Dynamical fluctuations near the QCD critical point

Shanjin Wu(吴善进)

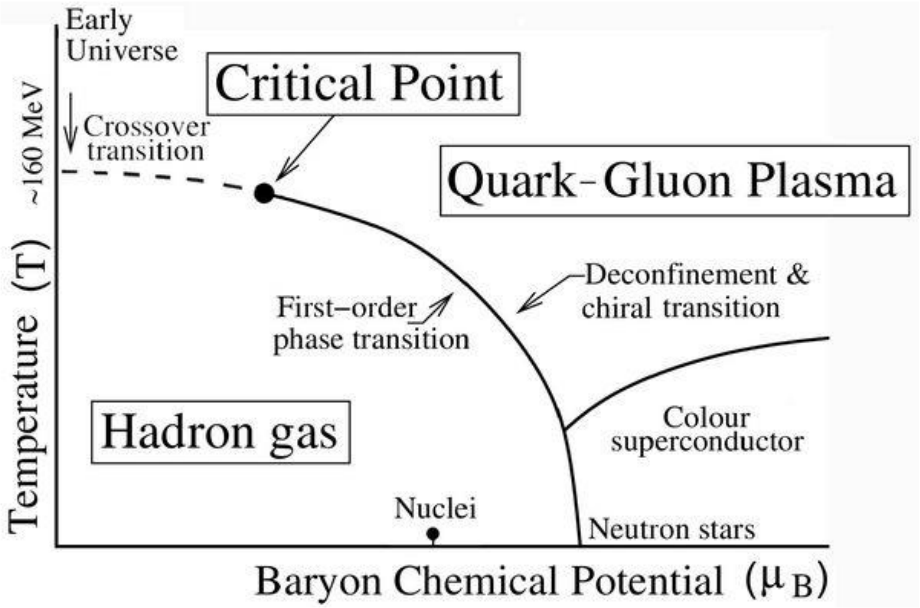
QCD phase diagram

- **Lattice QCD** (small μ_B finite T): Y. Aoki et al, Nature 443, 675 (2006).
  - Crossover (2nd order phase transition)
- **Effective models** (large μ_B) W.J.Fu et al., PRD 77, 014006 S.X.Qin et al., PRL. 106, 172301 K. Fukushima et al, PPNP. 72, 99
  - 1st order phase transition

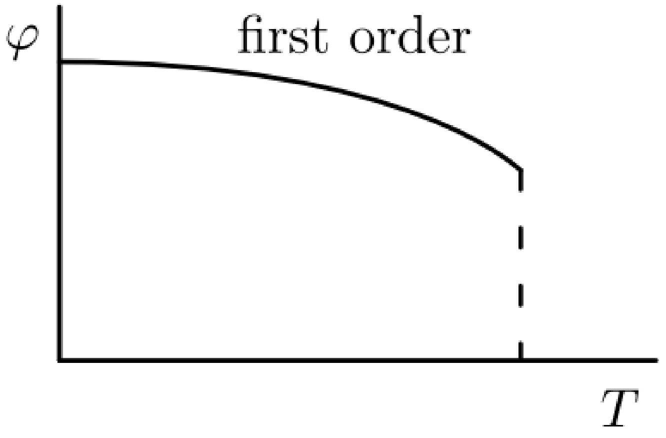
→ Critical point

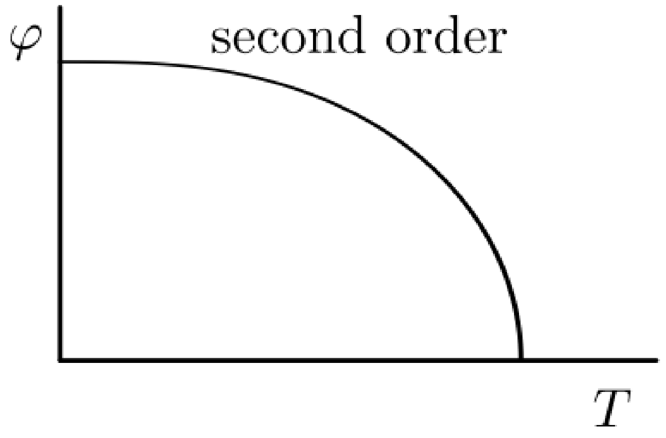
→ **Heavy-ion collisions :**

- tuning $\sqrt{s_{NN}}$,mapping $T-\mu$ phase diagram: RHIC(BES),NICA,FAIR,J_PARC,HIAF….



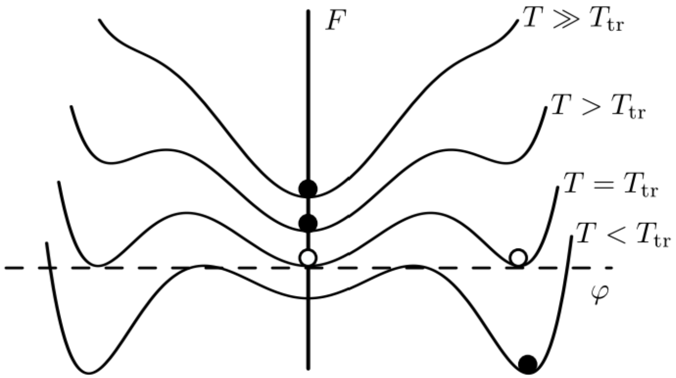
Theory of phase transition

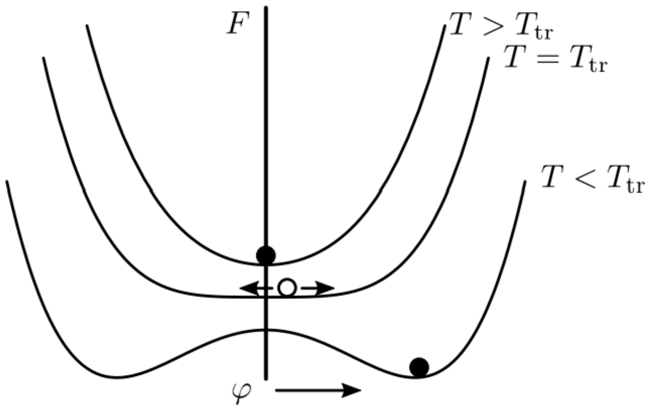




Lev Landau

Order parameter: identify symmetry and symmetry breaking

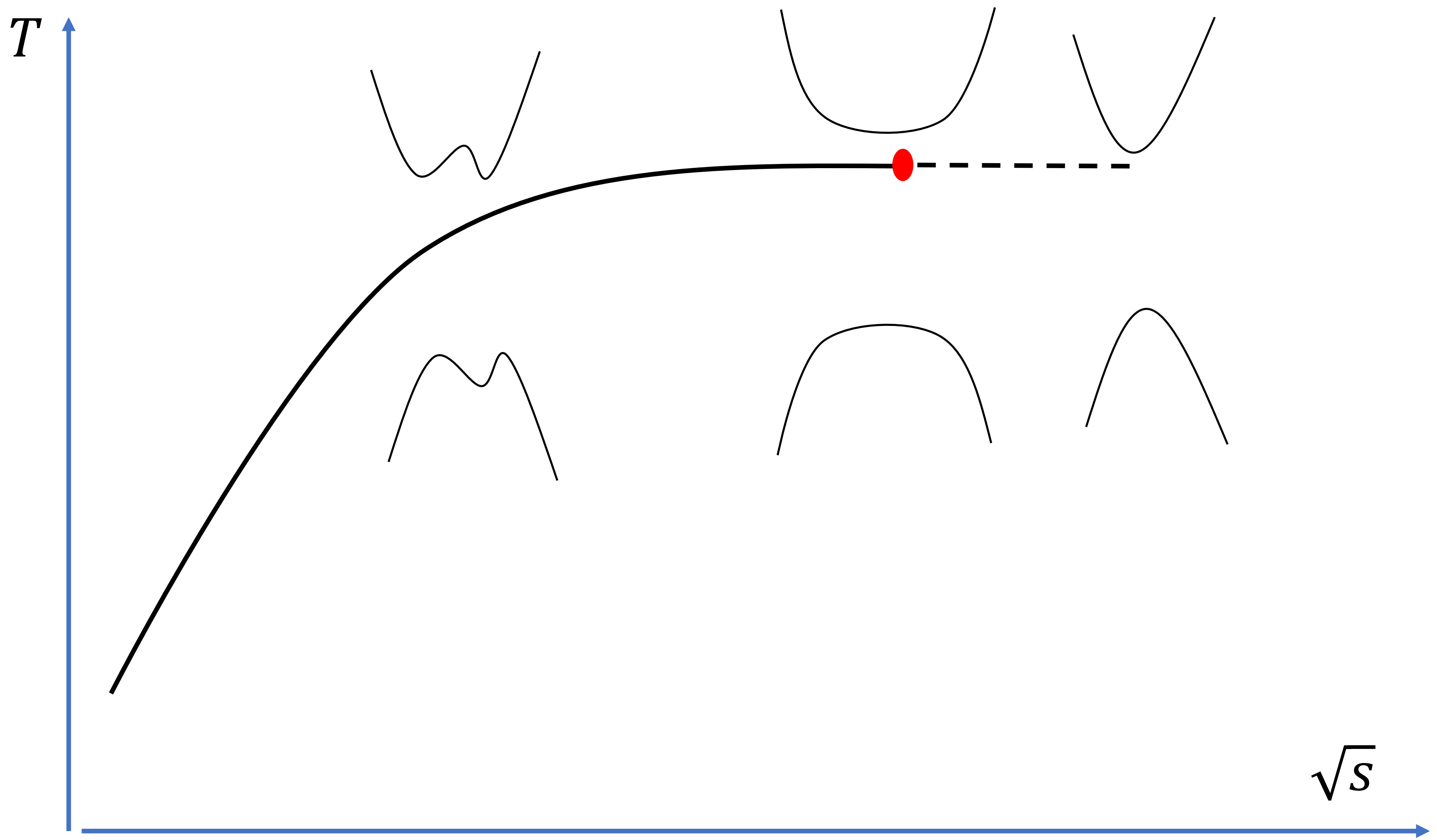




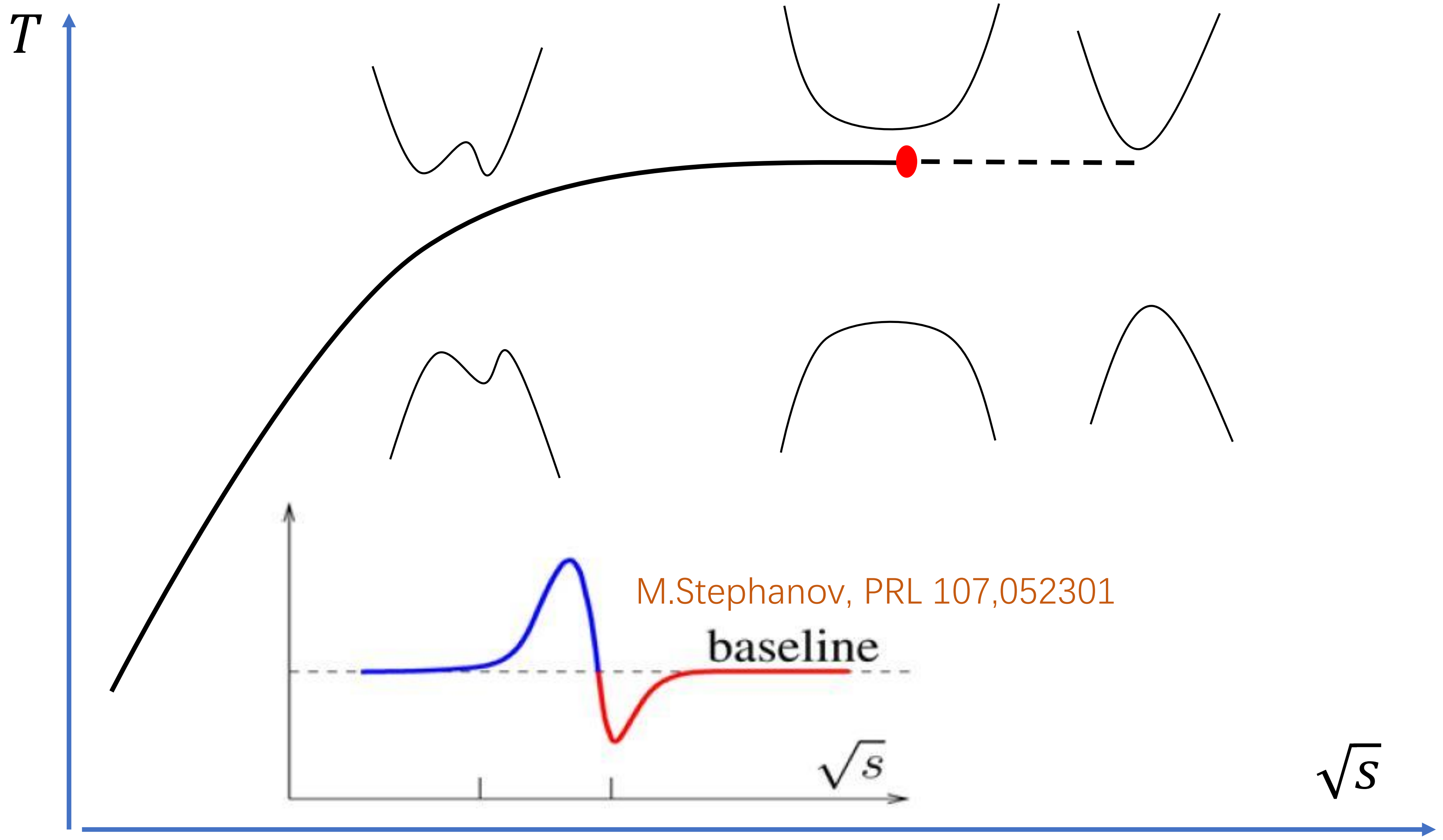
Lessons:

1. Different shape of distribution(free energy) in different region of phase diagram
2. Large fluctuations near critical point

Seach the QCD critical point with Beam Energy Scan

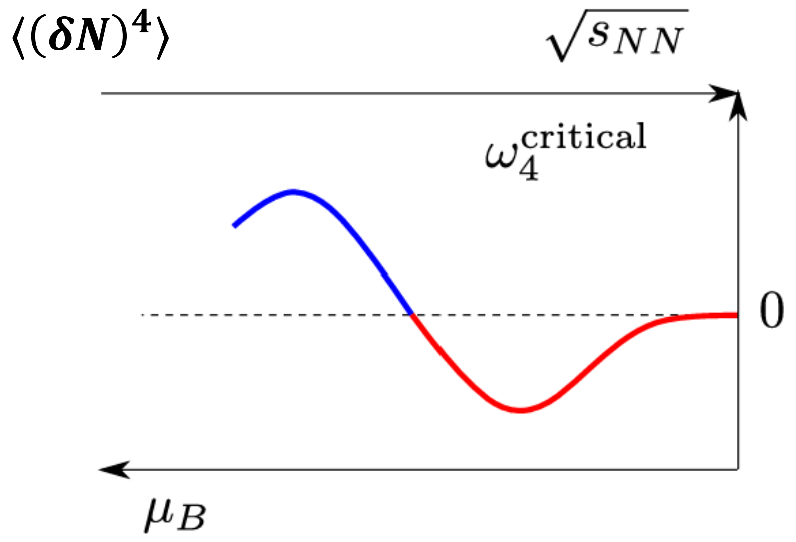


Seach the QCD critical point with Beam Energy Scan

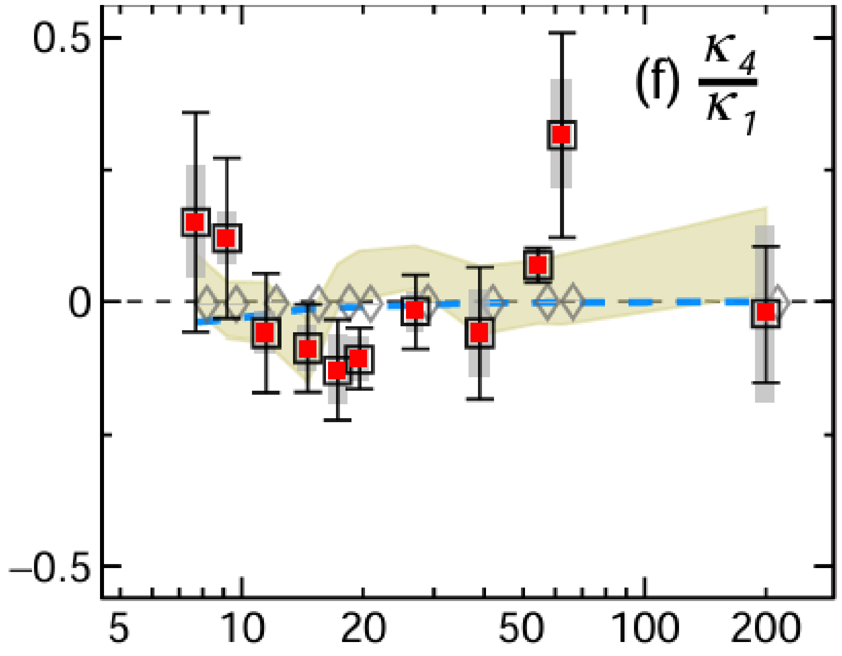


Net-proton fluctuations with Beam Energy Scan

- Characteristic feature of critical point:
  - long range correlation
  - large fluctuations
- **Non-monotonicity** of factorial Cumulant



M.Stephanov, 2410.02861



STAR, Phys.Rev.Lett. 135, 142301

QGP fireball is an expanding system

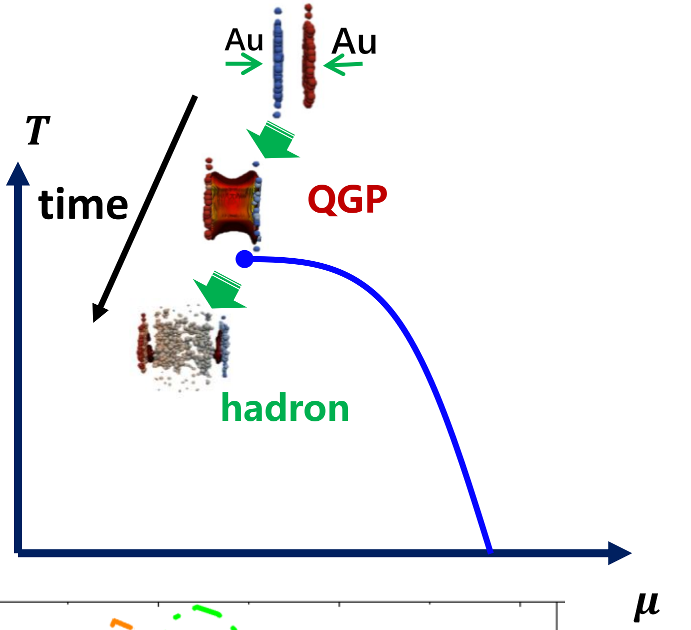
Dynamical effects modifies the critical fluctuations

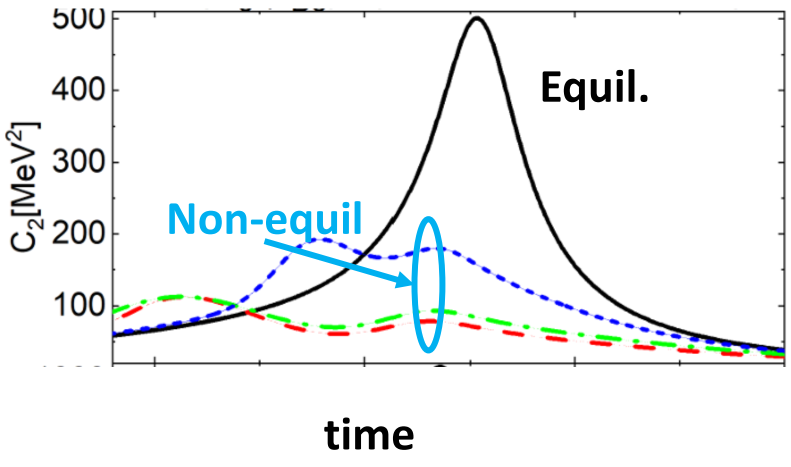
- Expanding QGP fireball forces the fluid cell swipes the critical regions with finite time τ_{expand};
- Long range correlation requires long time to correlate with each other $\tau_{relax} \sim \xi^z$;
- In heavy-ion collision, near the QCD critical point,

$$\tau_{relax} \gg \tau_{expand}$$

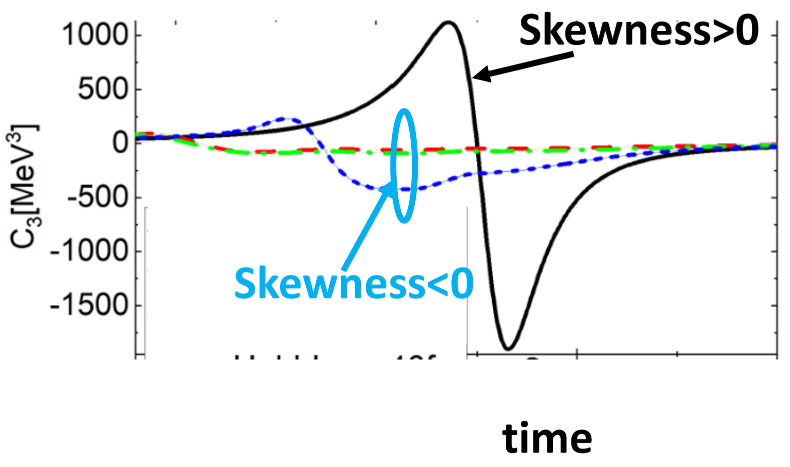
- Expansion drives the system has no enough time to relax to equilibrium

S.Tang, S.Wu, H.Song, PRC(2023)

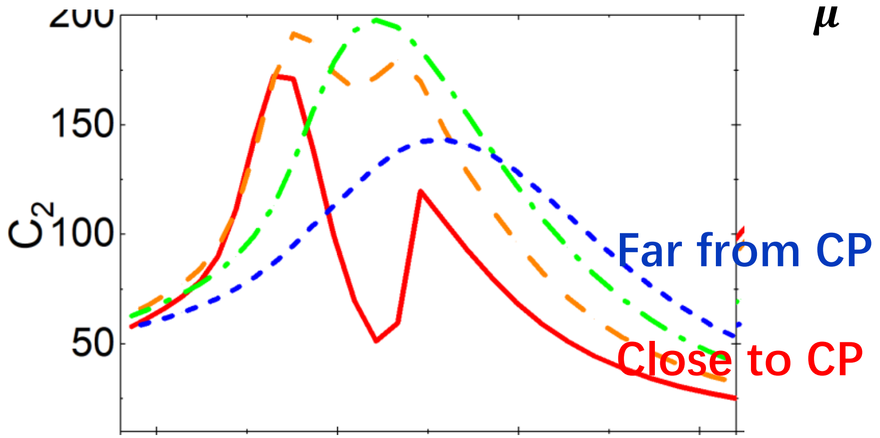




Suppression of the fluctuations



Reverse the sign of skewness



Largest $C_2 \neq$ closet to CP

Dynamical models near QCD critical point

- **QCD critical point belongs to model H** (Symmetry and dynamical analysis)**:** D.T.Son,Stephanov 04’
- **Model H** (conserved baryon $\mathbf{n} \approx (\boldsymbol{n}+\boldsymbol{\sigma})$ + transverse momentum density)**:**

$$\frac{\partial n}{\partial t} = \Gamma[\mathrm{n}, \pi^T] + \text{noise}, \qquad \frac{\partial \pi^T}{\partial t} = \eta[n, \pi^T] + noise$$

Numerical simulation is time consuming, not in (τ, x, y, η) frame C,Chattopadhyay et al 25’.

- **Simplified models:**
  - **Model A** (order parameter field $\boldsymbol{\sigma}$) S.Mukherjee et al 15’ 16’, L.Jiang et al17’, S.Wu et al 19’, S.Tang et al 23’, N.Attieh et al 24’

$$\frac{\partial \sigma}{\partial t} = \Gamma[\sigma] + \text{noise}$$

  - **Model B** (conserved field $\mathbf{n} \approx (\boldsymbol{n}+\boldsymbol{\sigma})$) M.Sakaida et al 17’, S.Wu et al 19’, M.Nahrgang et al 19’, G.Pihan et al 22’, S.Wu 25’, A.Sakai et al., 25’

$$\frac{\partial n}{\partial t} = \nabla^2\Gamma[n] + \text{noise}$$

Dynamical models near QCD critical point

More efficient modeling in heavy-ion collisions: **Hydrodynamics + Critical fluctuations**

- **Non-equilibrium chiral hydrodynamics** M. Nahrgang et al 11'12'14'16'19'

$$\partial_\mu T^{\mu\nu}_{fluid} = \partial_\mu \mathrm{T}^{\mu\nu}_{\sigma}, \qquad \frac{\partial\sigma}{\partial t} = \Gamma[\sigma] + noise$$

- **Fluctuating hydrodynamics** J.Kapusta et al 12',12', K.Murase et al 13', X.An et al 19',21'…

$$\partial_\mu T^{\mu\nu}_{fluid} = noise\ , \partial_\mu N^{\mu} = noise$$

- **Hydro+, hydro++…** (hydro + slow modes) M. Stephanov et al 18'19'20', N. Abbasi et al 22', L. Du et al 20',……

$$\partial_\mu T^{\mu\nu}_{*} = 0, \partial_\mu N^{\mu} = 0, \qquad \frac{\partial\phi}{\partial t} = \Gamma[\phi]$$

- **Hydro-kinetics** D.Teaney et al 17'18'19'22'…

$$\partial_\mu T^{\mu\nu} = 0, \partial_\mu N^{\mu} = 0, \quad \partial_t(noise\ correlator) = \#$$

See reviews: e.g. Shanjiu Wu, et al.,2104.13250; Lipei Du et al. 2402.10183; Xin An et al., 2108.13867; Marcus Bluhm et al., 2001.08831; Adam Bzdak et al.,1906.00936;M.Asakawa et al.,1512.05038

Dynamical models near QCD critical point

- **QCD critical point belongs to model H** (Symmetry and dynamical analysis)**:** D.T.Son,Stephanov 04'
- **Model H** (conserved baryon $\mathbf{n} \approx (\boldsymbol{n}+\boldsymbol{\sigma})$ + transverse momentum density)**:**

$$\frac{\partial n}{\partial t} = \Gamma[\mathbf{n}, \pi^T] + \mathbf{noise}, \qquad \frac{\partial \pi^T}{\partial t} = \eta[n, \pi^T] + noise$$

Numerical simulation is time consuming, not in (τ, x, y, η) frame C,Chattopadhyay et al 25', Y.-R.Chen et al, 23'

- **Simplified models:**
  - **Model A** (order parameter field $\boldsymbol{\sigma}$) S.Mukherjee et al 15' 16', L.Jiang et al17', S.Wu et al 19', S.Tang et al 23', N.Attieh et al 24'

$$\frac{\partial \sigma}{\partial t} = \Gamma[\sigma] + \mathbf{noise}$$

  - **Model B** (conserved field $\mathbf{n} \approx (\boldsymbol{n}+\boldsymbol{\sigma})$) M.Sakaida et al 17', S.Wu et al 19', M.Nahrgang et al 19', G.Pihan et al 22', S.Wu 25', A.Sakai et al., 25'

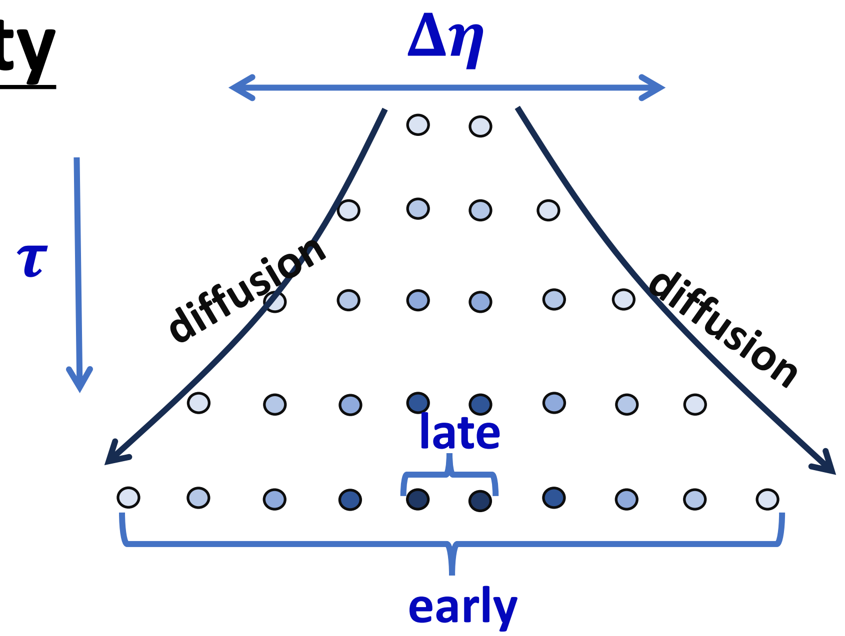
$$\frac{\partial n}{\partial t} = \nabla^2\Gamma[n] + \mathbf{noise}$$

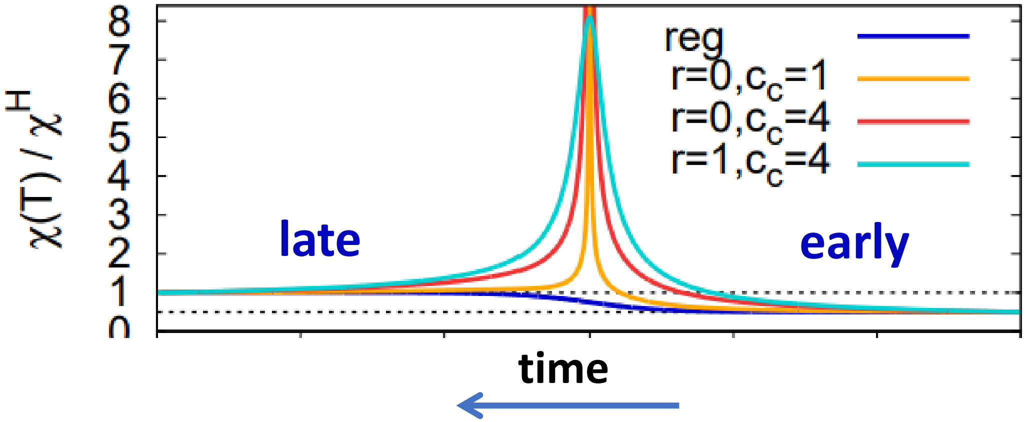
Dynamics of conserved net-baryon density

- Diffusion of conserved baryon near critical point:

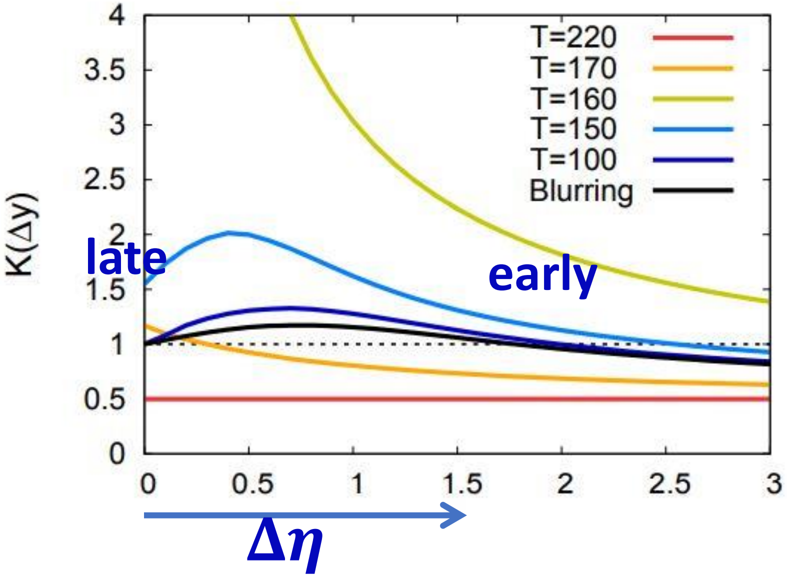
$$\partial_\tau n = \#\nabla^2 n + \text{noise}$$

- The process of diffusion consumes time.
- Larger $\Delta\eta \sim$ the early evolution.

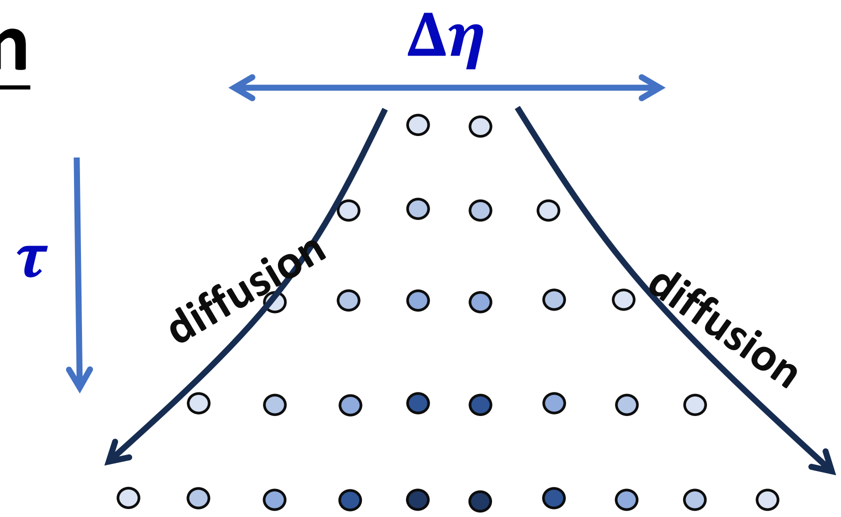




Sakaida et al, PRC.95.064905(2017)

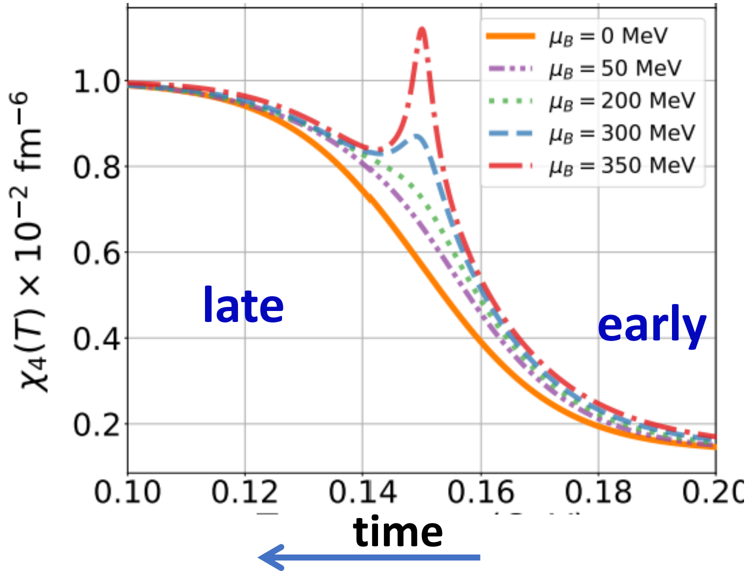


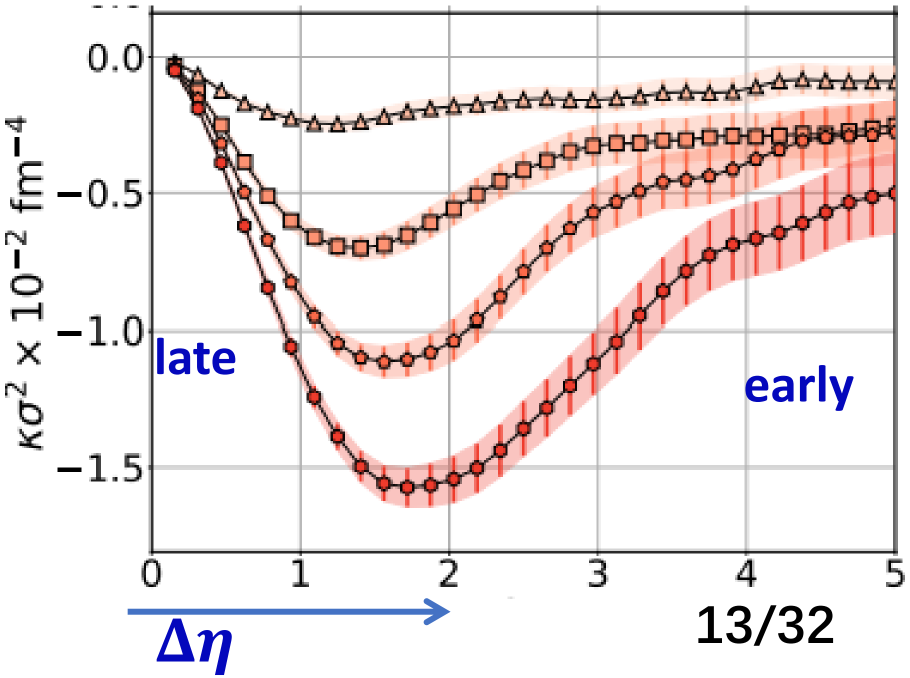
Conserved net-baryon with non-Gaussian



- Diffusion of conserved baryon near critical point:
$$\partial_\tau n = \#\nabla^2(n + \#n^2 + n^3/\chi_4) + \text{noise}$$
- The process of diffusion consumes time.
- Larger $\Delta\eta \sim$ the early evolution.

G.Pihan et al., PRC.107.014908(2022)

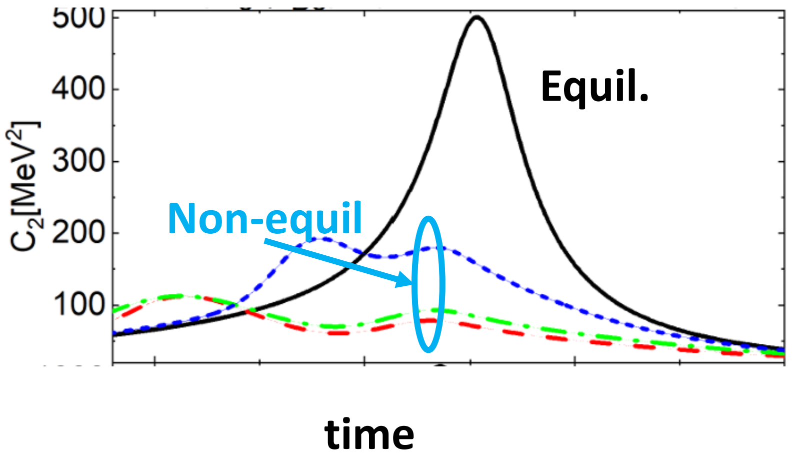


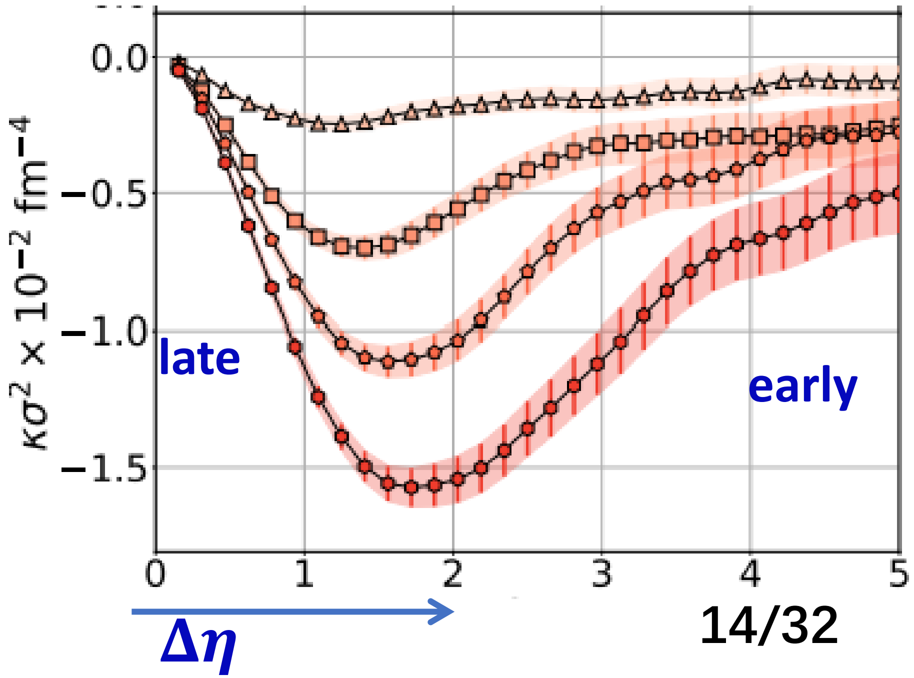


Short summary

- Fluctuations are modified in the expanding system

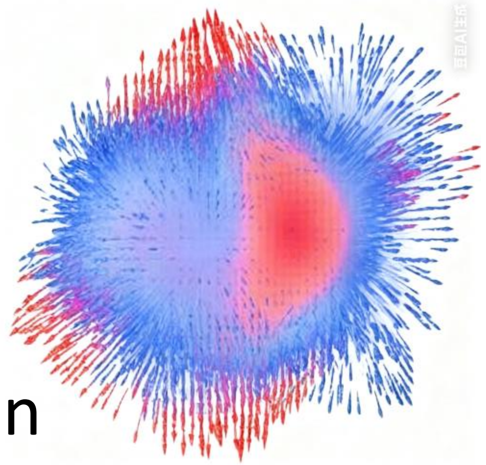
- The critical fluctuations results in non-monotonic behavior in η direction





QGP fireball has inhomogeneous T and μ profile

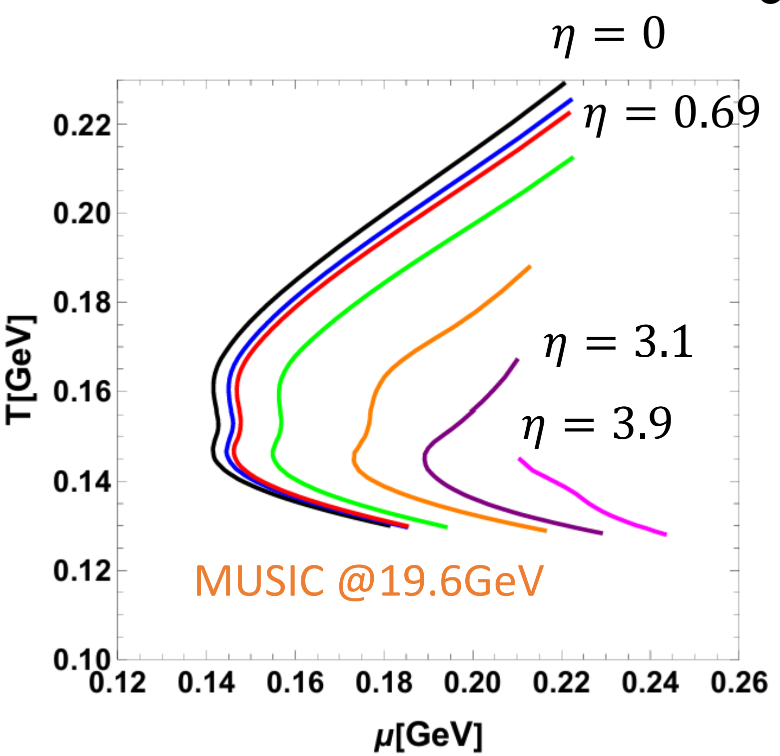
Inhomogeneous QGP profile



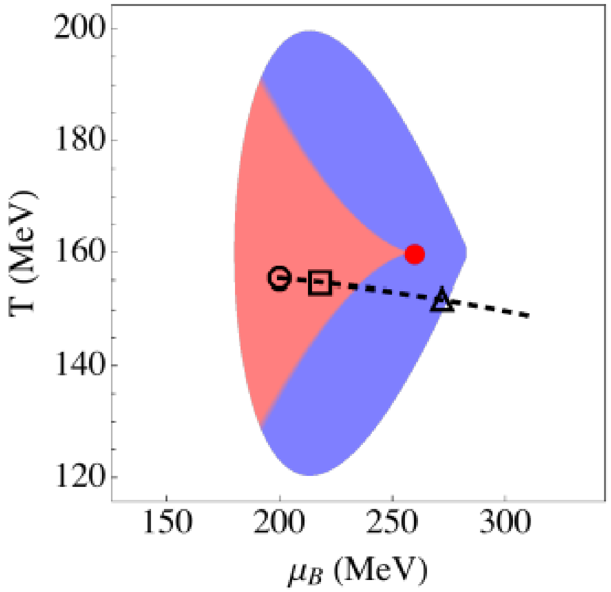
- Different rapidity ~ different trajectories
- ~ detect different parts of critical region
- ~ different critical behavior

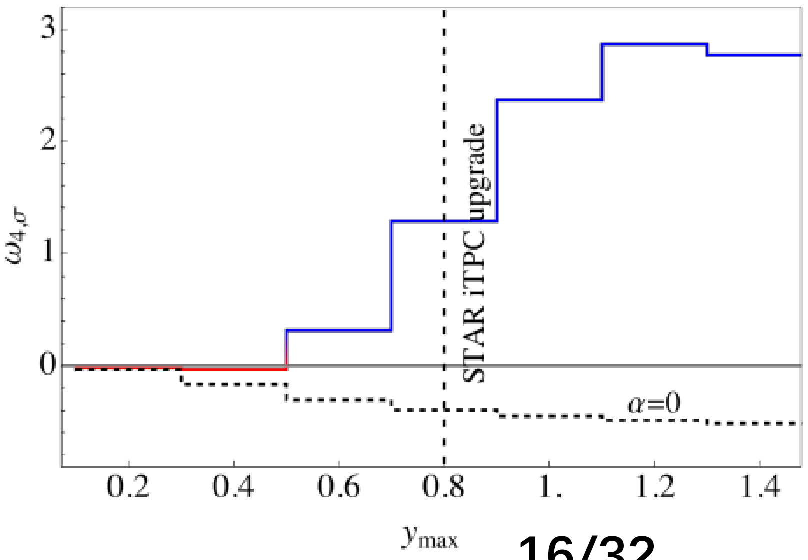
See Shuzhe's talk for correlator in inhomogeneous medium

J.Brewer et al., PhysRevC.98.061901



Different η in one QGP profile

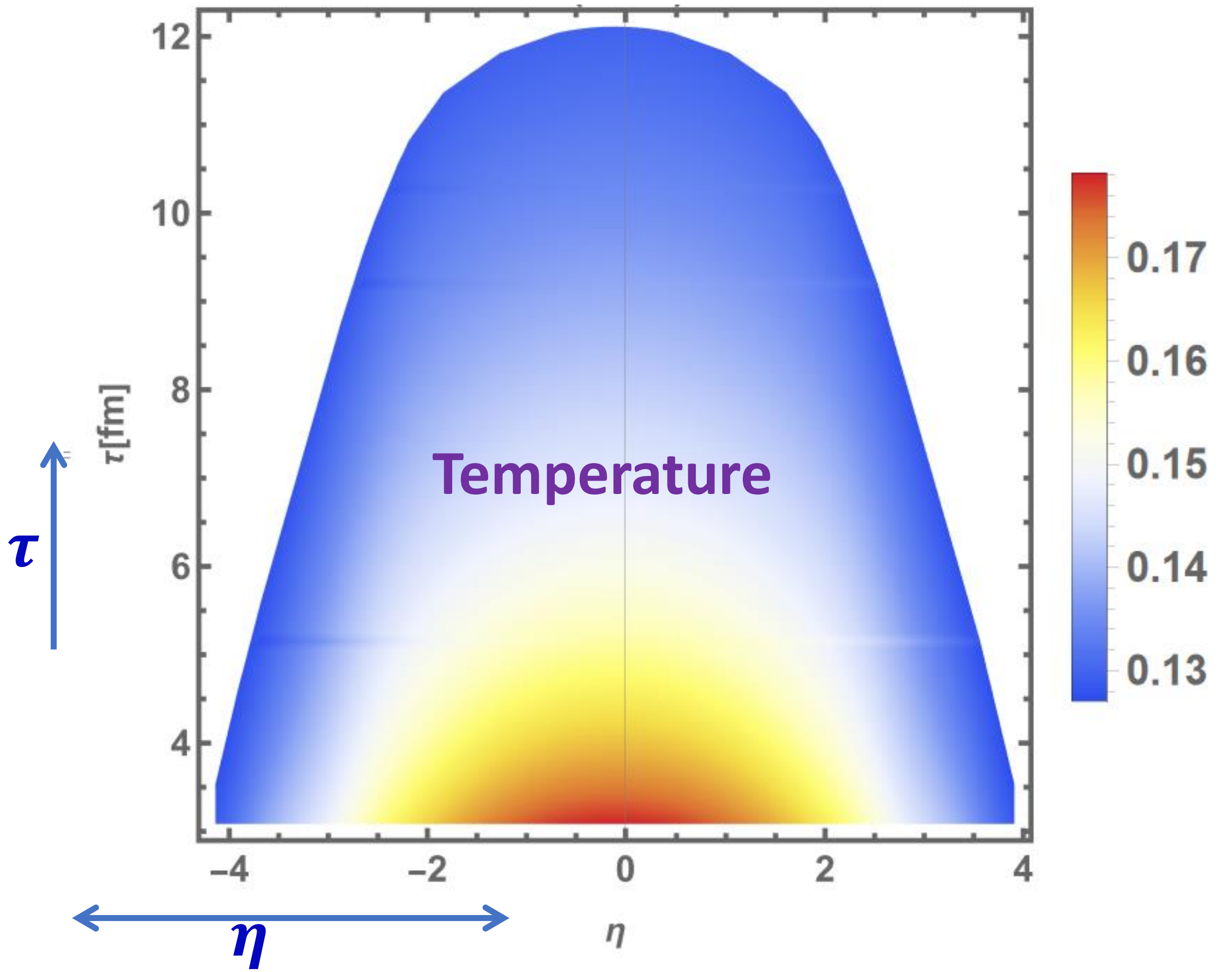


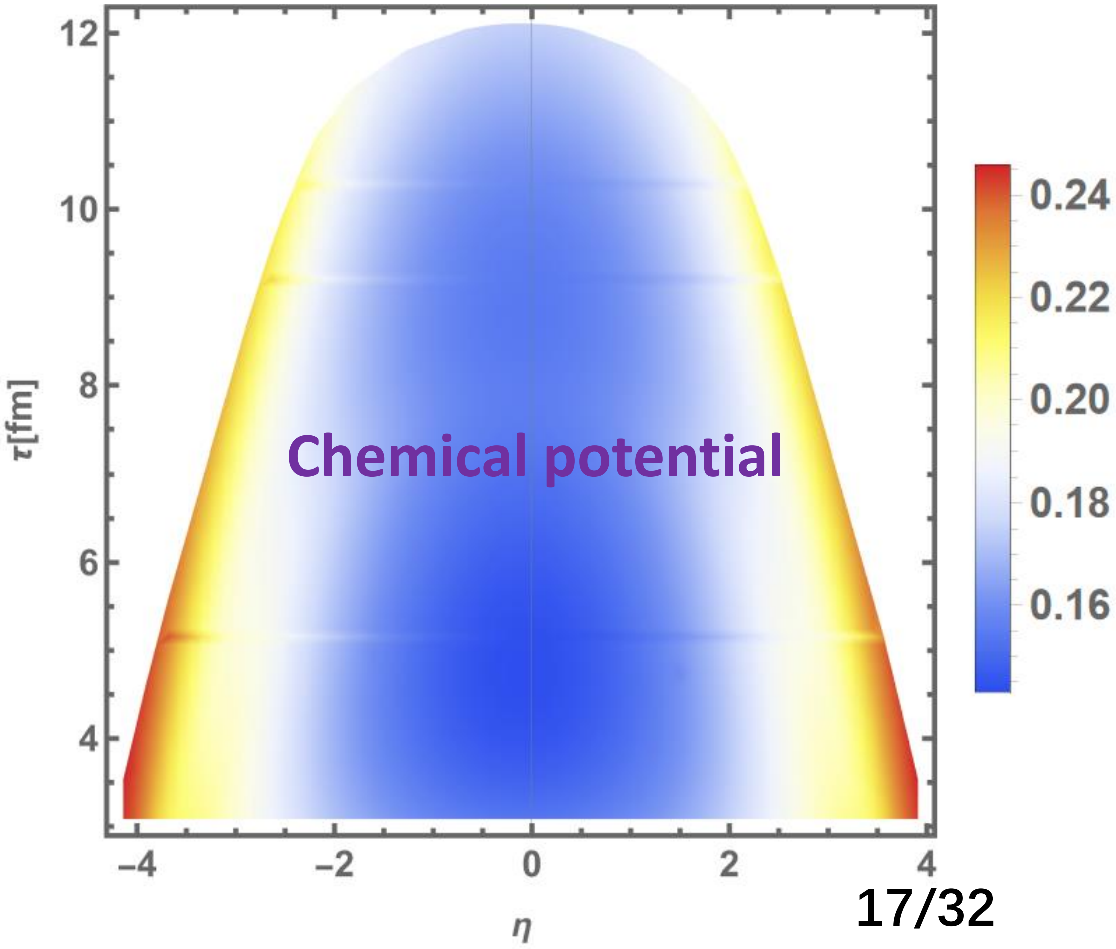


Temperature and chemical potential profile from hydro

- Hydro simulation: AMPT+MUSIC@19.6GeV

Shanjin Wu PRC 111,014915

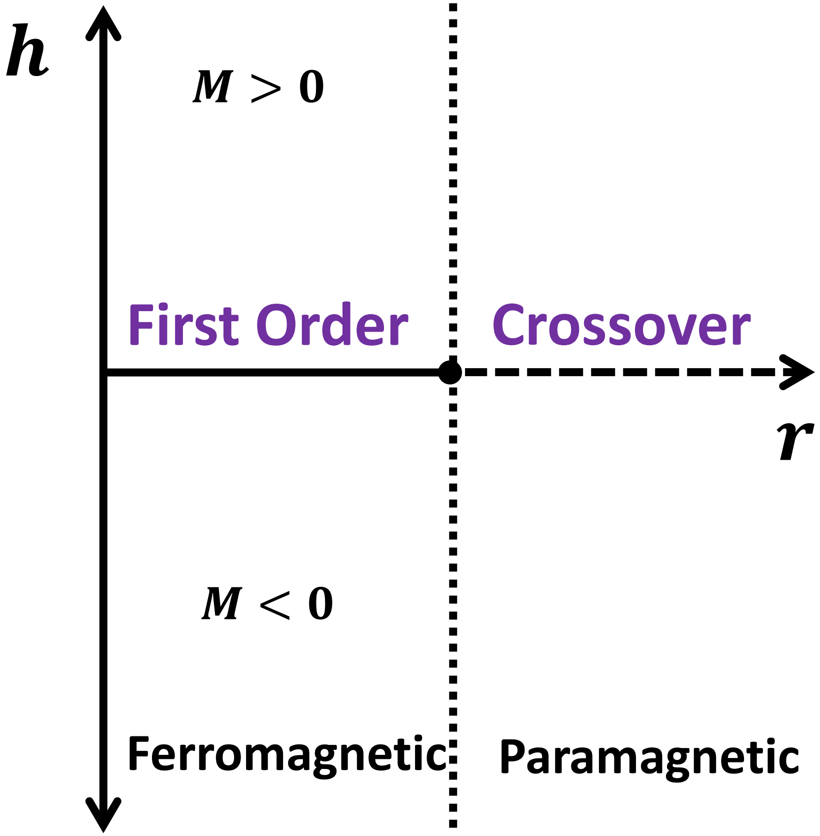




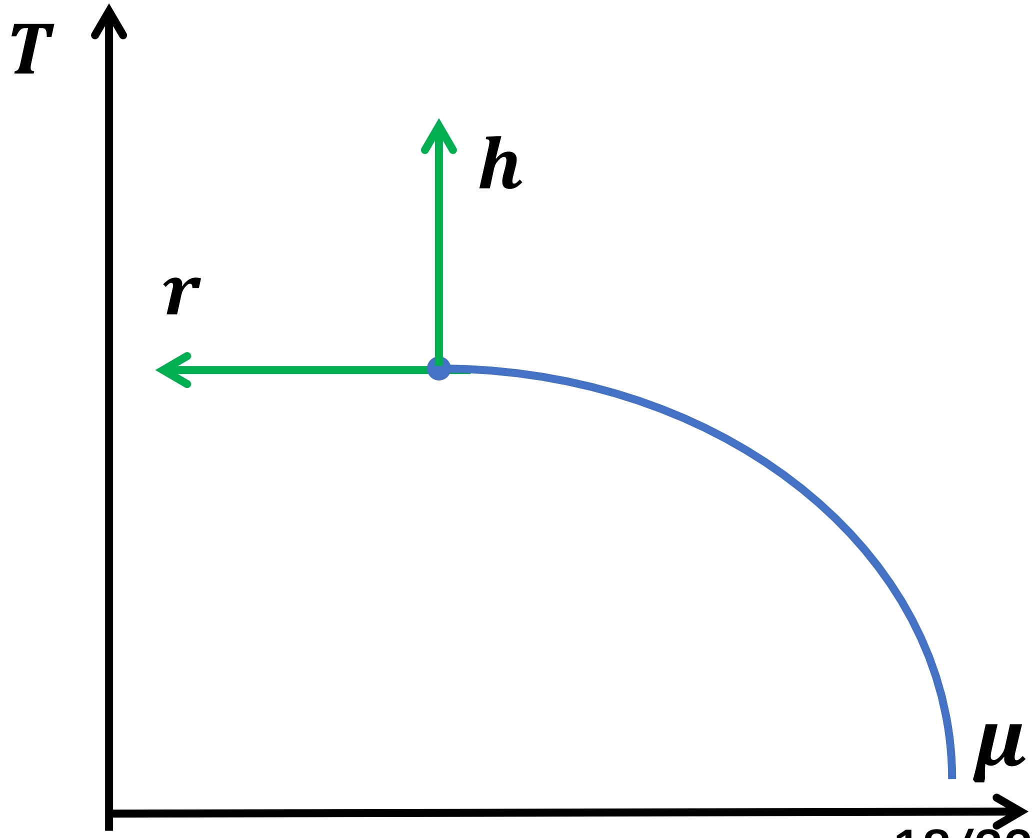
Map EoS from Ising model

Shanjin Wu PRC 111,014915

χ_2, χ_4 parameterized from Ising

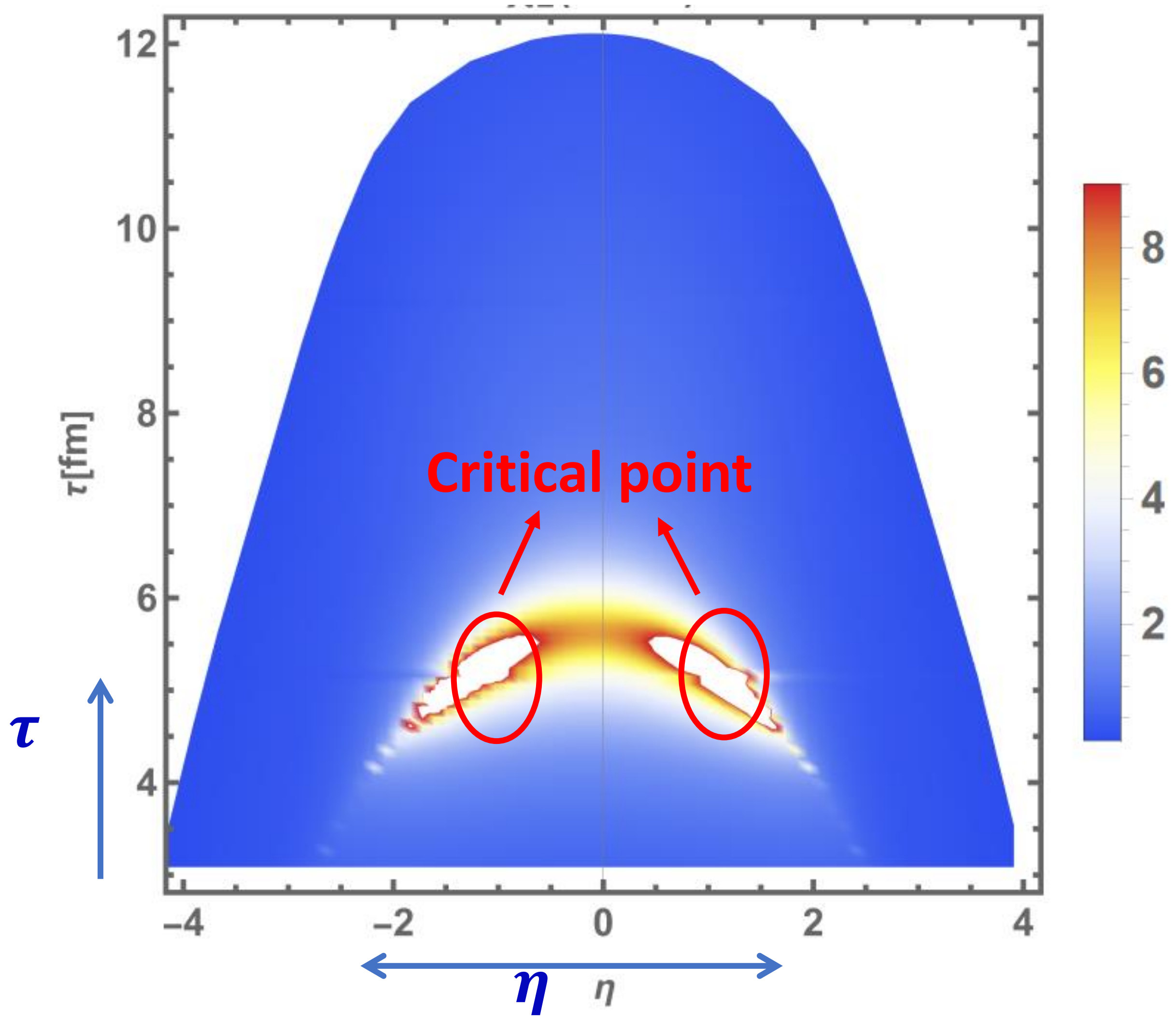


χ_2, χ_4 mapped to (T, μ) plane



Profile of susceptibilities $\chi_2(\tau,\eta)$

Shanjin Wu PRC 111,014915



- Mapping:

$$\chi_2(h,r),\chi_4(h,r) \to \chi_2(T,\mu),\chi_4(T,\mu)$$

- Hydro: $T(\tau,\eta),\mu(\tau,\eta)$

- $\chi_2(\tau,\eta),\chi_4(\tau,\eta)$

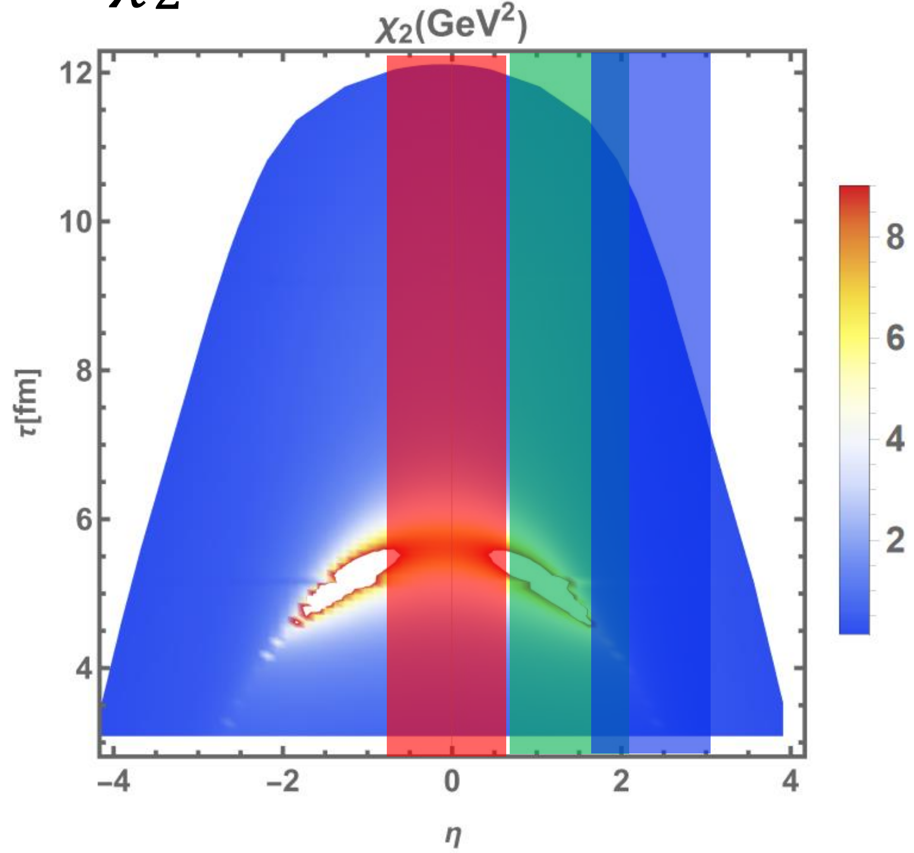
Critical point exist at finite space and time

Time evolution of conserved net-baryon fluctuations

G.Pihan et al., PRC.107.014908(2022); Sakaida et al, PRC.95.064905(2017)

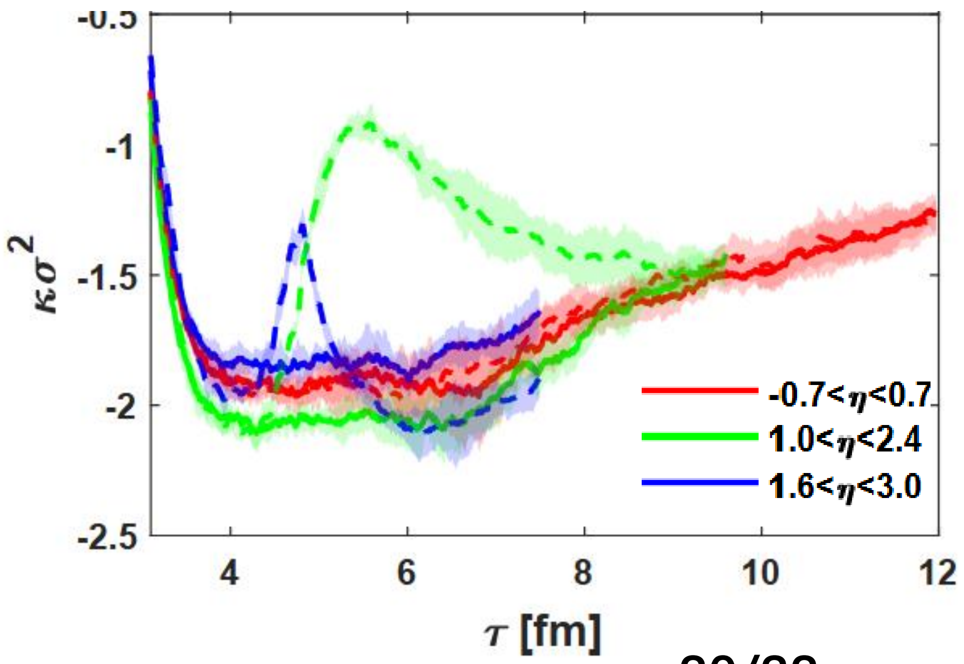
- Diffusion of conserved baryon (1+1D):

$$\partial_\tau n = \nabla^2\left(\frac{n}{\chi_2} + \lambda_3 n^2 + \lambda_4\, n^3\right) + \text{noise}$$



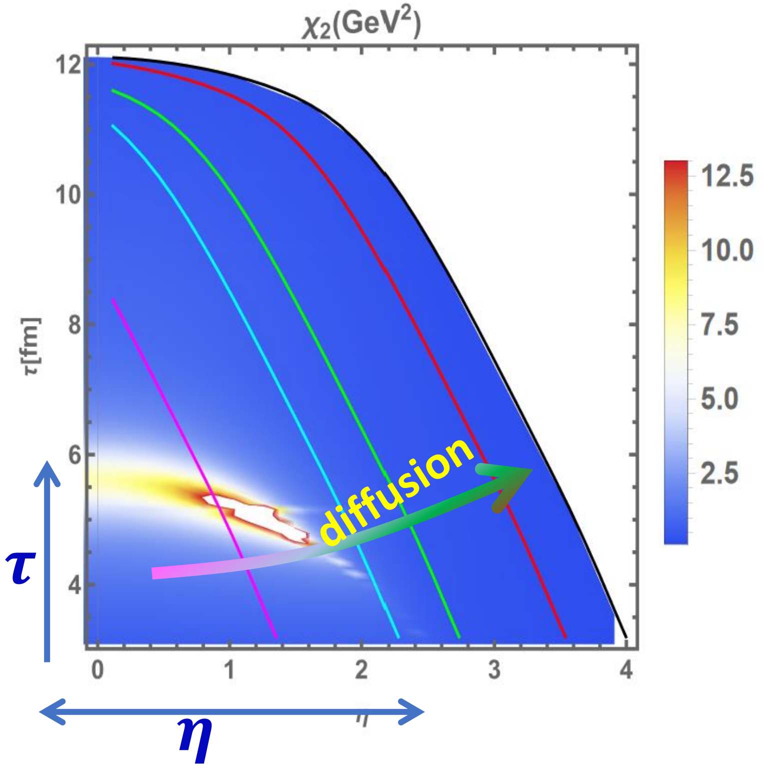
Shanjin Wu PRC 111,014915

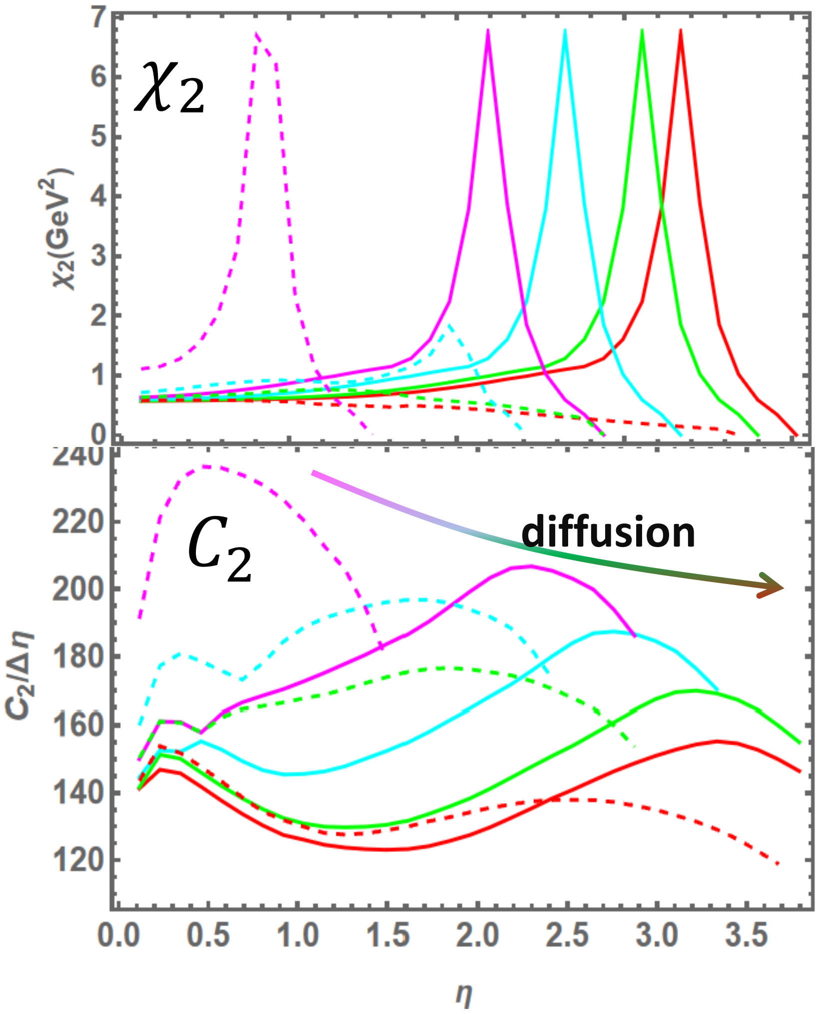
Inhomogeneous profile effects is significant at large rapidity



Conserved net-baryon fluctuations before freeze-out surface

Shanjin Wu PRC 111,014915





χ_2 driven by (T, μ) profile

C_2 relax as:

$$\partial_\tau C_2 \sim \frac{1}{\tau_{relax}}(C_2 - \chi_2)$$

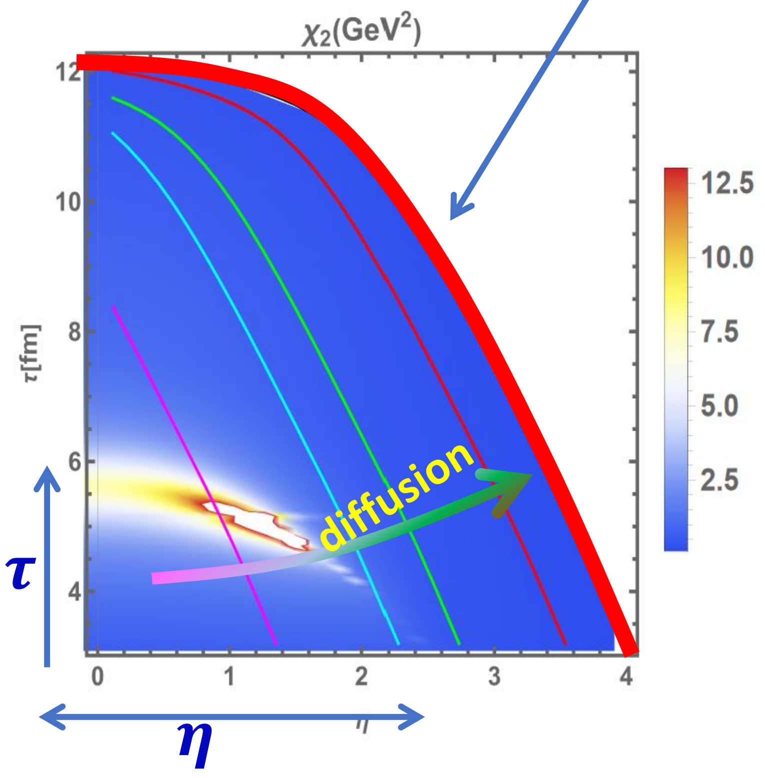
C_2 decreases slower than χ_2

$$C_2 = \langle(\delta n_B)^2\rangle$$

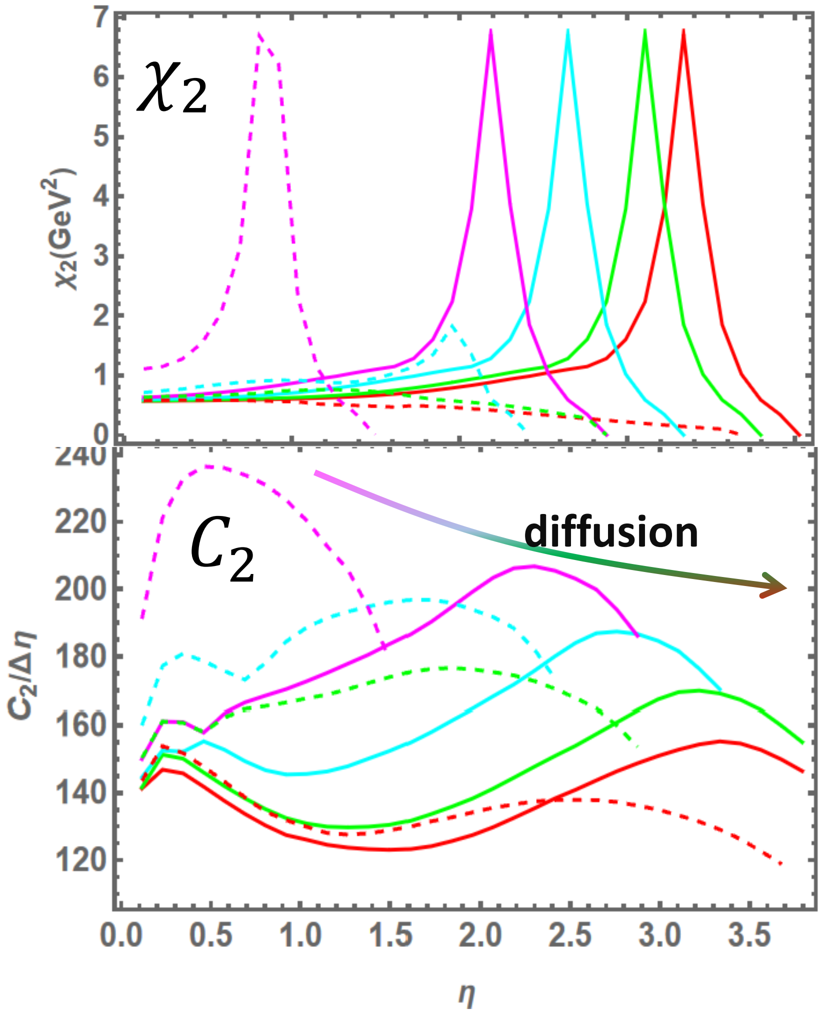
Solid: uniform profile; Dashed: inhomogeneous profile

Conserved net-baryon fluctuations at freeze-out surface

Freeze-out surface



$$C_2 = \langle (\delta n_B)^2 \rangle$$



Shanjin Wu PRC 111,014915

At freeze-out surface:

- Uniform:

$$C_2 \sim \chi_2$$

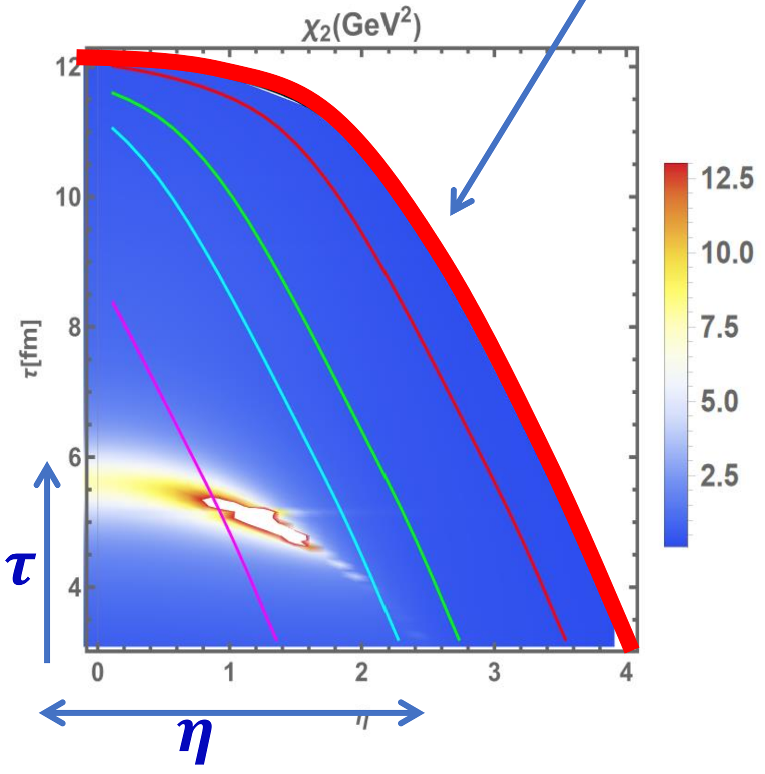
- Inhomogeneous:

$$C_2 \gg \chi_2$$

Solid: uniform profile; Dashed: inhomogeneous profile

Conserved net-baryon fluctuations at freeze-out surface

Freeze-out surface



Shanjin Wu PRC 111,014915

Net-baryon number at freeze-out surface

$$N_B = g \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^0} \int d\sigma_\mu p^\mu f(\boldsymbol{x}, \boldsymbol{p})$$

Net-baryon fluctuations at freeze-out surface

$$\delta N_B \sim \int \exp\left(\frac{\mu}{T}\right) \delta\mu \sim \int exp\left(\frac{\mu}{T}\right) \frac{\delta n_B}{\chi_2}$$

At freeze-out surface:

- Uniform:

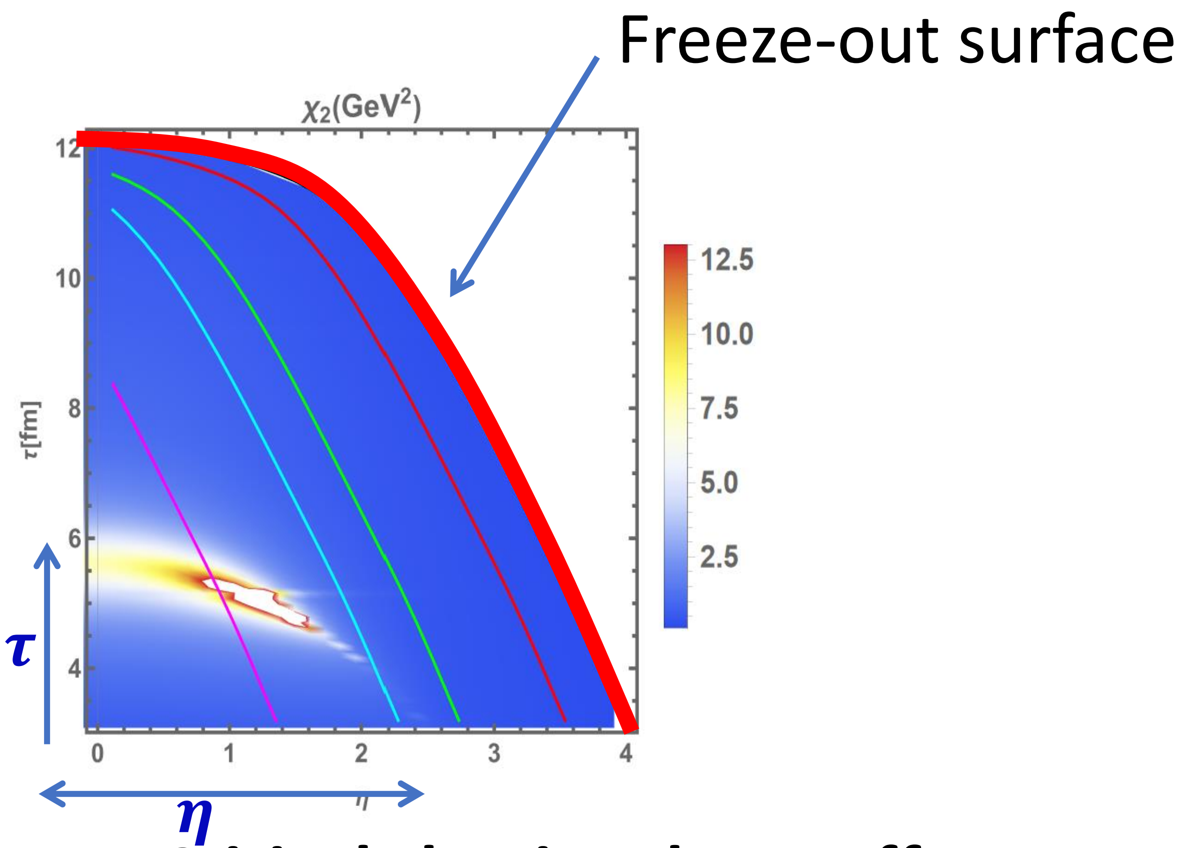
$$C_2 \sim \chi_2$$

- Inhomogeneous:

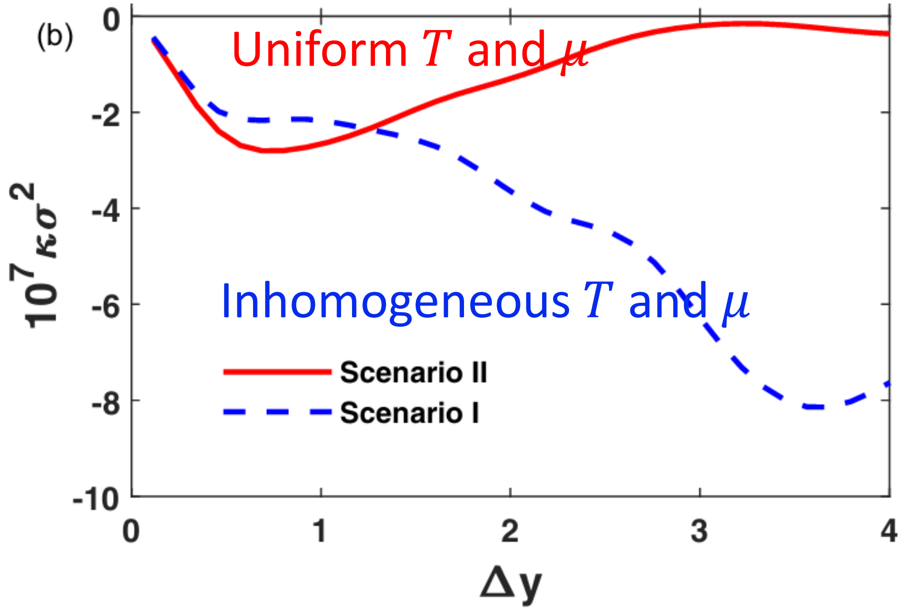
$$C_2 \gg \chi_2$$

Large enhancement of δN_B for inhomogeneous case

Conserved net-baryon fluctuations at freeze-out surface



Shanjin Wu PRC 111,014915



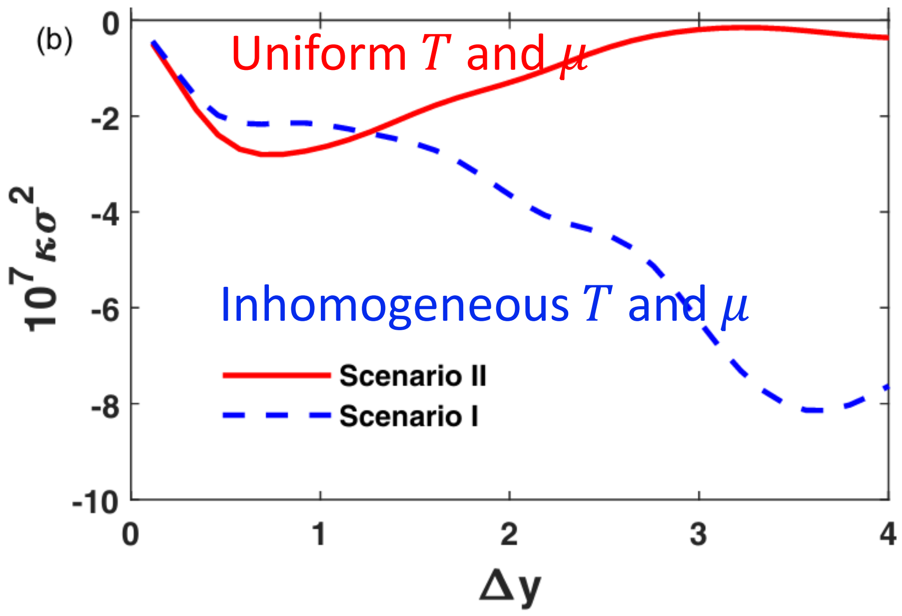
Critical slowing down effects:

cumulants at large rapidity preserves the larger fluctuations (critical effects) at early evolution history

Short summary

- Fluctuations at large rapidity preserved early evolution history

- Enhancement of fluctuations at large rapidity because of Inhomogeneous profile and critical slowing down effects



Causal diffusion near the QCD critical point

Causality for diffusion equation

- **Conventional diffusion:**

$$\partial_\tau n = D\nabla^2 \mathrm{n}$$

Speed of diffusion:

$$v = \frac{\partial \omega}{\partial q} = iDq$$

Will diverge and become acausal when $q \to \infty$.

- **Maxwell-Catteneo diffusion:**

$$(1 + \tau_R \partial_\tau)\partial_\tau n = D\nabla^2 \mathrm{n}$$

Then the speed of diffusion(causal):

$$v = \frac{\partial \omega}{\partial q} \approx \left(\frac{D}{\tau_R}\right)^{1/2}$$

Diffusion equation with non-Gaussian fluctuations

- **Model B:**

$$\partial_\tau n = \nabla^2\left(\frac{n}{\chi_2} + \lambda_3 n^2 + \lambda_4\, n^3\right) + \text{noise}$$

- **Model B in deterministic form:**

X.An,G.Basar, M.Stephanov, H.-U.Yee, Phys.Rev.Lett.127,072301

- $\partial_\tau C_2(q) = -2Dq^2[C_2(q) - T\chi_2]$
- $\partial_\tau C_3(q) = -3Dq^2[C_3(q) + \#C_2(q)^2 + \#C_2(q)]$
- $\partial_\tau C_4(q) = -4Dq^2[C_4(q) + \#C_2(q)C_3(q) + \#C_2(q)^3 + \cdots]$

Diffusion equation with non-Gaussian fluctuations

- **Model B:**

N.Abbasi, X.An, S.Wu, in progress

$$\partial_\tau n = \nabla^2\left(\frac{n}{\chi_2} + \lambda_3 n^2 + \lambda_4\, n^3\right) + \text{noise}$$

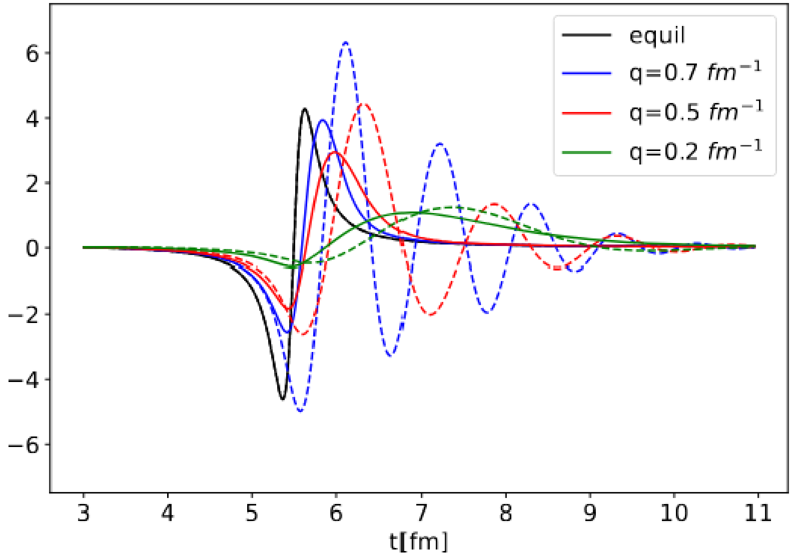
- **Causal Model B:**

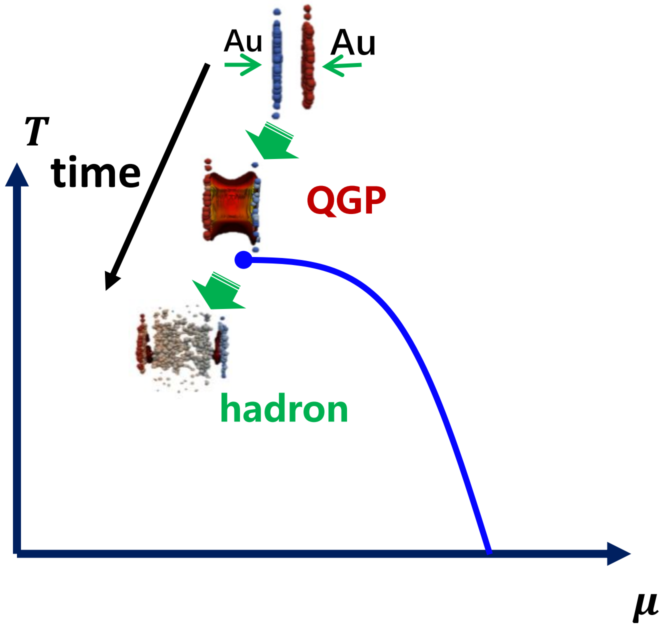
- $(1+\tau_R\partial_\tau)\,\partial_\tau C_2(q) = -2Dq^2[C_2(q) - T\chi_2]$
- $(1+\tau_R\partial_\tau)\,\partial_\tau C_3(q) = -3Dq^2[C_3(q) + \#C_2(q)^2 + \#C_2(q)]$
- $(1+\tau_R\partial_\tau)\partial_\tau C_4(q) = -4Dq^2[C_4(q) + \#C_2(q)C_3(q) + \#C_2(q)^3 + \cdots]$

Diffusion equation with non-Gaussian fluctuations

N.Abbasi, X.An, S.Wu, in progress

- **Causal Model B:**
- $(1+\tau_R\partial_\tau)\,\partial_\tau C_3(q) = -3Dq^2[C_3(q) + \#C_2(q)^2 + \#C_2(q)]$





Causality induce: enhancement of amplitude, flip the sign

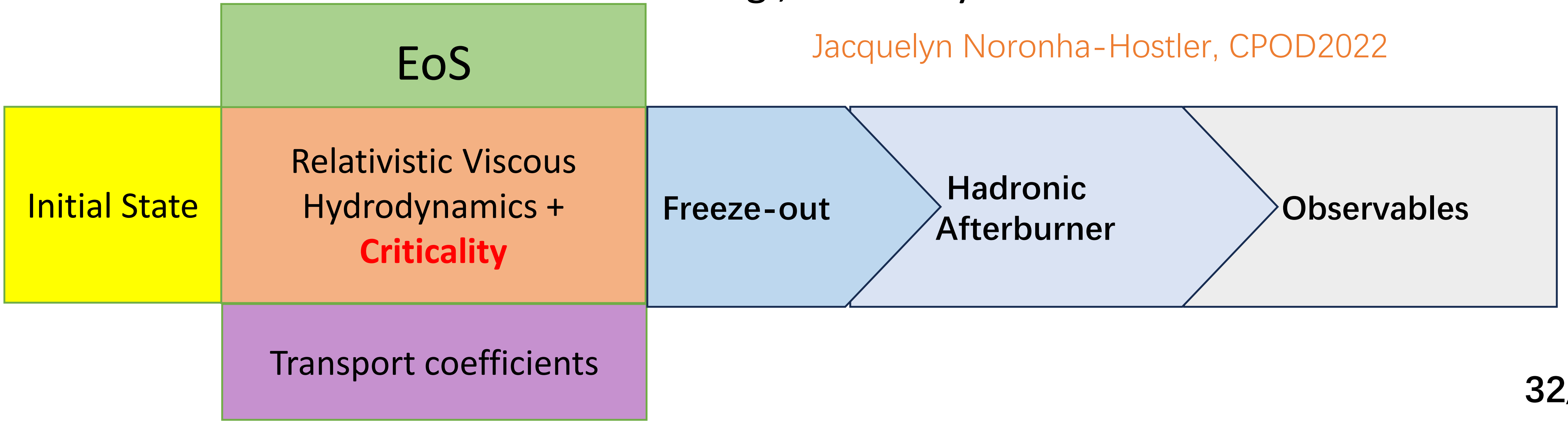
Summary

- Dynamical modeling the QGP evolution near the QCD critical point is essential for the study of fluctuations in heavy-ion experiments;
- Considering the inhomogeneous T and μ profile has significant effects at large rapidity.
- Causality needed to taken into account in the dynamical modeling of the conserved baryon fluctuations

Outlook

- QGP fireball system in heavy-ion experiments is not an ideal system:
  - Fast expanding
  - Finite size
  - Inhomogeneous temperature and chemical potential
  - Volume fluctuation and initial fluctuations
  - Conservation contamination
  - Correction from detector: e.g., efficiency correction

See e.g., M.Asakawa M. Kitazawa, PPNP, arXiv: 1512.05038
X.Luo, N.Xu, NST, arXiv: 1701.02105
S.Wu, C.Shen, H.Song, CPL, arXiv: 2104.13250



Jacquelyn Noronha-Hostler, CPOD2022

Thank You!