



THE QCD PHASE DIAGRAM: FROM THEORY TO EXPERIMENTAL SIGNATURES

Dalian, China
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LEADING-ORDER **QCD** EOS IN STRONG MAGNETIC FIELDS AT NONZERO BARYON CHEMICAL POTENTIAL



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Based on *arXiv:2508.07532 [hep-lat]*
to soon appear in *Phys. Rev. D. (accepted)*

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Nuclear Science
Computing Center at CCNU

LEADING-ORDER **QCD** EOS IN STRONG MAGNETIC FIELDS AT NONZERO BARYON CHEMICAL POTENTIAL

QCD EQUATION OF STATE

Equilibrium properties of strong interacting matter

$$p, \epsilon, \sigma, \dots \equiv f(T, \mu, eB, \dots)$$

thermodynamic obs.

control parameters

EARLY UNIVERSE

energy, evolution $\rightarrow$ Friedmann eq.

NEUTRON STARS

mass-radii relations $\rightarrow$ TOV eq.

HEAVY ION-COLLISIONS

QGP $\rightarrow$ hadronization $\rightarrow$ freeze-out

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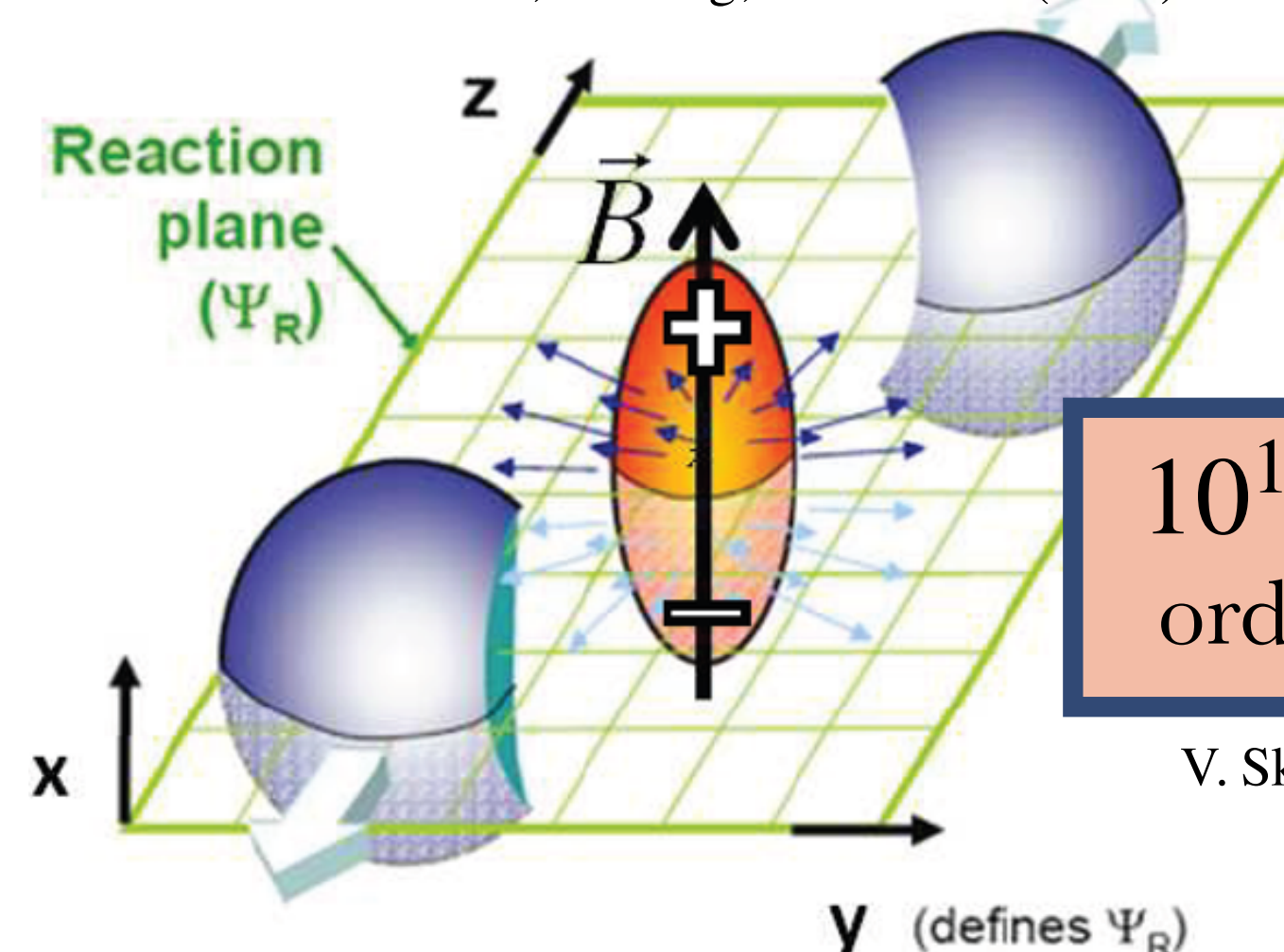
STRONG MAGNETIC FIELDS

orders comparable to
intrinsic scales of interaction

(non-perturbative effects)

**EOS IN MAGNETIC FIELDS
FROM LATTICE QCD**

J. Zhao, F. Wang, **PPNP** 107 (2019) 200-236



W.-T. Deng, X.-G. Huang
PRC 85 (2012) 044907

$10^{17} - 10^{18}$ G
orders of Λ_{QCD}^2

RHIC : $5m_\pi^2$
LHC : $70m_\pi^2$

V. Skokov, A. Illarionov, V. Toneev,
IJMPA 24 (2009) 5925

LEADING-ORDER QCD EoS IN STRONG MAGNETIC FIELDS AT

NONZERO BARYON CHEMICAL POTENTIAL

Finite- μ : SIGN-PROBLEM IN LATTICE QCD $\longrightarrow$ TAYLOR EXPAND

Pressure can be Taylor expanded as fluctuations of conserved charges $\mathcal{C} \in \{B, Q, S\}$:

$$P(T, eB, \mu) \equiv \frac{P}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}_{GC}(T, eB, V, \mu_{\mathcal{C}})$$

$$= \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{\text{BQS}} \mu_B^i \mu_Q^j \mu_S^k \quad \text{LO: } i+j+k=2$$

(Lattice computable & Experimentally measurable)

$$\chi_{ijk}^{\text{BQS}} \equiv \chi_{ijk}^{\text{BQS}}(T, eB) = \frac{\partial^{i+j+k}}{\partial \mu_B^i \partial \mu_Q^j \partial \mu_S^k} P(T, eB, \mu) \Big|_{\mu=0}$$

$$\mu_f \longleftrightarrow \mu_{\mathcal{C}} \quad \chi_{ijk}^{uds} \longleftrightarrow \chi_{ijk}^{\text{BQS}}$$

$$\begin{aligned} \mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \\ \mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S \end{aligned} \quad \text{(2+1)-f QCD}$$

C. R. Allton, et.al, **PRD** 66 (2002), 074507

R. V. Gavai, S. Gupta, **PRD** 68 (2003), 034506

QCD EoS USING FLUCTUATIONS OF CONSERVED CHARGES

LATTICE (2+1)-f QCD INGREDIENTS

- HISQ & Symanzik action with physical pion mass
- Lattice: $32^3 \times 8, 48^3 \times 12 \rightarrow$ continuum estimates

Control parameter space:

- Temperature: focused around QCD transition

$$T \equiv [145 - 166) \text{ MeV}$$

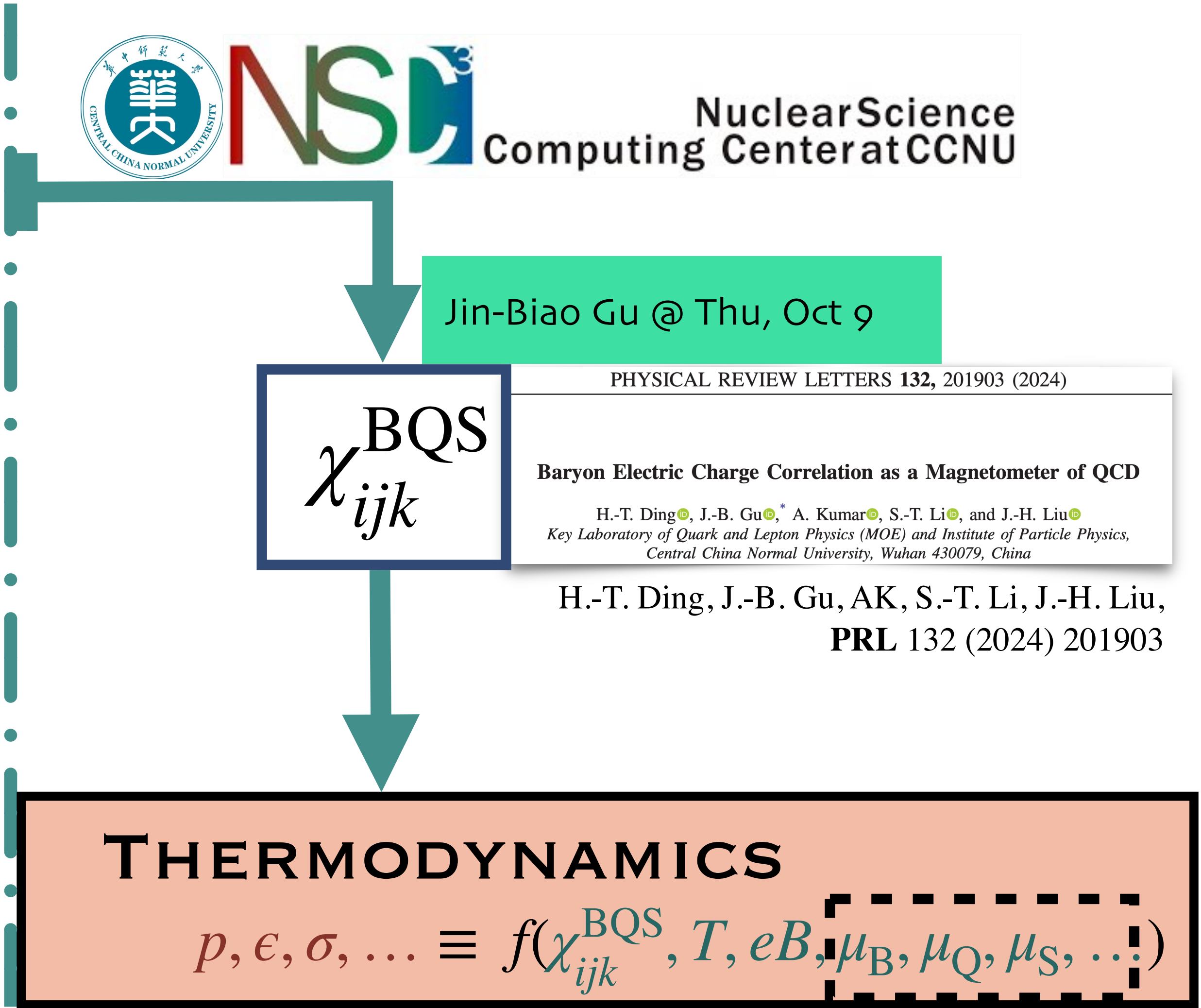
- Magnetic field (no sign-problem!)

$$\vec{B} = \vec{\nabla} \times \vec{A} \rightarrow \text{Landau gauge: uniform } B_z$$

PBC + Stokes theorem $\rightarrow$ Quantization:

$$eB = 6\pi N_b / a^2 N_\sigma^2 \quad \text{M. D'Elia, S. Mukherjee, F. Sanfilippo, PRD 82 (2010), 051501}$$

$$N_b = [1 - 32] \rightarrow eB \lesssim 0.8 \text{ GeV}^2 \sim 45 M_\pi^2$$



STRANGENESS NEUTRAL : INITIAL NUCLEI CONDITIONS

$$\neq (\mu_S = 0)$$

$$P_2 \equiv \hat{p}_{\text{LO}} = \frac{1}{2!} \left(\chi_2^{\text{B}} \hat{\mu}_{\text{B}}^2 + \chi_2^{\text{Q}} \hat{\mu}_{\text{Q}}^2 + \chi_2^{\text{S}} \hat{\mu}_{\text{S}}^2 \right) \\ + \chi_{11}^{\text{BQ}} \hat{\mu}_{\text{B}} \hat{\mu}_{\text{Q}} + \chi_{11}^{\text{BS}} \hat{\mu}_{\text{B}} \hat{\mu}_{\text{S}} + \chi_{11}^{\text{QS}} \hat{\mu}_{\text{Q}} \hat{\mu}_{\text{S}}$$

*Vanishing
chemical potential*

$$\hat{\mu}_{\text{Q}} \equiv \hat{\mu}_{\text{S}} = 0$$

$$P_2 \equiv \chi_2^{\text{B}}/2$$

Strangeness neutrality : $n^{\text{S}} = 0$

+

Isospin asymmetry : $n^{\text{Q}}/n^{\text{B}} = r$

$$\hat{\mu}_{\text{Q/S}} \equiv \hat{\mu}_{\text{Q/S}}(T, eB, \hat{\mu}_{\text{B}})$$



q_{2k-1}, s_{2k-1}

$$\mu_{\text{Q}}/\mu_{\text{B}} = q_1 + q_3 \hat{\mu}_{\text{B}}^2 + \mathcal{O}(\hat{\mu}_{\text{B}}^4) + \dots$$

$$\mu_{\text{S}}/\mu_{\text{B}} = s_1 + s_3 \hat{\mu}_{\text{B}}^2 + \mathcal{O}(\hat{\mu}_{\text{B}}^4) + \dots$$

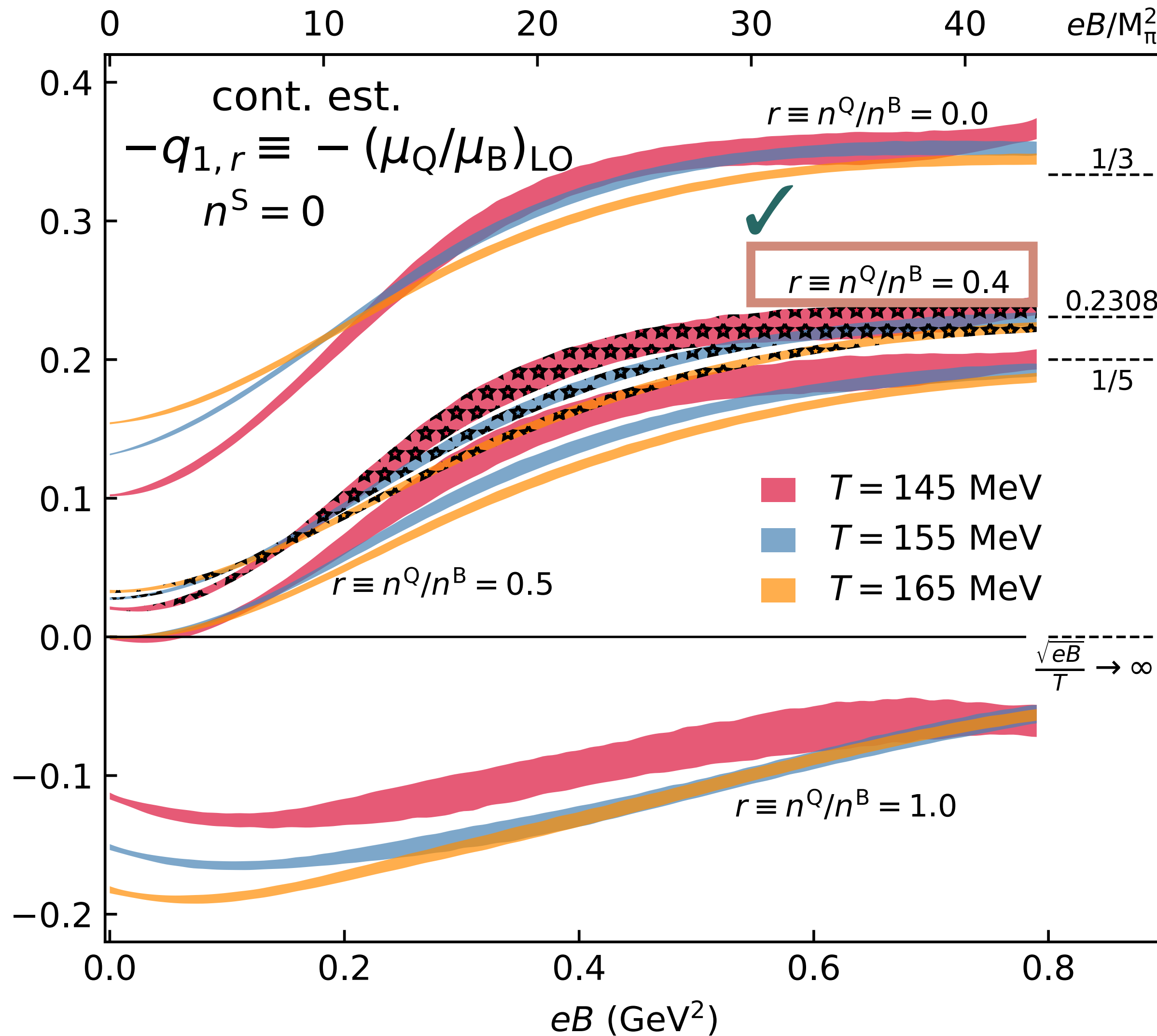
$(\mu_{\text{Q}}/\mu_{\text{B}})_{\text{LO}}$

$$q_1 = \frac{r (\chi_2^{\text{B}} \chi_2^{\text{S}} - \chi_{11}^{\text{BS}} \chi_{11}^{\text{BS}}) - (\chi_{11}^{\text{BQ}} \chi_2^{\text{S}} - \chi_{11}^{\text{BS}} \chi_{11}^{\text{QS}})}{(\chi_2^{\text{Q}} \chi_2^{\text{S}} - \chi_{11}^{\text{QS}} \chi_{11}^{\text{QS}}) - r (\chi_{11}^{\text{BQ}} \chi_2^{\text{S}} - \chi_{11}^{\text{BS}} \chi_{11}^{\text{QS}})}$$

$(\mu_{\text{S}}/\mu_{\text{B}})_{\text{LO}}$

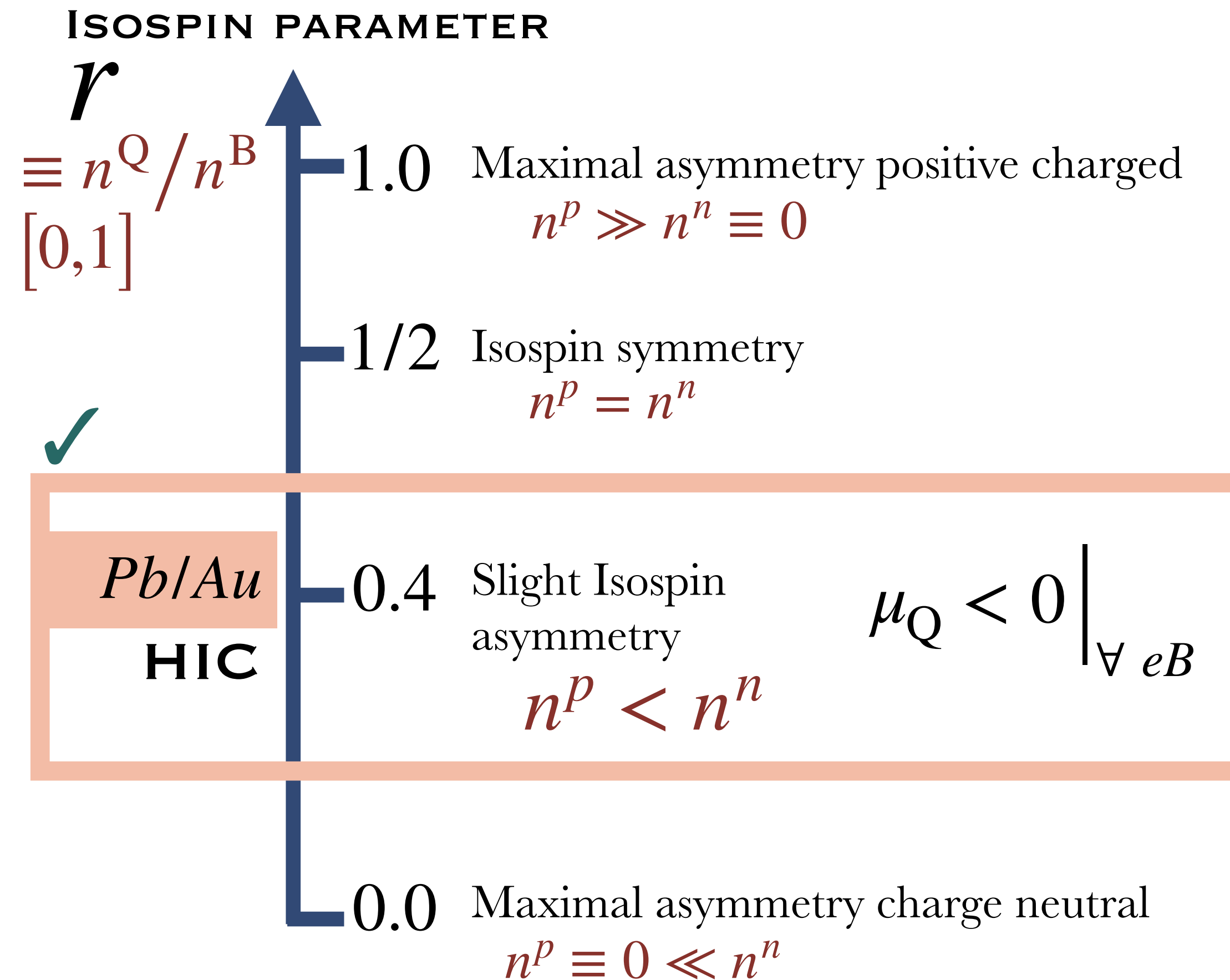
$$s_1 = - \frac{(\chi_{11}^{\text{BS}} + q_1 \chi_{11}^{\text{QS}})}{\chi_2^{\text{S}}}$$

$(\mu_Q/\mu_B)_{LO} \equiv q_1$: STRANGENESS NEUTRAL SYS

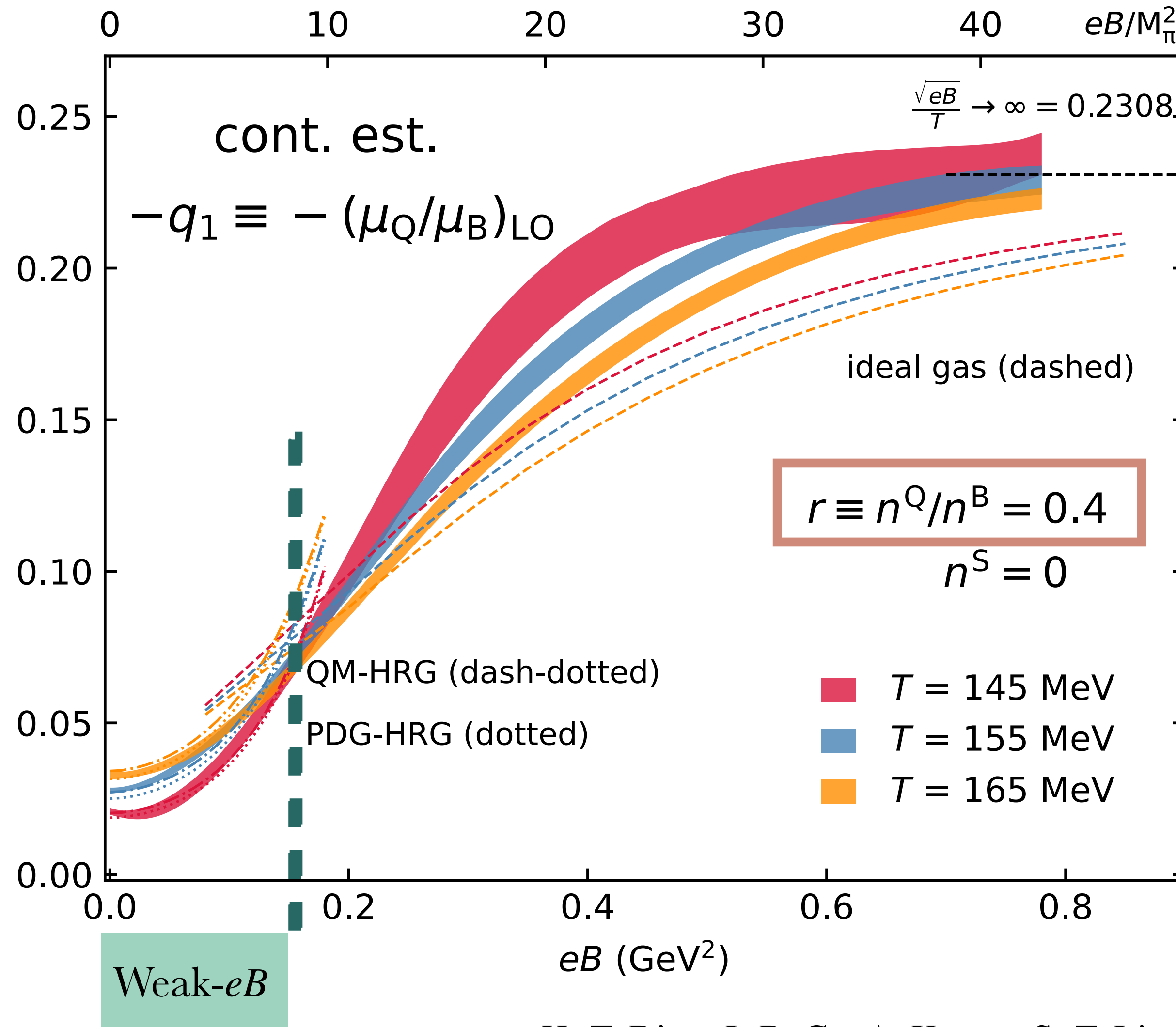


H.-T. Ding, J.-B. Gu, A. Kumar, S.-T. Li,
 arXiv:2508.07532 [hep-lat]

$$q_1 = \frac{r (\chi_2^B \chi_2^S - \chi_{11}^{BS} \chi_{11}^{BS}) - (\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}{(\chi_2^Q \chi_2^S - \chi_{11}^{QS} \chi_{11}^{QS}) - r (\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}$$



$(\mu_Q/\mu_B)_{\text{LO}} \equiv q_1$: INITIAL *Pb*–NUCLEI CONDITIONS



Negative! Further $-q_1$ as eB grows

Isospin asymmetry : $n^p < n^n \longrightarrow \mu_Q < 0 \Big|_{\forall eB}$



Weak- eB : Lattice QCD and HRG agree

Enhanced degeneracy

$$g_R \int \frac{d^3p}{(2\pi)^3} \xrightarrow{eB \text{ modified}} \frac{|q_R| B}{2\pi} \sum_{s_z=-s_R}^{s_R} \sum_{l=0}^{\infty} \int \frac{dp_z}{2\pi}$$

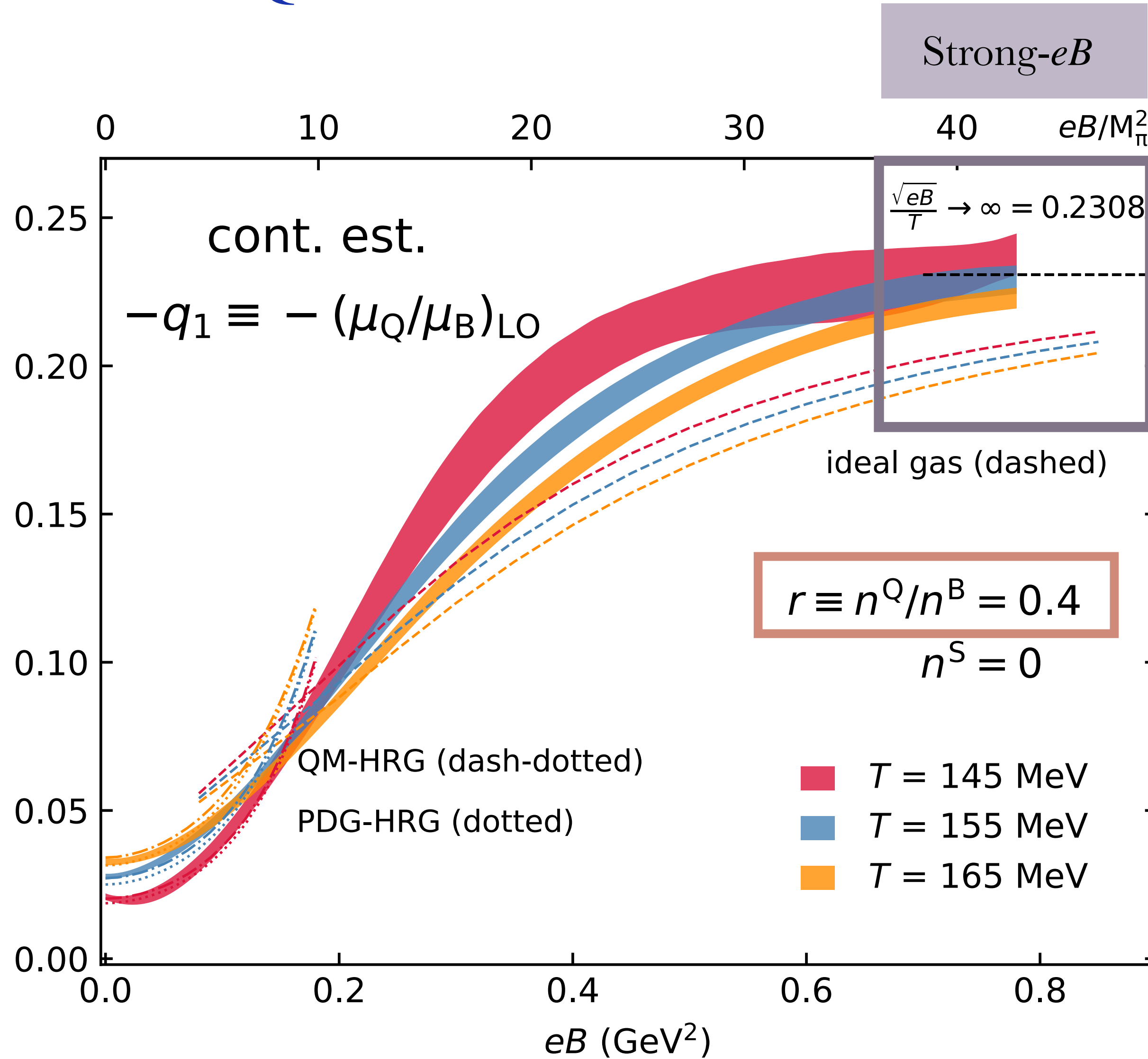
Lowered LLL energy

$$E_R^c = \sqrt{m_R^2 + p_z^2 + 2|q_R|B(l + 1/2 - s_z)}$$

K. Fukushima, Y. Hidaka, **PRL** 117 (2016), 102301
 H.-T. Ding, S.-T. Li, Q. Shi, X.-D. Wang,
EPJA 57 (2021) 6, 202

H.-T. Ding, J.-B. Gu, A. Kumar, S.-T. Li,
arXiv:2508.07532 [hep-lat]

$(\mu_Q/\mu_B)_{\text{LO}} \equiv q_1$: INITIAL *Pb*–NUCLEI CONDITIONS



$$q_1 = \frac{r (\chi_2^B \chi_2^S - \chi_{11}^{BS} \chi_{11}^{BS}) - (\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}{(\chi_2^Q \chi_2^S - \chi_{11}^{QS} \chi_{11}^{QS}) - r (\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}$$



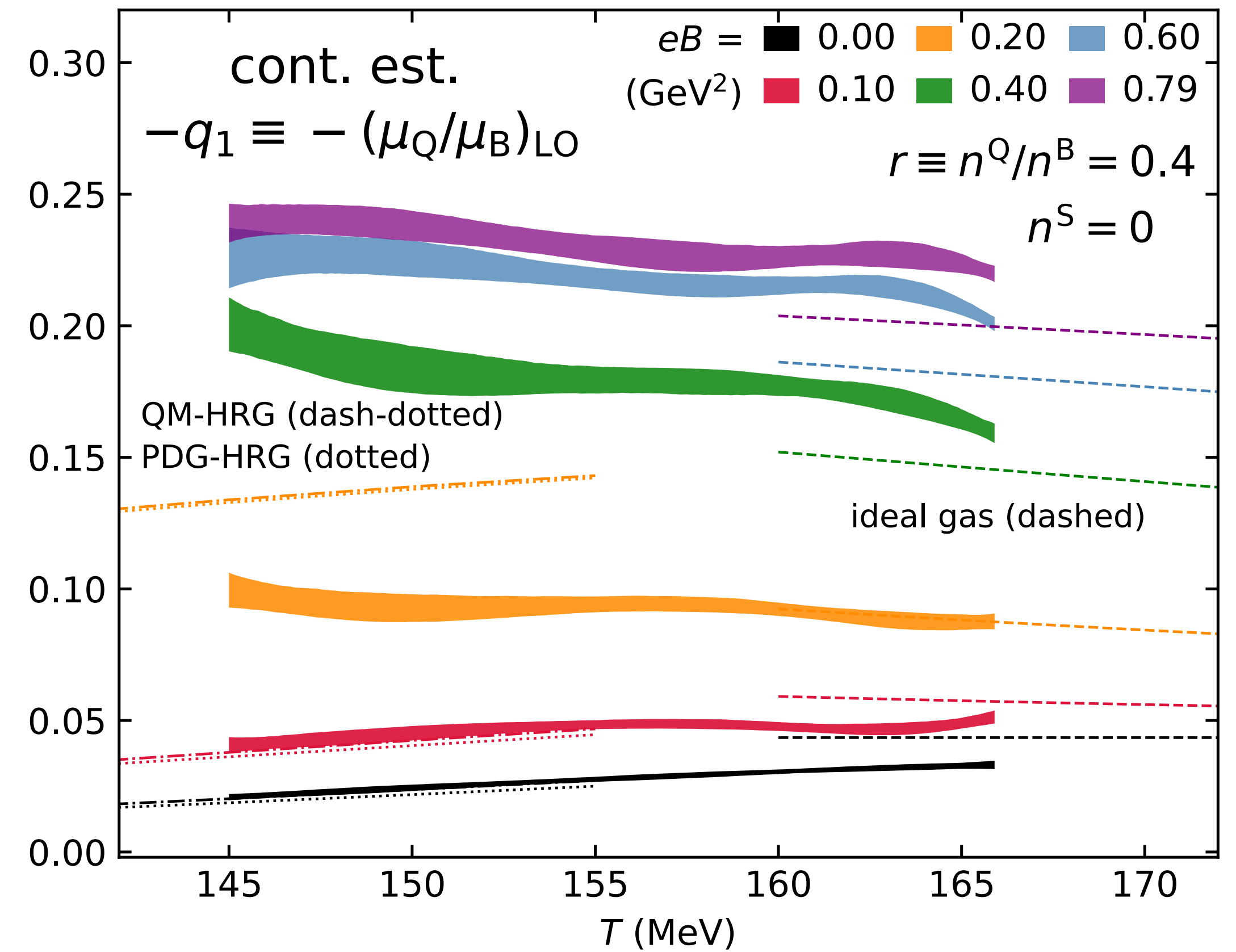
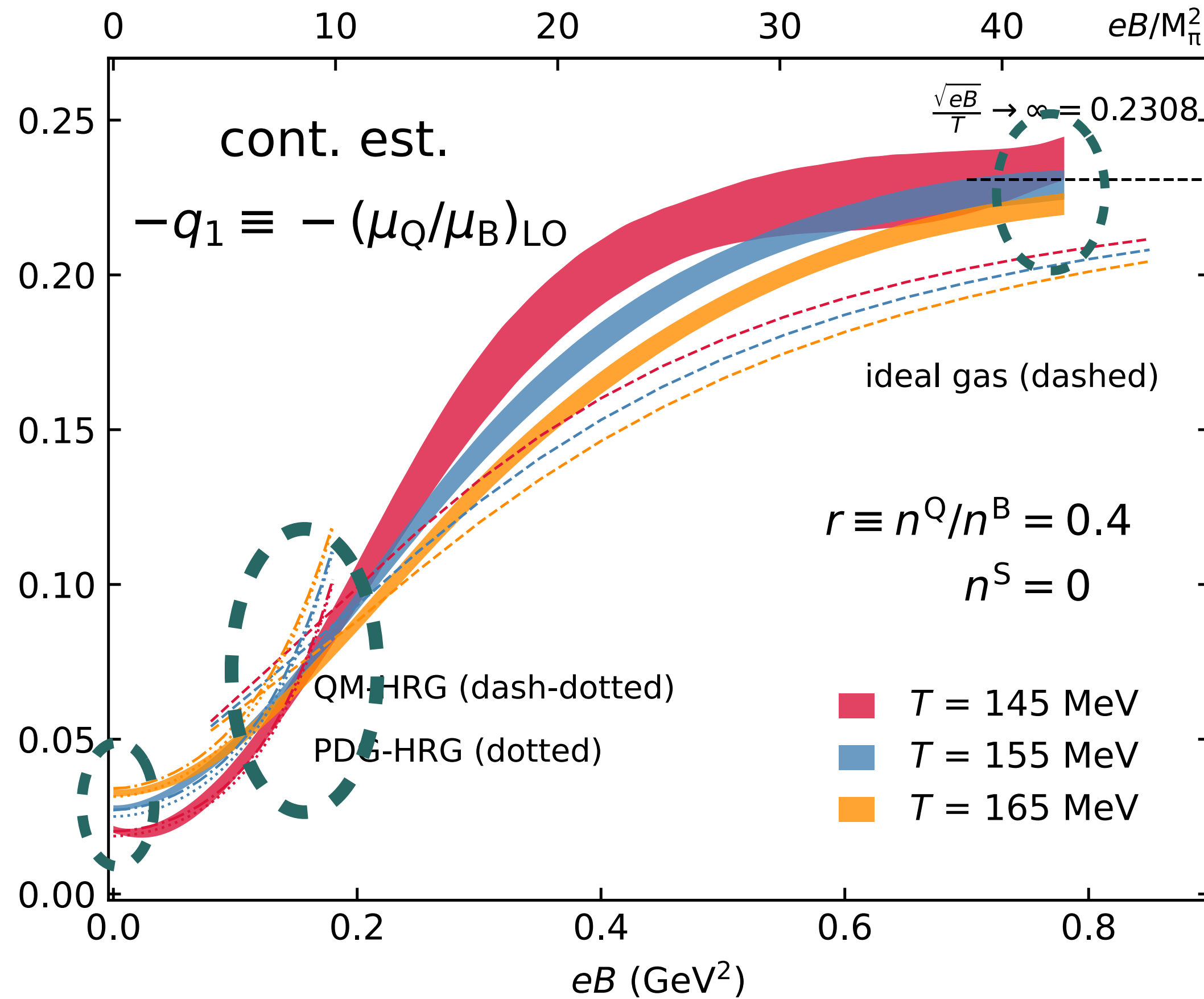
Strong- eB : saturation to free limit

$$\sqrt{eB}/T \rightarrow \infty, T \rightarrow \infty$$

$$\frac{p_{\text{ideal}}}{T^4} = \frac{8\pi^2}{45} + \sum_f \frac{3|q_f|B}{\pi^2 T^2} \left[\frac{\pi^2}{12} + \underbrace{\frac{\hat{\mu}_f^2}{4}}_{\text{LLL}} + p_f^{ll}(B) \right]$$

H.-T. Ding, S.-T. Li, Q. Shi, X.-D. Wang, *EPJA* 57 (2021) 6, 202

$(\mu_Q/\mu_B)_{\text{LO}} \equiv q_1$: INITIAL *Pb*–NUCLEI CONDITIONS



vs- eB : Crossing among T -bands

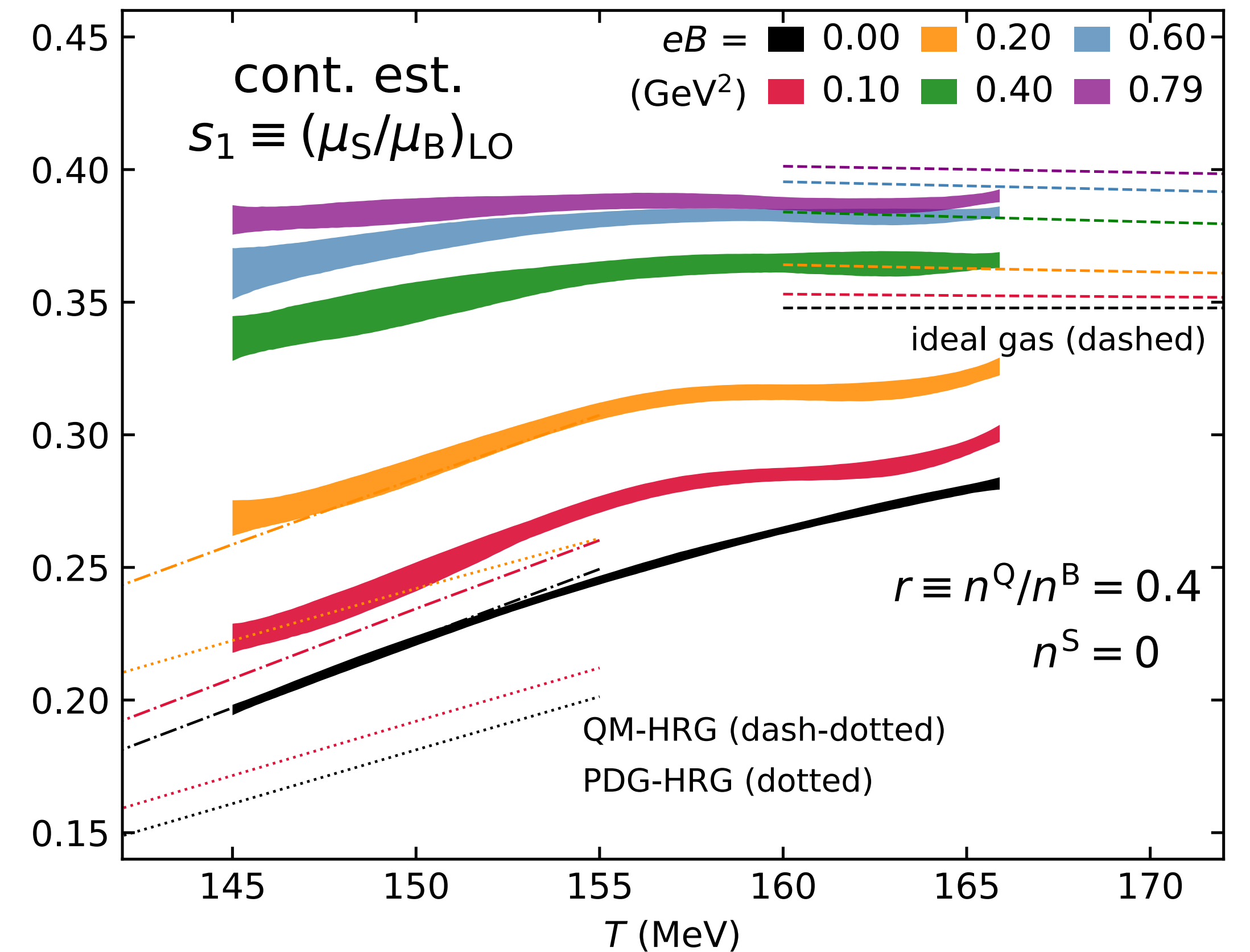
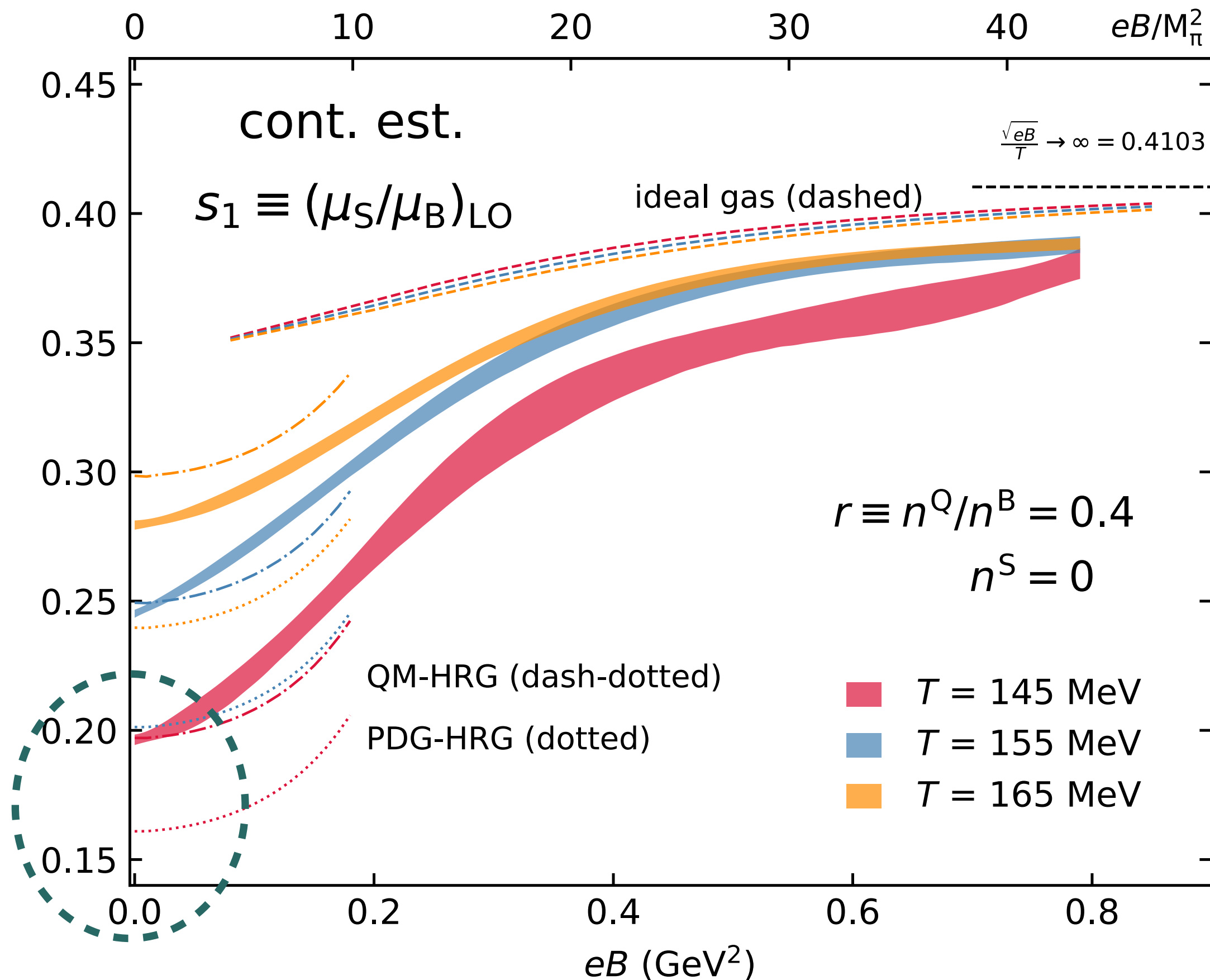


vs- T : Slope sign changes

$(\mu_S/\mu_B)_{\text{LO}} \equiv s_1$: INITIAL *Pb*–NUCLEI CONDITIONS

$$s_1 = - \frac{(\chi_{11}^{\text{BS}} + q_1 \chi_{11}^{\text{QS}})}{\chi_2^{\text{S}}}$$

- ☑ Positive s_1 and grows with eB . Indirect isospin asymmetry effects!
- ☑ Lattice results agreement better with QM-HRG than PDG-HRG



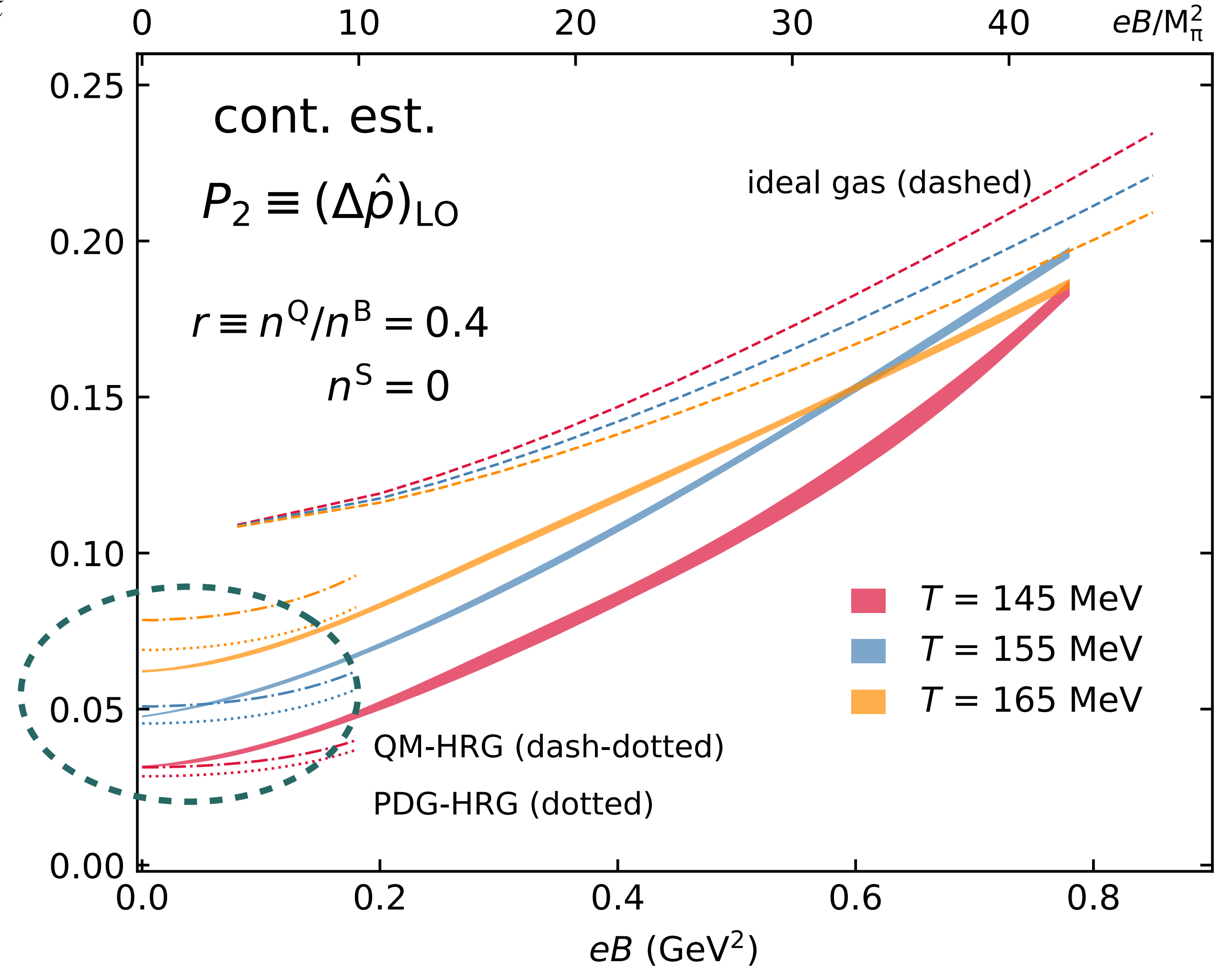
PRESSURE $\Delta\hat{p}_{\text{LO}} \equiv P_2$: EOS IN MAGNETIC FIELDS

$$\Delta\hat{p} \equiv \hat{p}(T, eB, \mu_B) - \hat{p}(T, eB, 0) = \sum_{k=1}^{\infty} P_{2k}(T, eB) \hat{\mu}_B^{2k}$$

$$P_2(T, eB) = \frac{1}{2} \left(\chi_2^B + \chi_2^Q q_1^2 + \chi_2^S s_1^2 \right) + \chi_{11}^{BQ} q_1 + \chi_{11}^{BS} s_1 + \chi_{11}^{QS} q_1 s_1$$

- ☑ HRG agreement at weak- eB and low- T .
 P_2 increases as eB grows, more dominant contributions from LLL.

$$p_{R, \text{HRG}}^c = \frac{|q_R| B}{(2\pi)^2 T^3} \sum_{s_z=-s_R}^{s_R} \sum_{l=0}^{\infty} \int_0^{\infty} dp_z \times \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k} e^{-k(E_R^c - \mu_R)/T}$$



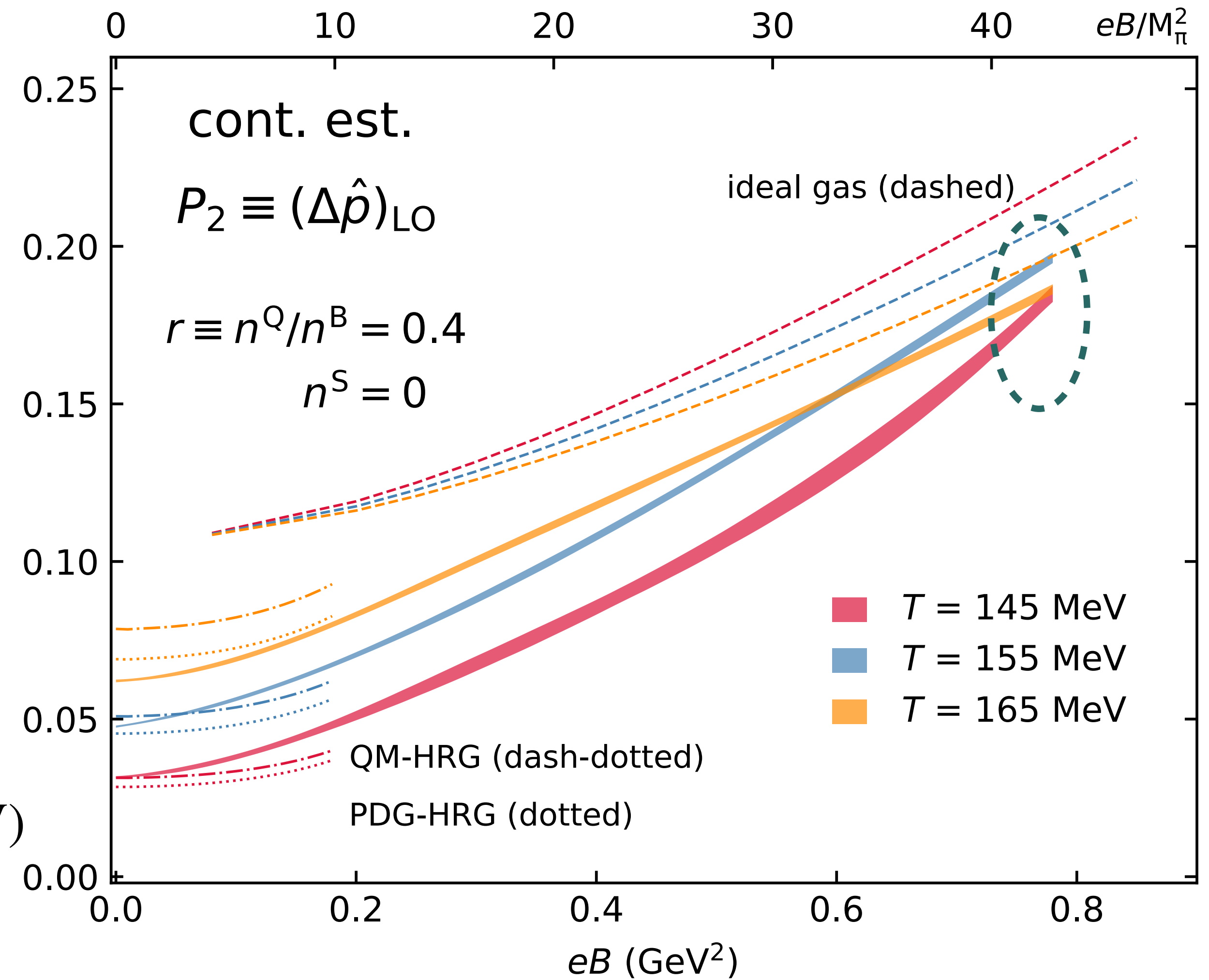
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- ☑ HRG agreement at weak- eB and low- T .
 P_2 increases as eB grows, more dominant contributions from LLL.
- ☑ Strong- eB : Lattice results approach ideal gas but no saturation

$$\frac{P_2(T, eB \simeq 0.8 \text{ GeV}^2)}{P_2(T, eB = 0)} \equiv \begin{array}{c} \text{165} \\ \text{---} \\ \sim 3 \end{array} \quad \begin{array}{c} \text{155} \\ \text{---} \\ \sim 4 \end{array} \quad \begin{array}{c} \text{145} \\ \text{---} \\ \sim 6 \end{array} \equiv T \text{ (MeV)}$$

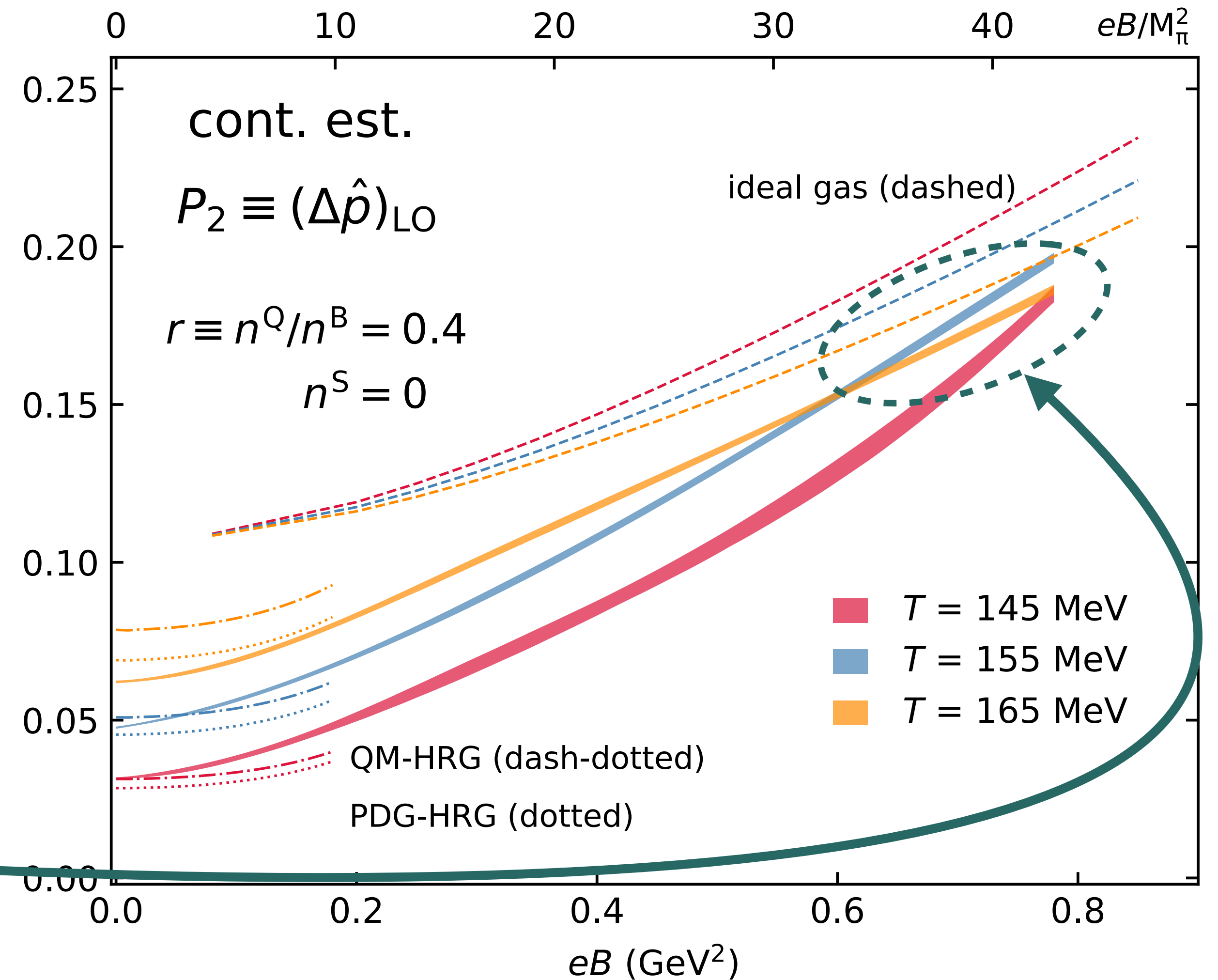


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- ☑ Strong- eB : Lattice results approach ideal gas but no saturation
- ☑ For $eB \gtrsim 0.6 \text{ GeV}^2$: Crossings among T -bands

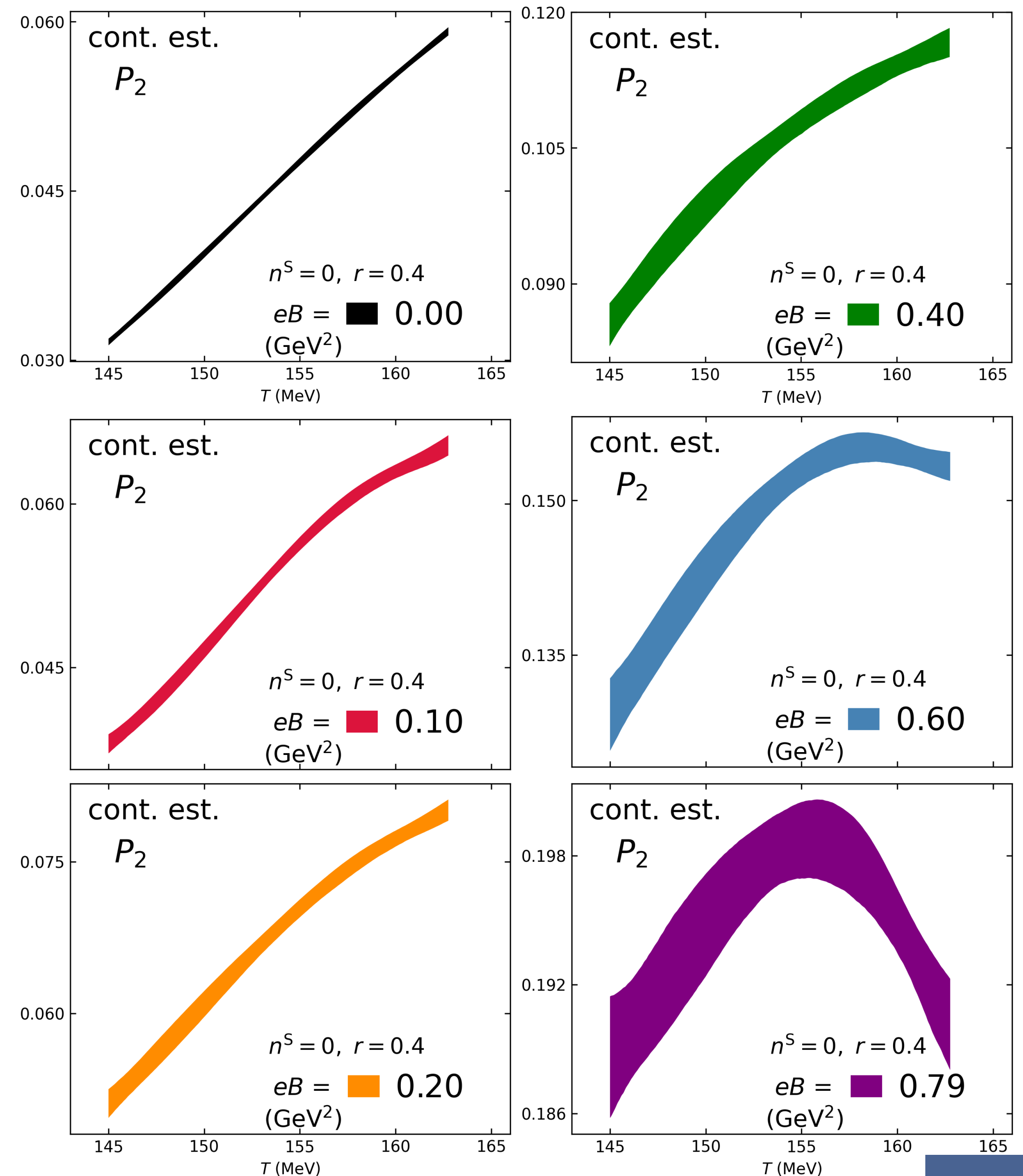
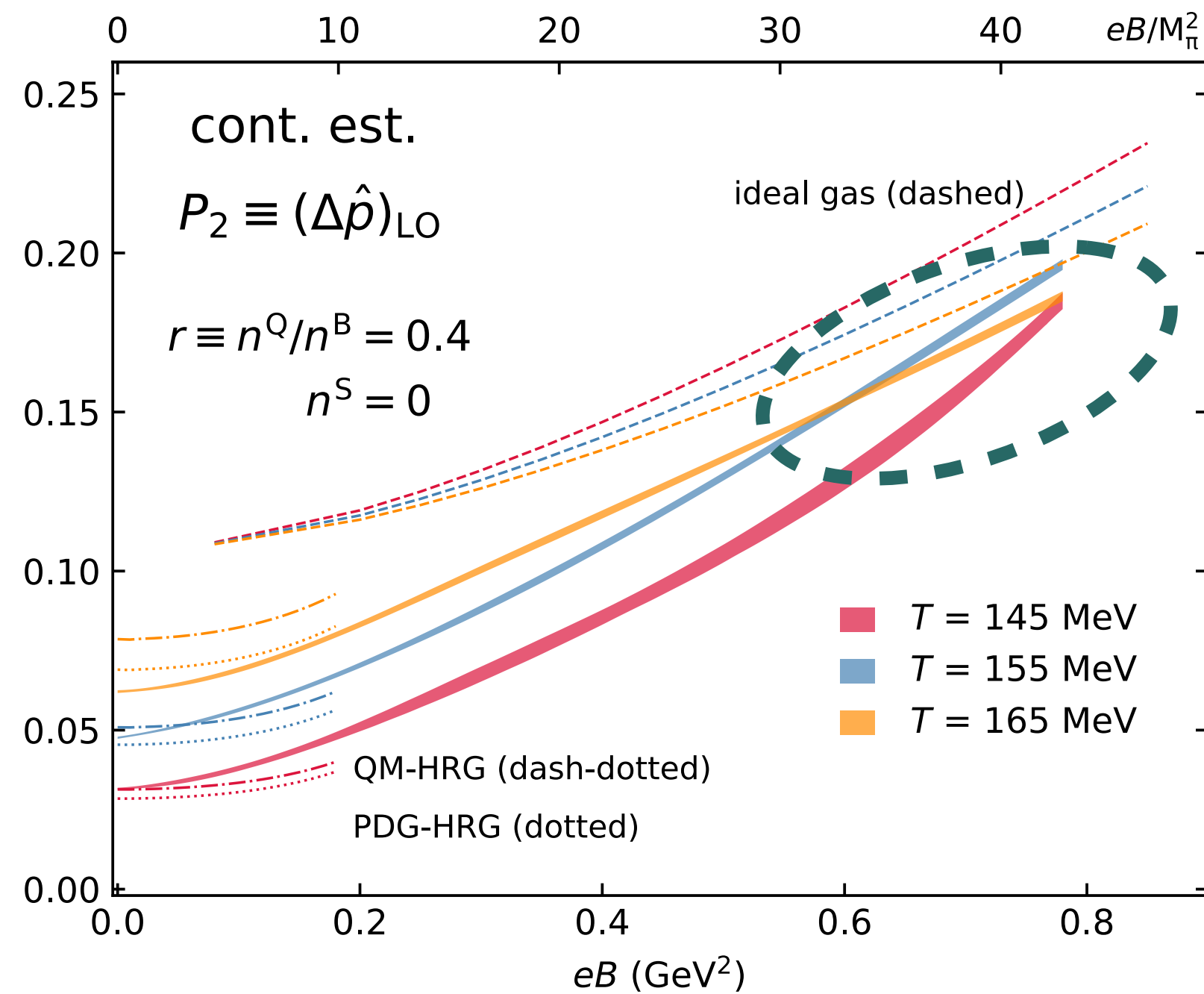


PRESSURE $\Delta\hat{p}_{\text{LO}} \equiv P_2$: EOS IN MAGNETIC FIELDS

☑ For $eB \gtrsim 0.6 \text{ GeV}^2$:

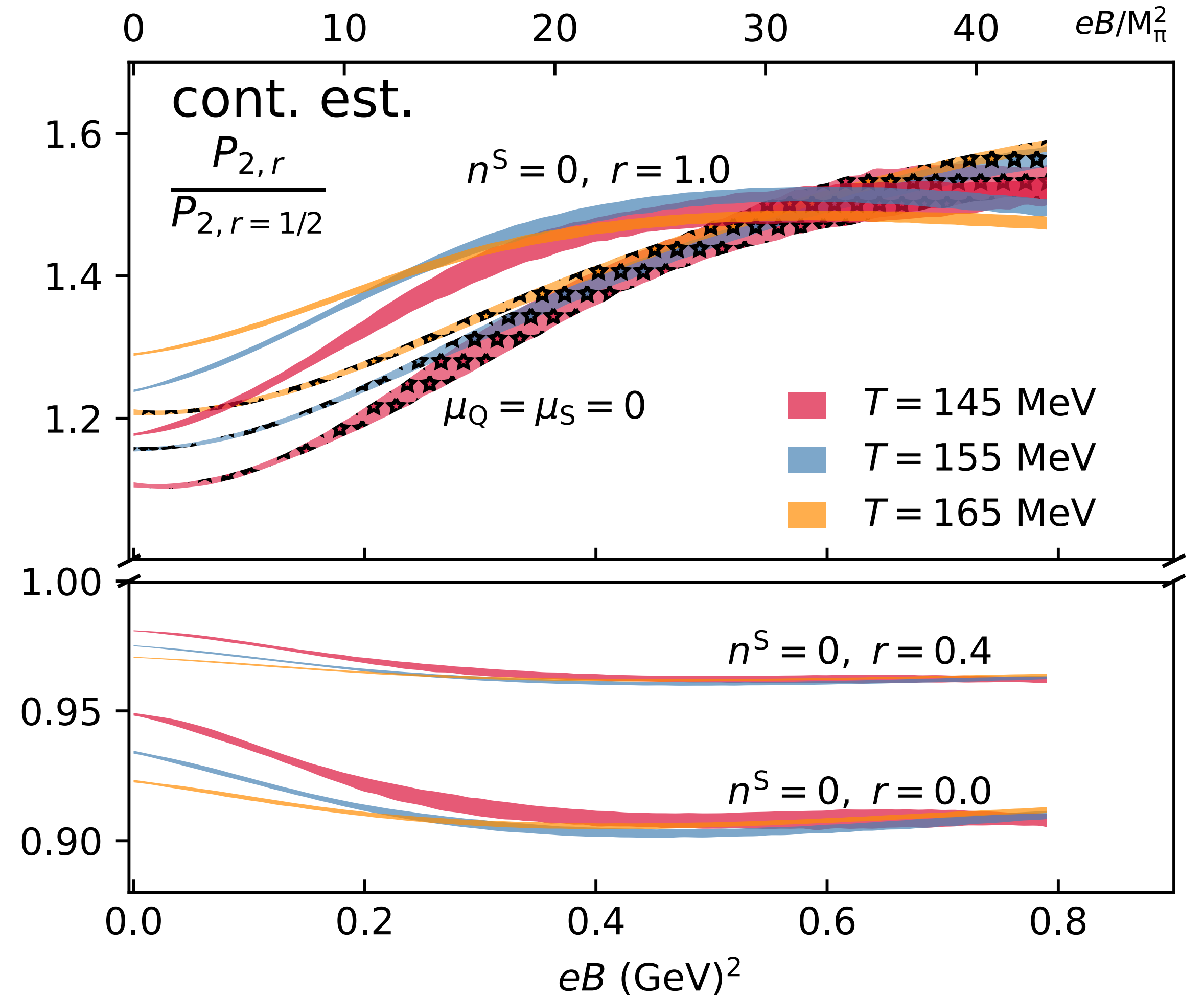
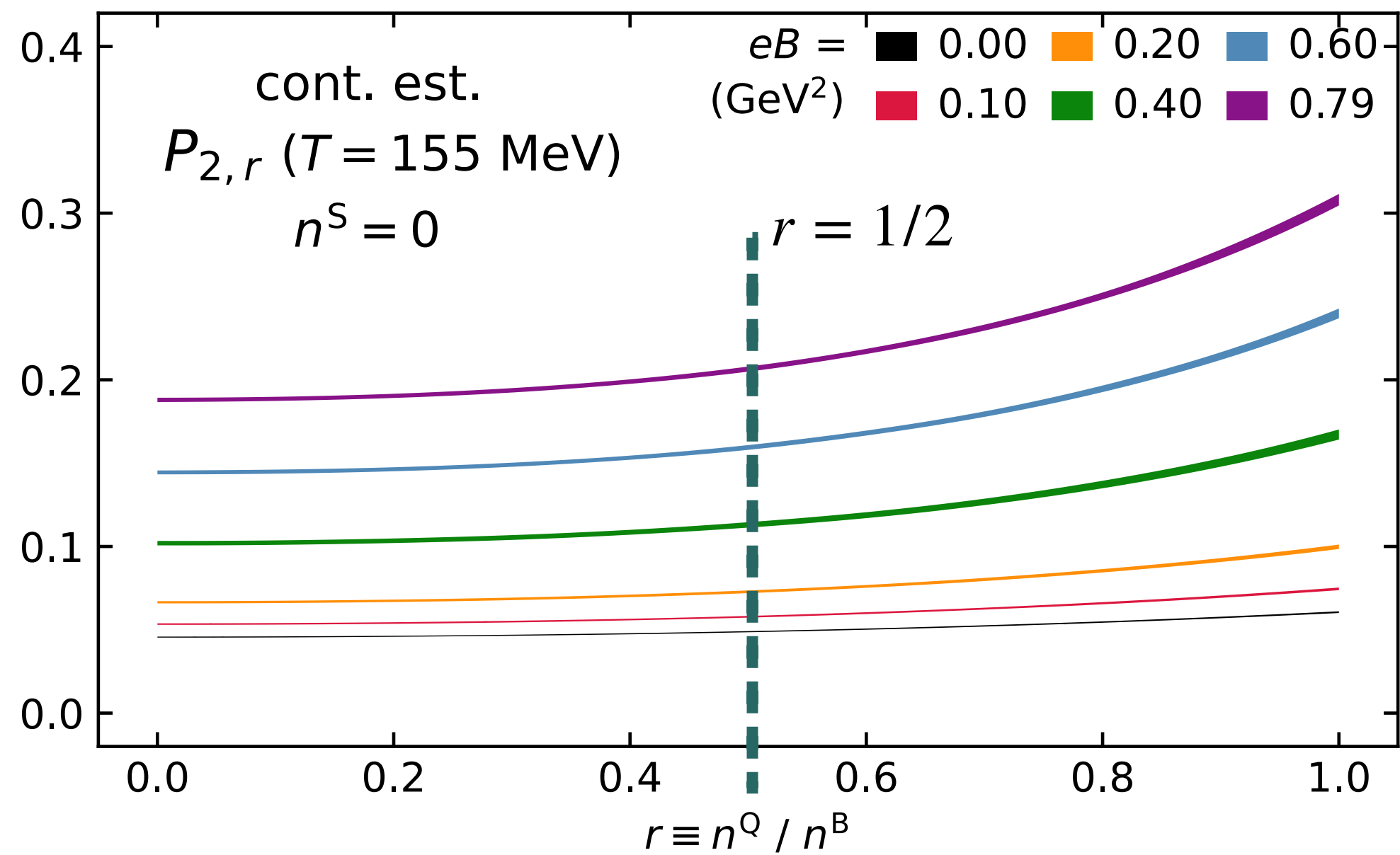
crossings among T -bands $\longleftrightarrow$ non-monotonicity
mild peak formation

☑ Peak structure shifts towards low- T : T_{pc} -lowering



PRESSURE $\Delta\hat{p}_{\text{LO}} \equiv P_2$: EOS ACROSS SYSTEMS

- ☑ Within strangeness neutral sys:
 P_2 increases as charged content r increases
- ☑ Deviation wrt isospin symmetric case:
 $r < 0.5$: $\sim 10\%$ and $r > 0.5$: $\sim 50\%$ at most



H.-T. Ding, J.-B. Gu, A. Kumar, S.-T. Li, *arXiv:2508.07532 [hep-lat]*

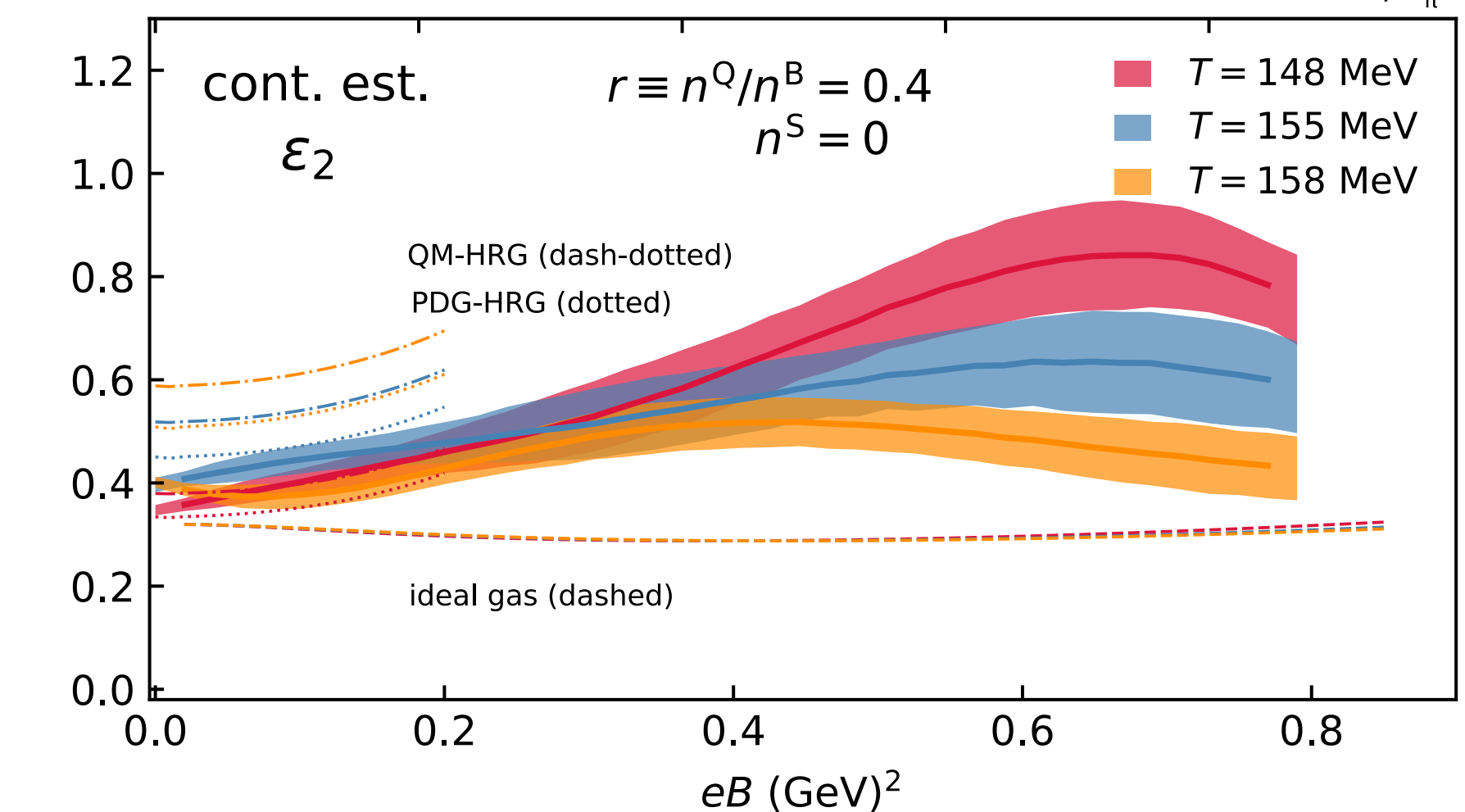
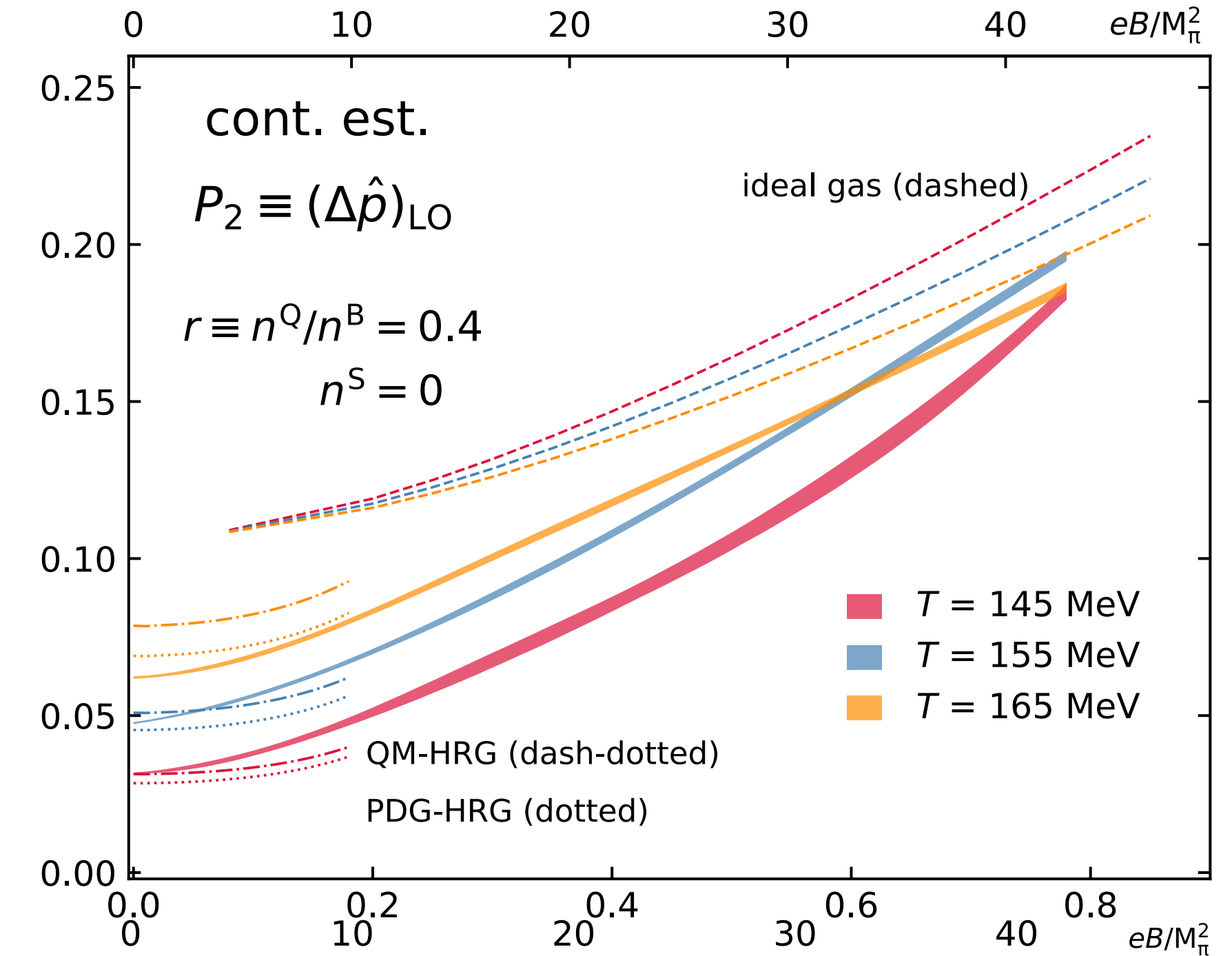
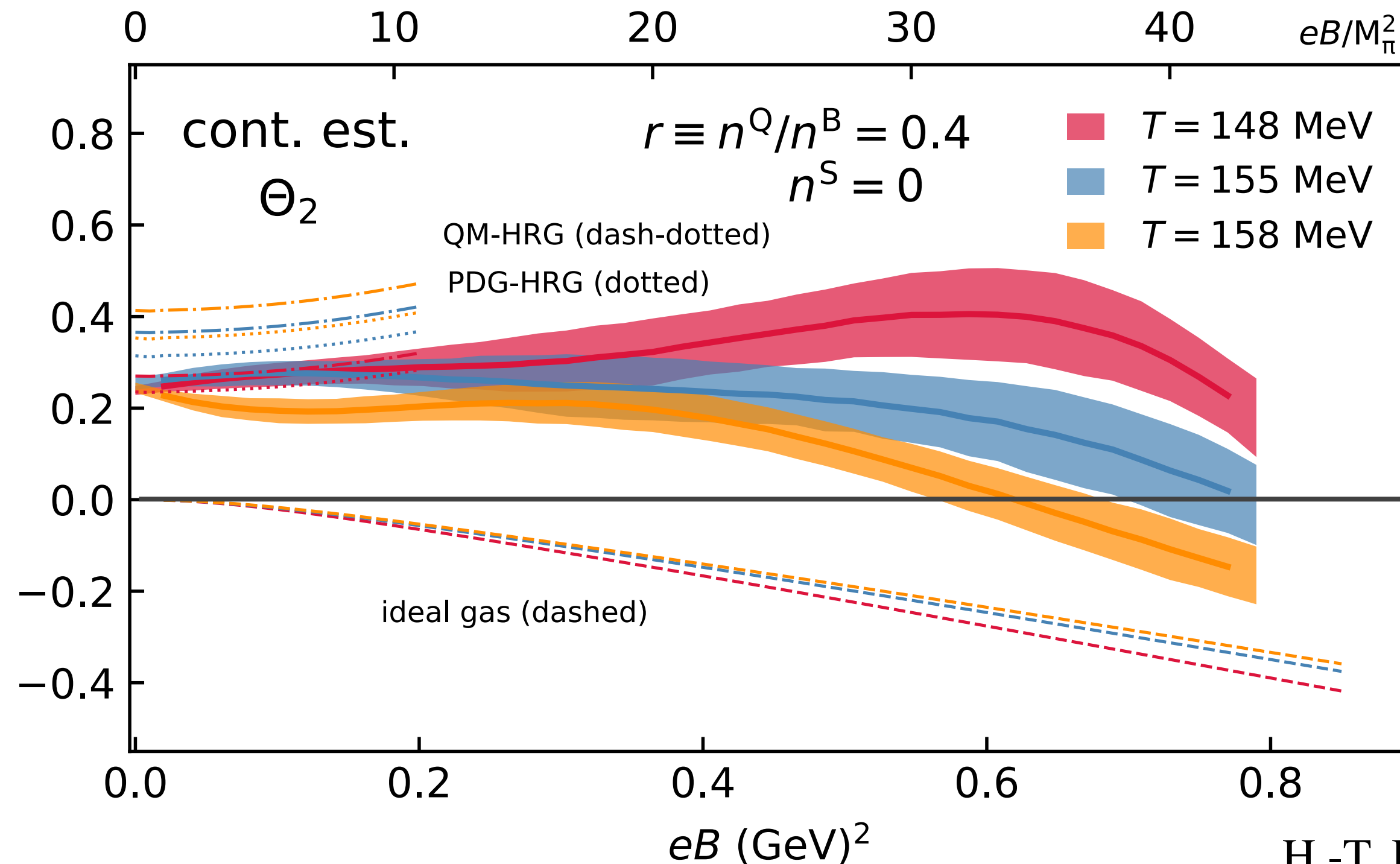
TRACE ANOMALY Θ_2 , ENERGY ϵ_2 & ENTROPY σ_2 DENSITY

- ☑ T - eB dep. of Θ_2 , ϵ_2 , σ_2 intricate: P_2 , q_1 , s_1 and ∂_T

$$\Theta_2(T, eB) = T(\partial_T P_2) - rT(\partial_T q_1)N_1^B$$

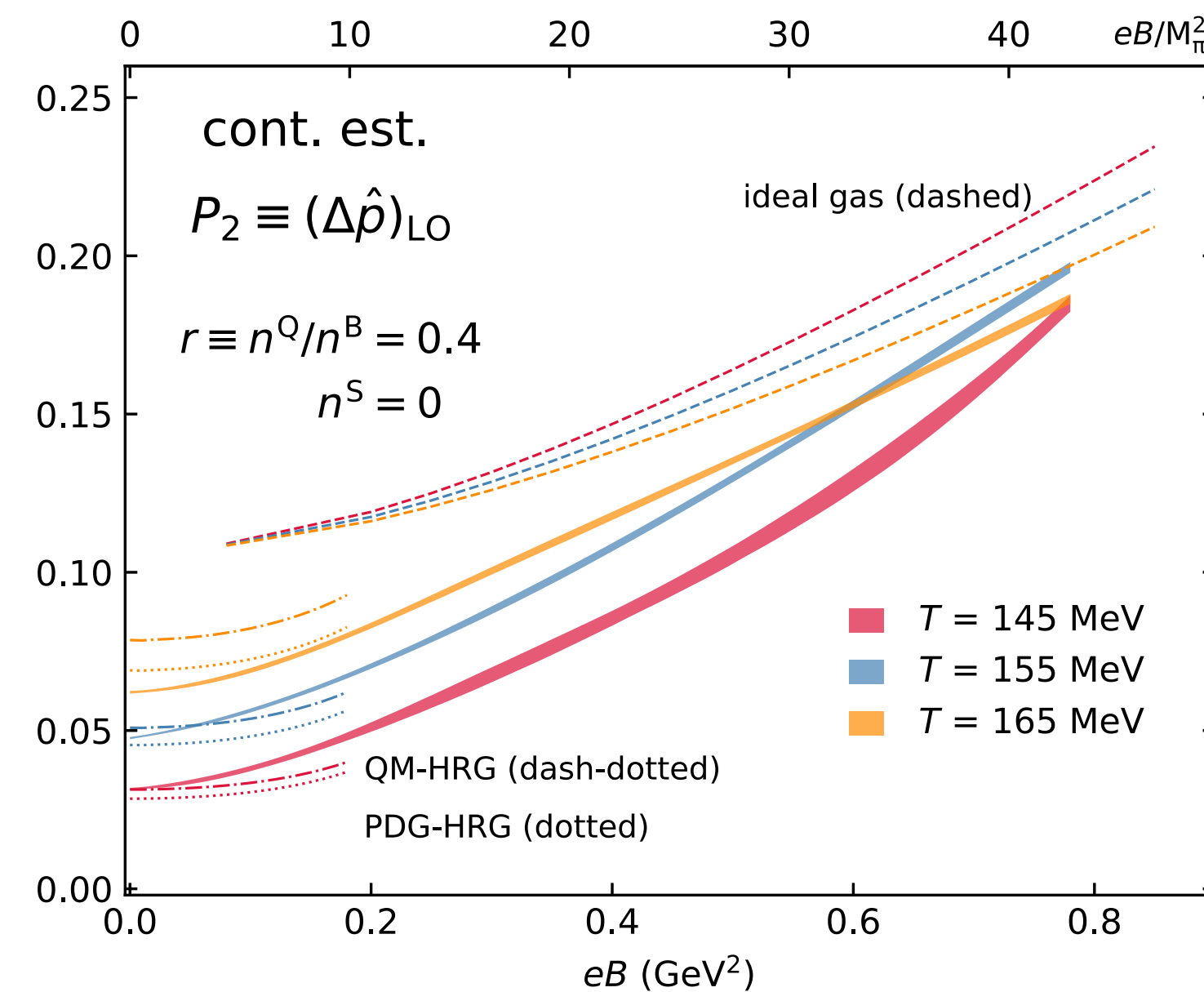
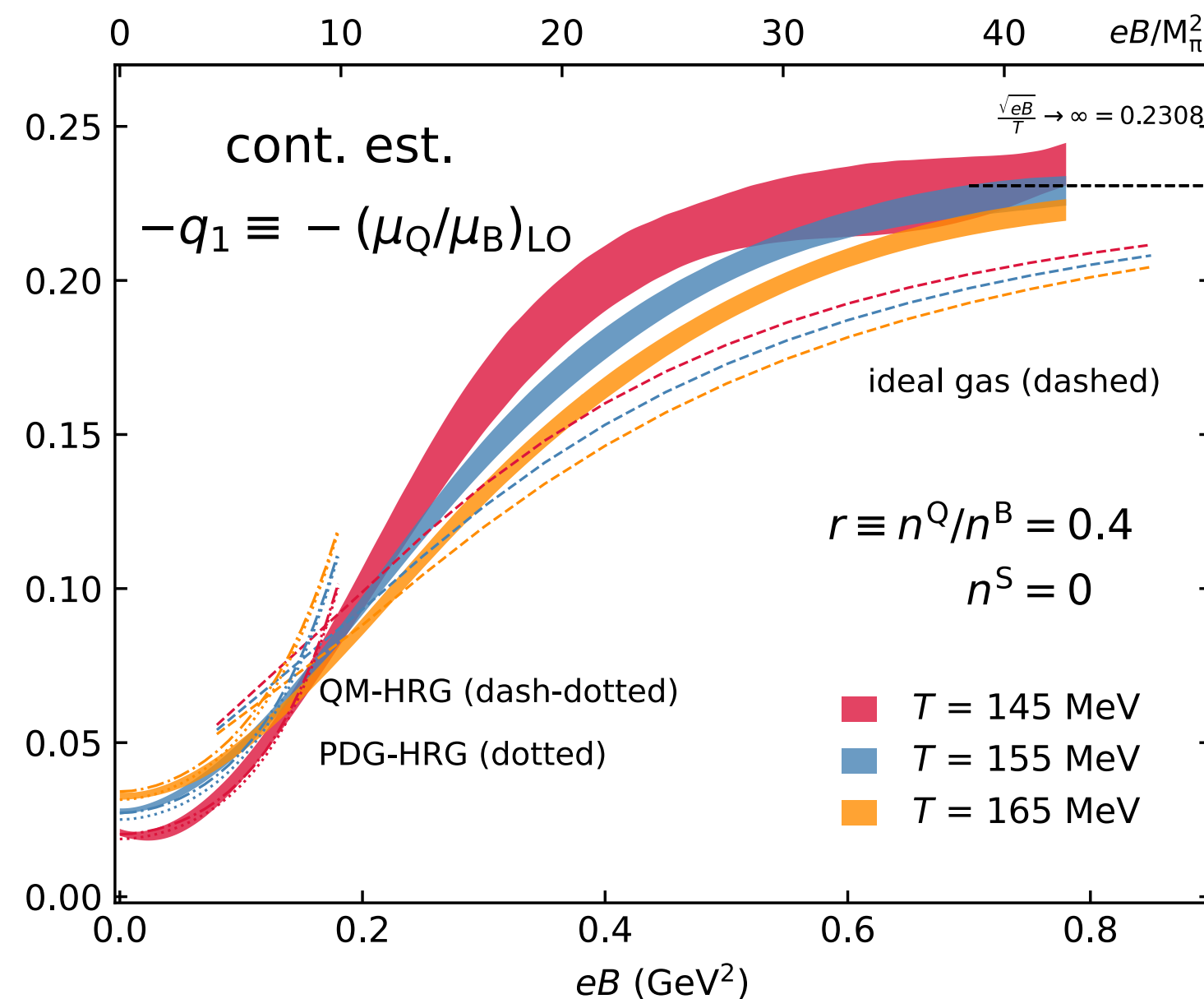
$$\epsilon_2(T, eB) = 3P_2 + \Theta_2 \quad \sigma_2(T, eB) = \epsilon_2 + P_2 + T(\partial_T P_2) - (1 + rq_1)N_1^B$$

- ☑ P_2 $\longleftrightarrow$ $\Theta_2 < 0 : 3P_2 > \epsilon_2$
crossings, peak structure *reduced and negative*



TAKE HOME MESSAGE

- Leading-order $(2 + 1)$ -f **QCD EoS** in strong magnetic fields using fluctuations of conserved charges
- q_1, s_1 : HRG breaks down and approach saturation to magnetized ideal gas
- P_2 : Crossing among fixed T bands $\rightarrow$ Non-monotonic/peak T -growth $\rightarrow T_{pc}$ -lowering



THANK YOU FOR YOUR TIME & ATTENTION!

ARPITH KUMAR, CCNU WUHAN

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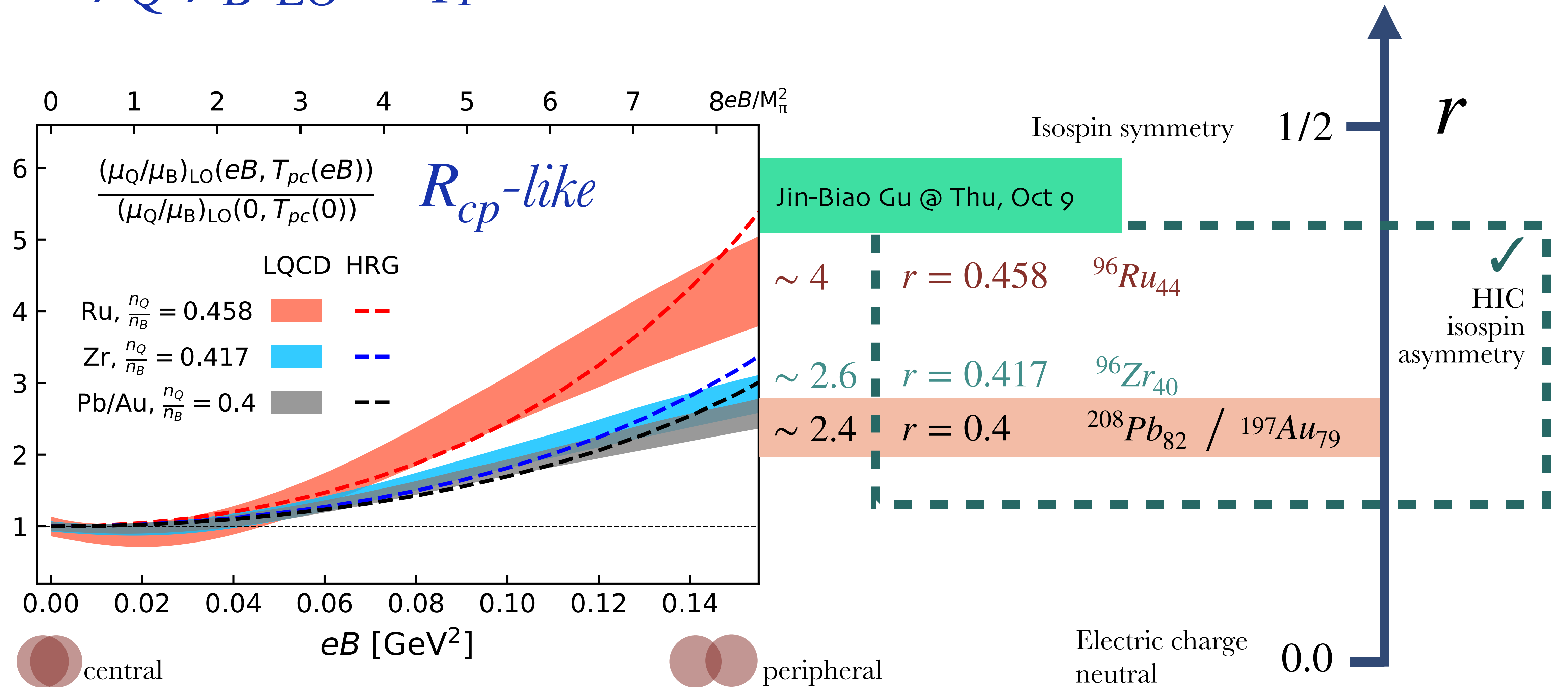
SOME BACKUPS!



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$(\mu_Q/\mu_B)_{LO} \equiv q_1$: QCD MAGNETOMETER



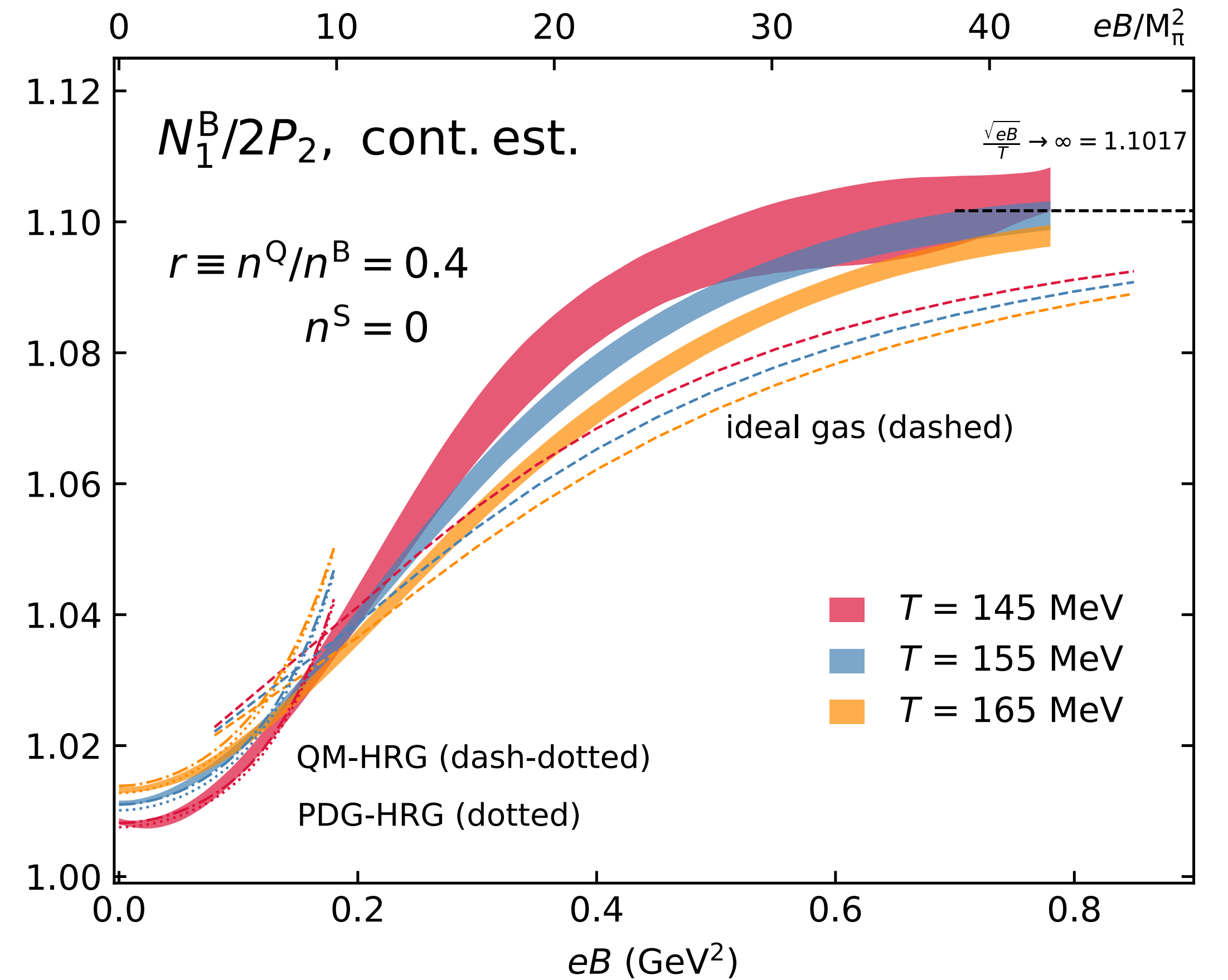
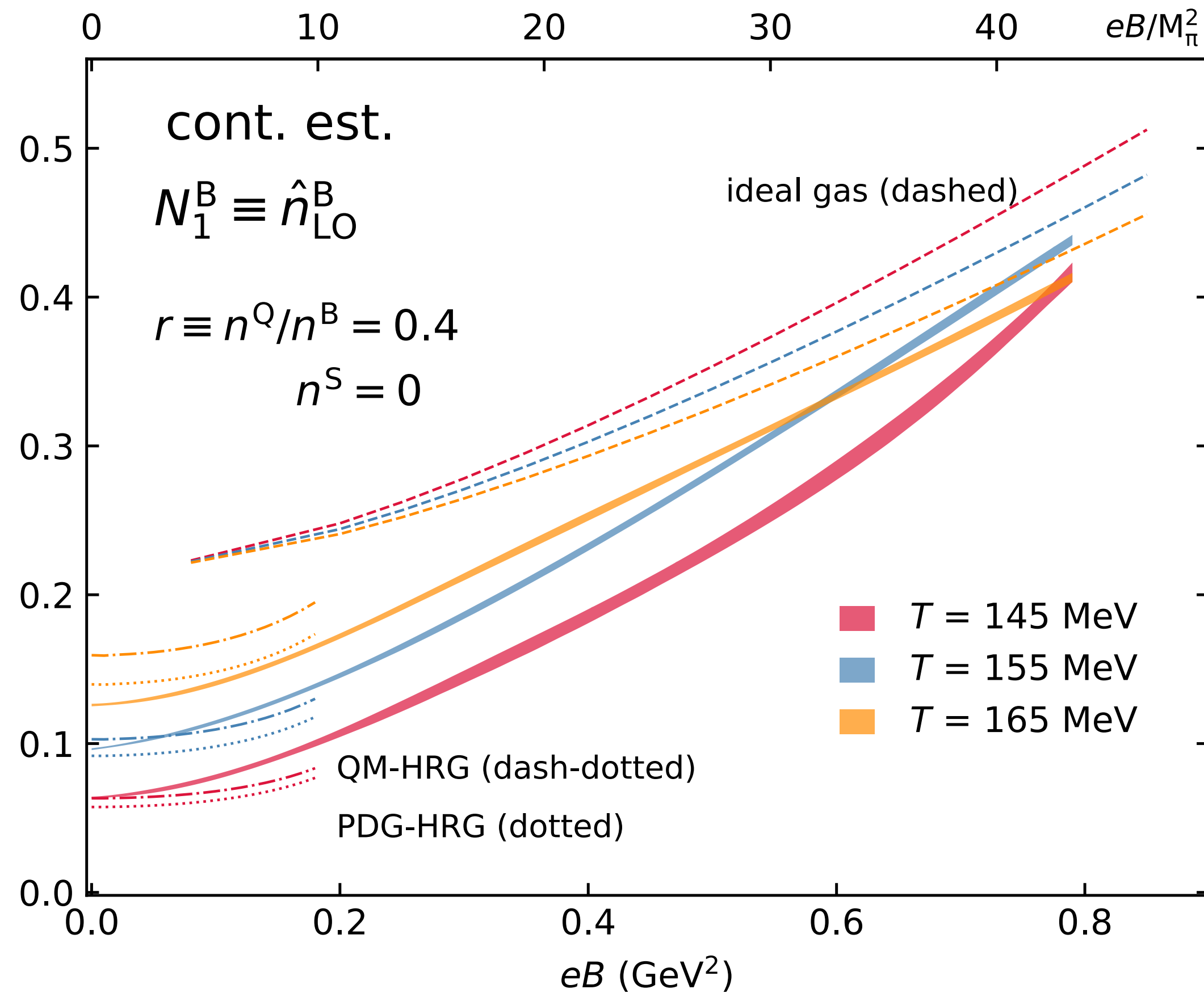
H.-T. Ding, J.-B. Gu, A. K, S.-T. Li, J.-H. Liu,
PRL 132 (2024) 201903

$\hat{n}_{\text{LO}}^{\text{B}} \equiv N_1^{\text{B}} : \text{BARYON DENSITY}$

$$N_1^{\text{B}} = \chi_2^{\text{B}} + q_1 \chi_{11}^{\text{BQ}} + s_1 \chi_{11}^{\text{BS}}$$

$$\frac{N_1^{\text{B}}}{2P_2} = \frac{1}{1 + rq_1}$$

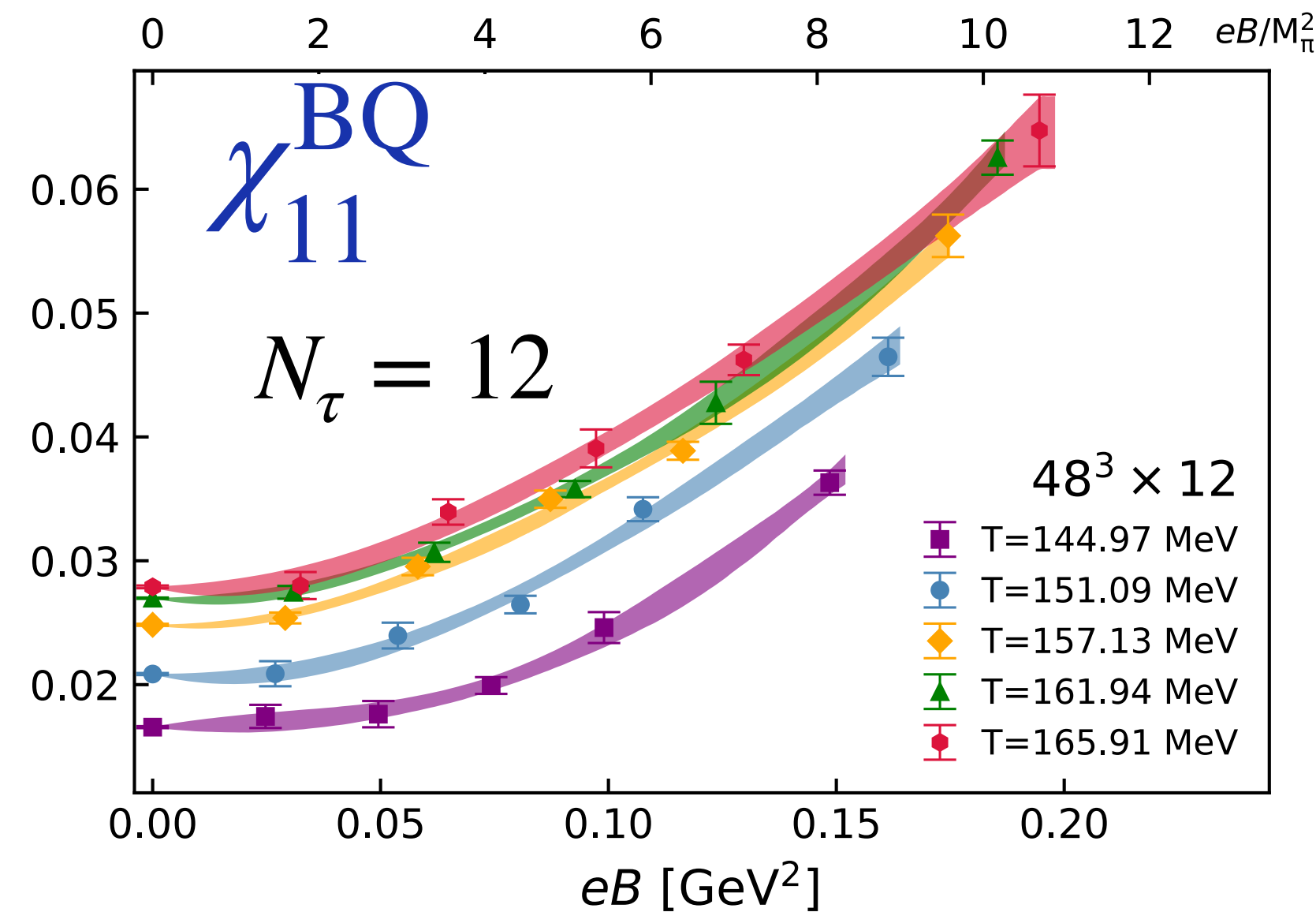
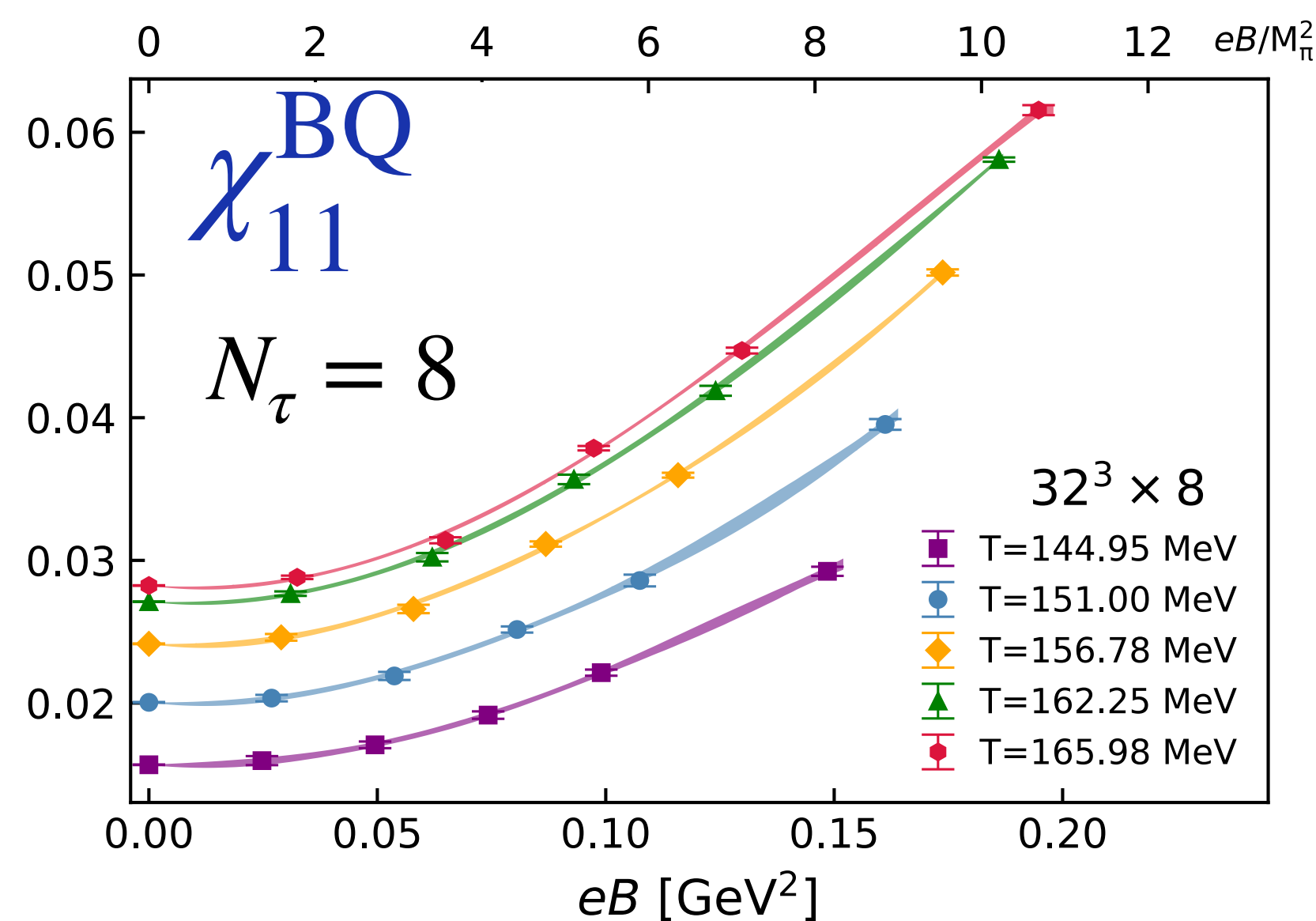
- ☑ Similar T - eB dependence to pressure; magnitude $\sim 2P_2$
- ☑ $N_1^{\text{B}}/2P_2$ deviation from unity: isospin symmetry breaking



LATTICE DATA AND SETUP AT $N_\tau = 8, 12$ ($N_\sigma/N_\tau = 4$)



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Computing Center at CCNU



H.-T. Ding, J.-B. Gu, **AK**, S.-T. Li, J.-H. Liu, **PRL 132 (2024) 201903**

SETUP:

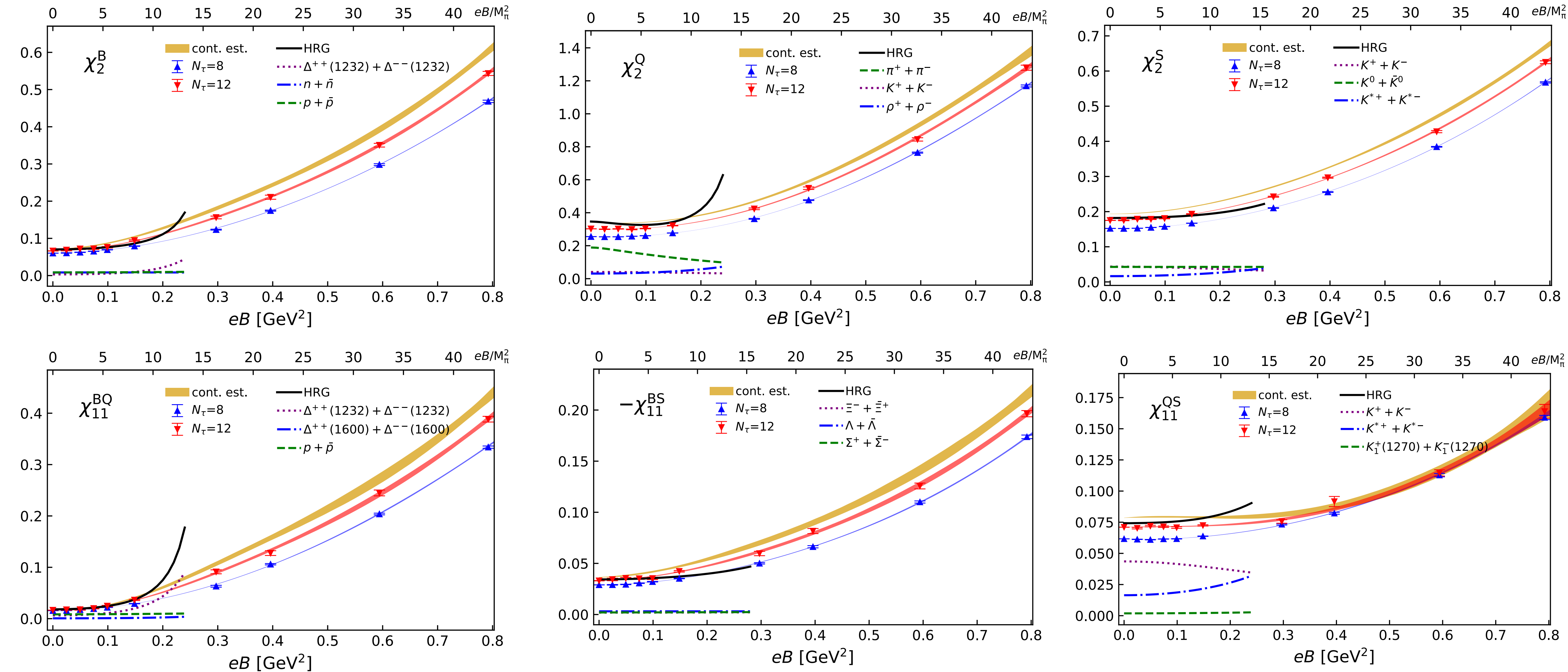
- **HISQ** & tree action with **physical pion mass**
- **Temperature** : focused around QCD transition $T \equiv [145 - 166) \text{ MeV}$

- **Magnetic field** : Quantized: $eB = 6\pi N_b a^{-2} N_\sigma^{-2}$

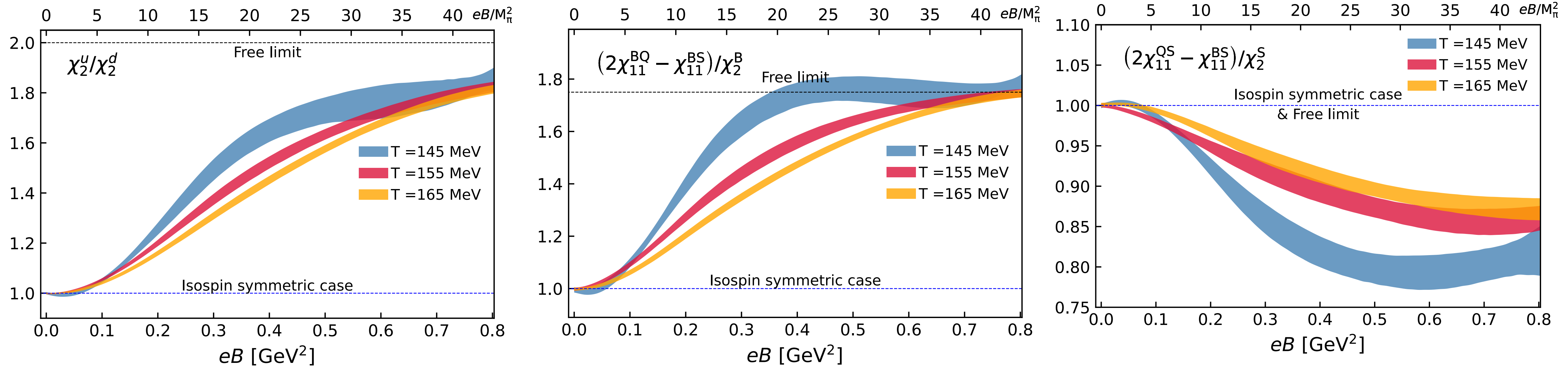
$$N_b = [1 - 6] \rightarrow eB \lesssim 0.15 \text{ GeV}^2 \sim 8M_\pi^2$$

M. D'Elia, S. Mukherjee, F. Sanfilippo,
PRD 82 (2010), 051501

2ND ORDER FLUCTUATIONS STRONG- eB AT $T = 145$ MeV

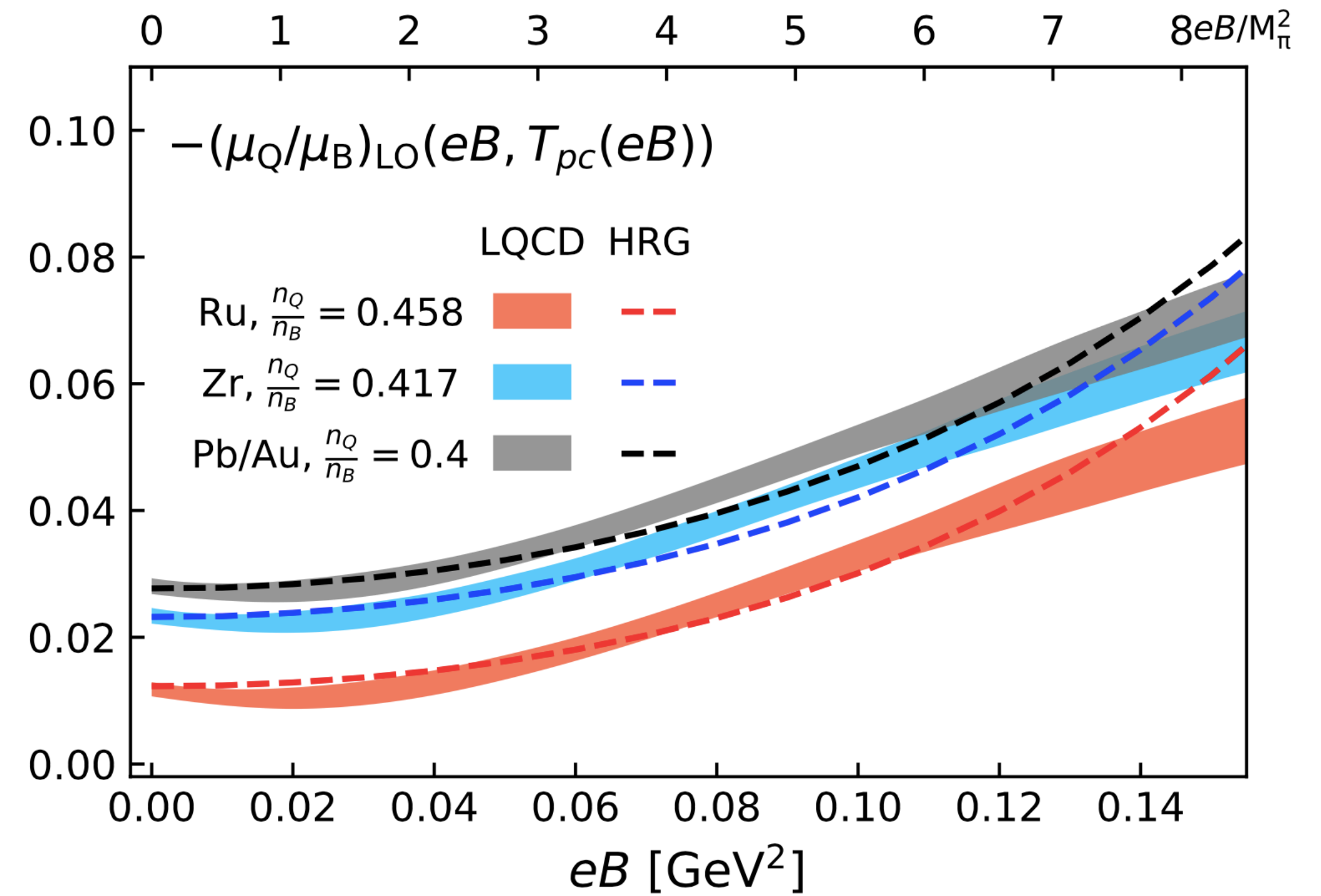
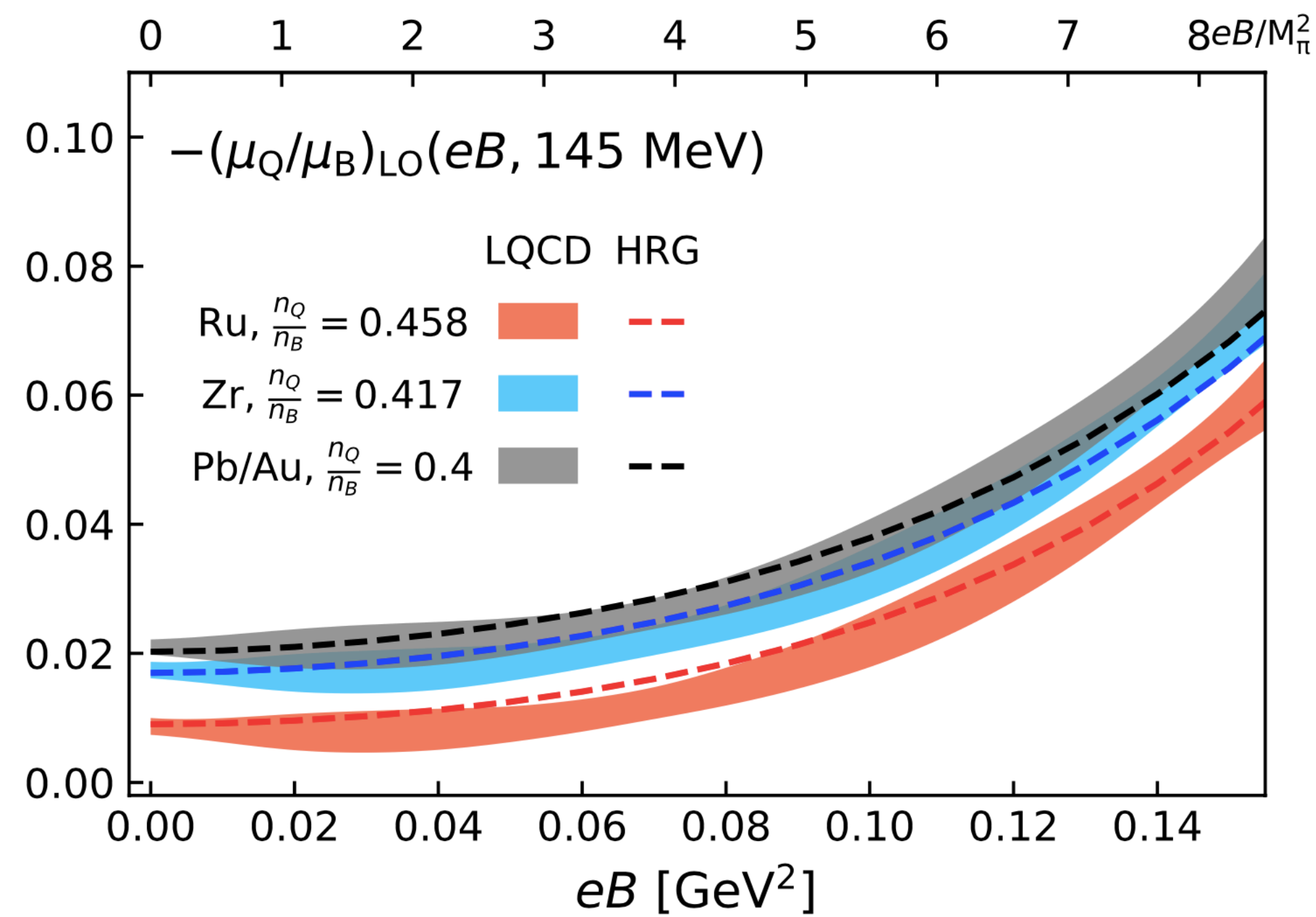


ISOSPIN SYMMETRY BREAKING IN STRONG FIELDS



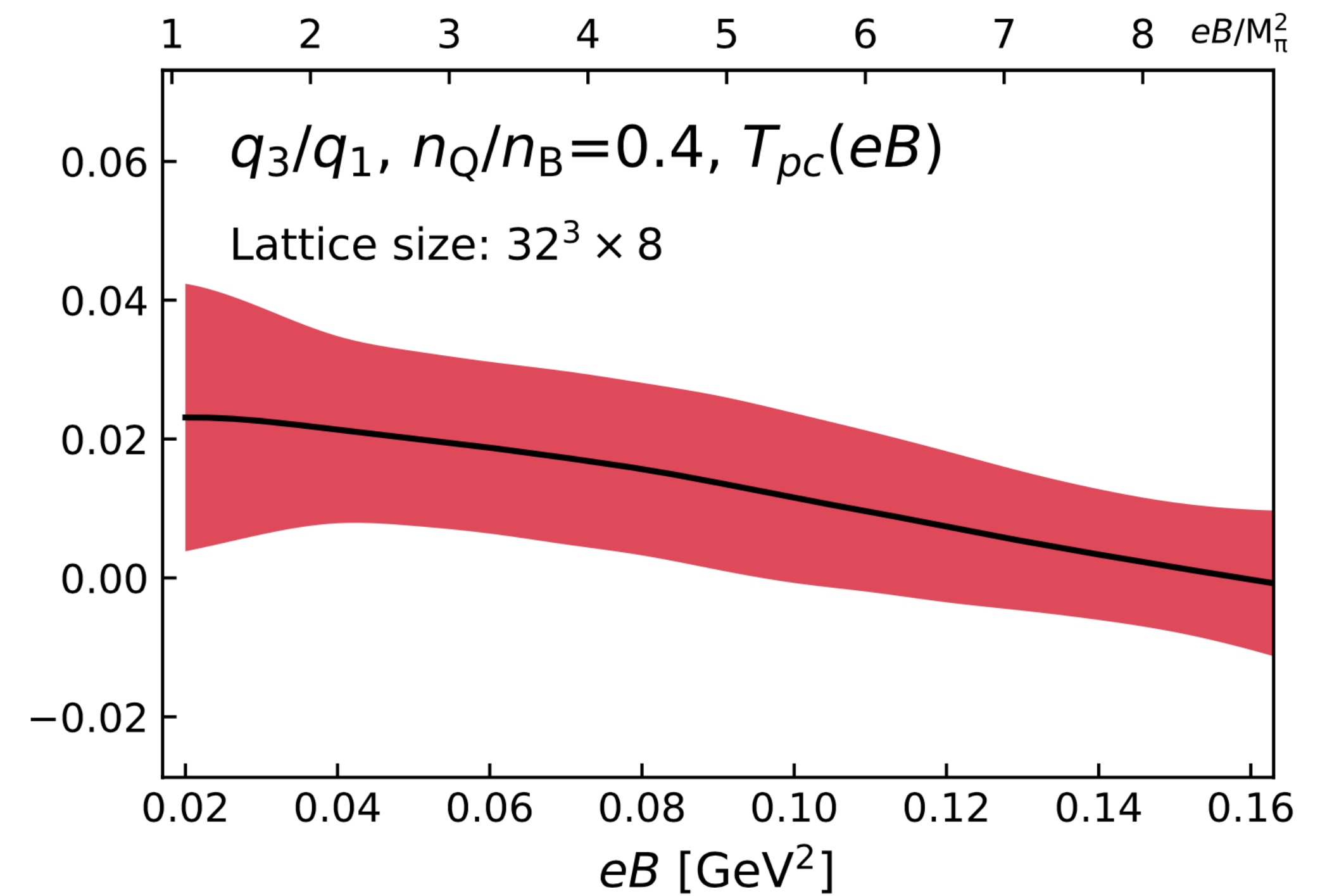
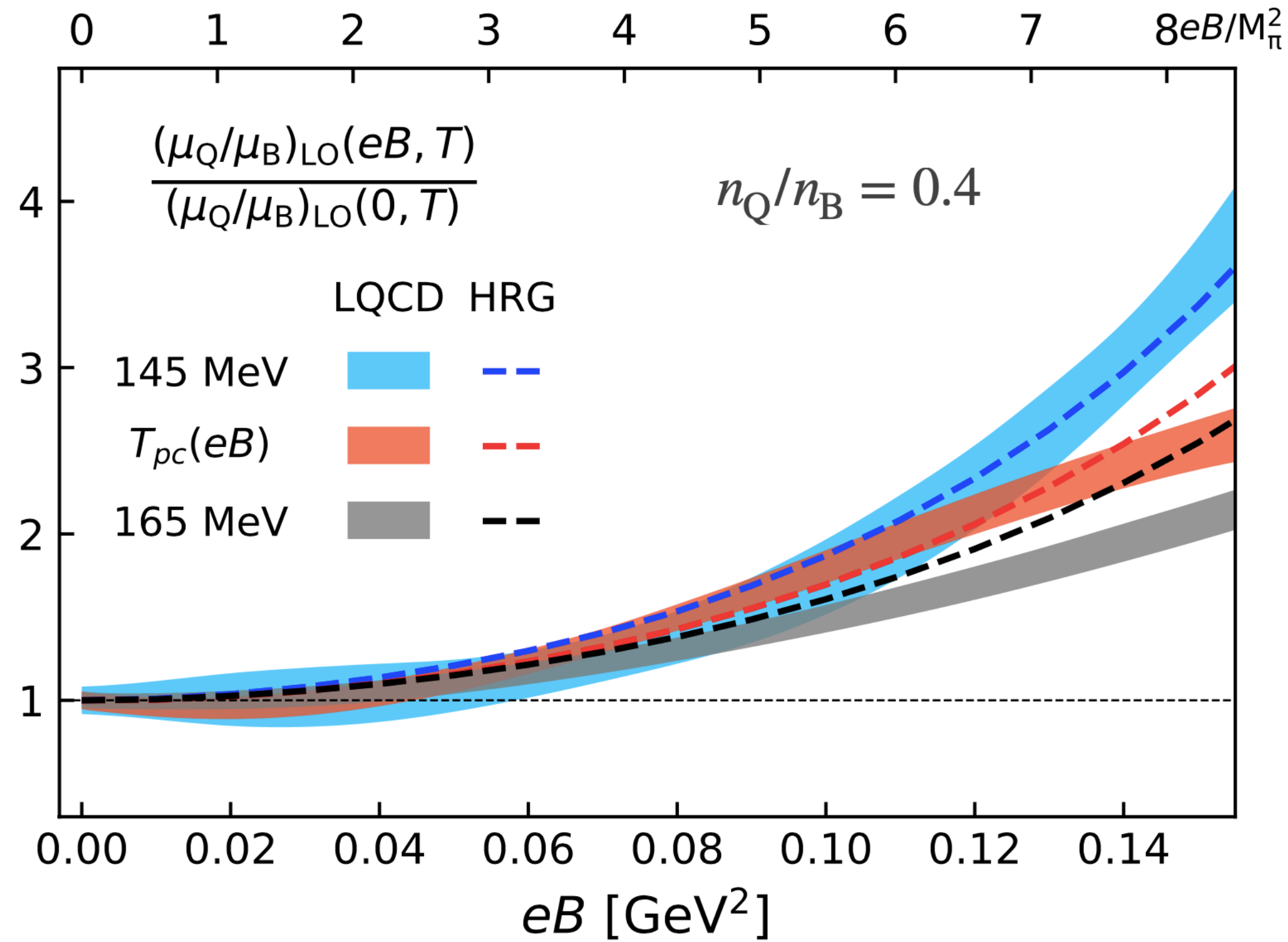
H.-T. Ding, J.-B. Gu, **AK**, S.-T. Li, **PRD** 111 (2025) 11, 114522

$(\mu_Q/\mu_B)_{\text{LO}} \equiv q_1$: COLLISION SYSTEMS



H.-T. Ding, J.-B. Gu, **AK**, S.-T. Li, J.-H. Liu, **PRL 132 (2024) 201903**

$(\mu_Q/\mu_B)_{\text{LO}} \equiv q_1$: COLLISION SYSTEMS

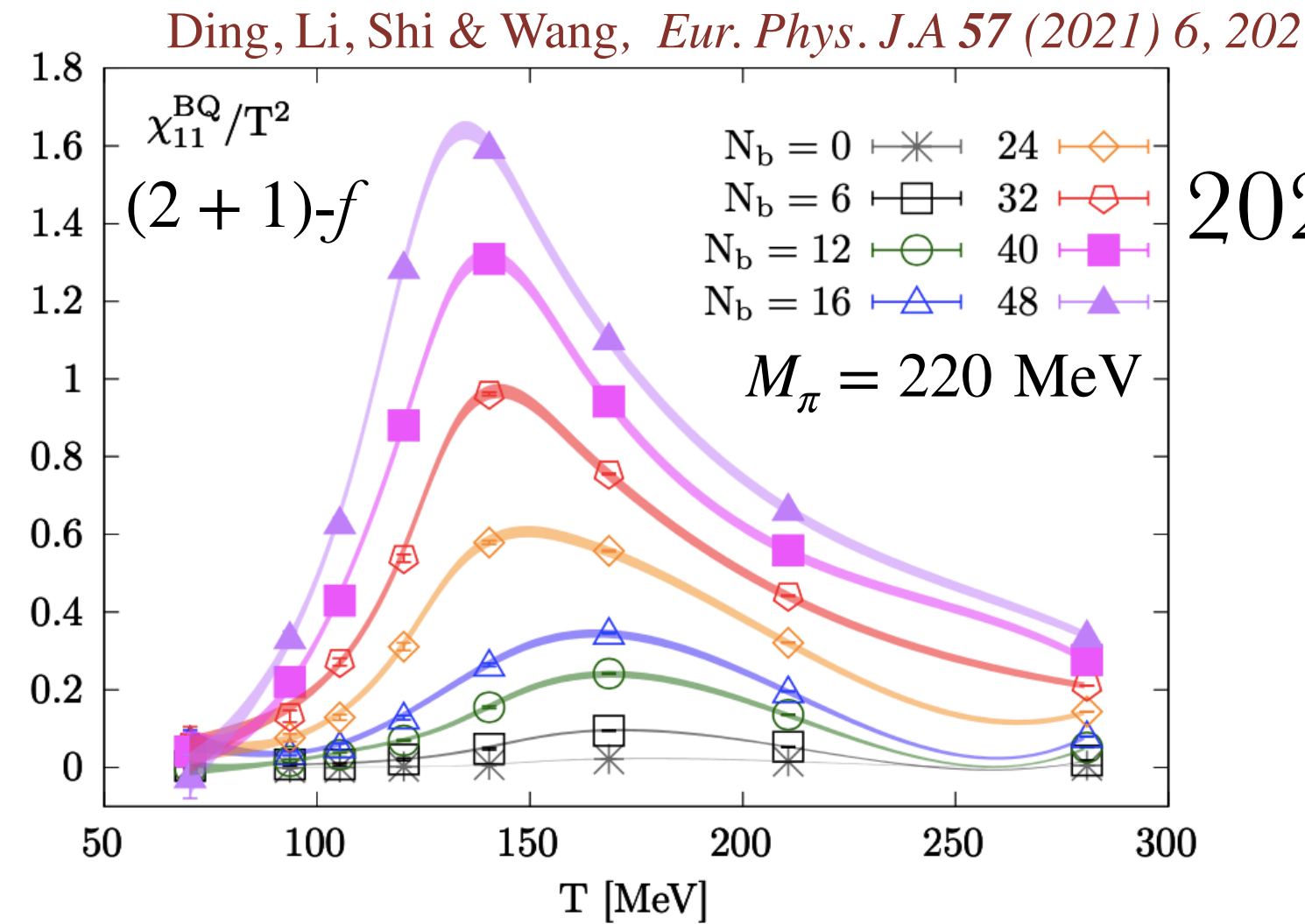


H.-T. Ding, J.-B. Gu, **AK**, S.-T. Li, J.-H. Liu, **PRL 132 (2024) 201903**

RECENT LATTICE WORKS:

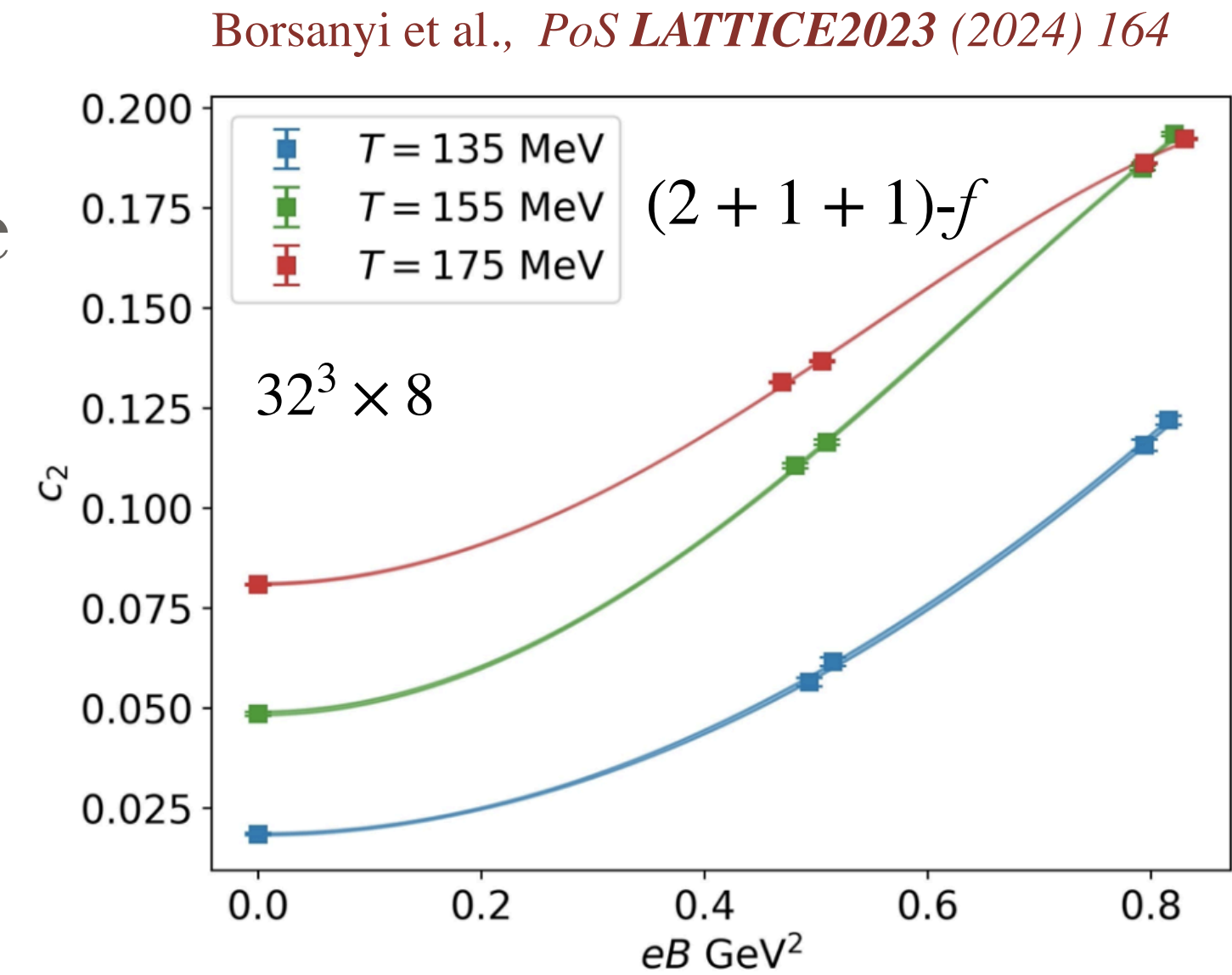
CONSERVED CHARGES IN MAGNETIC FIELDS

χ_{ijk}^{BQS}



2021

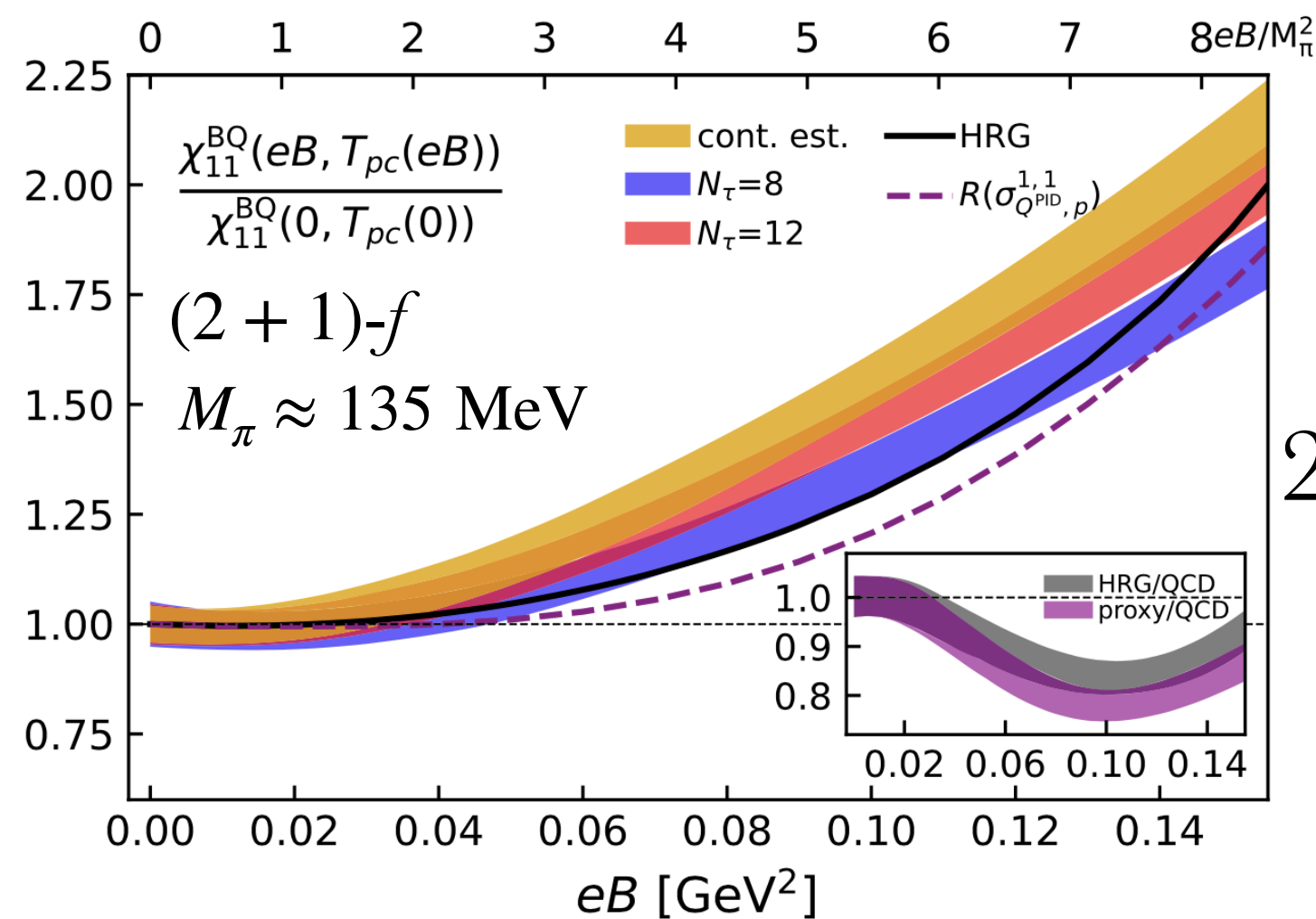
EoS:
Pressure



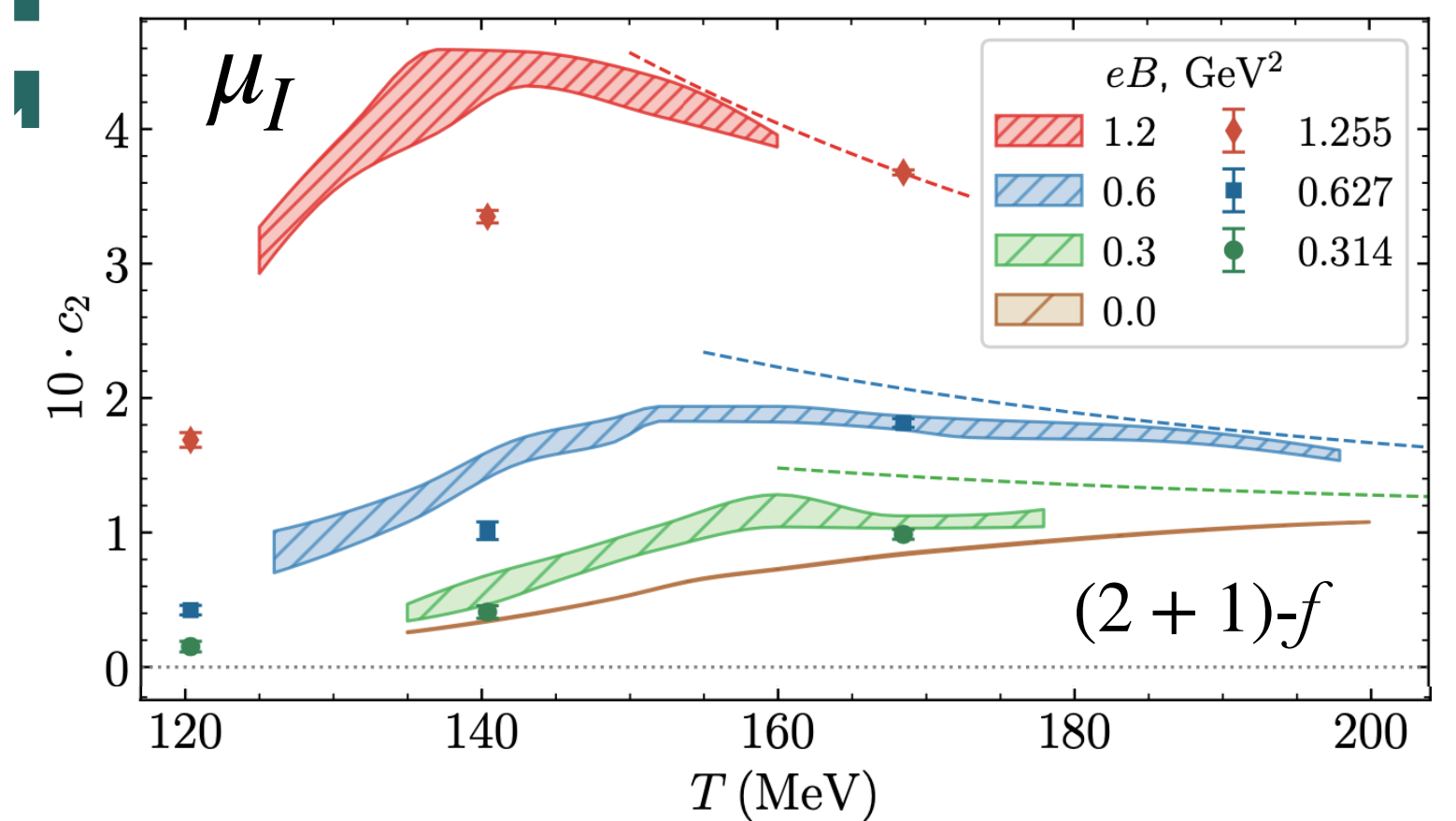
2023

★ Recent review article:
“QCD with background
electromagnetic fields on the
lattice: a review”

Endrodi, arXiv:2406.19780



2023



2024

Astrakhantsev et al., *Phys.Rev.D* **109** (2024) 9, 094511

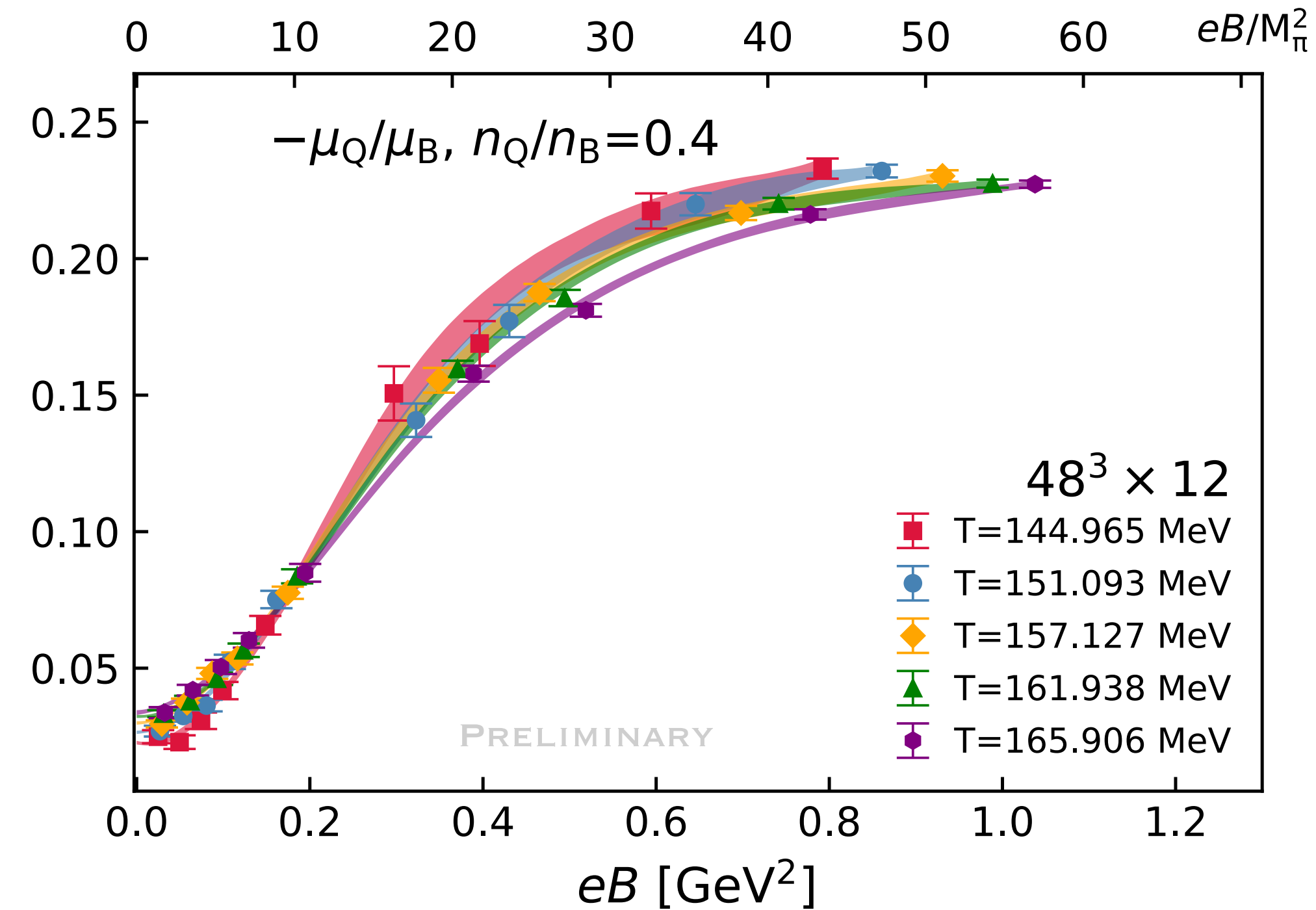
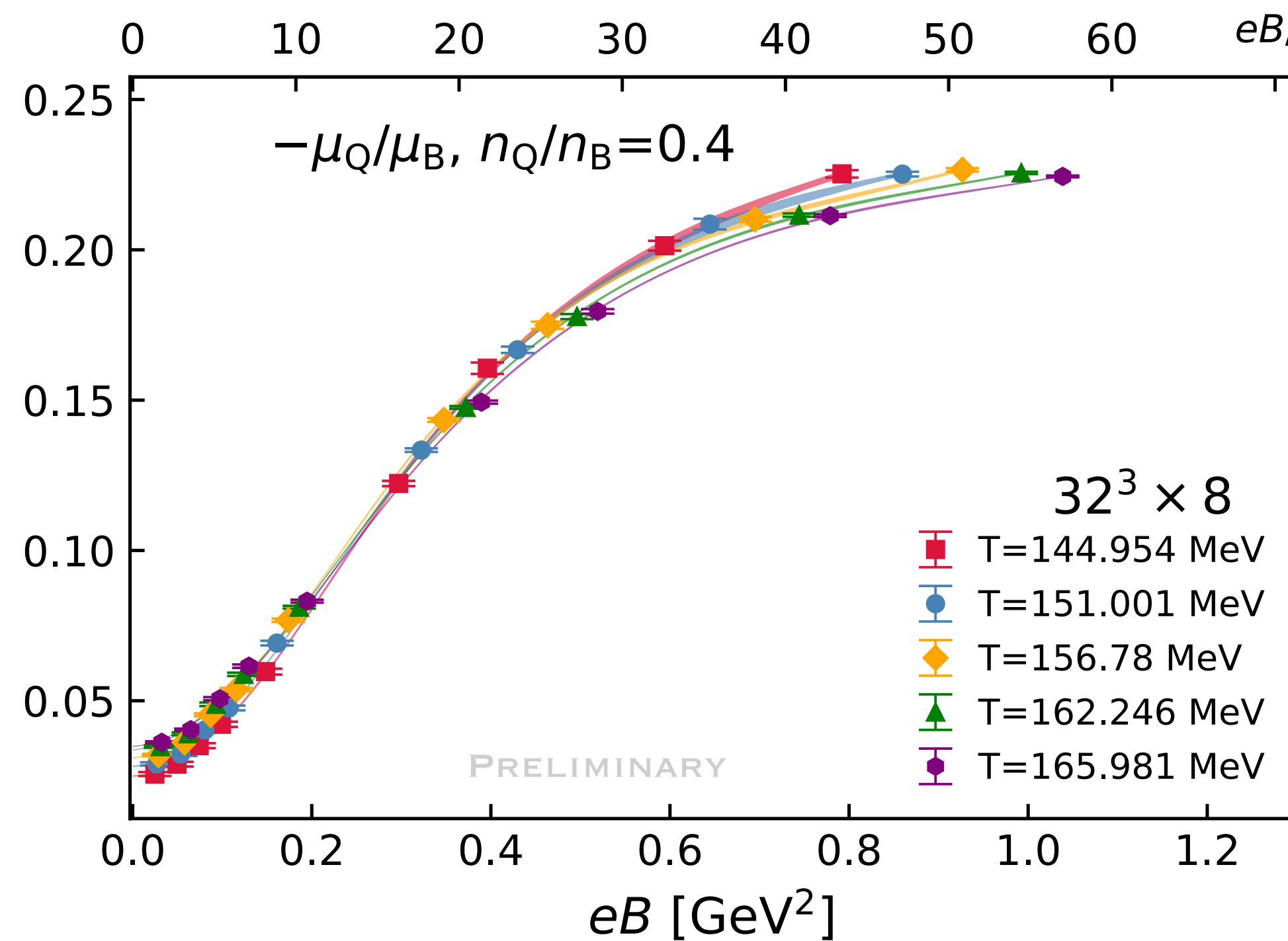
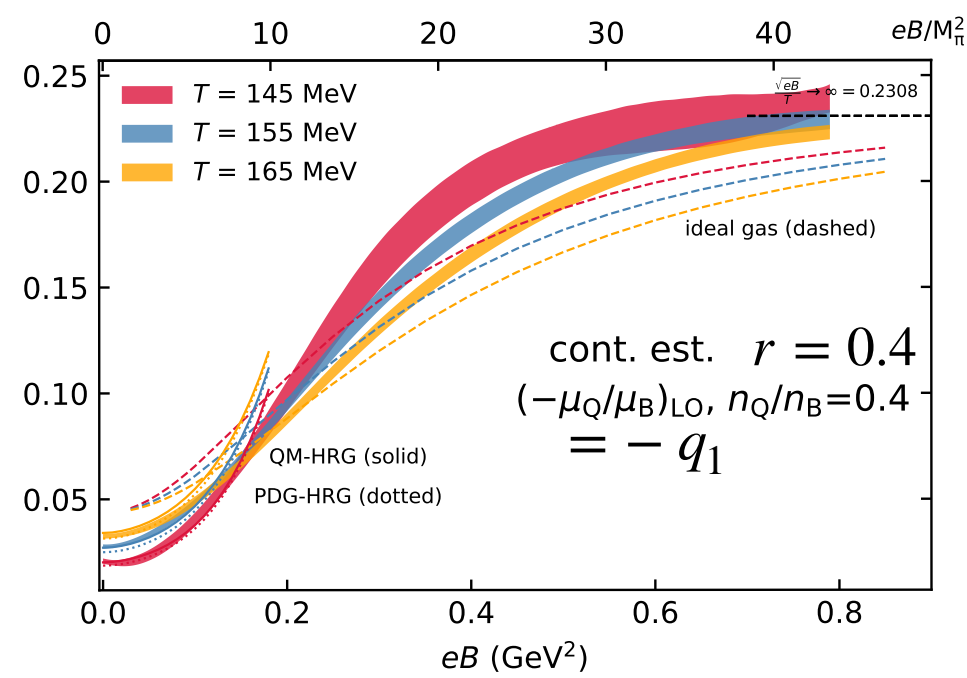
Ding, Gu, Kumar, Li & Liu, *Phys. Rev. Lett.* **132**, 201903 (2024)

μ_Q/μ_B IN PRESENCE OF eB

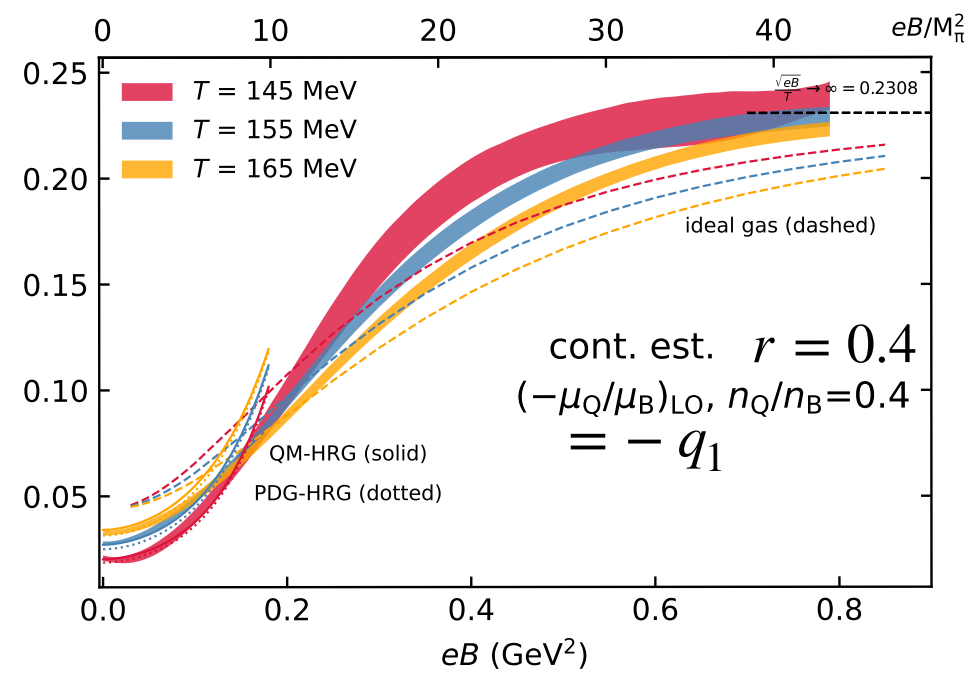
$$q_1 = \frac{r (\chi_2^B \chi_2^S - \chi_{11}^{BS} \chi_{11}^{BS}) - (\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}{(\chi_2^Q \chi_2^S - \chi_{11}^{QS} \chi_{11}^{QS}) - r (\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}$$

A. Lattice data + spline interpolation

$T \in [145 - 166) \text{ MeV}$
 $eB \in [0.0 - 0.8) \text{ GeV}^2$



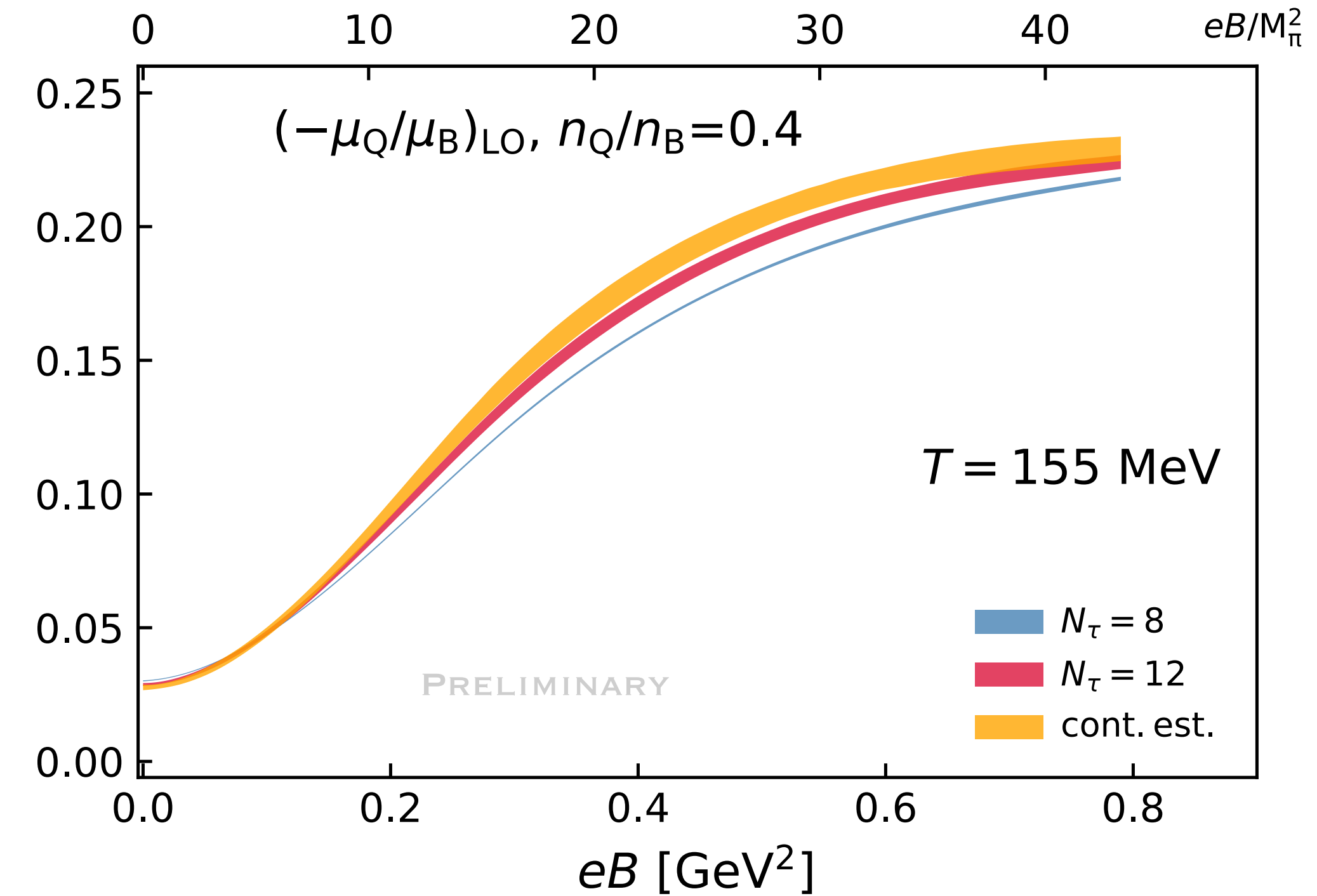
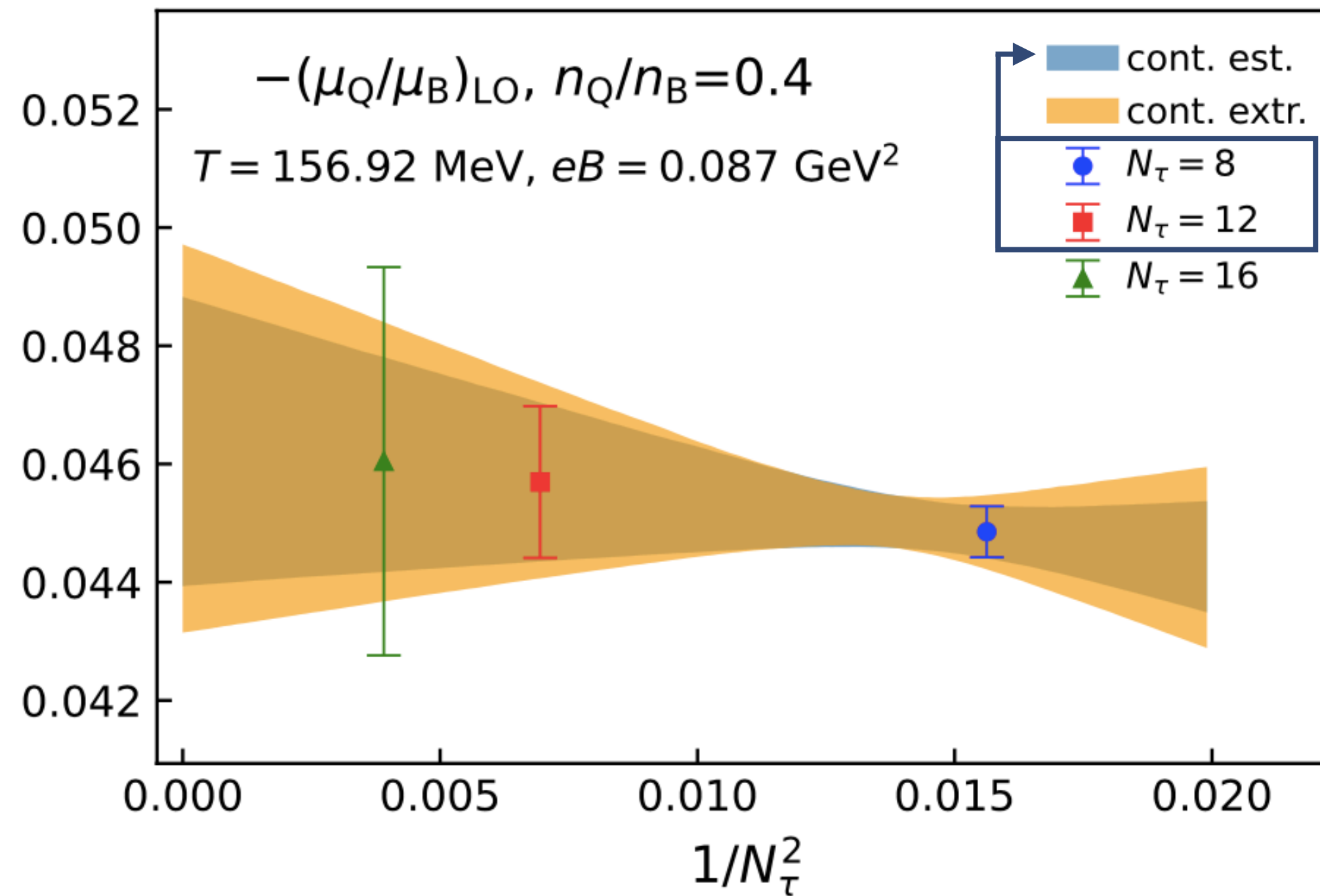
μ_Q/μ_B IN PRESENCE OF eB



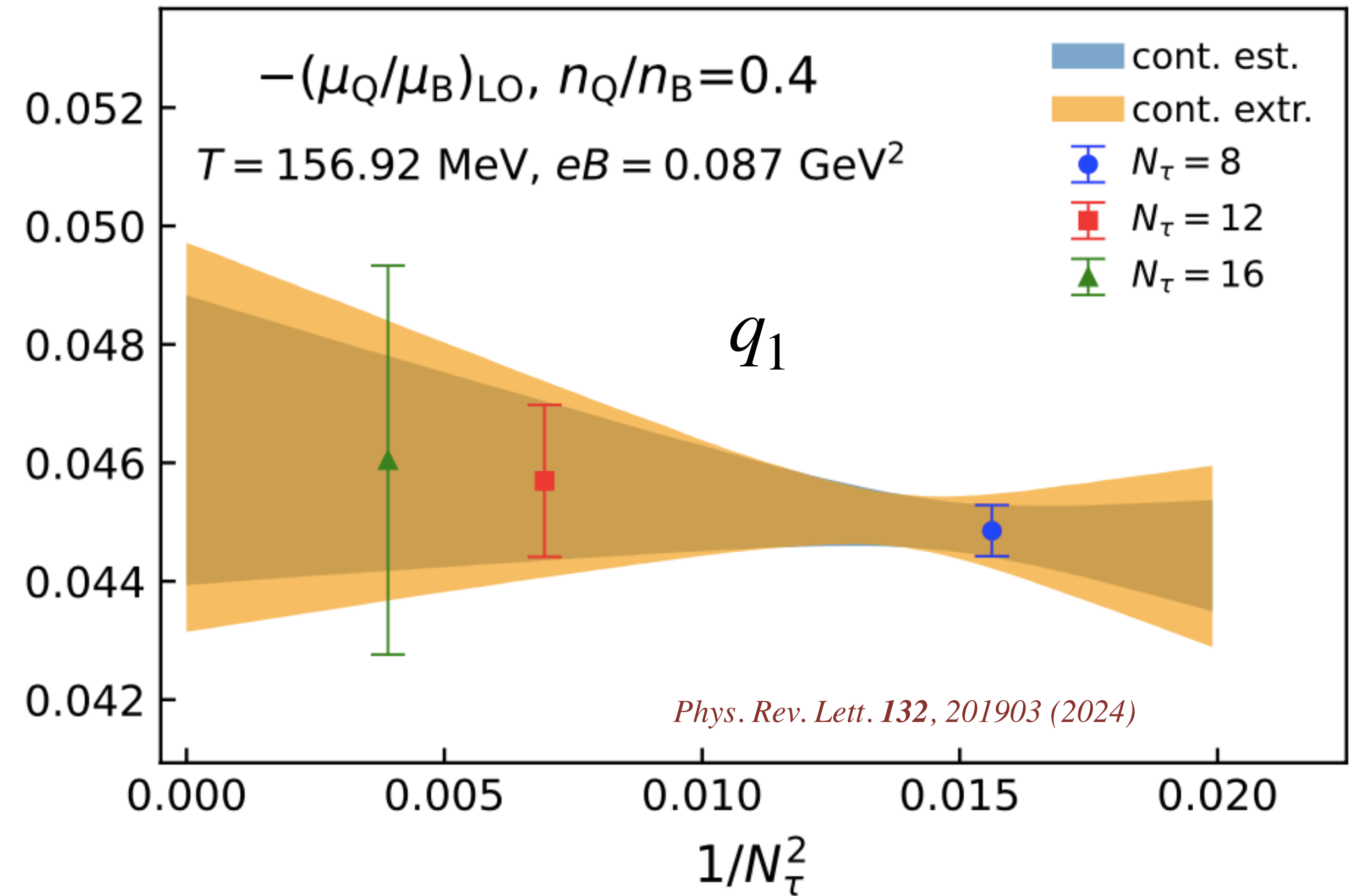
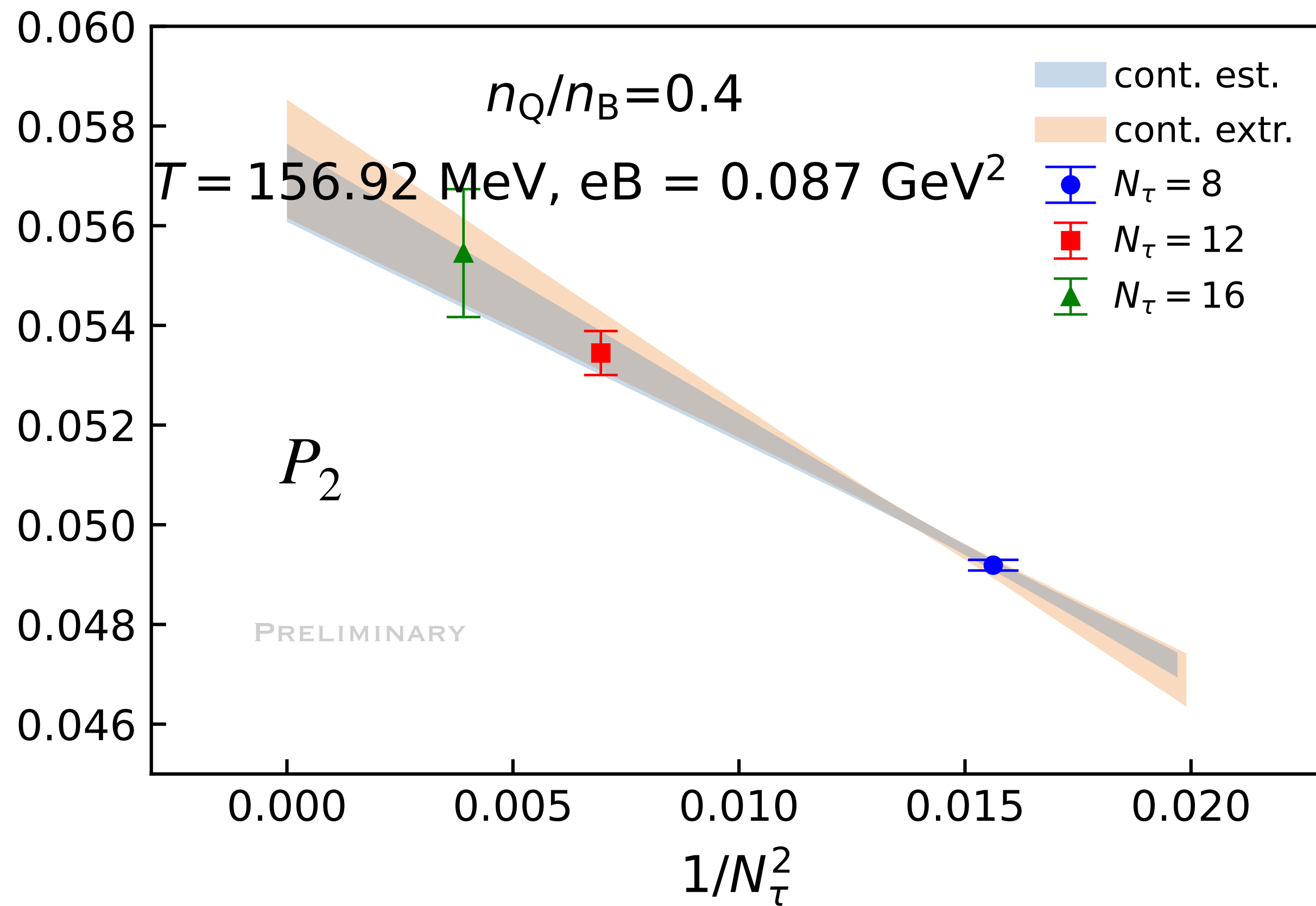
$$q_1 = \frac{r (\chi_2^B \chi_2^S - \chi_{11}^{BS} \chi_{11}^{BS}) - (\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}{(\chi_2^Q \chi_2^S - \chi_{11}^{QS} \chi_{11}^{QS}) - r (\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}$$

B. Continuum estimates

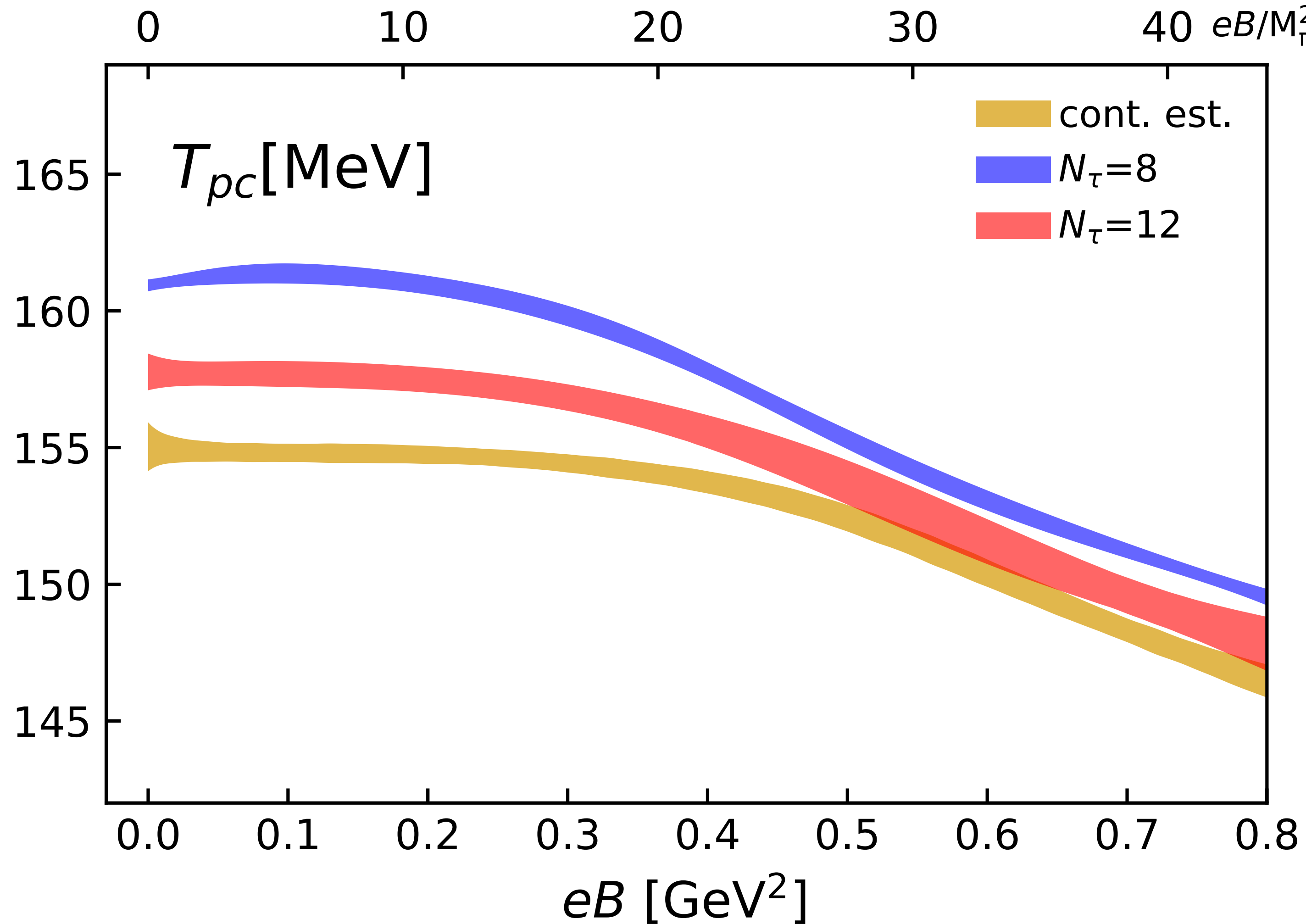
$T \in [145 - 166)$ MeV
 $eB \in [0.0 - 0.8)$ GeV²



CONTINUUM ESTIMATES VS EXTRAPOLATIONS



TRANSITION LINE AND CHIRAL SUSCEPTIBILITY



Peak location of $\chi_M \rightarrow T_{pc}(eB)$

$$M = \frac{1}{f_K^4} \left[m_s \left(\langle \bar{\psi} \psi \rangle_u + \langle \bar{\psi} \psi \rangle_d \right) - (m_u + m_d) \langle \bar{\psi} \psi \rangle_s \right]$$

$$\begin{aligned} \chi_M &= m_s \left(\partial_{m_u} + \partial_{m_d} \right) M \Big|_{m_u=m_d=m_l} \\ &= \frac{1}{f_K^4} \left[m_s \left(m_s \chi_l - 2 \langle \bar{\psi} \psi \rangle_s - 4 m_l \chi_{su} \right) \right], \end{aligned}$$

NEXT-TO-LEADING ORDER

Ongoing work: insights on next-to-leading order contributions.

n^B dominant to Δp (factor ~ 2)

Interestingly as eB grows contributions reduce drastically.

