

Improving the AMPT Model with FRG for the search of QCD critical endpoint



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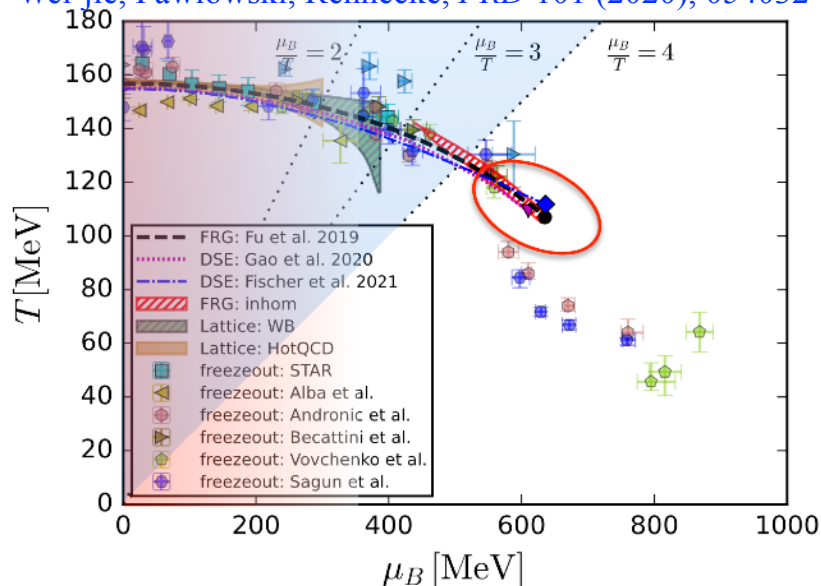
- [1] Qian Chen, Guo-Liang Ma, Phys.Rev.C 106 (2022) , 014907, arXiv:2207.11736
- [2] Han-Sheng Wang, Guo-Liang Ma, Zi-Wei Lin, Wei-jie Fu, Phys.Rev.C 105 (2022) , 034912, arXiv:2102.06937
- [3] Qian Chen, Rui Wen, Shi Yin, Wei-jie Fu, Zi-Wei Lin, Guo-Liang Ma, arXiv:2402.12823

Outline

- **Introduction**
 - QCD phase structure
 - AMPT model
- **AMPT studies on QCD phase structure**
 - Baryon number conservation effect
 - Nuclear thickness effect
 - Deep learning AMPT data
 - Hadronic effect
- **Summary and outlook**

QCD Phase Structure

Wei-jie, Pawłowski, Rennecke, *PRD* 101 (2020), 054032



FRG result:

$$(T, \mu_B)_{\text{CEP}} = (107, 635) \text{ MeV}$$

- Critical end point is the key feature of QCD phase structure
- Some hints from **RHIC-BES** experiment: net-baryon (proton) cumulants, directed flow, HBT, light nuclei...
- Experimental programs: **RHIC-BESII**, **FAIR**, **NICA**, **HIAF**

- Small baryon chemical potential: **Smooth Crossover Transition**
- Large baryon chemical potential: **First-order Phase Transition**
- QCD **critical endpoint**: where first-order phase transition ends
- Increase chemical potential by **lowering beam energy**

Why Conserved Charges Fluctuations

$$\langle (\delta N)^4 \rangle \sim \xi^7$$

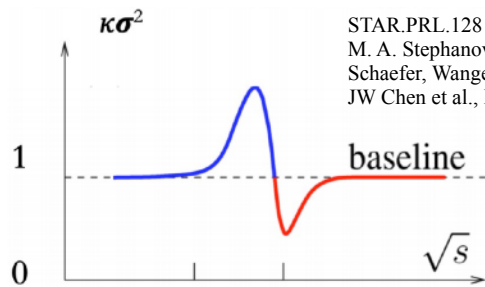
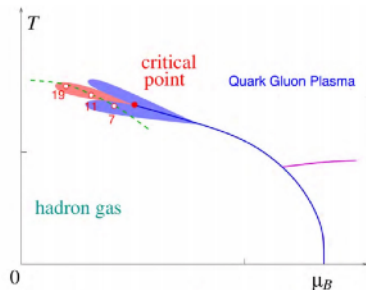
$$\langle (\delta N)^3 \rangle \sim \xi^{4.5}$$

➤ correlation length (ξ)
diverges at CEP

➤ fluctuations of conserved
charges are related to
susceptibility (χ)

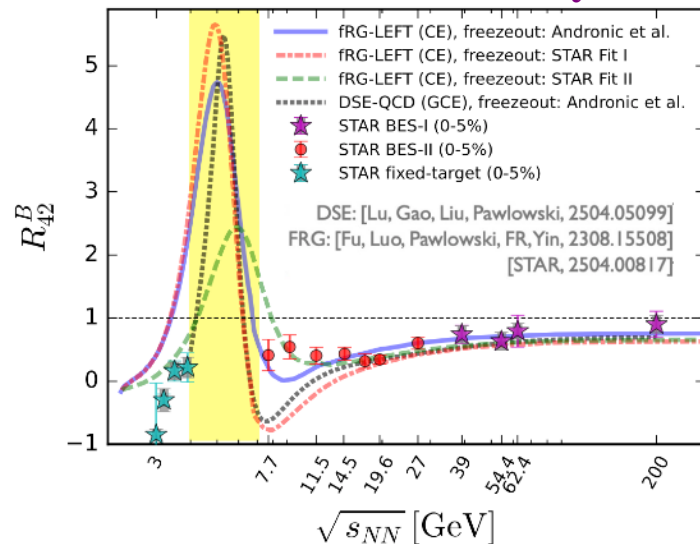
$$\kappa\sigma^2 = \frac{C_{4,q}}{C_{2,q}} = \frac{\chi_q^{(4)}}{\chi_q^{(2)}}$$

$$S\sigma = \frac{C_{3,q}}{C_{2,q}} = \frac{\chi_q^{(3)}}{\chi_q^{(2)}}$$



STAR.PRL.128 (2022) 20, 202303
M. A. Stephanov, PRL 107.052301(2011).
Schaefer, Wanger, PRD 85, 034027 (2012)
JW Chen et al., PRD 93, 034037 (2016);

Fluctuations measured by STAR



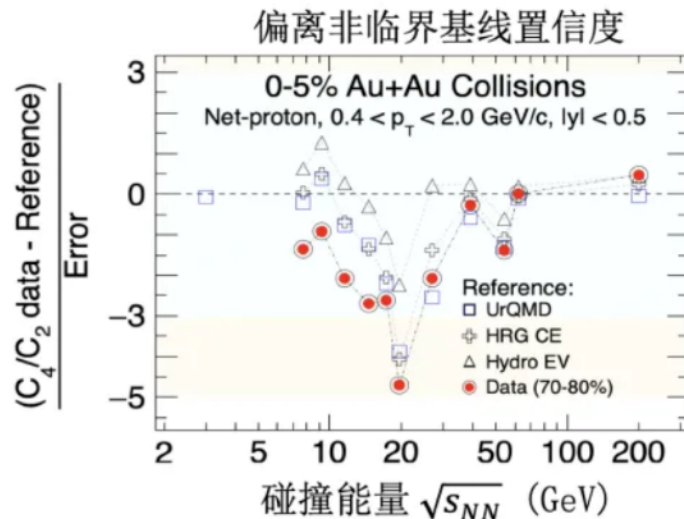
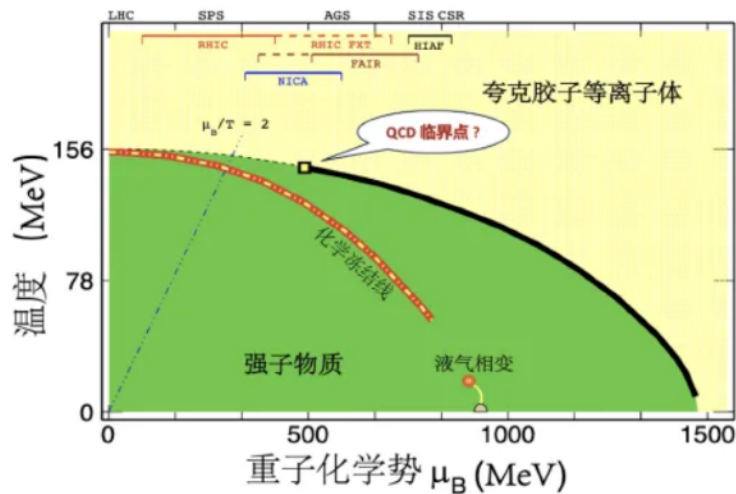
**Non-monotonic energy
dependence of the $\kappa\sigma^2$**



hint of entering critical region.

New Data of Conserved Charges Fluctuations

Phys. Rev. Lett. 135, 142301 (2025)



- For C_4/C_2 , a minimum around 19.6 GeV with a significance of deviation at $\sim 2-5\sigma$

Conserved Charges Fluctuations && Multi-particle Correlation

Cumulants:

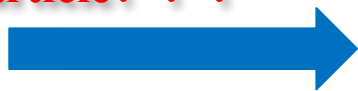
$$C_1 = \kappa_1,$$

$$C_2 = \kappa_2 + \kappa_1,$$

$$C_3 = \kappa_3 + 3\kappa_2 + \kappa_1,$$

$$C_4 = \kappa_4 + 6\kappa_3 + 7\kappa_2 + \kappa_1.$$

Only for one kind of particle! ! !



Correlation Functions:

$$\kappa_1 = C_1 = \langle N \rangle,$$

$$\kappa_2 = -C_1 + C_2,$$

$$\kappa_3 = 2C_1 - 3C_2 + C_3,$$

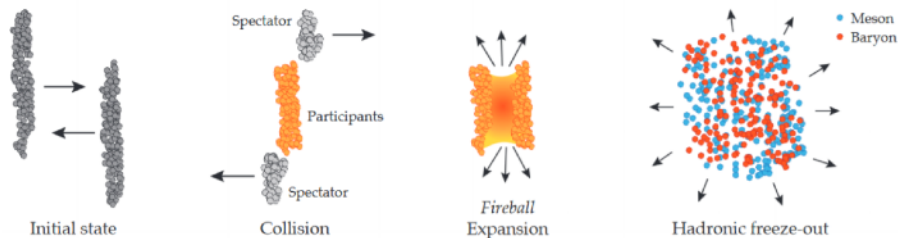
$$\kappa_4 = -6C_1 + 11C_2 - 6C_3 + C_4.$$

Factorial moments:

$$F_3 = \int dy_1 dy_2 dy_3 \rho_3(y_1, y_2, y_3) = F_1^3 + 3F_1 C_2 + C_3$$
$$\rho_3(y_1, y_2, y_3) = \rho_1(y_1)\rho_1(y_2)\rho_1(y_3) + \rho_1(y_1)\underline{C_2(y_2, y_3)}$$
$$+ \rho_1(y_2)\underline{C_2(y_1, y_3)} + \rho_1(y_3)\underline{C_2(y_1, y_2)}$$
$$+ \underline{C_3(y_1, y_2, y_3)}$$

Bzdak, Adam et al. Phys.Rev. C95 (2017)5,054906.

The AMPT Model



new quark coalescence:

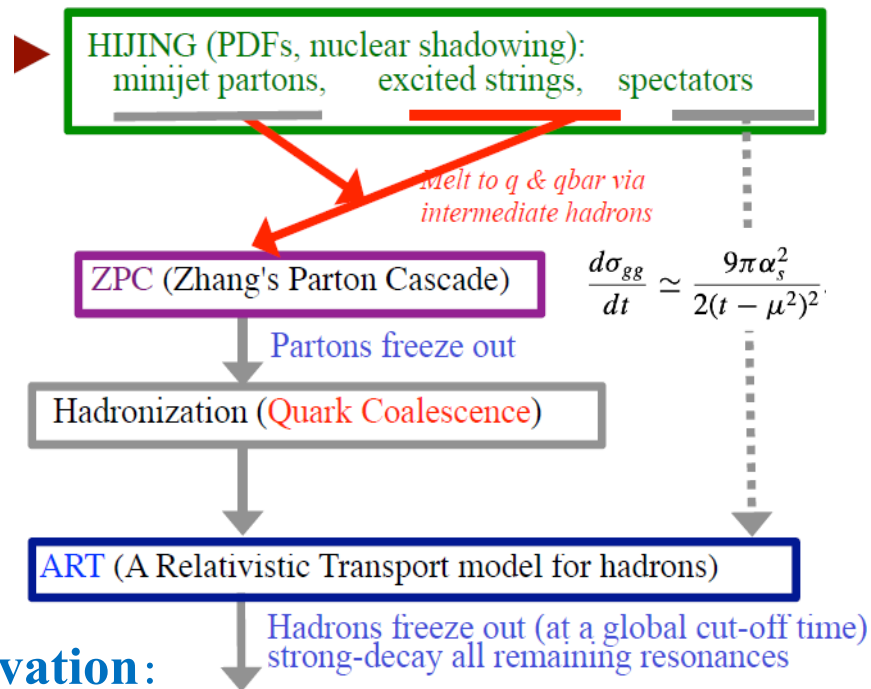
Quarks form either meson or baryon, depending on the distance to its coalescence partner(s) (r_{BM})

$d_B < d_M * r_{BM}$:form baryon
otherwise: form meson

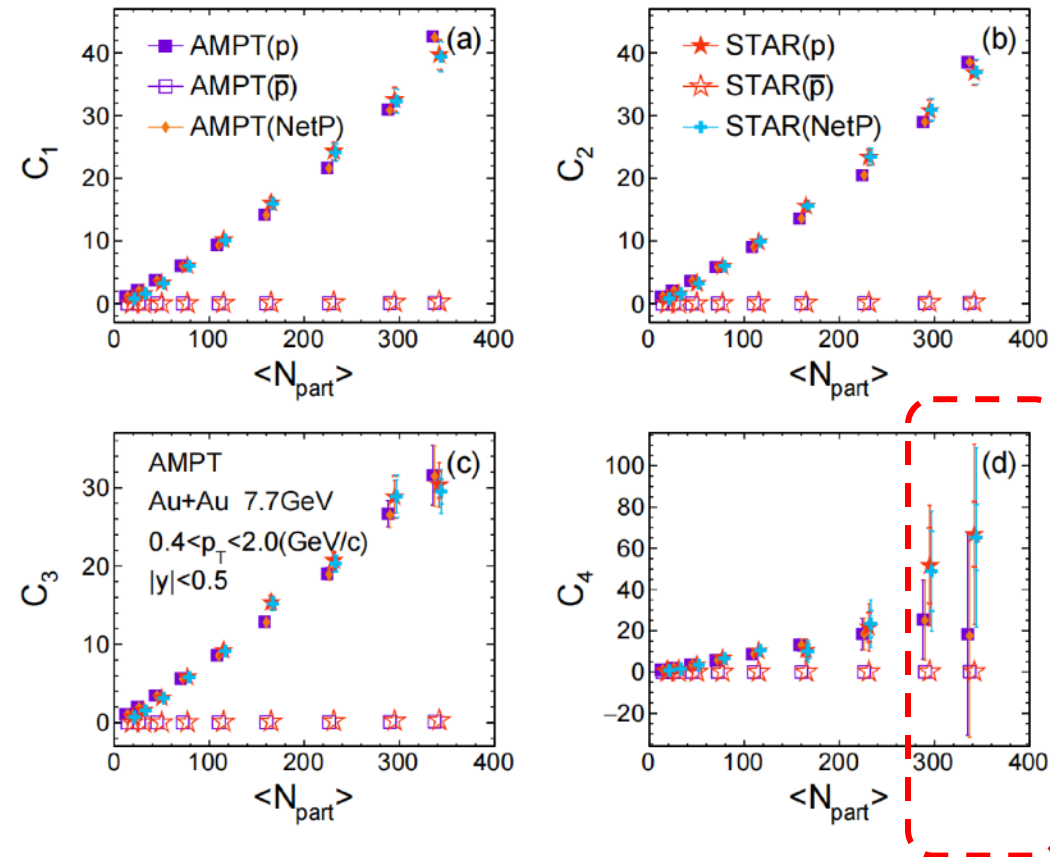
Y. He and Z.-W. Lin, Phys. Rev. C 96, 014910 (2017).

charge conservation:

The AMPT model ensures conservation of conserved charges (including electric charge, baryon number, and strangeness) for all hadronic reaction channels during hadronic rescatterings



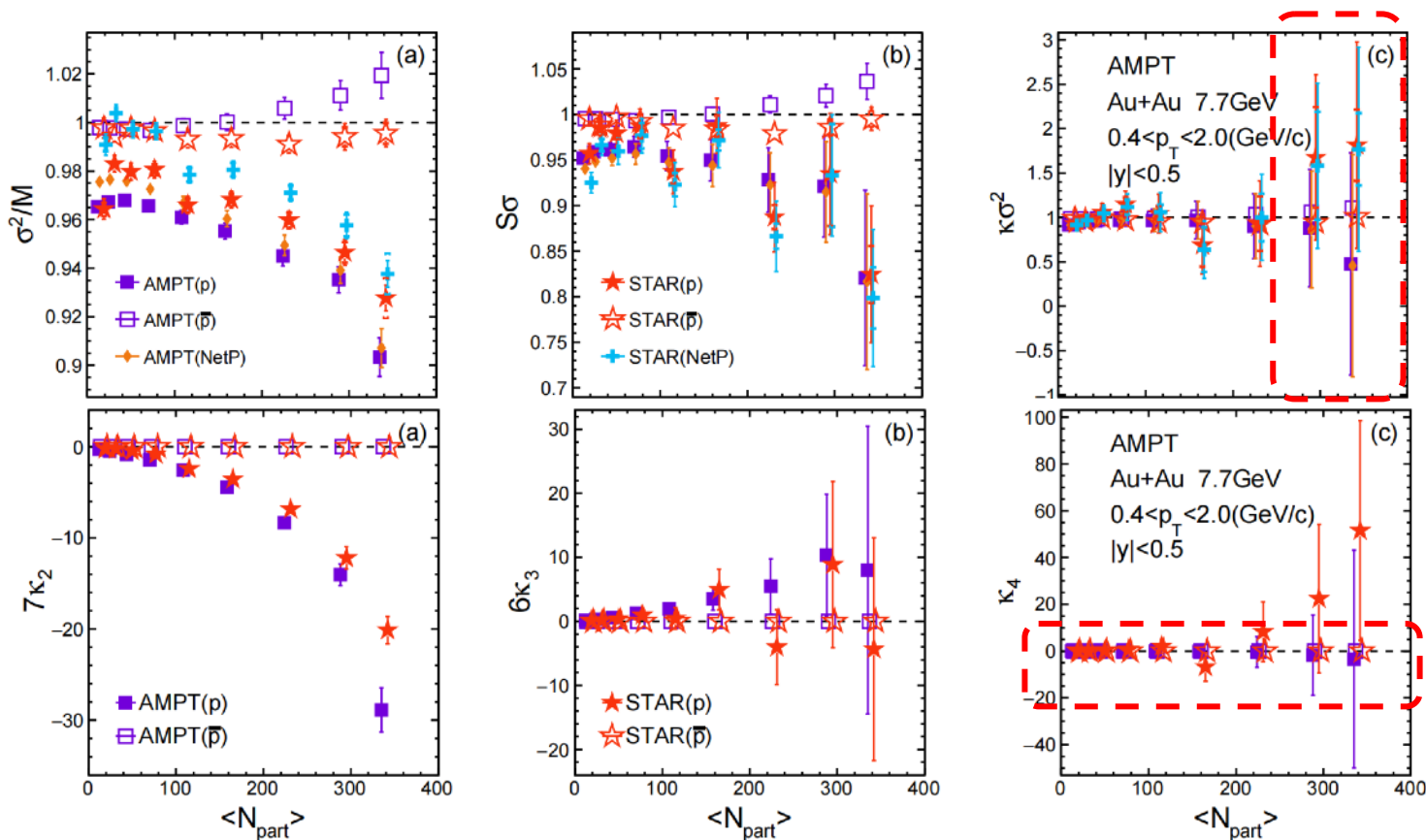
Proton Number Fluctuations from AMPT



- The cumulants C_n of proton, antiproton, and net-proton all exhibit an increasing dependence of $\langle N_{part} \rangle$
- In the 0-5% and 5-10% centrality ranges, the fourth-order cumulant (C_4) in the AMPT model notably **underestimates** STAR results

Qian Chen, Guo-Liang Ma, Phys.Rev.C 106 (2022) 014907

Multi-proton Correlations from AMPT



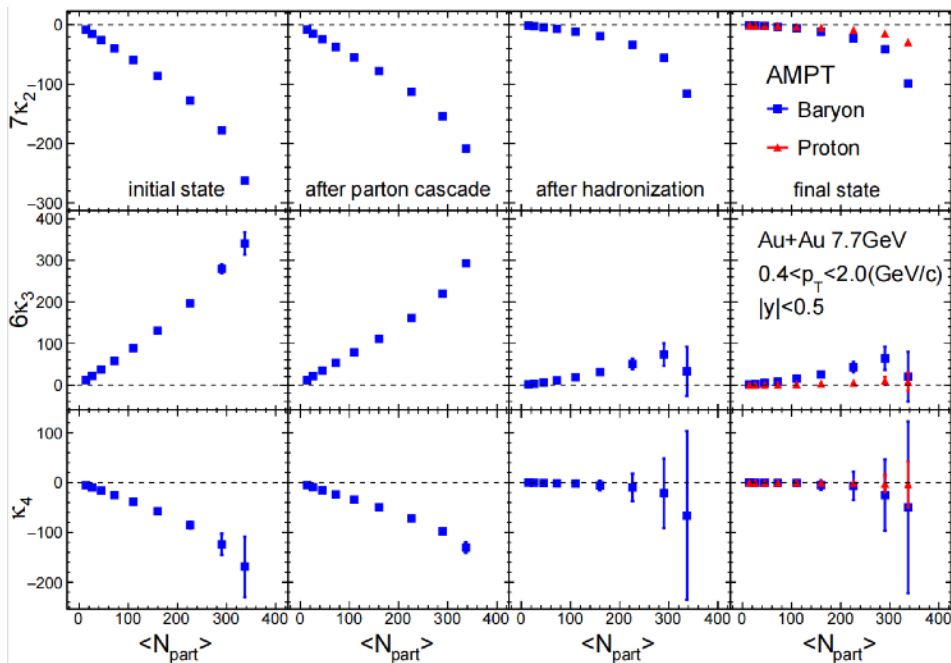
Four-proton correlation from the AMPT model consistent with **zero**.

No long-range correlation, lack of the CEP ?

How Baryon Number Conservation Breaks Down

Binomial distribution holds if baryon number conservation:

$$P(N) = \frac{B!}{N!(B-N)!} p^N (1-p)^{(B-N)}$$



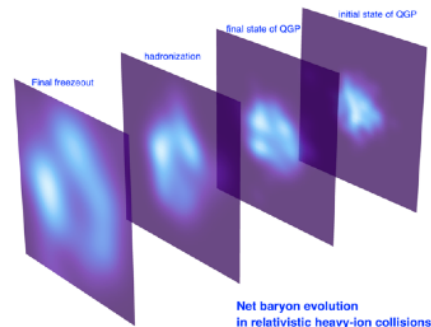
n-baryon correlations:

$$\begin{aligned} \kappa_1 &= \langle N \rangle \\ &= pB \\ \kappa_2 &= -\frac{\langle N \rangle^2}{B} \end{aligned}$$

$$\kappa_3 = 2\frac{\langle N \rangle^3}{B^2}$$

$$\kappa_4 = -6\frac{\langle N \rangle^4}{B^3}$$

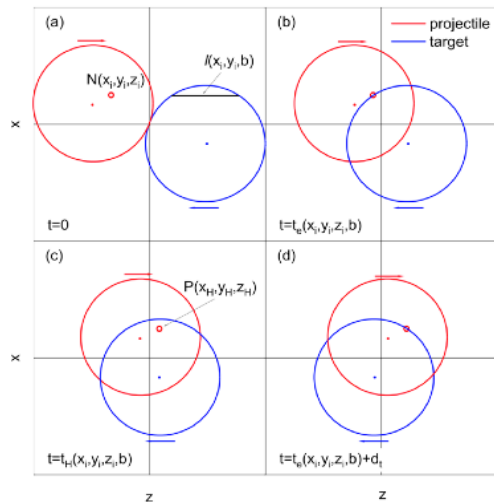
- κ_n with the sign of $(-1)^{n+1}$ and the strength proportional to $\langle N \rangle^n$
- κ_n are getting weaker with the stage evolution of heavy-ion collisions.



(3) Nuclear thickness effect

[2] Han-Sheng Wang, Guo-Liang Ma, Zi-Wei Lin, Wei-jie Fu, Phys.Rev.C 105 (2022) , 034912, arXiv:2102.06937

Two Trains Problem: How to Introduce Nuclear Thickness



**z-position of
parton formation:**

$$z_H = z_i \pm t_H \sinh y_{CM},$$

**time of
parton formation:**

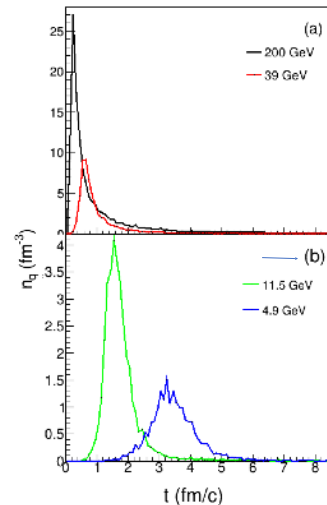
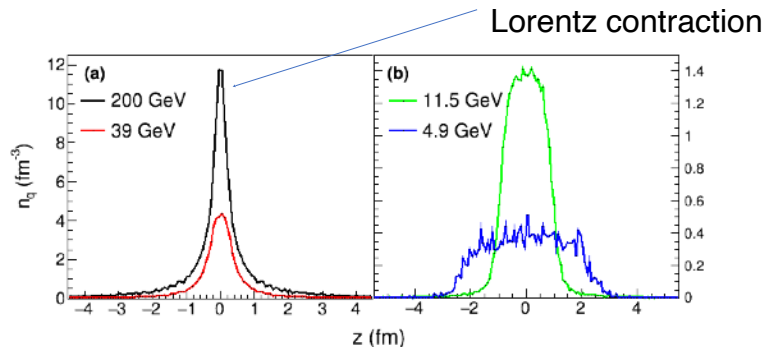
$$t_e(x_i, y_i, z_i, b) = \frac{\sqrt{R^2 - b^2/4} - [l(x_i, y_i, b)/2 \pm z_i]}{2 \sinh y_{CM}},$$

$$\frac{d^2 E_T}{dy dt_H} = a_n [(t_H - t_e)(t_e + d_t - t_H)]^n \frac{dE_T}{dy}, \quad t_H \in [t_e, t_e + d_t],$$

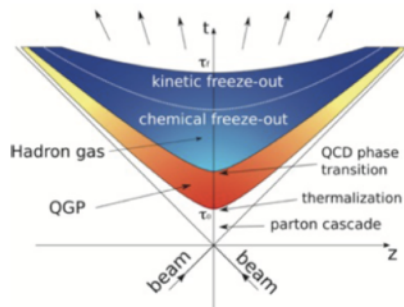
$$d_t = \frac{2R_A}{\sinh y_{CM}},$$

Normalization
factor:

$$a_n = \frac{1}{d_t^{2n+1} B(n+1, n+1)} \xrightarrow{\text{Beta dis.}}$$

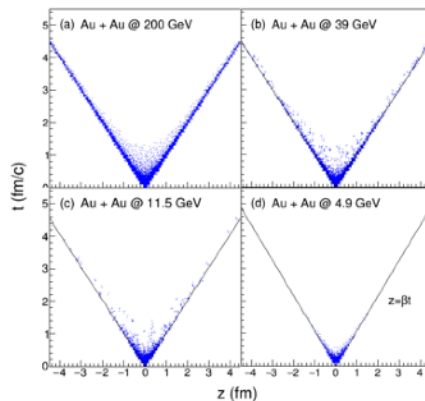
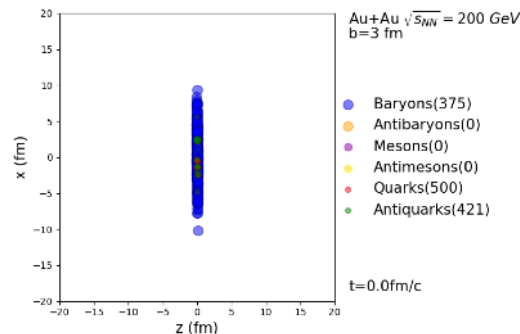


AMPT Space-time Picture without vs with Thickness

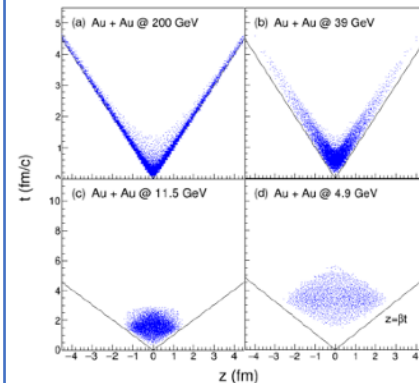
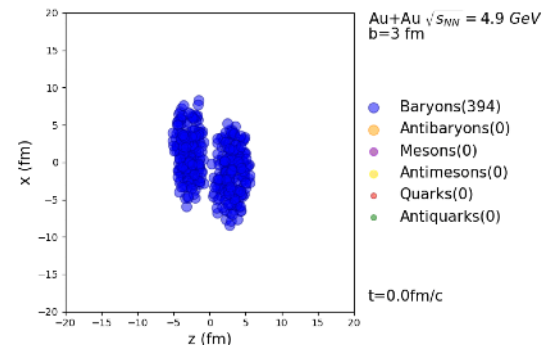


At low energy, the longitudinal thickness of nucleus should be considered

w/o thickness



with thickness



Thickness Effect on Maximum Density of Central Cell

net-Baryon's Maximum Energy and Density in the Bjorken Picture:

$$n_{Bj}^{Max}(t) = \frac{1}{\tau_F A_t} \frac{dN_{netB}}{dy} \quad \epsilon_{Bj}^{Max}(t) = \frac{1}{\tau_F A_t} \frac{dE_T}{dy}$$

Maximum energy density in the triangular picture:

$$\epsilon_{tri}^{max} = \frac{2}{A_T t_{21}} \frac{dm_T}{dy}(0) \left[-1 - \frac{\tau_F}{t_{21}} + \sqrt{\frac{\tau_F}{t_{21}}} \sqrt{2 + \frac{\tau_F}{t_{21}}} + 2 \ln \left(\frac{1 + \sqrt{1 + 2 t_{21}/\tau_F}}{2} \right) \right].$$

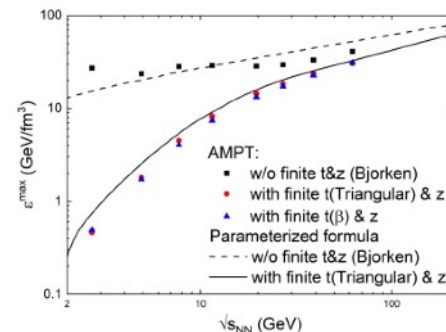
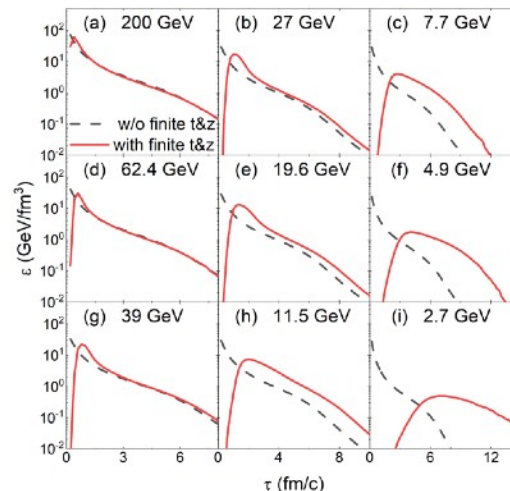
T. Mendenhall and Z. W. Lin, Phys. Rev. C103, 024907 (2021)

If assuming Critical energy density:

$$\epsilon_c = 3P_c = 3B \approx 0.6 GeV$$

then QGP can be achieved at:

$$\sqrt{s_{NN}} > 2.7 GeV, E_{Kbeam} > 3.5 GeV$$



Extracting T & μ under Boltzmann & Quantum Statistics

$$f_{\text{Boltz}}[\mu, T] = \frac{1}{e^{(\epsilon - \mu_q)/T}}, \quad (q = u, d, s)$$

$$n_q = N_c / \pi 2 \left(\int_{m_q}^{\infty} \frac{\epsilon^2 d\epsilon}{e^{\frac{\epsilon - \mu_q}{T}}} - \int_{m_q}^{\infty} \frac{\epsilon^2 d\epsilon}{e^{\frac{\epsilon + \mu_q}{T}}} \right), \quad \epsilon_q = N_c / \pi 2 \left(\int_{m_q}^{\infty} \frac{\epsilon^3 d\epsilon}{e^{\frac{\epsilon - \mu_q}{T}}} + \int_{m_q}^{\infty} \frac{\epsilon^3 d\epsilon}{e^{\frac{\epsilon + \mu_q}{T}}} \right), \quad \epsilon_g = (N_c^2 - 1) / \pi 2 \int_0^{\infty} \frac{\epsilon^3 d\epsilon}{e^{\frac{\epsilon}{T}}}$$

$$m_u = 0.006, m_d = 0.010, m_s = 0.199, N_c = 3 \quad N_c: \text{ color number}$$

$$f_{\text{BE}}[\mu, T] = \frac{1}{e^{(\epsilon - \mu_q)/T} - 1}, \quad f_{\text{FD}}[\mu, T] = \frac{1}{e^{(\epsilon - \mu_q)/T} + 1}, \quad (q = u, d, s)$$

$$n_q = N_c / \pi 2 \left(\int_{m_q}^{\infty} \frac{\epsilon^2 d\epsilon}{e^{\frac{\epsilon - \mu_q}{T}} + 1} - \int_{m_q}^{\infty} \frac{\epsilon^2 d\epsilon}{e^{\frac{\epsilon + \mu_q}{T}} + 1} \right), \quad \epsilon_q = N_c / \pi 2 \left(\int_{m_q}^{\infty} \frac{\epsilon^3 d\epsilon}{e^{\frac{\epsilon - \mu_q}{T}} + 1} + \int_{m_q}^{\infty} \frac{\epsilon^3 d\epsilon}{e^{\frac{\epsilon + \mu_q}{T}} + 1} \right),$$

$$\epsilon_g = (N_c^2 - 1) / \pi 2 \int_0^{\infty} \frac{\epsilon^3 d\epsilon}{e^{\frac{\epsilon}{T}} - 1}$$

Extracting T & μ under Boltzmann & Quantum Statistics

Relations between different chemical potentials:

$$\mu_u \rightarrow \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q, \mu_d \rightarrow \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q, \mu_s \rightarrow \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$$

$$\left\{ \begin{array}{l} n_B = (n_u + n_d + n_s)/3 = f_{n_B}(T, \mu_B, \mu_s, \mu_Q) \\ n_Q = 2/3 n_u - 1/3 n_d - 1/3 n_s = f_{n_Q}(T, \mu_B, \mu_s, \mu_Q) \\ n_S = -n_s = f_{n_S}(T, \mu_B, \mu_s, \mu_Q) \\ \varepsilon = \varepsilon_u + \varepsilon_d + \varepsilon_s + \varepsilon_g = f_\varepsilon(T, \mu_B, \mu_s, \mu_Q) \end{array} \right.$$

$$\left\{ \begin{array}{l} n_B = (n_u + n_d + n_s)/3 = f_{n_B}(T, \mu_Q) \\ \varepsilon = \varepsilon_u + \varepsilon_d + \varepsilon_s + \varepsilon_g = f_\varepsilon(T, \mu_Q) \end{array} \right., \quad (\mu_q = 0, \mu_s = 0)$$

Extracting T & μ under Boltzmann & Quantum Statistics

Boltzmann Statistics:

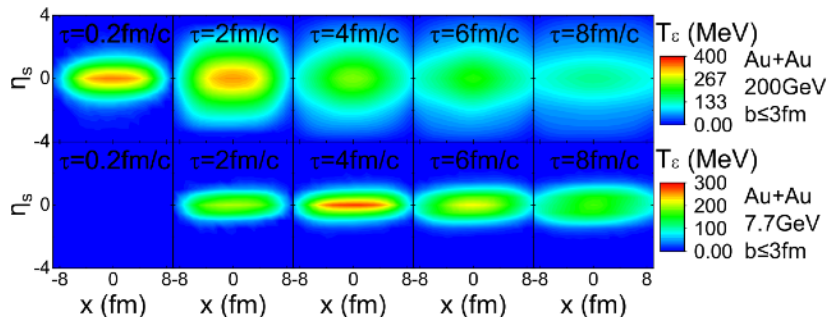
$$\begin{aligned}
 n_B &= \frac{2}{\pi^2} T \left[m_d^2 K_2 \left(\frac{m_d}{T} \right) \sinh \left(\frac{\mu_B - \mu_Q}{3T} \right) + m_s^2 K_2 \left(\frac{m_s}{T} \right) \sinh \left(\frac{\mu_B - \mu_Q - 3\mu_S}{3T} \right) \right. \\
 &\quad \left. + m_u^2 K_2 \left(\frac{m_u}{T} \right) \sinh \left(\frac{\mu_B + 2\mu_Q}{3T} \right) \right], \\
 n_Q &= -\frac{2}{\pi^2} T \left[m_d^2 K_2 \left(\frac{m_d}{T} \right) \sinh \left(\frac{\mu_B - \mu_Q}{3T} \right) + m_s^2 K_2 \left(\frac{m_s}{T} \right) \sinh \left(\frac{\mu_B - \mu_Q - 3\mu_S}{3T} \right) \right. \\
 &\quad \left. - 2m_u^2 K_2 \left(\frac{m_u}{T} \right) \sinh \left(\frac{\mu_B + 2\mu_Q}{3T} \right) \right], \\
 n_S &= -\frac{6}{\pi^2} m_s^2 T K_2 \left(\frac{m_s}{T} \right) \sinh \left(\frac{\mu_B - \mu_Q - 3\mu_S}{3T} \right), \\
 \epsilon_{total} &= \frac{6}{\pi^2} T \left[8T^3 + m_u^3 K_1 \left(\frac{m_u}{T} \right) \cosh \left(\frac{\mu_B + 2\mu_Q}{3T} \right) + 3m_u^2 T K_2 \left(\frac{m_u}{T} \right) \cosh \left(\frac{\mu_B + 2\mu_Q}{3T} \right) \right. \\
 &\quad + m_s^3 K_1 \left(\frac{m_s}{T} \right) \cosh \left(\frac{\mu_B - \mu_Q - 3\mu_S}{3T} \right) + 3m_s^2 T K_2 \left(\frac{m_s}{T} \right) \cosh \left(\frac{\mu_B - \mu_Q - 3\mu_S}{3T} \right) \\
 &\quad \left. + m_d^3 K_1 \left(\frac{m_d}{T} \right) \cosh \left(\frac{\mu_B - \mu_Q}{3T} \right) + 3m_d^2 T K_2 \left(\frac{m_d}{T} \right) \cosh \left(\frac{\mu_B - \mu_Q}{3T} \right) \right],
 \end{aligned}$$

Quantum Statistics:

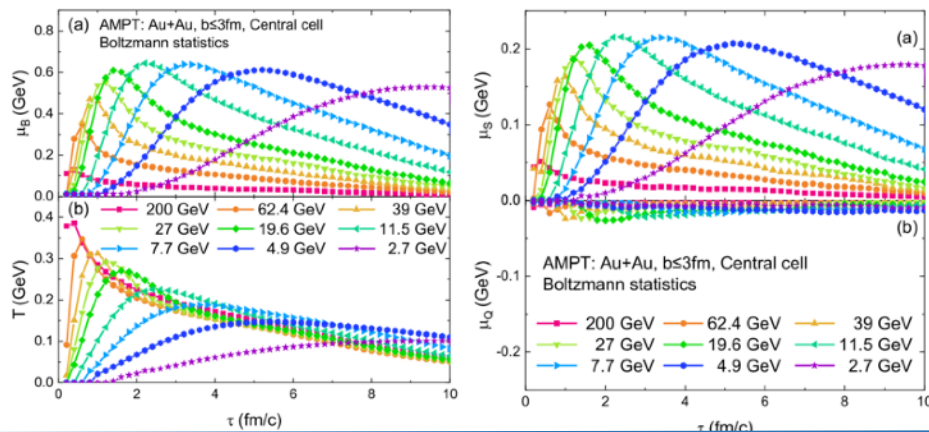
$$\begin{aligned}
 n_B &= \frac{1}{27\pi^2} \mu_B^3 - \frac{1}{9\pi^2} \mu_B^2 \mu_S + \frac{1}{9\pi^2} \mu_B [3(\pi^2 T^2 + \mu_S^2) + 2\mu_Q \mu_S + 2\mu_Q^2] \\
 &\quad - \frac{1}{27\pi^2} (9\pi^2 \mu_S T^2 - 2\mu_Q^3 + 3\mu_Q^2 \mu_S + 9\mu_Q \mu_S^2 + 9\mu_S^3), \\
 n_Q &= \frac{1}{9\pi^2} \mu_Q (6\pi^2 T^2 + 2\mu_B^2 - 2\mu_B \mu_S + 3\mu_S^2) \\
 &\quad + \frac{1}{9\pi^2} \mu_S (3\pi^2 T^2 + \mu_B^2 - 3\mu_B \mu_S + 3\mu_S^2) + \frac{1}{9\pi^2} \mu_Q^2 (2\mu_B + \mu_S) + \frac{2}{9\pi^2} \mu_Q^3, \\
 n_S &= -\frac{1}{27\pi^2} \mu_B^3 + \frac{1}{9\pi^2} \mu_B^2 (\mu_Q + 3\mu_S) - \frac{1}{9\pi^2} \mu_B (3\pi^2 T^2 + \mu_Q^2 + 6\mu_Q \mu_S + 9\mu_S^2) \\
 &\quad + \frac{1}{27\pi^2} (\mu_Q + 3\mu_S) (9\pi^2 T^2 + \mu_Q^2 + 6\mu_Q \mu_S + 9\mu_S^2), \\
 \epsilon_{total} &= \frac{1}{36\pi^2} \mu_B^4 - \frac{1}{9\pi^2} \mu_B^3 \mu_S + \frac{1}{6\pi^2} \mu_B^2 (3\pi^2 T^2 + 2\mu_Q^2 + 2\mu_Q \mu_S + 3\mu_S^2) \\
 &\quad - \frac{1}{9\pi^2} \mu_B (9\pi^2 \mu_S T^2 - 2\mu_Q^3 + 3\mu_Q^2 \mu_S + 9\mu_Q \mu_S^2 + 9\mu_S^3) \\
 &\quad + \frac{1}{36\pi^2} [3\pi^2 T^2 (19\pi^2 T^2 + 12\mu_Q^2 + 12\mu_Q \mu_S + 18\mu_S^2) \\
 &\quad + 6\mu_Q^4 + 4\mu_Q^3 \mu_S + 18\mu_Q^2 \mu_S^2 + 27\mu_S^4 + 36\mu_Q \mu_S^3].
 \end{aligned}$$

- In the AMPT model, partons obey Boltzmann statistics, with quantum statistics serving only as a reference.

AMPT results on Time evolution of T & μ



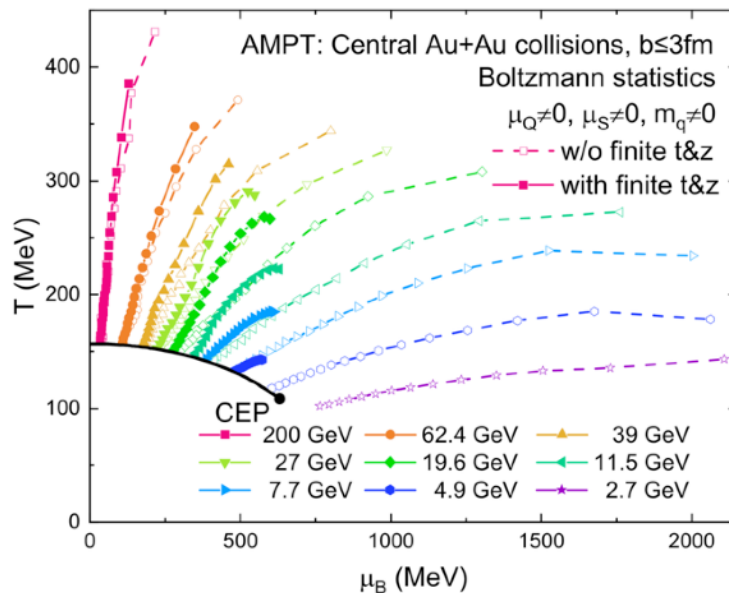
T & μ at the center of the overlapping region:



- A+A system was first compressed and heated, then cooled and dilated due to expansion.
- The center of the overlapping region reached its maximum temperature ($\tau \approx 0.2$ and 4 fm/c) at energies of 200 and 7.7 GeV, respectively.
- The baryon chemical potential reaches maximum at an energy of 7.7 GeV.

Trajectories of the QGP at different collision energies

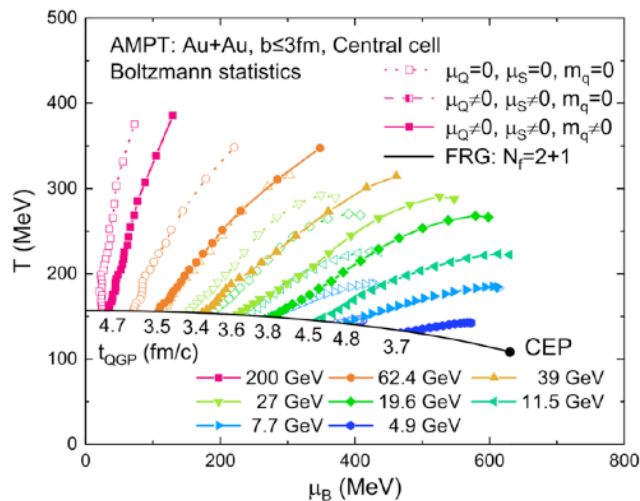
Trajectories of the QGP in center cell produced at different collision energies:



- As collision energy decreases, difference caused by nuclear thickness becomes increasingly pronounced.
- For lower energies, neglecting the finite thickness effect will overestimate temperature and baryon chemical potential.

Trajectories of the QGP at different collision energies

Trajectories of the QGP in center cell produced at different collision energies:

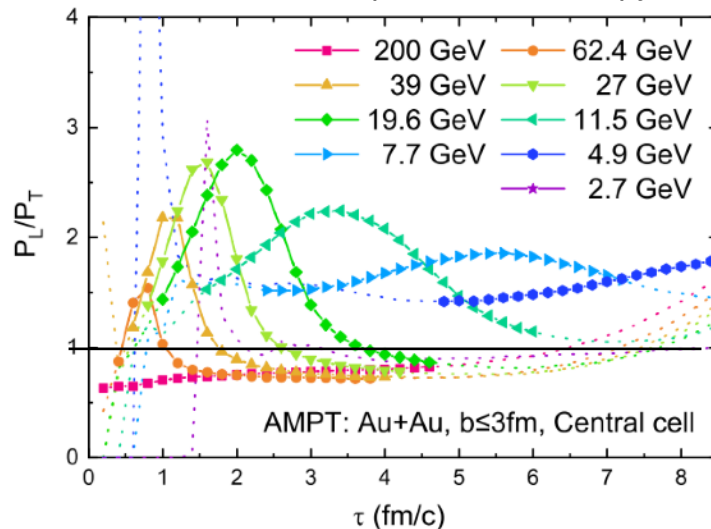


- Partonic phase can last 3.4–4.8 fm/c
- μ_Q 、 μ_S play a not negligible role in QGP evolution.
- Beam energies below 4.9 GeV are likely to reach the CEP from FRG, which is achievable in fixed-target experiments at RHIC.

The degree to which it reaches thermal equilibrium

$$T^{\mu\nu} = \frac{1}{V} \sum_i \frac{p_i^\mu p_i^\nu}{E_i}, \quad P_L = T_{33} \quad P_T = (T_{11} + T_{22})/2$$

The evolution of pressure anisotropy



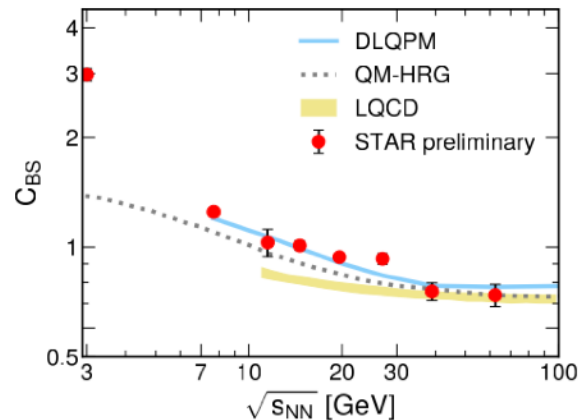
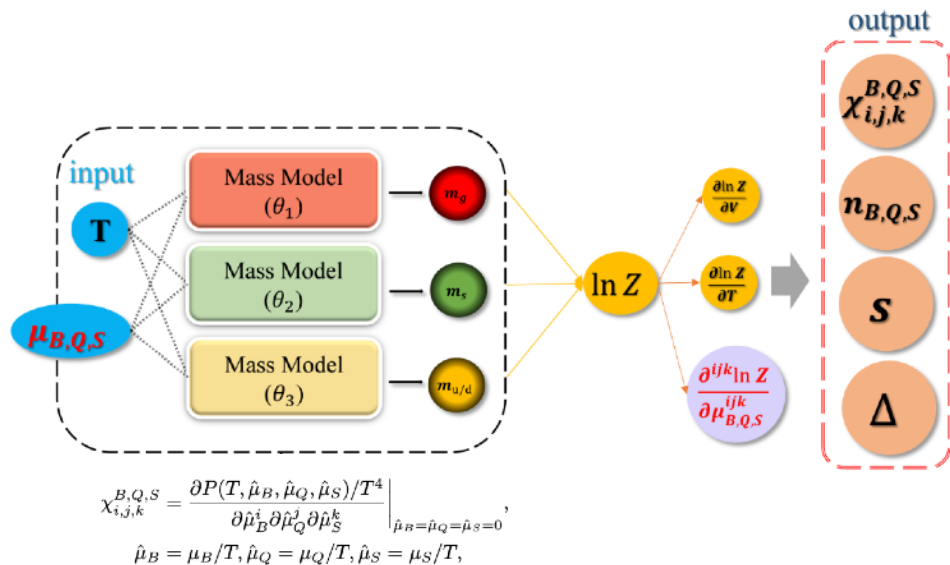
- When some systems approach the QCD phase boundary, they do not undergo complete thermalization.
- At lower energies, partons deviate more from thermal equilibrium.

(4) Deep learning AMPT data

Deep Learning Quasi Particle Model

F. P. Li, L-G. Pang, G-Y. Qin,

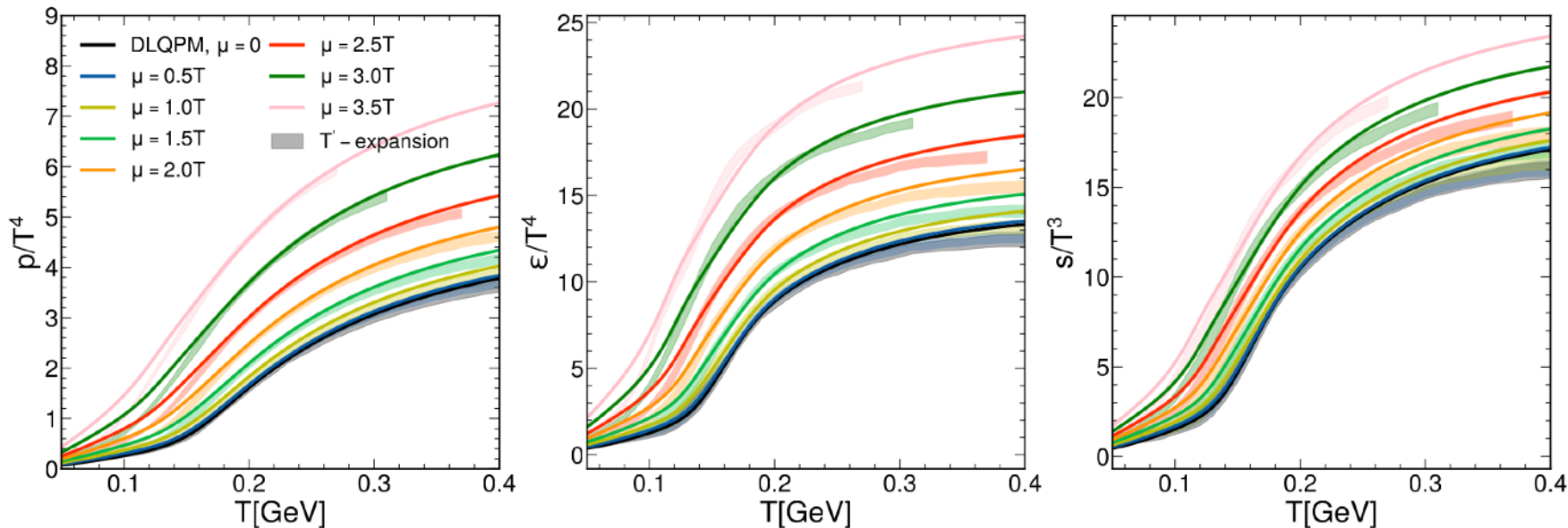
- 1) Deep-learning quasi-particle masses from QCD equation of state, arXiv:2211.07994
- 2) QCD equation of state at finite μ_B using deep learning assisted quasi-parton model, arXiv:2501.10012
- 3) Extending the Deep-Learning Quasi-Parton Gas Model to 4D QCD Equation of State, arXiv:25XX.XXXX



```
Tem, Mud, Ms, Mgluon = DLQPM(T["11.5GeV"], muB["11.5GeV"], muQ["11.5GeV"], muS["11.5GeV"],
                                path_data, device, mode = "save")
```

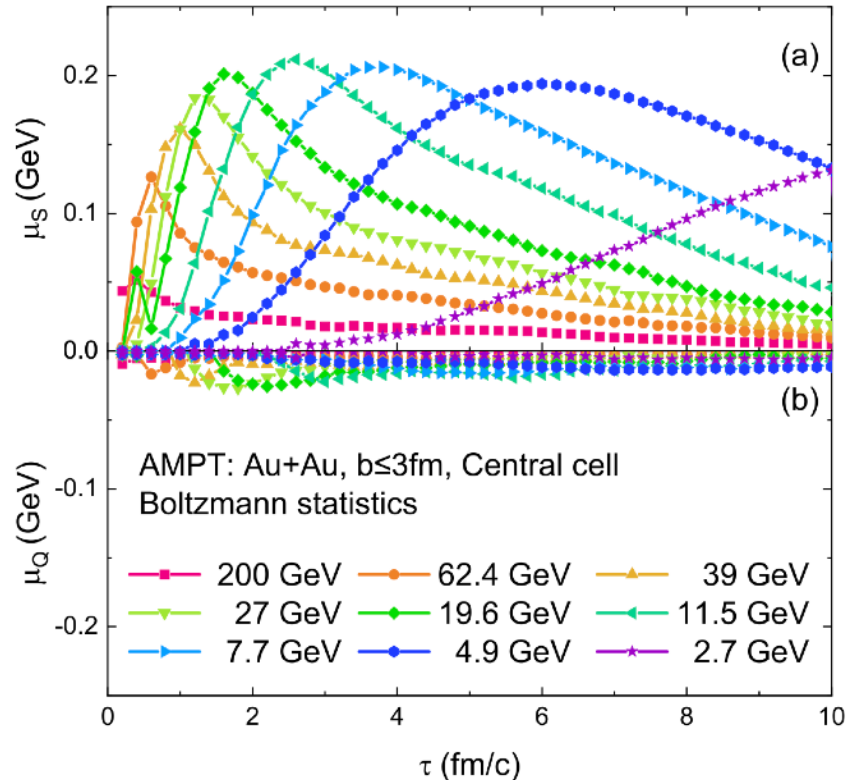
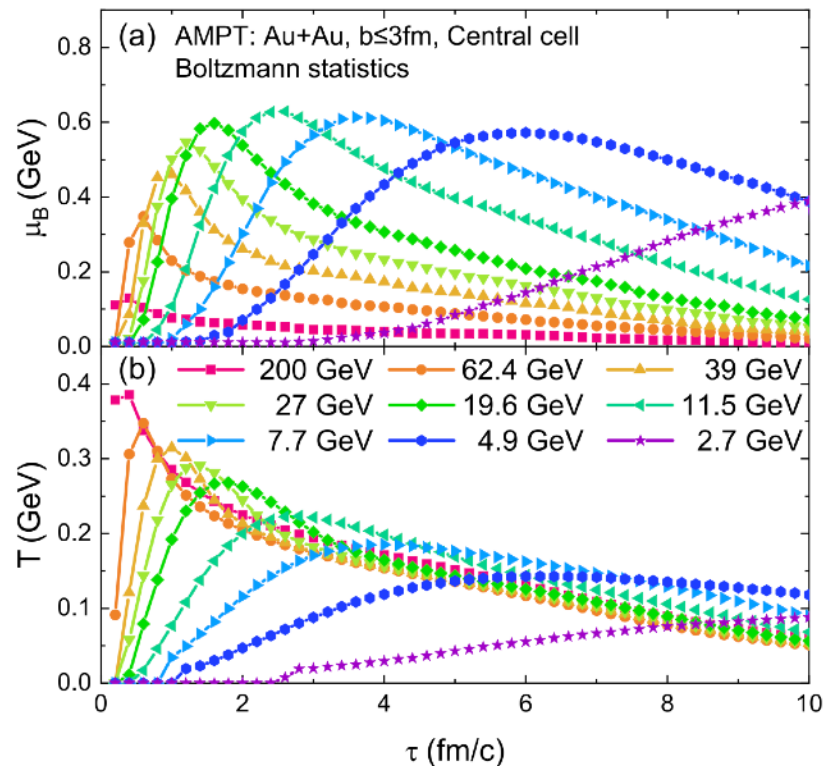
DLQP Model's Good Fit for EOS

F. P. Li, L-G. Pang, G-Y. Qin, Extending the Deep-Learning Quasi-Parton Gas Model to 4D QCD Equation of State, arxiv:25XX.XXXX

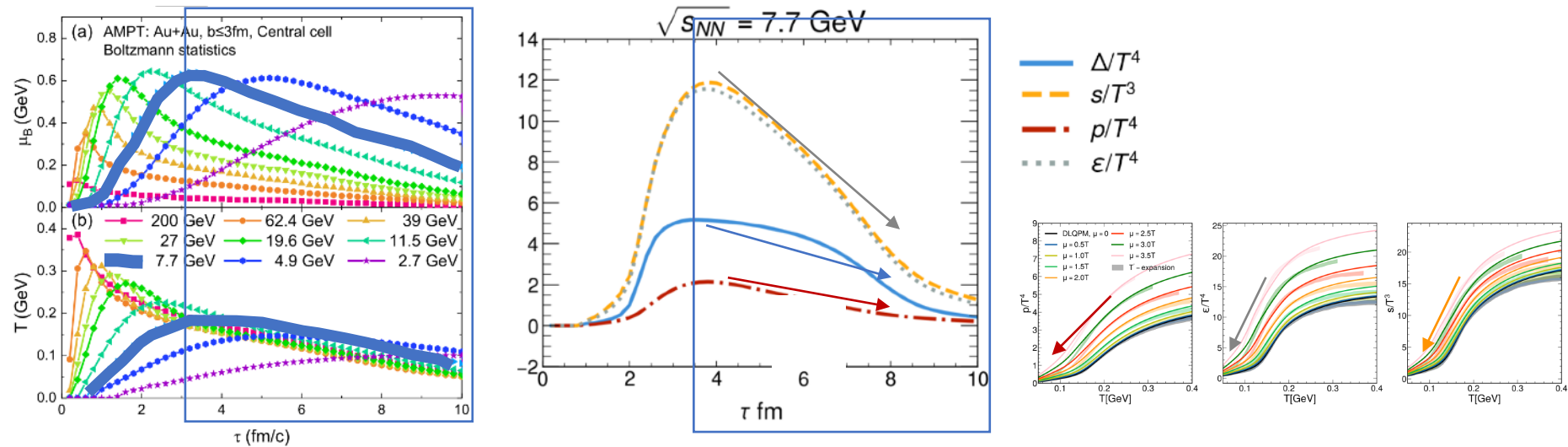


$$\mu = \sqrt{\mu_B^2 + \mu_Q^2 + \mu_S^2}, \text{ under the constraint that } \mu_B/\sqrt{2} = \mu_Q = \mu_S.$$

AMPT Input to DLQP Model

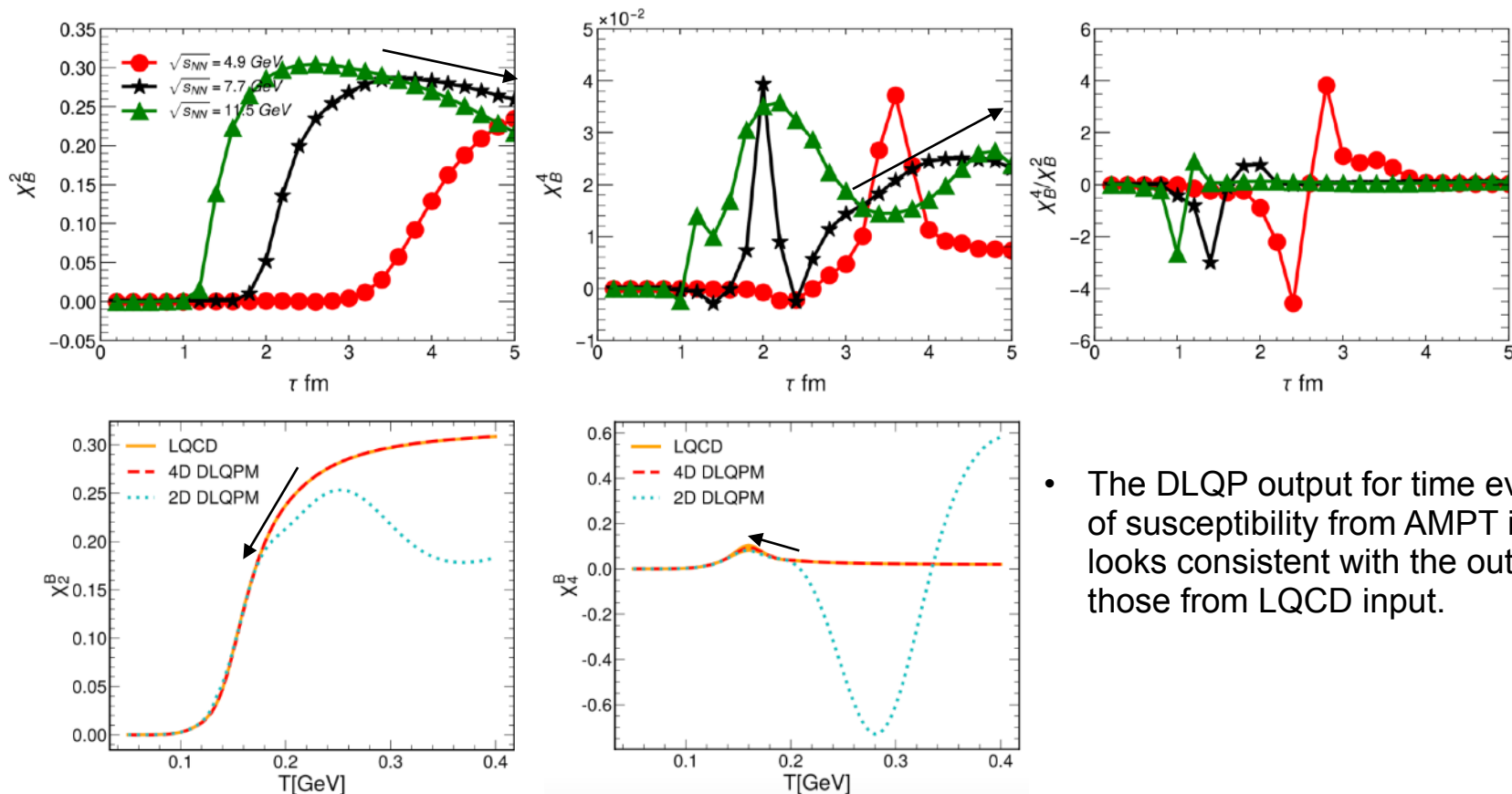


DLQP model output generated by AMPT input



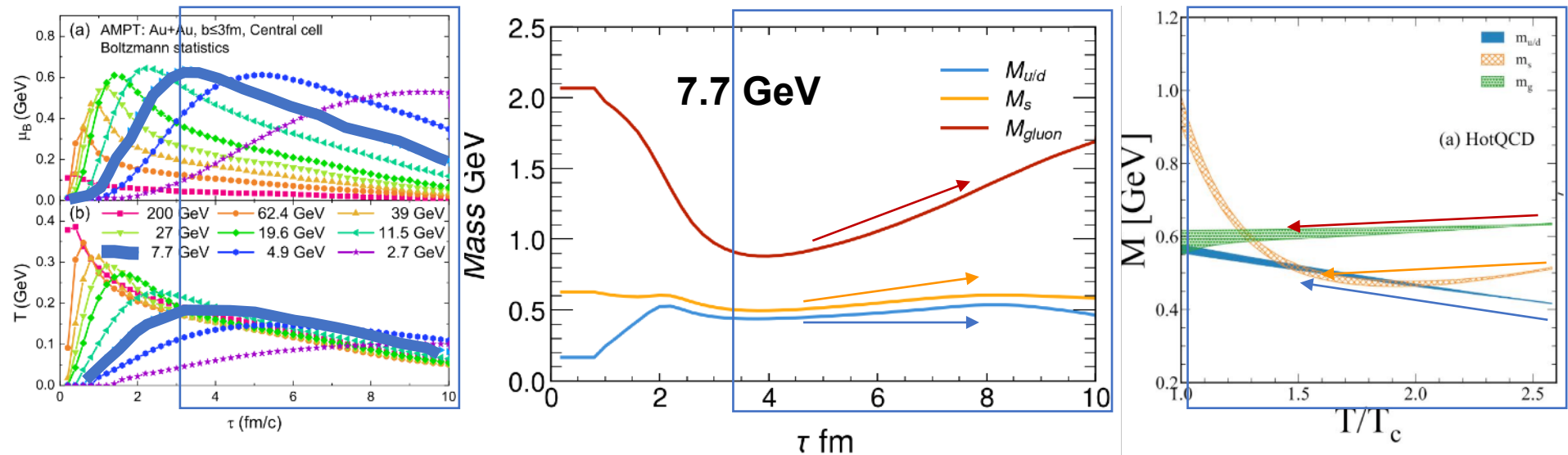
- The DLQP output for time evolution of EOS from AMPT input looks consistent with the output of those from LQCD input.

DLQP model output generated by AMPT input



- The DLQP output for time evolution of susceptibility from AMPT input looks consistent with the output of those from LQCD input.

DLQP model output generated by AMPT input



FuPeng Li, HL Lu, LG Pang, GY Qin, PLB 2023

- But the DLQP output for time evolution of m_i from AMPT input is opposite to the output of those from LQCD input. **Why?**

How to incorporate FRG interaction into AMPT

$$Z(p^2) \xrightarrow[\text{缀饰 VS 返璞}]{\text{dressed VS plain}} \mathcal{M} \xrightarrow{|\mathcal{M}|^2} \sigma$$

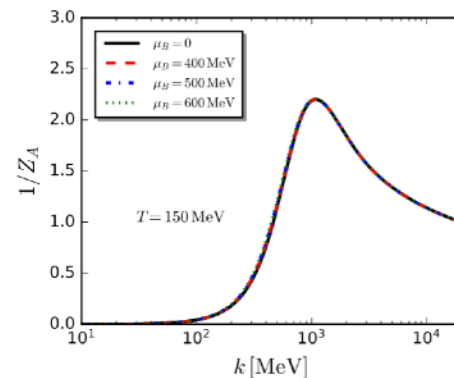
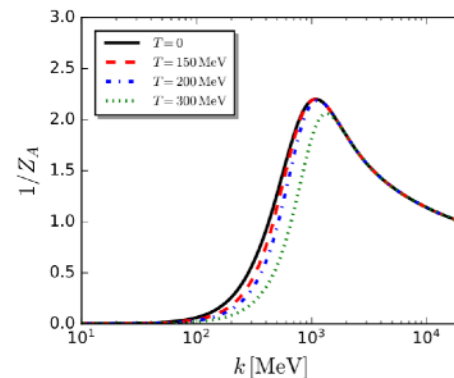
$$D_{\mu\nu}^{ab}(p) = \delta^{ab} \frac{Z(p^2)}{p^2} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)$$

$$|\mathcal{M}|^2 \propto \left| \frac{Z(t)}{t} \right|^2 \Rightarrow \frac{d\sigma}{dt} \propto \frac{Z^2(t)}{t^2}$$

$$\frac{d\sigma_{gg}}{dt} \simeq \frac{9\pi\alpha_s^2}{2(t - \mu^2)^2} Z(p^2)$$

Fu-Peng Liu, Shi Yin, Wei-jie Fu, Zi-Wei Lin and GLM, in progress

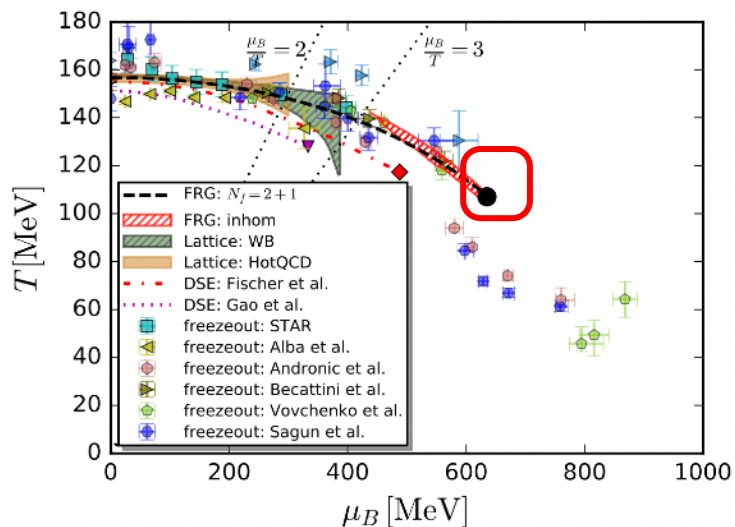
Gluon propagator at finite T and muB



W. Fu, J.M. Pawłowski, F. Rennecke, arXiv:1909.02991

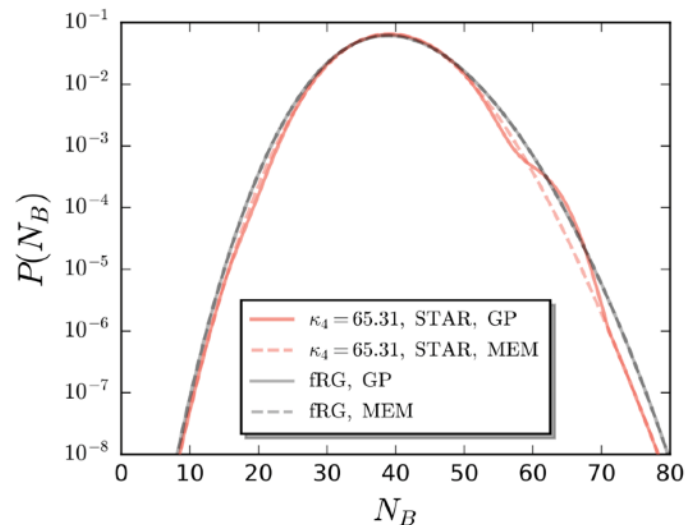
(5) Hadronic rescattering effect

The Functional Renormalization group approach



Fu, Pawłowski, Rennecke, PRD 101 (2020) 5, 054032

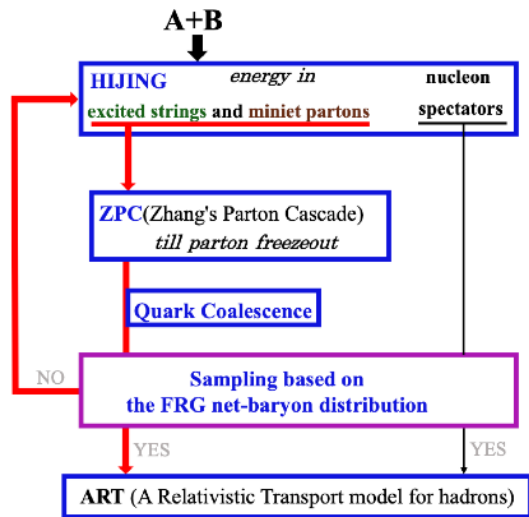
FRG can be applied to high baryon chemical potential (μ_B).



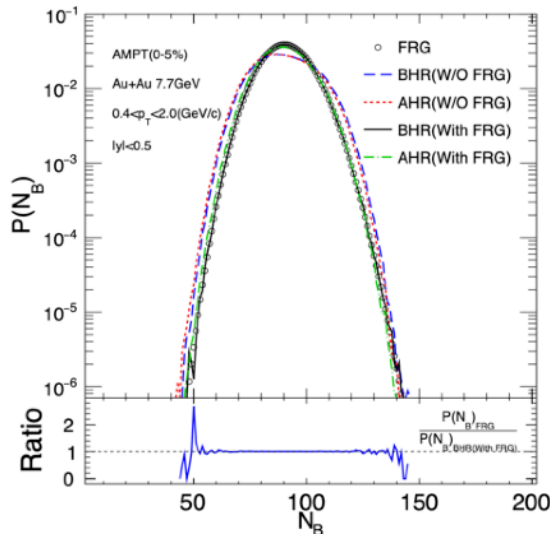
Huang, Chuang and Fu, Wei-jie, *et al.* arxiv: 2303.10869

FRG neglects interactions between hadrons and decay processes but incorporates the critical fluctuations mechanism.

Incorporating FRG Into The AMPT Model

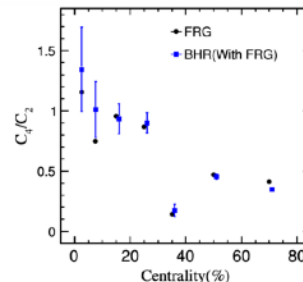
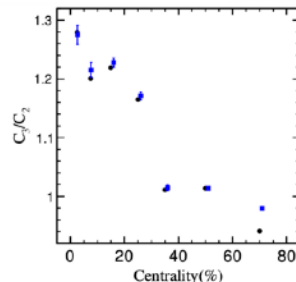
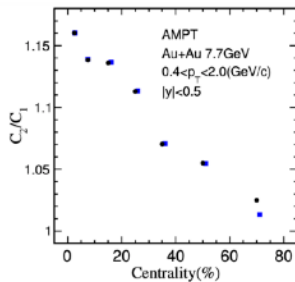


AMPT output:



FRG input:

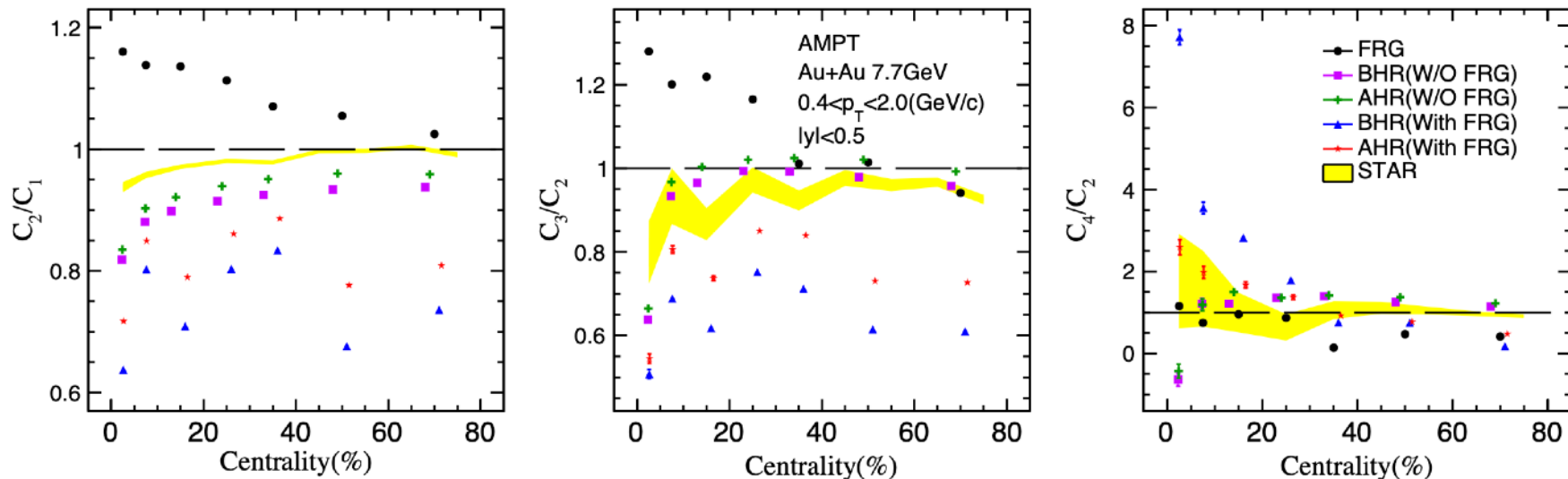
Centrality	μ_B (MeV)	T_{pc} (MeV)	V (fm ³)
0-5%	399	139	980
5-10%	395	140	701
10-20%	391	140	508
20-30%	382	141	340
30-40%	376	143	212
40-60%	357	144	126
60-80%	337	146	48



Qian Chen, Rui Wen, Shi Yin, Wei-jie Fu, Zi-Wei Lin et al., arXiv:2402.12823

Incorporating FRG into the AMPT model

Qian Chen, Rui Wen, Shi Yin, Wei-jie Fu, Zi-Wei Lin et al., arXiv:2402.12823



- Apparent influence on net-baryon fluctuations from critical fluctuations
- Hadronic rescatterings have a Poissonization effect on critical fluctuations.

Summary

- ◆ We are developing the AMPT model to understand the experimental data of net-proton number fluctuations.
- ◆ The current AMPT results without critical fluctuations are consistent with the expectation from baryon number conservation.
- ◆ The future incorporation of FRG interactions into AMPT model will reveal the real role of critical fluctuations.

Researchers moves from confusion, through the long pursuit, toward a sudden joy.

—from Three Stages of Great Undertakings by pioneering Chinese scholar Wang Guowei.



Happy 60th birthday to Prof. Jan M. Pawlowski

Thank you!