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Transport properties and phase diagram of nuclear matter

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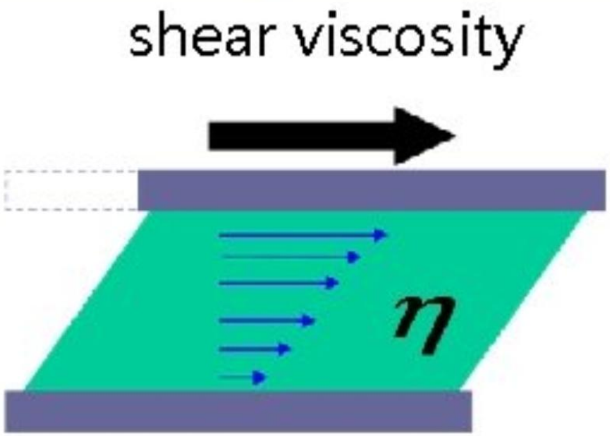
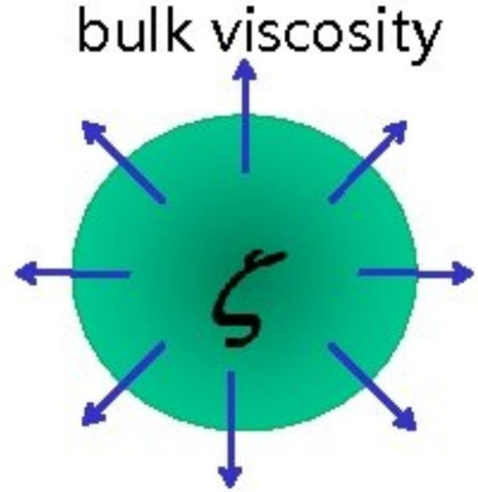
Collaborators：Lei-Ming Hua

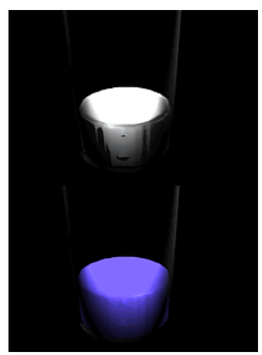
Transport properties of asymmetric nuclear matter in the spinodal region
L.M. Hua and J. Xu*, Phys. Rev. C 109, 034614 (2024)
Shear viscosity of nuclear matter in the spinodal region
L.M. Hua and J. Xu*, Phys. Rev. C 107, 034601 (2023)

Shear and bulk viscosity



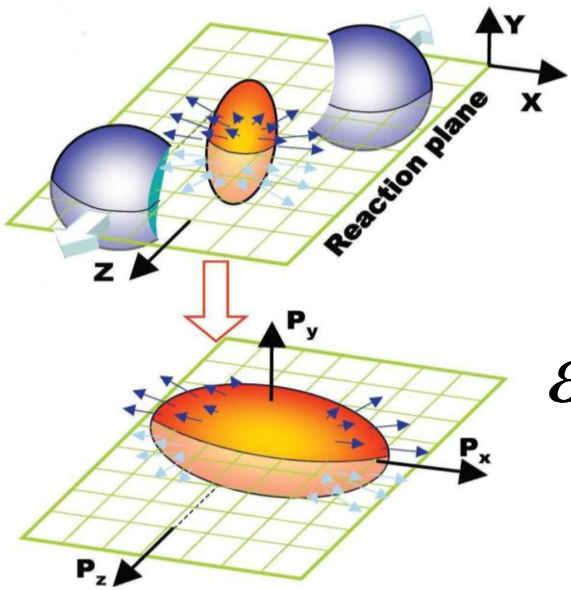


$$\tau = \frac{F}{A} = \eta \frac{\partial u}{\partial y}$$



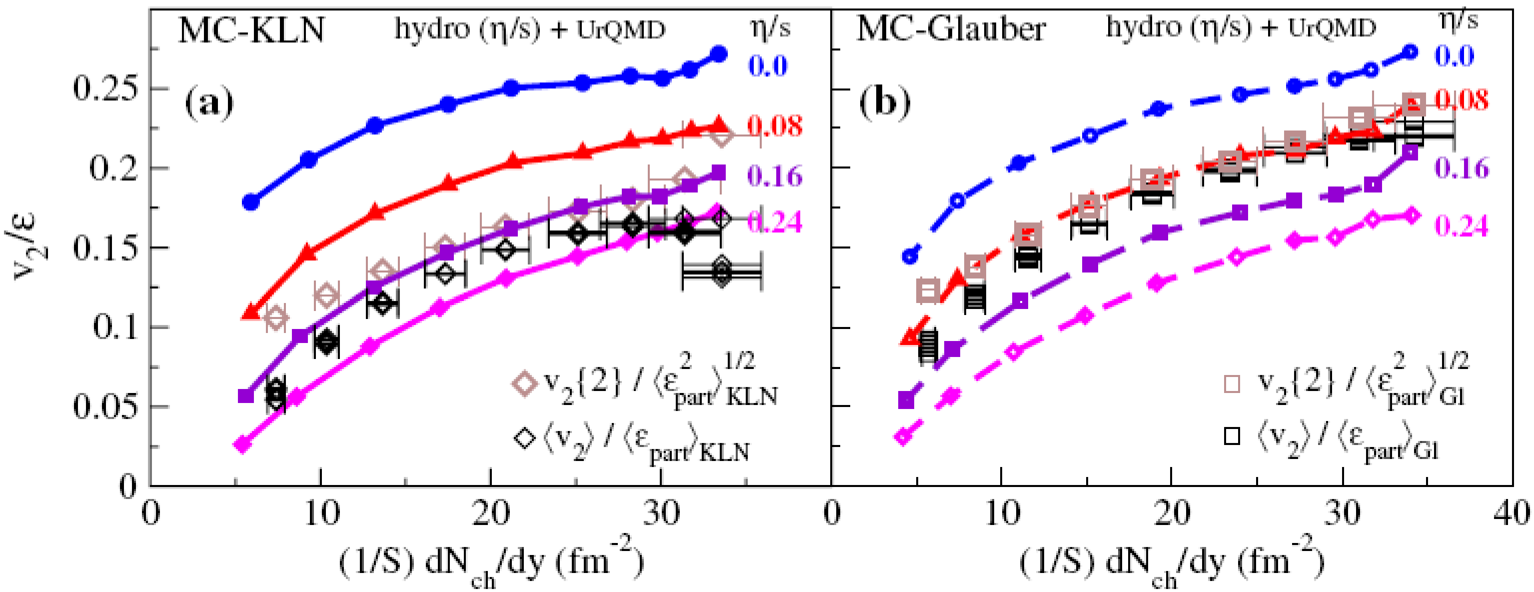
$$P - P_0 = \zeta \nabla \cdot \vec{u}$$

sQGP: a nearly ideal fluid



Eccentricity **Elliptic flow**

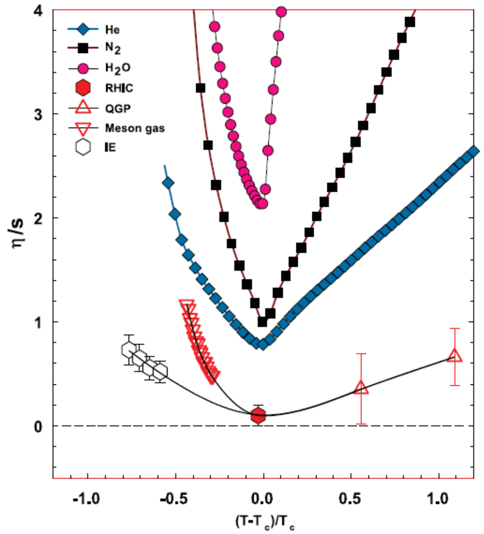
$$\varepsilon_2 = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle} \xrightarrow{\eta/s} v_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$



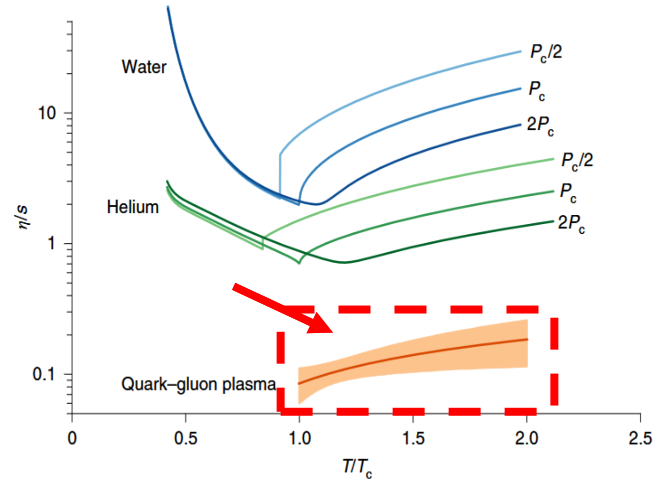
$$1 < 4\pi(\eta/s)_{QGP} < 2.5$$

Song et al., PRL (2011)

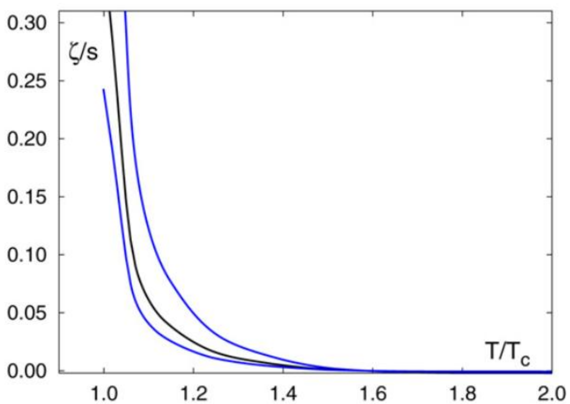
Viscosity and phase transition



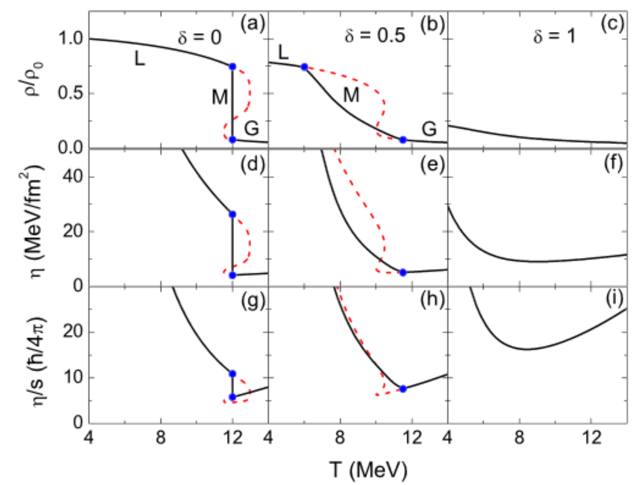
Lacey et al., PRL (2007)



Bernhard, Moreland, Bass, Nature Physics (2019)



Karsch, Kharzeev, Tuchin, PLB (2008)



Xu, Chen, Ko, Li, Ma, PLB (2013)

Viscosity calculation methods

- **Classical method** $\eta \sim 1/\sigma$
- **Relaxation time approximation**
- **Champman-Enskog**
- **Green-Kubo (GK) method**

Plumari, Puglisi, F. Scardina, V. Greco, PRC (2012)

$$\eta = \frac{1}{T}\int d^3r \int_{t_0}^{\infty} dt \langle \pi^{xy}(\vec{0}, t_0)\pi^{xy}(\vec{r}, t)\rangle_{\text{equil}} \qquad \pi^{xy} = \frac{1}{V_c}\sum_i \frac{p_i^x p_i^y}{E_i}$$

$$\zeta = \frac{1}{T}\int d^3r \int_{t_0}^{\infty} dt \langle \Delta\pi(\vec{0}, t_0)\Delta\pi(\vec{r}, t)\rangle_{\text{equil}} \qquad \Delta\pi = \pi - \pi_{\text{eq}}$$

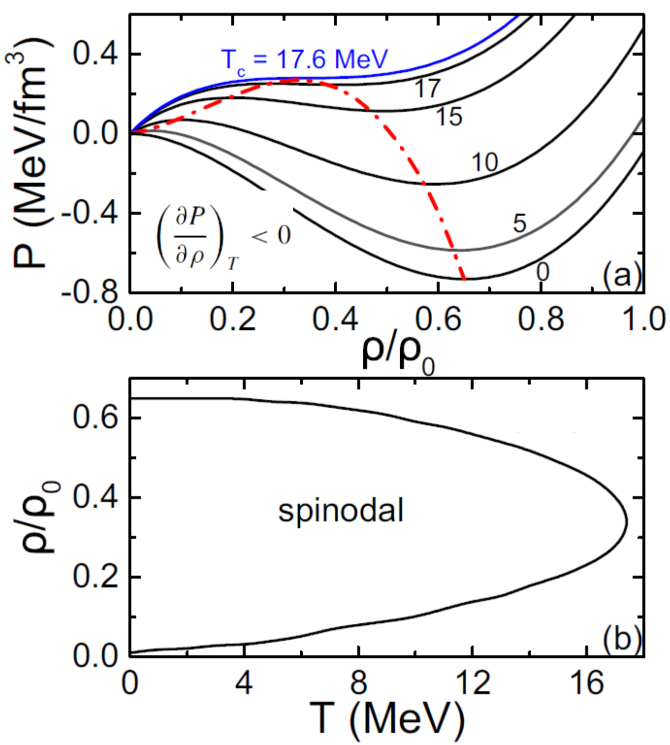
$$\pi = \frac{1}{3}(\pi^{xx} + \pi^{yy} + \pi^{zz})$$

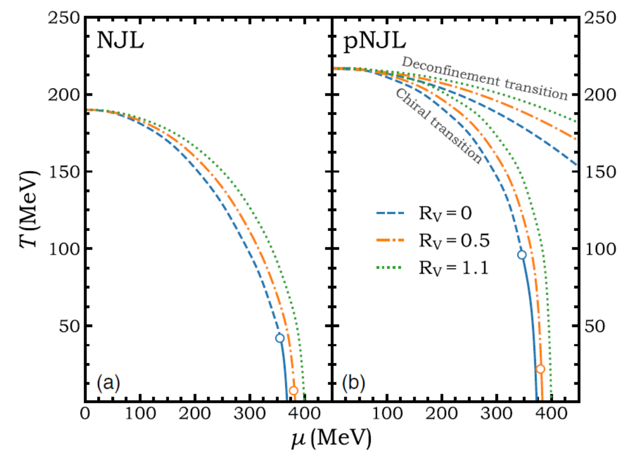
Symmetric nuclear matter (SNM) phase diagram

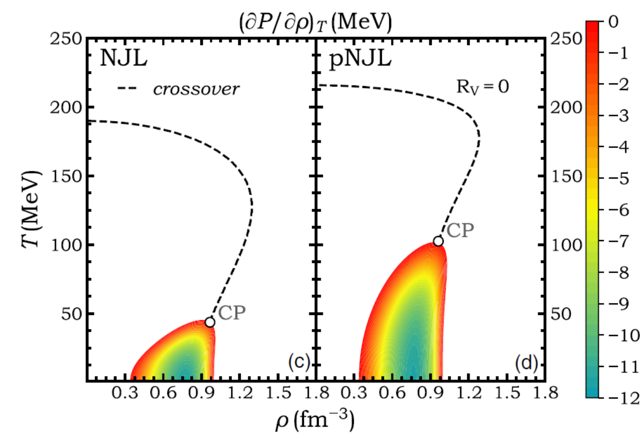
$$U(\rho) = \alpha\left(\frac{\rho}{\rho_0}\right) + \beta\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

$$P = Ts - \epsilon + \mu\rho$$

compared with Nambu--Jona-Lasinio model

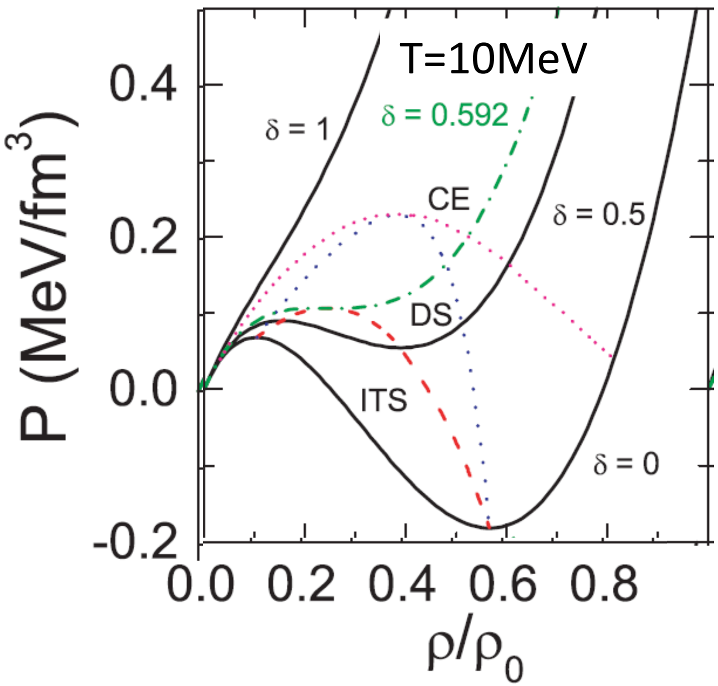


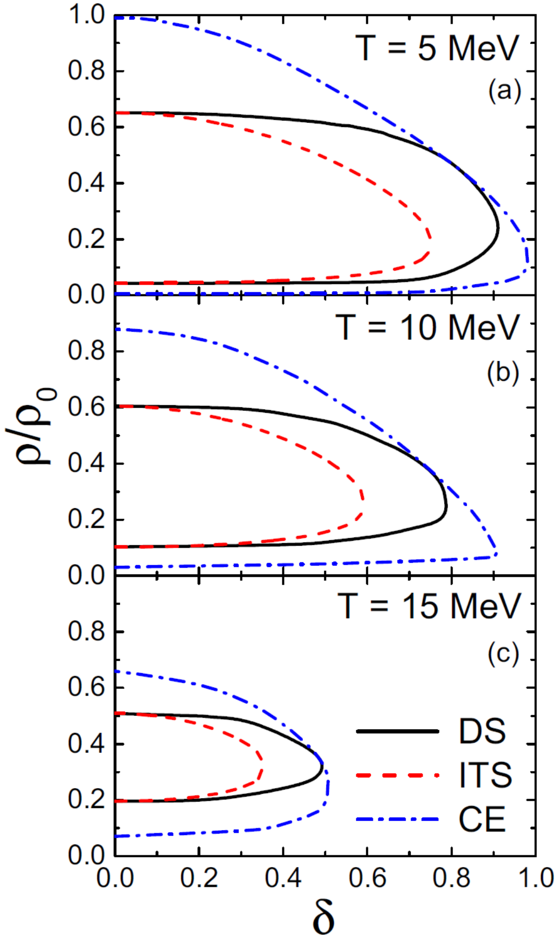




Asymmetric nuclear matter (ANM) phase diagram

$$U_{n,p}(\rho,\delta) = \alpha\left(\frac{\rho}{\rho_0}\right) + \beta\left(\frac{\rho}{\rho_0}\right)^{\gamma} \pm 2E_{\rm sym}^{\rm pot}\left(\frac{\rho}{\rho_0}\right)^{\gamma_{\rm sym}}\delta \qquad \delta = (\rho_n - \rho_p)/\rho$$





isothermal spinodal (ITS): $(\partial P/\partial\rho)_{T,\delta} < 0$

diffusive spinodal (DS): $(\partial\mu_n/\partial\delta)_{P,T} < 0$ or $(\partial\mu_p/\partial\delta)_{P,T} > 0$

phase coexistence (CE): $(\mu_n,\mu_p,P,T)^L = (\mu_n,\mu_p,P,T)^G$

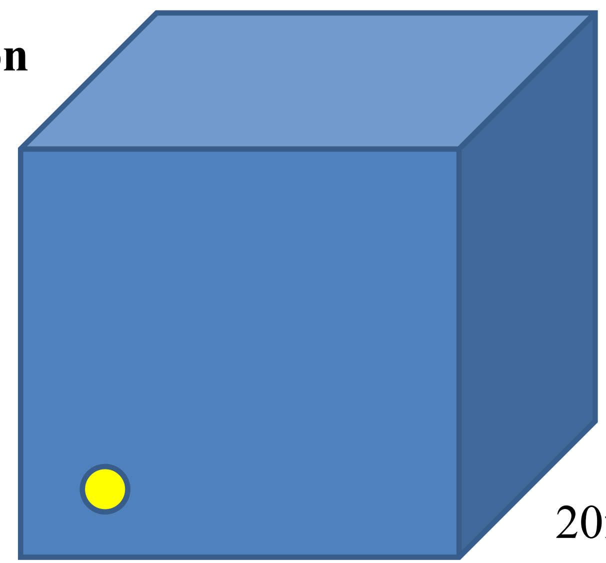
Box calculation of IBUU

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\cdot\nabla_r-\nabla_r U\cdot\nabla_p\right)f(\vec{r},\vec{p};t)=\frac{1}{(2\pi\hbar)^6}\int d^3p_2d^3p_3d\Omega\, v_{rel}\frac{d\sigma_{12}}{d\Omega}(2\pi\hbar)^3\delta(\vec{p}+\vec{p}_2-\vec{p}_3-\vec{p}_4)$$
$$\times[f_3f_4(1-f)(1-f_2)-ff_2(1-f_3)(1-f_4)].$$

equations of motion

$$\frac{d\vec{r}_i}{dt}=\frac{\vec{p}_i}{m}$$
$$\frac{d\vec{p}}{dt}=-\nabla U$$

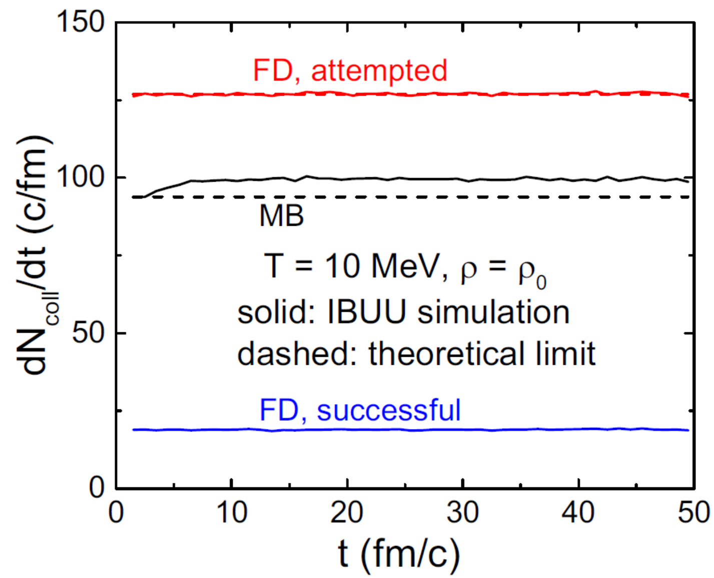
box system with periodic boundary condition



20fm

20fm

isotropic and constant $\sigma = 40\ mb$



theoretical limit

$$\frac{dN_{coll}}{dt}=\frac{1}{2}V\rho^2\sigma\int d^3p_1d^3p_2v_{mol}\tilde{f}(p_1)\tilde{f}(p_2)$$

Nucleon-nucleon collisions

- **Bertsch's approach (Phys. Rep. 160, 189 (1988))**
  – Go into the C.M. frame of the two nucleons $\delta t = \alpha \Delta t$
  – Collision can happen if $\alpha \approx 1/\gamma$

$$b = \sqrt{(\Delta \mathbf{r})^2 - (\Delta \mathbf{r}\cdot \mathbf{p}/p)^2} < \sqrt{\sigma/\pi} \quad \text{and} \quad \left|\frac{\Delta \mathbf{r}\cdot \mathbf{p}}{p}\right| < \left(\frac{p}{\sqrt{p^2+m_1^2}} + \frac{p}{\sqrt{p^2+m_2^2}}\right)\boxed{\delta t}/2$$

  – If collision happen, change the direction of P_{cm} in the C.M. frame
  – Boost the momenta of the two nucleons to lab frame
  – Check phase space density; if Pauli blocked, return to the initial momenta
- **Pauli blocking probability** $1-(1-n_i)(1-n_j)$

Occupation probability

$$n_i = \frac{(2\pi\hbar)^3}{g_N V_r V_p}\int_{i\in V_r, V_p} f(\vec{r},\vec{p})\, d^3r\, d^3p$$

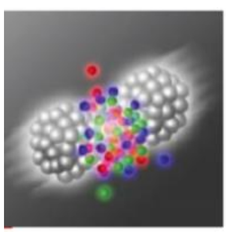
Transport Model Evaluation Project (TMEP)

Transport simulation
- initialization
- collision
  - elastic
  - inelastic
  - Pauli blocking
- mean-field

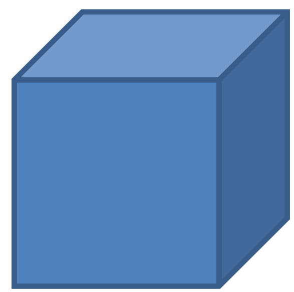
14 Boltzmann-Uehling-Uhlenbeck
12 Quantum Molecular Dynamics

Model dependence

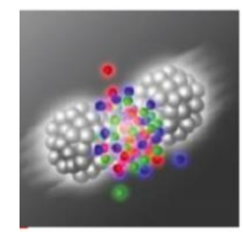
HIC simulation



box simulation
(theoretical limit)



HIC simulation



JX et al.,
PRC (2016)

initialization;
theoretical error
of directed flow

Y.X. Zhang et al., PRC (2018)
elastic collisions, Pauli blocking

A. Ono, JX, et al., PRC (2019)
inelastic collisions, π production

M. Colonna et al., PRC (2021)
mean-field evolution

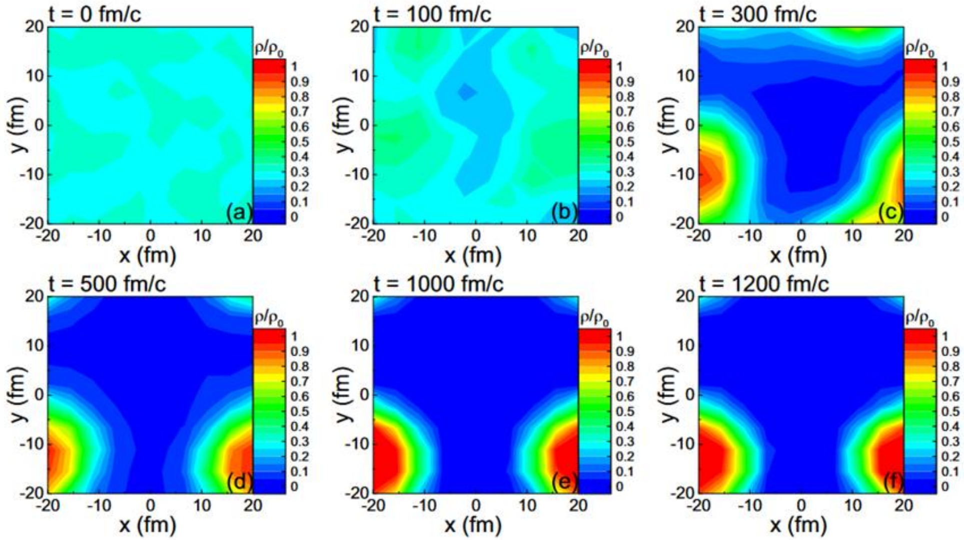
JX et al.,
PRC (2024)

reduce uncertainty of
π^-/π^+ from 5.2%
to 1.6%

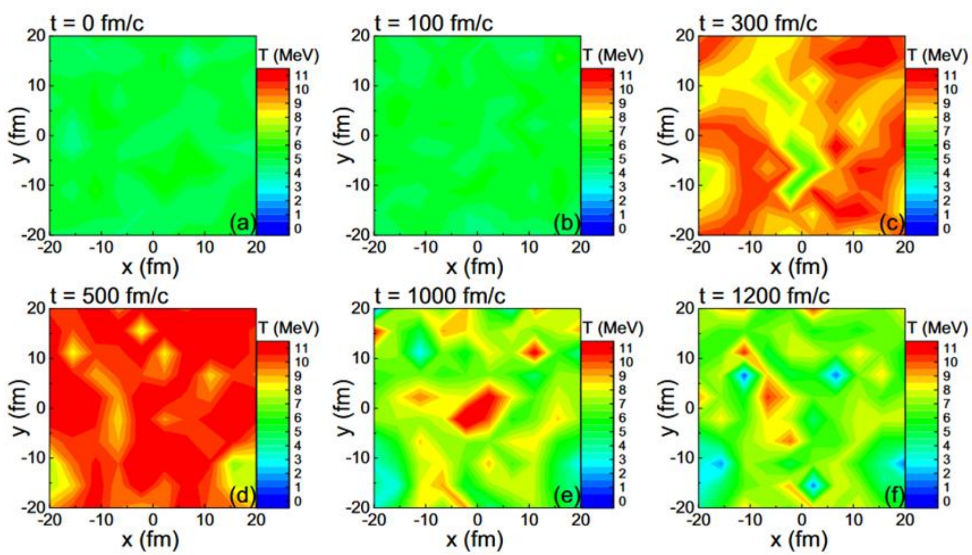
main uncertainties:
mean-field, Pauli blocking

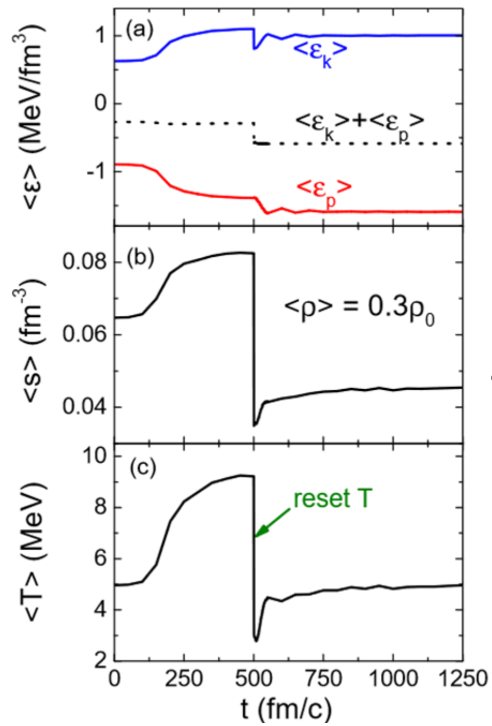
Preparation of dynamically equilibrated SNM system

density evolution



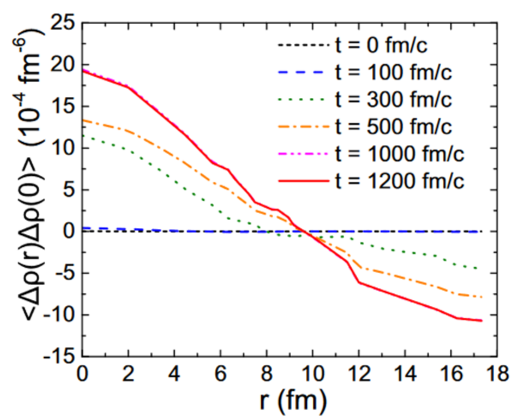
temperature evolution





$<\rho>=0.3\rho_0$

$<T>=5\text{MeV}$



Validity of GK method in non-uniform system

$$\eta = \frac{1}{T}\int d^3r \int_{t_0}^{\infty} dt\,\langle \pi^{xy}(\vec{0}, t_0)\pi^{xy}(\vec{r}, t)\rangle_{\text{equil}} \quad \Rightarrow \quad \eta = \frac{V}{T}\int_{t_0}^{\infty} dt\,\langle \Pi^{xy}(t_0)\Pi^{xy}(t)\rangle_{\text{equil}}$$

$$\pi^{xy}(\vec{0}, t_0) = \frac{1}{N_c}\sum_{c_1}\left(\sum_{i_{c_1}}\frac{p^x_{i_{c_1}}p^y_{i_{c_1}}}{E_{i_{c_1}}}\right)_{t_0}$$

$$\Pi^{xy}(t_0)\Pi^{xy}(t)$$

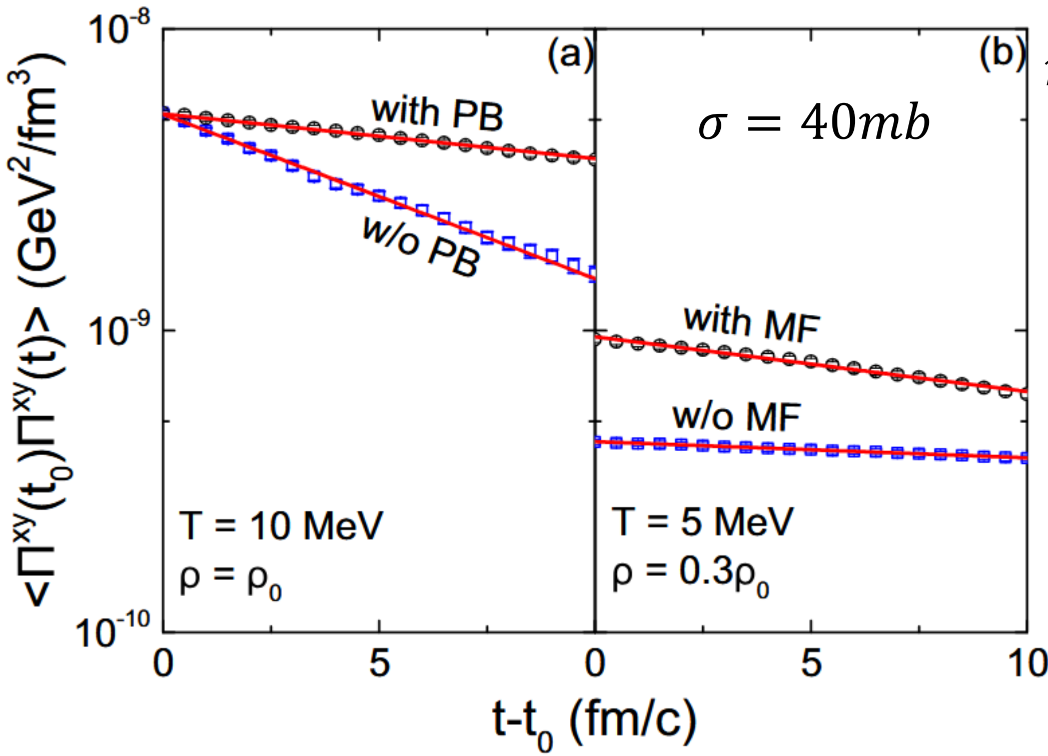
$$= \frac{1}{V}\left(\sum_i \frac{p^x_i p^y_i}{E_i}\right)_{t_0}\frac{1}{V}\left(\sum_j \frac{p^x_j p^y_j}{E_j}\right)_t$$

$$= \frac{1}{(N_cV_c)^2}\left(\sum_{c_1}\sum_{i_{c_1}}\frac{p^x_{i_{c_1}}p^y_{i_{c_1}}}{E_{i_{c_1}}}\right)_{t_0}\left(\sum_{c_2}\sum_{j_{c_2}}\frac{p^x_{j_{c_2}}p^y_{j_{c_2}}}{E_{j_{c_2}}}\right)_t$$

$$= \frac{1}{(N_cV_c)^2}\sum_{c_1}\sum_{c_2}\left(\sum_{i_{c_1}}\frac{p^x_{i_{c_1}}p^y_{i_{c_1}}}{E_{i_{c_1}}}\right)_{t_0}\left(\sum_{j_{c_2}}\frac{p^x_{j_{c_2}}p^y_{j_{c_2}}}{E_{j_{c_2}}}\right)_t$$

choosing different cell of r = 0 is identical to choosing different starting time t_0 or parallel events once the system has reached dynamic equilibrium

Shear viscosity from GK method



$$\eta = \frac{V}{T}\int_{t_0}^{\infty} dt < \Pi^{xy}(t_0)\Pi^{xy}(t) >_{equil}$$

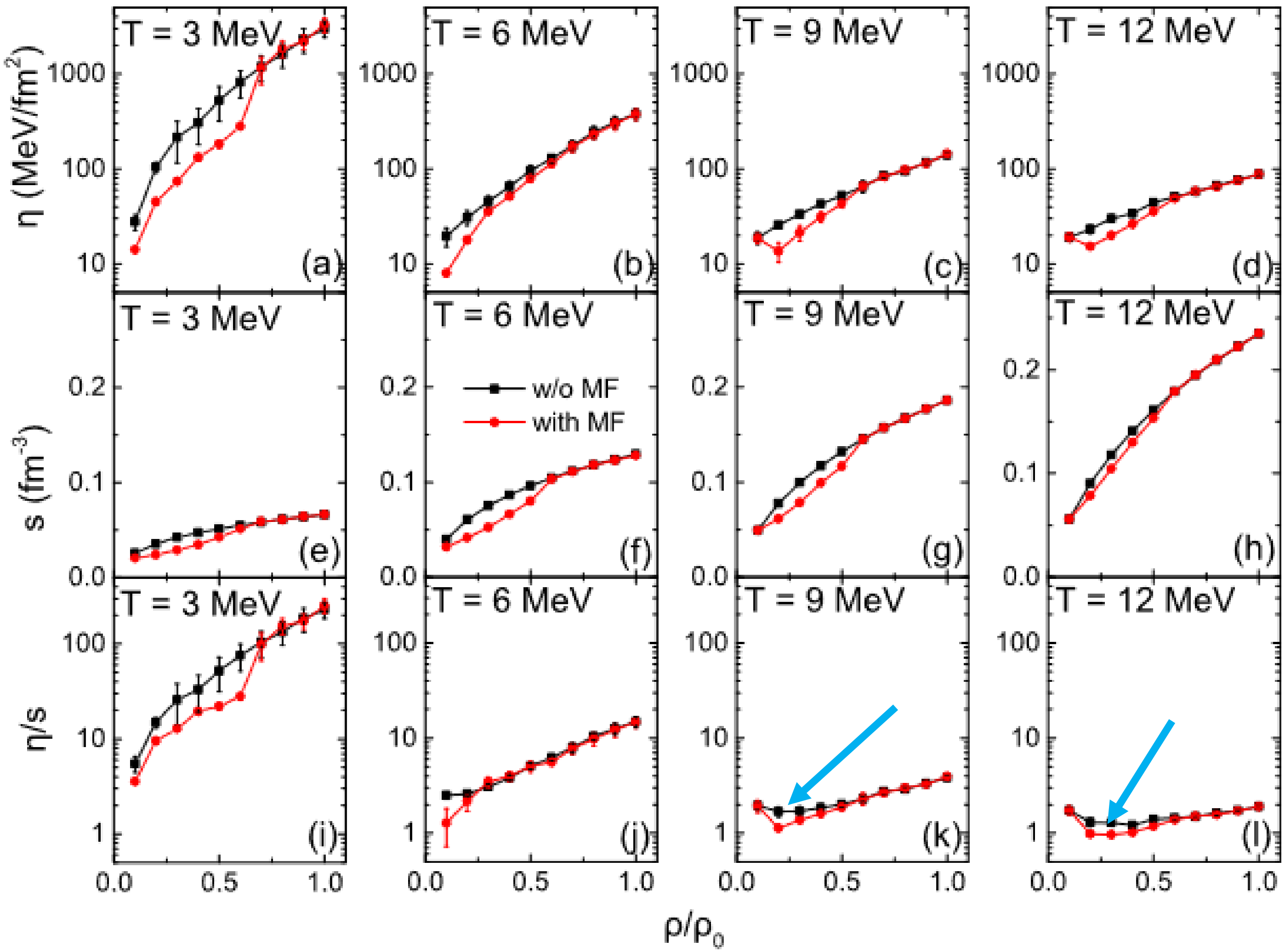
$$\pi^{xy} = \frac{1}{V_c}\sum_i \frac{p_i^x p_i^y}{E_i}$$

$$\langle \Pi^{xy}(t_0)\Pi^{xy}(t)\rangle = Ae^{-B(t-t_0)}$$

$$\eta = \frac{AV}{BT}$$

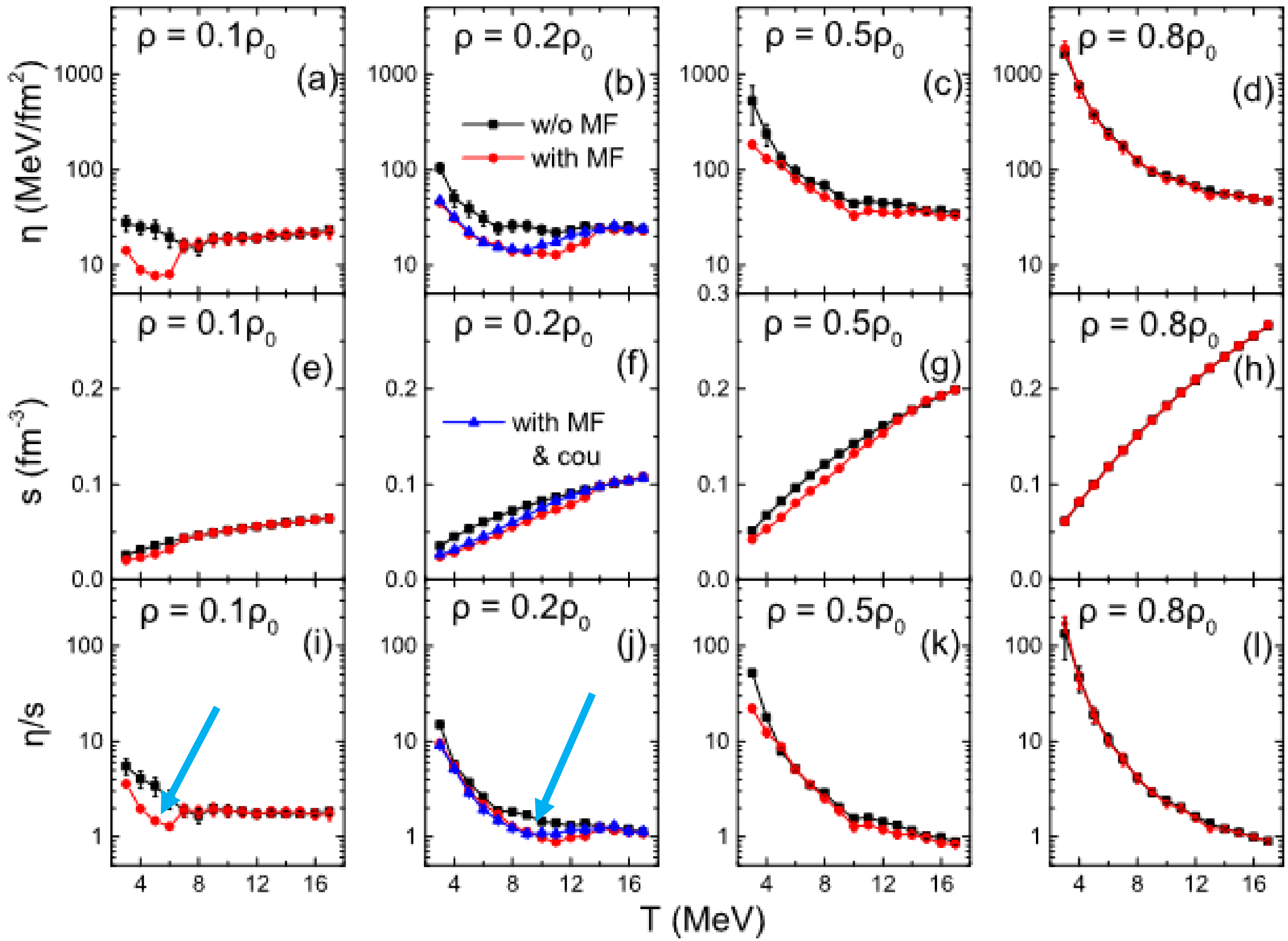
with MF (nonuniform): stronger initial correlations, more collisions

Results of shear viscosity I



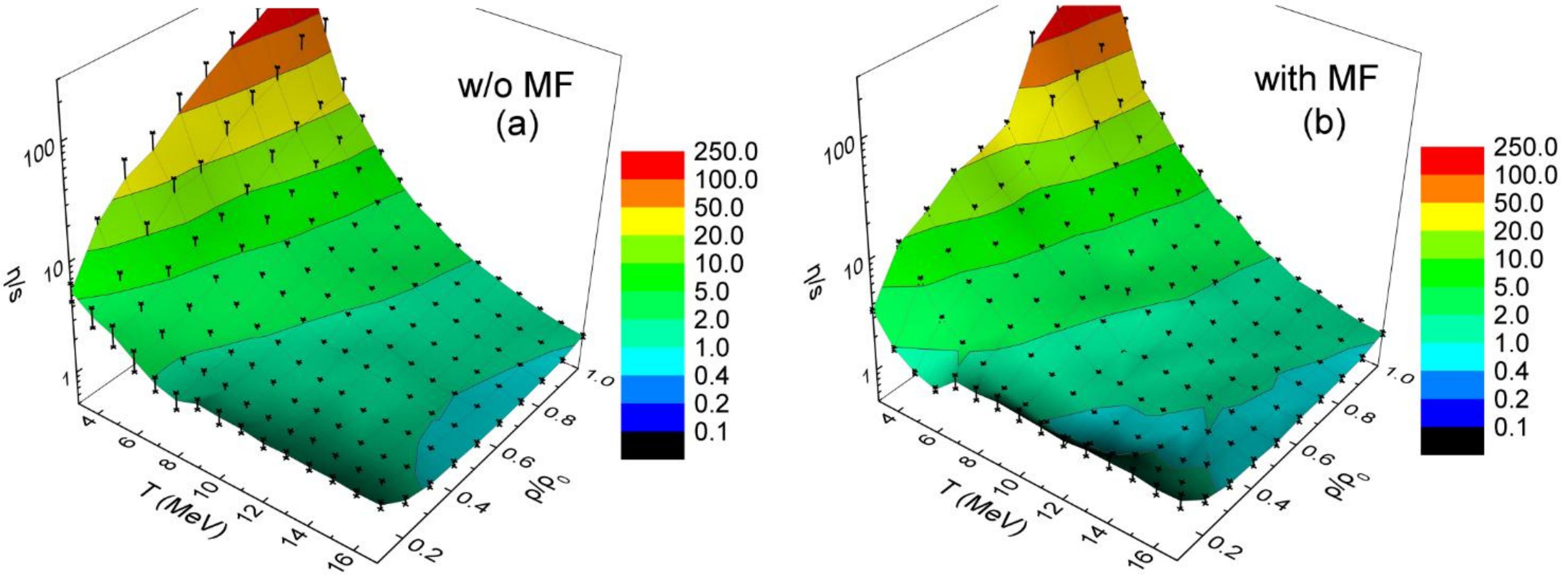
For a constant σ , clustering reduces η due to more collisions

Results of shear viscosity II

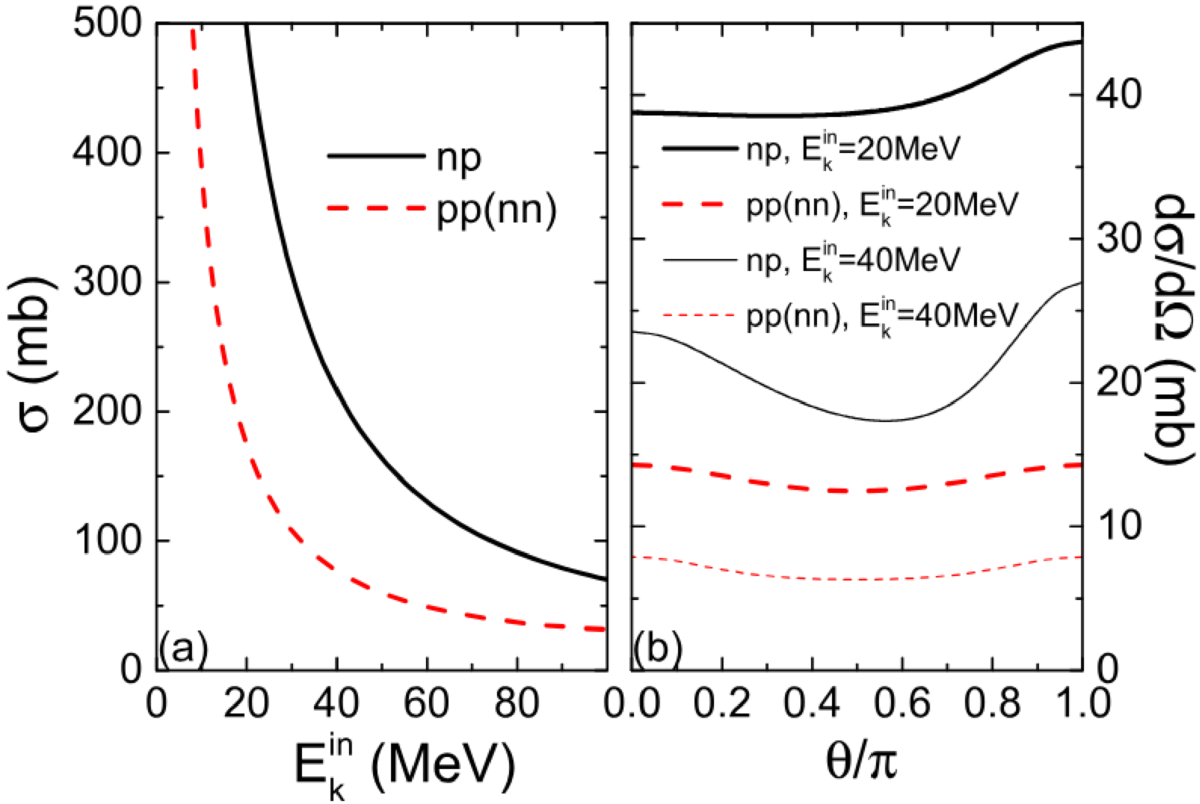


For a constant σ , clustering leads to a minimum η/s in dilute NM

Results of shear viscosity III



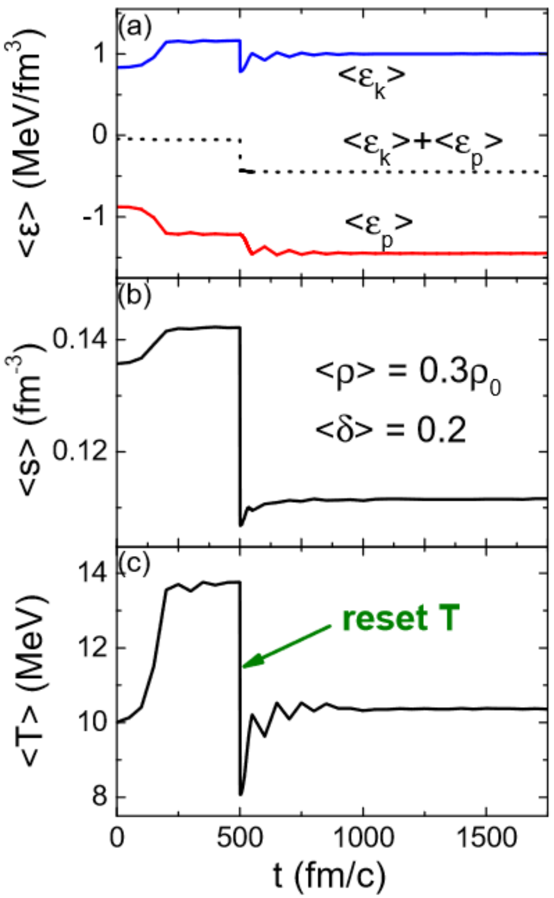
Energy- and angular-dependent σ



from proton-proton and proton-neutron phase-shift data

Arndt, Hackman, Roper, PRC (1977)

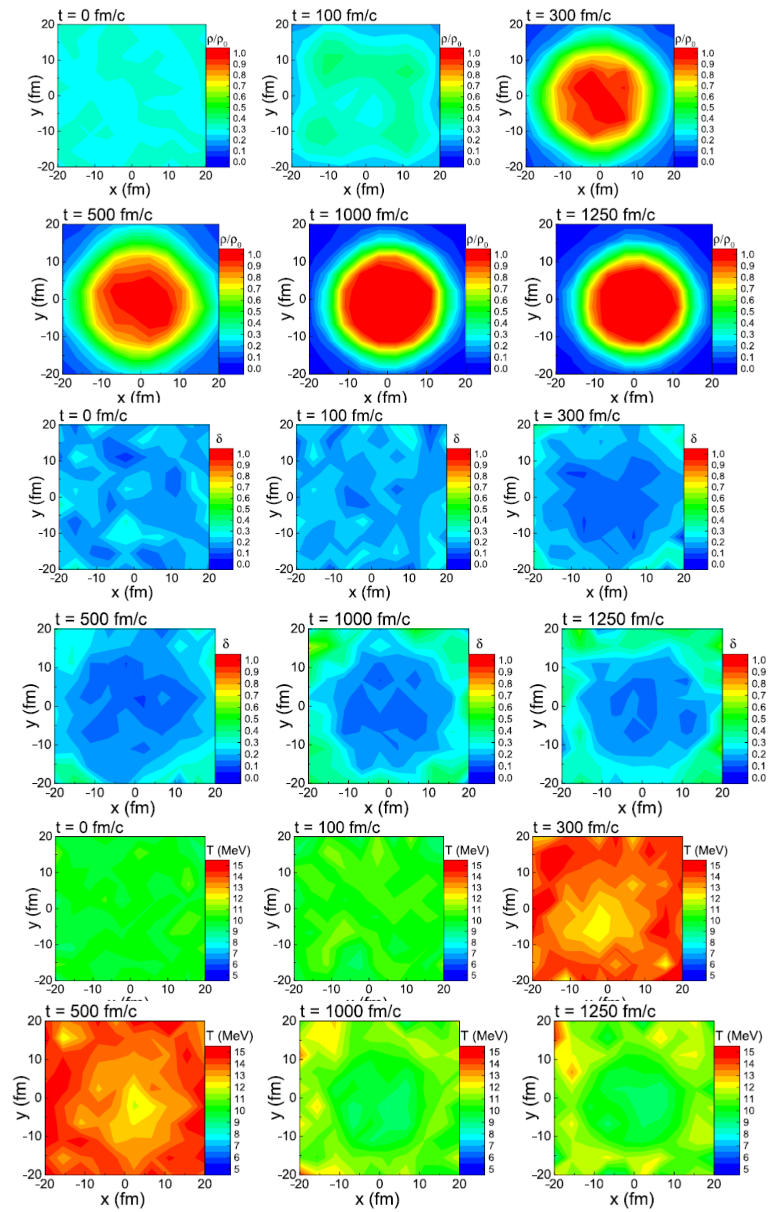
Preparation of dynamically equilibrated ANM system



density evolution

δ evolution

temperature evolution

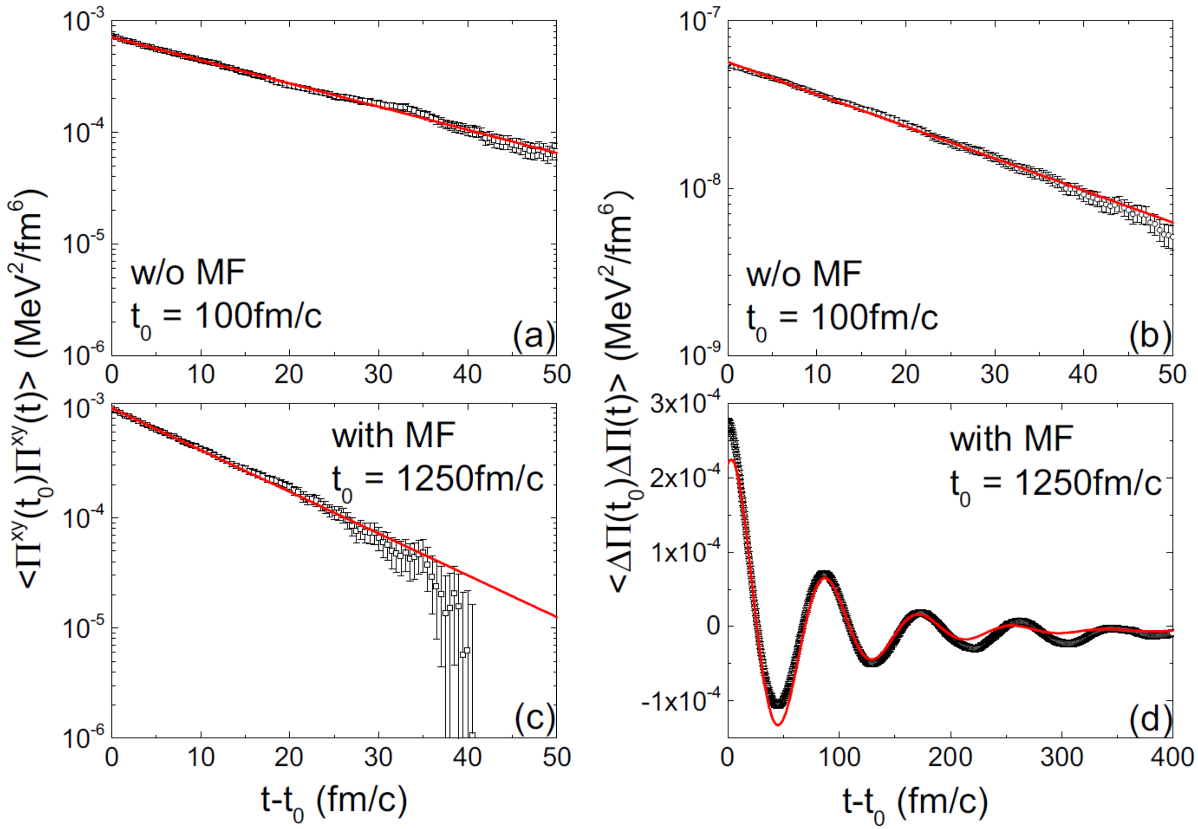


$\langle \rho \rangle = 0.3\rho_0$

$\langle \delta \rangle = 0.2$

$\langle T \rangle = 10\text{MeV}$

Shear and bulk viscosity from GK method



$$\eta = \frac{V}{T}\int_{t_0}^{\infty} dt \langle \Pi^{xy}(t_0)\Pi^{xy}(t)\rangle_{\text{equil}}$$

$$\zeta = \frac{V}{T}\int_{t_0}^{\infty} dt \langle \Delta\Pi(t_0)\Delta\Pi(t)\rangle_{\text{equil}}$$

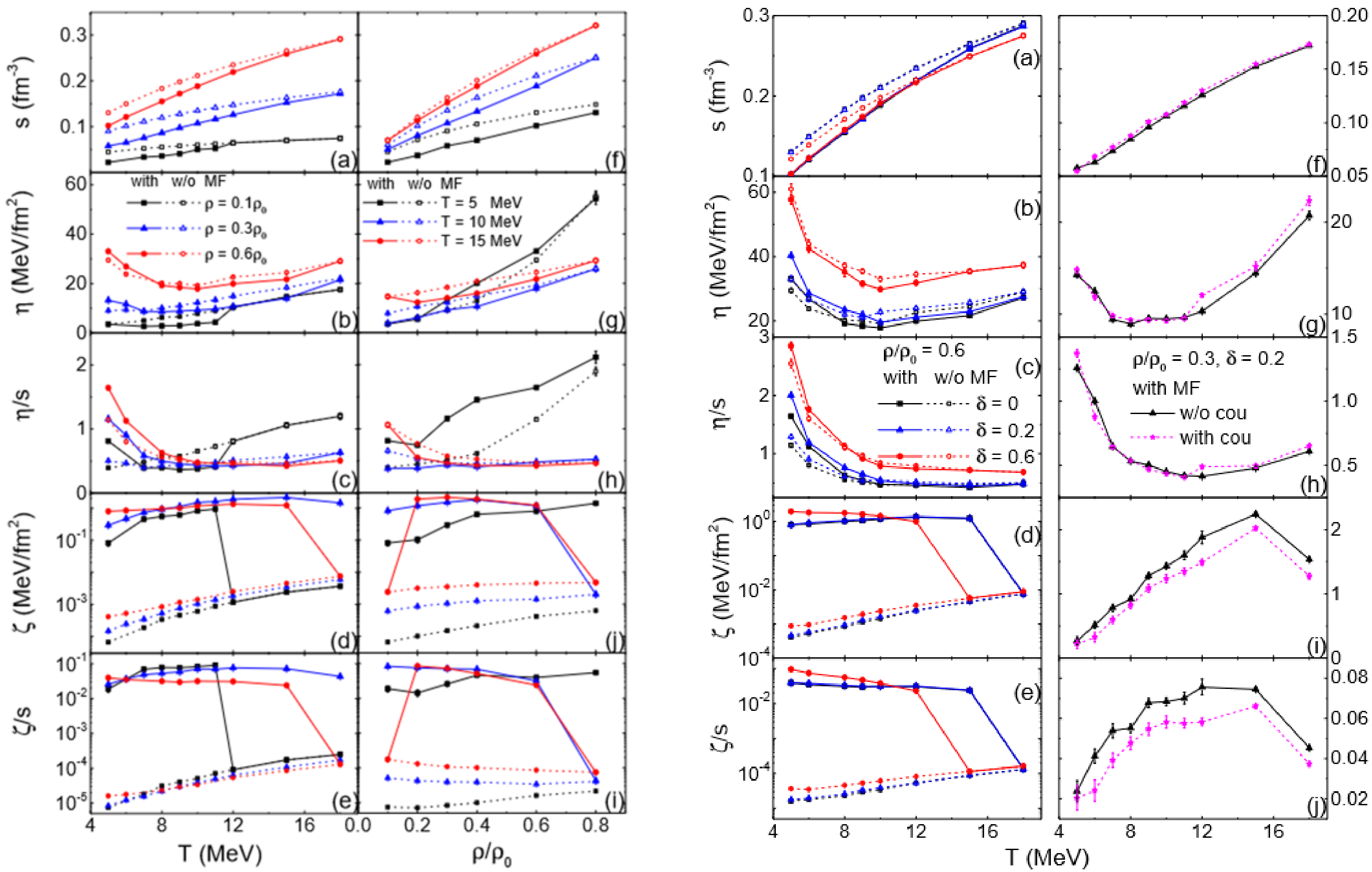
$$\langle \Pi^{xy}(t_0)\Pi^{xy}(t)\rangle = Ae^{-B(t-t_0)}$$

$$\int_{t_0}^{\infty} Ae^{-B(t-t_0)}dt = \frac{A}{B}$$

$$\langle \Delta\Pi(t_0)\Delta\Pi(t)\rangle = Ae^{-B(t-t_0)}\sin[C(t-t_0)+D]$$

$$\int_{t_0}^{\infty} Ae^{-B(t-t_0)}\sin[C(t-t_0)+D]dt = \frac{AB\sin(D)+AC\cos(D)}{B^2+C^2}.$$

Results of shear and bulk viscosity



For a more realistic σ , clustering reduces or enhances η due to competitions between initial correlations and collisions, and the minimum of η/s is shifted; Bulk viscosity is more sensitive to the phase diagram but less sensitive to σ.

Conclusions

- **Clustering enhances <$\pi^{xy}\pi^{xy}$> correlations and collisions, and reduces η and η/s for a constant σ**
- **Minimum of η/s (T) appears at low densities**
- **Minimum of η/s (T) depends on $\sigma(E)$**
- **Clustering significantly enhances <$\pi\pi$> correlations and thus ζ and ζ/s**
- **ζ/s (T) is more sensitive to clustering than to $\sigma(E)$**

Outlook

- In-medium NN cross section
- Other hadronic system
  - hadron resonance gas
  - hot neutron-star matter
- Partonic system
  - NJL transport in spinodal

Thank you!

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