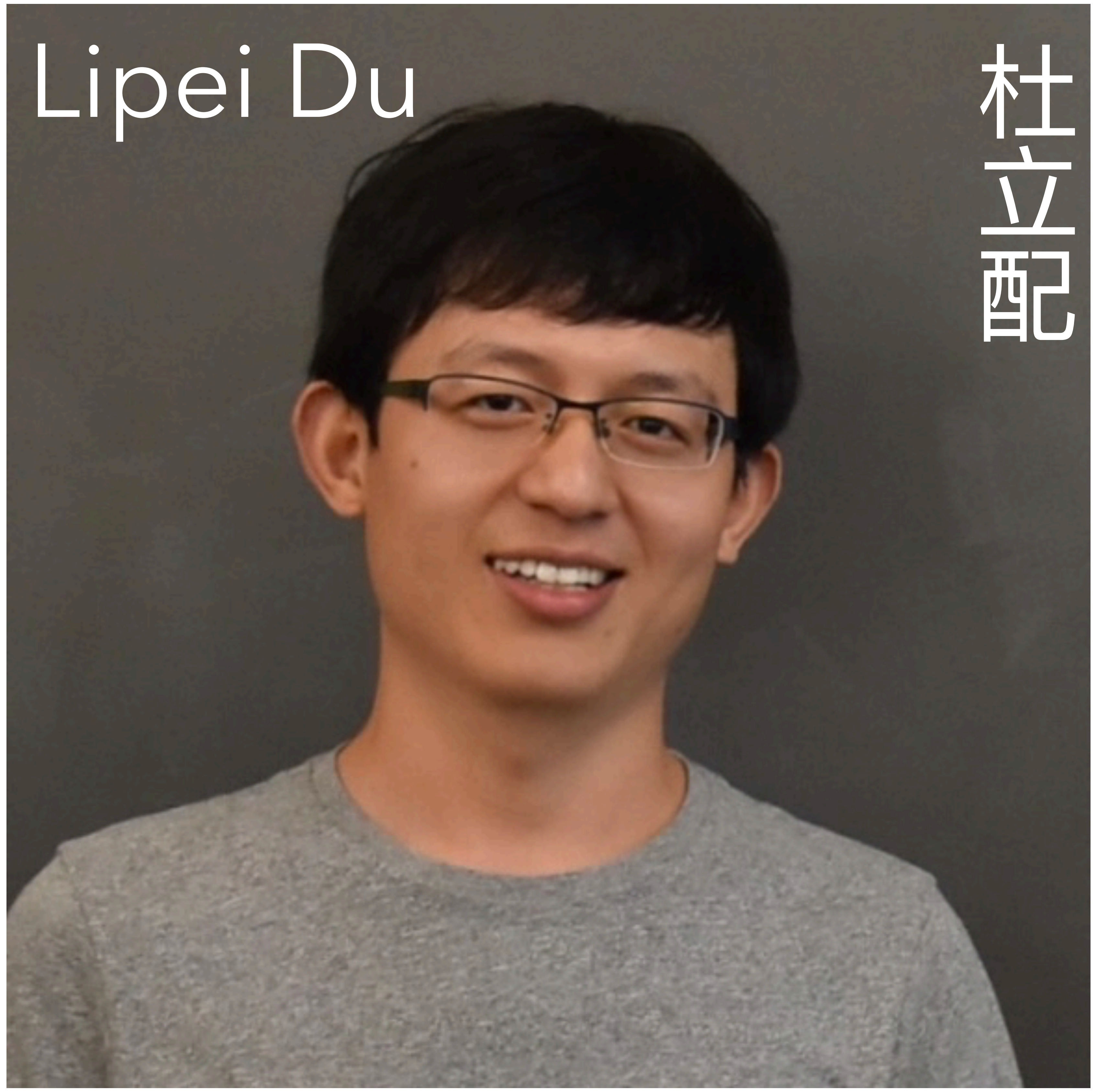


Correlations in an inhomogeneous medium

Shuzhe Shi (施舒哲)
Tsinghua University

w/Jianing Li & Lipei Du, PhysRevC.109.034906 + in preparation

measuring correlation length with cumulants

$$\kappa_2 \equiv \langle (\delta N)^2 \rangle = \frac{T}{V}\xi^2 \, ,$$

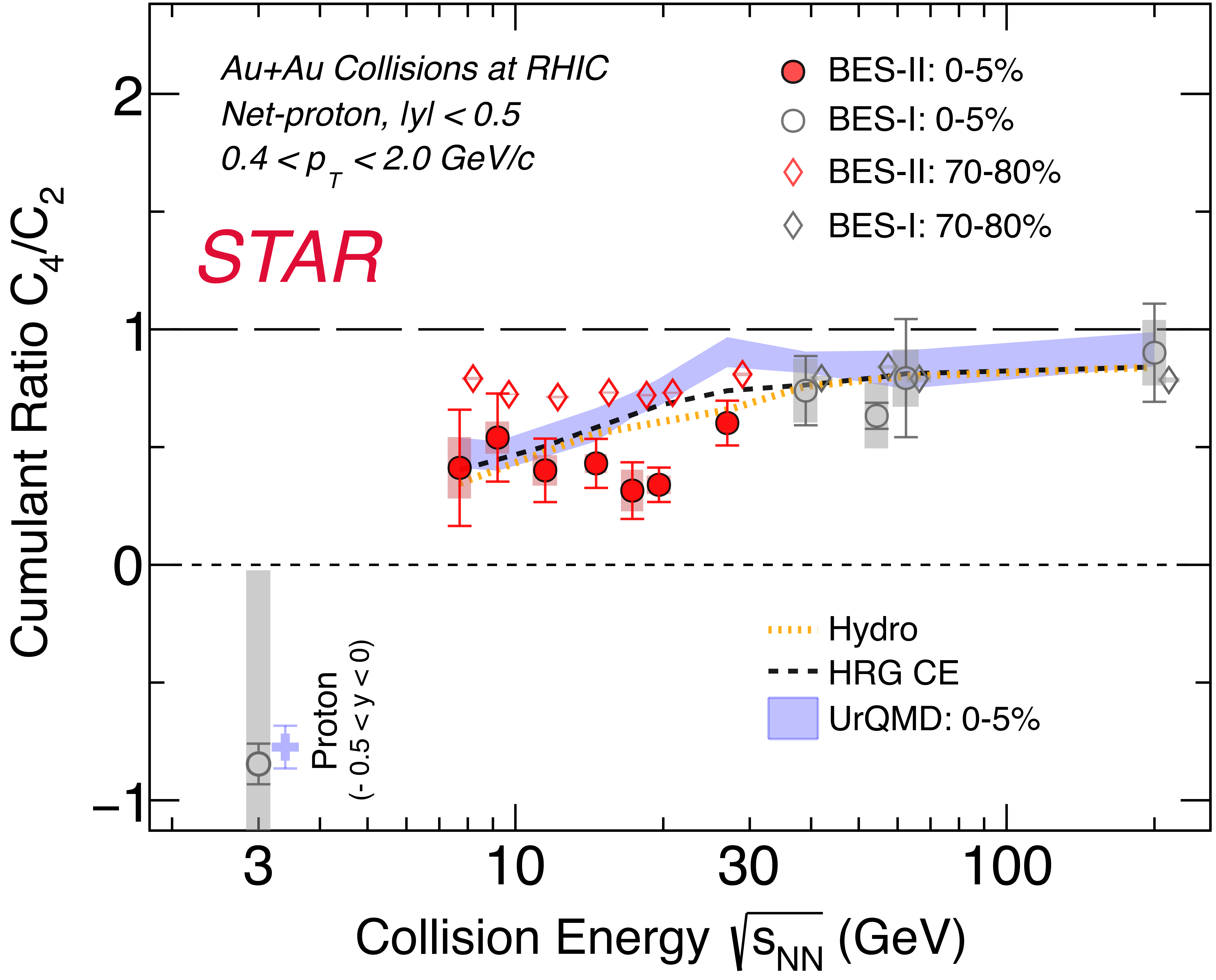
$$\kappa_4 \equiv \langle (\delta N)^4 \rangle - 3(\kappa_2)^2 = -\, 6\tilde{\lambda}_4 \frac{T}{V}\xi^7 \, .$$

M. Stephanov,

Phys.Rev.Lett. 102 (2009) 032301

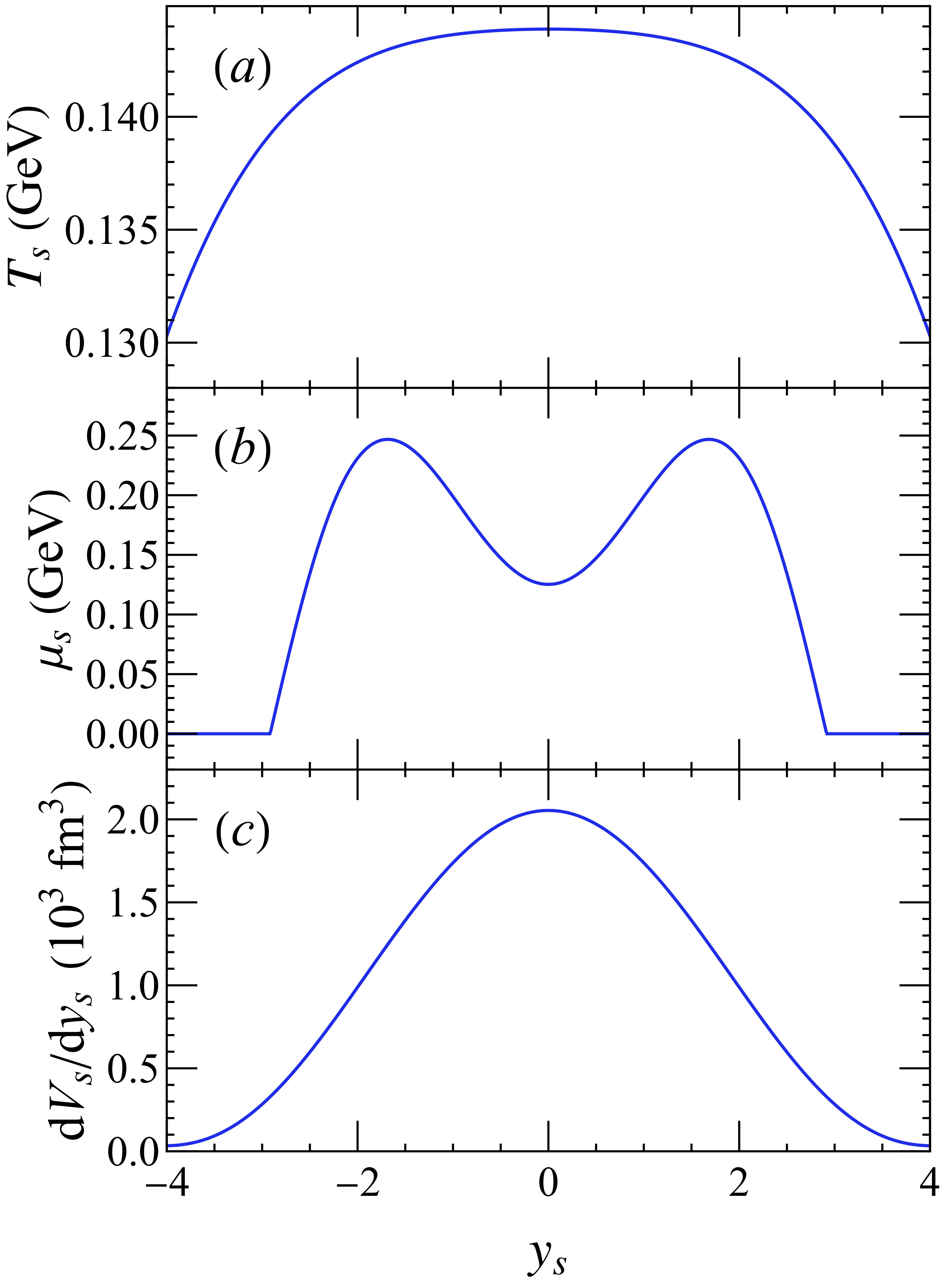
Presumptions:

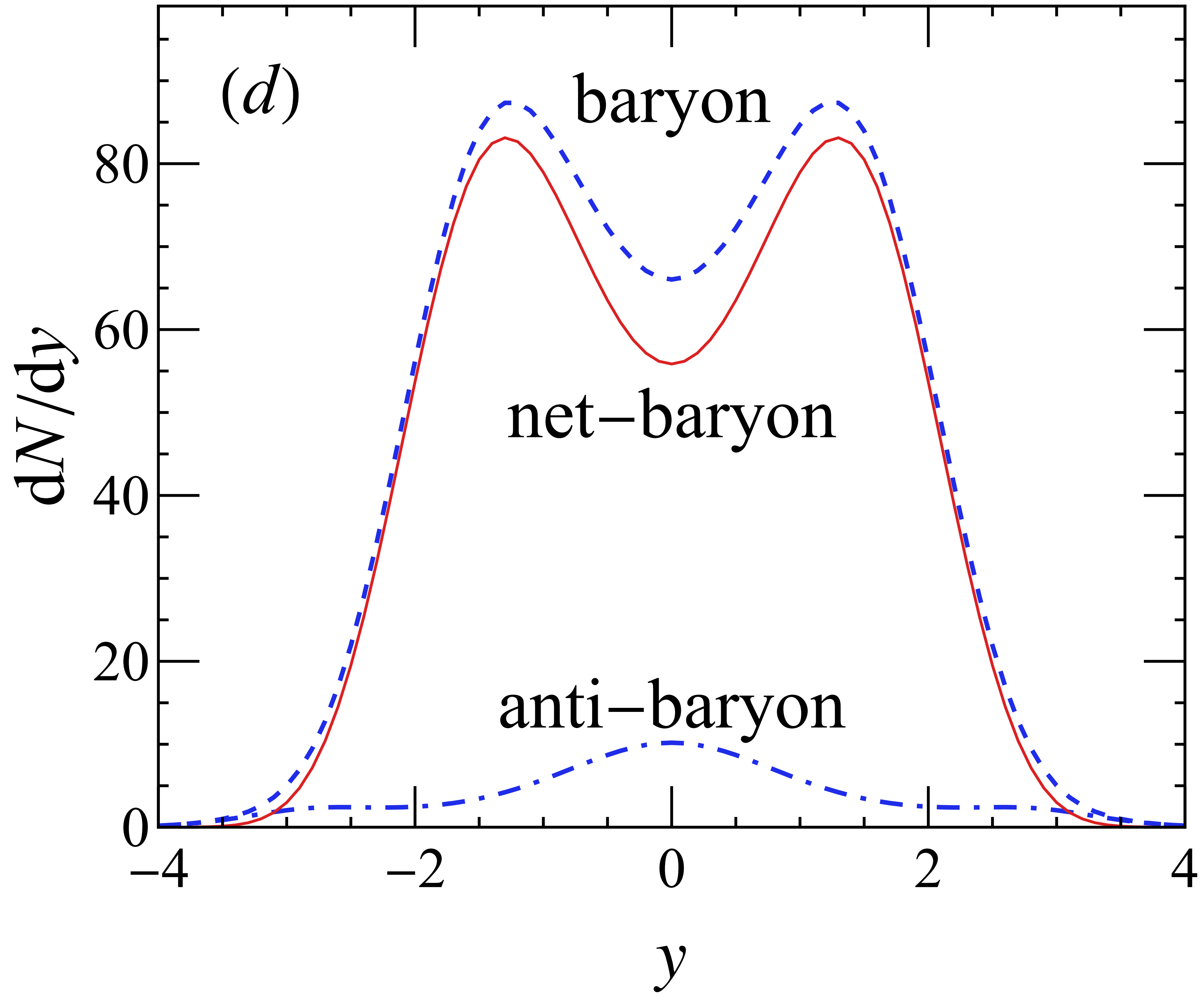
1. *homogeneous*
2. *open thermal system*



STAR collaboration, *Phys.Rev.Lett.* 135 (2025) 142301 *inhomogeneous rapidity-dependent hydro*

needed to understand experiment





L. Du, H. Gao, S. Jeon, C. Gale, PhysRevC.109.014907

J. Li, L. Du, ***SS***, PhysRevC.109.034906

non-critical correlations baseline w/ inhomogeneous + conservation effects

conservation effect

$$P\left(B_D\right) = \sum_{N_D, \bar{N}_D, N_{\not{D}}, \bar{N}_{\not{D}}=0}^{\infty} \left(\delta_{B_D, N_D-\bar{N}_D} P_A\left(N_D\right) P_A\left(\bar{N}_D\right)\right) \times \left(\delta_{B_{\not{D}}, N_{\not{D}}-\bar{N}_{\not{D}}} P_{\not{D}}\left(N_{\not{D}}\right) P_{\not{D}}\left(\bar{N}_{\not{D}}\right)\right) \times \delta_{B, B_D+B_{\not{D}}}.$$

D : detected, $\quad \not{D}$: not detected; $\qquad N$: baryon, $\quad \bar{N}$: anti − baryon;

J. Li, L. Du, ***SS***, PhysRevC.109.034906

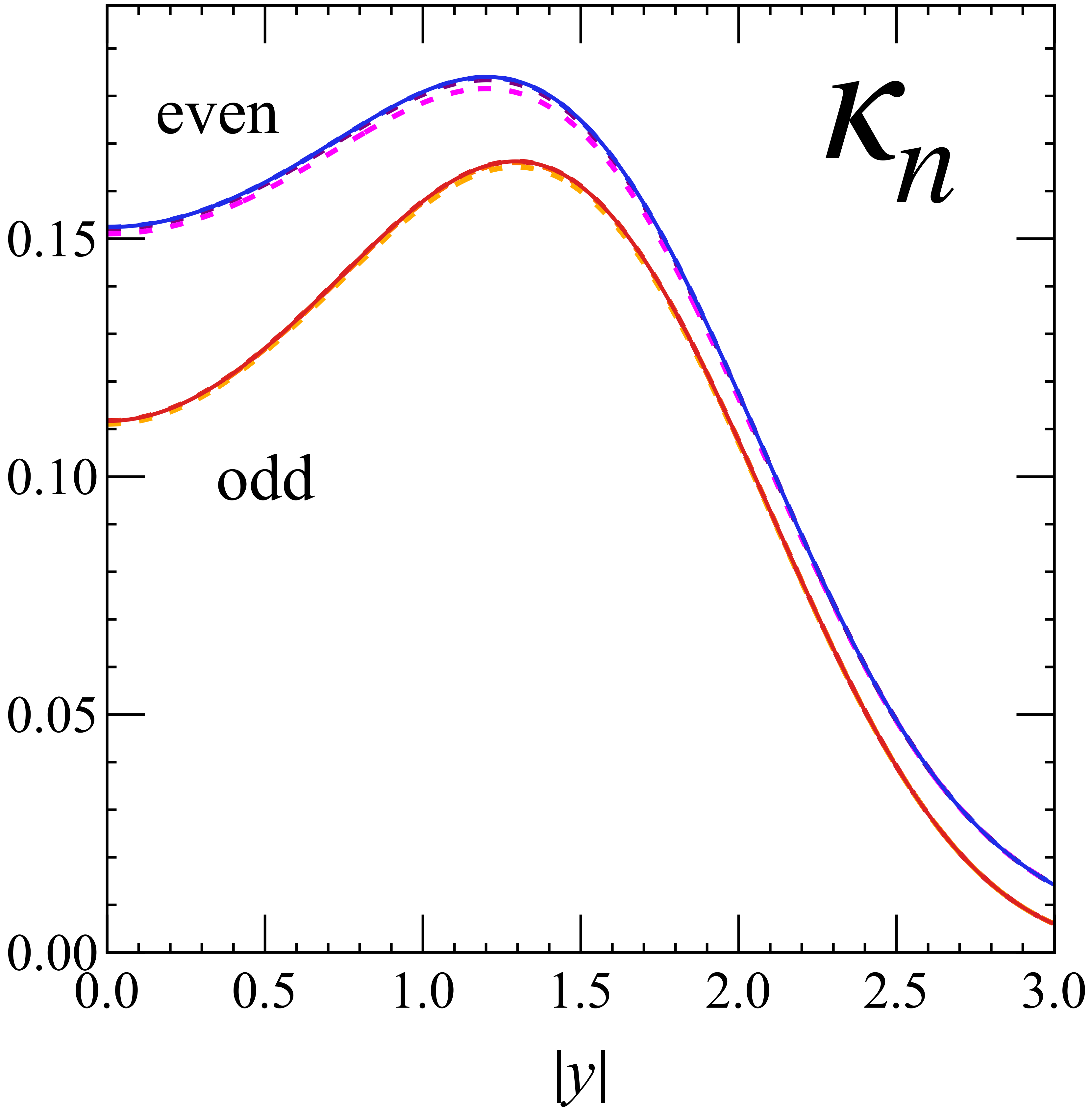
see also:

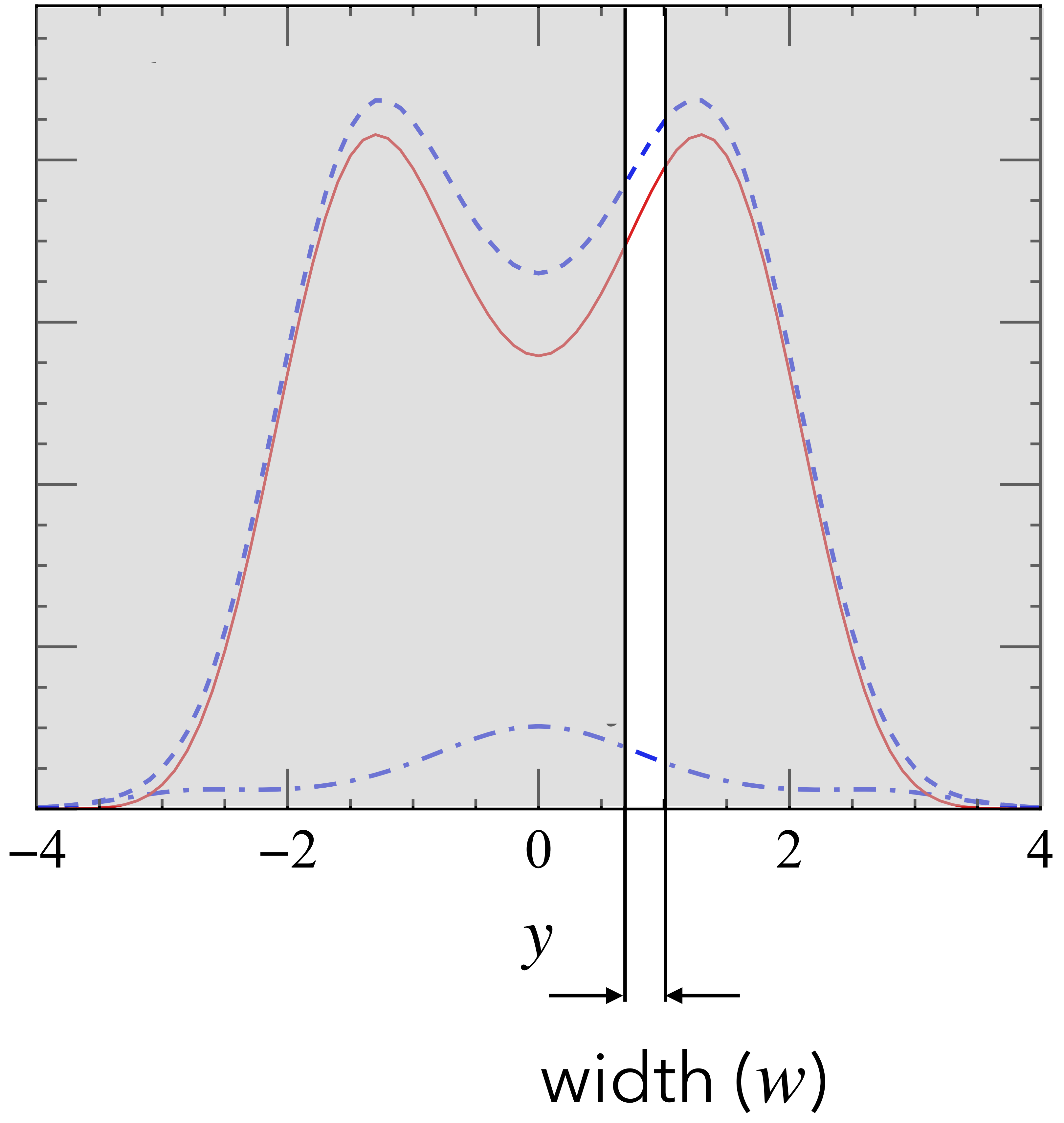
Vovchenko, Koch, PhysRevC.103.044903

Braun-Munzinger, Friman, Karsch, Redlich, Skokov, PhysRevC.84.064911

…

non-critical correlations baseline w/ inhomogeneous + conservation effects



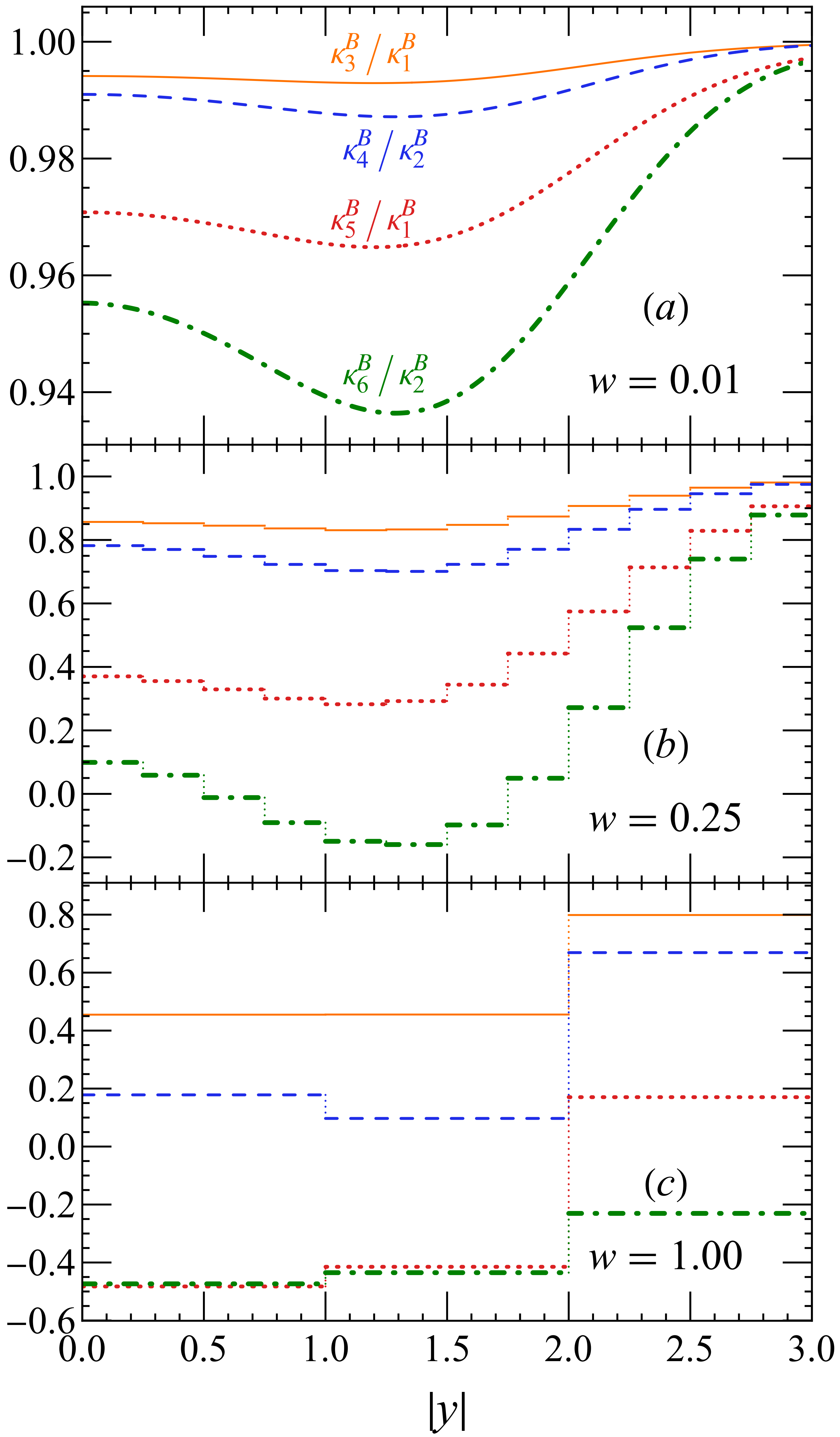


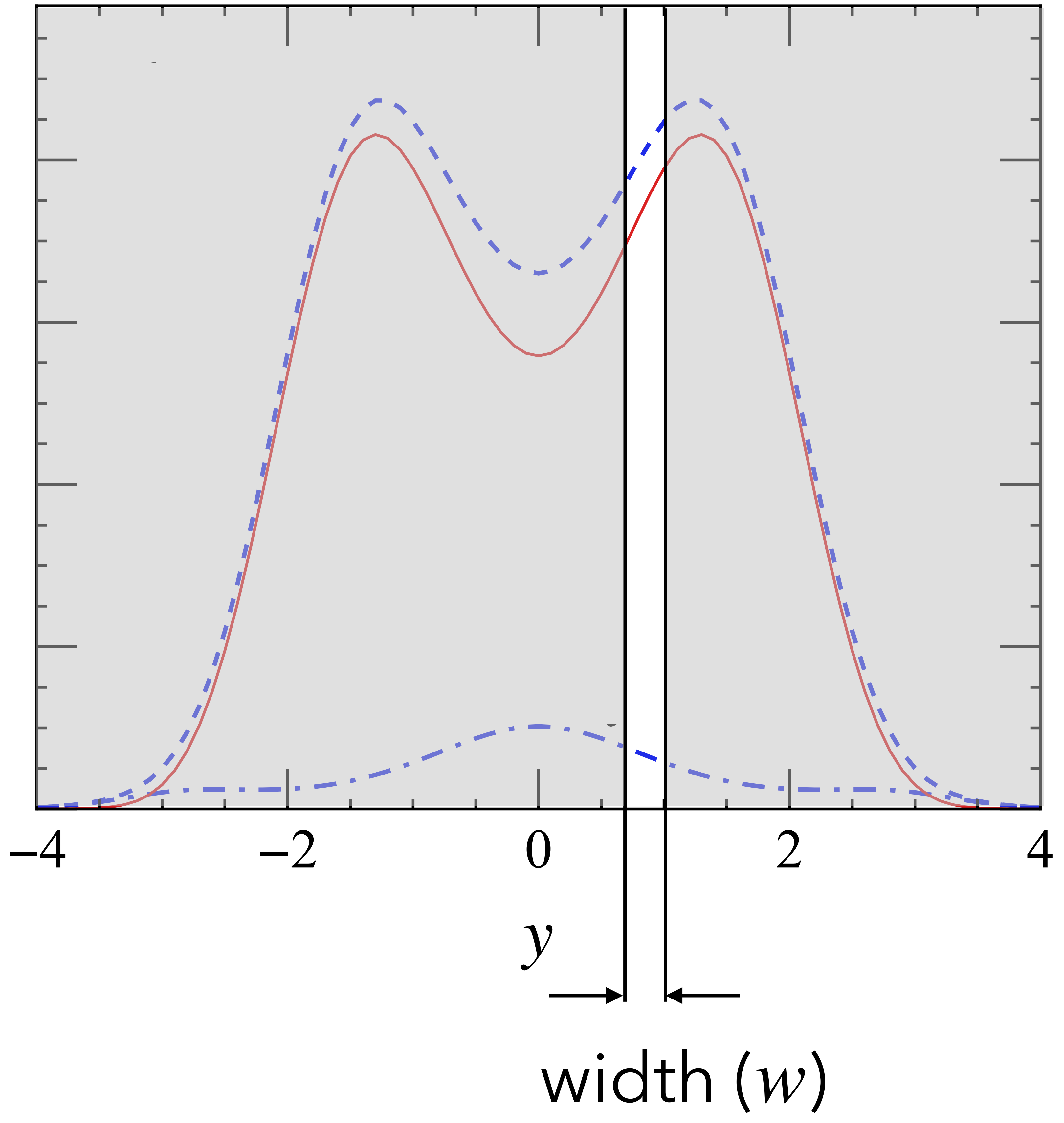
solid: open thermal system

dashed: closed thermal system with small width ($w = 0.002$)

J. Li, L. Du, ***SS***, PhysRevC.109.034906

non-critical correlations baseline w/ inhomogeneous + conservation effects



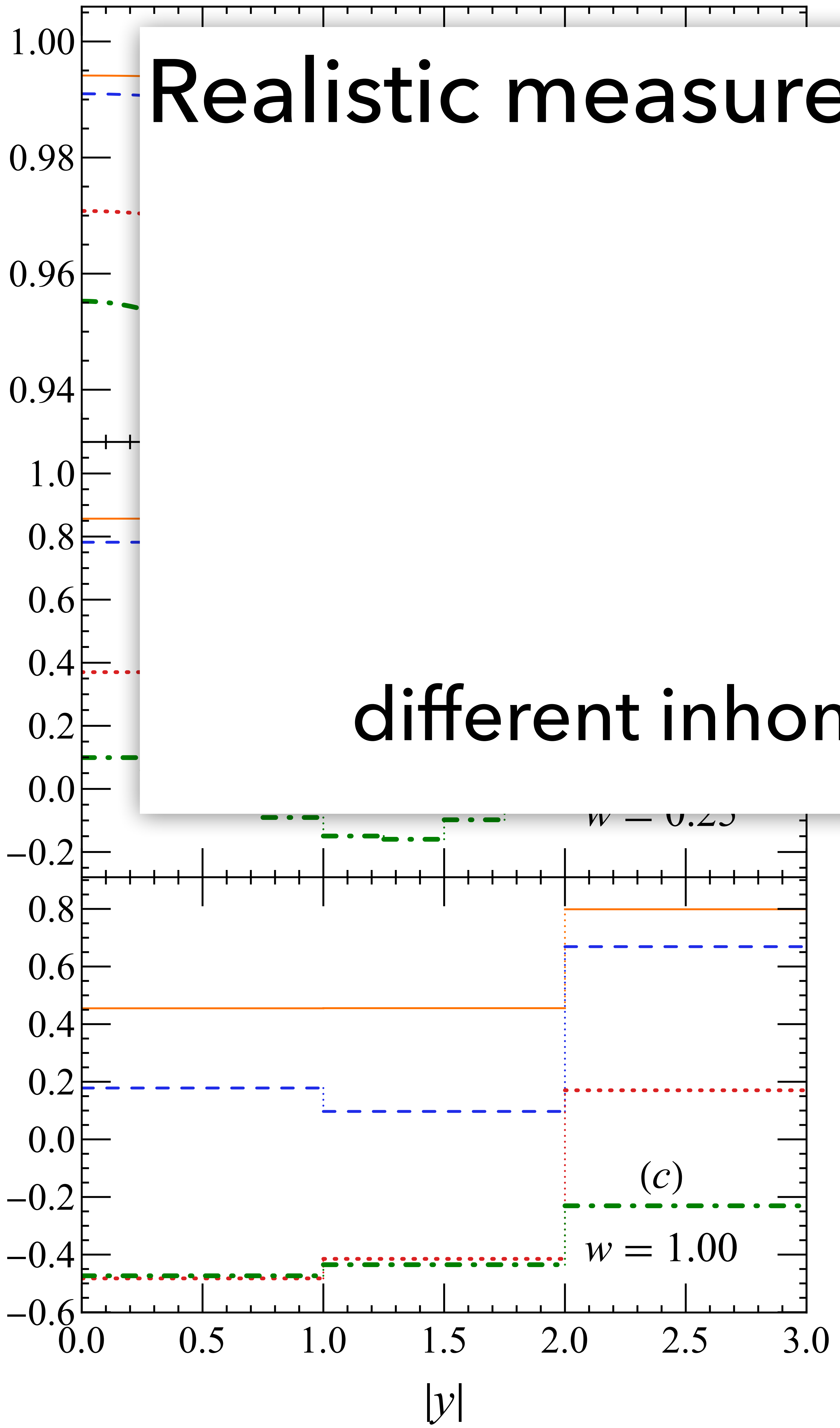


non-Poisson distribution ⇐ conservation effect

non-monotonic rapidity dependence ⇐ inhomogeneity

J. Li, L. Du, ***SS***, PhysRevC.109.034906

non-critical correlations baseline w/ inhomogeneous + conservation effects



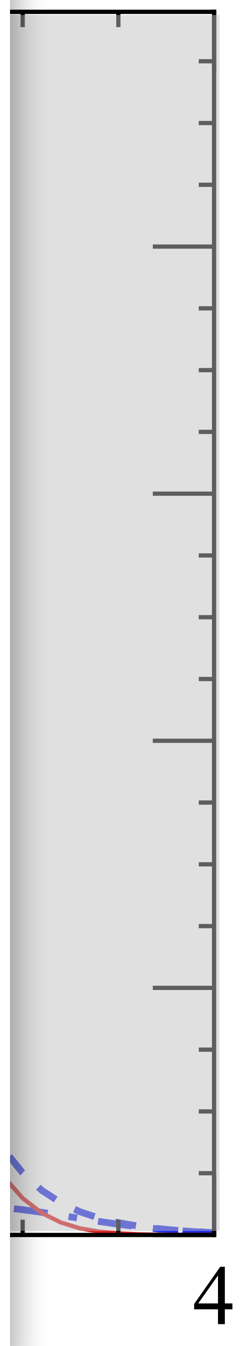
Realistic measurements in BES:

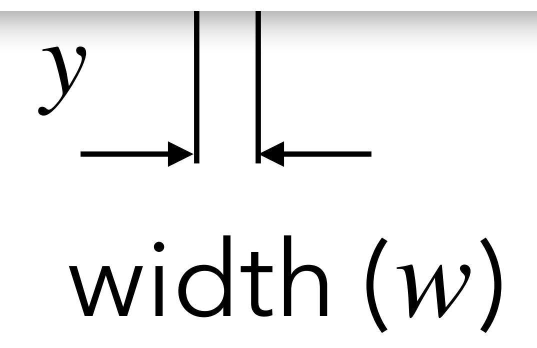
different beam energy

different inhomogeneity

different conservation correction

fixed rapidity window
different beam rapidity





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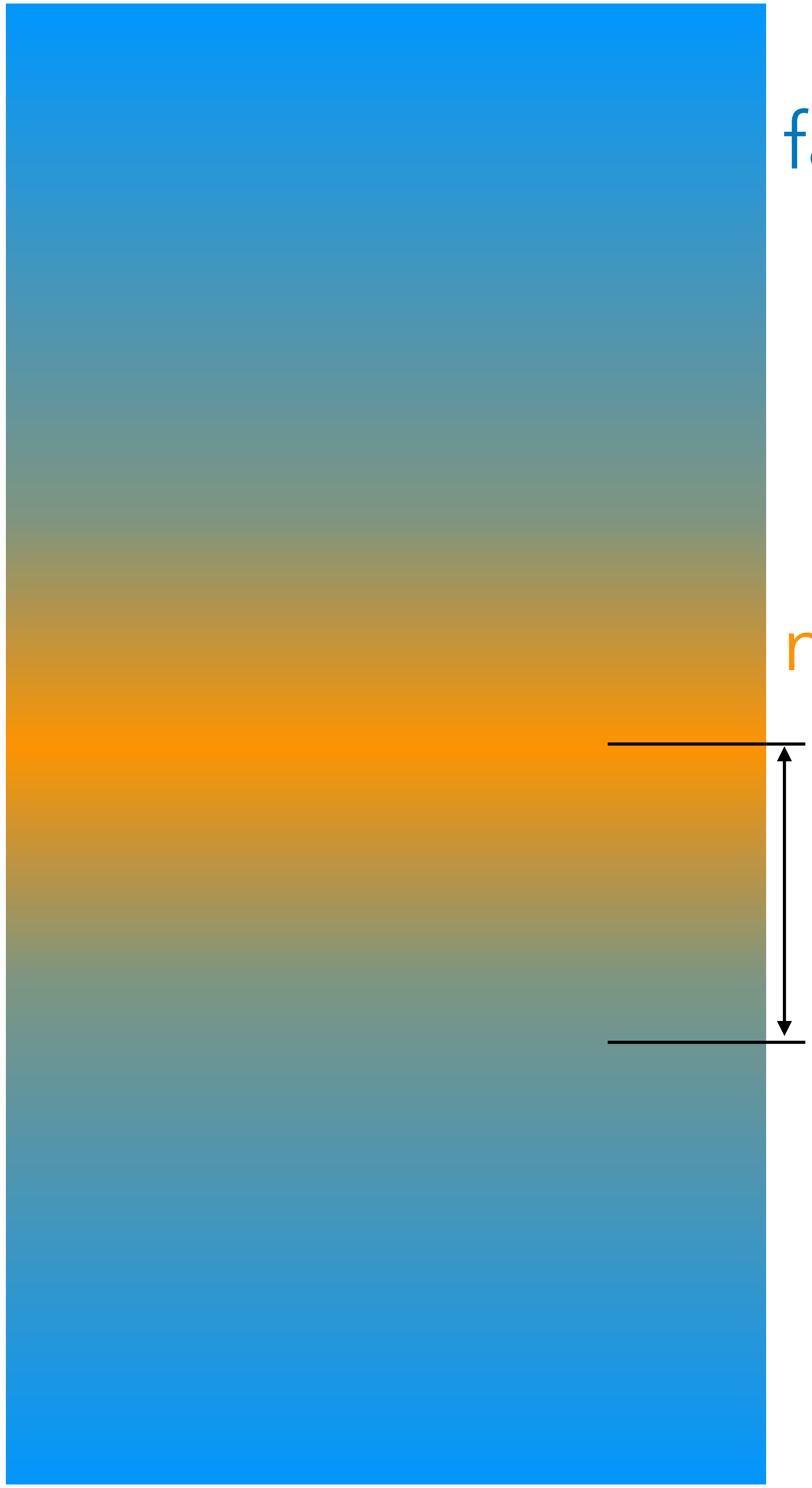
J. Li, L. Du, ***SS***, PhysRevC.109.034906

critical correlations in an inhomogeneous medium?

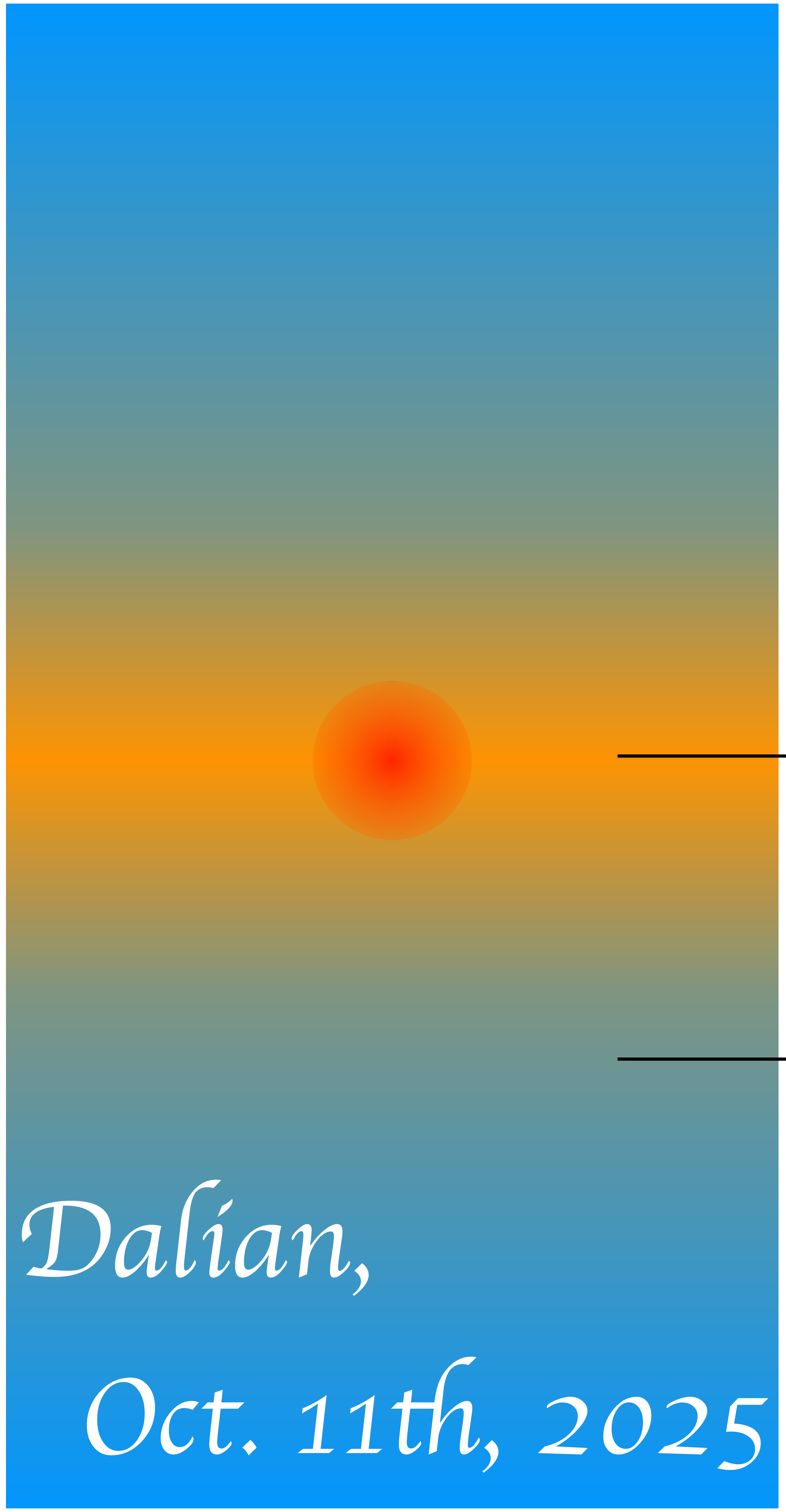


far from CP

near CP

$$\ell_{\text{inhomo}} \lesssim \xi_{\text{therm}}$$

critical correlations in an inhomogeneous medium?



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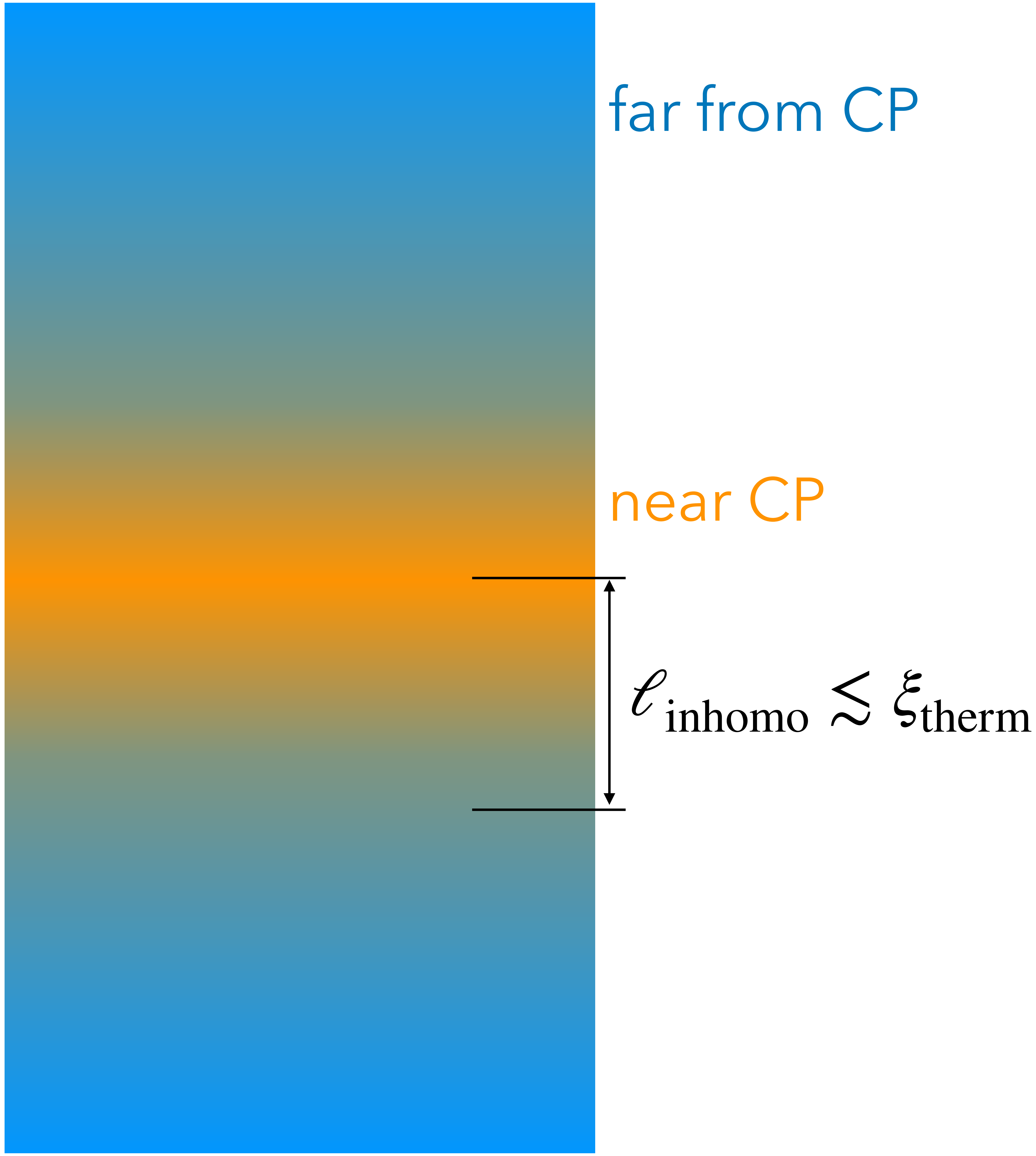

Review: Criticality in a *homogeneous* system

Hadrons coupled with a scalar critical field

Free energy density:

$$\Omega[\sigma(\boldsymbol{x})] = -\frac{1}{2}\sigma(\boldsymbol{x})\,\nabla^2\sigma(\boldsymbol{x}) + \frac{1}{2}m^2\sigma^2(\boldsymbol{x})\,.$$

$$m = 1/\xi(T,\mu)$$

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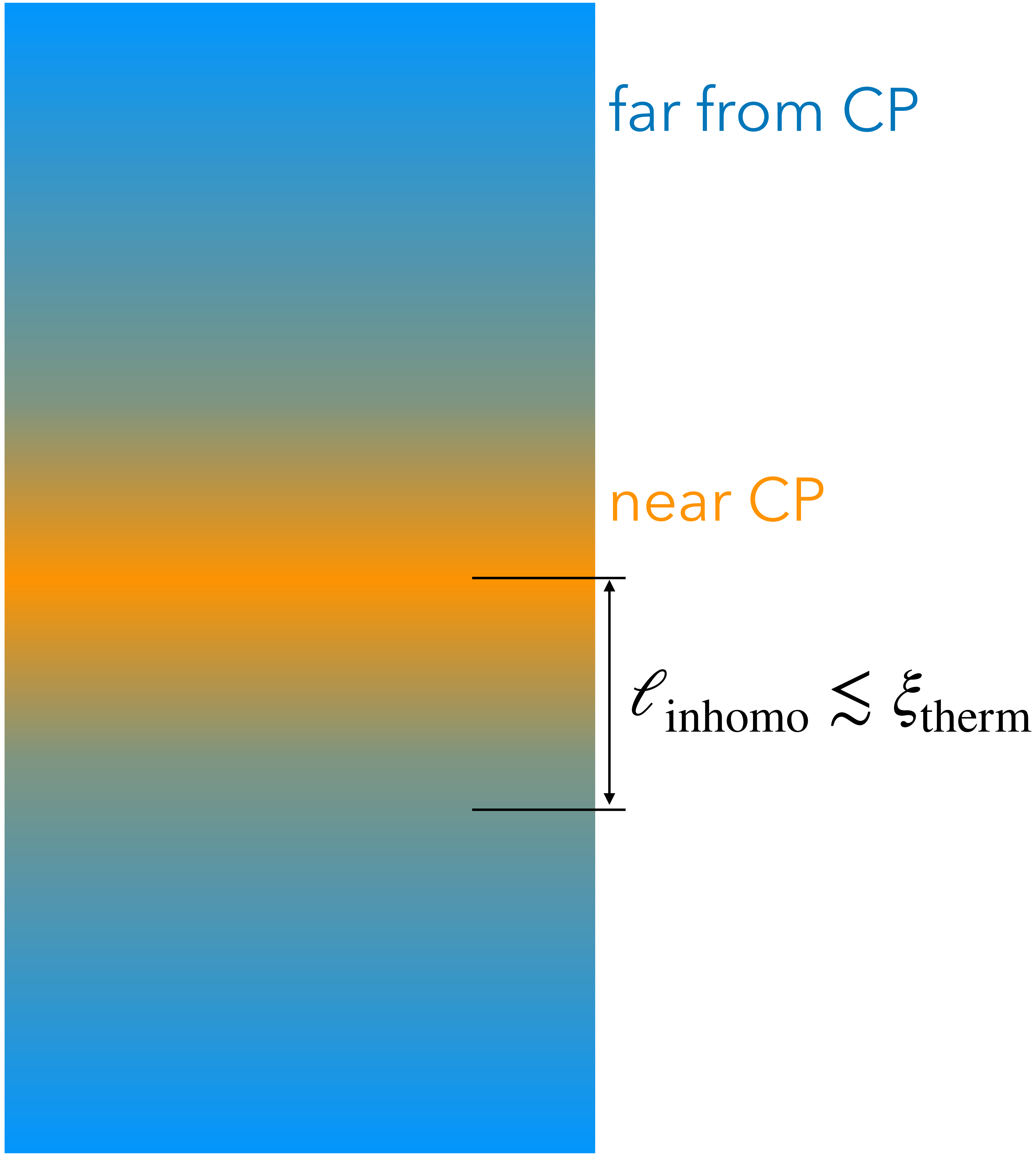

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Density matrix:

$$\mathscr{W}[\sigma(\boldsymbol{x})] = \frac{e^{-\int \beta\,\Omega[\sigma(\boldsymbol{x})]\,\mathrm{d}^3x}}{\mathcal{Z}}\,,$$

Two-point correlation:

$$\langle \sigma(\boldsymbol{x}_1)\sigma(\boldsymbol{x}_2)\rangle = \int \mathscr{D}\sigma\;\mathscr{W}[\sigma(\boldsymbol{x})]\;\sigma(\boldsymbol{x}_1)\sigma(\boldsymbol{x}_2)\,.$$

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$$\int \Omega[\sigma(\boldsymbol{x})]d^3x = \int (p^2+m^2)\tilde{\sigma}^*(\boldsymbol{p})\tilde{\sigma}(\boldsymbol{p})\frac{d^3p}{(2\pi)^3}\,,$$

$$\sigma(\boldsymbol{x}) = \int \tilde{\sigma}(\boldsymbol{p})e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\frac{d^3p}{(2\pi)^3}$$

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$$\downarrow$$

$$m = 1/\xi(T,\mu)$$

$$e^{-r/\xi}$$

$$\langle\sigma(\boldsymbol{x}_1)\sigma(\boldsymbol{x}_2)\rangle = \frac{T\,\boxed{e^{-m\,|\boldsymbol{x}_1-\boldsymbol{x}_2|}}}{4\pi|\boldsymbol{x}_1-\boldsymbol{x}_2|}\,.$$

$$\boxed{\int\Omega[\sigma(\boldsymbol{x})]d^3x = \int(p^2+m^2)\tilde{\sigma}^*(\boldsymbol{p})\tilde{\sigma}(\boldsymbol{p})\frac{d^3p}{(2\pi)^3}\,,}$$

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critical correlations in an inhomogeneous medium?

Criticality in an *inhomogeneous* system

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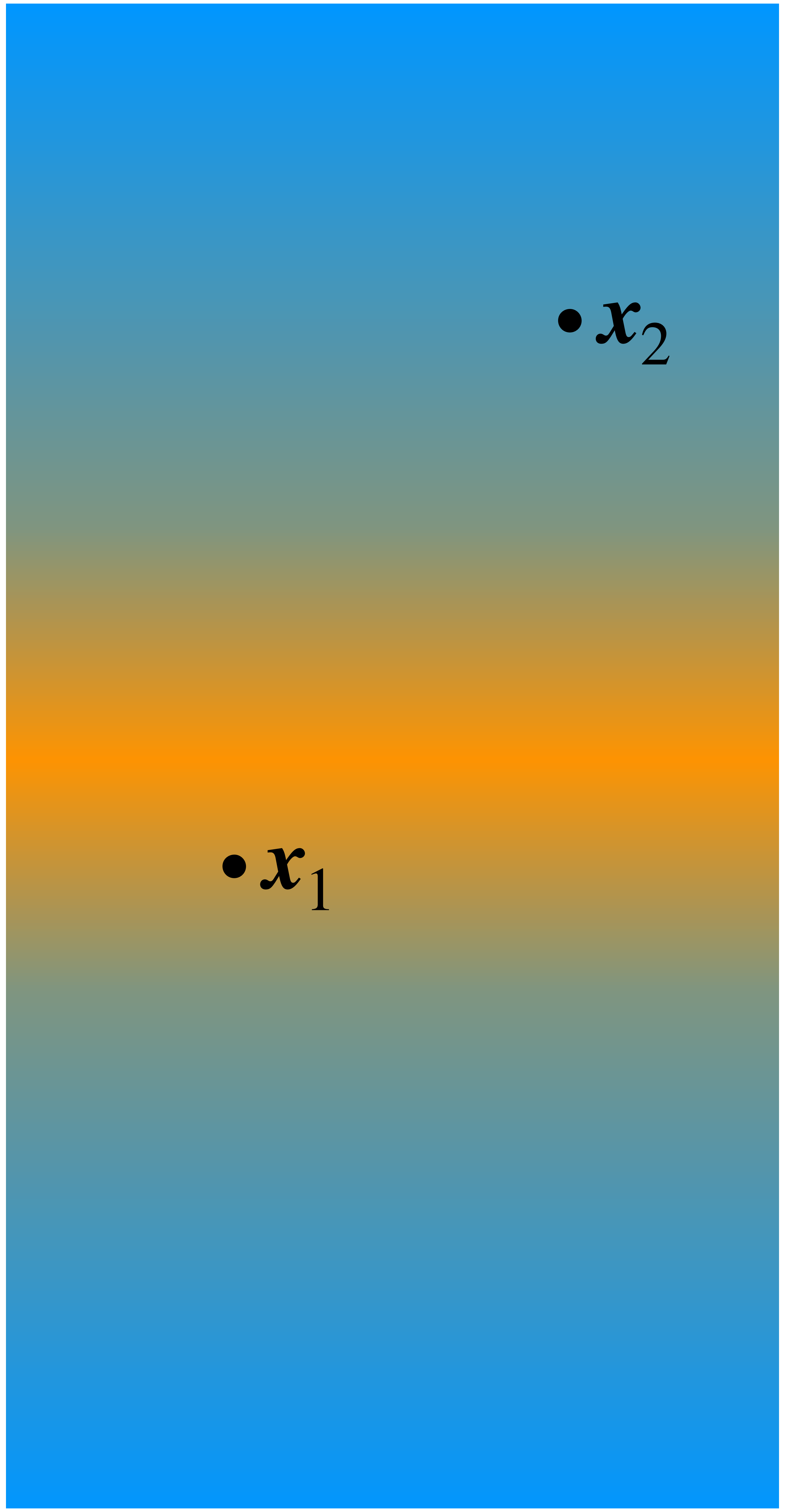
$$\sigma(\boldsymbol{x}) = \sum_n s_n\,\psi_n(\boldsymbol{x})$$

$$\int \beta(\boldsymbol{x})\Omega[\sigma(\boldsymbol{x})]d^3x = \sum_n \frac{1}{2}\lambda_n s_n^2\,,$$

$$\left(-\nabla^2 - \frac{\nabla\beta}{\beta}\cdot\nabla + m^2(\boldsymbol{x}) - \frac{\nabla^2\beta(\boldsymbol{x})}{2\beta(\boldsymbol{x})}\right)\psi_n(\boldsymbol{x}) = \lambda_n\psi_n(\boldsymbol{x})$$

$$\langle\sigma(\boldsymbol{x}_1)\sigma(\boldsymbol{x}_2)\rangle = \sum_n \frac{\psi_n(\boldsymbol{x}_1)\psi_n(\boldsymbol{x}_2)}{\lambda_n}\,.$$

example: harmonic oscillator



$$m^2(\boldsymbol{x}) = m_0^2 + \boxed{m_0^2\omega^2 z^2}, \qquad \beta(\boldsymbol{x}) = \beta_0 \,.$$

Eigenfunctions can be solved analytically:

$$\psi_{n,+}(\boldsymbol{x}\,|\,\boldsymbol{p}_\perp) = \cos(\boldsymbol{p}_\perp \cdot \boldsymbol{x}_\perp)\,\phi_n(z)\,,$$

$$\psi_{n,-}(\boldsymbol{x}\,|\,\boldsymbol{p}_\perp) = \sin(\boldsymbol{p}_\perp \cdot \boldsymbol{x}_\perp)\,\phi_n(z)\,,$$

$$\lambda_n(\boldsymbol{p}_\perp) = \frac{(2n+1)\,m_0\,\omega + m_0^2 + |\boldsymbol{p}_\perp|^2}{T}\,,$$

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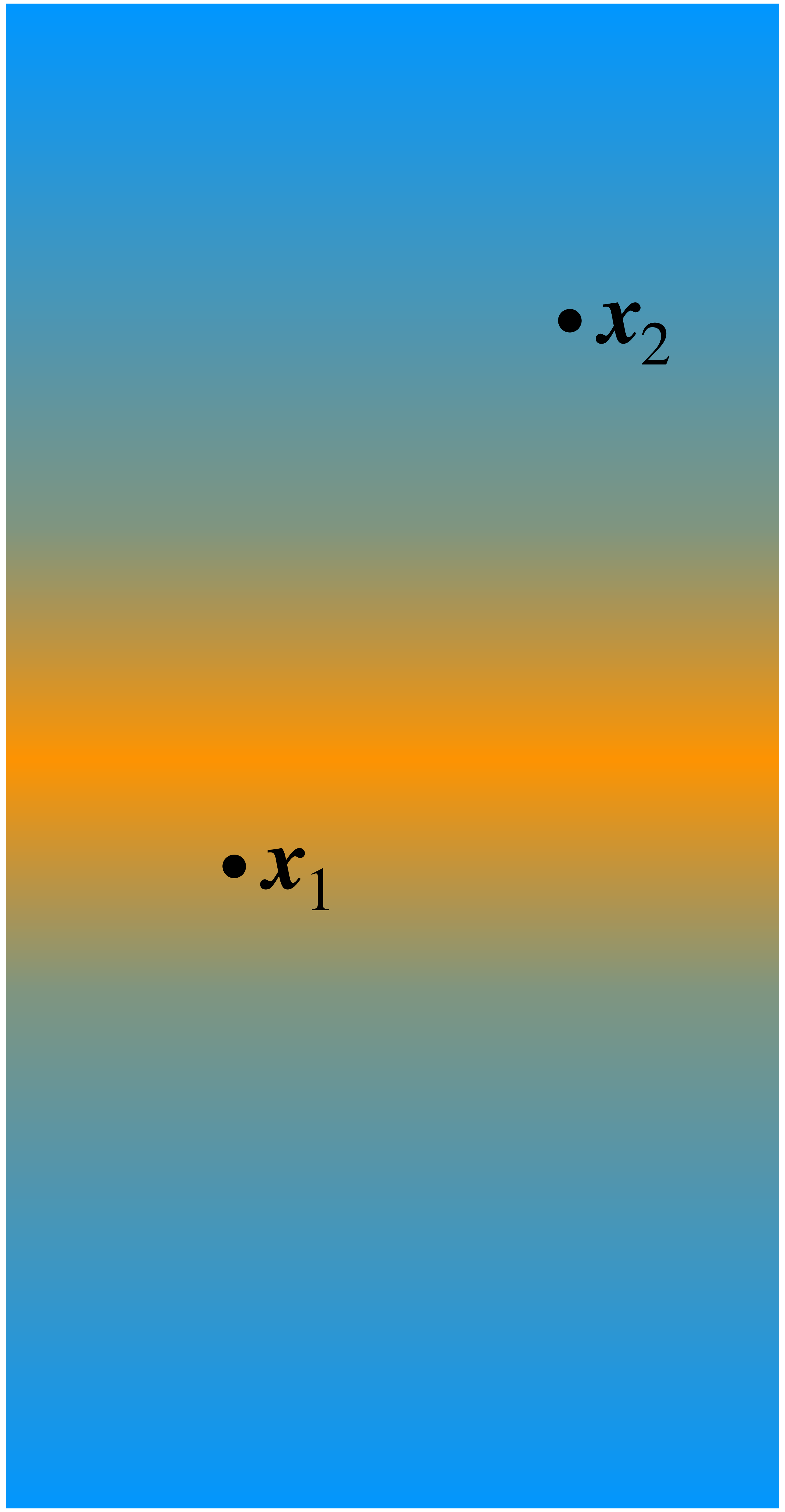

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inhomogeneity correction

$$\langle \sigma(\boldsymbol{x}_1)\sigma(\boldsymbol{x}_2)\rangle$$
$$= \frac{T}{4\pi\,|\boldsymbol{x}_1 - \boldsymbol{x}_2|} e^{-m_0\left(1+\omega^2 \frac{z_1^2 + z_1 z_2 + z_2^2}{6}\right)|\boldsymbol{x}_1 - \boldsymbol{x}_2|} + \mathcal{O}(\omega^4)\,.$$

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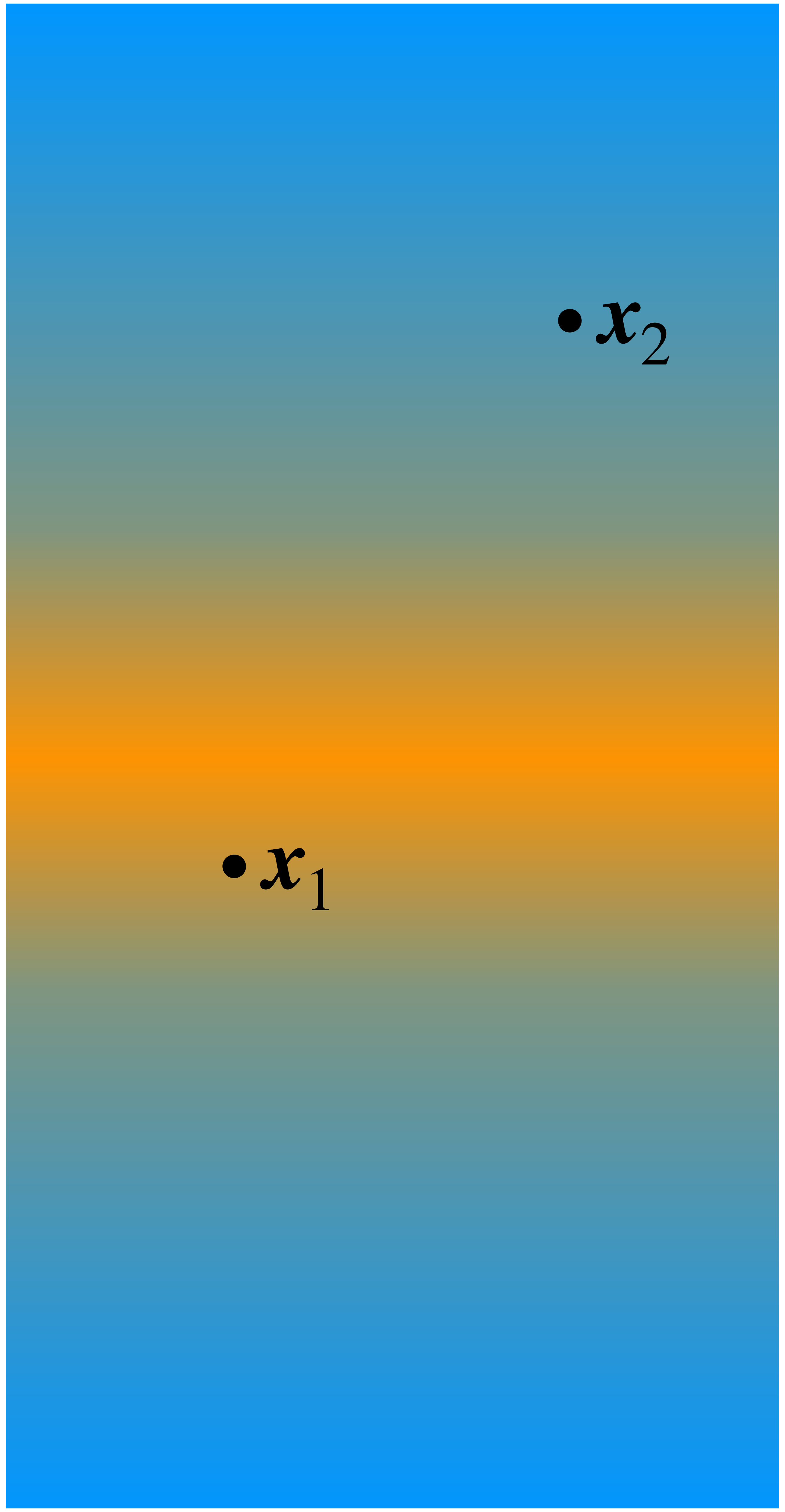

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reproduces the homogenous result if $\omega \to 0$

summary and outlooks

conservation effect

inhomogeneous corrections

- modifies the baseline w/o criticality
- corrects the critical correlation function
  - can be calculated (semi)analytically

disentangling finite-size effect from the critical correlations?