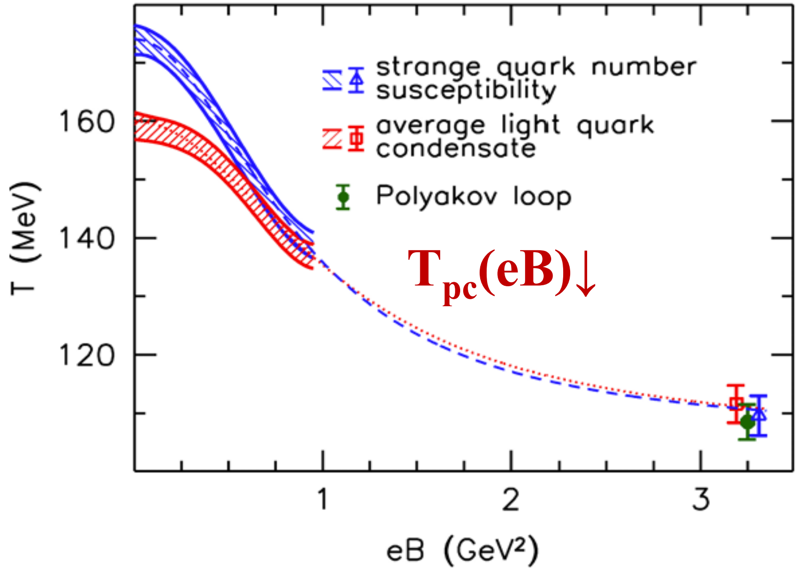


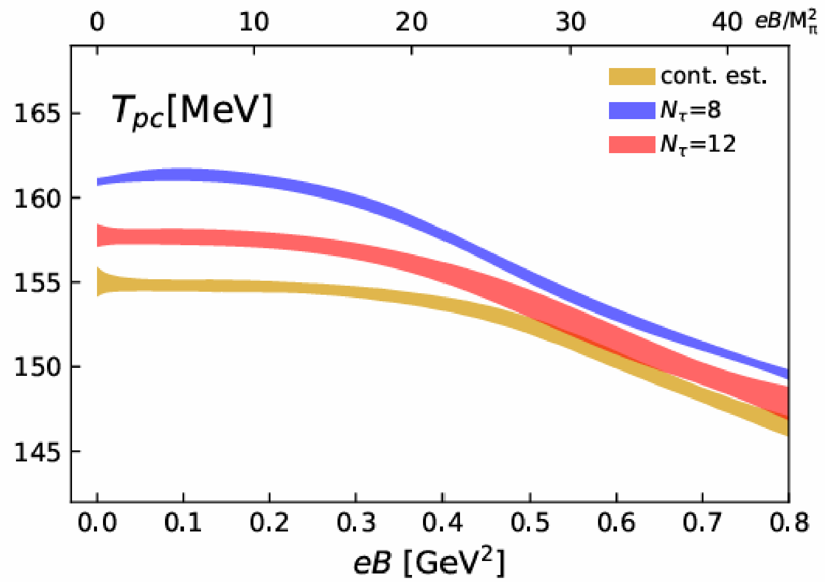
Case	Included mesons
0	None
I	π_0 only
II	$\pi_\pm$ only
III	π_0, $\pi_\pm$



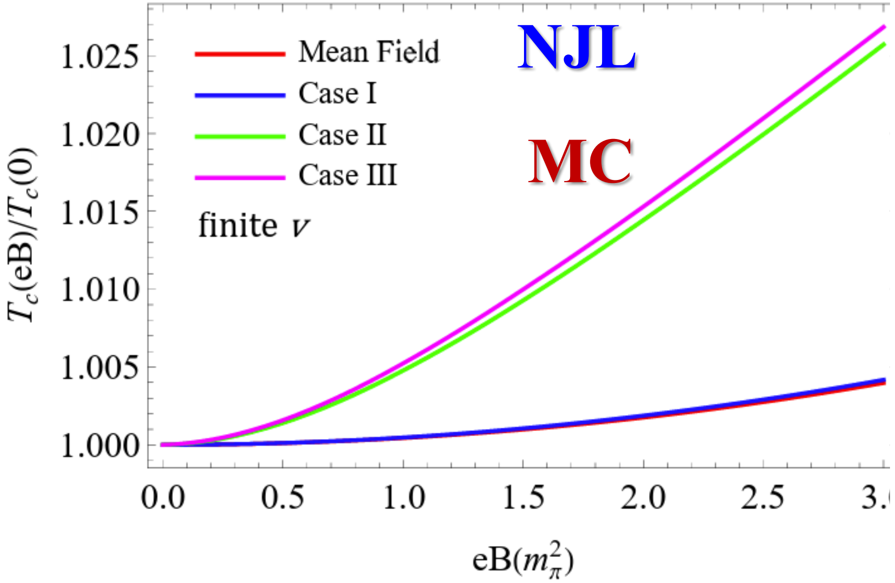
stronger eB case, &3-flavor case, under consideration

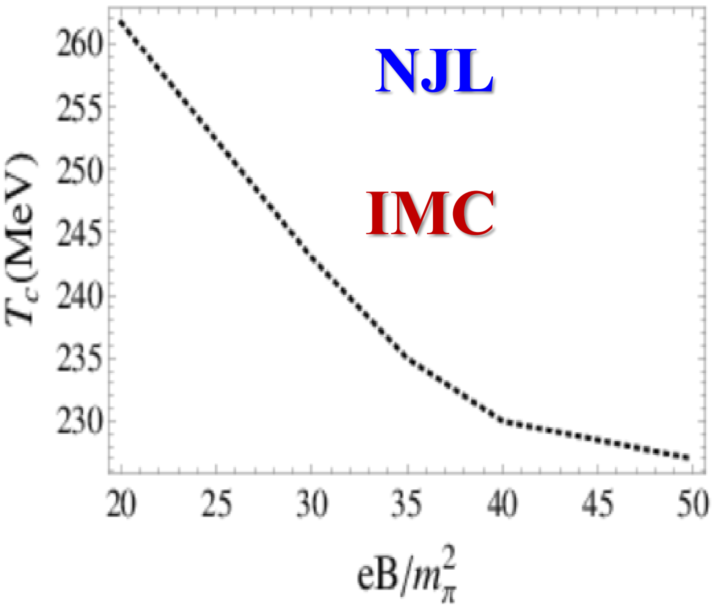
LQCD

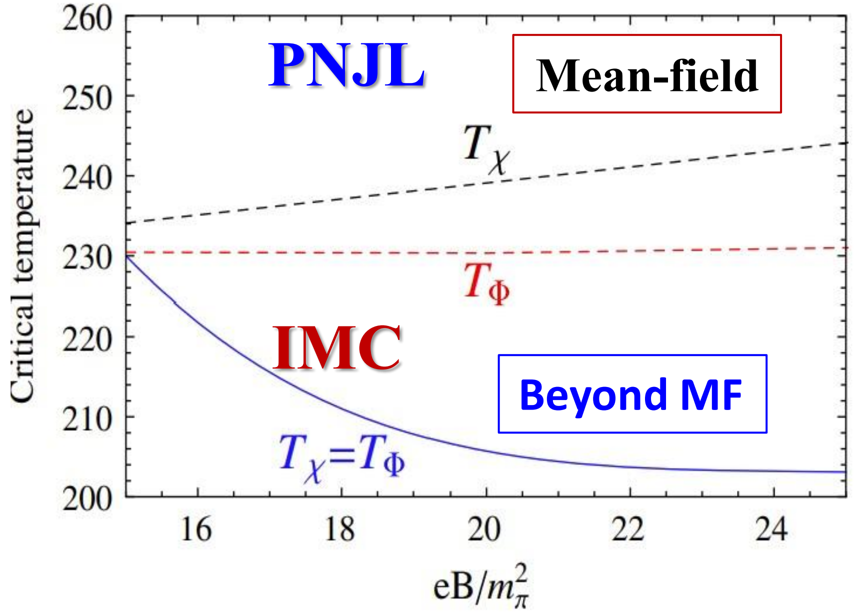




(P)NJL model beyond mean field



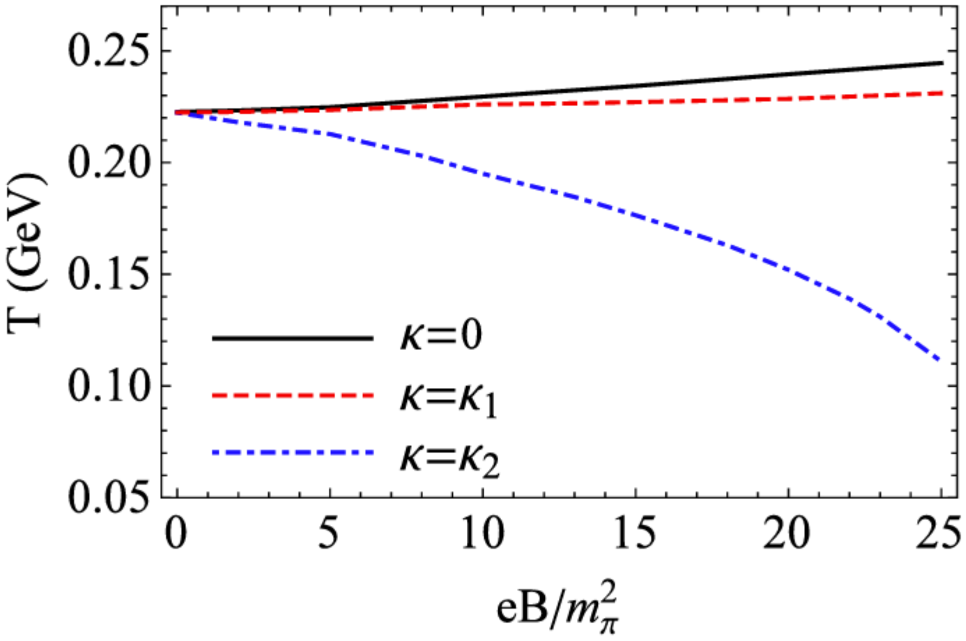




2.1.2 Analytical efforts for IMC

(4) quark anomalous magnetic moment:

Chaudhuri et al. , PRD99, 116025 (2019); EPJA 56, 213 (2020); PRD103, 096021 (2021);
Mao et al. , PRD102, 114035 (2020); PRD106, 034018 (2022); Ghosh, PRD103, 116008 (2021);
Xu, Chao, Huang, PRD103,076015(2021); Feng, PRD107, 076004(2023); Tavares, PRD109, 016011 (2024) .



$$\mathcal{L} = \bar{\psi}(x)\left(i\gamma^\mu D_\mu + \frac{1}{2}a\sigma^{\mu\nu}F_{\mu\nu}\right)\psi(x) + \frac{G}{2}\{[\bar{\psi}(x)\psi(x)]^2 + [\bar{\psi}(x)i\gamma_5\vec{\tau}\psi(x)]^2\} - \mathcal{U}(\Phi,\bar{\Phi})$$

AMM $a = Q\kappa$

$$\kappa_2 > \kappa_1 > \kappa_0$$

large quark AMM， $T_{pc}(B)\downarrow$

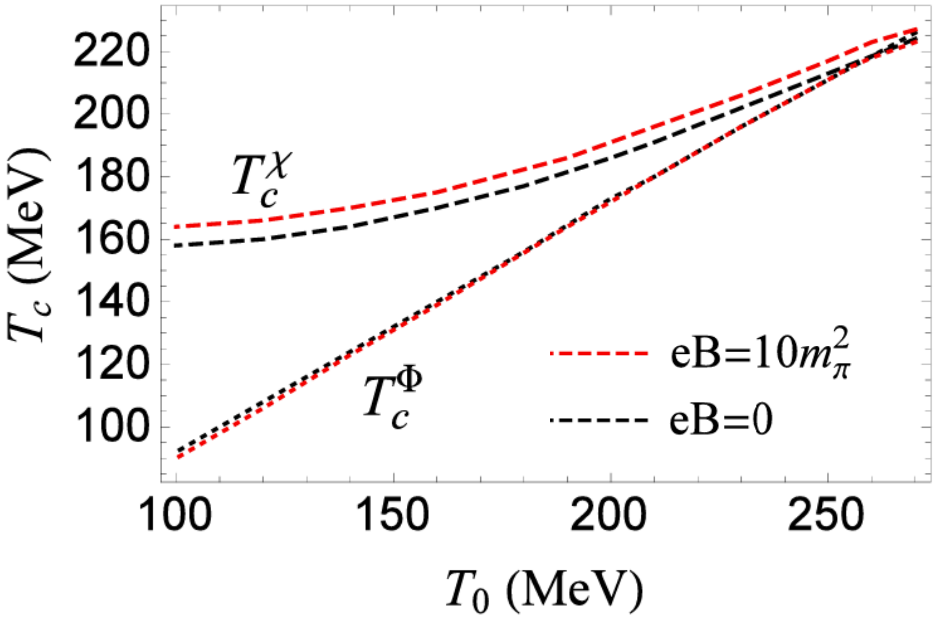
2.1.3 B influence the quark-gluon interaction

(5) eB-dependent interaction between Polyakov loop and quarks

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu D^\mu - \hat{m}_0)\psi + \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2\right] - \mathcal{U}(\Phi, \bar{\Phi}) \qquad \mathbf{B} = (0, 0, B)$$

$$\frac{\mathcal{U}(\Phi, \bar{\Phi})}{T^4} = -\frac{b_2(t)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\bar{\Phi}^3 + \Phi^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2 \qquad t = T_0/T$$

$$b_2(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

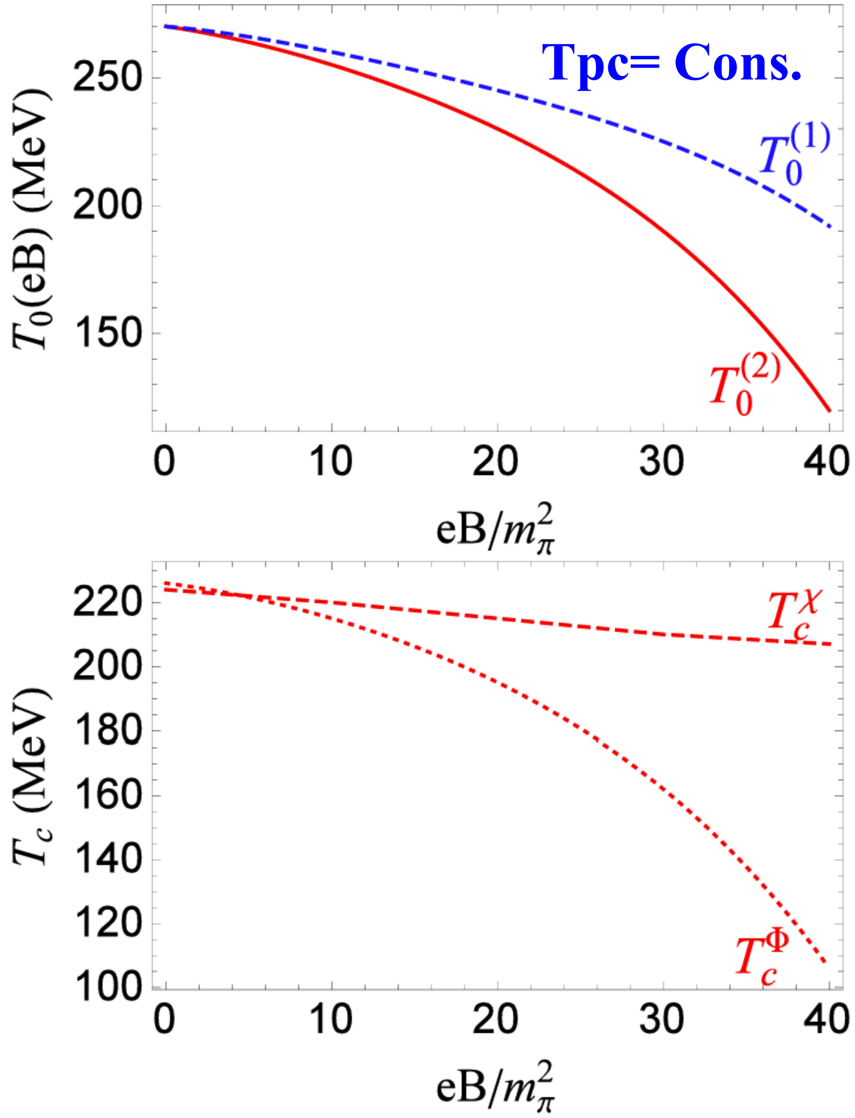


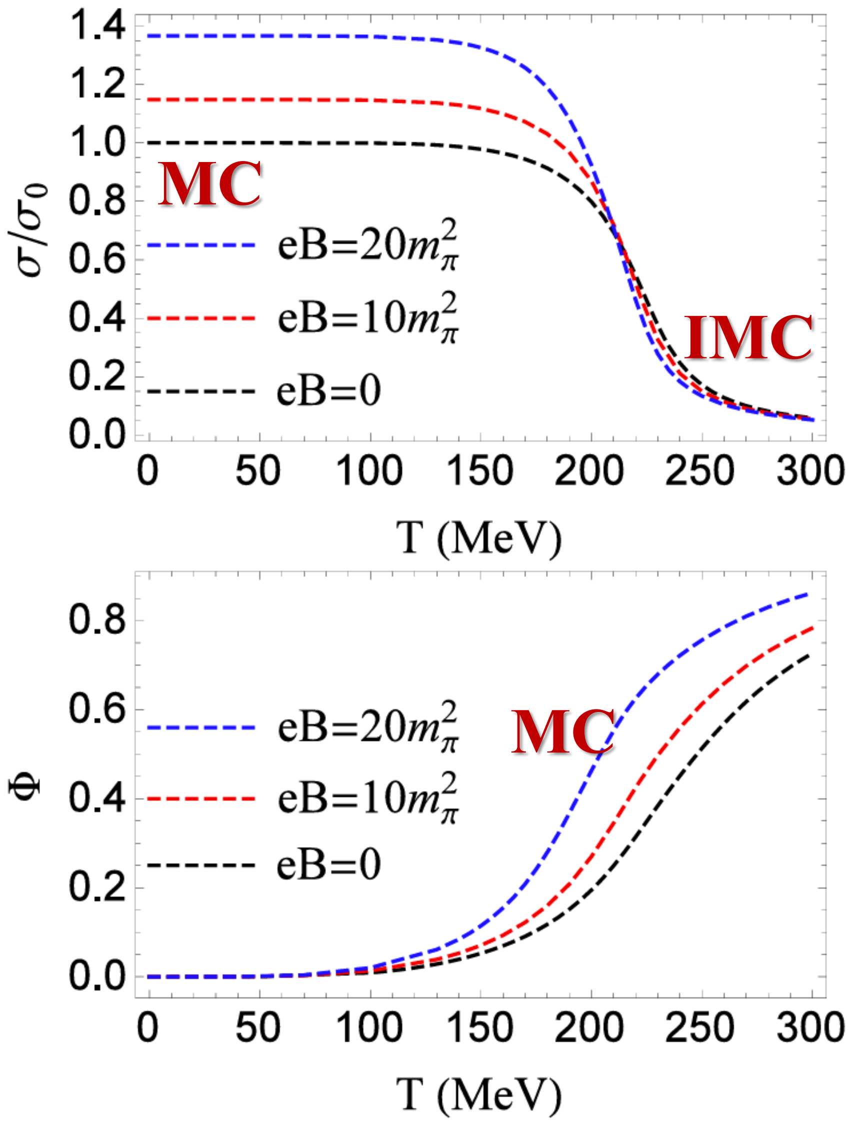
- T_0= 270MeV critical temperature of deconfinement in pure gauge field.
- coupling with quarks decreases T_0
- $\boxed{T_0\,(eB)\downarrow} \Longrightarrow T_{pc}(eB)\downarrow$

F.Bruckmann, et.al, JHEP04, 112(2013);
E.S.Fraga,et.al, PLB731, 154 (2014) ;Mao, PRD110, 054002(2024).

2.1.3 T_0 (eB) --- interaction between Polyakov loop and quarks

u, d quarks



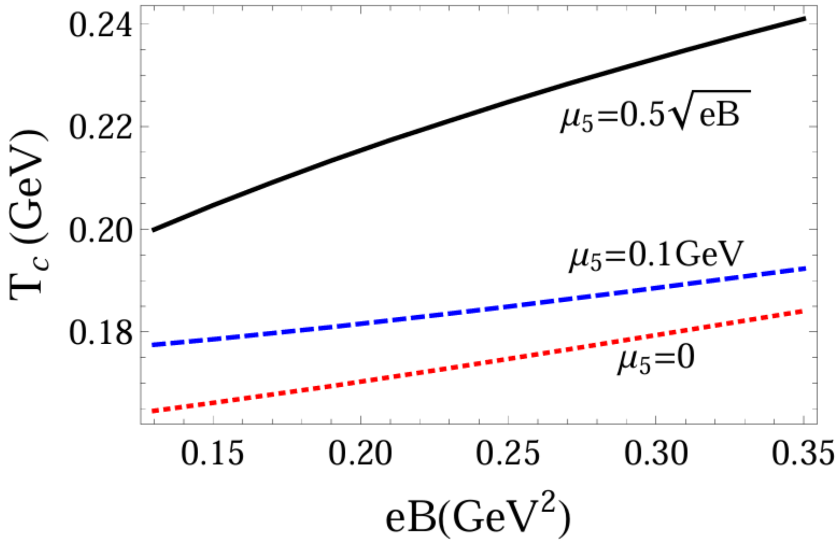


2.1.4 Analytical efforts for IMC

(6) sphalerons & instantons :chirality imbalance, $U_A(1)$ anomaly

Chao, Huang et al. , PRD88, 054009 (2013); Fukushima PRD81, 114031 (2010);
Mao et.al JPS Conf. Proc. 20, 011009 (2017);Zhou, Mao, arXiv:2025.XXXXX.

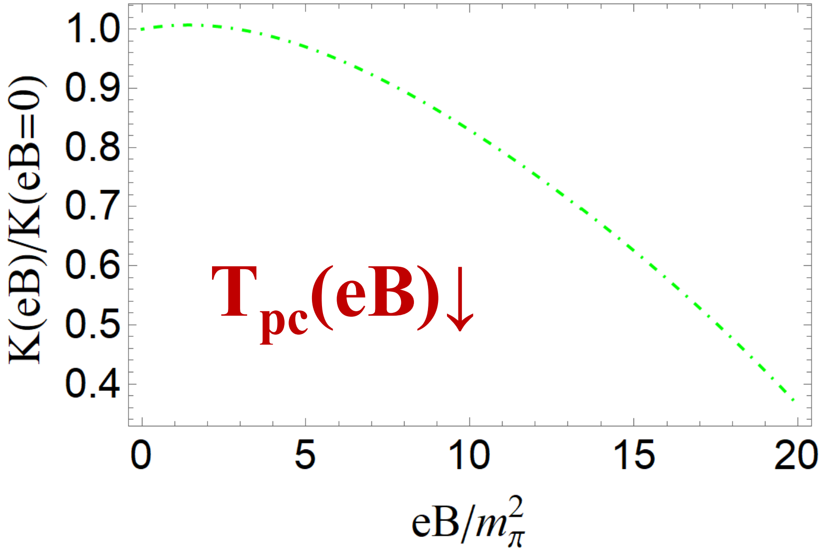
$$\mathcal{L} = \bar{\psi}\left(i\gamma_\nu D^\nu + \mu_5\gamma^0\gamma^5\right)\psi + \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2\right]$$



$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - \hat{m}_0)\psi + \mathcal{L}_S + \mathcal{L}_{KMT},$$

$$\mathcal{L}_S = G\sum_{\alpha=0}^{8}[(\bar{\psi}\lambda_\alpha\psi)^2 + (\bar{\psi}i\gamma_5\lambda_\alpha\psi)^2],$$

$$\mathcal{L}_{KMT} = -K[\det\bar{\psi}(1+\gamma_5)\psi + \det\bar{\psi}(1-\gamma_5)\psi].$$



eB effect on $U_A(1)$ anomaly is under consideration.

2.1.4 Analytical efforts for IMC

multi-factors

(1) magnetic inhibition: feedback from pion to quarks

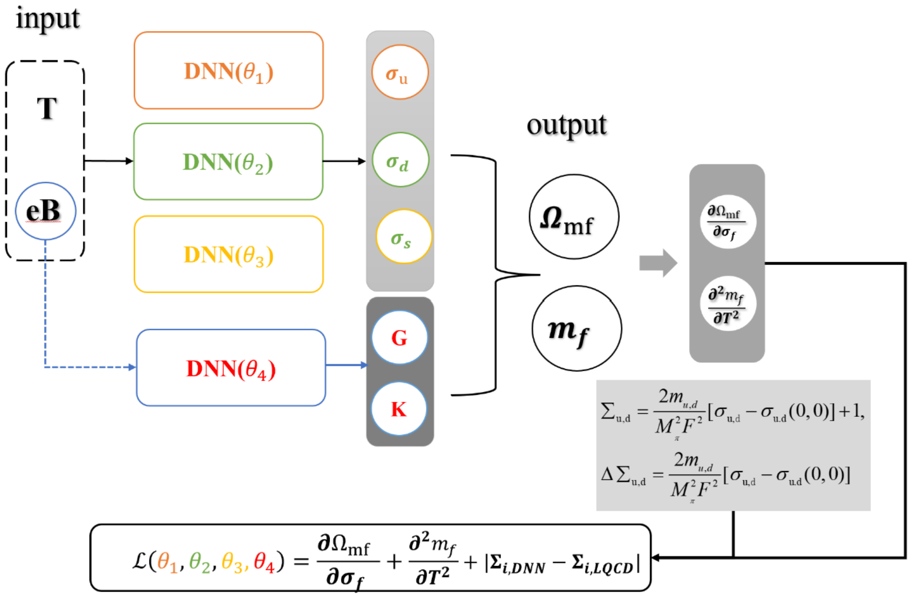
(2) thermo-magnetic effect: fluctuations @(P)QM

(3) weakening (eB dependence) of coupling between quarks

(4) quark anomalous magnetic moment

(5) eB-dependent interaction between Polyakov loop and quarks

(6) sphalerons & instantons :chirality imbalance, $U_A(1)$ anomaly



F.P. Li, L.X. Wang and S.J. Mao, deep learning study of IMC effect, under preparation.

2.2 What is the influence of IMC to QCD matter

Answer: IMC leads to no qualitative difference, but causes quantitative changes.

2.2.1 fluctuations and correlations with/without IMC

Fluctuations & correlations:

$$\chi_{i,j,k}^{B,Q,S} = -\frac{\partial^{i+j+k}(\Omega/T^4)}{\partial\hat{\mu}_B^i\partial\hat{\mu}_Q^j\partial\hat{\mu}_S^k}\Big|_{\mu_X=0}$$

chiral crossover

PNJL Lagrangian:

$$\begin{aligned}
\mathcal{L} &= \bar{\psi}\left(i\gamma^\mu D_\mu - \hat{m}_0\right)\psi + \mathcal{L}_4 + \mathcal{L}_6 - \mathcal{U}(\Phi,\bar{\Phi}),\\
\mathcal{L}_4 &= G\sum_{\alpha=0}^{8}\left[(\bar{\psi}\lambda_\alpha\psi)^2 + (\bar{\psi}i\gamma_5\lambda_\alpha\psi)^2\right],\\
\mathcal{L}_6 &= -K\left[\mathrm{det}\bar{\psi}(1+\gamma_5)\psi + \mathrm{det}\bar{\psi}(1-\gamma_5)\psi\right],\\
\mathcal{U}(\Phi,\bar{\Phi}) &= T^4\left[-\frac{b_2(t)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\bar{\Phi}^3+\Phi^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2\right].
\end{aligned}$$

Thermodynamical potential:

$$\begin{aligned}
\Omega &= 2G(\sigma_u^2+\sigma_d^2+\sigma_s^2) - 4K\sigma_u\sigma_d\sigma_s + \mathcal{U}(\Phi,\bar{\Phi}) + \Omega_q,\\
\Omega_q &= -\sum_{f=u,d,s}\frac{|Q_fB|}{2\pi}\sum_l \alpha_l\int\frac{dp_z}{2\pi}\Big[3E_f\\
&+ T\ln\left(1+3\Phi e^{-\beta E_f^+} + 3\bar{\Phi}e^{-2\beta E_f^+} + e^{-3\beta E_f^+}\right)\\
&+ T\ln\left(1+3\bar{\Phi} e^{-\beta E_f^-} + 3\Phi e^{-2\beta E_f^-} + e^{-3\beta E_f^-}\right)\Big],
\end{aligned}$$

2.2.1 introducing IMC: G(eB), T_0(eB)

Lagrangian density:

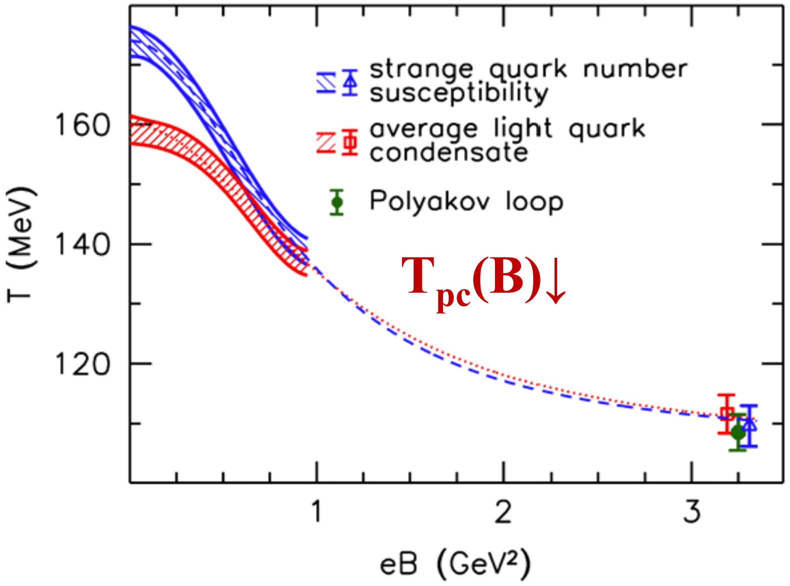
$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu D_\mu - \hat{m}_0\right)\psi + \mathcal{L}_4 + \mathcal{L}_6 - \mathcal{U}(\Phi,\bar{\Phi}), \quad (1)$$

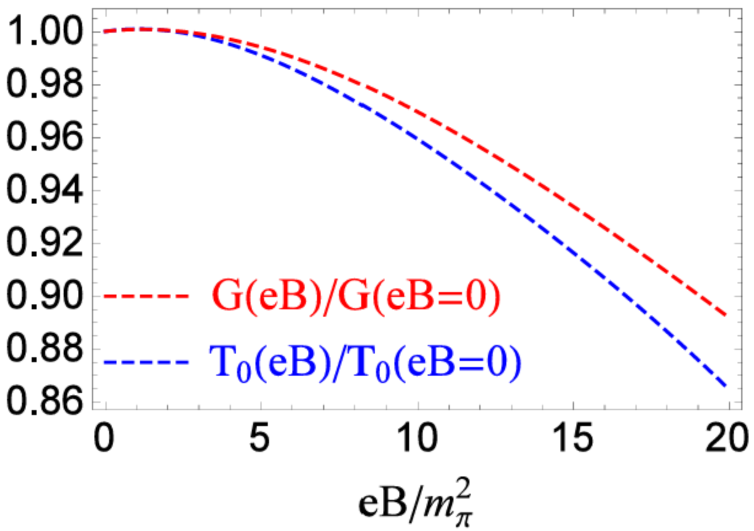
$$\mathcal{L}_4 = G\sum_{\alpha=0}^{8}\left[(\bar{\psi}\lambda_\alpha\psi)^2 + (\bar{\psi}i\gamma_5\lambda_\alpha\psi)^2\right],$$

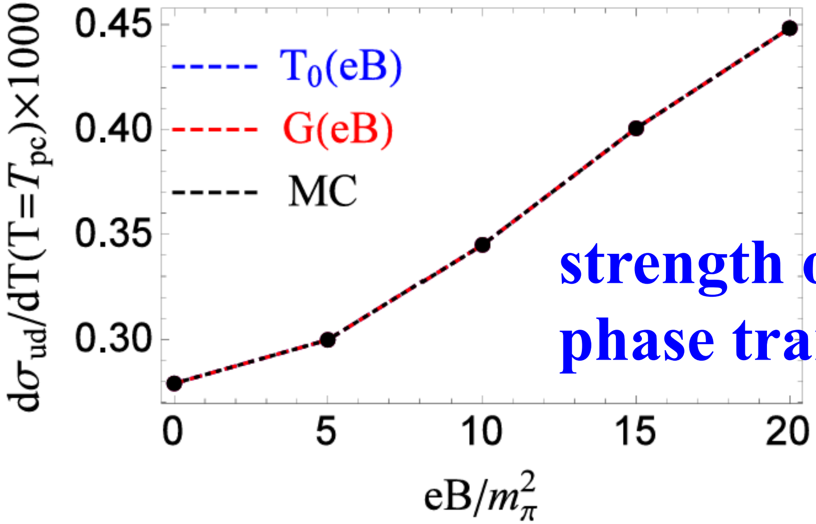
$$\mathcal{L}_6 = -K\left[\det\bar{\psi}(1+\gamma_5)\psi + \det\bar{\psi}(1-\gamma_5)\psi\right],$$

$$\mathcal{U}(\Phi,\bar{\Phi}) = T^4\left[-\frac{b_2(t)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\bar{\Phi}^3+\Phi^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2\right].$$

$$t' = T_0/T$$



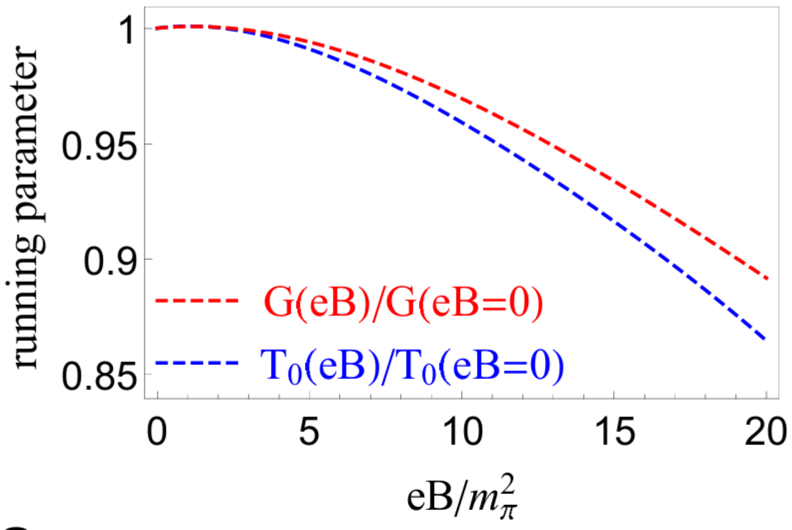


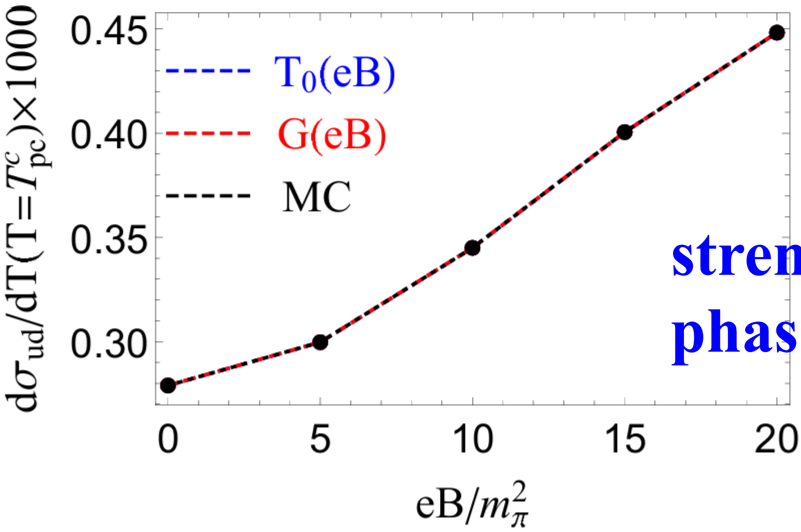


strength of phase transition

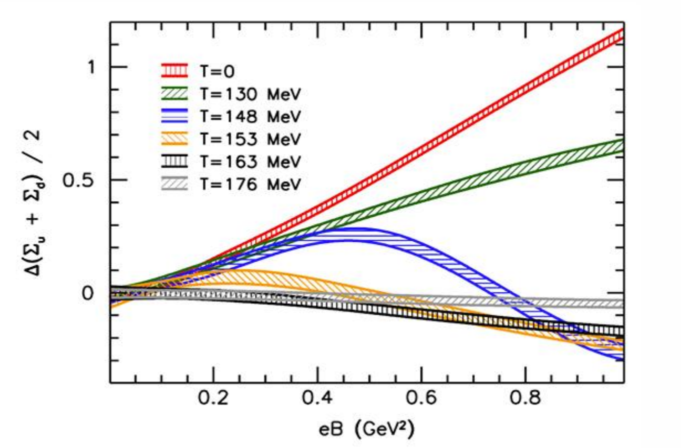
Mao,Yang, PRD112, 014026 (2025);CPC49, 063106(2025) .

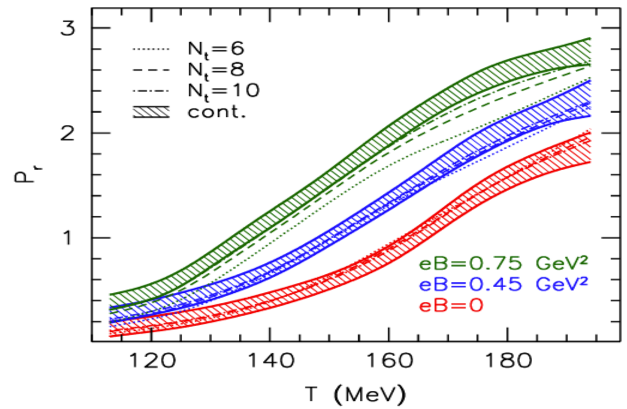
2.2.1 Comparision with LQCD results

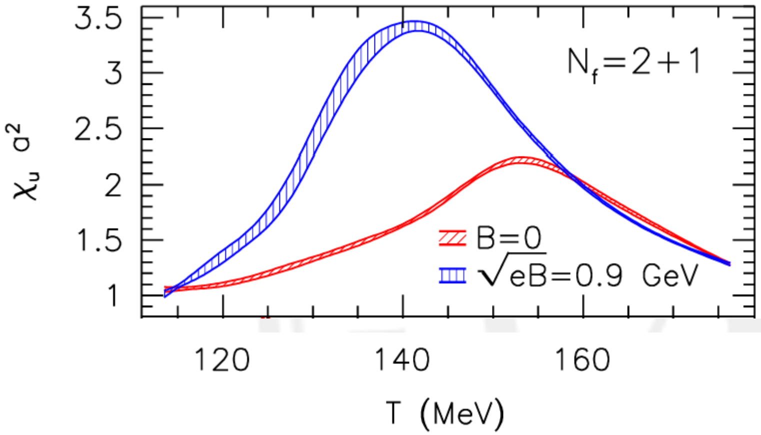




strength of phase transition





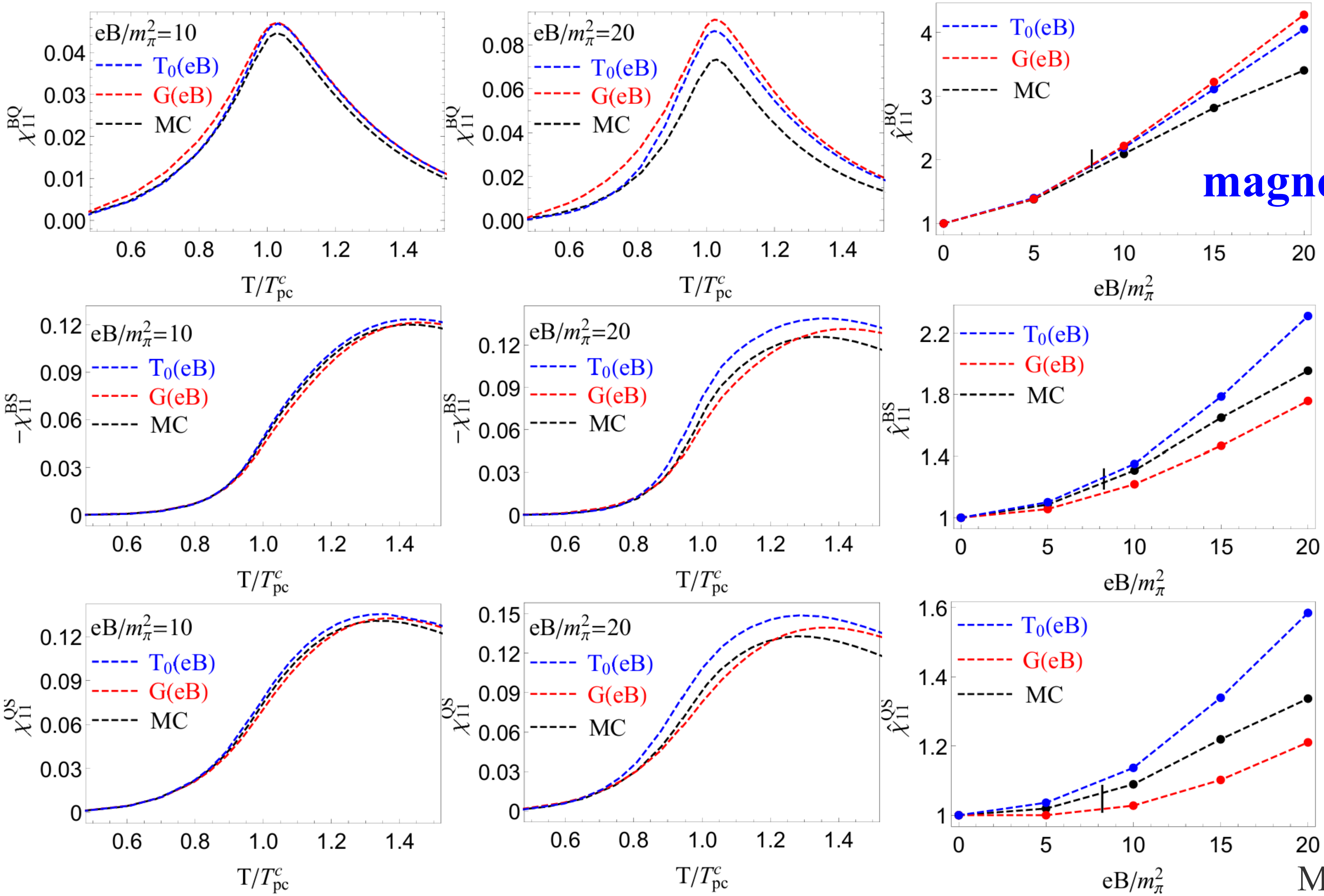


Mao, Yang, PRD112, 014026 (2025).

G. Bali, et.al, PRD86, 071502(2012);JHEP02,044(2012);

2.2.1 correlations with IMC

$$\chi^{B,Q,S}_{i,j,k} = -\frac{\partial^{i+j+k}(\Omega/T^4)}{\partial\hat{\mu}^i_B\partial\hat{\mu}^j_Q\partial\hat{\mu}^k_S}\bigg|_{\mu_X=0}$$



magnetometer of QCD

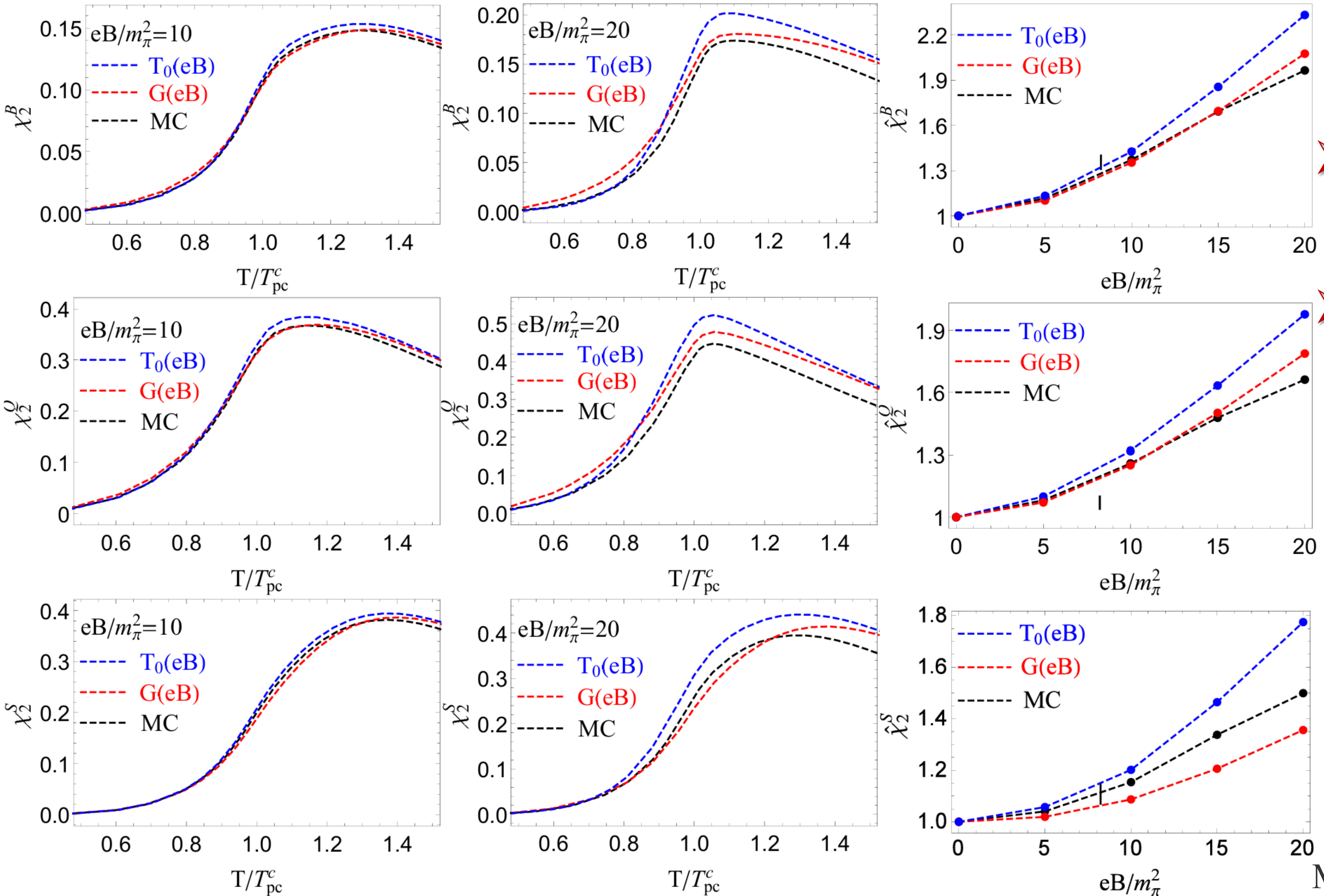
(next talk of Gu)

- **IMC leads to no qualitative difference**
- **consistent with LQCD**

Mao, Yang, PRD112, 014026 (2025)

2.2.1 fluctuations with IMC

$$\chi_{i,j,k}^{B,Q,S} = -\frac{\partial^{i+j+k}(\Omega/T^4)}{\partial\hat{\mu}_B^i\partial\hat{\mu}_Q^j\partial\hat{\mu}_S^k}\Big|_{\mu_X=0}$$

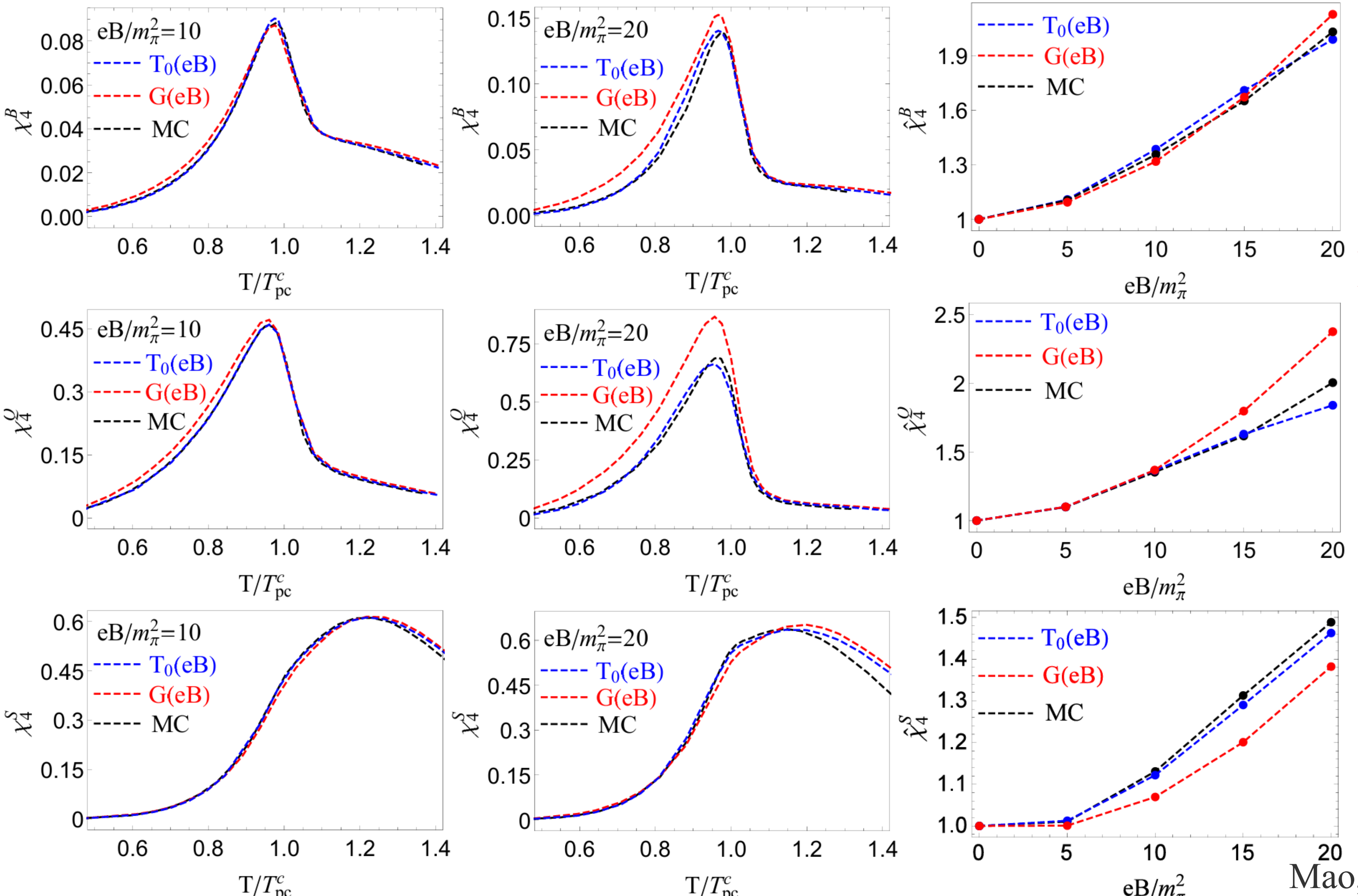


- **IMC leads to no qualitative difference**
- **consistent with LQCD**

Mao, Yang, PRD112, 014026 (2025)

2.2.1 fluctuations with IMC

$$\chi_{i,j}^{B,Q,S} = -\frac{\partial^{i+j+k}(\Omega/T^4)}{\partial\hat{\mu}_B^i\partial\hat{\mu}_Q^j\partial\hat{\mu}_S^k}\bigg|_{\mu_X=0}$$



- **IMC leads to no qualitative difference**

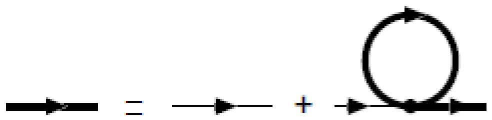
Mao, Yang, PRD112, 014026 (2025)

2.2.2 Mass spectra of meson @ eB

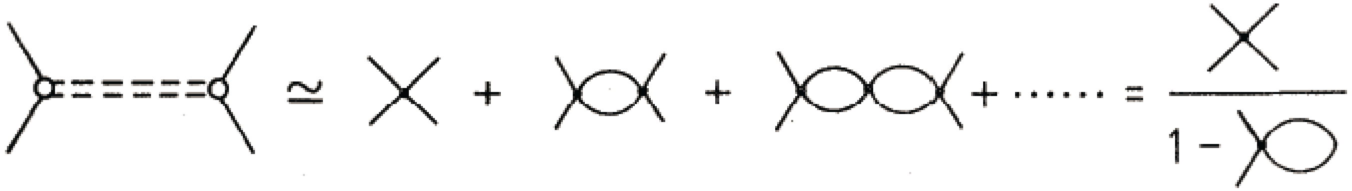
(P)NJL model beyond mean field

$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}\,(i\gamma^{\mu}D_{\mu}-\hat{m}_0)\,\psi+\mathcal{L}_4+\mathcal{L}_6-\mathcal{U}(\Phi,\bar{\Phi}),\\
\mathcal{L}_4 &= G\sum_{\alpha=0}^{8}\left[(\bar{\psi}\lambda_{\alpha}\psi)^2+(\bar{\psi}i\gamma_5\lambda_{\alpha}\psi)^2\right],\\
\mathcal{L}_6 &= -K\left[\det\bar{\psi}(1+\gamma_5)\psi+\det\bar{\psi}(1-\gamma_5)\psi\right],\\
\mathcal{U}(\Phi,\bar{\Phi}) &= T^4\left[-\frac{b_2(t)}{2}\bar{\Phi}\Phi-\frac{b_3}{6}\left(\bar{\Phi}^3+\Phi^3\right)+\frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2\right].
\end{aligned}
$$

$$\mathbf{B}=(0,0,B)$$

(1)Quarks： mean field 

(2)**Mesons**: RPA resummation (quantum fluctuation)

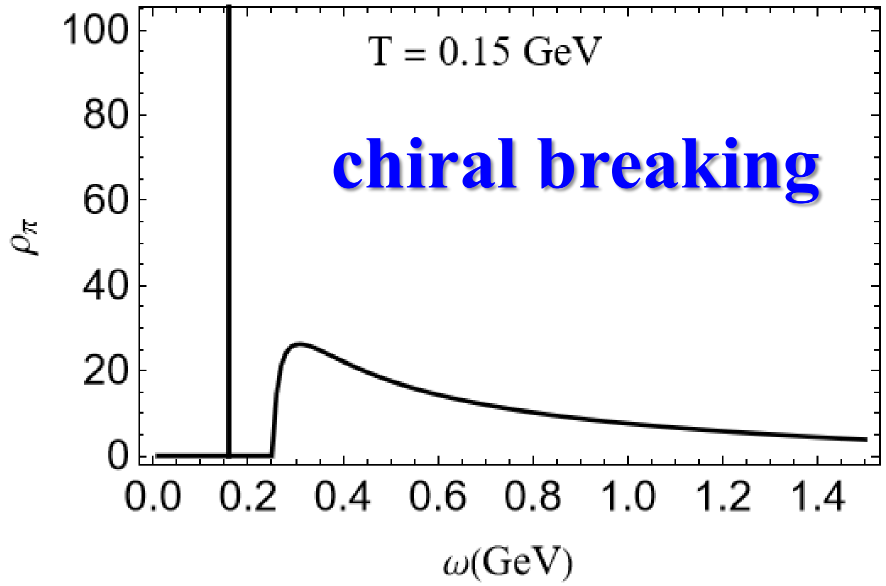


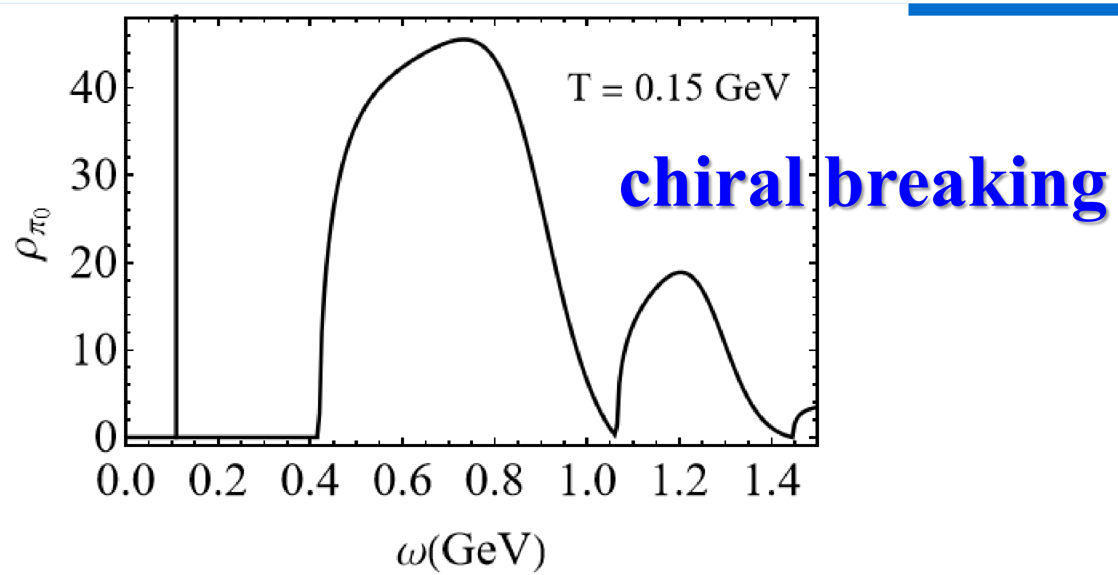
S.J. Mao and Y.X. Wange, PRD96, 034004(2017); L.Y. Li and S.J. Mao, PRD108, 054001(2023); J.Mei, T.Xia, S.J. Mao, PRD107,074018(2023); arXiv: 2025.XXXXX

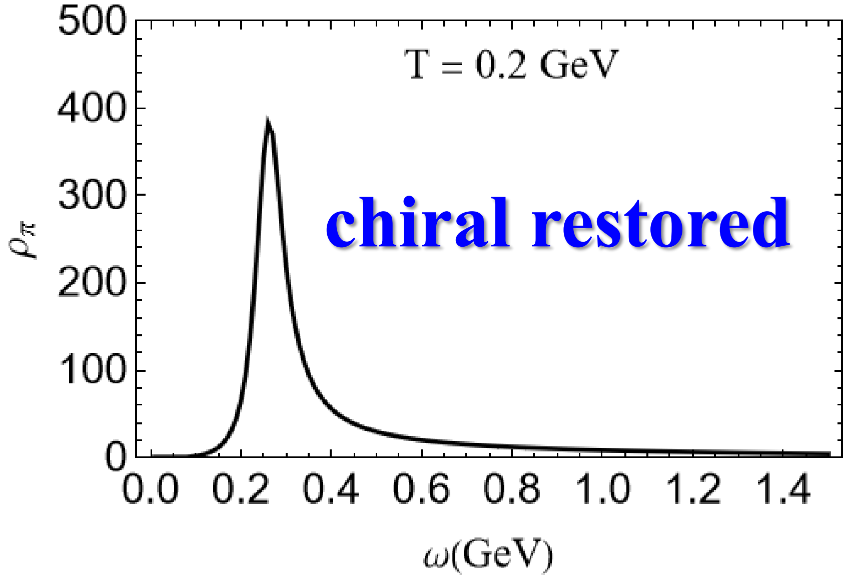
Spectral function of π_0 @ eB

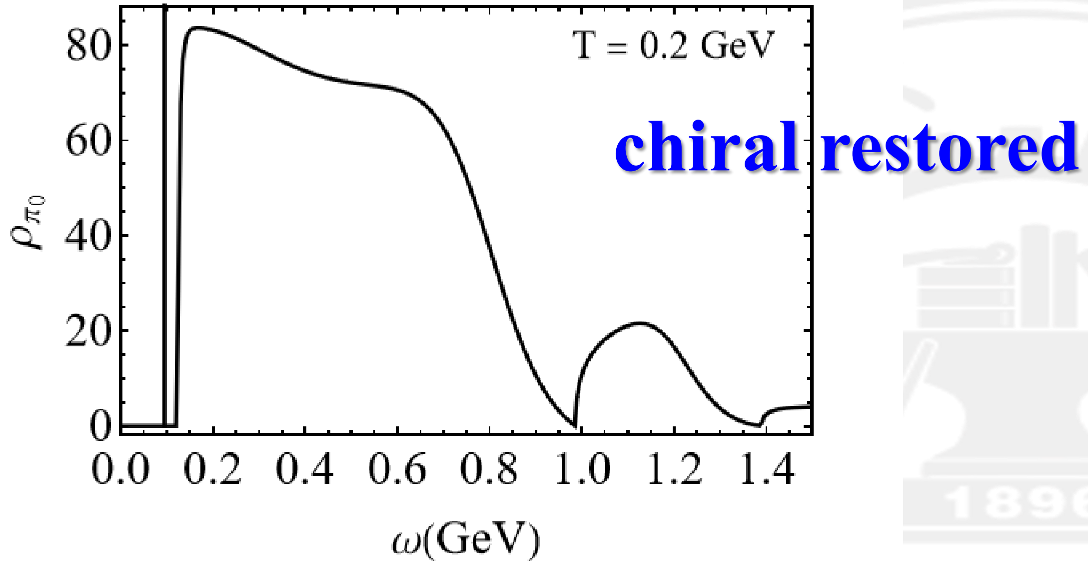
eB=0

finite eB (Landau levels)

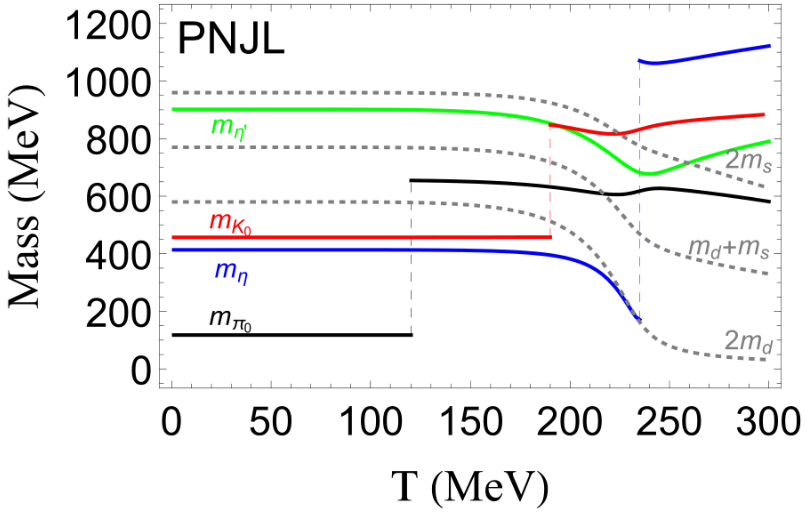


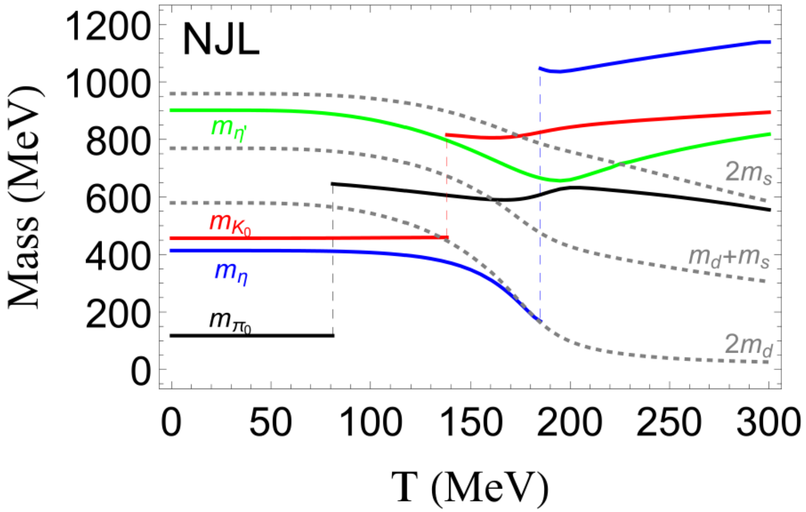


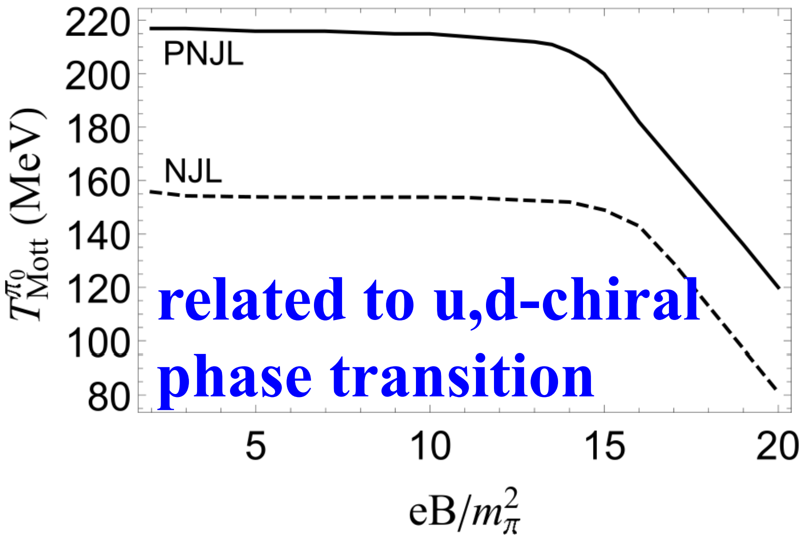


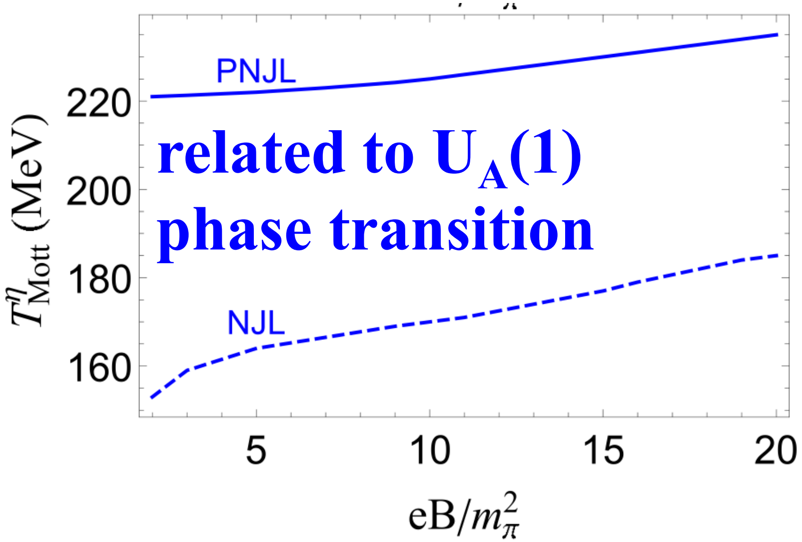


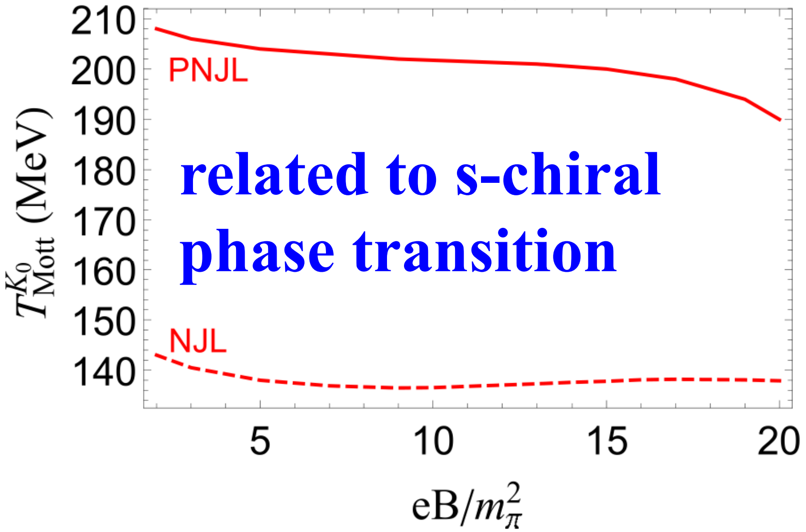
2.2.2 Mott transition of meson @ eB





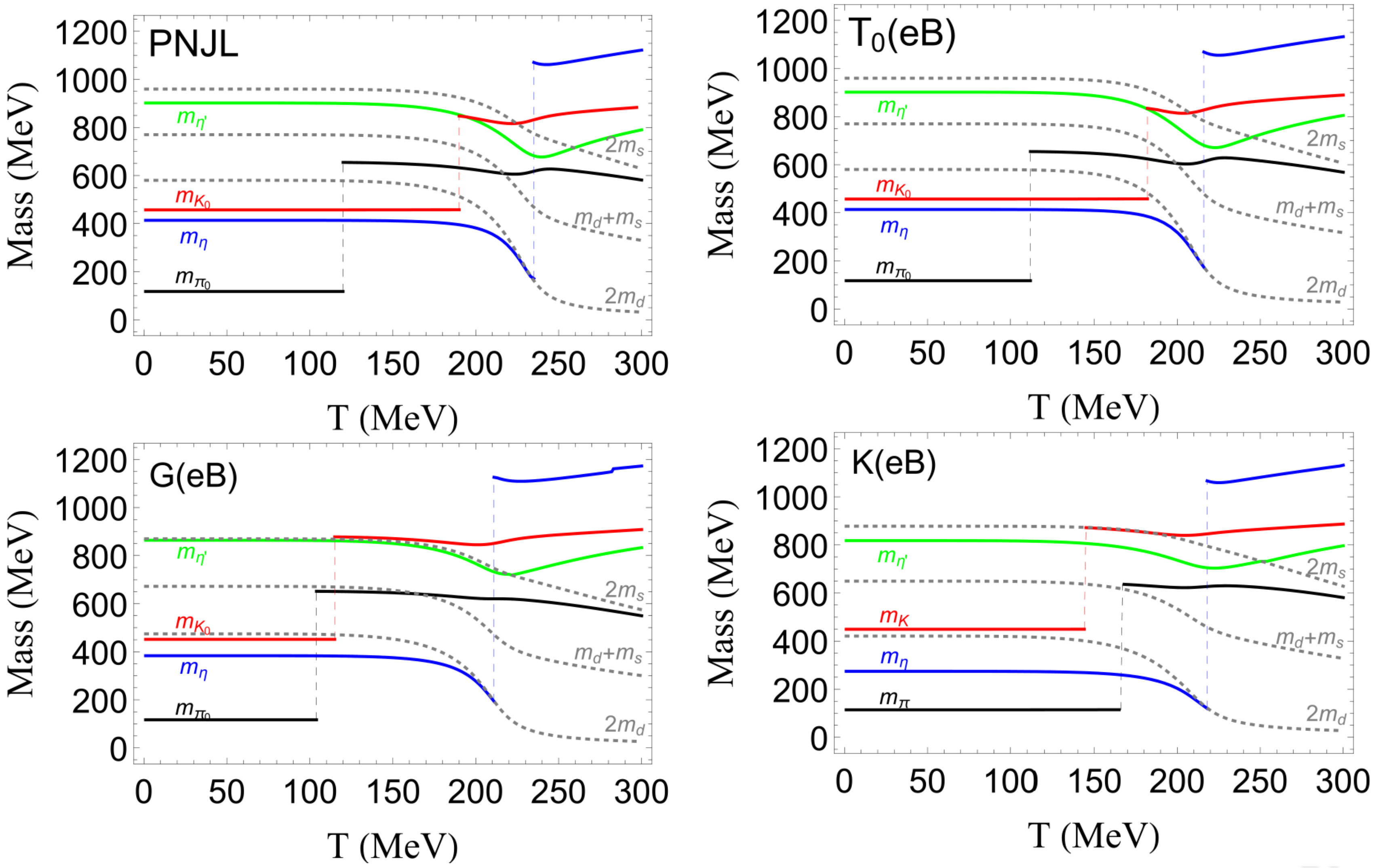


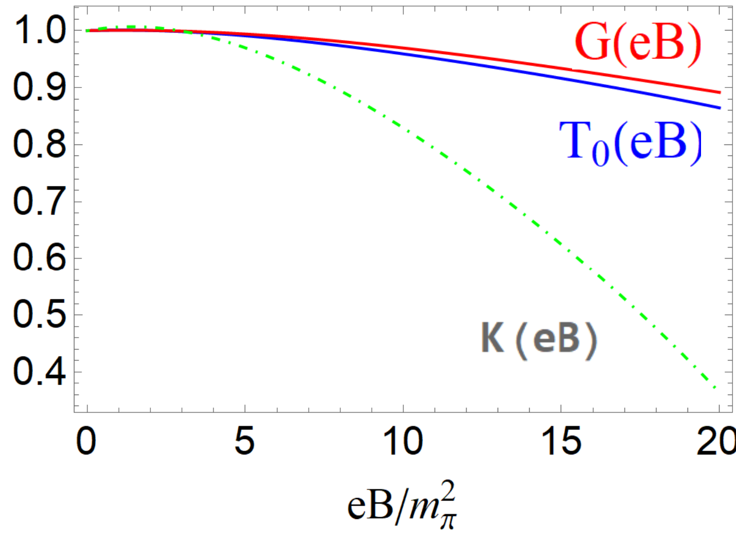




- **IMC leads to no qualitative difference**

2.2.2 Mass spectra of meson @ eB

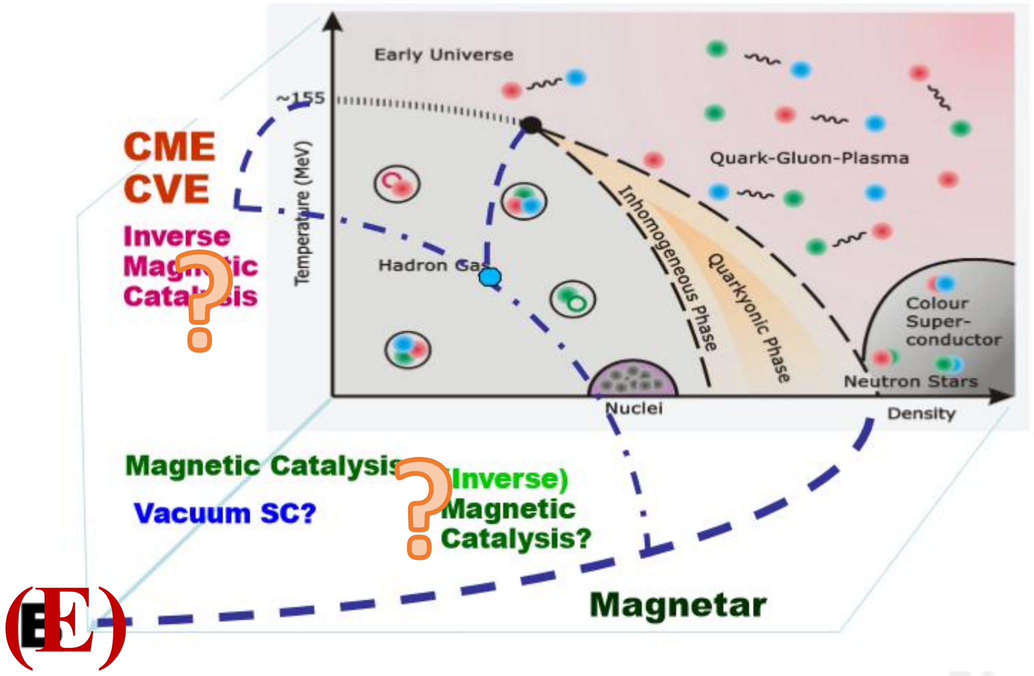




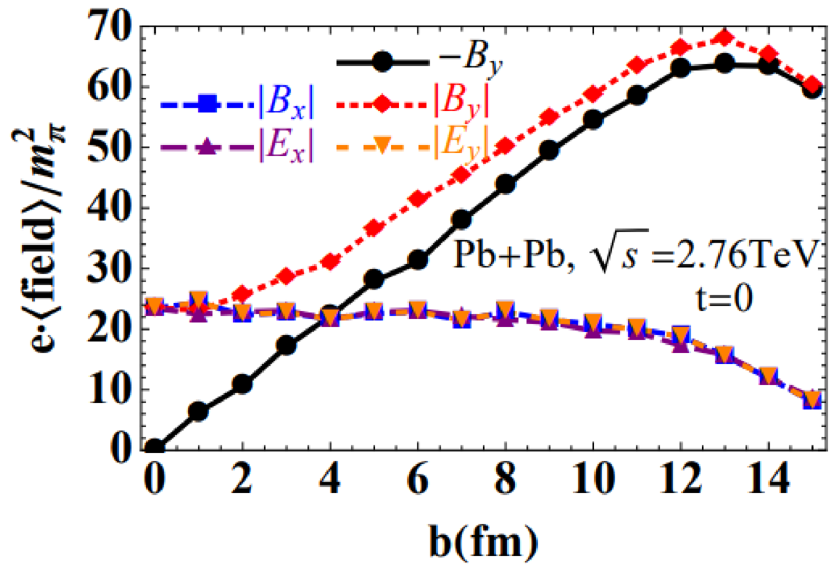
➢ **IMC leads to no qualitative difference**

S.J. Mao and Y.X. Wange, PRD96, 034004(2017); L.Y. Li and S.J. Mao, PRD108, 054001(2023); arXiv: 2025.XXXXX

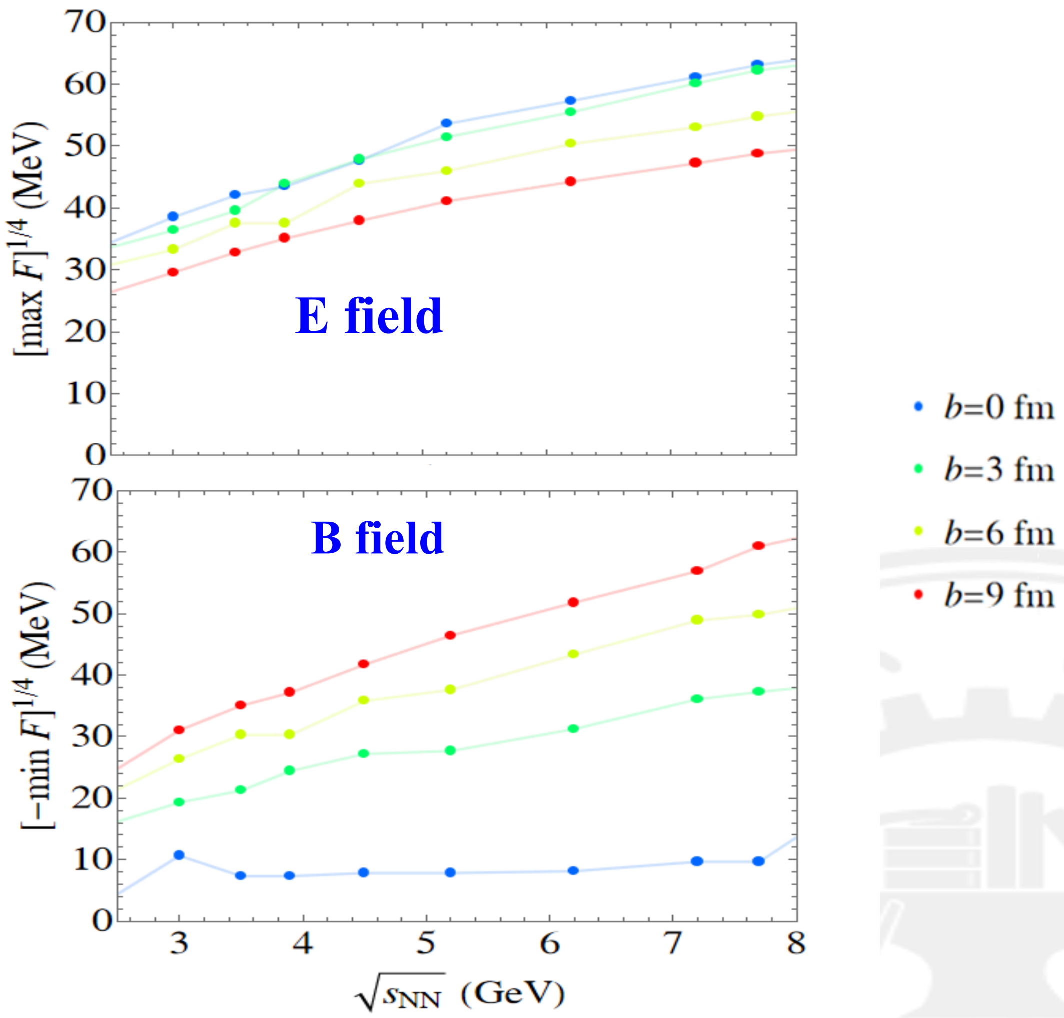
3 Electric Catalysis (EC)



3 E field needs more attention.



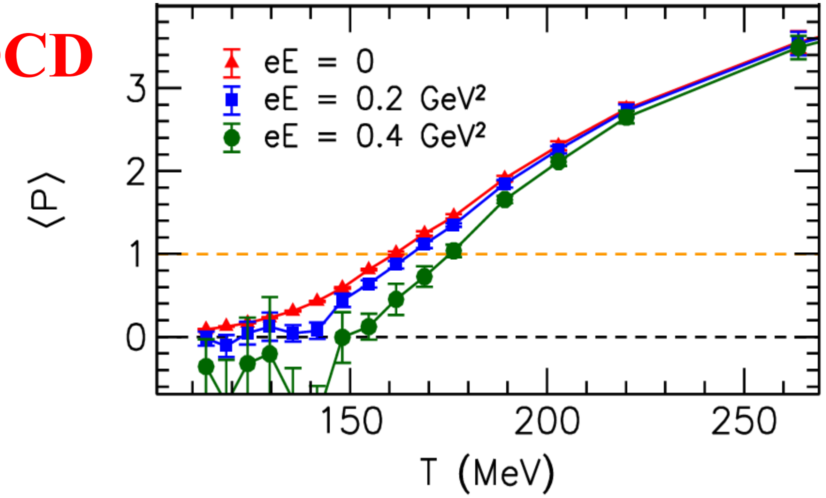
Deng&Huang, PRC85, 044907(2012)

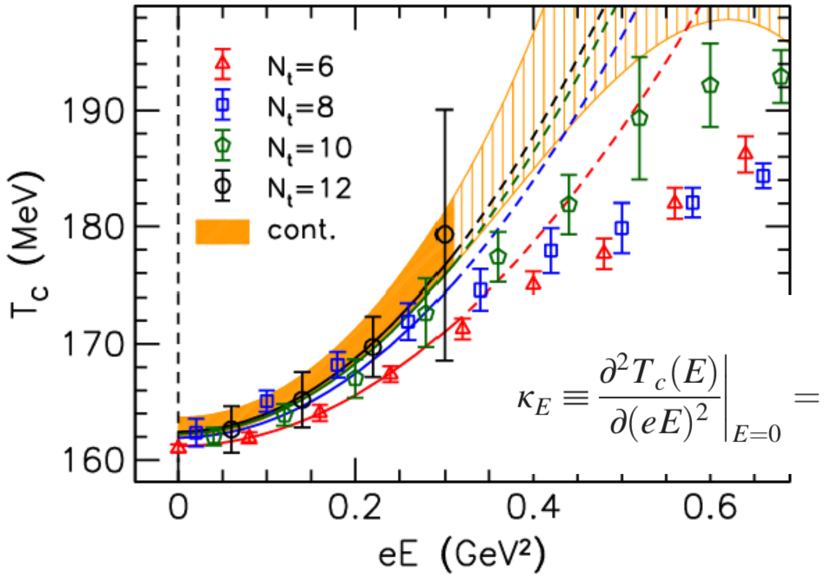


Taya,et.al,2402.17136; 2409.07685

3.1 Electric Catalysis (EC)

LQCD



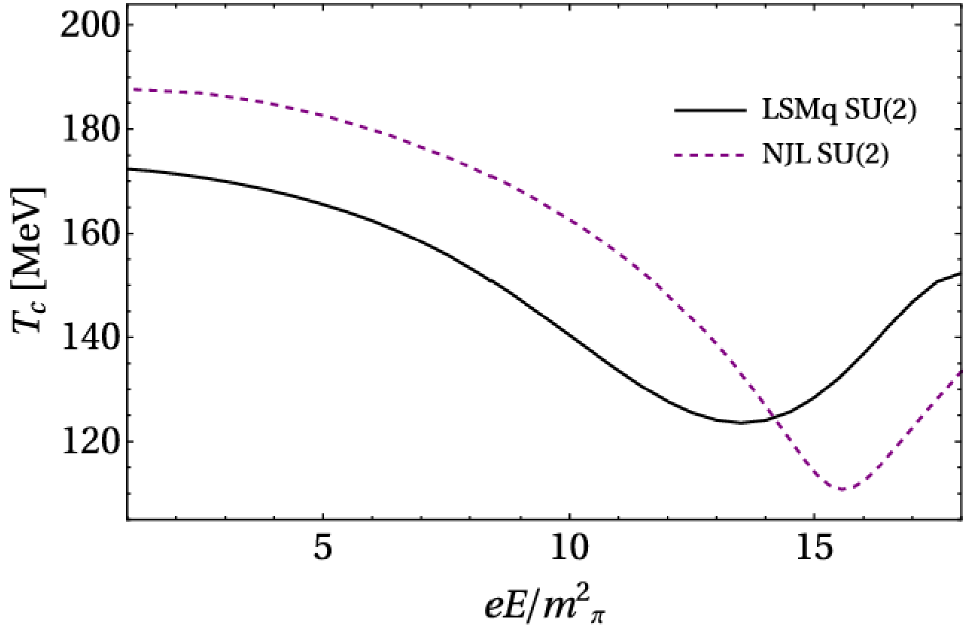


$$\kappa_E \equiv \left.\frac{\partial^2 T_c(E)}{\partial (eE)^2}\right|_{E=0} = 0.37(9)\ \mathrm{GeV}^{-3}$$

G.Endroedi and G. Marko, PRD109, 034506(2024).

Analytical results in mean field

???

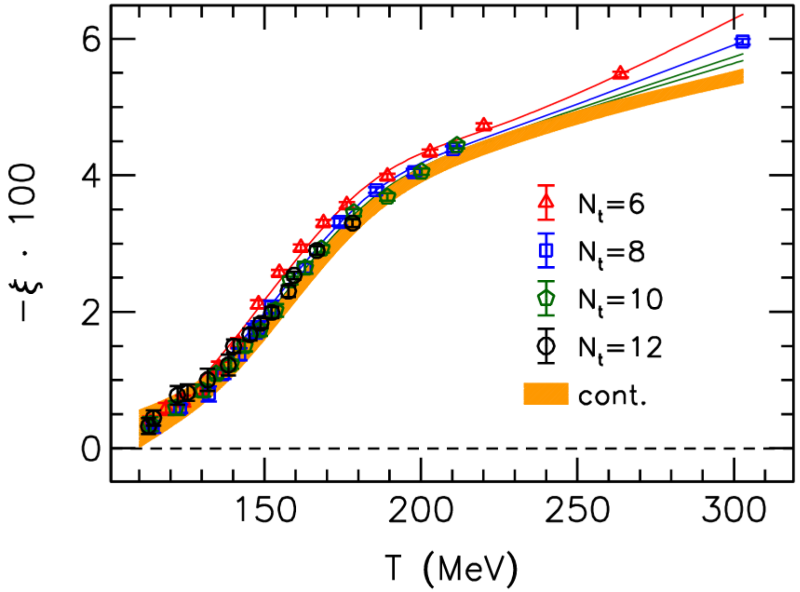


G.Q. Cao, X.G. Huang, PRD93, 016007(2016);
W.R. Tavares, et.al, PRD108, 016017(2023).

3.2 Electric susceptibility

LQCD

electric susceptibility



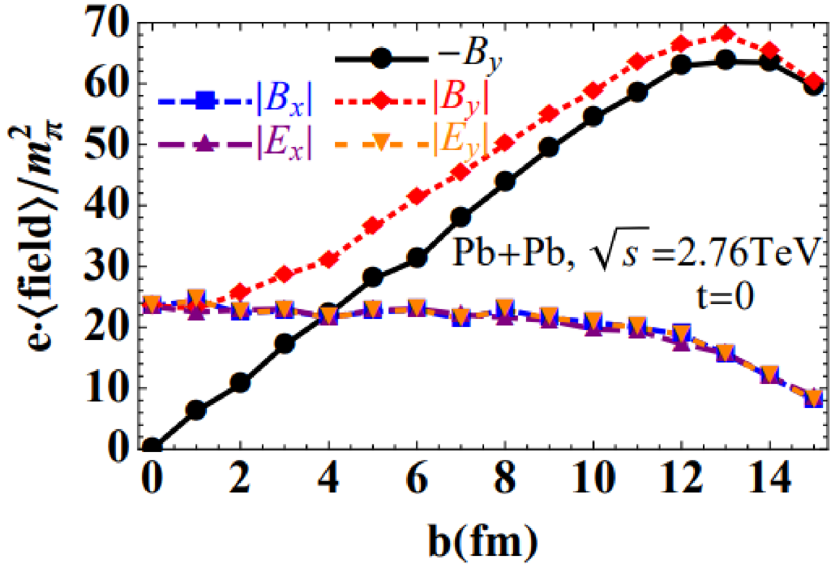
G.Endroedi, G. Marko, PRD109, 034506(2024)

Schwinger mechanism:
quark pair production???

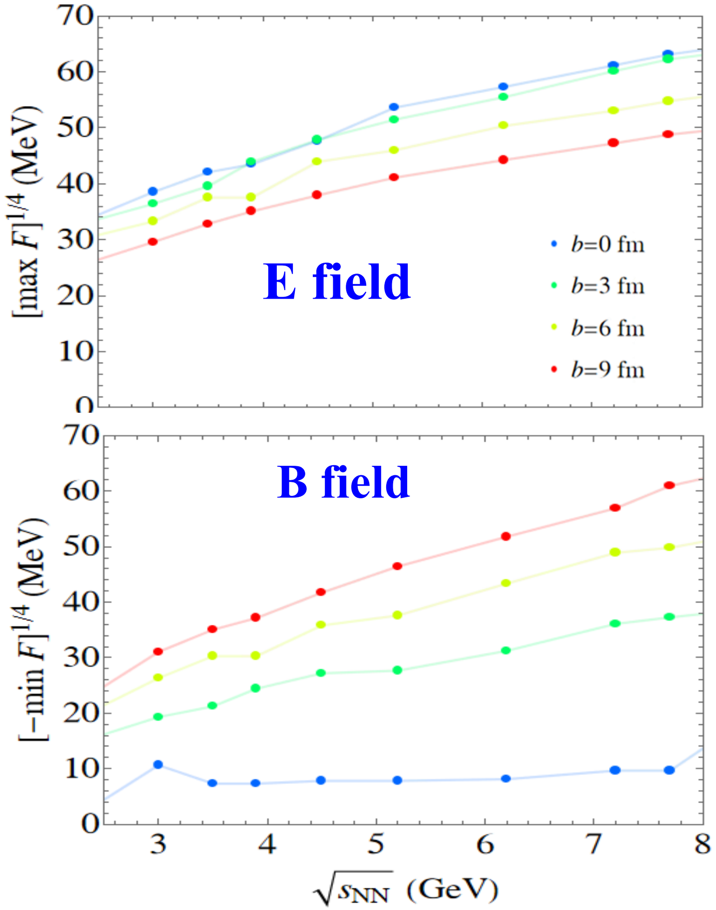
Electric field effect on QCD matter is under progress.

summary and outlook

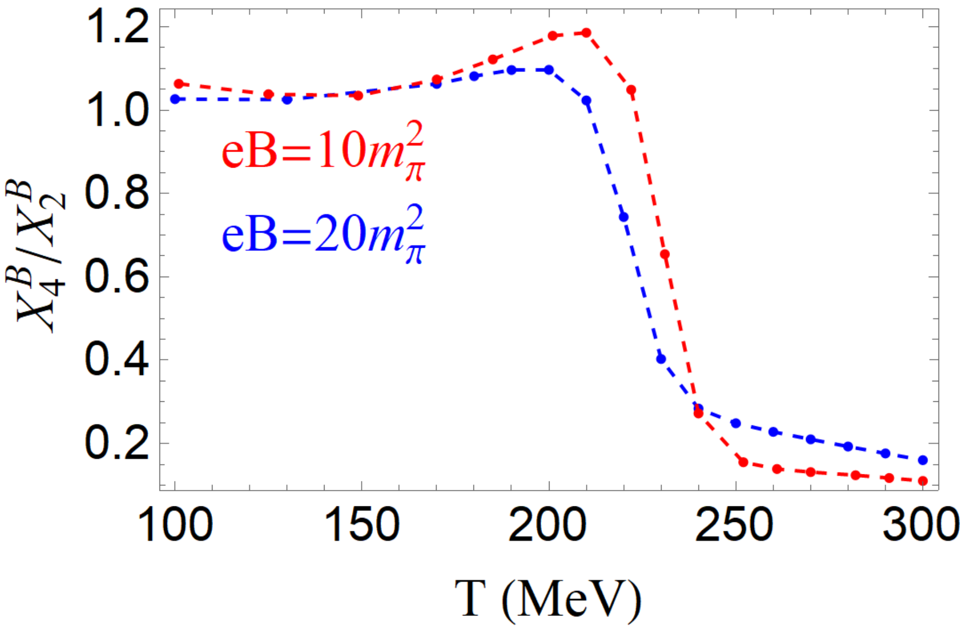
- ➢ **Not a single factor can explain IMC effect**
- ➢ $\chi_{i,j}^{B,Q,S}$ **and mesons do not relay on IMC effect too much.**
- ➢ **E field needs more attention.**



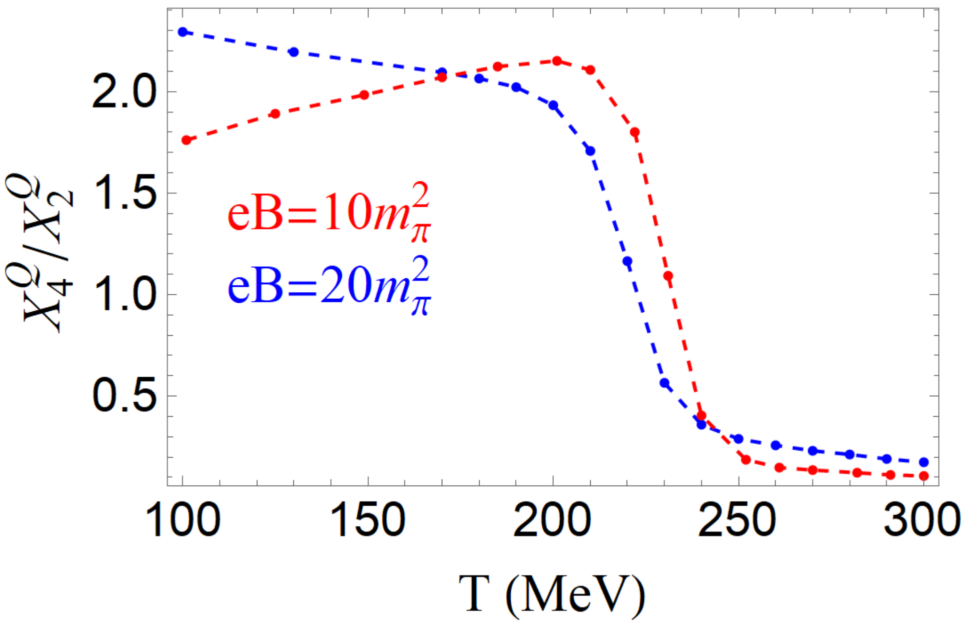
Deng&Huang, PRC85, 044907(2012)



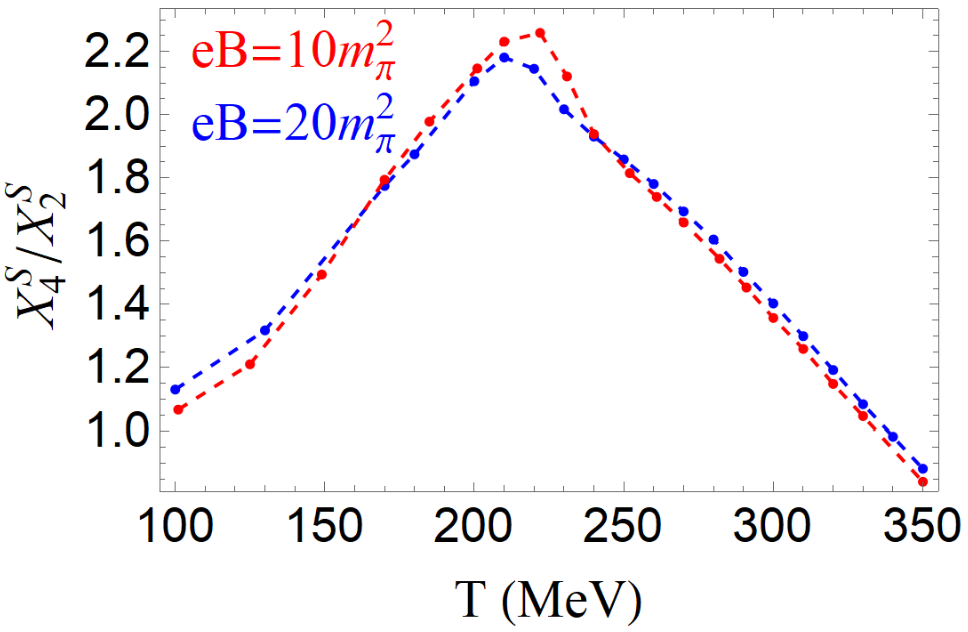
Taya,et.al,2402.17136; 2409.07685



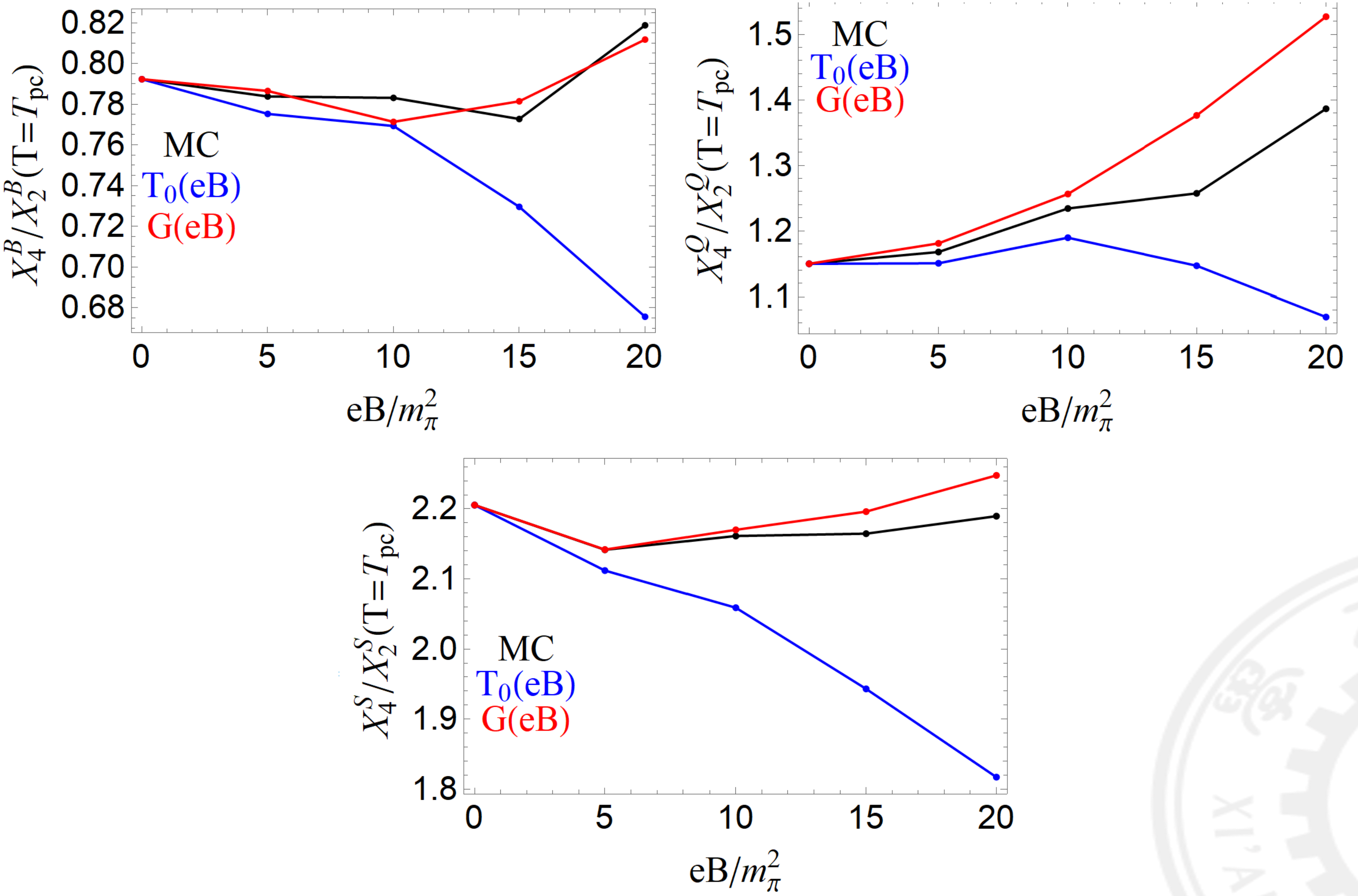
eB=10m_π^2
eB=20m_π^2
X_4^B/X_2^B
T (MeV)



eB=10m_π^2
eB=20m_π^2
X_4^Q/X_2^Q
T (MeV)



eB=10m_π^2
eB=20m_π^2
X_4^S/X_2^S
T (MeV)



$X_4^B/X_2^B(\mathrm{T}=T_{\mathrm{pc}})$
0.82
0.80
0.78
0.76
0.74
0.72
0.70
0.68
MC
$\mathrm{T_0(eB)}$
G(eB)
0
5
10
15
20
eB/m_π^2
$X_4^Q/X_2^Q(\mathrm{T}=T_{\mathrm{pc}})$
1.5
1.4
1.3
1.2
1.1
MC
$\mathrm{T_0(eB)}$
G(eB)
0
5
10
15
20
eB/m_π^2
$X_4^S/X_2^S(\mathrm{T}=T_{\mathrm{pc}})$
2.2
2.1
2.0
1.9
1.8
MC
$\mathrm{T_0(eB)}$
G(eB)
0
5
10
15
20
eB/m_π^2