Dissecting the moat regime at low energies I: Renormalization and the phase structure

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Based on: 2510.06712, Fabian Rennecke and **SY**

Outline

- **Introduction**
  - QCD phase diagram
  - Moat regime
- **Moat regime at low energies**
  - Quark-Meson Model
  - Renormalization and phase structure
  - Yukawa Coupling
- **Summary and Outlook**

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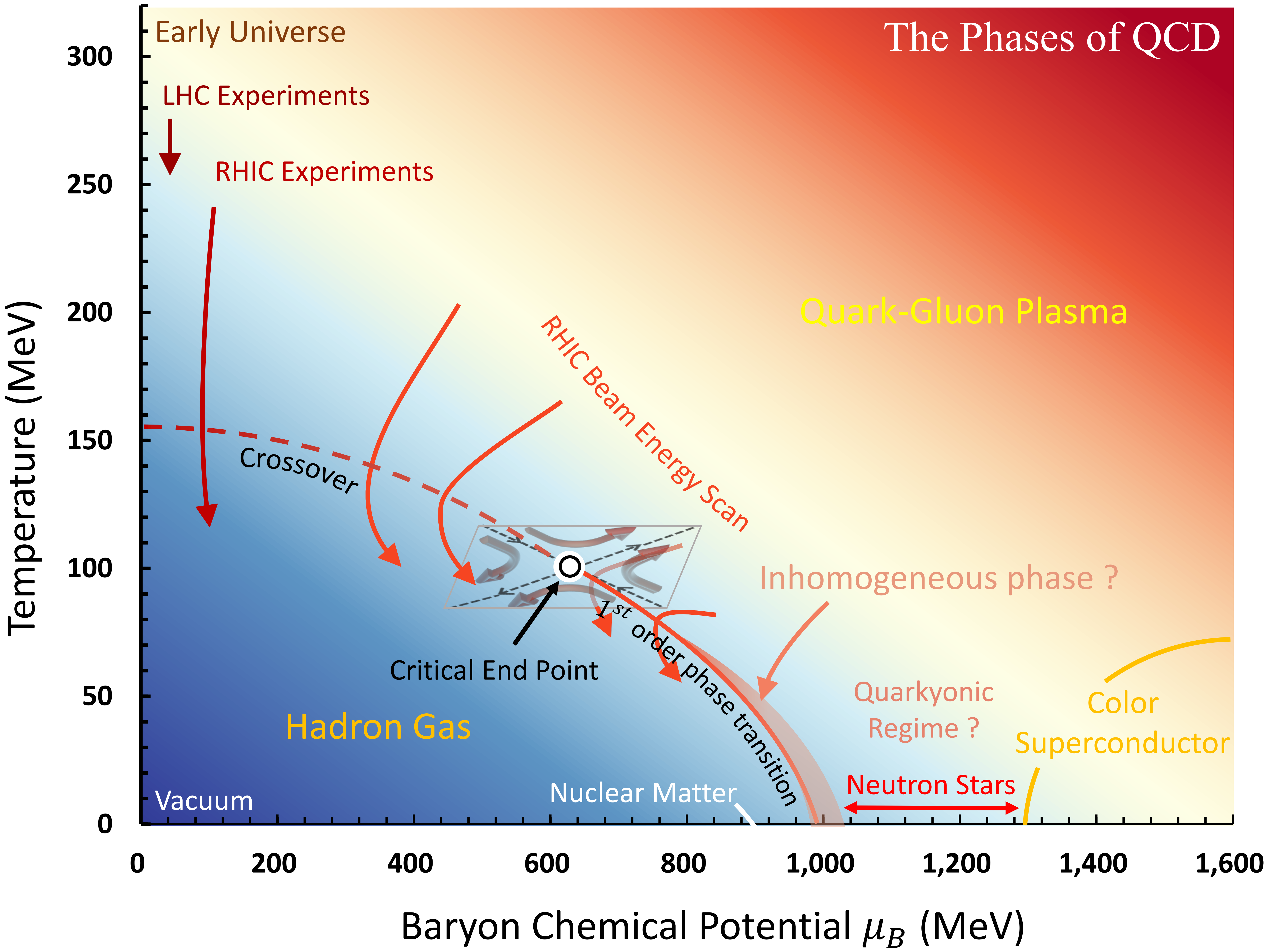
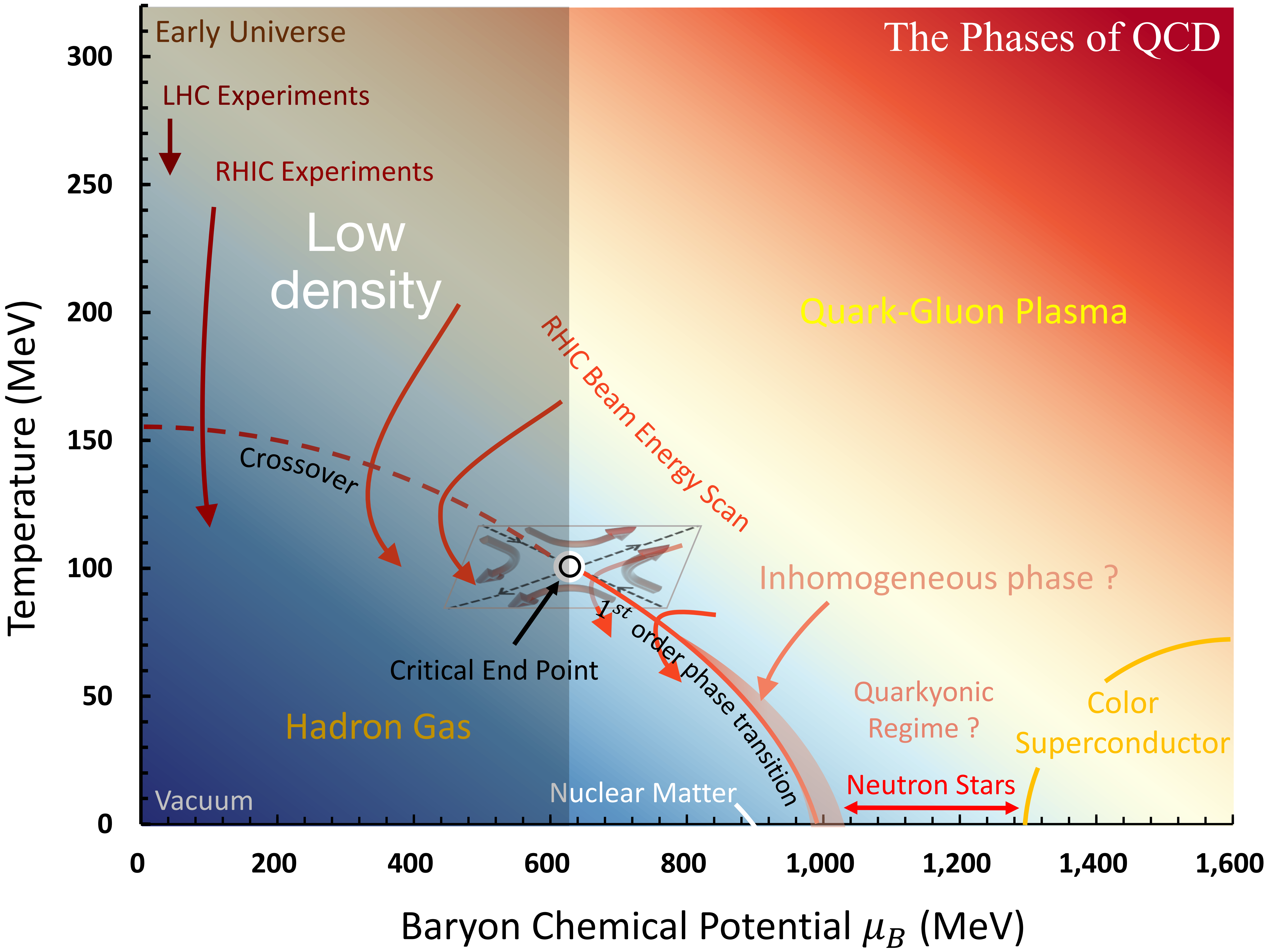
QCD phase diagram



SY, Tan, Fu, Nucl. Tech. 46 (2023) 04, 040002

QCD phase diagram

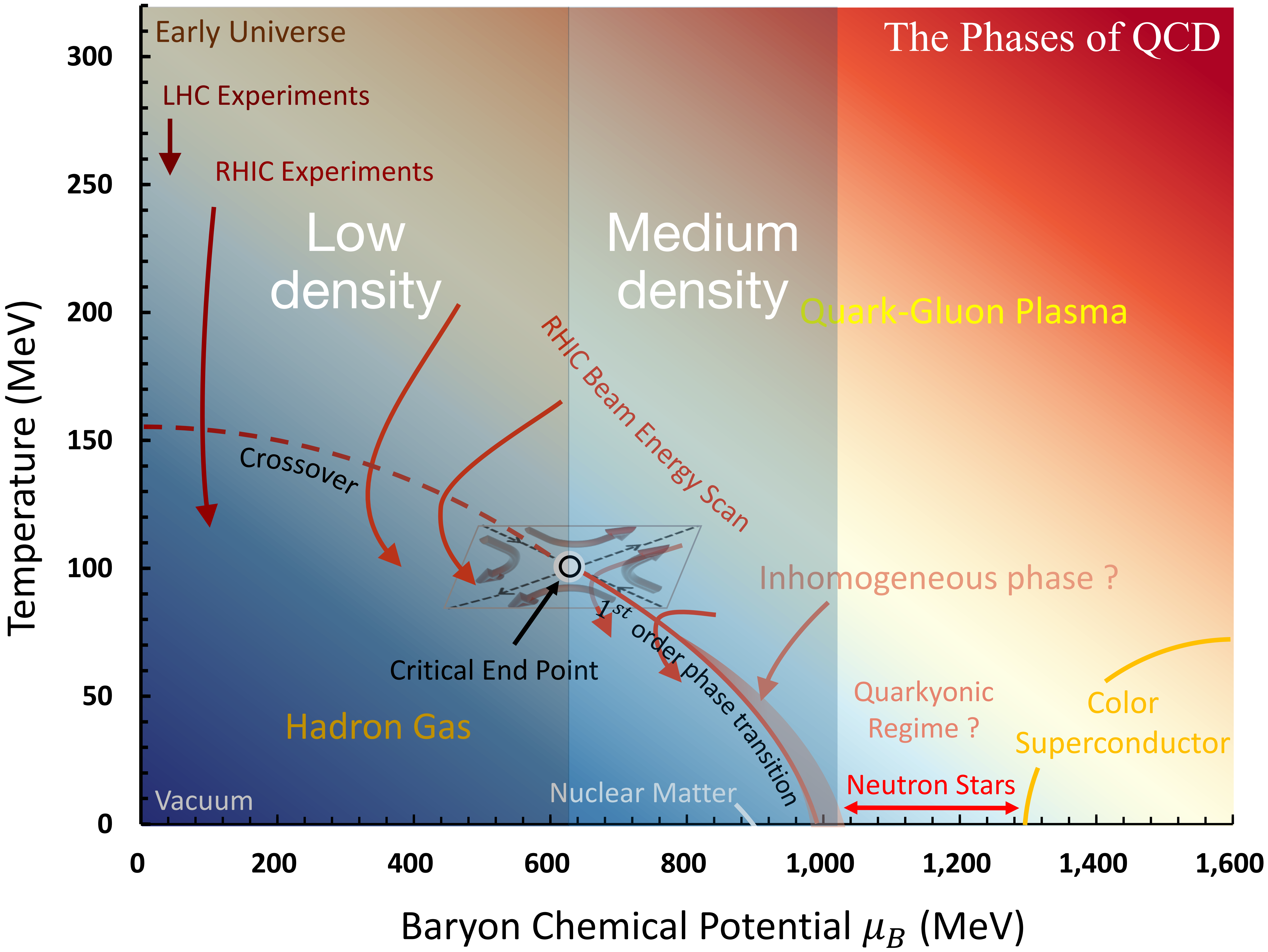


SY, Tan, Fu, Nucl. Tech. 46 (2023) 04, 040002

Low density:

- Chiral crossover
- Critical End Point

QCD phase diagram



SY, Tan, Fu, Nucl. Tech. 46 (2023) 04, 040002

Low density:

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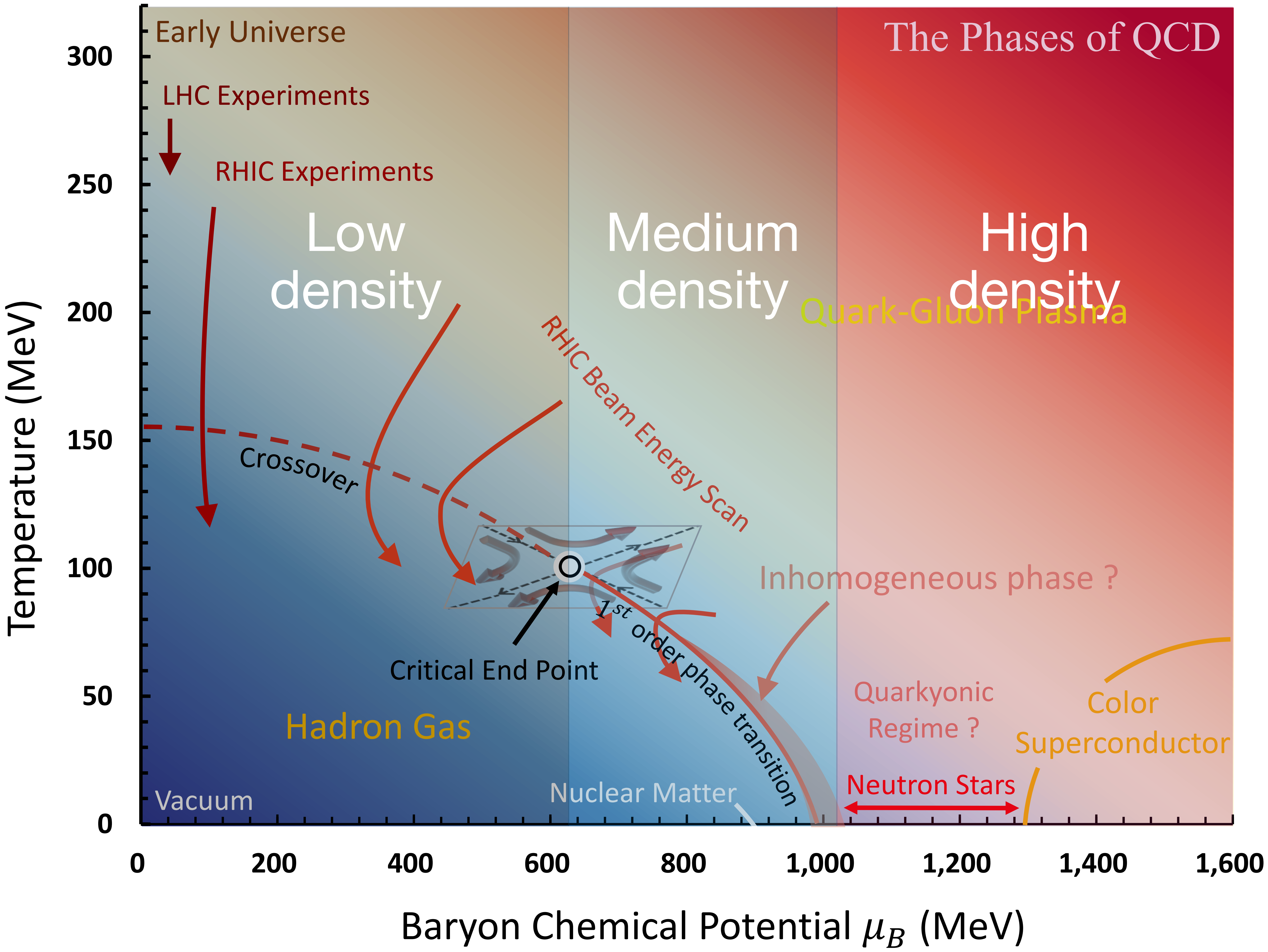
Medium density:

- First-order phase transition
- Onset of new phases (Moat regime, Inhomogeneous phase, Spatial modulation, Liquid crystal phase…)

QCD phase diagram



SY, Tan, Fu, Nucl. Tech. 46 (2023) 04, 040002

Low density:

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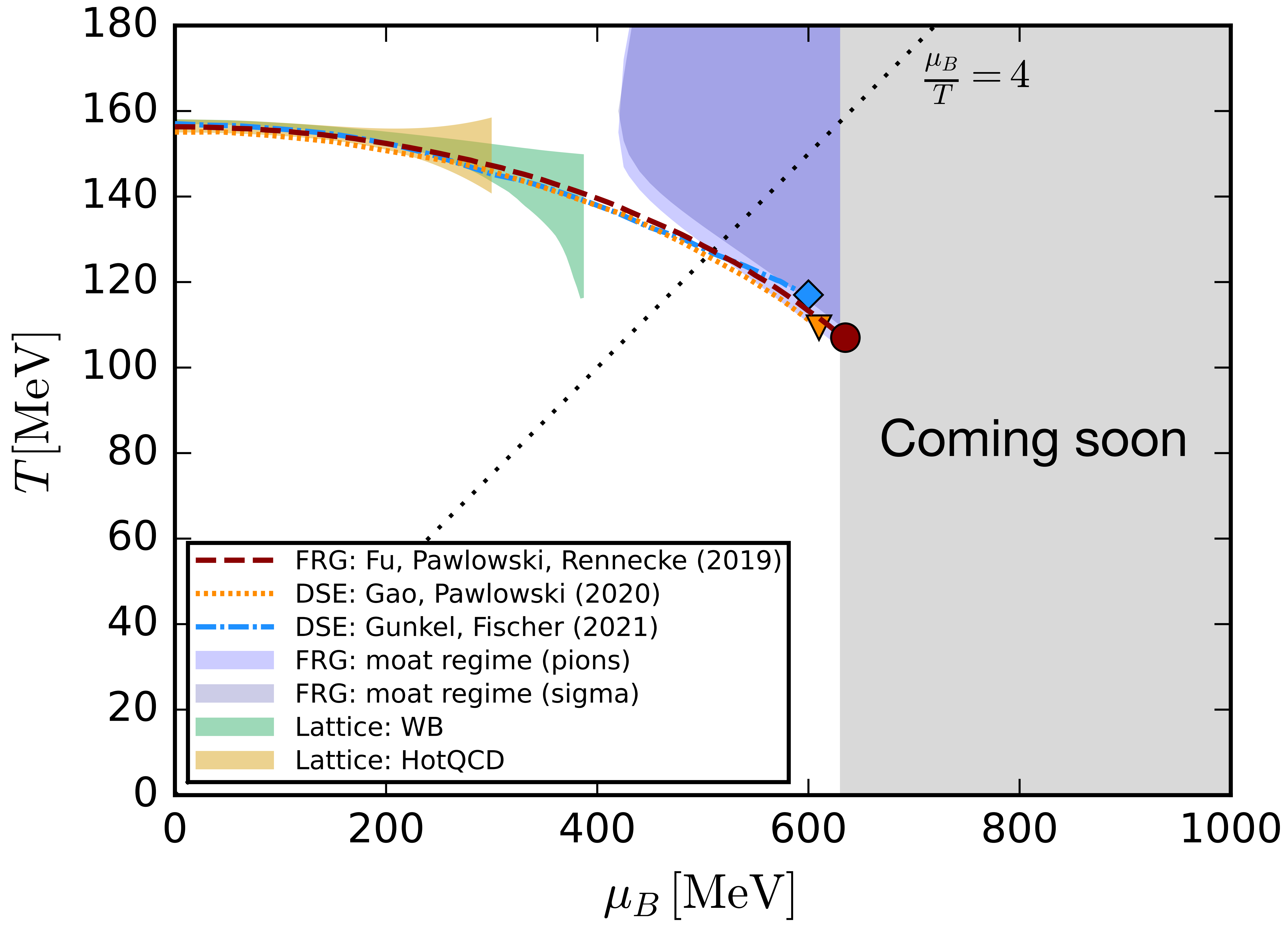
Medium density:

- First-order phase transition
- Onset of new phases
  (Moat regime,
  Inhomogeneous phase,
  Spatial modulation,
  Liquid crystal phase…)

High density:

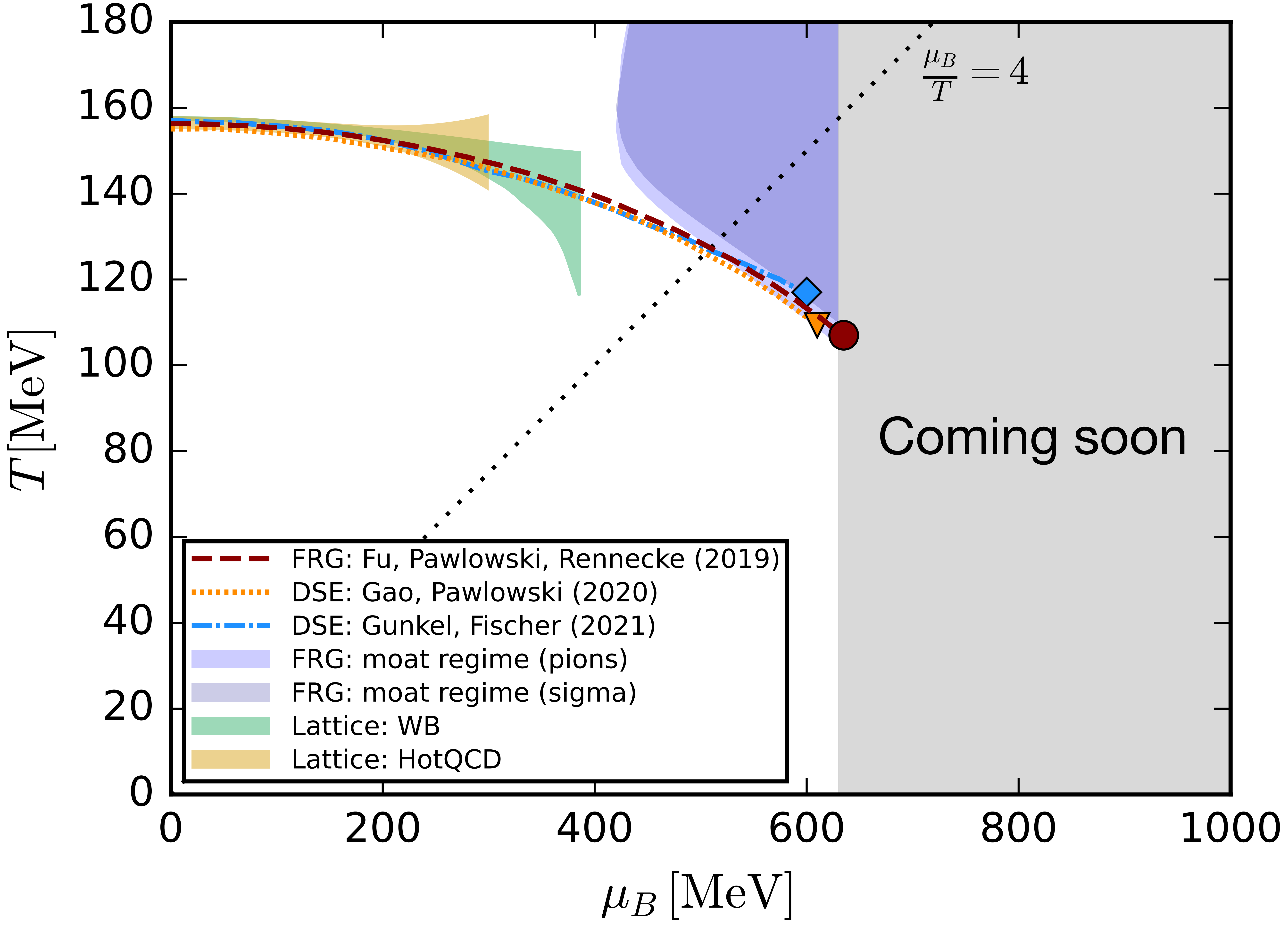
- Neutron Stars
- Color Superconductor
- Color-flavor locking

Prediction of functional methods



fQCD (2025)

Prediction of functional methods



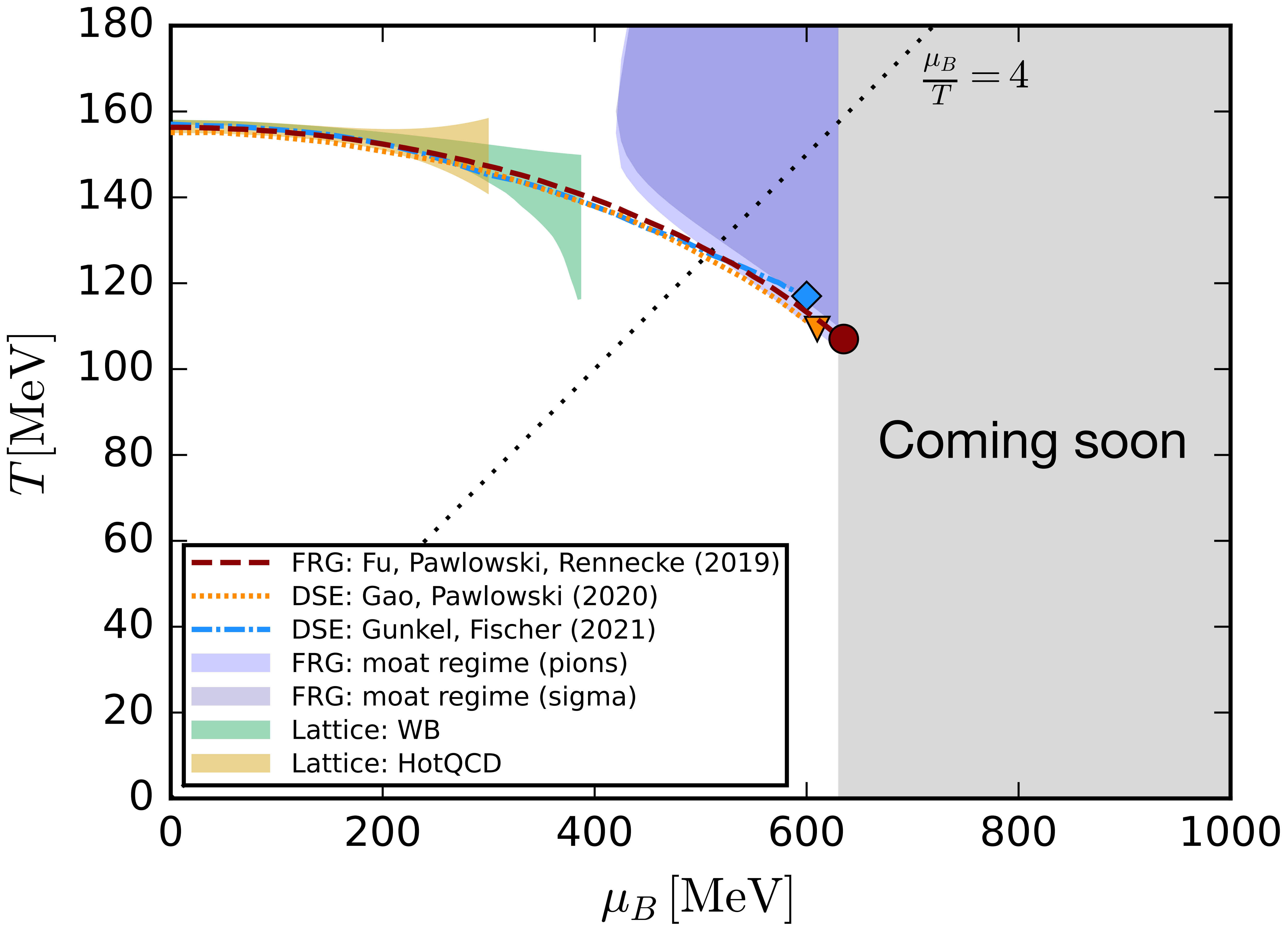
fQCD (2025)

- Negative meson wave function observed in fRG-QCD

$$Z_\phi^\perp = \frac{\partial \Gamma^{(2)}_{\phi\phi}(p^2)}{\partial \mathbf{p}^2}\bigg|_{p_0=0,\mathbf{p}=0}$$

$$\Gamma^{(2)}_{\phi\phi}(p^2) = Z_\phi^\parallel\, p_0^2 + Z_\phi^\perp\, \mathbf{p}^2 + m_\phi^2$$

Prediction of functional methods



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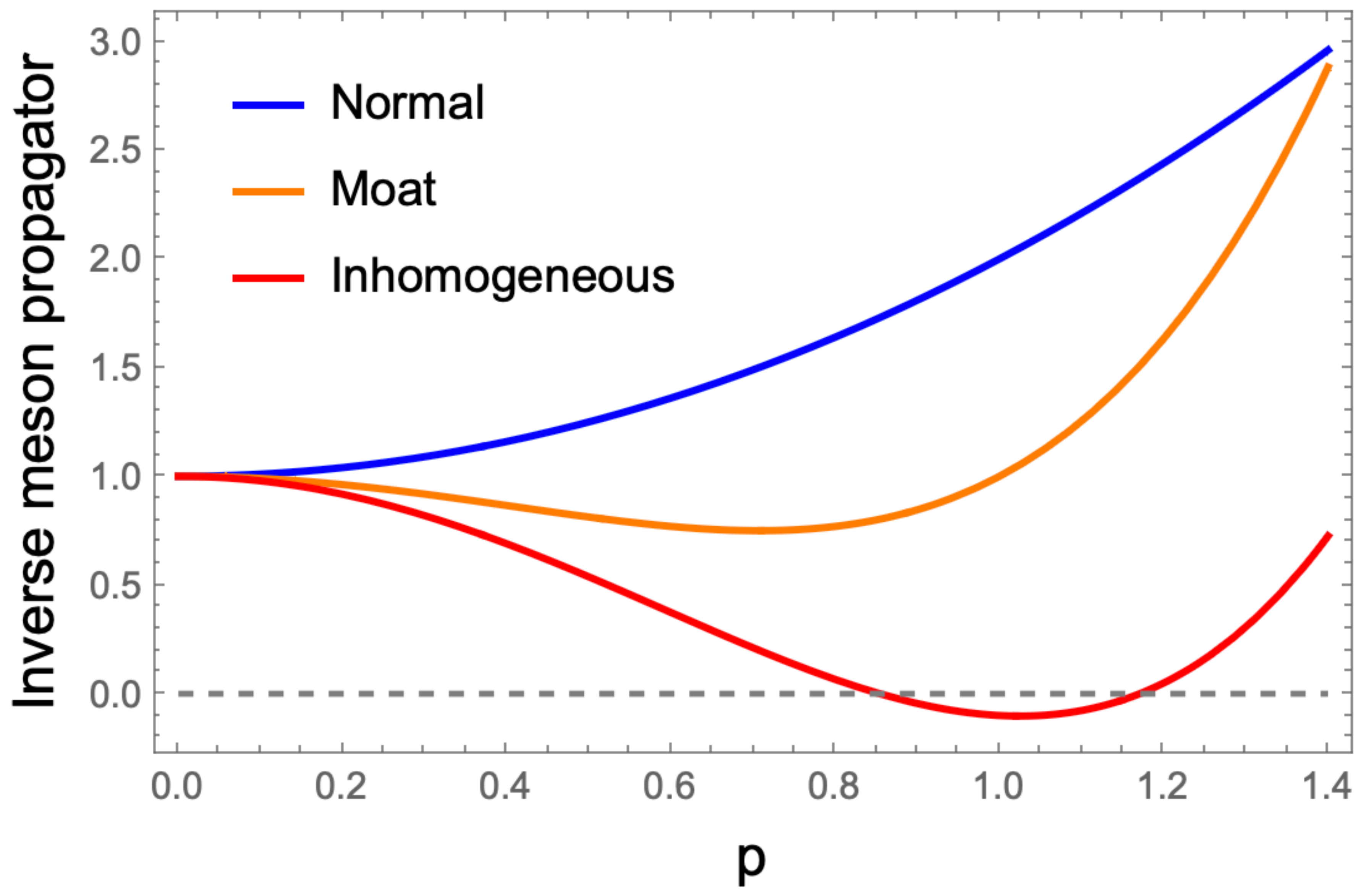
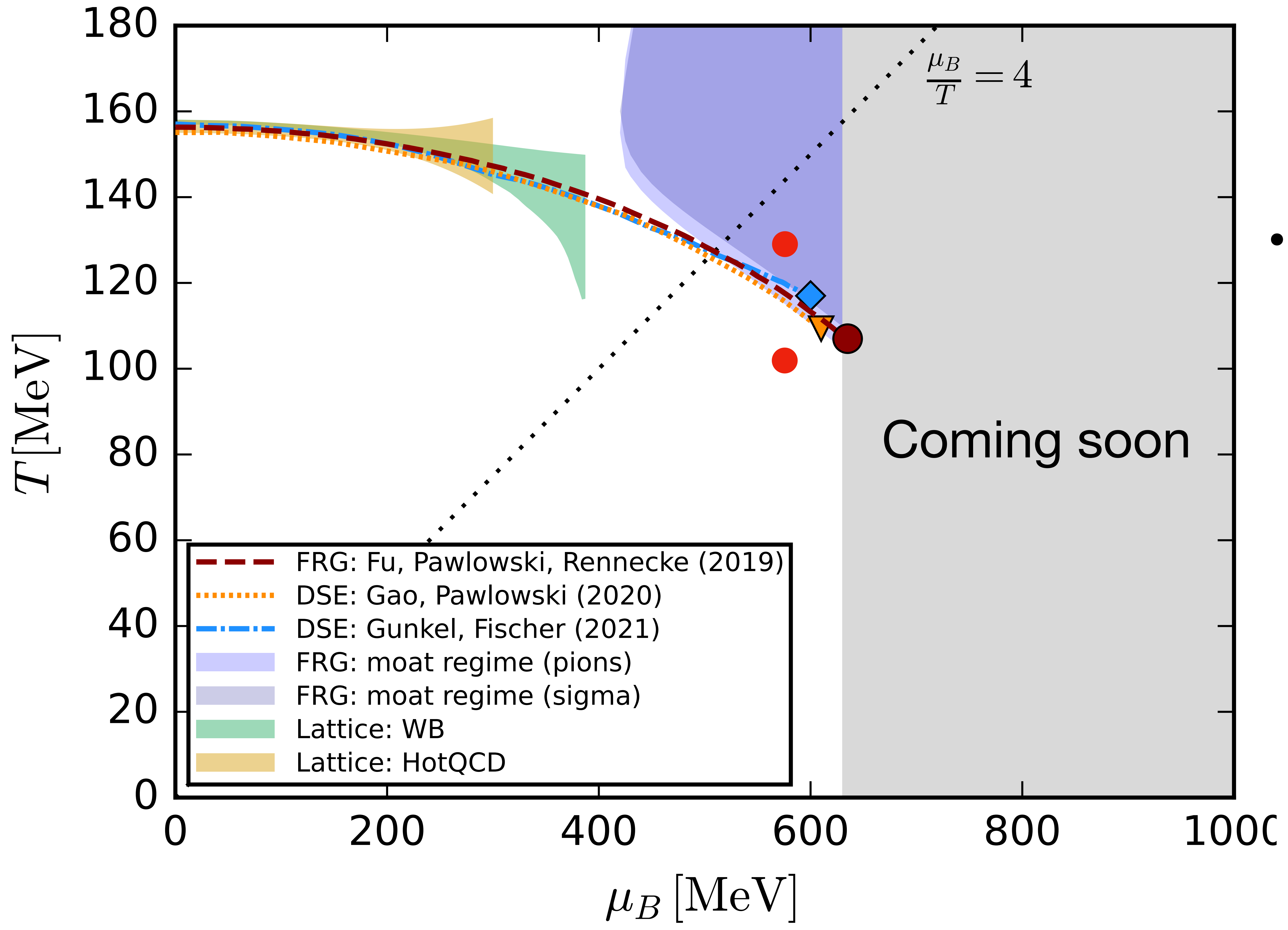
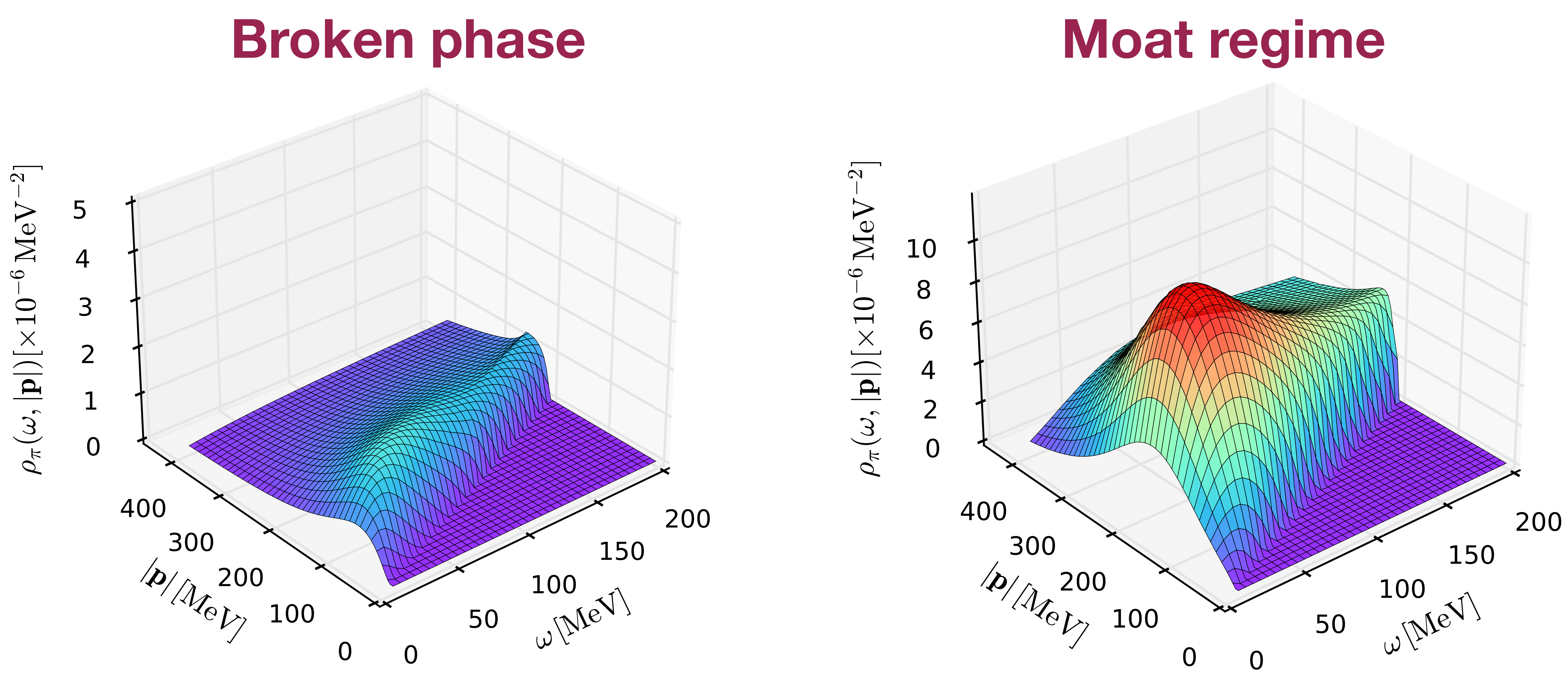


Prediction of functional methods



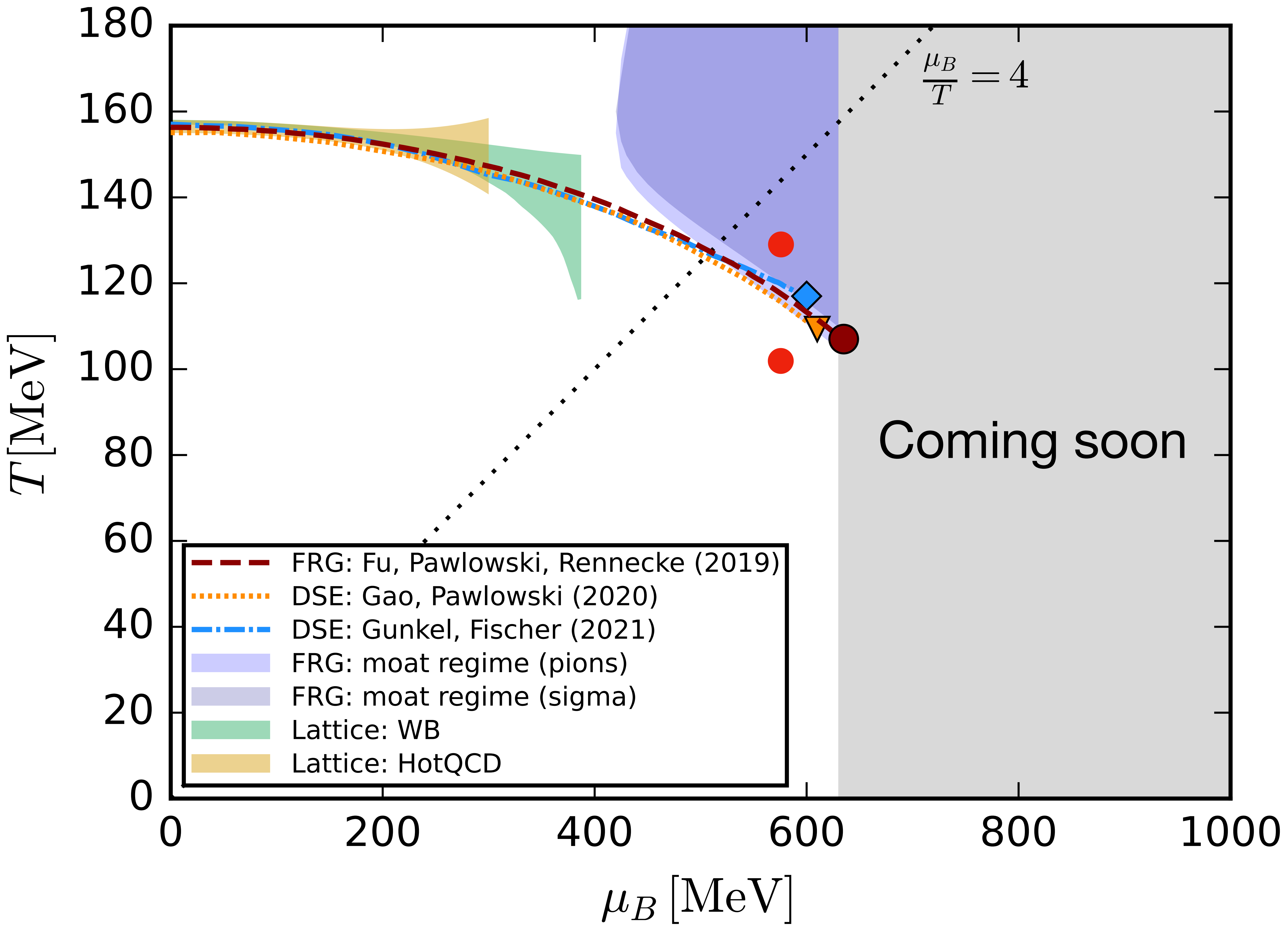
fQCD (2025)

- **Enhance the space-like region of spectral function**



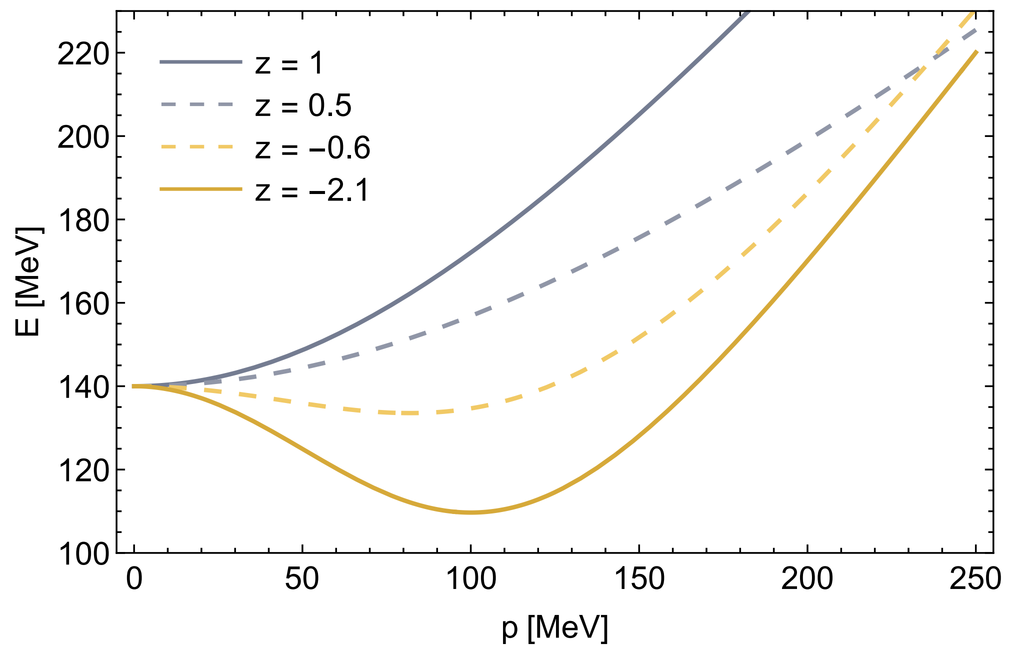
Fu, Pawlowski, Pisarski, Rennecke, Wen, **SY**, arXiv: 2412.15949

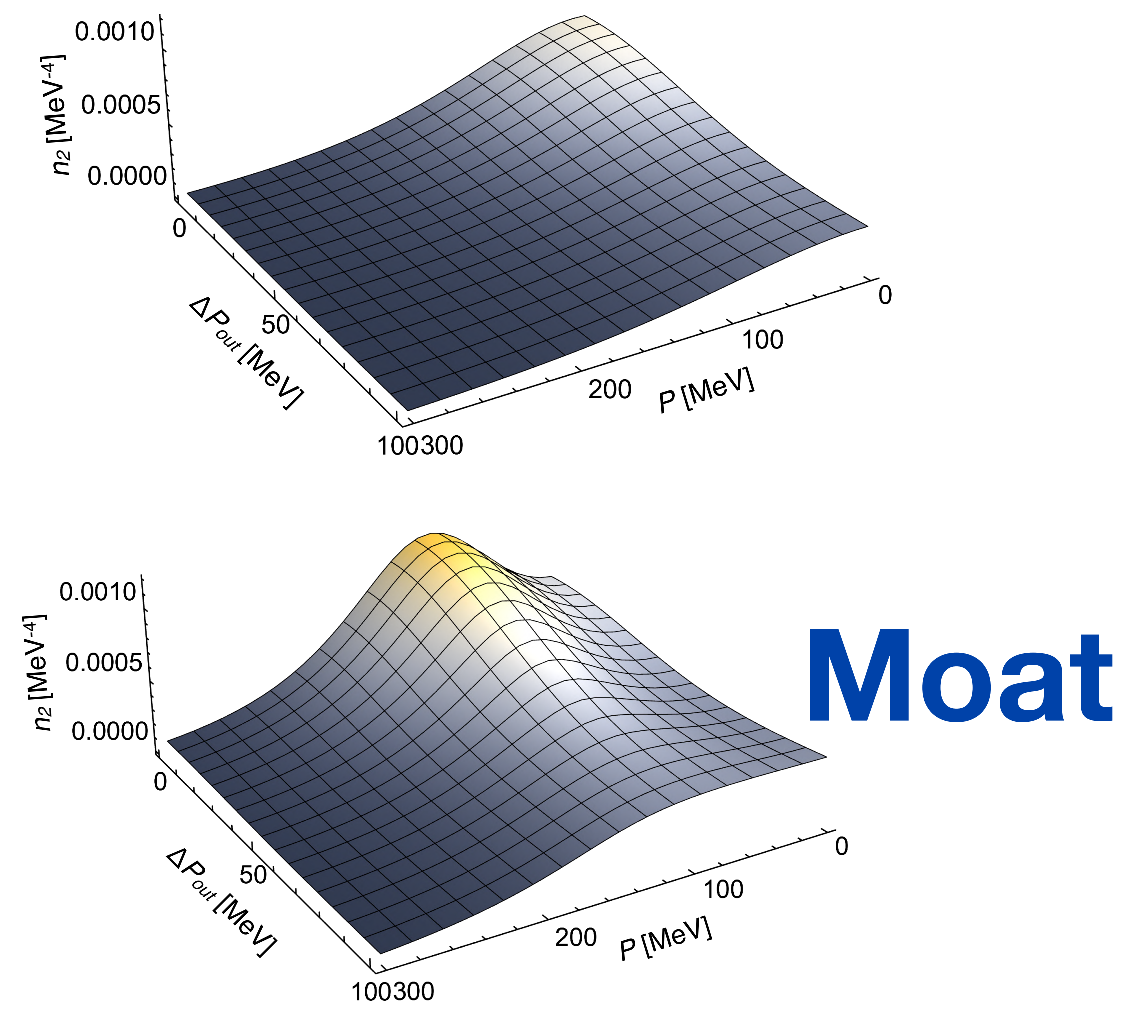
Prediction of functional methods



fQCD (2025)

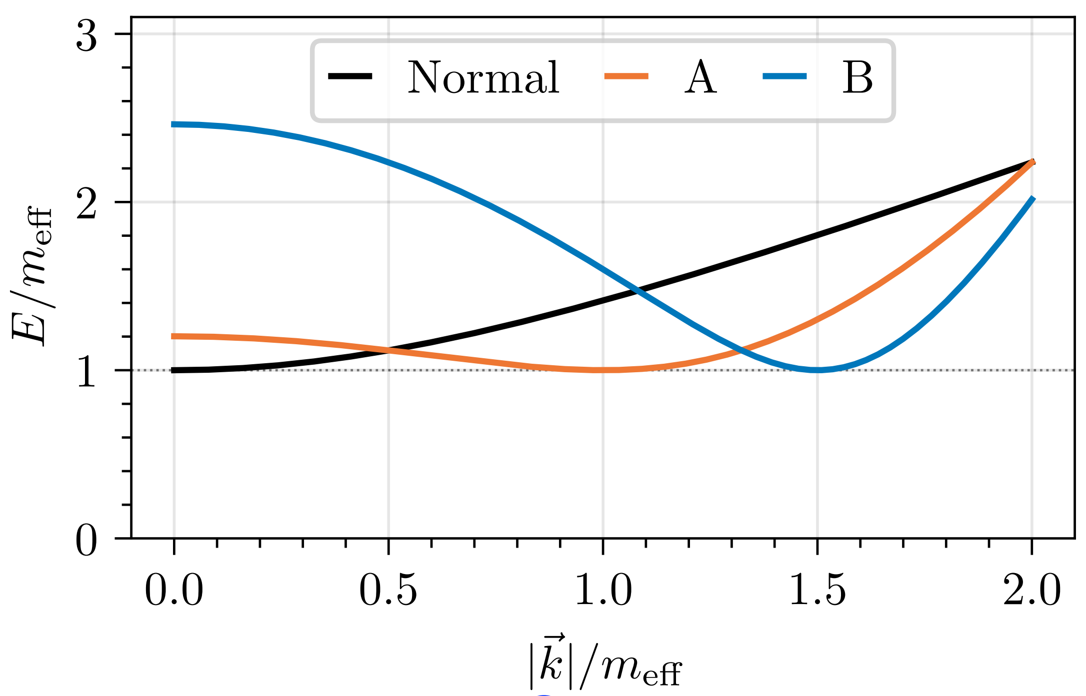
- **Peak in HBT correlation**

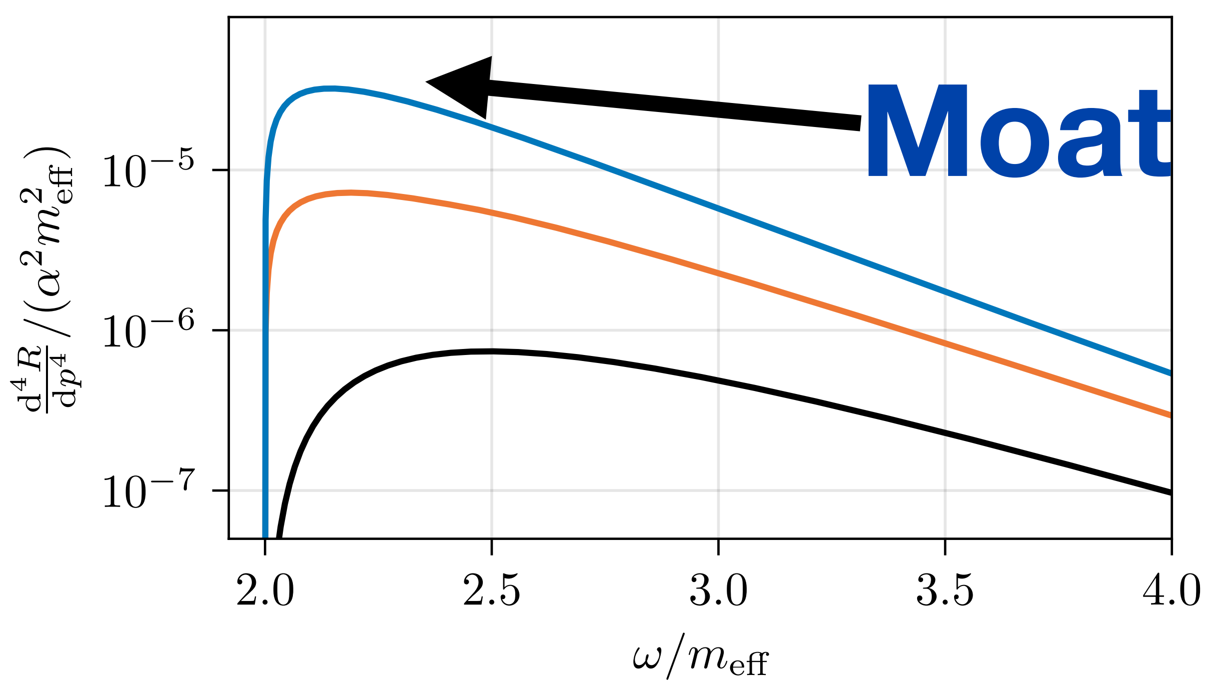




Rennecke, Pisarski, Rischke
Phys. Rev. D 107 (2023) 11, 116011

- **Enhance the dilepton production rate**

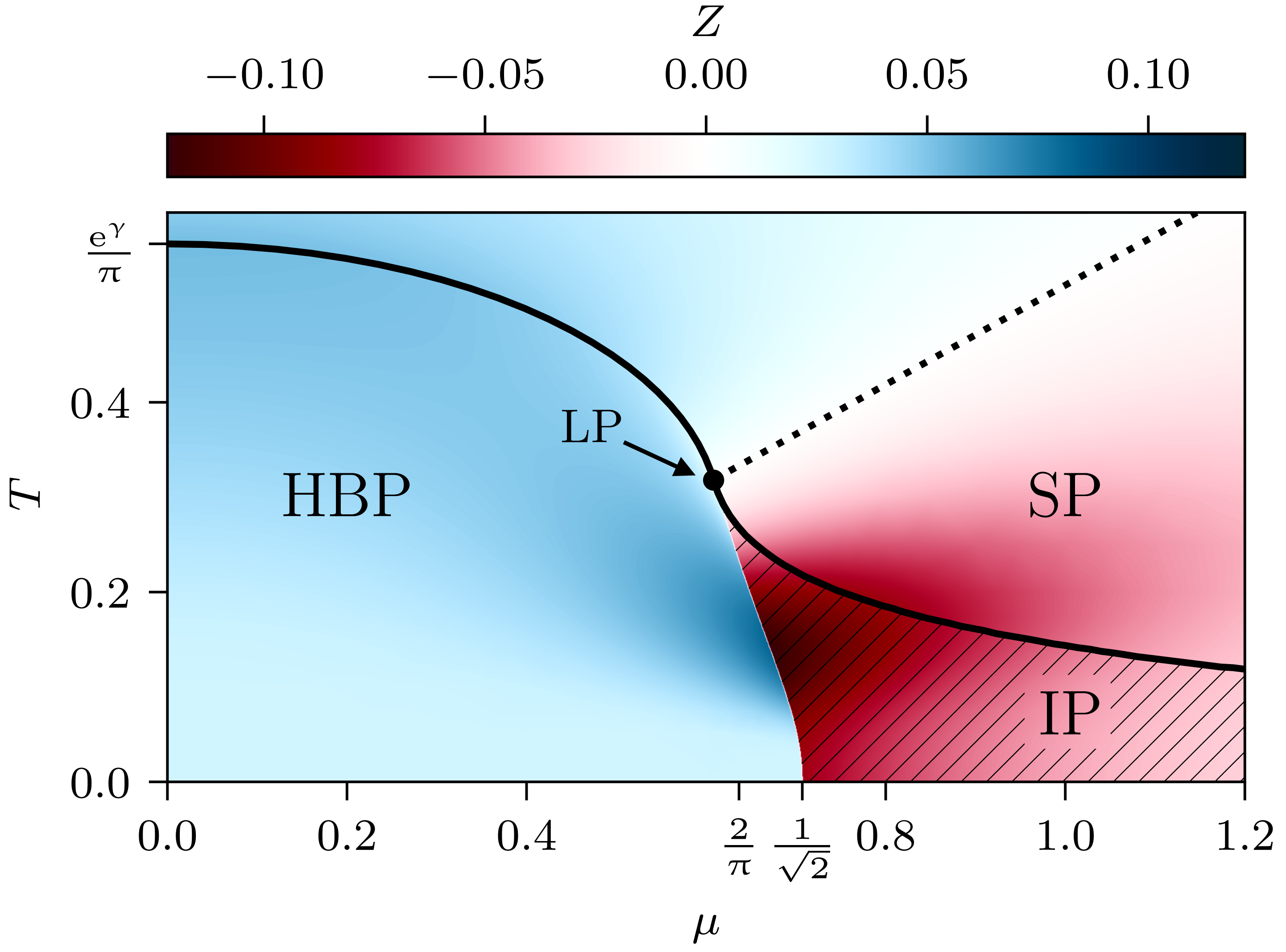




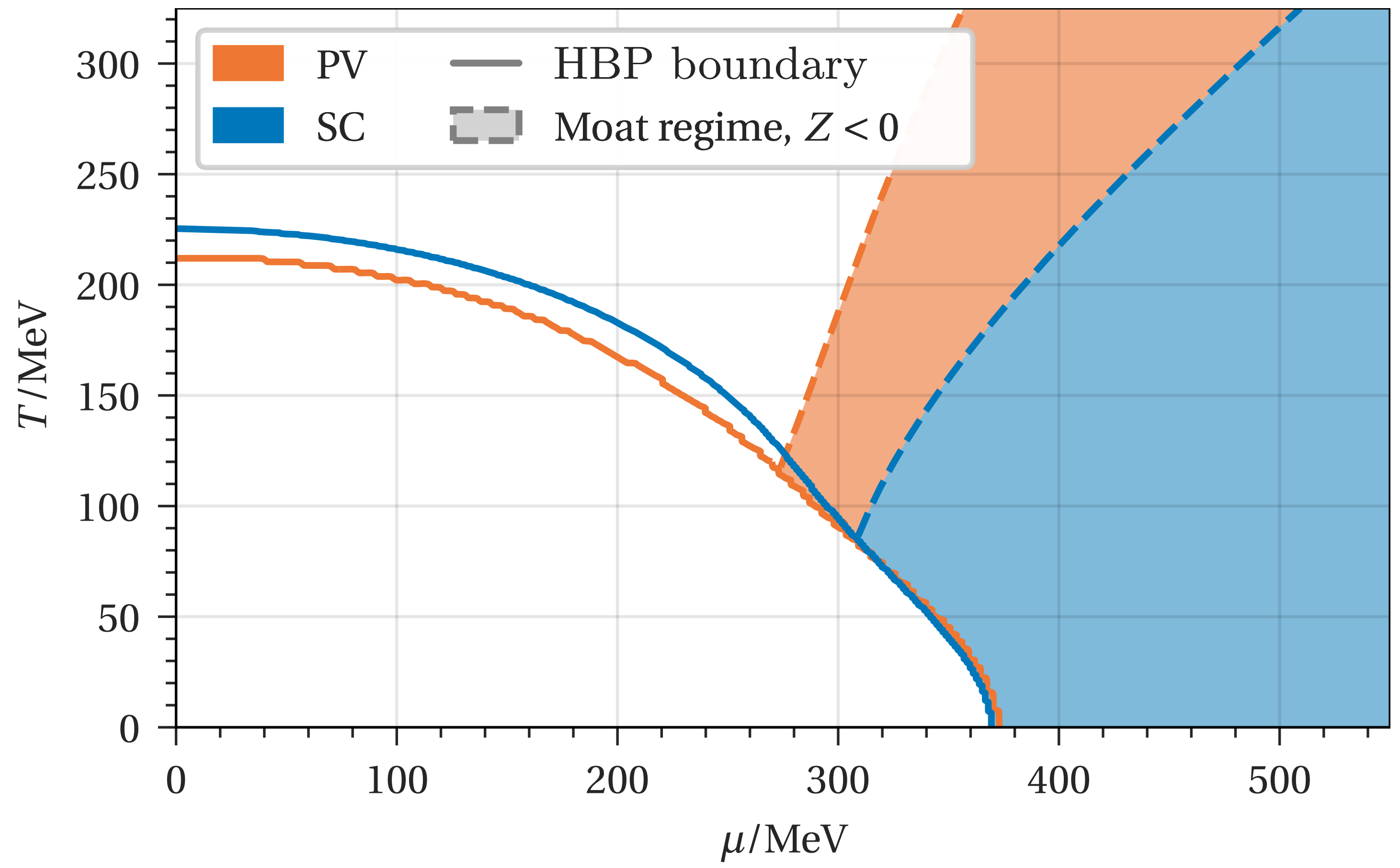
Nussinov, Ogilvie, Pannullo, Pisarski, Rennecke, Schindler, Winstel,
Phys. Rev. Lett. 135 (2025) 10, 101904

Moat regime in low energy models

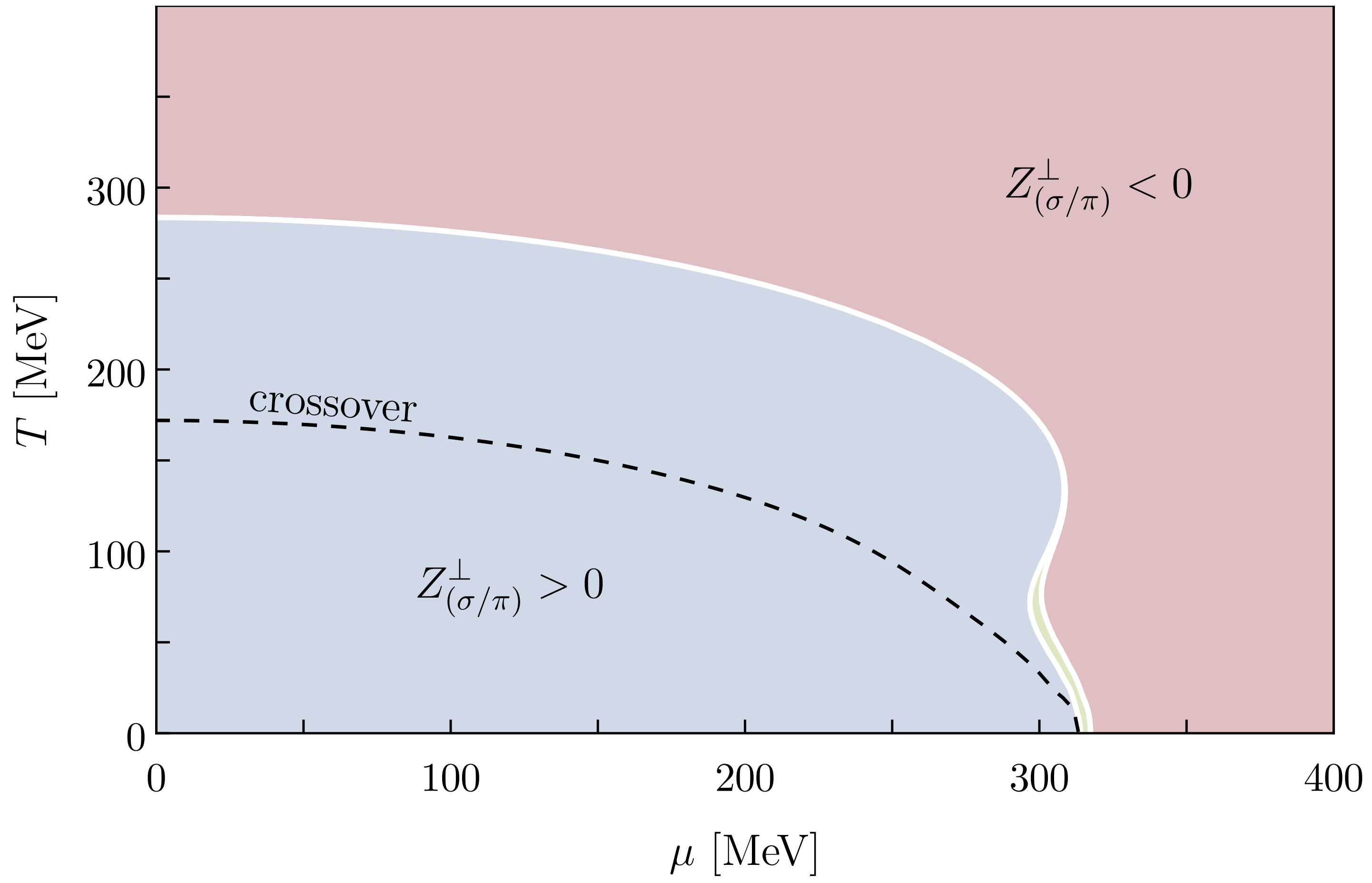
(1+1)-dimensional Gross-Neveu model
arXiv:2112.07024



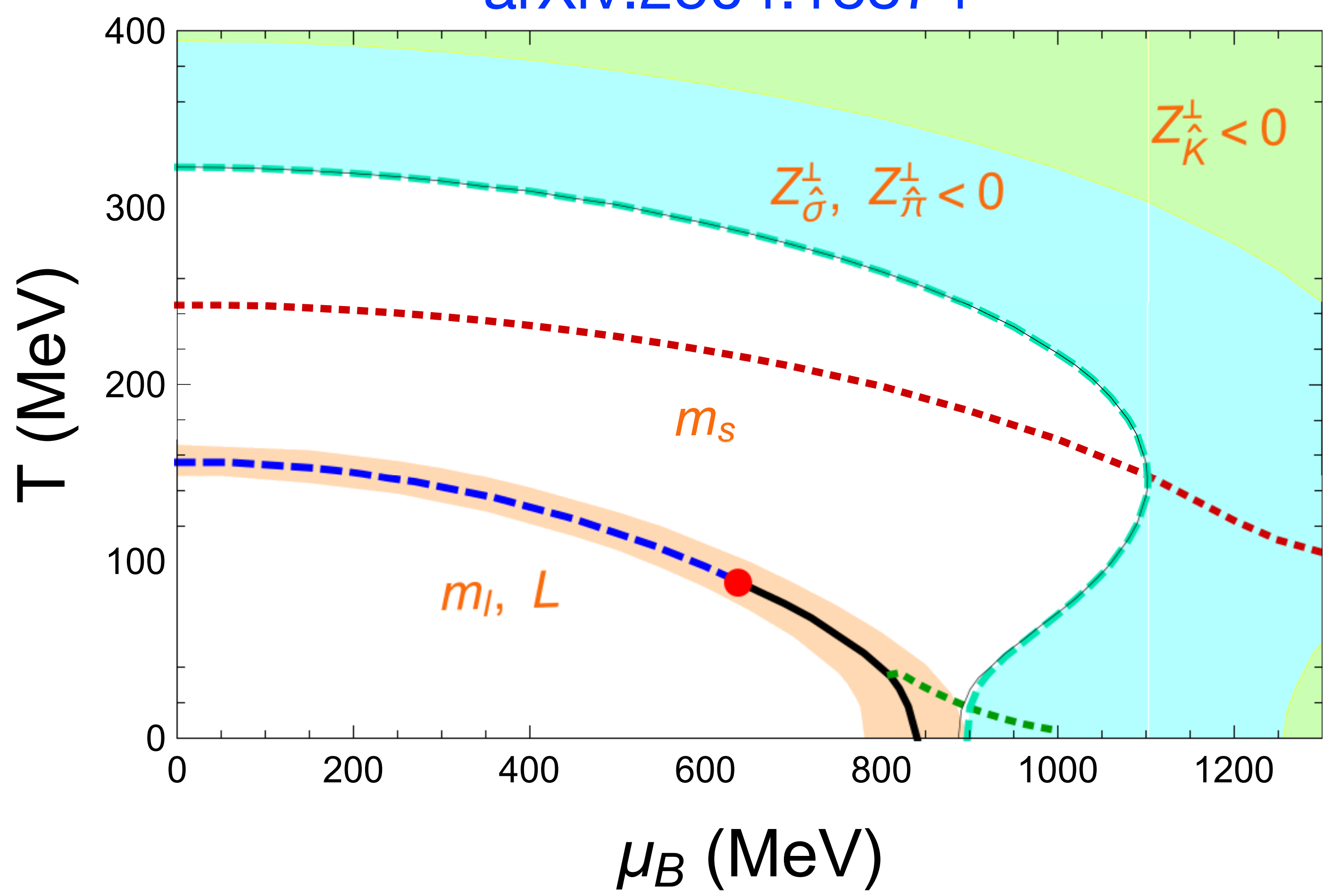
NJL model
arXiv:2406.11312



Quark-Meson model within fRG
arXiv:2412.16059



2+1 flavor Polyakov-Quark-Meson Model
arXiv:2504.18874



Questions to answer:

- Size of Moat regime depends on regularization schemes of model?
- What is the mechanism of Moat regime formation?
- Why shape of Moat regime in low energy models is different from QCD?

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Quark-Meson Model

Two-flavor Quark-Meson model:

$$\mathcal{L}[\phi, q, \bar{q}] = \bar{q}\left[\gamma_\mu \partial_\mu - \gamma_0 \hat{\mu}\right] q + \frac{1}{2}\left(\partial_\mu \phi\right)^2$$

$$+ h\, \bar{q}(T^0 \sigma + i\gamma_5 \mathbf{T} \cdot \boldsymbol{\pi}) q + U(\rho) - c\sigma$$

$$+ \mathcal{L}_{\mathrm{ct}}$$

Random Phase Approximation (RPA):

$$V(\rho) = U(\rho) - \frac{T}{\mathcal{V}} \ln \det \mathcal{M}(\sigma)$$

$$\mathcal{M}(\sigma) = \gamma_\mu \partial_\mu - \gamma_0 \mu + h T^0 \sigma$$

Only quark contribution

Full effective potential:

$$V(\rho) = V_{\mathrm{vac}}(\rho) + V_{\mathrm{thermal}}(\rho)$$

Mesonic part:

$$U(\rho) = -\nu\rho + \lambda\rho^2/2$$

Dimensional regularization:

$$V_{\mathrm{vac}}(\rho) = \frac{N_f N_c m_f^4}{16\pi^2}\left[\frac{1}{\epsilon} - 2\ln\left(\frac{m_f}{M}\right) + C + \mathcal{O}(\epsilon)\right]$$

$$+ \mathcal{L}_{\mathrm{ct}}\big|_{\phi=\mathrm{const.}}$$

Counter term:

$$\mathcal{L}_{\mathrm{ct}} \supset \delta_\lambda \phi^4\,, \qquad \delta_\lambda = -\frac{N_c N_f h^4}{2^8 \pi^2}\left(\frac{1}{\epsilon} + C\right)$$

Regularized vacuum part:

$$V_{\mathrm{vac}}^{\overline{\mathrm{MS}}}(\rho) = -\frac{N_c N_f m_f^4}{8\pi^2} \ln\left(\frac{m_f}{M}\right)$$

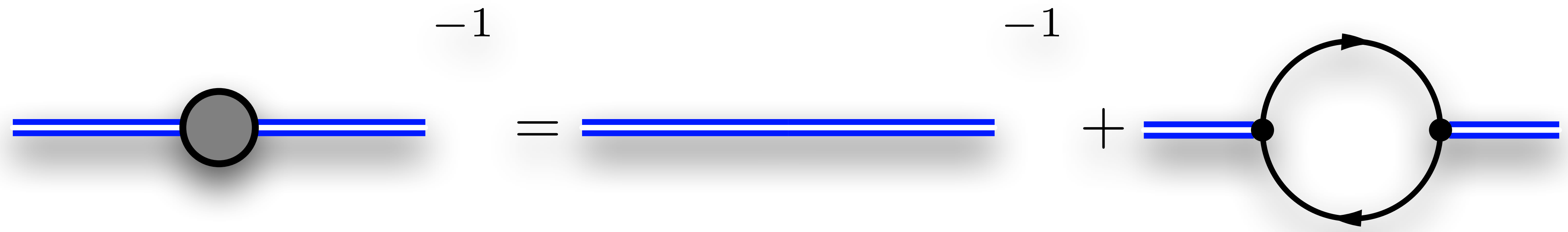
Renormalization condition
at vacuum (curvature masses):

$$M = 300\,\mathrm{MeV}$$

$$\sigma_0 = 92\,\mathrm{MeV}, \quad \bar{m}_\pi^{\mathrm{vac}} = 136\,\mathrm{MeV}, \quad \bar{m}_\sigma^{\mathrm{vac}} = 480\,\mathrm{MeV}, \quad m_f^{\mathrm{vac}} = 300\,\mathrm{MeV}$$

$$\longrightarrow \quad \nu = (482\,\mathrm{MeV})^2, \quad \lambda = 75.8, \quad c = 0.0017\,\mathrm{GeV}^3, \quad h = 6.5$$

Pion two-point function



$$\Sigma_\pi(p^2;T,\mu) = p^2 + m_\pi^2 + \Pi^\pi_{\rm RPA}(p^2;T,\mu)$$

$$\Sigma_\pi(p^2;T,\mu) = Z_\pi^{\parallel}(p^2;T,\mu)\,p_0^2 + Z_\pi^{\perp}(p^2;T,\mu)\,\boldsymbol{p}^2 + \bar{m}_\pi^2(T,\mu)$$

$$Z_\pi^{\perp}(p=0;T,\mu) = \frac{\partial}{\partial \boldsymbol{p}^2}\Sigma_\pi(p_0=0,\boldsymbol{p};T,\mu)\Big|_{\boldsymbol{p}^2=0}$$

Dimensional regularization:

$$Z^{\perp}_{\pi,\rm vac}(0) = \frac{h^2N_c}{16\pi^2}\left[\frac{1}{\epsilon} - \gamma_E + \ln 4\pi - 2\ln\left(\frac{m_f}{M}\right)\right]$$

$$+1+\delta_Z$$

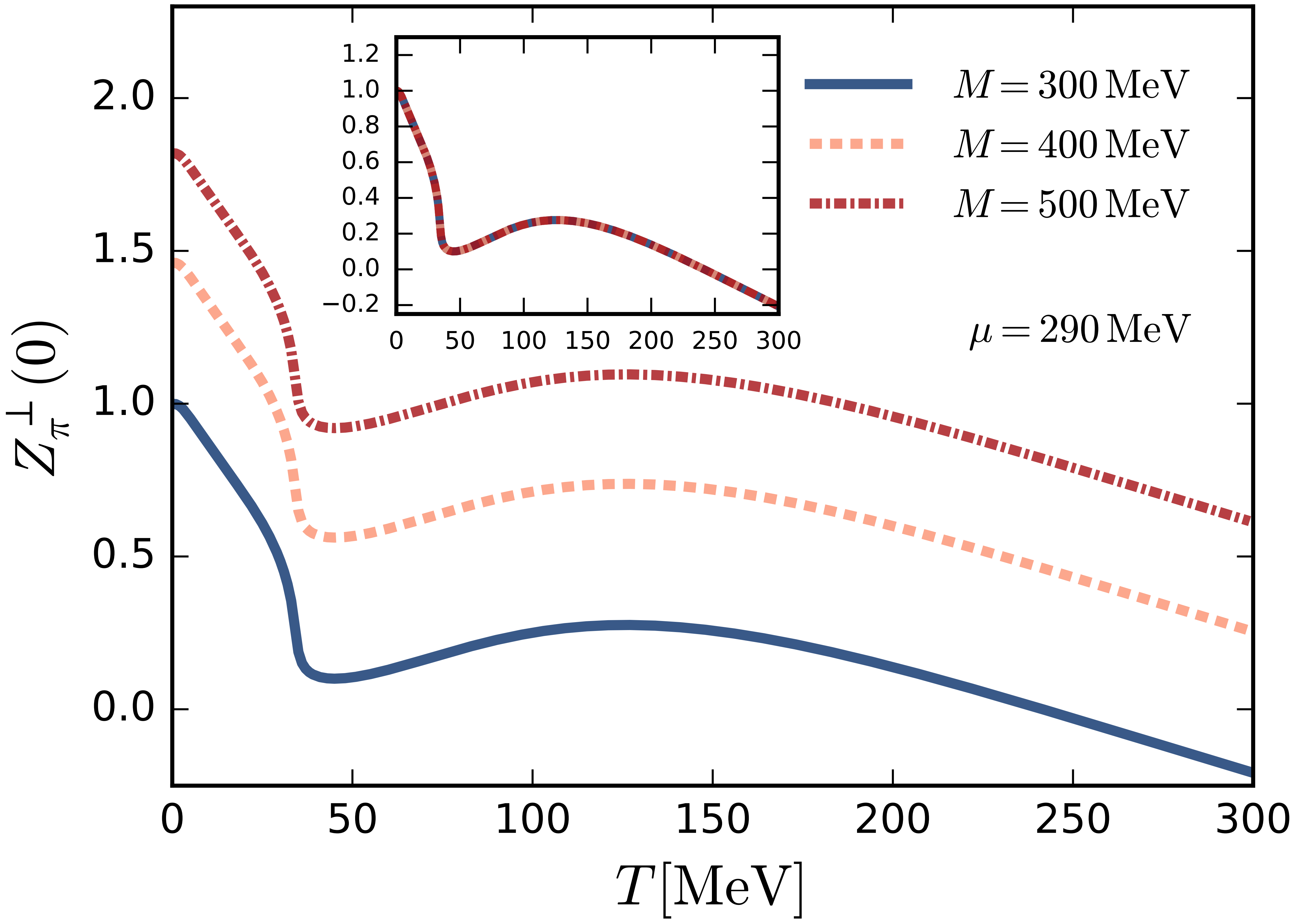
Counter term:

$$\delta_Z = -\frac{h^2N_c}{16\pi^2}\left[\frac{1}{\epsilon} - \gamma_E + \ln 4\pi - 2\bar{C}\right]$$

Regularized vacuum part:

$$Z^{\perp,\rm re}_{\pi,\rm vac}(0) = 1 - \frac{h^2N_c}{8\pi^2}\left[\bar{C} + \ln\left(\frac{m_f}{M}\right)\right]$$

Need a renormalization condition!



Renormalization of wave function

Pole mass or screening mass can be used as renormalization condition:

$$\Sigma(p_0 = im_p, \boldsymbol{p} = 0; T, \mu) = 0 \;\rightarrow\; m^2_{\pi,p} \simeq \frac{\bar{m}^2_\pi}{Z^{\parallel}_\pi(p=0)}$$

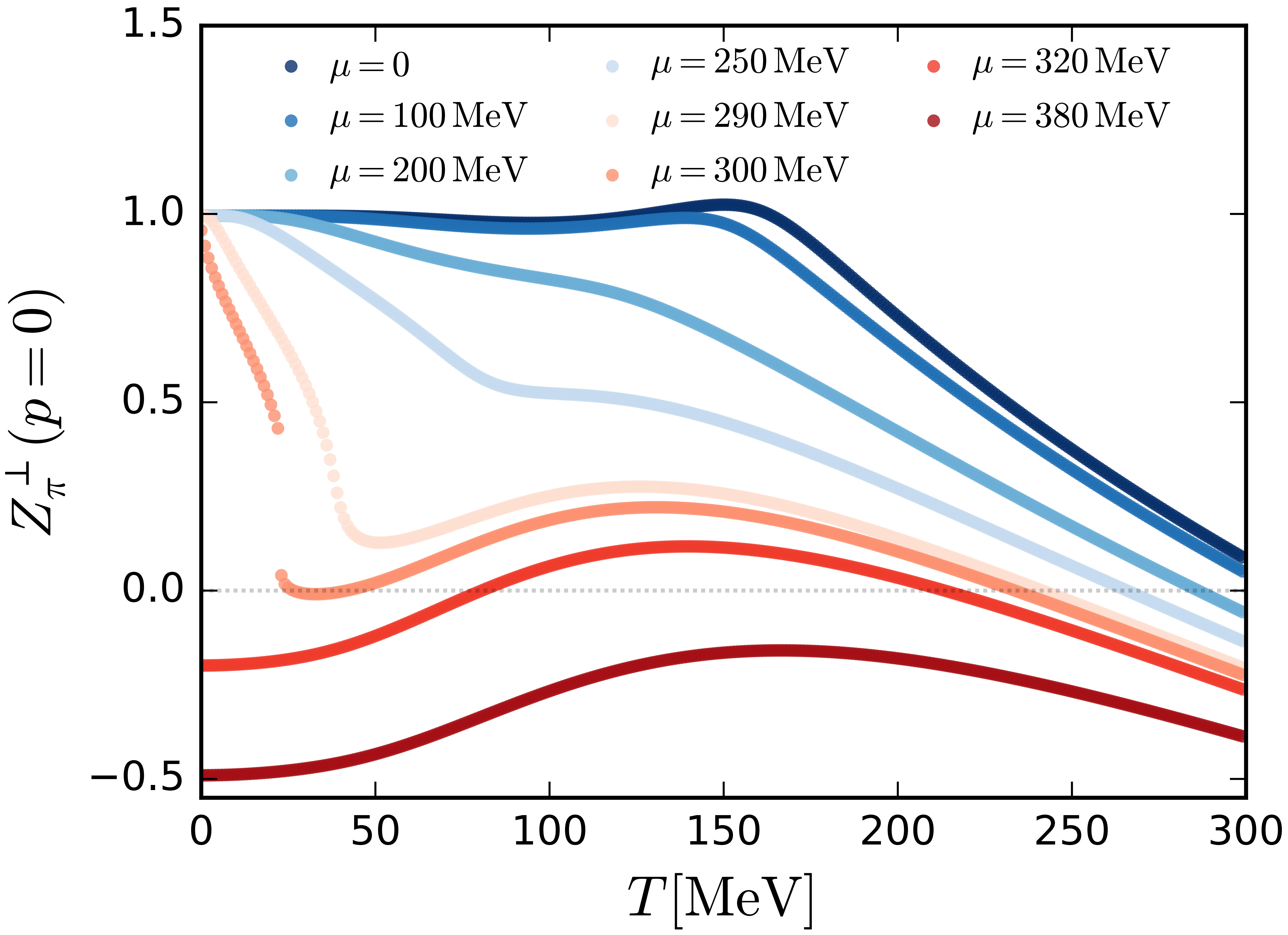
or

$$\Sigma(p_0 = 0, \boldsymbol{p} = im_s; T, \mu) = 0 \;\rightarrow\; m^2_{\pi,s} \simeq \frac{\bar{m}^2_\pi}{Z^{\perp}_\pi(p=0)}$$

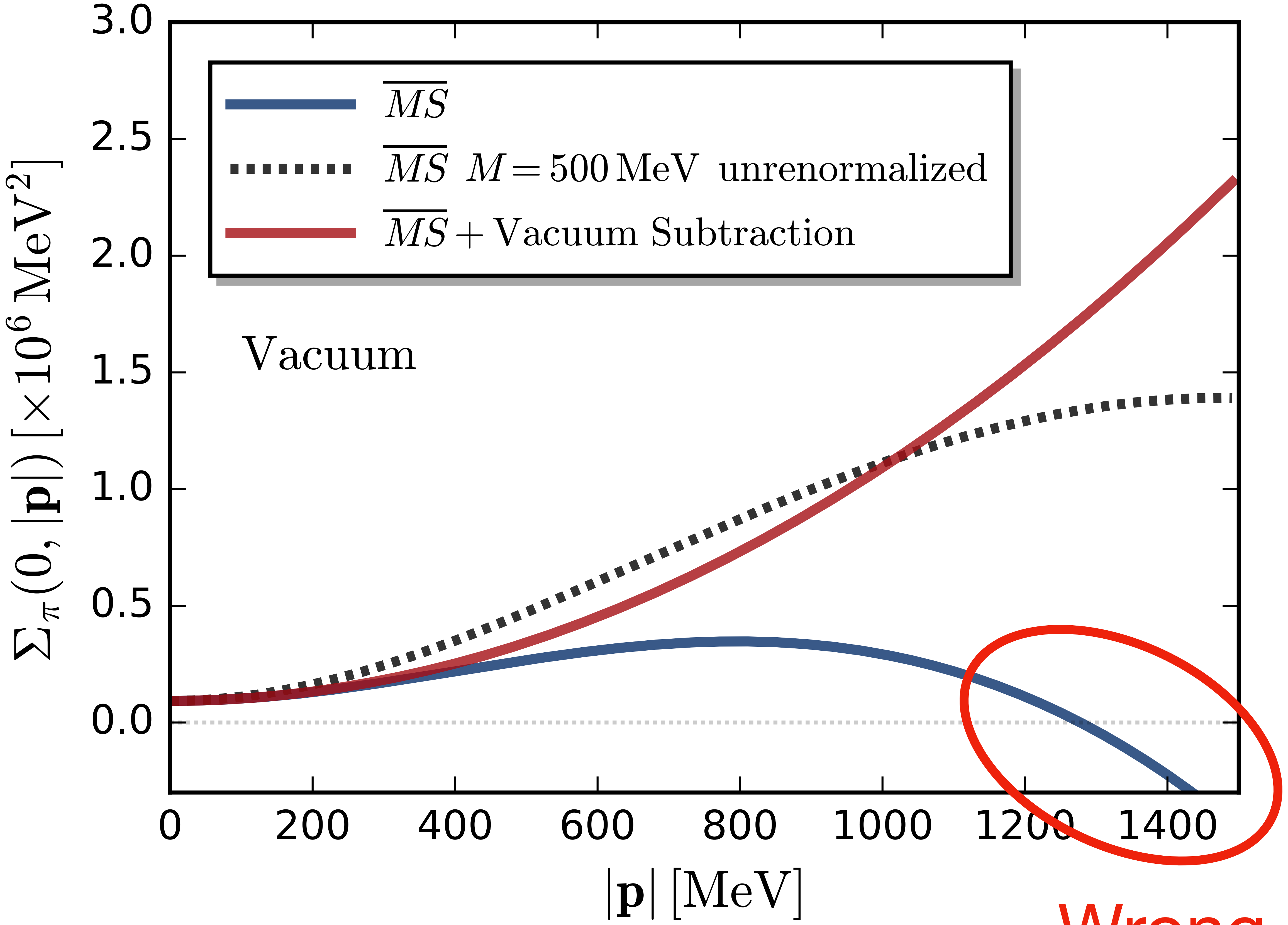
Here we take a simple condition:

$$\bar{m}^{\text{vac}} = m_s^{\text{vac}} = m_p^{\text{vac}}$$

$$Z^{\perp}_\pi(p=0; T=0, \mu=0) = 1$$



Renormalization of finite momentum



Wrong !

- Unphysical results at high spatial momentum for $\overline{MS}$
- We introduce a renormalization condition for whole two-point function

Trivial two-point function at vacuum:

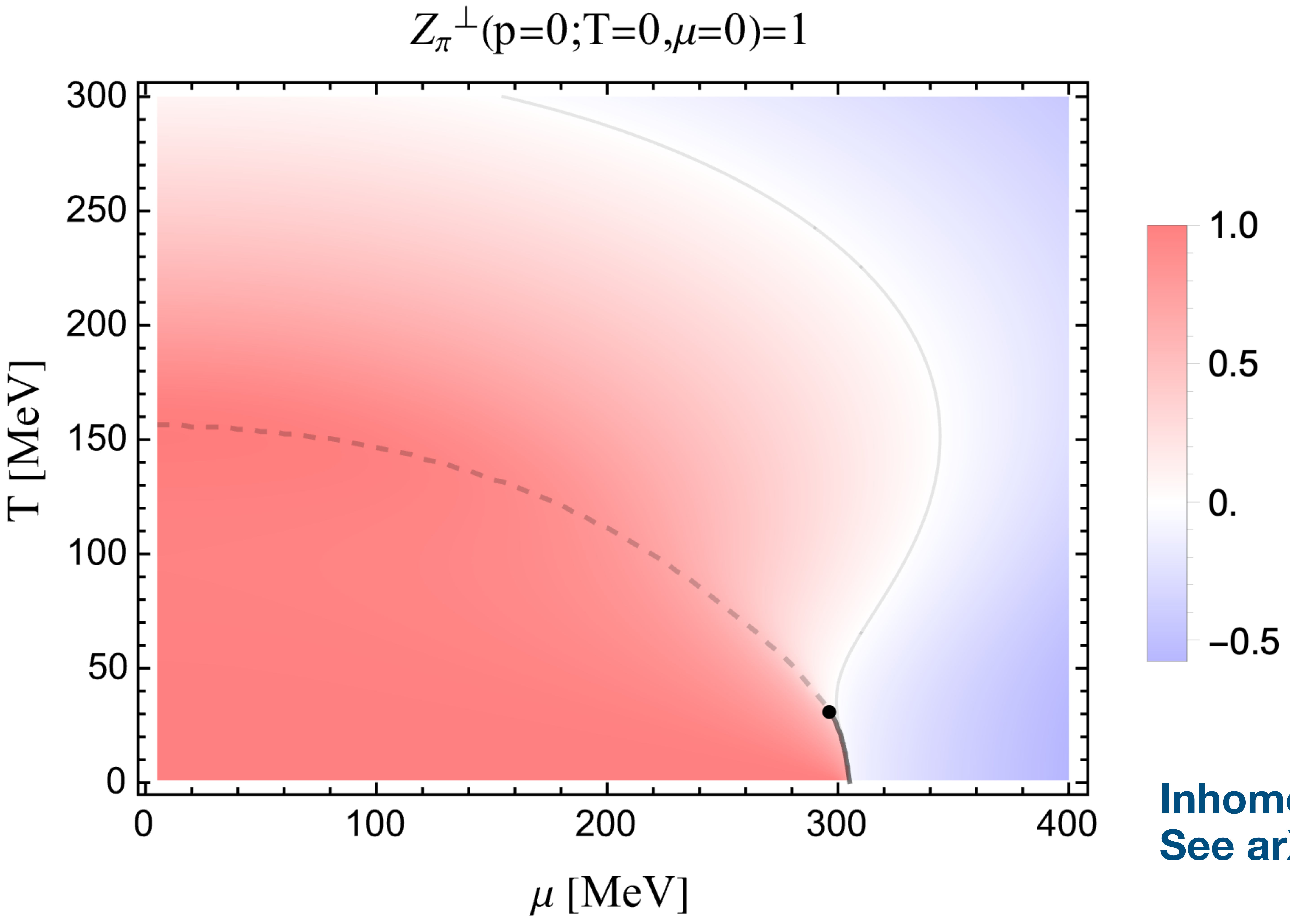
$$\Sigma^{\pi}_{\mathrm{vac}}(p^2; m_f) = p^2 + \bar{m}^2_\pi$$

$$\Pi^{\pi,\mathrm{vs}}_{\mathrm{vac}}(p^2; m_f) = \Pi^{\pi,\mathrm{re}}_{\mathrm{vac}}(p^2; m_f)$$ Self-energy

$$- \Pi^{\pi,\mathrm{re}}_{\mathrm{vac}}(p_0^2 = 0, \boldsymbol{p}^2; m_f^{\mathrm{vac}})$$ Vacuum self-energy

$$+ \Pi^{\pi,\mathrm{re}}_{\mathrm{vac}}(0; m_f^{\mathrm{vac}})$$ Self-energy mass term

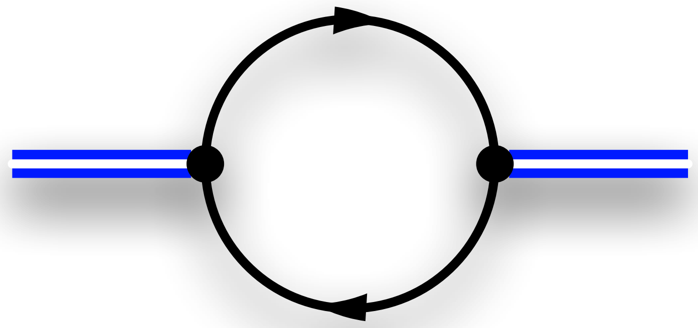
Phase structure



- Moat regime also exists at low density
- The shape of moat regime is different from QCD results
- No inhomogeneous instability of the pion has been observed

Inhomogeneous phase depends on sigma meson mass
See arXiv:1404.0057 by Carignano, Buballa, Schaefer

CA and PH fluctuations

Self-energy 

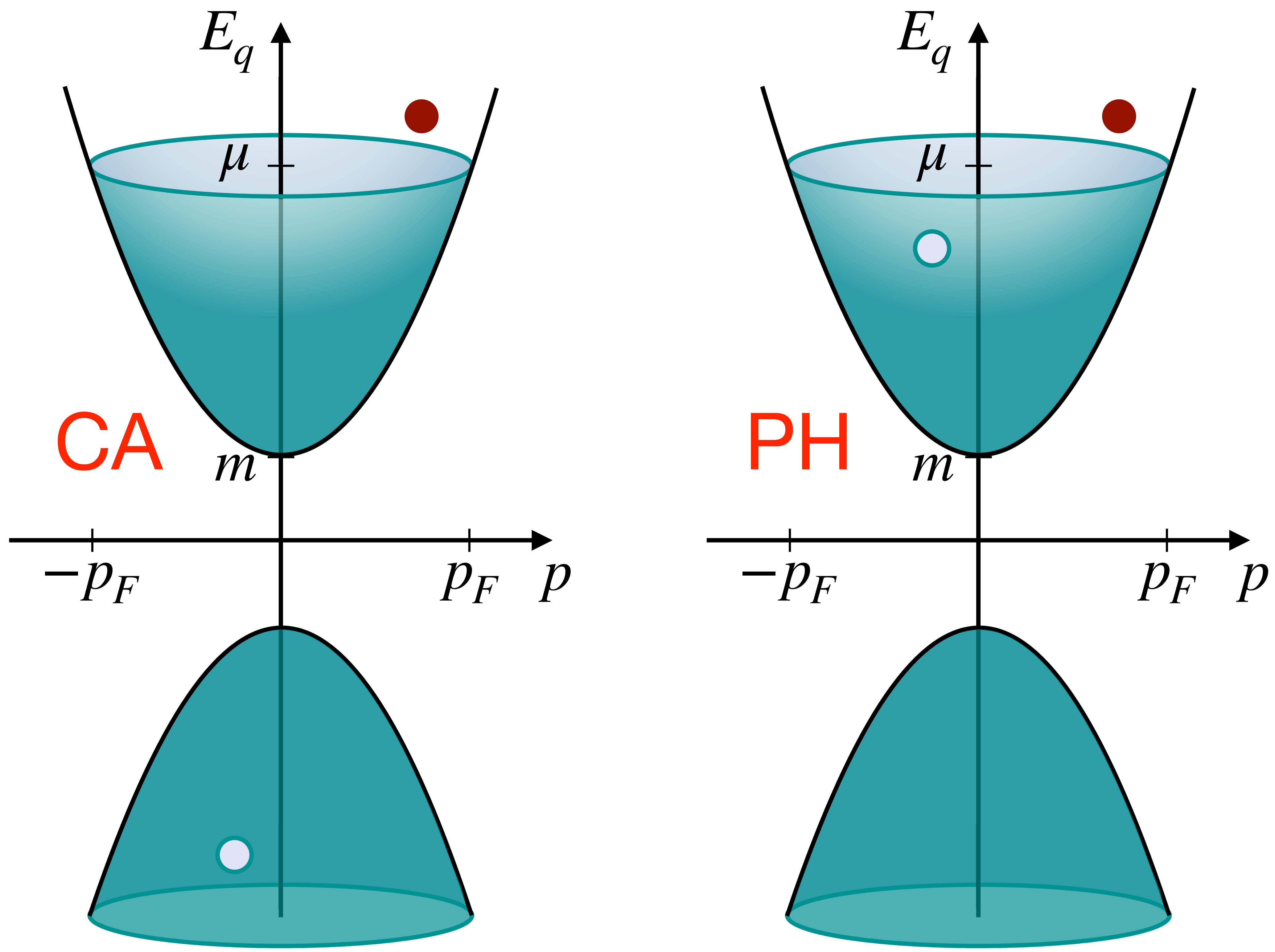
Particle Creation and annihilation (CA):

$$\frac{-1+n_F(E_q(q);T,\pm\mu)+n_F(E_q(q);T,\mp\mu)}{E_q(q)+E_q(q-p)}$$

Particle-Hole (PH):

$$\frac{n_F(E_q(q);T,\pm\mu)-n_F(E_q(q-p);T,\pm\mu)}{E_q(q)-E_q(q-p)}$$

Dirac cone



Fu, Pawlowski, Pisarski, Rennecke, Wen, SY,
arXiv:2412.15949

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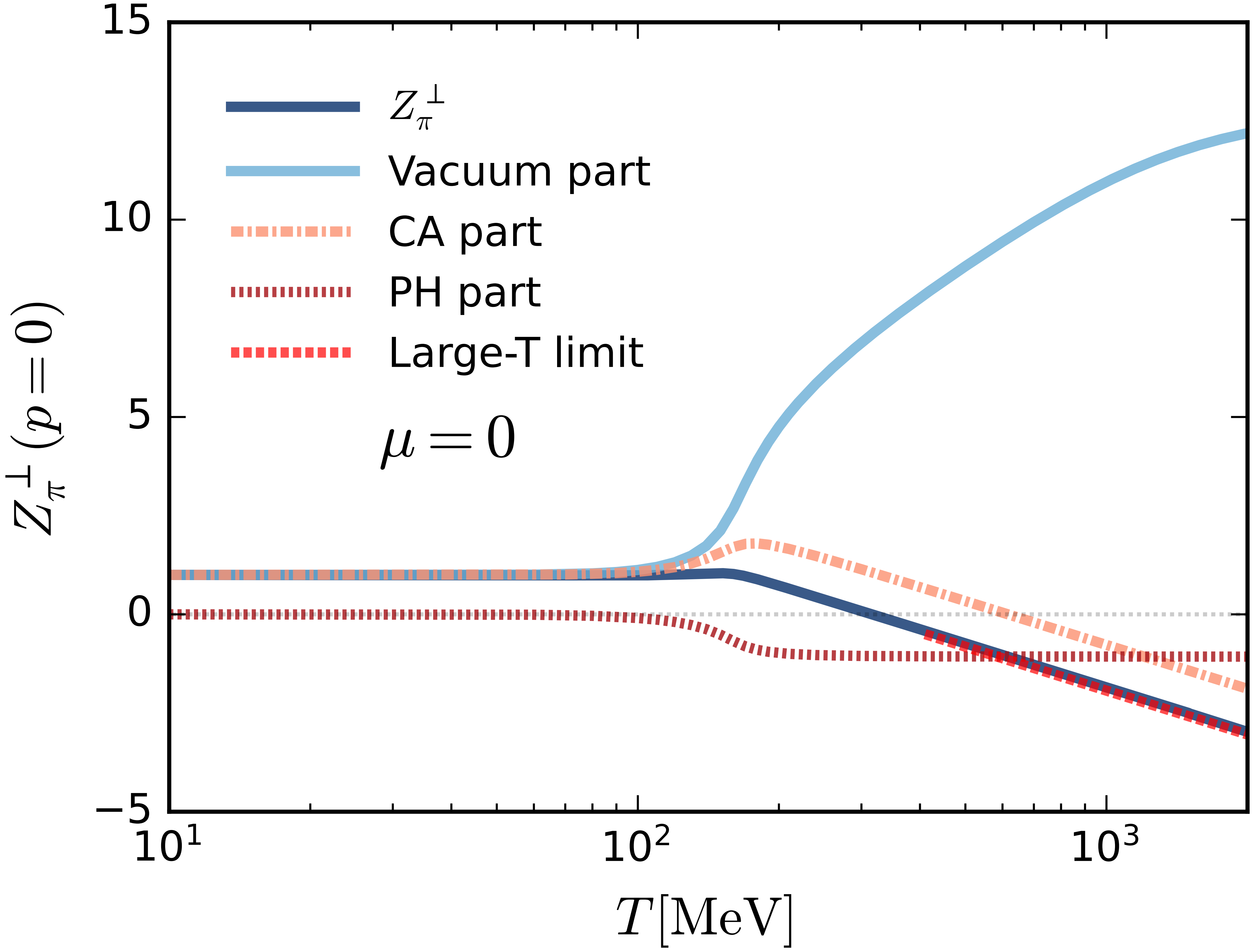
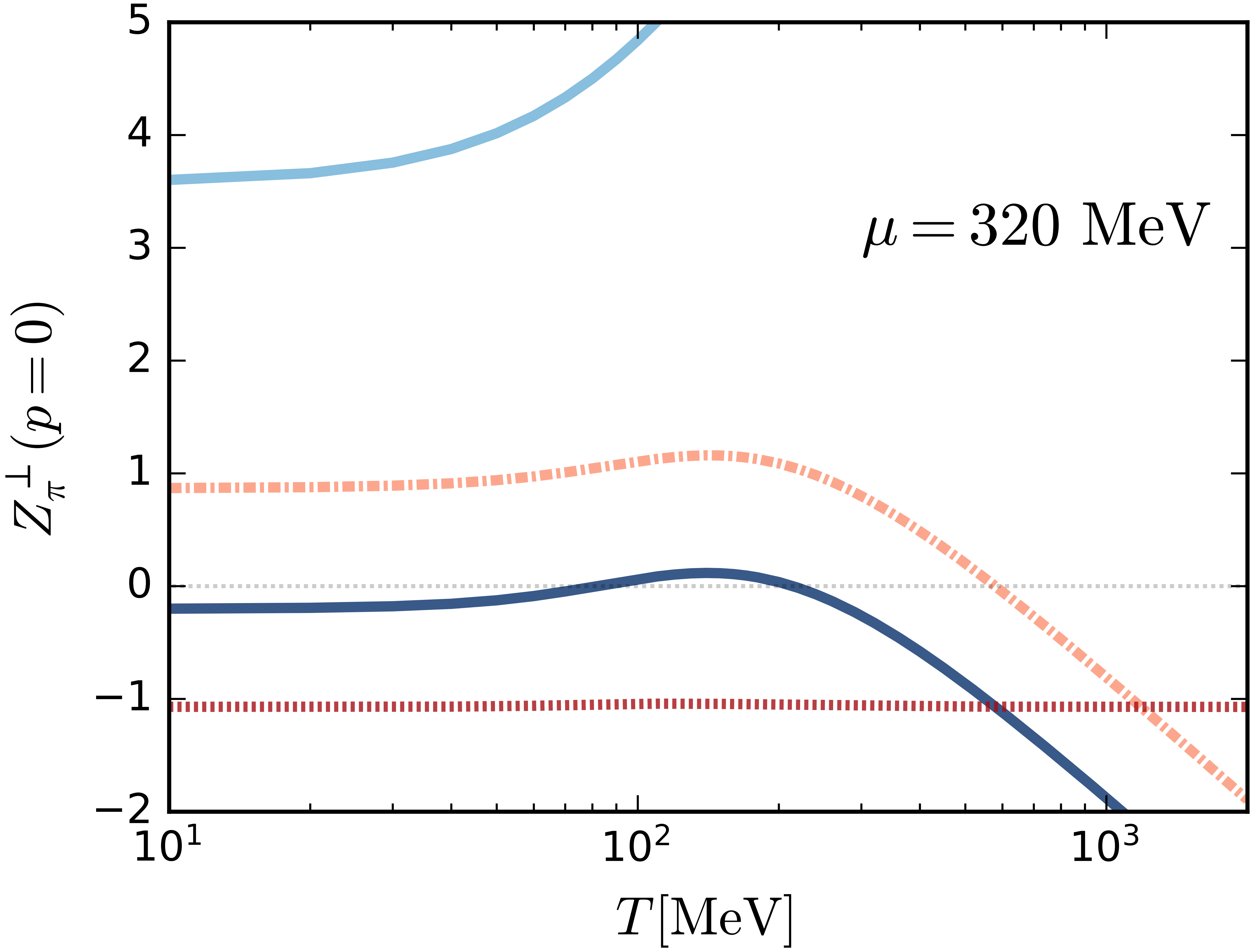


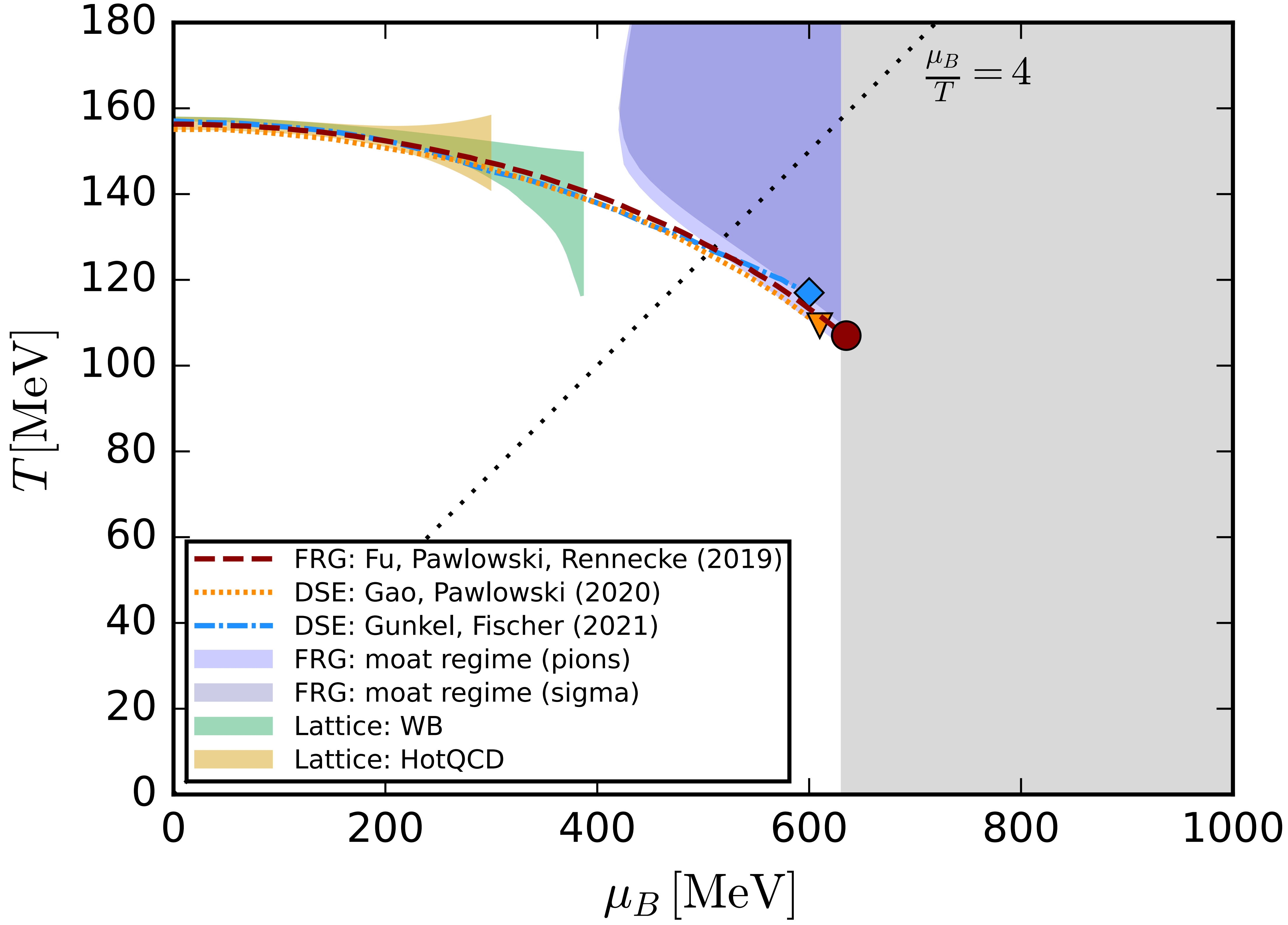


High T limit at vanishing density:

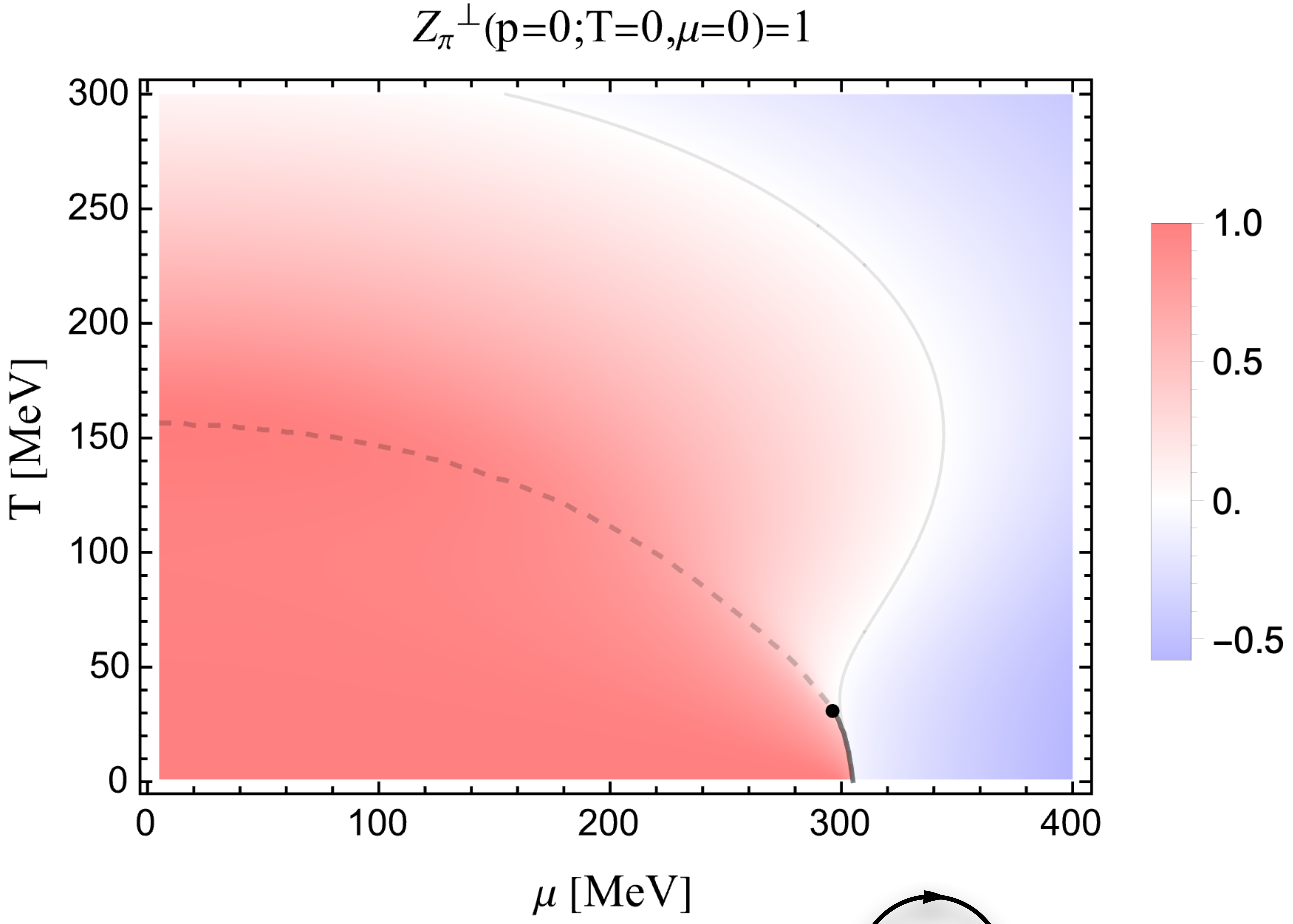
$$Z_\pi^\perp(0;T) \xrightarrow{T\to\infty} -\frac{h^2 N_c}{8\pi^2}\ln\left(\frac{T}{M}\right)$$

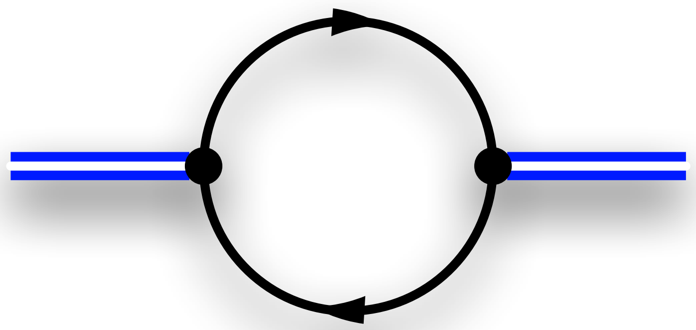
- **CA fluctuation drives $Z_\pi^\perp$ negative at high T**
- **PH fluctuation drives $Z_\pi^\perp$ negative at low T**

Shape of moat regime



fQCD (2025)

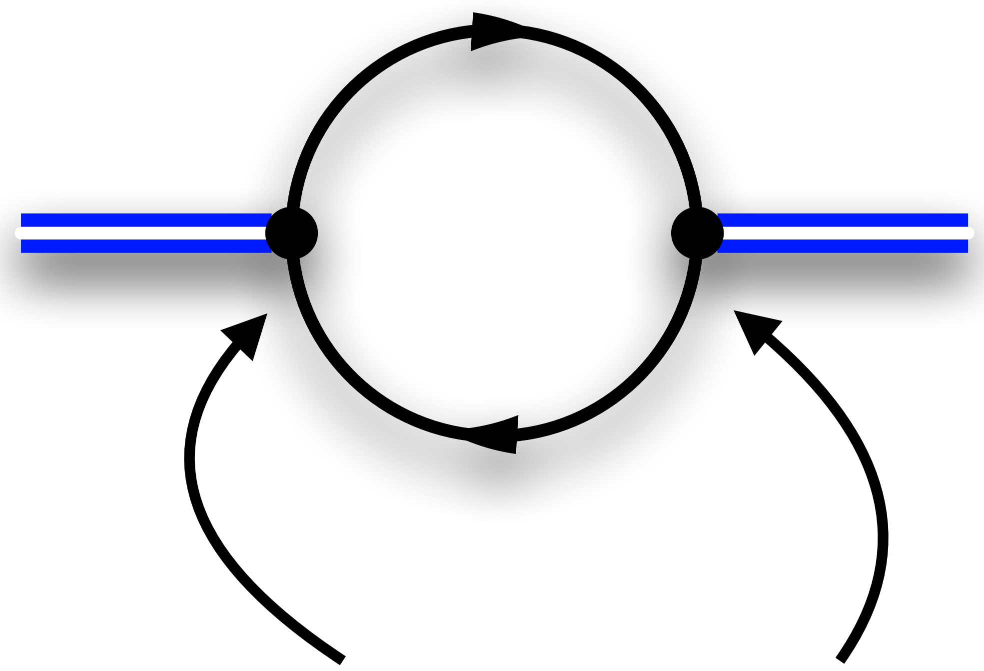




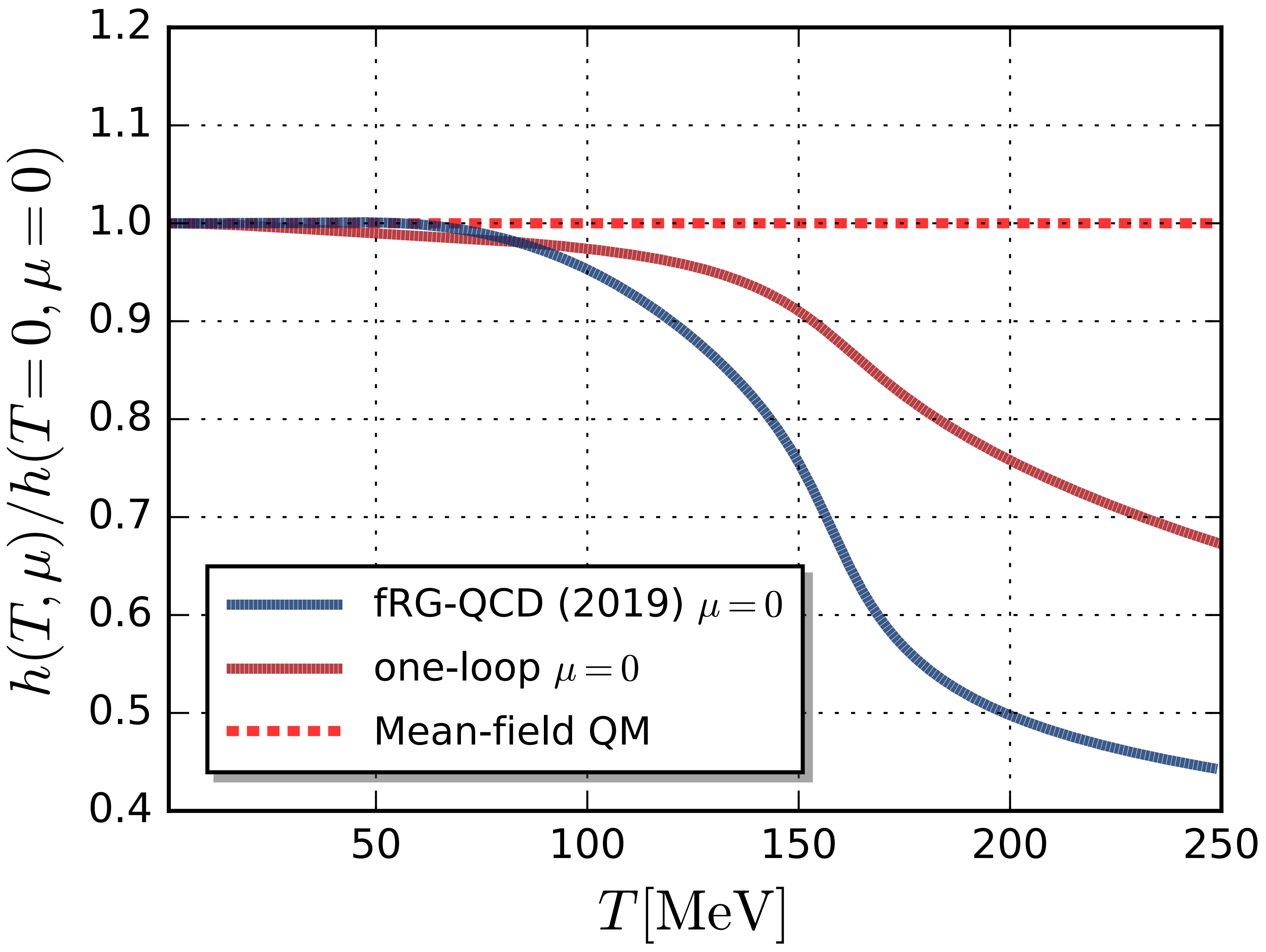
- **Moat regime in both QCD and models is given by quark-loop**
- **Shape of moat region from QCD calculations differs from that obtained in models**

What do we miss?

Yukawa Coupling

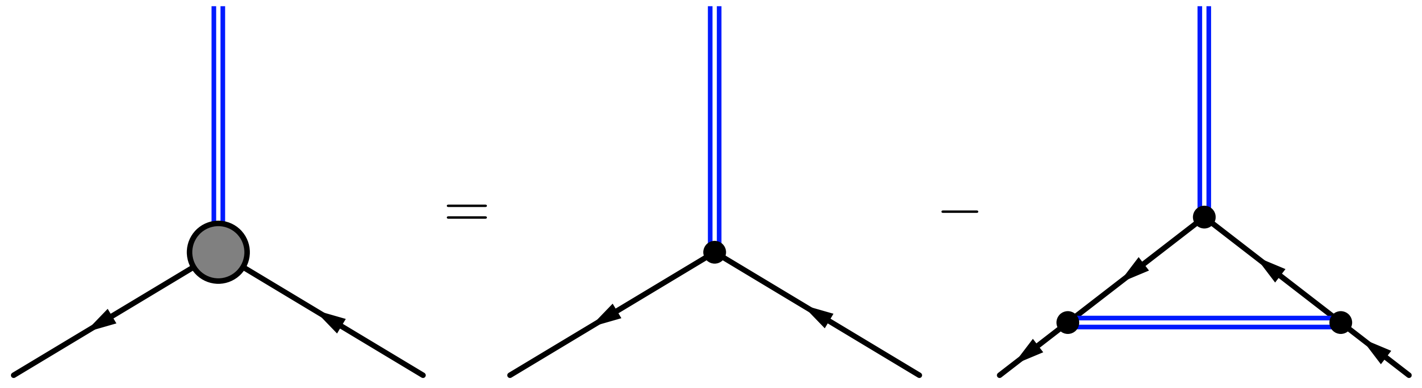


Fluctuations of quark-meson interaction are missing

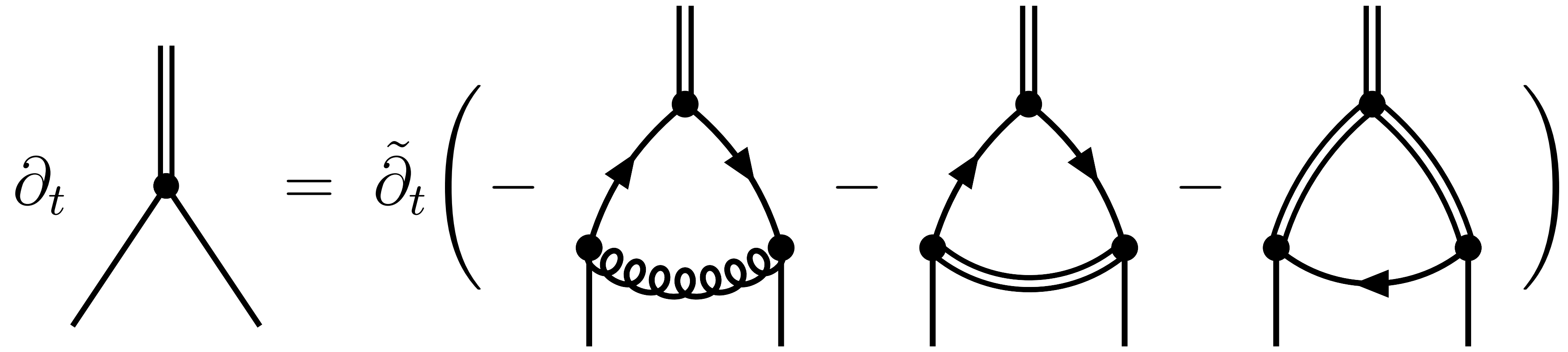


Mean-field: constant

One-loop in model:

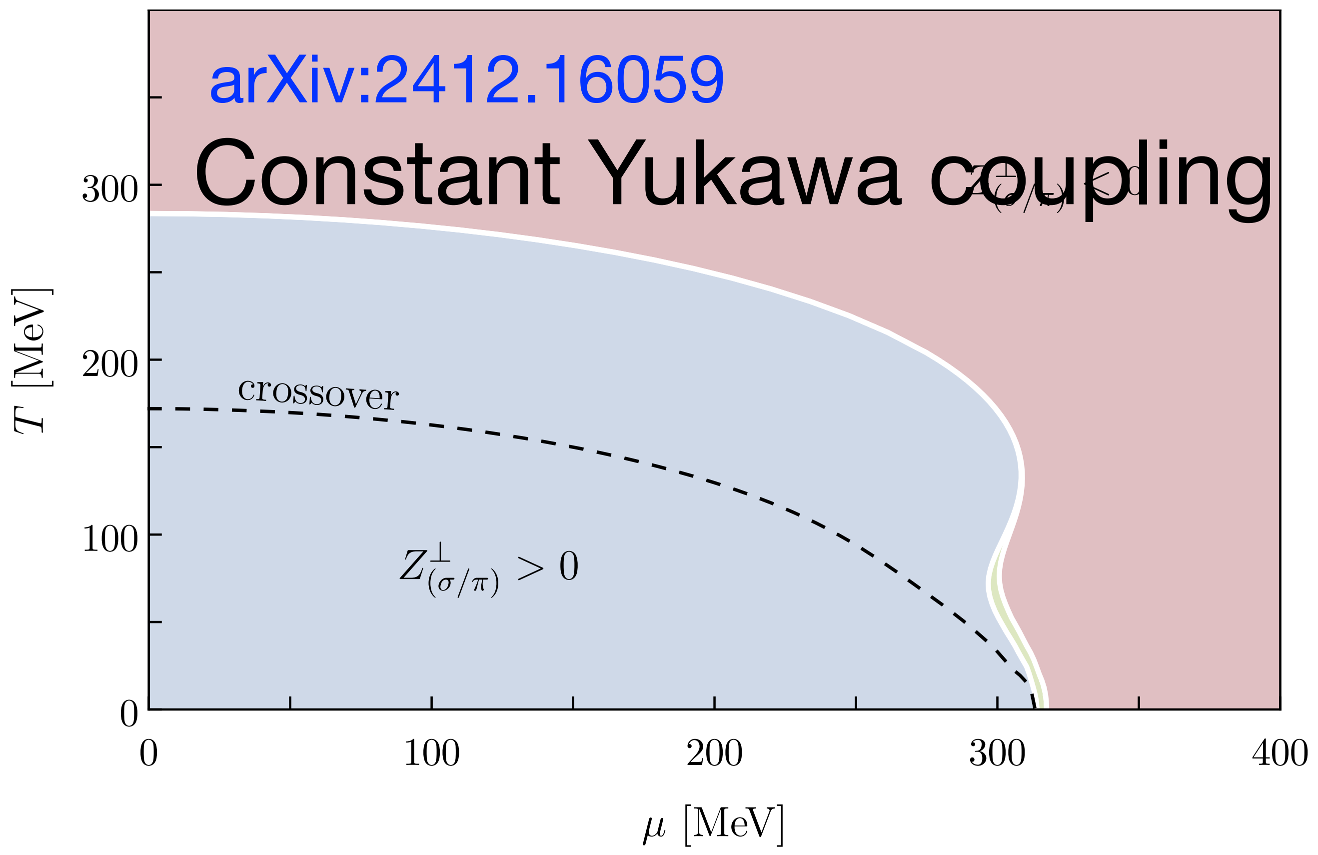


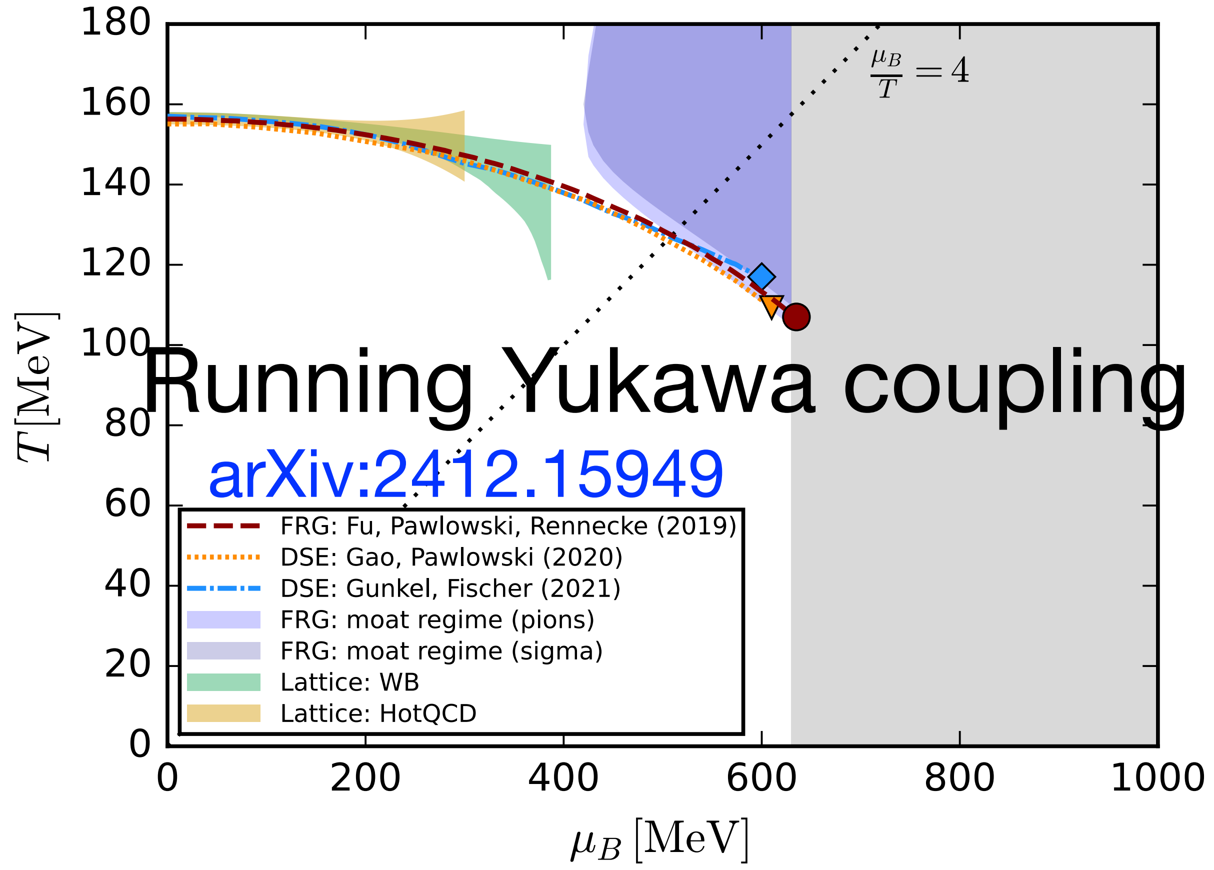
fRG-QCD:

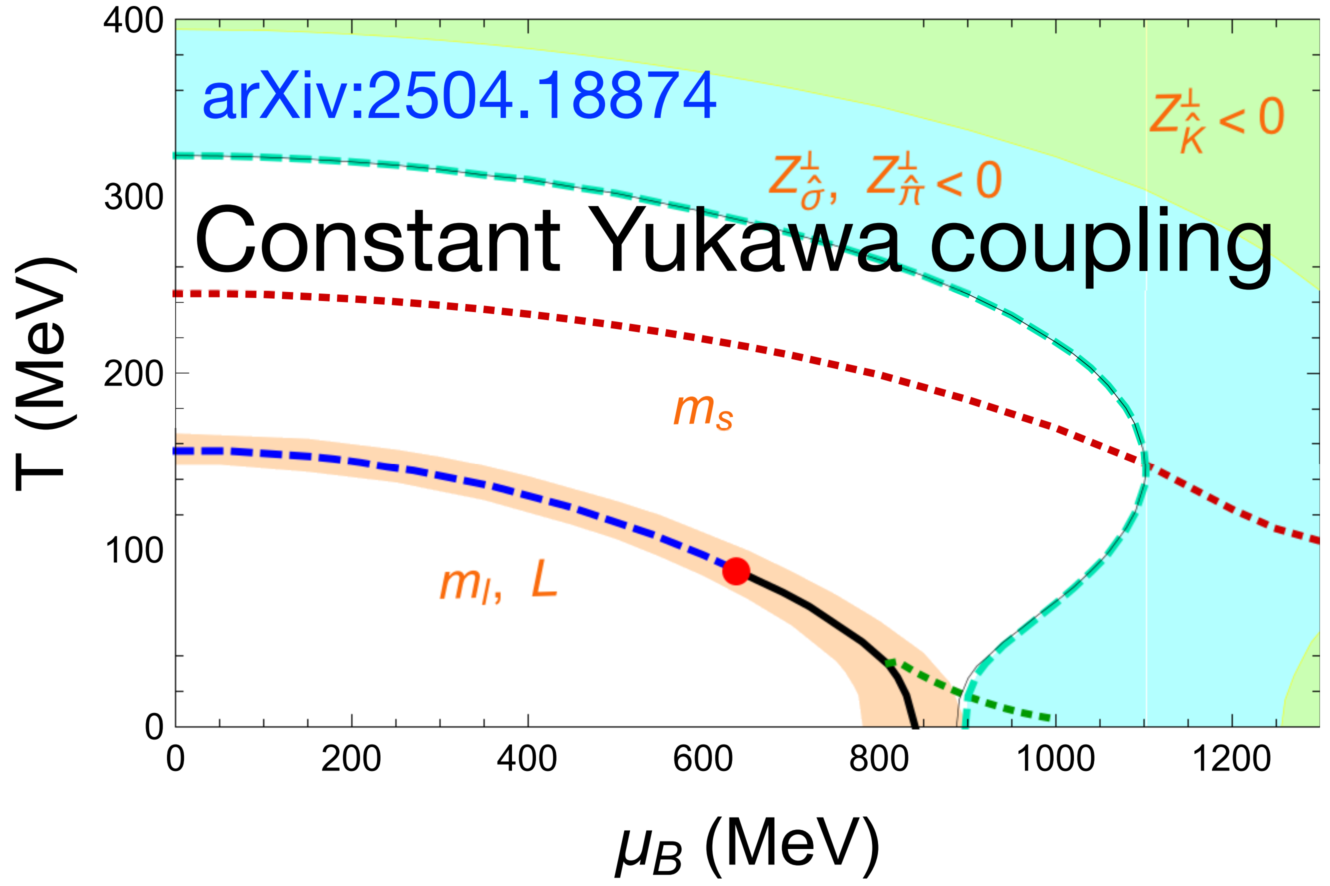


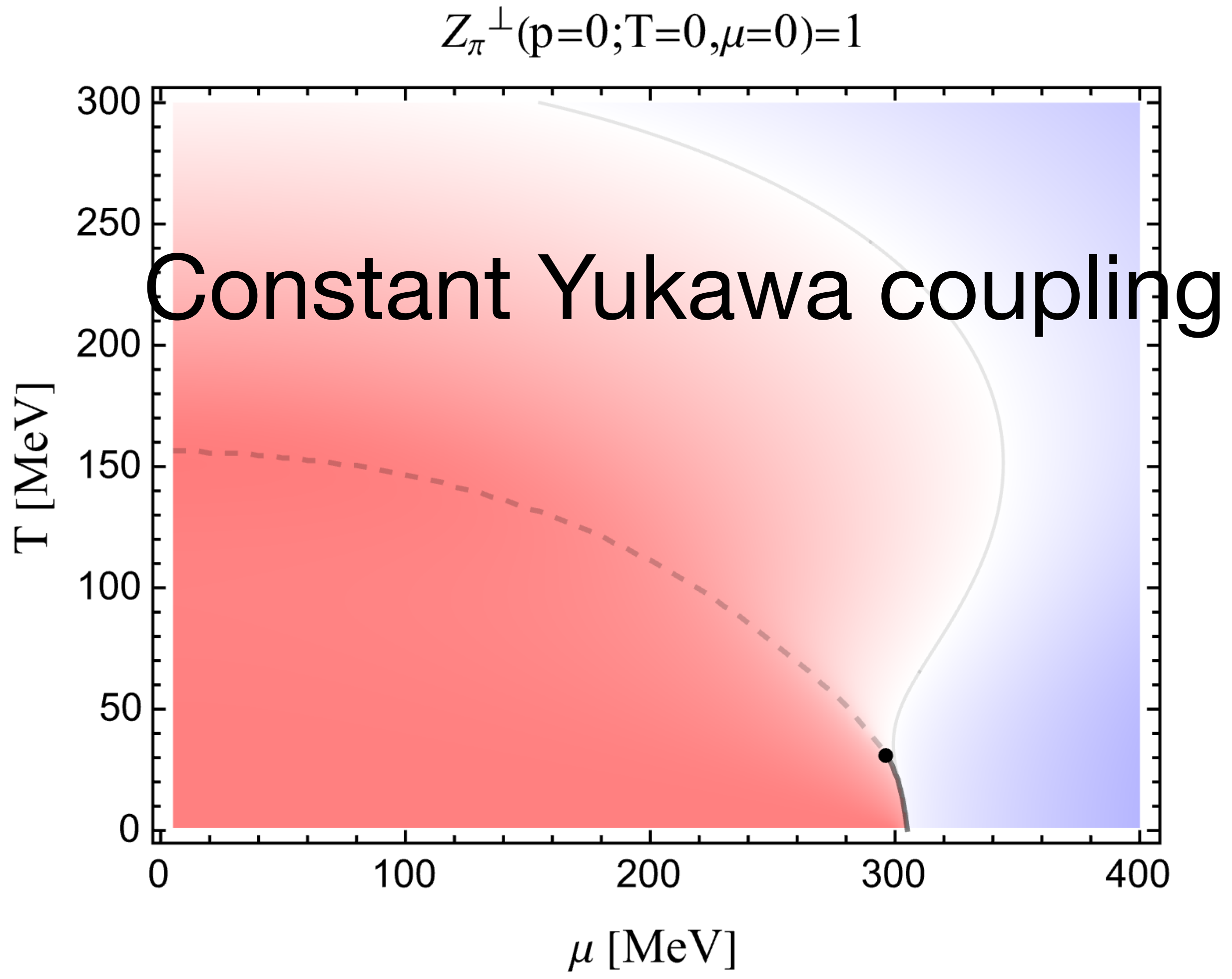
- **Fluctuation of Yukawa coupling will suppress CA and PH fluctuations at high temperature**
- **Different approximation can give different Yukawa coupling behavior**

Yukawa Coupling

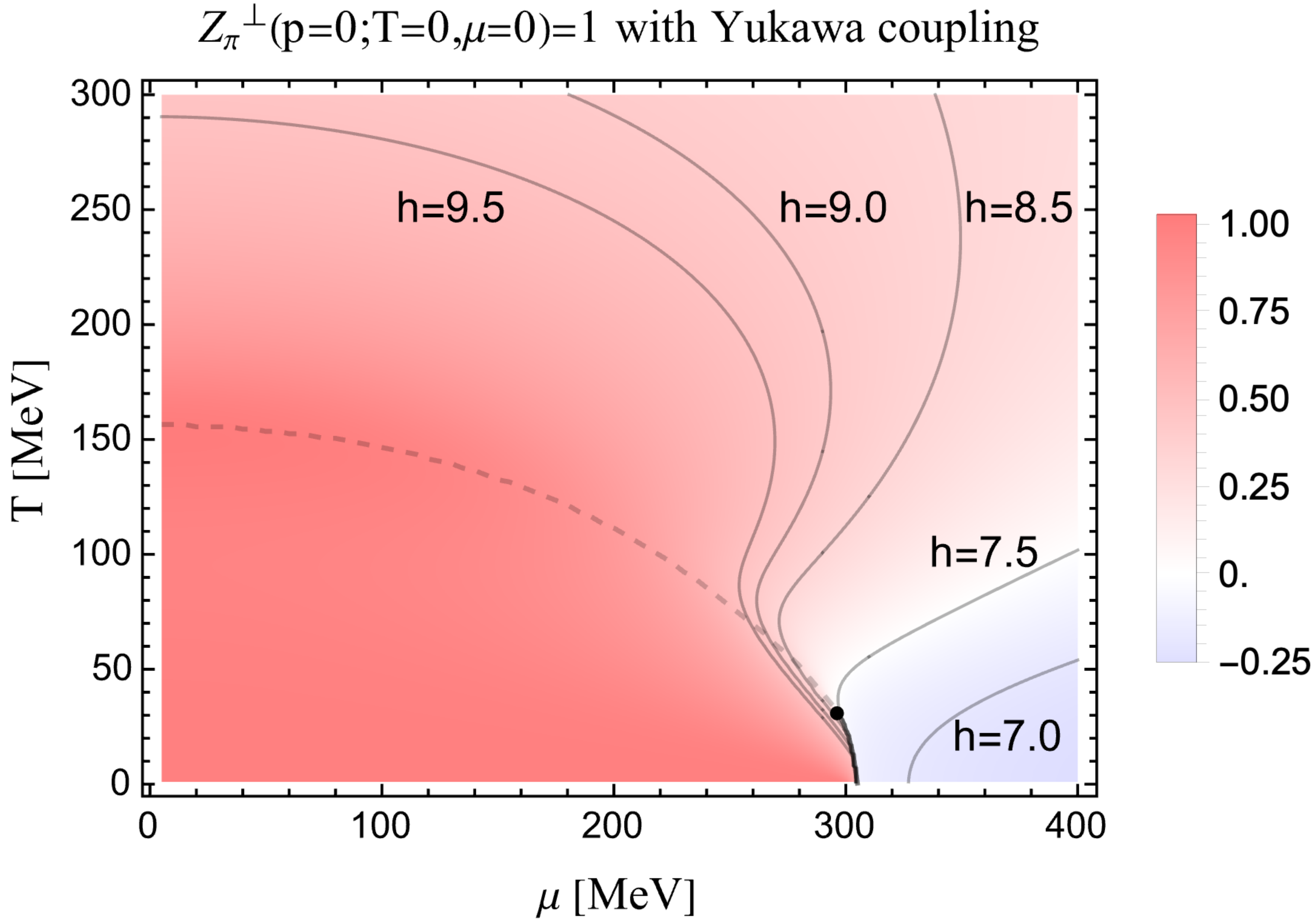








One-loop Yukawa coupling



Fluctuation of Yukawa coupling controls the moat regime !

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Summary and Outlook

- Moat regime is studied within a quark-meson model
- In addition to regularization, renormalization condition of meson wave function renormalization should also be specified
- CA processes drive moat at high T, PH fluctuations drive moat at both low and high T
- Shape of moat regime is controlled by Yukawa coupling

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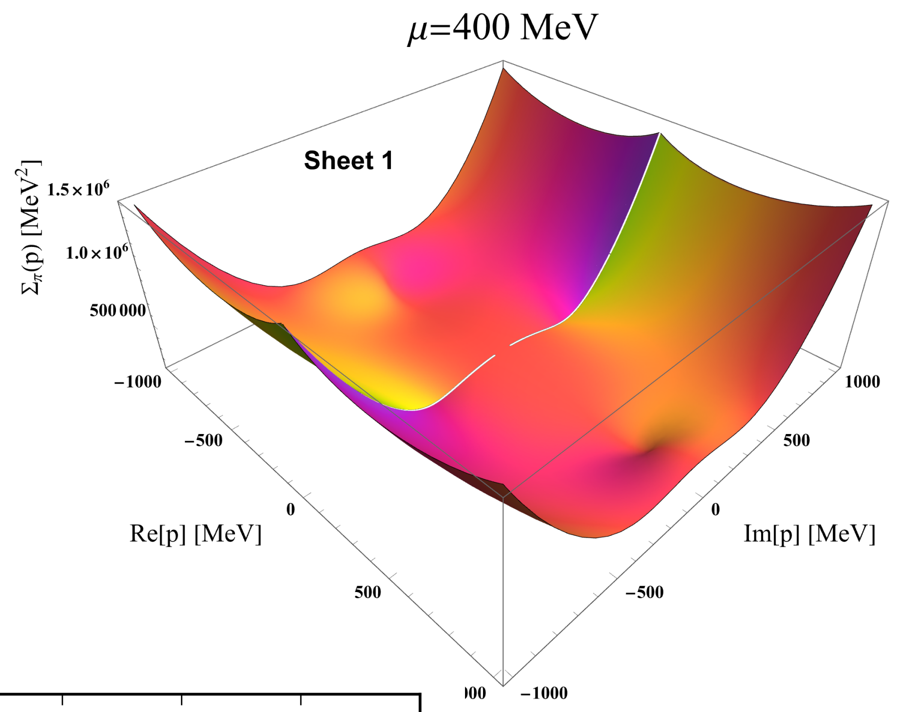

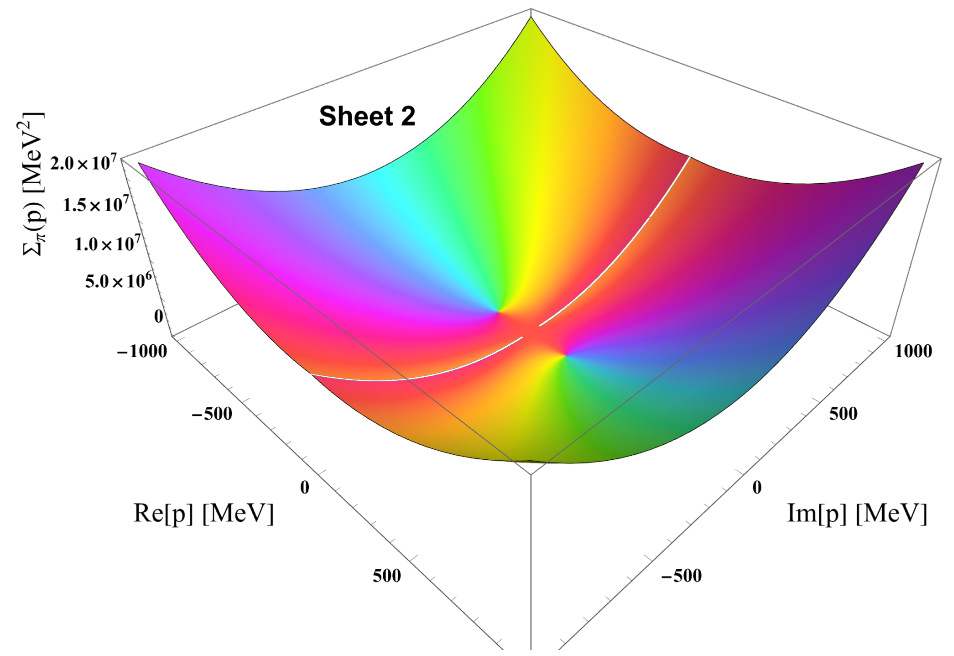

Dissecting the moat regime at low energies II :

- ✦ Analytic structure of meson two-point function
- ✦ Friedel Oscillations
- ✦ Spectral functions
- ✦ Inhomogeneous instability
- ✦ …