Universal Pseudo-Goldstone Damping from the Real-Time Functional Renormalization Group

The QCD Phase Diagram: From Theory to Experimental Signatures, Dalian

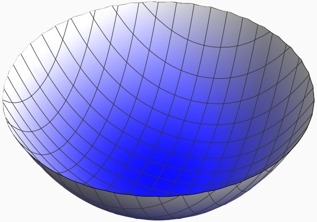
Yang-yang Tan

Based on: Nature Commun. 16 (2025)2916,
Collaborate with Yong-rui Chen & Wei-jie Fu & Wei-jia Li.

October 10, 2025

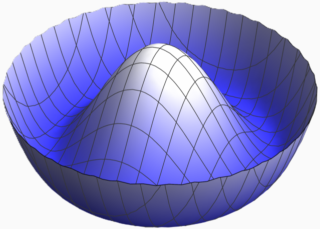
fQCD Collaboration: J. Braun, Y.-r. Chen, W.-j. Fu, F. Gao, A. Geißel, C. Huang, F. Ihssen, Y. Lu, J. M. Pawlowski, F. Rennecke, F. Sattler, B. Schallmo, Y.-y. Tan, S. Töpfel, R. Wen, J. Wessely, S. Yin, Z.-n. Wang, and N. Zorbach. (2025)

Symmetry Breaking Patterns and pseudo-Goldstone Bosons



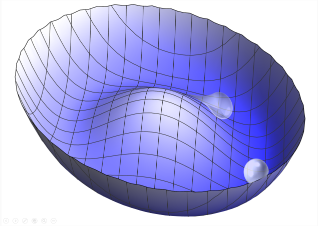
$SU(2)_L \times SU(2)_R \simeq O(4)$

Symmetric phase
Two-flavor QCD with
$m_q \to 0$



$O(4) \to O(3) \times Z_2 \ (\vec{\pi}, \sigma)$

Spontaneous symmetry breaking
π- Goldstone bosons with $m_\pi = 0$



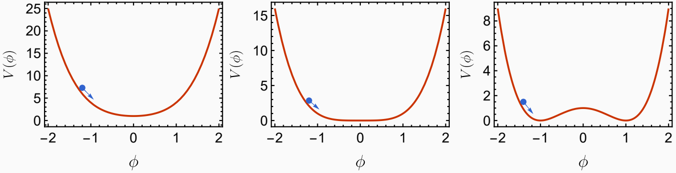
Approximate symmetry spontaneous breaking m_q is small but $\neq 0$
or $c \neq 0$
π- pseudo-Goldstone bosons with $m_\pi \neq 0$

Other low energy physics examples: U(1) (superfluid), translations (crystals, charge density waves), rotations (nematic phases)

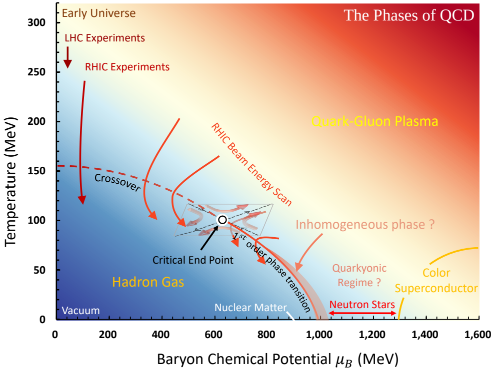
Outline

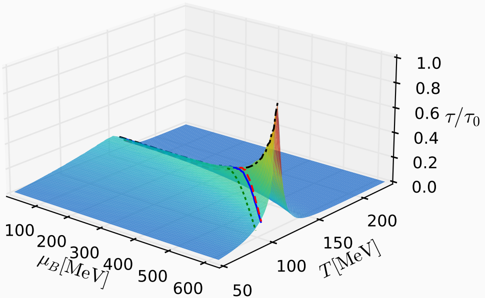
Dynamic Critical Phenomena

Dynamic Critical Phenomena



Critical slowing down
with the relaxation time:
$\tau = \xi^{z} f(k\xi)$
z:dynamic critical exponent

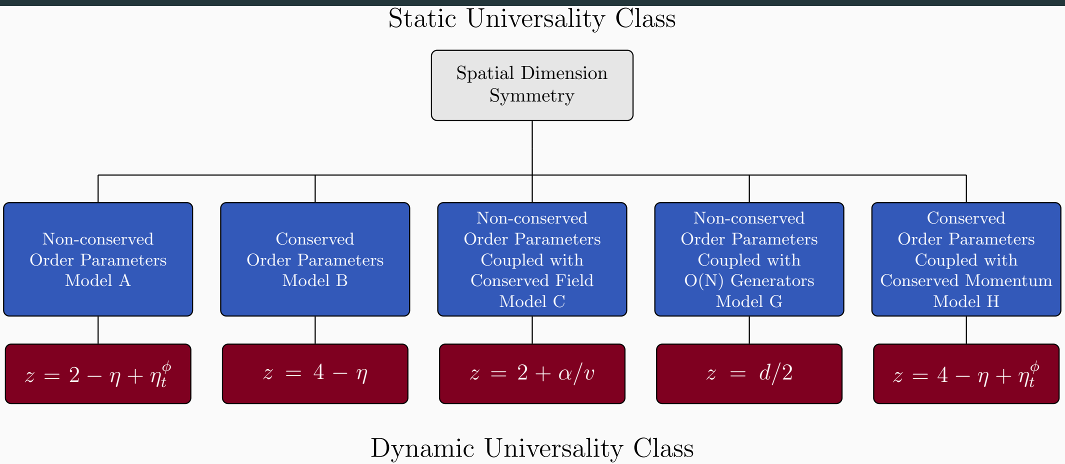




YT, Yin, Chen, Huang, Fu, in preparation

- Nonperturbative approach
- Real-time description

Dynamic Universality Class



Hohenberg and Halperin, Rev. Mod. Phys. 49 (1977) 435.

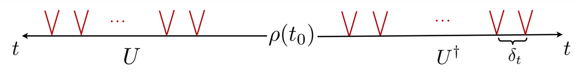
Chiral Phase Transition Model G

Critical End Point Model H

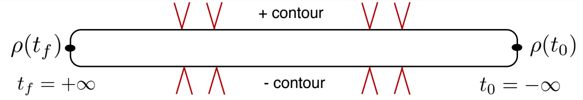
Real-Time Functional Renormalization Group

Schwinger-Keldysh path integral

Schrödinger equation: $i\partial_t|\psi(t)\rangle = H|\psi(t)\rangle \longrightarrow |\psi(t)\rangle = U\left(t, t_0\right)\left|\psi\left(t_0\right)\right\rangle$



von Neumann equation: $i\frac{\partial}{\partial t}\rho(t) = [H(t), \rho(t)] \longrightarrow \rho(t) = U\left(t, t_0\right)\rho\left(t_0\right)U^\dagger\left(t, t_0\right)$



Schwinger, J. Math. Phys. 2, 407 (1961);
Keldysh, Zh. Eksp. Teor. Fiz. 47, 1515 (1964);
Chou, Su, Hao, Yu, Phys. Rept. 118, 1 (1985).

Keldysh partition function: $Z = \mathrm{Tr}\rho$

Sieberer, Buchhold, Diehl. Rept.Prog.Phys. 79 (2016) 9, 096001.

Functional renormalization group approach

$$Z_k[J] = \int \mathcal{D}\varphi \exp\left(-S[\varphi] + J\cdot\varphi - \Delta S_k[\varphi]\right)$$

where$\Delta S_k[\varphi] = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}\varphi(-q)R_k(q)\varphi(q)$

The scale dependent effective action:

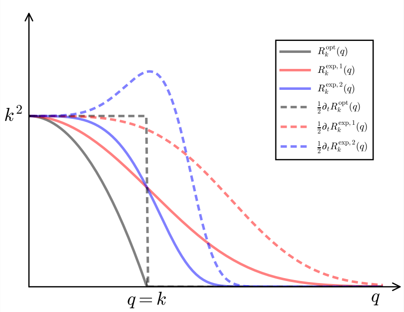
$$\Gamma_k[\phi] = -\ln Z_k[J] + \int d^d x J(x)\phi(x) - \Delta S_k[\phi]$$

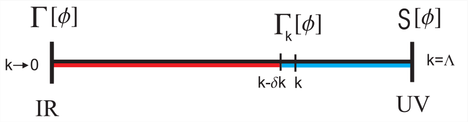
Wetterich Equation :

$$k\partial_k\Gamma_k[\phi] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)}[\phi] + R_k\right)^{-1} k\partial_k R_k\right]$$

The scale evolution equation for n- point function(one loop exact)

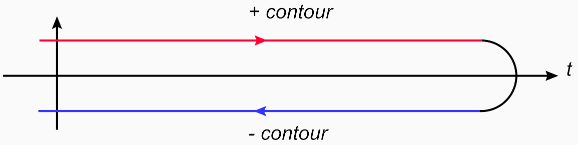
$$\partial_t\Gamma_k^{(n)}[\phi] = \tilde{\partial}_t\left(\begin{array}{c}\text{all one - loop correction}\\ \text{diagrams of } \Gamma_k^{(n)}[\phi]\end{array}\right)$$





- Derivative expansion
- Vertex expansion

fRG in Keldysh path integral



$$\varphi_c = \frac{1}{\sqrt{2}}(\varphi_+ + \varphi_-)\,, \quad \varphi_q = \frac{1}{\sqrt{2}}(\varphi_+ - \varphi_-)$$

The flow equation in the closed time path :

$$\partial_\tau \Gamma_k[\Phi] = \frac{i}{2}\mathrm{Tr}\left[(\partial_\tau R_k^*)\,(\Gamma_k^{(2)}[\Phi] + R_k)^{-1}\right]$$
$$\equiv \frac{i}{2}\mathrm{Tr}\left[(\partial_\tau R_k^*)\,G_k\right]$$

YT, Chen, Fu, SciPost Phys. 12 (2022) 026

- Propagator

$$G_k = \begin{pmatrix} G_k^K & G_k^R \\ G_k^A & 0 \end{pmatrix}$$

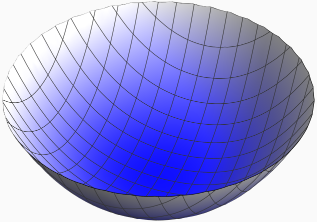
- Regulator

$$R_k = \begin{pmatrix} 0 & R_k^A \\ R_k^R & R_k^F \end{pmatrix}$$

$G^R = (G^A)^*$ and the Regulator $R_k^F = 0$. We use frequency-independent flat regulator to maintain the causal structure: $R_k^R(\omega, \boldsymbol{q}) = R_k^A(\omega, \boldsymbol{q}) = -Z_k(k^2 - \boldsymbol{q}^2)\theta(k^2 - \boldsymbol{q}^2)$

Universal damping of pseudo-Goldstone

Symmetry breaking of chiral phase transition



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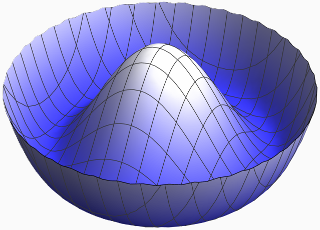
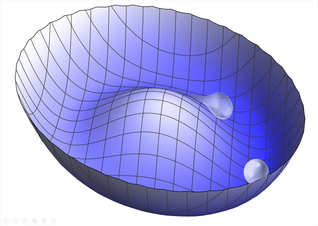


$O(4) \to O(3) \times Z_2 \;\; (\vec{\pi}, \sigma)$

Spontaneous symmetry breaking
π- Goldstone bosons with $m_\pi = 0$



Approximate symmetry spontaneous breaking m_q is small but $\neq 0$
or $c \neq 0$
π- pseudo-Goldstone bosons with $m_\pi \neq 0$

$O(N)$ Effective action on the S-K contour

$$\Gamma[\phi_c, \phi_q] = \int d^4x \left(Z_a^{(t)} \phi_{a,q} \, \partial_t \, \phi_{a,c} - Z_a^{(i)} \phi_{a,q} \, \partial_i^2 \, \phi_{a,c} + V'(\rho_c) \, \phi_{a,q} \, \phi_{a,c} - 2 \, Z_a^{(t)} \, T \, \phi_{a,q}^2 - \sqrt{2} c \, \sigma_q \right)$$

Universal damping of pseudo-Goldstone

The real-time dispersion relation of pseudo-Goldstone:

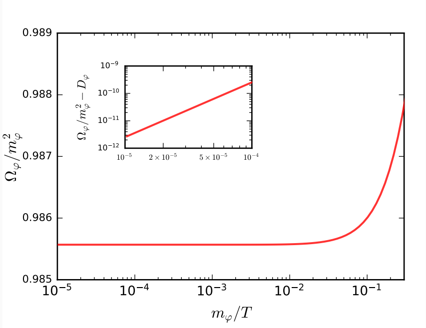
$$\omega(\boldsymbol{p}) = \pm c_s \sqrt{m_\varphi^2 + \boldsymbol{p}^2} - \frac{i}{2}\left[\Omega_\varphi + (D_Q + D_\varphi)\,\boldsymbol{p}^2\right] + \cdots$$

Ω_φ: relaxation/damping rate, D_φ: Goldstone diffusivity,

D_Q:charge diffusivity, m_φ: screening mass, c_s:speed of sound

$$\frac{\Omega_\varphi}{m_\varphi^2} \simeq D_\varphi + \mathcal{O}\left(m_\varphi^2/T^2\right)$$

$T = 0.3T_c$



$$\frac{\Omega_\varphi}{m_\varphi^2} - D_\varphi(T) \propto \left(\frac{m_\varphi}{T}\right)^\alpha$$

$$\alpha = 2.00021(2)$$

Holographics:
Amoretti, Areán, Goutéraux, Musso, PRL 123 (2019) 211602; Amoretti, Areán, Goutéraux, Musso, JHEP 10 (2019) 068; Ammon et al., JHEP 03 (2022) 015; Cao, Baggioli, Liu, Li, JHEP 12 (2022) 113
Hydrodynamics:
Delacrétaz, Goutéraux, Ziogas, PRL 128 (2022) 141601
EFT:
Baggioli, Phys. Rev. Res. 2 (2020) 022022;
Baggioli, Landry, SciPost Phys. 9 (2020) 062

Universal damping of pseudo-Goldstone near the critical point

The relaxation/damping rate in Model A:

$$G^R_{\varphi\varphi}(\omega,q)=\frac{1}{-\mathrm{i}Z^{(t)}_\varphi\omega+Z^{(i)}_\varphi\left(q^2+m^2_\varphi\right)}\to\ \frac{\Omega_\varphi}{m^2_\varphi}=\frac{Z^{(i)}_\varphi}{Z^{(t)}_\varphi}$$

scaling function

$$Z^{(i)}_\varphi=t^{-\nu\eta}f^{(i)}(\mathbb{z})\,,\quad Z^{(t)}_\varphi=t^{-\nu\eta_t}f^{(t)}(\mathbb{z})$$

with scaling variable $\mathbb{z}\equiv tc^{-1/(\beta\delta)}$

$$\frac{Z^{(i)}_\varphi}{Z^{(t)}_\varphi}\propto t^{\nu(\eta_t-\eta)}\,,\quad \frac{Z^{(i)}_\varphi}{Z^{(t)}_\varphi}\propto c^{\frac{\nu}{\beta\delta}(\eta_t-\eta)}$$

Gell-Mann–Oakes–Renner relation

$$m^2_\varphi=\frac{V'(\rho_0)}{Z^{(i)}_\varphi}=\frac{c}{\sigma_0 Z^{(i)}_\varphi}$$

$$\frac{\Omega_\varphi}{m^2_\varphi}\propto m_\varphi^{\Delta_\eta},\ \ \Delta_\eta\equiv\eta_t-\eta>0$$

$$\Delta_\eta\simeq 0.0172$$

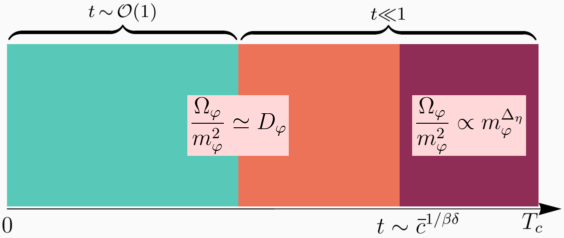
GMOR also breaks down with(only relates to the static properties):

$$m^2_\varphi\propto c^{\frac{2\nu}{\beta\delta}}\simeq c^{0.806}$$

mean-field theory with:

$$m^2_\varphi\simeq c^{2/3}$$

Universal damping of pseudo-Goldstone near the critical point

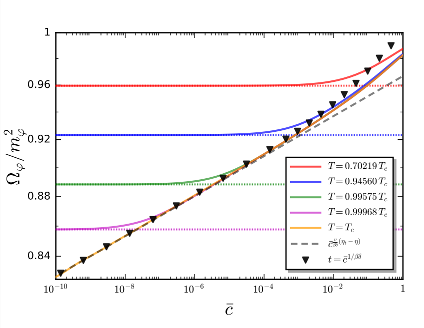


$$\frac{\Omega_\varphi}{m_\varphi^2} \propto m_\varphi^{\Delta_\eta}, \;\; \Delta_\eta \equiv \eta_t - \eta > 0$$

$$\Delta_\eta \simeq 0.0172$$

Critical region of chiral phase transition:

$$m_{\pi 0} \lesssim 0.1 \sim 1\mathrm{MeV}$$



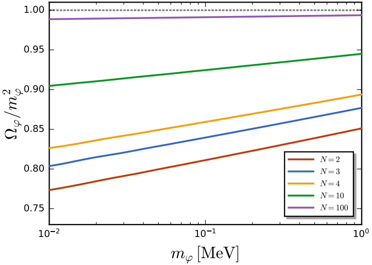
Large N limit

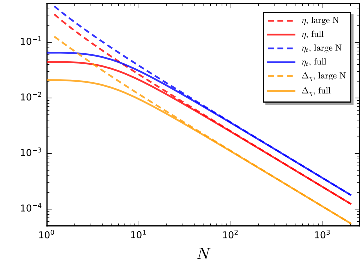
Neglect sigma mode in the fRG flow equation we have:

$$\eta = \frac{5}{N-1}\frac{(1+\eta)(1-2\eta)^2}{(5-\eta)(2-\eta)^2}$$

$$\eta_t = \frac{1}{9(N-1)}\frac{(1-2\eta)^2\left(13+15\eta-2\eta^3\right)}{(2-\eta)^2}$$

In the large N limit $\eta \propto 1/N$,$\eta_t \propto 1/N$ and $\Delta_\eta \propto 1/N$





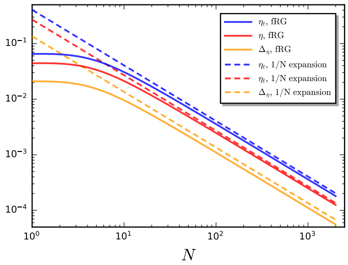
The large N expansion of η and η_t from Wilson's RG:

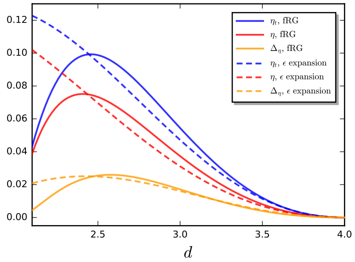
$$\eta = 4N^{-1}\left(\frac{4}{d} - 1\right) S_d + \mathcal{O}\left(N^{-2}\right)$$

with

$$S_d = \frac{\sin\left[\pi\left(\frac{d}{2} - 1\right)\right]}{\pi\left(\frac{d}{2} - 1\right) B\left(\frac{d}{2} - 1, \frac{d}{2} - 1\right)},$$

$$\eta_t = \left[\left(\frac{4}{4-d}\right)\left(\frac{dB\left(\frac{d}{2} - 1, \frac{d}{2} - 1\right)}{8\int_0^{1/2} dx [x(2-x)]^{d/2-2}} - 1\right) + 1\right]\eta.$$





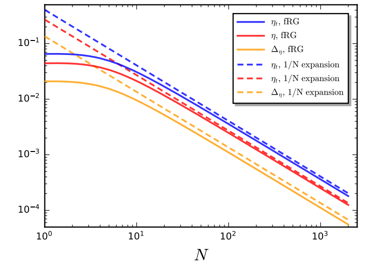
Comparison to the $1/N$ expansion and ϵ expansion

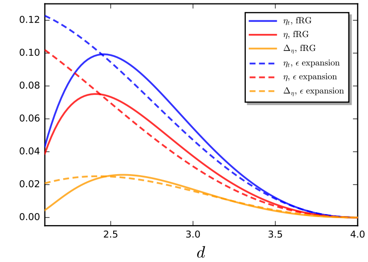
The $\epsilon = 4 - d$ expansion of η and η_t from Wilson's RG:

$$
\begin{aligned}
&\eta \\
&= \frac{N+2}{2(N+8)^2}\epsilon^2 + \frac{N+2}{2(N+8)^2}\left[\frac{6(3N+14)}{(N+8)^2} - \frac{1}{4}\right]\epsilon^3 \\
&\quad + \frac{N+2}{32(N+8)^6}\Big[-5N^4 - 230N^3 + 1124N^2 \\
&\quad - 768(5N+22)(N+8)\times 0.60103 \\
&\quad + 17920N + 46144\Big]\epsilon^4 + \mathcal{O}(\epsilon^5)
\end{aligned}
$$

and

$$
\begin{aligned}
\eta_t = &\Big[1 + \Big(1 - 0.188483417\epsilon - 0.099952926\epsilon^2 \\
&+ \mathcal{O}(\epsilon^3)\Big) \times \left(6\ln\left(\frac{4}{3}\right) - 1\right)\Big]\eta
\end{aligned}
$$





Pseudo-Goldstone in Model G

Model G

The equations of motion for Model G:

$$Z_\varphi^{(t)}\frac{\partial\phi_a}{\partial t} = -\frac{\delta F[\phi,n]}{\delta\phi_a} + Z_\varphi^{(t)}\frac{g}{2}\{\phi_a, n_{bc}\}\frac{\delta F[\phi,n]}{\delta n_{bc}} + \theta_a$$

$$Z_n^{(t)}\frac{\partial n_{ab}}{\partial t} = \partial_i^2\frac{\delta F[\phi,n]}{\delta n_{ab}} + Z_n^{(t)} g\{n_{ab},\phi_c\}\frac{\delta F[\phi,n]}{\delta\phi_c} + Z_n^{(t)}\frac{g}{2}\{n_{ab}, n_{cd}\}\frac{\delta F[\phi,n]}{\delta n_{cd}} + \partial_i\zeta_{ab}^i$$

with the free energy functional reads

$$F[\phi,n] = \int d^dx\left(\frac{1}{2}Z_\varphi^{(i)}(\partial_i\phi_a)(\partial_i\phi_a) + V(\rho) - c\sigma + \frac{1}{4}Z_n^{(i)} n_{ab}n_{ab}\right)$$

and the Poisson bracket:

$$\{\phi_a, n_{bc}\} = \delta_{ac}\phi_b - \delta_{ab}\phi_c\ ,$$

$$\{n_{ab}, n_{cd}\} = \delta_{ac}n_{bd} + \delta_{bd}n_{ac} - \delta_{ad}n_{bc} - \delta_{bc}n_{ad}\ .$$

Damping and propagation of pseudo-Goldstone in ModelG

O(N) Symmetry

Non conserved Fields: ϕ_a

Conserved charge densities: n_{ab}

$$\xrightarrow[\textit{Symmetry Breaking}]{\langle\phi_0\rangle = \sigma_0 \neq 0}$$

Decoupled Modes

Higgs: $\delta\sigma$

Vector charge: n_{ij}

(O($N-1$) generators)

Coupled Modes

Goldstone: φ_i

$g\sigma_0$ ↓ ↑ $g\sigma_0,\ c$

Axial charge: n_{0i}

(Broken generators)

Coupled (φ_i, n_{0i}) System:

Leads to damping and propagation for dispersion $\omega_\pm(q)$

Far below T_c ($T \ll T_c$):

Damping $\sim m_\varphi^2$, Propagation $\sim m_\varphi$ (Dominant)

Near T_c ($T \sim T_c$):

Damping rate $\Omega_\varphi \sim \frac{Z_\varphi^{(i)}}{Z_\varphi^{(t)}} m_\varphi^2$ Propagation depends on R

$(R > 1)$ Propagation, $(R < 1)$ Purely Damped

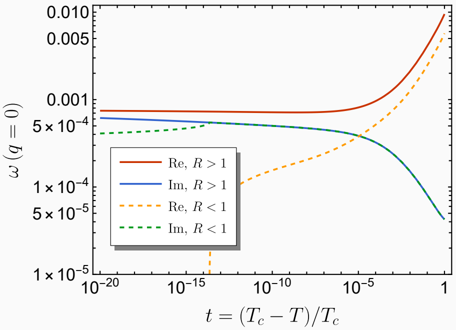
$$R = \frac{4{Z_\varphi^{(t)}}^2 Z_n^{(i)} g^2 \sigma_0^2}{Z_\varphi^{(i)} m_\varphi^2}$$

Damping and propagation of pseudo-Goldstone in ModelG

Model A

$$\frac{\Omega_\varphi}{m_\varphi^2} \propto m_\varphi^{\Delta_\eta}, \ \ \Delta_\eta \equiv \eta_t - \eta > 0$$

3d O(4) Model: $\Delta_\eta \simeq 0.0172$

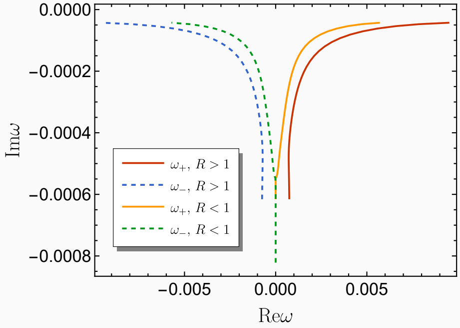


Model G

$$\frac{\Omega_\varphi}{m_\varphi^2} \propto m_\varphi^{\Delta_\eta^G}, \ \ \Delta_\eta^G = \eta_t^G - \eta < 0$$

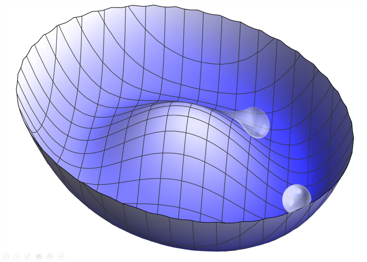
$d$dimensional $O(N)$ Model: $\Delta_\eta^G = (d-4)/2$

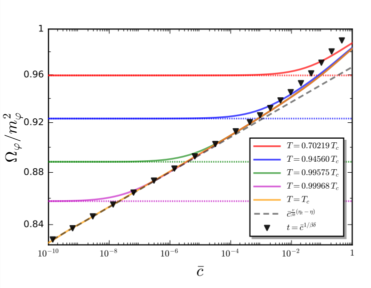
3d O(4) Model: $\Delta_\eta^G = -1/2$



Summary

Summary





- The pseudo-Goldstone damping has a universal relation $\frac{\Omega_\varphi}{m_\varphi^2} \simeq D_\varphi + \mathcal{O}\left(m_\varphi^2/T^2\right)$ in the hydrodynamic regime while breaks down to $\frac{\Omega_\varphi}{m_\varphi^2} \propto m_\varphi^{\Delta_\eta}$, $\Delta_\eta > 0$ in the critical regime.
- The pseudo-Goldstone damping in Model G has a same universal relation $\frac{\Omega_\varphi}{m_\varphi^2} \propto m_\varphi^{\Delta_\eta^G}$, $\Delta_\eta^G = -0.5 < 0$ in the critical regime.