

第六届粒子物理前沿研讨会

吉林大学

2025年7月15-21日

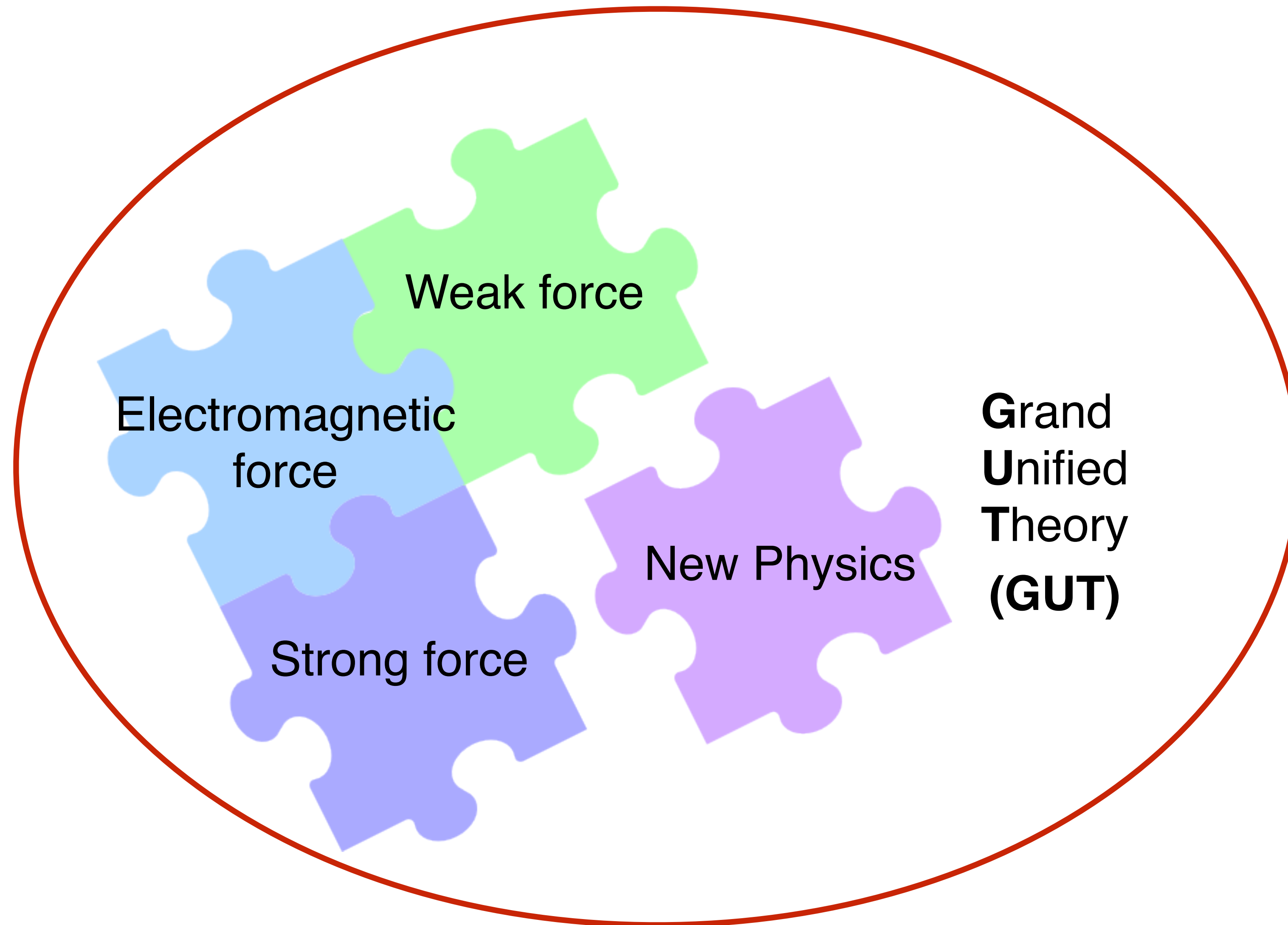
Asymptotic GUT in extra dimension

周也铃 杭高院 2025-07-16



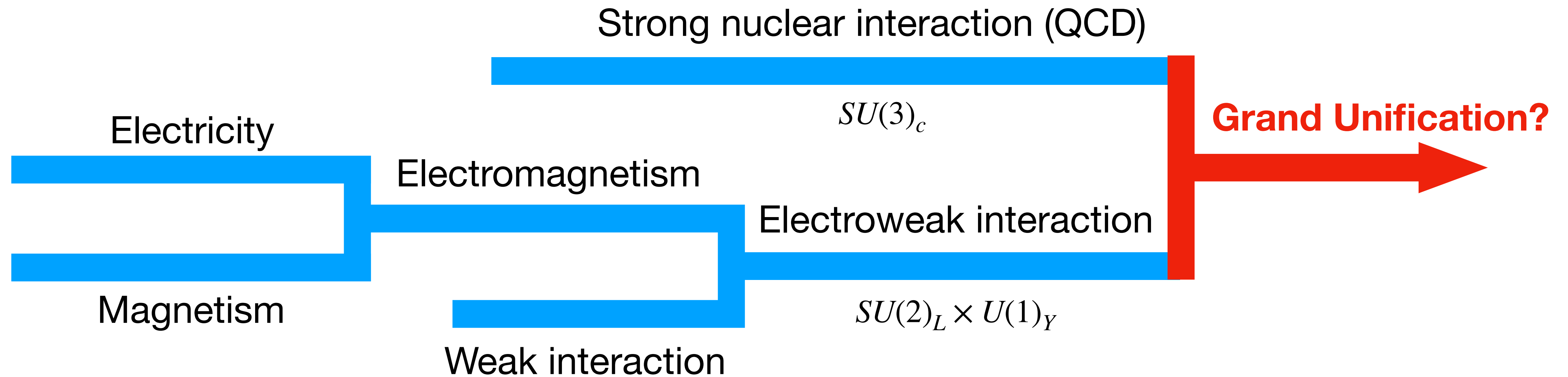
國科大杭州高等研究院
Hangzhou Institute for Advanced Study, UCAS

Introduction



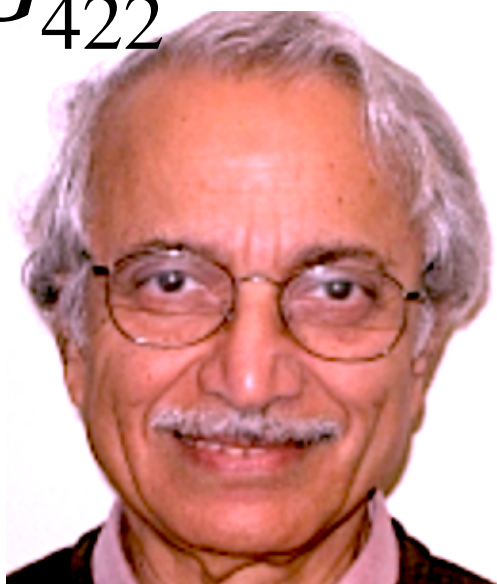
Gravity... not included

Road to unifications



GUT models

Pati-Salam (1973, 1974) $SU(4)_C \times SU(2)_L \times SU(2)_R := G_{422}$



J. Pati



A. Salam

PHYSICAL REVIEW D VOLUME 8, NUMBER 4 15 AUGUST 1973

Unified Lepton-Hadron Symmetry and a Gauge Theory of the Basic Interactions*

Jogesh C. Pati†

Department of Physics and Astronomy, University of Maryland, College Park, Maryland

Abdus Salam

*International Centre for Theoretical Physics, Miramare, Trieste, Italy
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(Received 5 February 1973)

An attempt is made to unify the fundamental hadrons and leptons into a common irreducible representation F of the same symmetry group G and to generate a gauge theory of strong, electromagnetic, and weak interactions. Based on certain constraints from the hadronic side, it is proposed that the group G is $SU(4') \times SU(4'')$, which contains a Han-Nambu-type $SU(3') \times SU(3'')$ group for the hadronic symmetry, and that the representation F is $(4, 4^*)$. There exist four possible choices for the lepton number L and accordingly four possible assignments of the hadrons and leptons within the $(4, 4^*)$. Two of these require

PHYSICAL REVIEW D VOLUME 10, NUMBER 1 1 JULY 1974

Lepton number as the fourth "color"

Jogesh C. Pati*

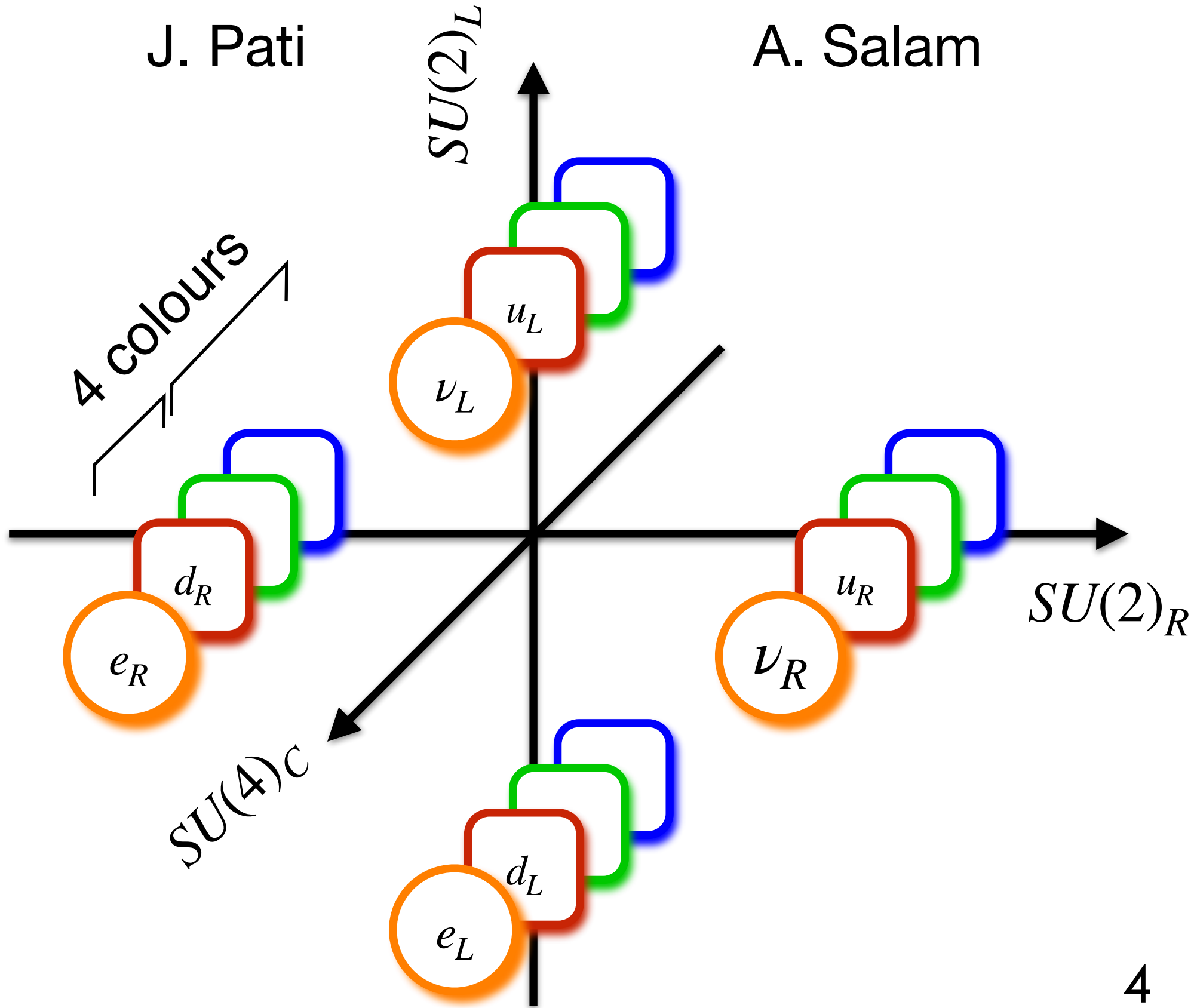
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

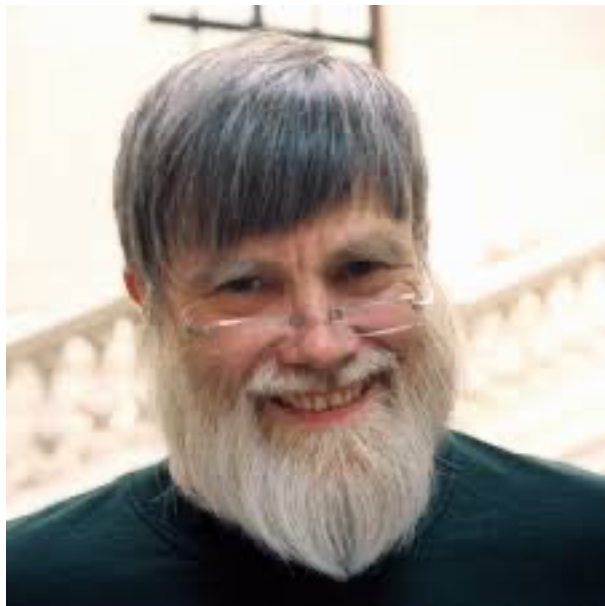
Abdus Salam

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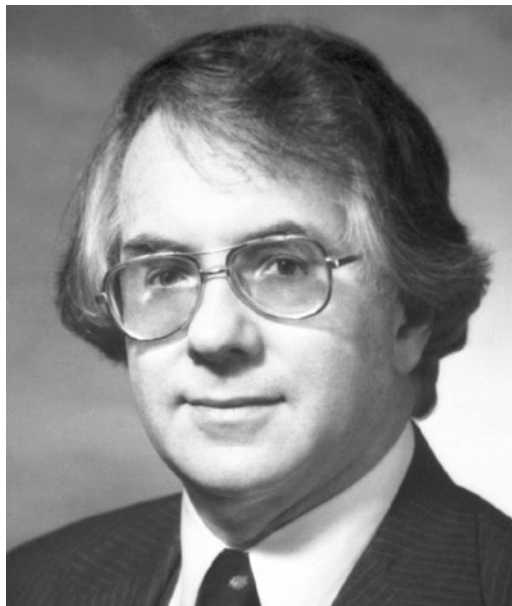
(Received 25 February 1974)

Universal strong, weak, and electromagnetic interactions of leptons and hadrons are generated by gauging a non-Abelian renormalizable anomaly-free subgroup of the fundamental symmetry structure $SU(4)_L \times SU(4)_R \times SU(4')$, which unites three quartets of "colored" baryonic quarks and the quartet of known leptons into 16-folds of chiral fermionic multiplets, with lepton number treated as the fourth "color" quantum number. Experimental consequences of this scheme are discussed. These include (1) the emergence and effects of exotic gauge mesons carrying both baryonic as well as leptonic quantum numbers, particularly in semileptonic processes, (2) the manifestation of anomalous strong interactions among leptonic and semileptonic processes at high energies, (3) the independent possibility of baryon-lepton number violation in quark and proton decays, and (4) the occurrence of $(V+A)$ weak-current effects.

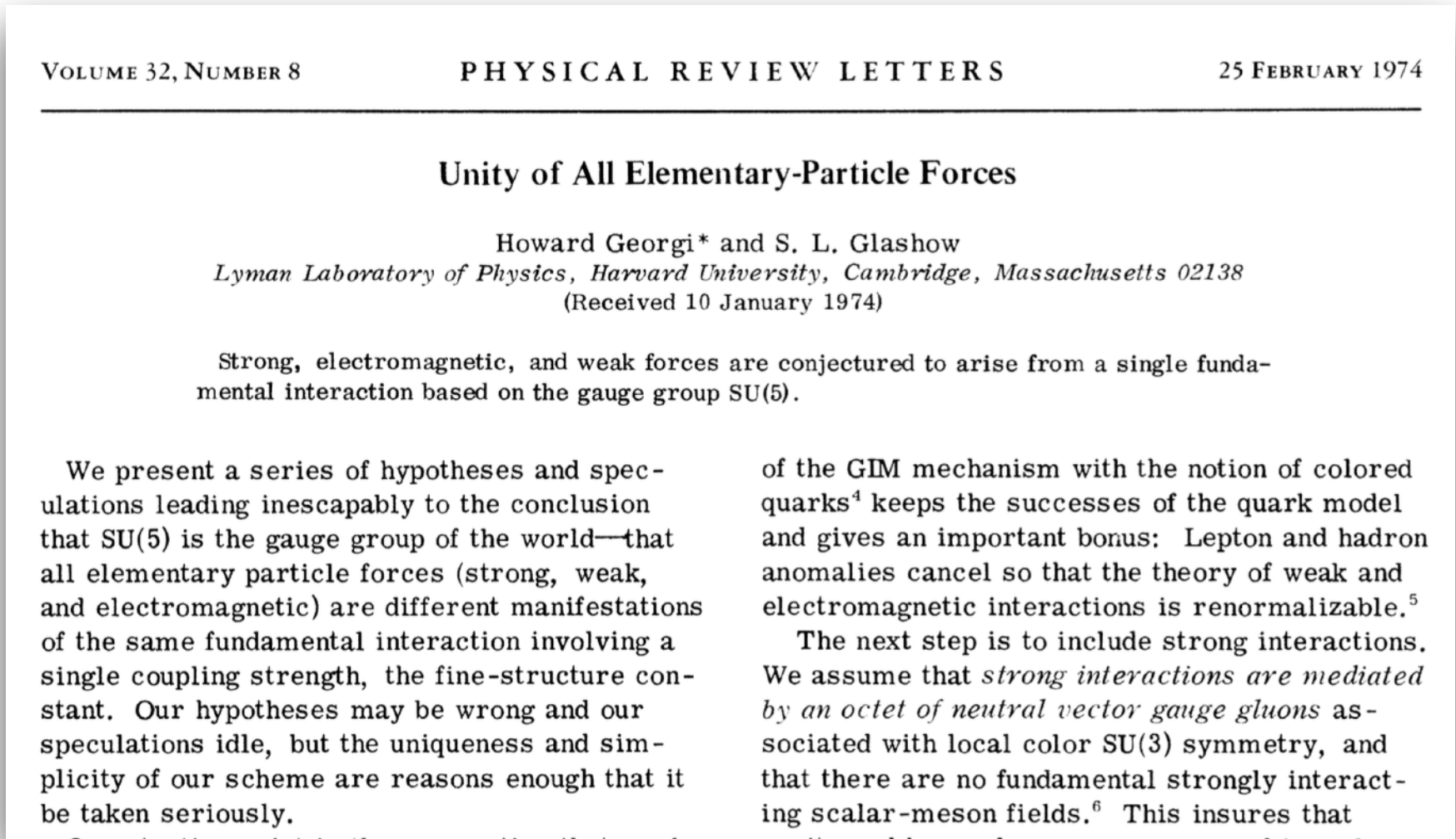




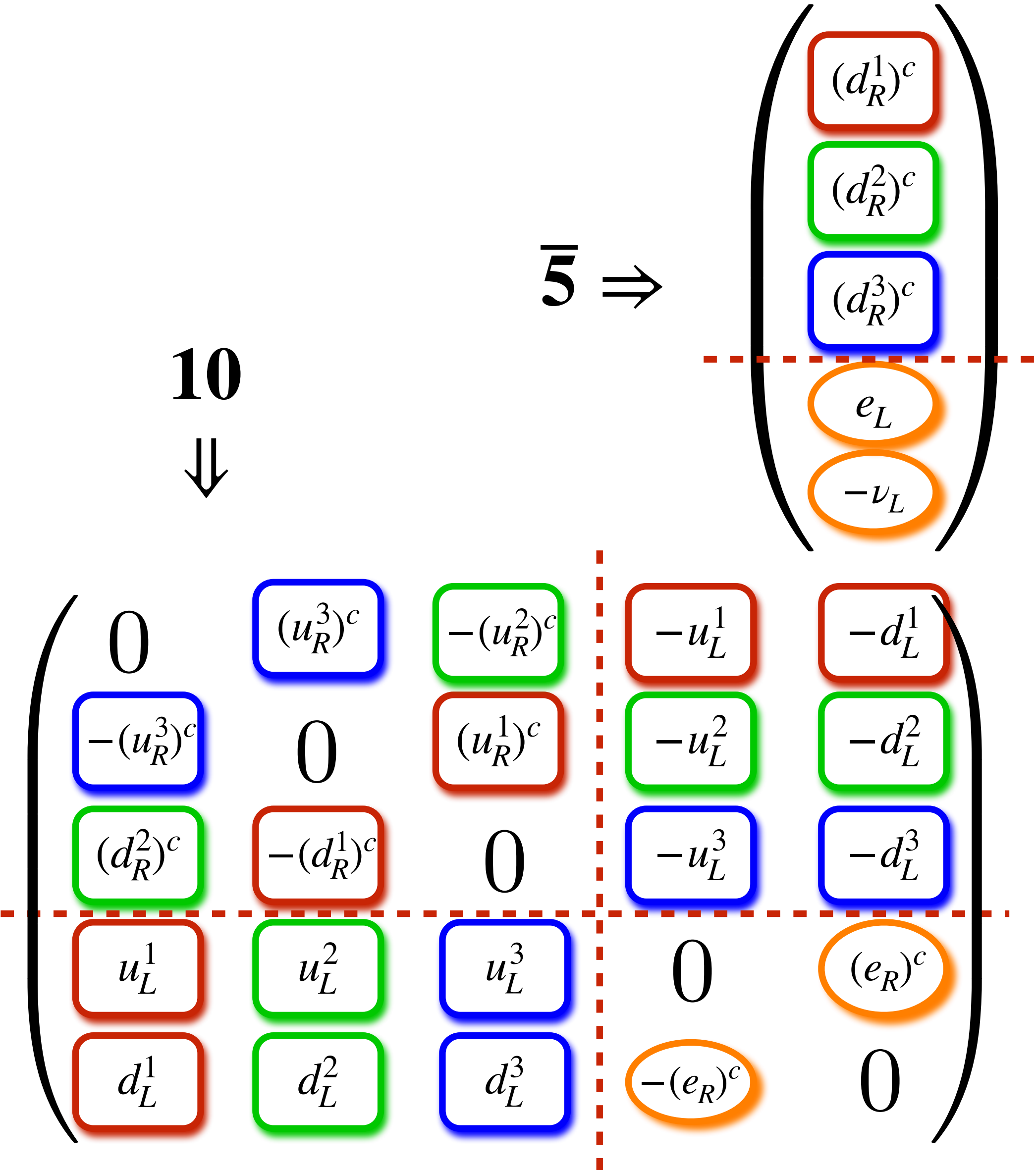
H. Georgi



S. Glashow



And Higgses 5, 45, 24.



GUT models

- $SO(10)$ GUTs Fritzsch, Minkowski (1975)

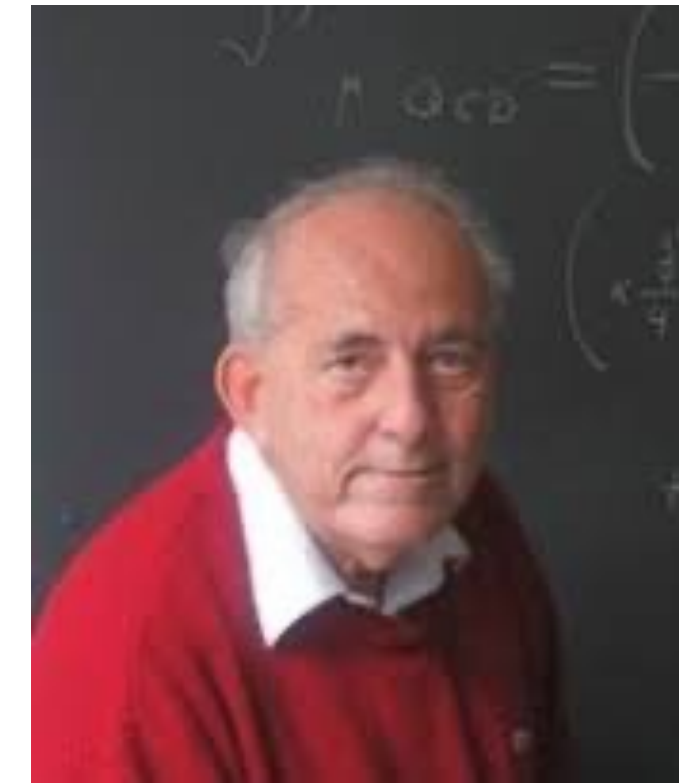
$$\mathbf{16} = \bar{\mathbf{5}} + \mathbf{10} + \mathbf{1} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}, \mathbf{1}, \mathbf{2})$$

$$SO(10) \quad SU(5) \quad SU(4)_c \times SU(2)_L \times SU(2)_R$$

Contains $SU(5)$ and Pati-Salam, and more ...



H. Fritzsch

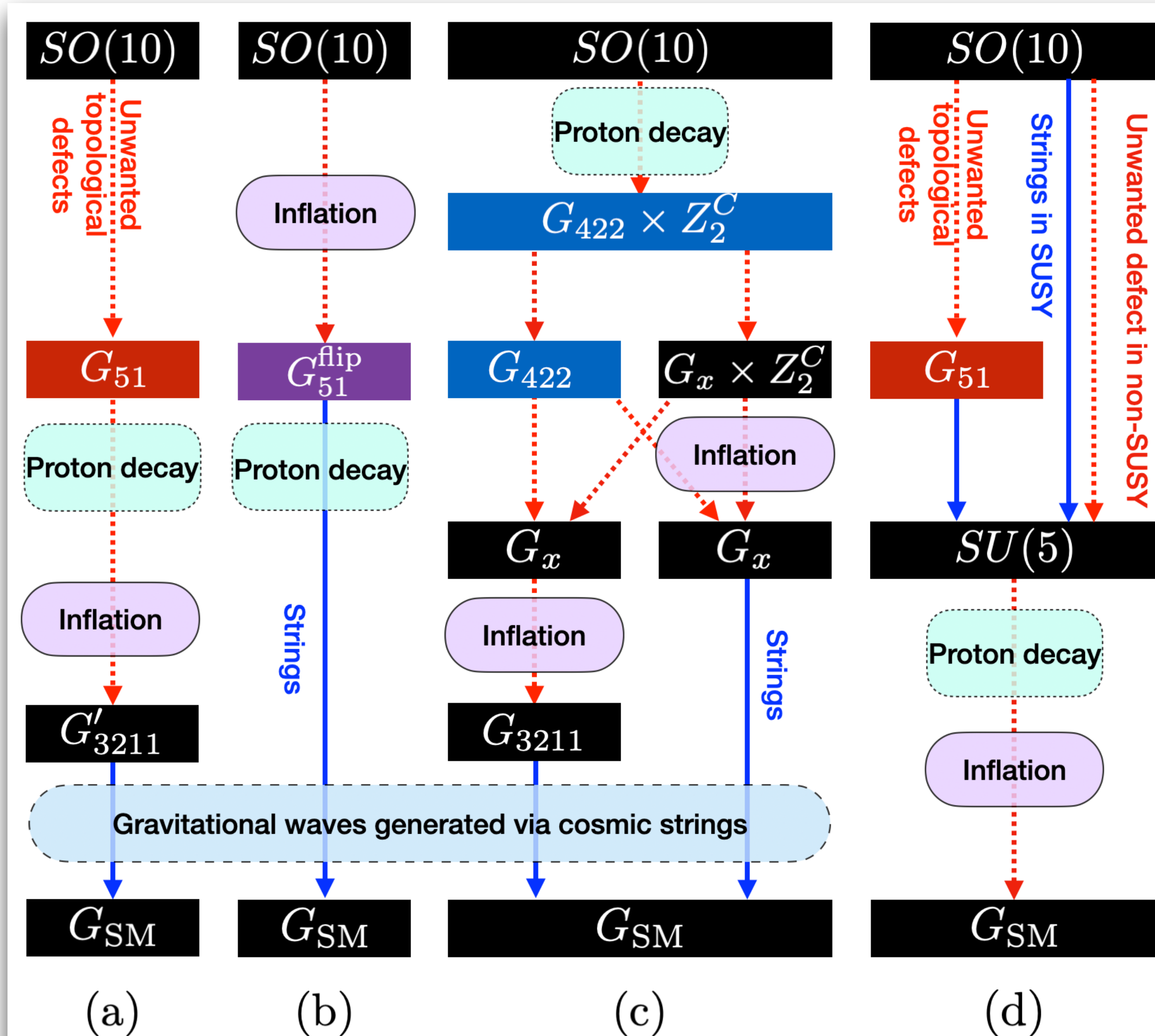


P. Minkowski

- Not minimal but realistic $SU(5)$ e.g., with extra $\mathbf{15}_H, \mathbf{24}_F$
- $SU(5) \times U(1)_{B-L} := G_{51}$ $\bar{\mathbf{5}} + \mathbf{10} + \mathbf{1}, \nu_R \sim \mathbf{1}$
- Flipped $SU(5) \times U(1)_X := G_{51}^{\text{flip}}$ $u \leftrightarrow d, \nu \leftrightarrow e$

Rujula, Georgi, Glashow (1980); Barr,(1982); Derendinger, Kim,
Nanopoulos (1984); Antoniadis, Ellis, Hagelin, Nanopoulos (1989)

GUT phenos



Unwanted topological defects:
monopoles and domain walls

In any breaking chains, inflation has to be introduced to inflate unwanted defects

$$G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R$$

$$G_{51} = SU(5) \times U(1)_X$$

$$G_{51}^{flip} = SU(5)_{flip} \times U(1)_{flip}$$

$$Z_2^C: \psi_L \leftrightarrow \psi_R^c$$

$$G_x = G_{421} \text{ or } G_{3221}$$

$$G_{3221} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$G_{421} = SU(4)_C \times SU(2)_L \times U(1)_Y$$

$$G_{3211} = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$$

$$G'_{3211} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$$

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

King, Pascoli, Turner, YLZ, 2005.13549

Basics of GUTs

- Unification of symmetries

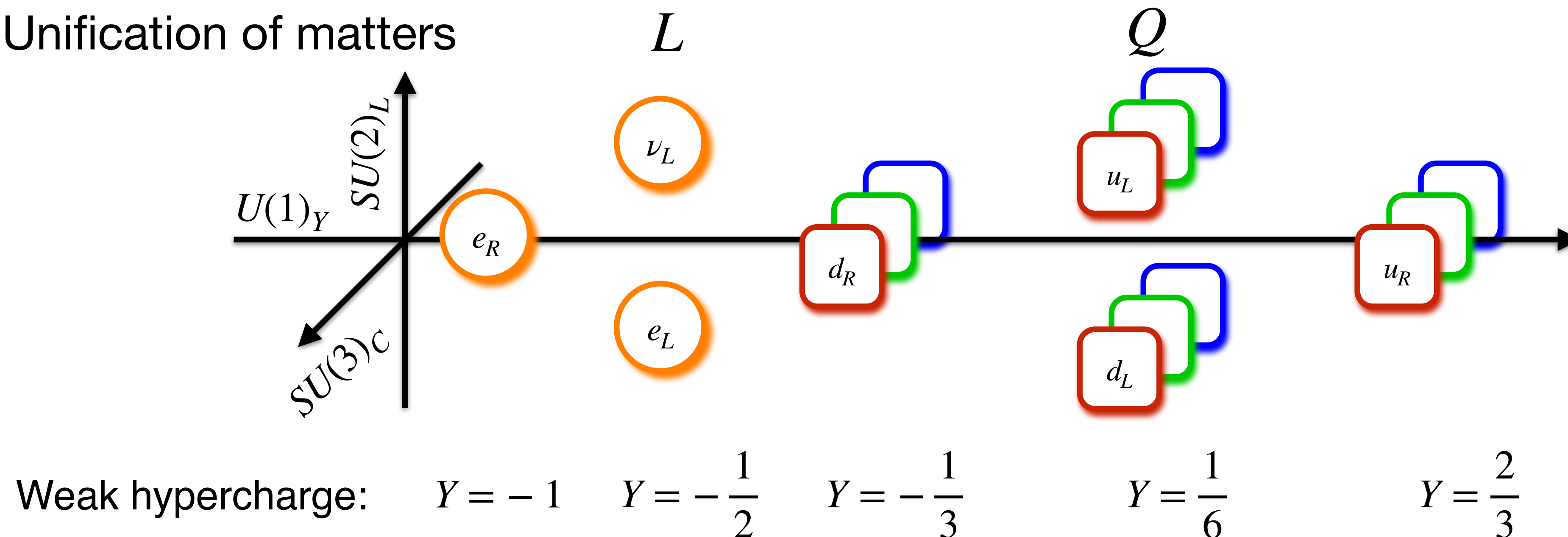
$$G_{\text{GUT}} \supset G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

- Unification of couplings

$$\begin{array}{c} \downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow \\ g_3 \quad = \quad g_2 \quad = \quad g_1 \end{array} \quad \begin{array}{c} \text{EW} \\ \text{up to a loop} \\ \text{correction factor} \end{array}$$

The scale where three gauge couplings are unified, denoted as M_{GUT} in this talk

- Unification of matters



RG running of gauge couplings

Given $\alpha_i = \frac{g_i^2}{4\pi}$ for gauge coupling g_i of group G_i

$G_i = SU(3)_c, SU(2)_L, U(1)_Y, \dots$

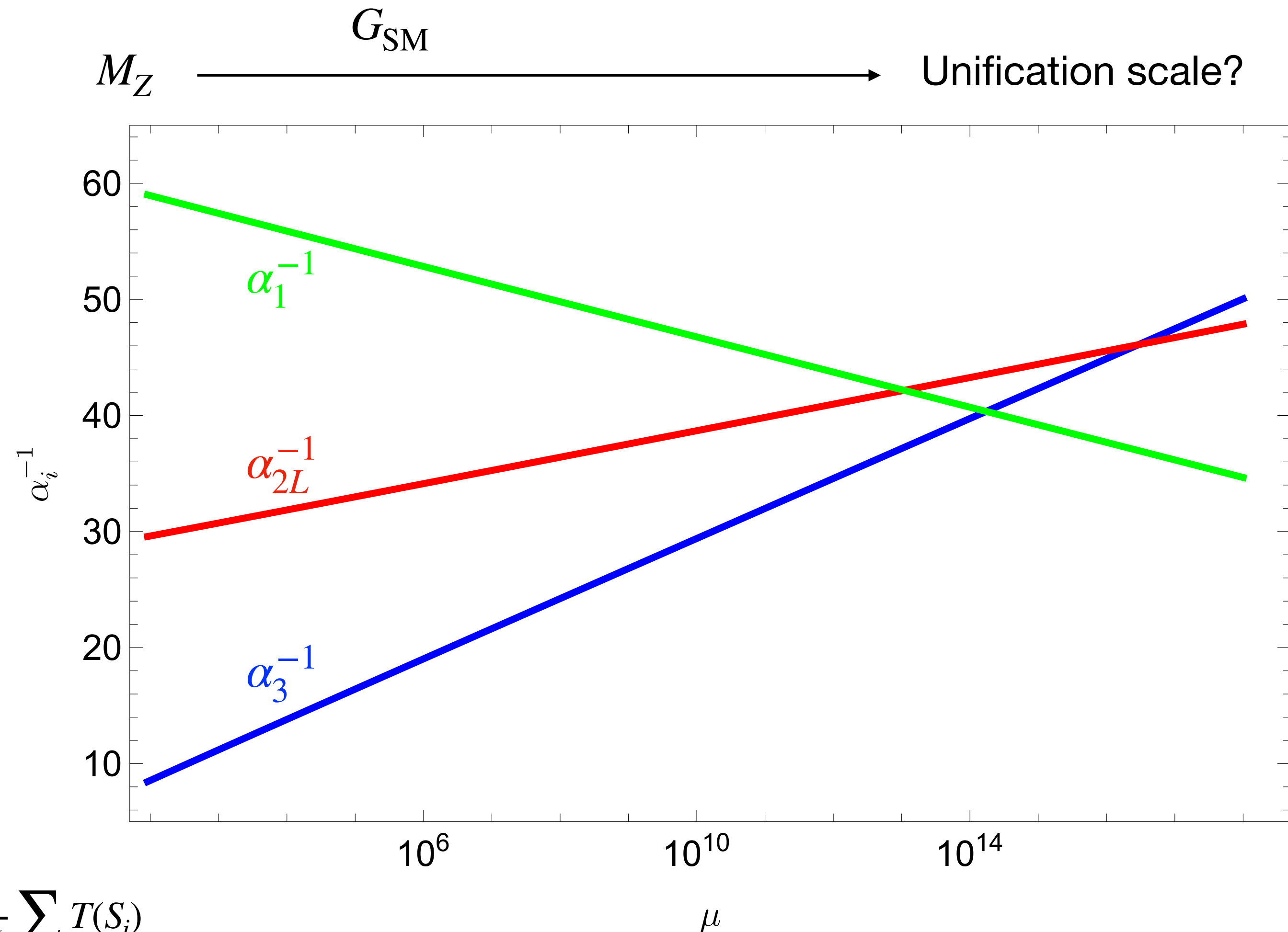
$$\frac{d\alpha_i}{dt} = \beta_i$$

$$t = \log \frac{\mu}{\mu_0}$$

β function at leading order

$$\beta_i = b_i \frac{\alpha_i^2}{2\pi} + \dots$$

$$b_i = -\frac{11}{3}C_2(G_i) + \frac{4}{3}\sum_F T(F_i) + \frac{1}{6}\sum_S T(S_i)$$



$$b_3 = -7$$

$$b_{2L} = -\frac{19}{6}$$

$$b_1 = \frac{41}{10}$$

RG running of gauge couplings

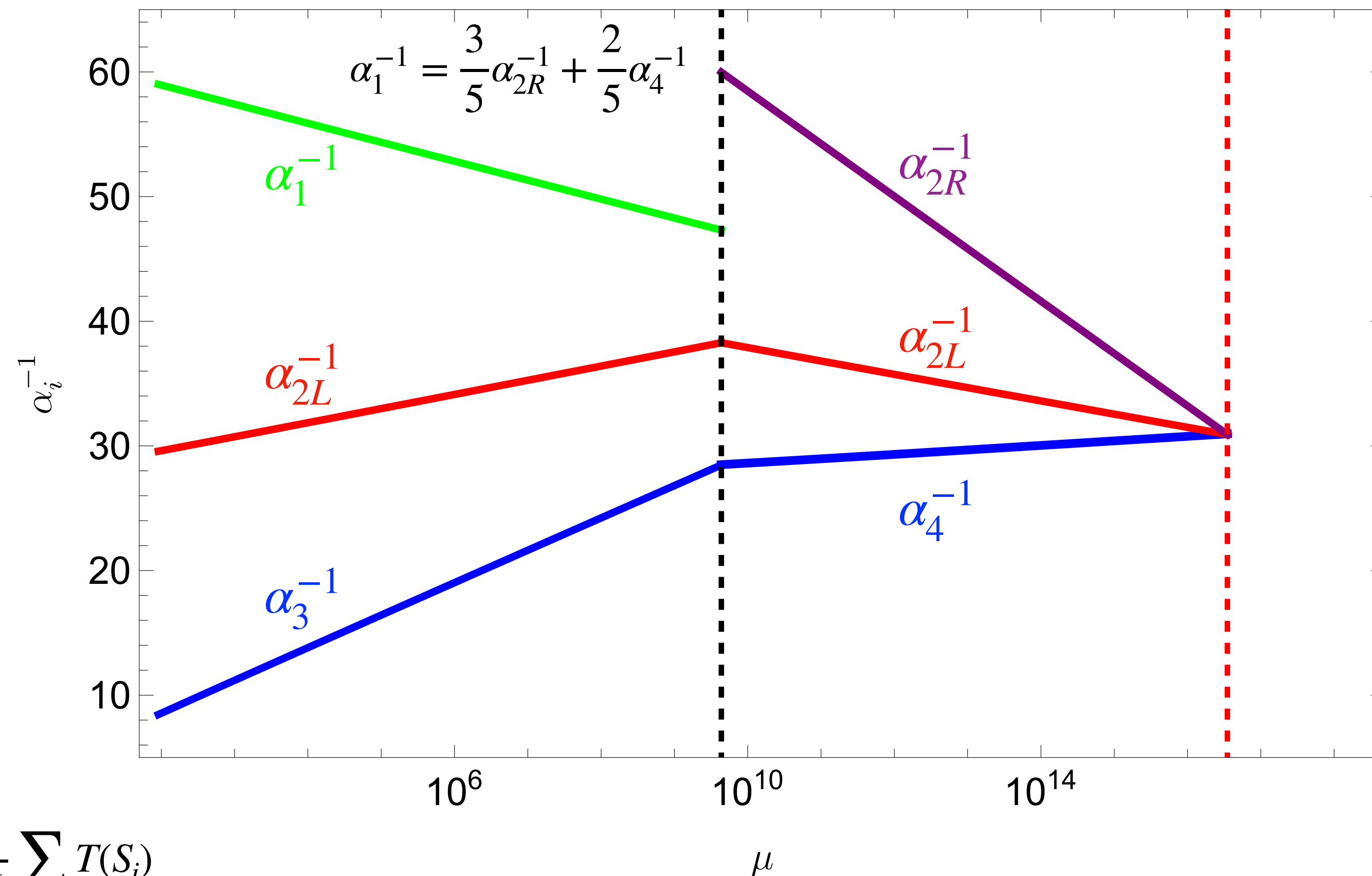
Given $\alpha_i = \frac{g_i^2}{4\pi}$ for gauge coupling g_i of group G_i

$G_i = SU(4)_c, SU(2)_L, SU(2)_R, \dots$

$M_Z \xrightarrow{G_{\text{SM}}} M_{\text{PS}} \xrightarrow{G_{422}} M_X \text{ (GUT scale)}$

$$G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$$

$$G_{\text{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y$$



$$b_4 = -\frac{7}{3}$$

$$b_{2L} = 2$$

$$b_{2R} = \frac{28}{3}$$

$$b_i = -\frac{11}{3}C_2(G_i) + \frac{4}{3}\sum_F T(F_i) + \frac{1}{6}\sum_S T(S_i)$$

UV behaviour of gauge couplings

- RGE of gauge couplings above the GUT scale

take SO(10) as an example, $\alpha_{10} = \frac{g_{10}^2}{4\pi}$

$$2\pi \frac{d\alpha_{10}}{dt} = b_{10} \alpha_{10}^2$$

$$\alpha_{10}(\mu > M_X) = \frac{\alpha_{10}(M_X)}{1 - \alpha_{10}(M_X) \frac{b_{10}}{2\pi} \log\left(\frac{\mu}{M_X}\right)}$$

→ $\alpha_{10} \rightarrow \infty$ for $b_{10} > 0$

→ $\alpha_{10} \rightarrow 0$ for $b_{10} < 0$

b_{10} : depending on fermion and Higgs particle contents of the model

Higgs contents	b_{10}
$(10_c, \overline{126}, 45_c)$	$-\frac{32}{3}$
$(10_r, 120_r, \overline{126}, 45_r)$	$-\frac{15}{2}$
$(10_c, 120_c, \overline{126}, 45_c)$	$-\frac{4}{3}$
$(10_c, 120_c, \overline{126}, 45_c, 54_c)$	$\frac{8}{3}$
$(10_c, 120_c, \overline{126}, 210_c)$	$\frac{44}{3}$

Landau pole at $\mu = M_X \exp\left(\frac{2\pi}{\alpha_{10} b_{10}}\right)$

asymptotically approach to $\frac{2\pi}{-b_{10} \log(\mu/M_X)}$

UV behaviour of gauge couplings: another possibility?

- A theory is said to be asymptotically safe if the 'essential' coupling parameters approach a fixed point as the momentum scale of their renormalization point goes to infinity.

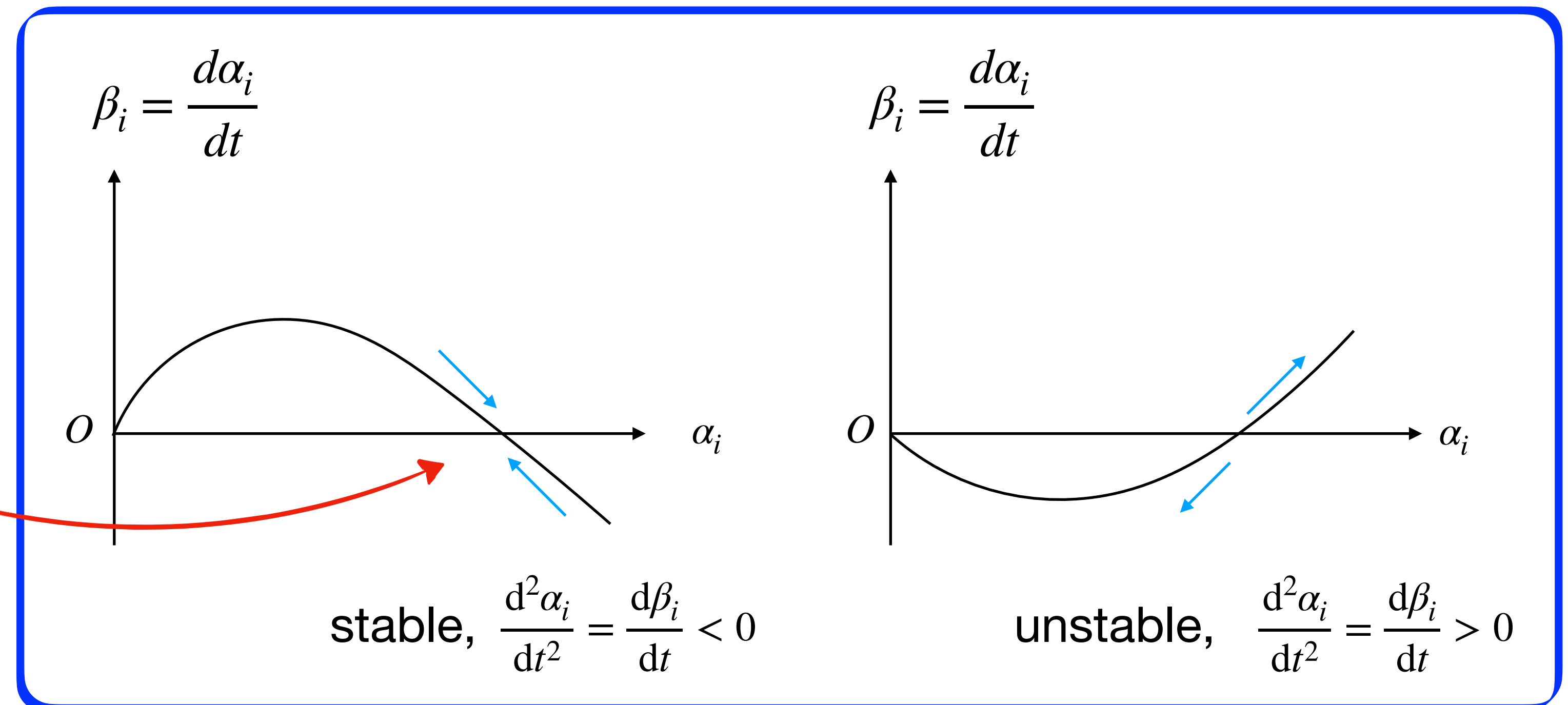
— — S. Weinberg in *Ultraviolet divergences in quantum theories of gravitation*, 1979

- Fixed points in β function

Asymptotic safety in the UV limit

α_{10} might approach to a fixed, non-vanishing and finite value.

⇒ The third phase for UV behaviour of gauge theories



Yukawa couplings in GUTs

Very model-dependent, take SO(10) as examples

- Field arrangements: fermion $\sim \mathbf{16}$, $\mathbf{16} \times \mathbf{16} = \mathbf{10}_S + \mathbf{120}_A + \mathbf{126}_S$
 $\Rightarrow$ scalars $\sim \mathbf{10}, \overline{\mathbf{126}}, \mathbf{120}$
- Yukawa couplings $\mathcal{L}_Y = \Psi_{16} \left[Y_{10} H_{10} + Y_{120} H_{120} + Y_{\overline{126}} H_{\overline{126}} \right] \Psi_{16}$
 $(+ \Psi_{16} \left[Y_{10} H_{10}^* + Y_{120}^{ij} H_{120}^* \right] \Psi_{16})$ can be forbidden by the Peccei-Quinn symm

- Correlation between quark and lepton Yukawa couplings
 $Y_d = r_1(h + f + i h')$ $Y_u = h + r_2 f + i r_3 h'$ $M_{\nu_R} = m_{\nu_R} f$ $h \propto Y_{10}$
 $Y_e = r_1(h - 3f + i c_e h')$ $Y_\nu = h - 3r_2 f + i c_\nu h'$ $M_\nu = -Y_\nu M_{\nu_R}^{-1} Y_\nu^T \nu_u^2 + f \nu_L$ $f \propto Y_{\overline{126}}$
 $h' \propto Y_{120}$

Inputs	a_1	a_2	c_ν	m_0	$(\eta_u, \eta_c, \eta_t; \eta_d, \eta_s, \eta_b)$
	63.57°	84.17°	-1.945	82.82 meV	$(+, +, -; +, -, +)$
Outputs	θ_{13}	θ_{12}	θ_{23}	δ	m_1
	8.53°	32.7°	41.9°	-125°	3.36 meV
$(\chi^2 = 0.33)$	$m_{\beta\beta}$	M_{N_1}		M_{N_2}	M_{N_3}
	5.83 meV	$4.23 \cdot 10^{11}$ GeV		$5.32 \cdot 10^{11}$ GeV	$1.66 \cdot 10^{13}$ GeV

Fu, King, Marsili, Pascoli,
Turner, YLZ,
2209.00021 (non-SUSY);
2308.05799 (SUSY)

Small hierarchy between RH neutrino masses

Yukawa couplings in GUTs

- $10_C + \overline{126}_C$ Bertolini, Schwetz, Malinsky, hep-ph/0605006
Bertolini, Di Luzio, Malinsky, 0903.4049; 0912.1796

- $10_R + 120_R + \overline{126}_C$ Babu, Bajc, Saad, 1612.04329

Most recently see: Babu, Di Bari, Fong, Saad, 2409.03840

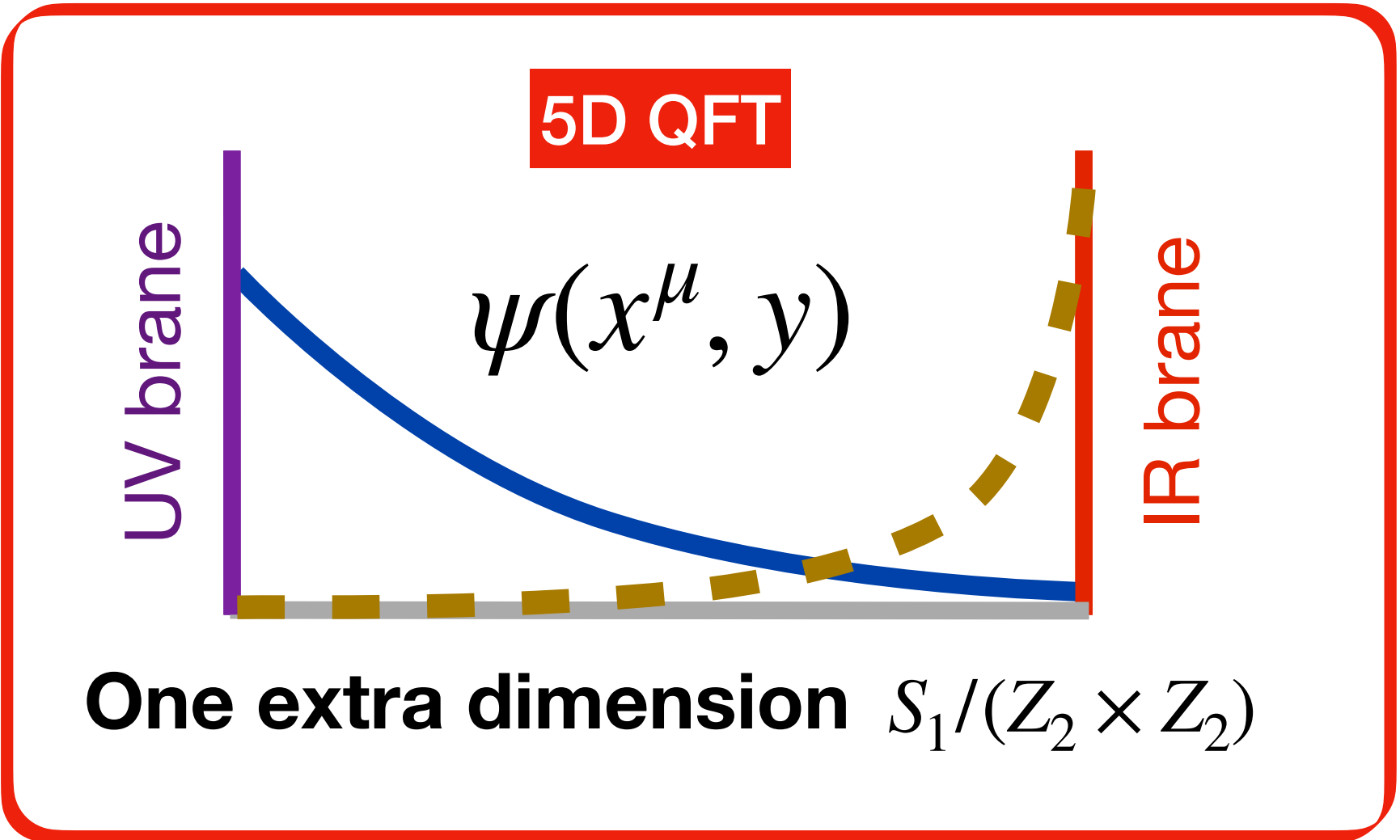
$$\text{NO} : (M_1, M_2, M_3) = (6.57 \times 10^4, 2.08 \times 10^{12}, 8.10 \times 10^{14}) \text{ GeV},$$

Large hierarchy between RH neutrino masses

See also

Joshipura, Patel, 1102.5148; Dueck, Rodejohann, 1306.4468...

5D GUTs



$\simeq$

4D EFT

SM particles KK modes

$$\psi_0(x^\mu) + \sum_{\text{KK}=1}^{\text{dof}} \psi_{\text{KK}}(x^\mu)$$

$$\text{dof} = \begin{cases} 1, & \mu < M_{\text{KK}} \\ [\mu/M_{\text{KK}}], & \mu > M_{\text{KK}} \end{cases}$$

$$S = \int d^4x dy \mathcal{L}_{5\text{D}} = \int d^4x \mathcal{L}_{4\text{D}} + \sum_{\text{KK}=1}^{\text{dof}} \int d^4x \mathcal{L}_{\text{KK}} \quad M_{\text{KK}} = 1/R$$

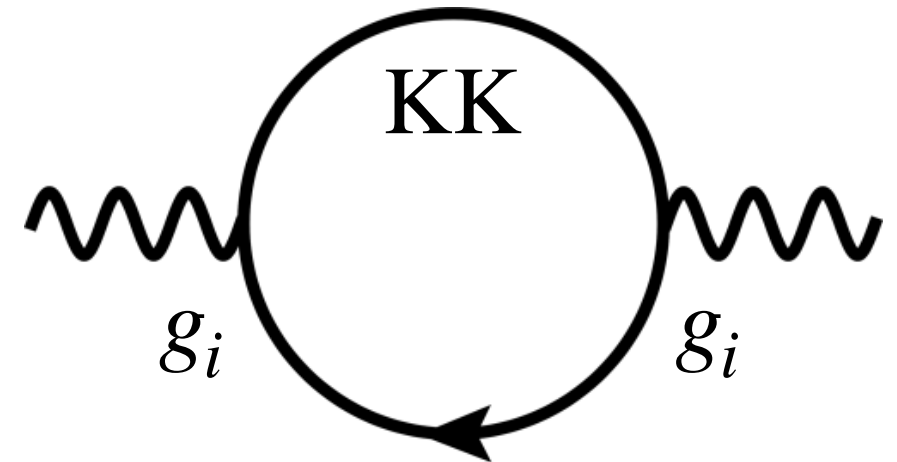
In 5D, it is useful to introduce 't Hooft coupling for the gauge coupling

$$\tilde{\alpha}_i = \text{dof} \times \alpha_i, \text{ for } \mu \gg M_{\text{KK}}$$

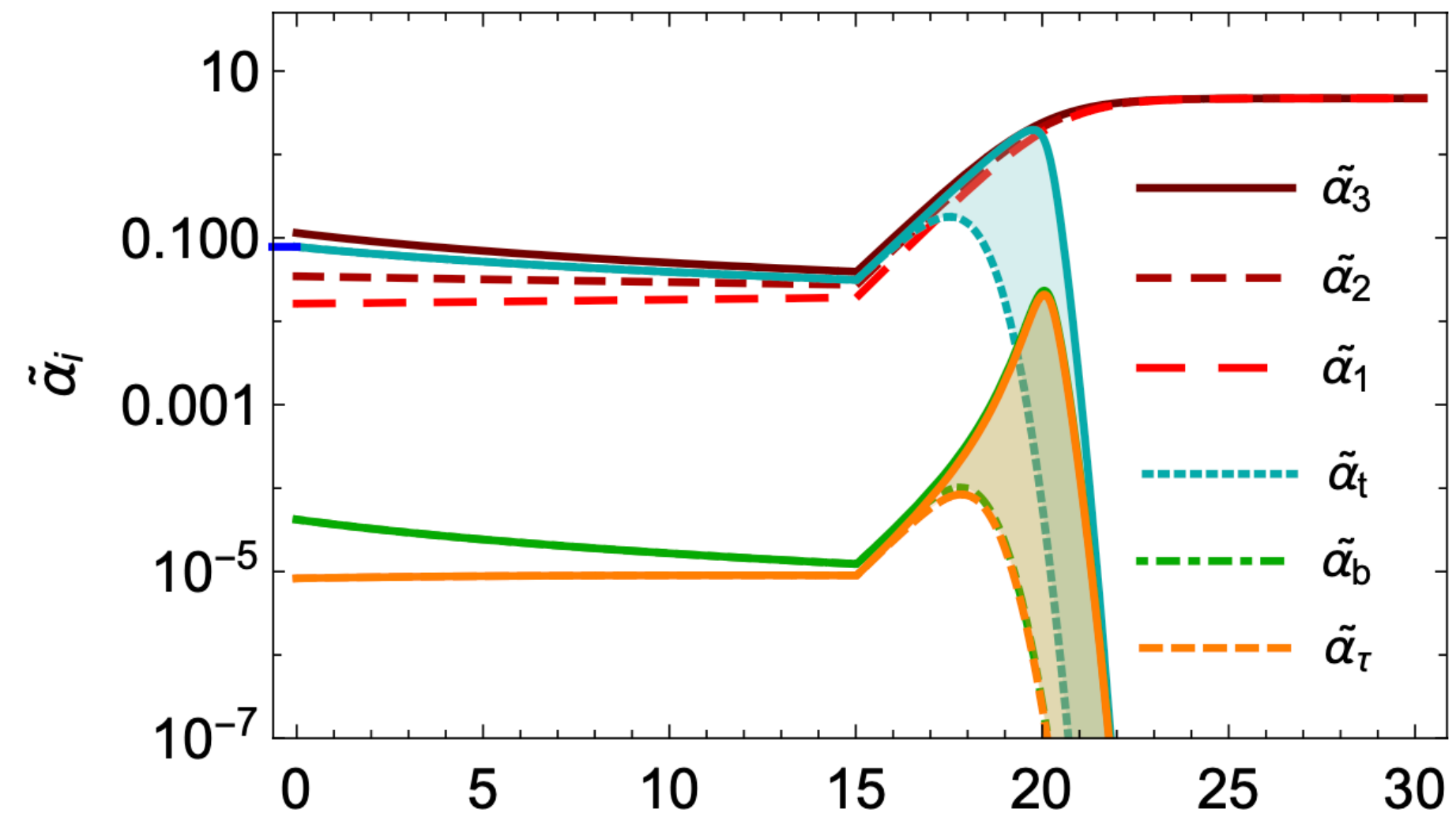
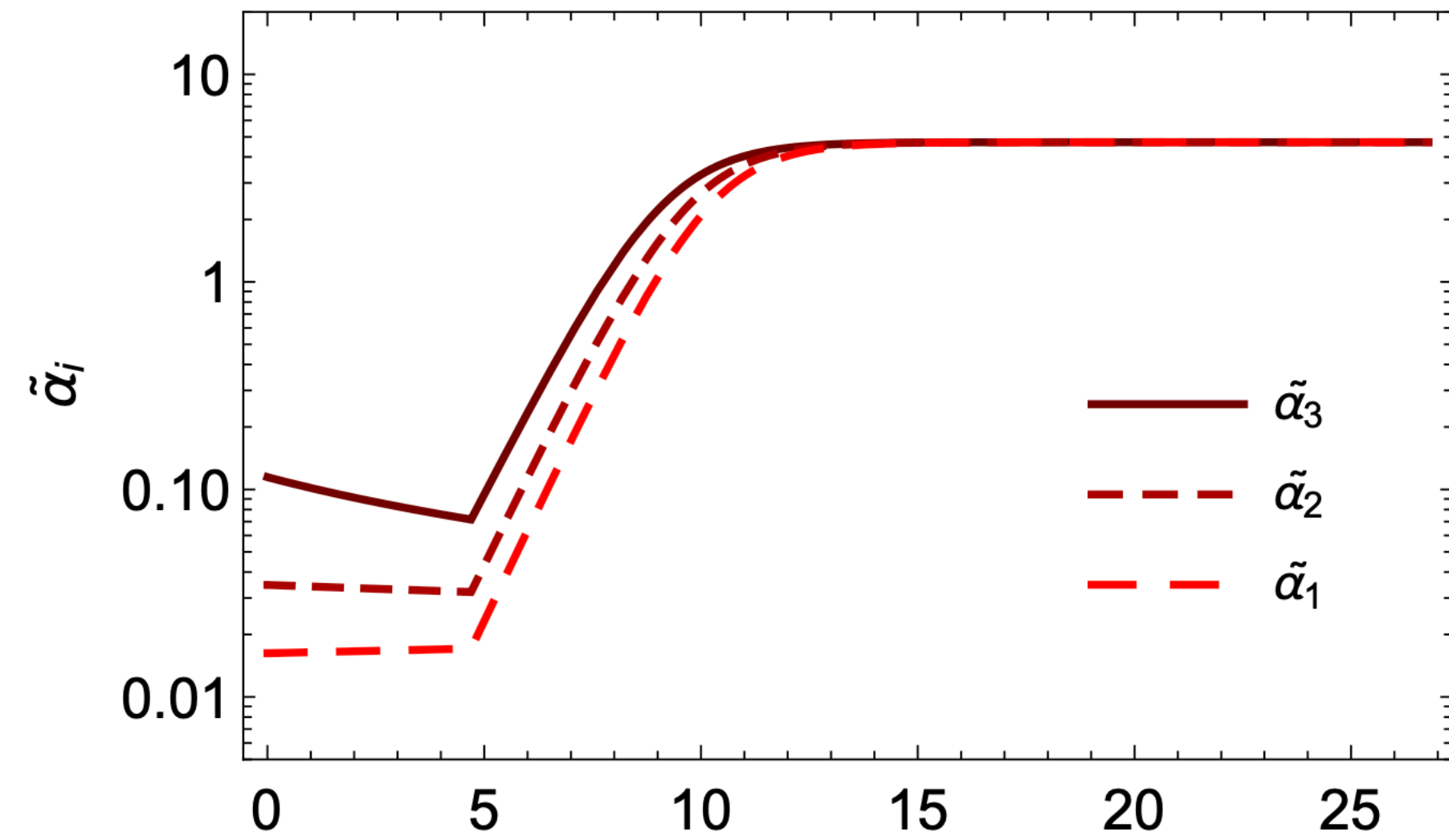
$$\tilde{g} = \sqrt{k} g_b$$

$$\tilde{\alpha}_i(t) = \alpha_i(t) S(t)$$

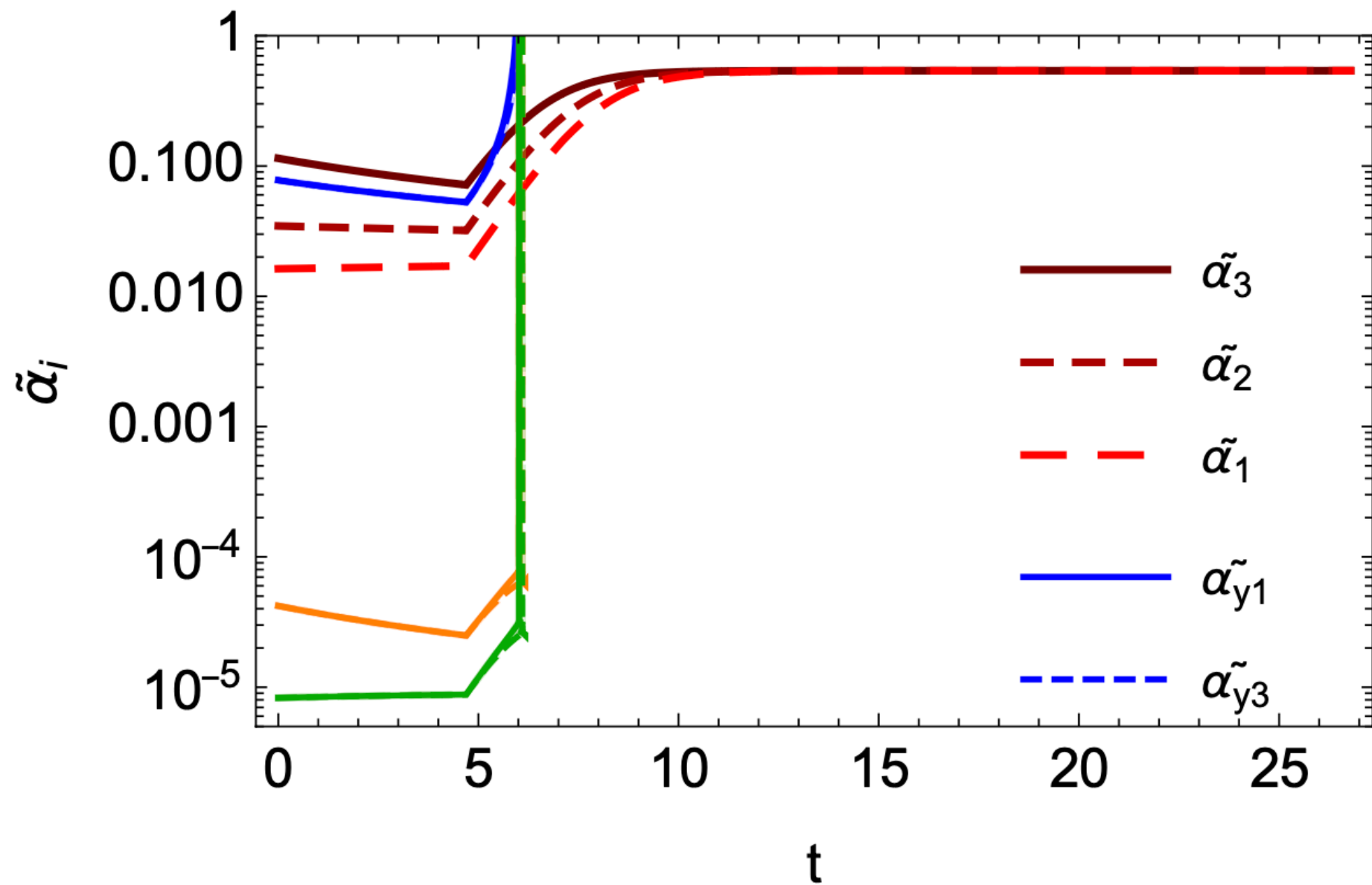
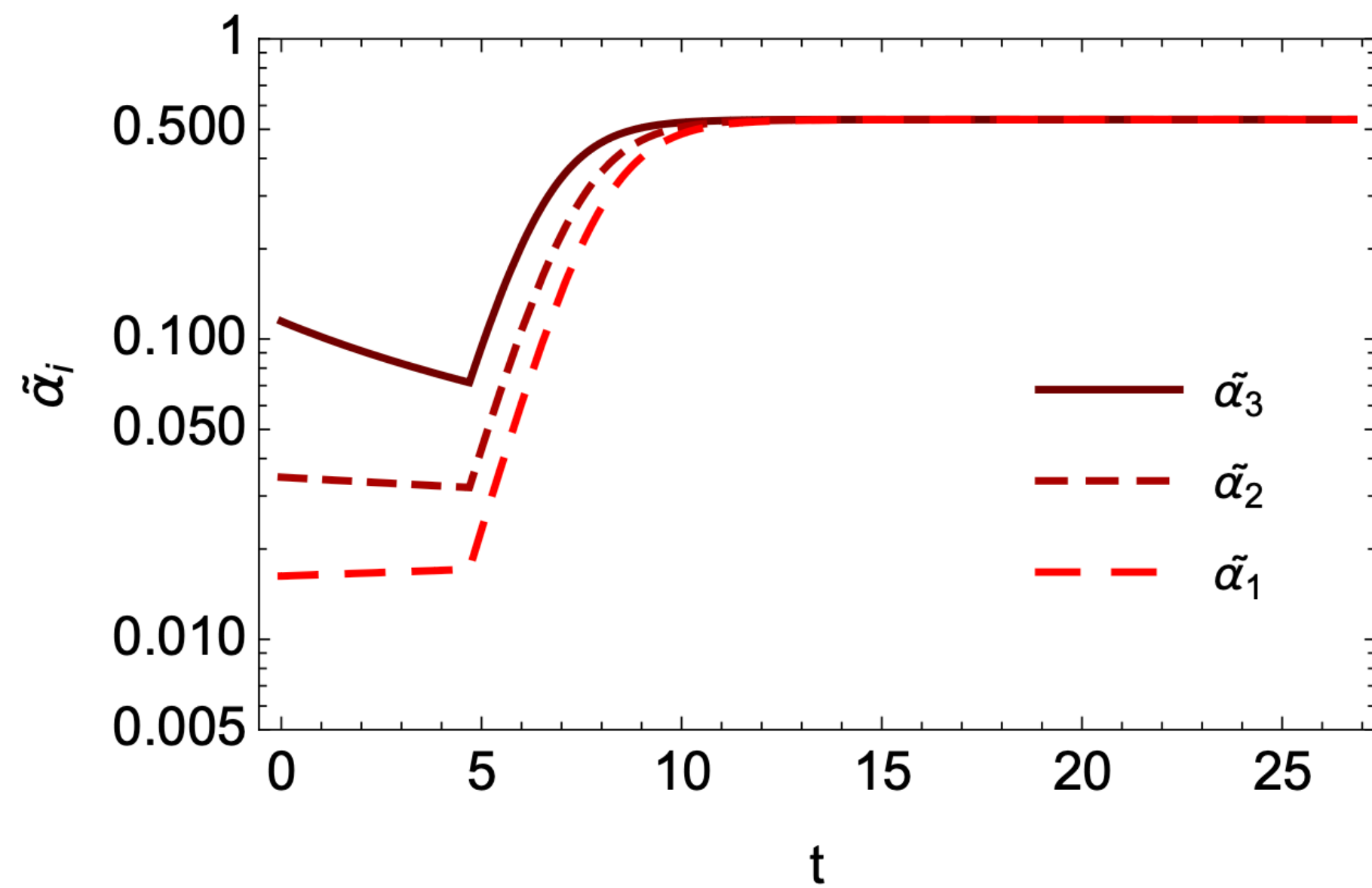
$$S(t) = \begin{cases} 1 & \text{for } \mu < 1/R, \\ \mu R = M_Z R e^t & \text{for } \mu > 1/R. \end{cases}$$



Holger Gies, hep-th/0305208, Tim Morris, hep-ph/0410142



5D SU(5)



5D SO(10)



A successful and realistic SO(10) aGUT



方高祥



王志伟

Asymptotic grand unification in SO(10) with one extra dimension

Fang, Wang, YLZ, arXiv:2505.08068

- Breaking chain

$$5\text{D } SO(10) \xrightarrow[\text{BC}]{M_{\text{KK}} \sim 10^{10} \text{ GeV}} 4\text{D Pati-Salam} \xrightarrow[\mathbf{16}]{M_{\text{PS}} \sim 10^6 \text{ GeV}} \text{SM}$$

- Particle contents

Energy scale	Symmetry	Fermion	Higgs
$\mu > M_{\text{KK}}$	$SO(10)$	$\Psi_{\mathbf{16}} \sim \mathbf{16}$	$H_{\mathbf{10}} \sim \mathbf{10}$, complex
		$\Psi_{\overline{\mathbf{16}}} \sim \overline{\mathbf{16}}$	$H_{\mathbf{120}} \sim \mathbf{120}$, real
		$\nu_{\text{S}} \sim \mathbf{1}$	$H_{\mathbf{16}} \sim \mathbf{16}$
$M_{\text{PS}} < \mu < M_{\text{KK}}$	G_{422}	$\psi_L \sim (4, 2, 1)$ $\psi_R \sim (4, 1, 2)$ $\nu_{\text{S}} \sim (1, 1, 1)$	$h_1, h'_1 \sim (1, 2, 2), h_{15} \sim (15, 2, 2)$ $h_{\overline{4}} \sim (\overline{4}, 1, 2)$
$M_Z \ll \mu < M_{\text{PS}}$	G_{SM}	q_L, d_R, u_R $l_L, \nu_R, e_R, \nu_{\text{S}}$	h_{SM}

No $\overline{\mathbf{126}}$ Higgs

$$\text{Symmetry breaking} \quad 5\text{D } SO(10) \xrightarrow[\text{BC}]{M_{\text{KK}} \sim 10^{10} \text{ GeV}} 4\text{D Pati-Salam} \xrightarrow[\mathbf{16}]{M_{\text{PS}} \sim 10^6 \text{ GeV}} \text{SM}$$

- Boundary Conditions (BCs) to achieve the first breaking: $SO(10) \rightarrow SO(6) \times SO(4)$

$$SO(6) \simeq SU(4)_c, \quad SO(4) \simeq SU(2)_L \times SU(2)_R$$

$$\begin{array}{cc} \text{BC on UV brane} & \left(\begin{array}{cccccccccc} + & + & + & + & + & + & - & - & - & - \\ + & + & + & + & + & + & - & - & - & - \\ + & + & + & + & + & + & - & - & - & - \\ + & + & + & + & + & + & - & - & - & - \\ + & + & + & + & + & + & - & - & - & - \\ + & + & + & + & + & + & - & - & - & - \\ - & - & - & - & - & - & + & + & + & + \\ - & - & - & - & - & - & + & + & + & + \\ - & - & - & - & - & - & + & + & + & + \\ - & - & - & - & - & - & + & + & + & + \end{array} \right) & \text{BC on IR brane} & \left(\begin{array}{cccccccccc} + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \end{array} \right) \end{array}$$

Fermions and Higgses BCs are arranged accordingly, such that those appearing in 4D PS model have zero mode, and

- 16** Higgs to used to achieve the second breaking: $G_{422} \rightarrow G_{\text{SM}}$

$$\begin{array}{ccccccc} \mathbf{16} & = (4,2,1) + (\bar{4},2,1) & = (1,1,0) + (1,1,1) + (1,2, -\frac{1}{2}) & + (\bar{3},1, -\frac{2}{3}) + (\bar{3},1,\frac{1}{3}) + (\bar{3},2,\frac{1}{3}) \\ & G_{422} & & G_{321} \end{array}$$

Yukawa coupling in 5D SO(10)

$$S \supset - \int d^4x dy \quad y_{10} \bar{\Psi}_{16} H_{10} \Psi_{16} + i y_{120} \bar{\Psi}_{16} H_{120} \Psi_{16} + y_{16} \bar{\nu}_S H_{16} \Psi_{16} + \frac{L}{2} \mu_M \bar{\nu}_S \nu_S^c \delta(y-L) + \text{h.c.}$$

$$S \supset - \int d^4x \quad y_1 \bar{\psi}_L h_1 \psi_R + \bar{\psi}_L (y'_1 h'_1 + y_{15} h_{15}) \psi_R + y_4 \bar{\nu}_S h_{\bar{4}} \psi_R + \frac{1}{2} \mu_M \bar{\nu}_S \nu_S^c + \text{h.c.}$$

$$\Psi_{16} = \psi_L + \Psi_R^c$$

$$\Psi_{\bar{16}} = \Psi_L^c + \psi_R$$

$$H_{10} \subset h_1, \quad H_{120} \supset h_{1'} + h_{15}, \quad H_{16} \supset h_{\bar{4}}$$

$$y_t = \sqrt{2} y_{10} c_{10}^u + \sqrt{2} y_{120} (c_{120}^{d'} + \frac{1}{\sqrt{3}} c_{120}^d)$$

$$y_b = \sqrt{2} y_{10} c_{10}^d + \sqrt{2} y_{120} (c_{120}^{d'} + \frac{1}{\sqrt{3}} c_{120}^d)$$

$$y_\tau = \sqrt{2} y_{10} c_{10}^d + \sqrt{2} y_{120} (c_{120}^{d'} - \sqrt{3} c_{120}^d)$$

$$y_\nu = \sqrt{2} y_{10} c_{10}^u + \sqrt{2} y_{120} (c_{120}^{d'} - \sqrt{3} c_{120}^d)$$

Inverse seesaw

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & m_S \\ 0 & m_S & \mu_M \end{pmatrix} \quad \begin{aligned} m_D &= y_\nu \langle H_{\text{SM}} \rangle \\ m_S &= y_{16} \langle H_{16} \rangle \end{aligned}$$

$$m_\nu = \frac{m_D^2}{m_S^2} \mu_M$$

UV behaviour of gauge coupling

- RGEs for gauge couplings in 4D

$$\beta_i = \frac{b_i}{2\pi} \alpha_i^2$$

Standard Model, $(b_3, b_{2L}, b_1) = \left(-7, -\frac{19}{6}, \frac{41}{10}\right)$

Pati-Salam Model, $(b_4, b_{2L}, b_{2R}) = \left(-5, \frac{7}{3}, 3\right)$

- RGEs for gauge couplings in 5D

$$\beta_i = \frac{b_i}{2\pi} \alpha_i^2 + \left(S(t) - 1\right) \frac{b_{10}}{2\pi} \tilde{\alpha}_i^2$$

't Hooft coupling: $\tilde{\alpha}_i(t) = \alpha_i(t)S(t)$

$$\frac{d\tilde{\alpha}_i}{dt} = \tilde{\beta}_i = \tilde{\alpha}_i + \frac{b_{10}}{2\pi} \tilde{\alpha}_i^2$$

β coefficient including KK contributions

$$b_{10} = \left(-\frac{11}{3} + \frac{1}{6}\right) C_2(SO(10)) + \frac{4}{3} \sum_F T(F) + \frac{1}{6} \sum_S T(S)$$

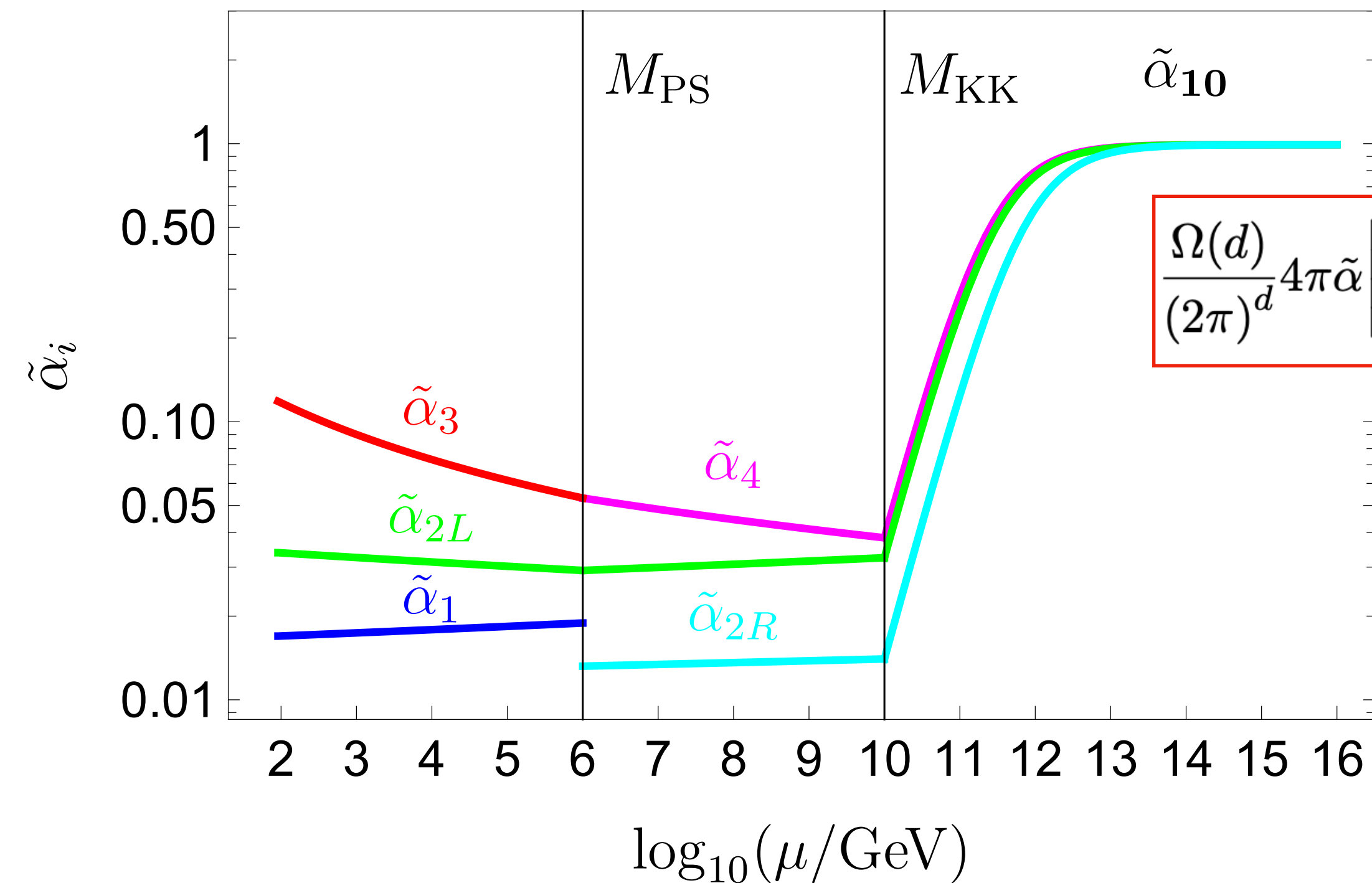
V_5 appears as scalar

$$b_{10} < 0 \quad \Rightarrow \quad \tilde{\alpha}_i = \frac{2\pi}{e^{-t+c_i} - b_{10}} \rightarrow \frac{2\pi}{-b_{10}}$$

UV fixed point

UV behaviour of gauge coupling

Gauge couplings asymptotically safe



Higgs contents	$\tilde{\alpha}_{10}^{\text{UV}}$
$(10_r, 120_r, 16)$	$\frac{4\pi}{13}$
$(10_c, 120_r, 16)$	$\frac{6\pi}{19}$
$(10_r, 120_c, 16)$	$\frac{12\pi}{11}$
$(10_c, 120_c, 16)$	$\frac{6\pi}{5}$
$(10_r, 120_r, 45_r, 16)$	$\frac{12\pi}{31}$
$(10_r, 120_r, 45_r, 16)$	$\frac{12\pi}{31}$
$(10_c, 120_r, 45_r, 16)$	$\frac{2\pi}{5}$
$(10_r, 120_r, 54_r, 16)$	$\frac{4\pi}{9}$
$(10_r, \overline{126})$	12π
$(10_c, \overline{126})$	∞

$$\begin{aligned}
 b_{10} = & -28 + \frac{16}{3}n_{\Psi_{16,c}} + \frac{1}{6}n_{H_{10,r}} + \frac{14}{3}n_{H_{120,r}} \\
 & + \frac{4}{3}n_{H_{45,r}} + 2n_{H_{54,r}} + \frac{35}{3}n_{H_{\overline{126},c}} + \frac{2}{3}n_{H_{16,c}} + \dots
 \end{aligned}$$

$\overline{126}$ Higgs is not recommended

Deriving Yukawa RGEs

- Gamma matrices (32×32) and chiral representation **16** in the gauge space of $SO(10)$

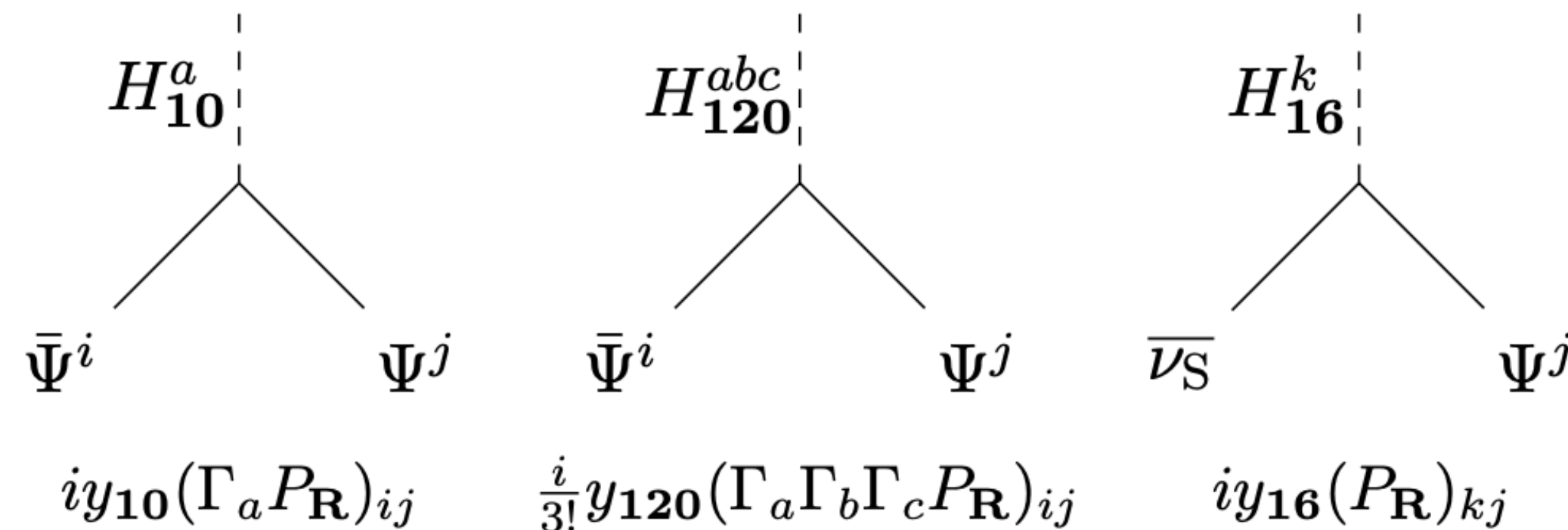
$$\{\Gamma_a, \Gamma_b\} = 2\delta_{ab} \quad \Gamma_\chi = i\Gamma_1\Gamma_2\cdots\Gamma_{10} = \begin{pmatrix} -I_{16} & 0 \\ 0 & I_{16} \end{pmatrix} \quad P_{\mathbf{L},\mathbf{R}} = \frac{1}{2}(I_{32} \mp \Gamma_\chi)$$

$$\Psi_{\mathbf{L}} = P_{\mathbf{L}}\Psi = \begin{pmatrix} \Psi_{\mathbf{16}} \\ 0 \end{pmatrix} \quad \Psi_{\mathbf{R}} = P_{\mathbf{R}}\Psi = \begin{pmatrix} 0 \\ \Psi_{\overline{\mathbf{16}}} \end{pmatrix} \quad 32 = \mathbf{16} + \overline{\mathbf{16}}$$

- Field arrangements: Fermion $\sim \mathbf{16} + \overline{\mathbf{16}}$, Scalars $\sim \mathbf{10} + \mathbf{120} + \mathbf{16}$ ($\overline{\mathbf{126}}$ is not included)

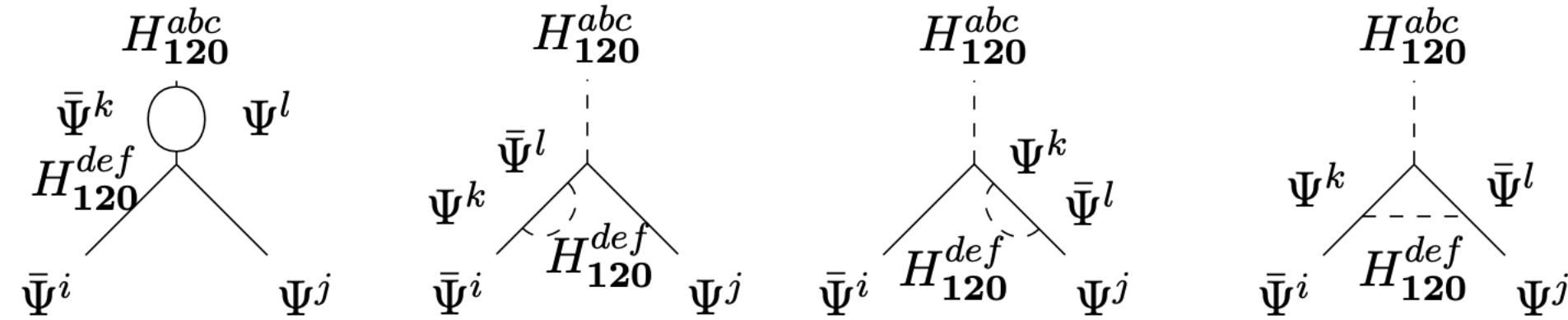
$$-\mathcal{L}_Y = \bar{\Psi}^i \left[y_{10}(\Gamma_a P_{\mathbf{R}})_{ij} H_{10}^a + \frac{1}{3!} y_{120}(\Gamma_a \Gamma_b \Gamma_c P_{\mathbf{R}})_{ij} H_{120}^{abc} \right] \Psi^j + y_{16} \bar{\nu}_S H_{16}^k (P_{\mathbf{R}})_{kj} \Psi^j + \text{h.c.}$$

- Feynman rules



Deriving Yukawa RGEs

- Calculate loop diagrams one by one



- RGE for Yukawa couplings

$$16\pi^2 \frac{dy_r}{dt} = 16\pi^2 \frac{dy_r}{dt} \Big|_{4D} + (S(t) - 1) 16\pi^2 \frac{dy_r}{dt} \Big|_{KK}$$

- 0-mode contribution

Assuming y_{16} small enough, thus its contribution can be ignored.

$$16\pi^2 \frac{dy_{10}}{dt} \Big|_{4D} = \eta_{10} \Big|_{4D} y_{10} - 96 Y_{120} y_{10}^\dagger y_{120} + 10 Y_{10} y_{10}^\dagger y_{10} + 60(y_{10} y_{120}^\dagger y_{120} + y_{120} y_{120}^\dagger y_{10}) \quad \eta_{10} \Big|_{4D} = -\frac{270}{8} g_{10}^2 + 8 \text{Tr}(y_{10} y_{10}^\dagger)$$

$$16\pi^2 \frac{dy_{120}}{dt} \Big|_{4D} = \eta_{120} \Big|_{4D} y_{120} + 104 y_{120} y_{120}^\dagger y_{120} + 5(y_{120} y_{10}^\dagger y_{10} + y_{10} y_{10}^\dagger y_{120}) \quad \eta_{120} \Big|_{4D} = -\frac{270}{8} g_{10}^2 + 8 \text{Tr}(y_{120} y_{120}^\dagger)$$

- KK-mode contribution

$$16\pi^2 \frac{dy_{10}}{dt} \Big|_{KK} = \eta_{10} \Big|_{KK} y_{10} + 10 y_{10} y_{10}^\dagger y_{10} + 60(y_{10} y_{120}^\dagger y_{120} + y_{120} y_{120}^\dagger y_{10}) + 60(y_{10} y_{120}^\dagger y_{120} + y_{120} y_{120}^\dagger y_{10}) \quad \eta_{10} \Big|_{KK} = -\frac{171}{8} g_{10}^2 + 16 \text{Tr}(y_{10} y_{10}^\dagger)$$

$$16\pi^2 \frac{dy_{120}}{dt} \Big|_{KK} = \eta_{120} \Big|_{KK} y_{120} + 104 y_{120} y_{120}^\dagger y_{120} + 5(y_{120} y_{10}^\dagger y_{10} + y_{10} y_{10}^\dagger y_{120}) \quad \eta_{120} \Big|_{KK} = -\frac{219}{8} g_{10}^2 + 16 \text{Tr}(y_{120} y_{120}^\dagger)$$

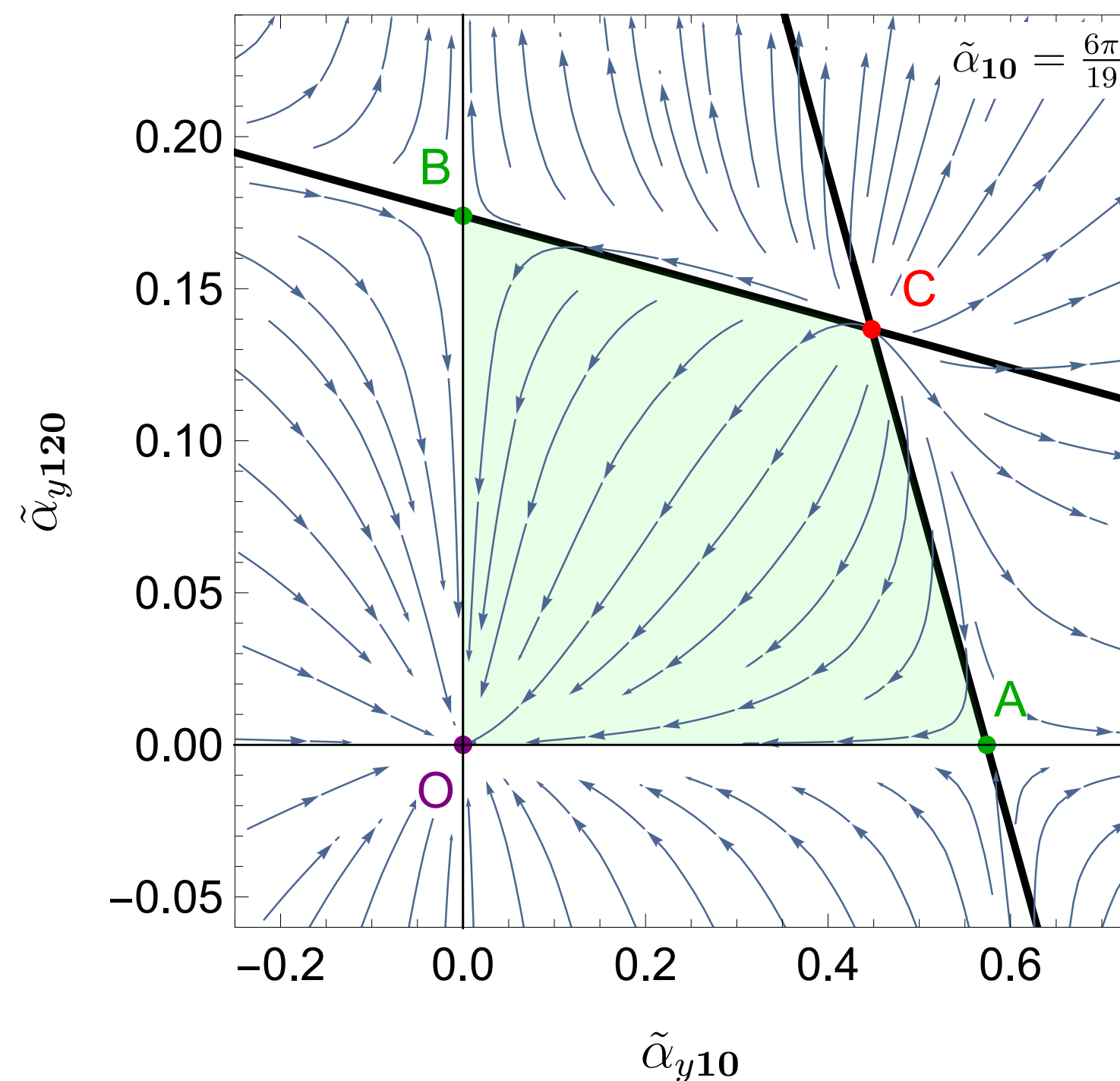
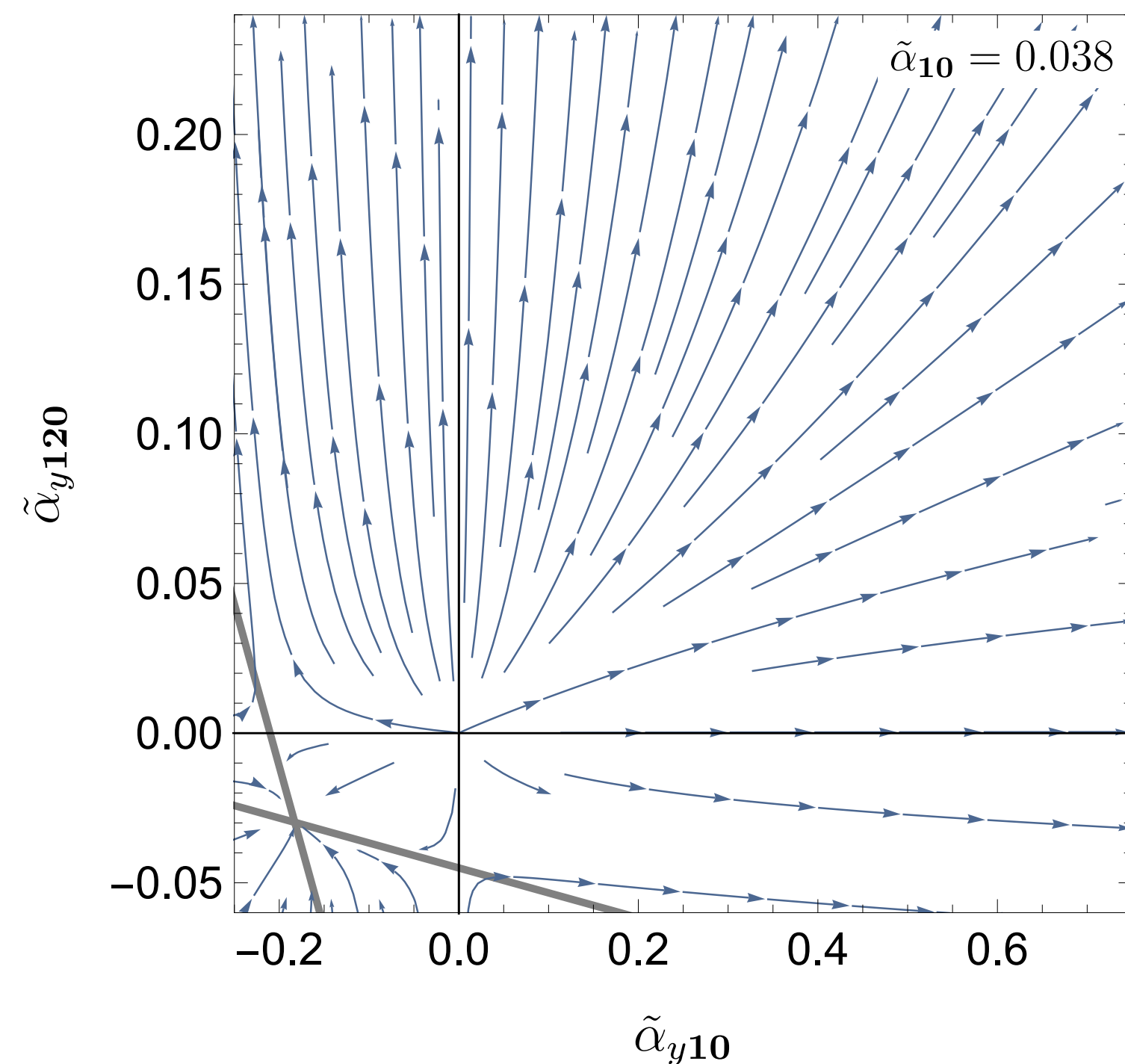
Deriving Yukawa RGEs

- RGs for 't Hooft couplings $\tilde{\alpha}_{yr}(t) = \alpha_{yr}(t)S(t), \quad \alpha_{yr} = \frac{y_r^2}{4\pi}, \quad r = \mathbf{10, 120}$

$$2\pi \frac{d\tilde{\alpha}_{y120}}{dt} = \left[2\pi + 10\tilde{\alpha}_{y10} + 120\tilde{\alpha}_{y120} - \frac{219}{8}\tilde{\alpha}_{10} \right] \tilde{\alpha}_{y120}$$

$$2\pi \frac{d\tilde{\alpha}_{y10}}{dt} = \left[2\pi + 26\tilde{\alpha}_{y10} + 24\tilde{\alpha}_{y120} - \frac{171}{8}\tilde{\alpha}_{10} \right] \tilde{\alpha}_{y10}$$

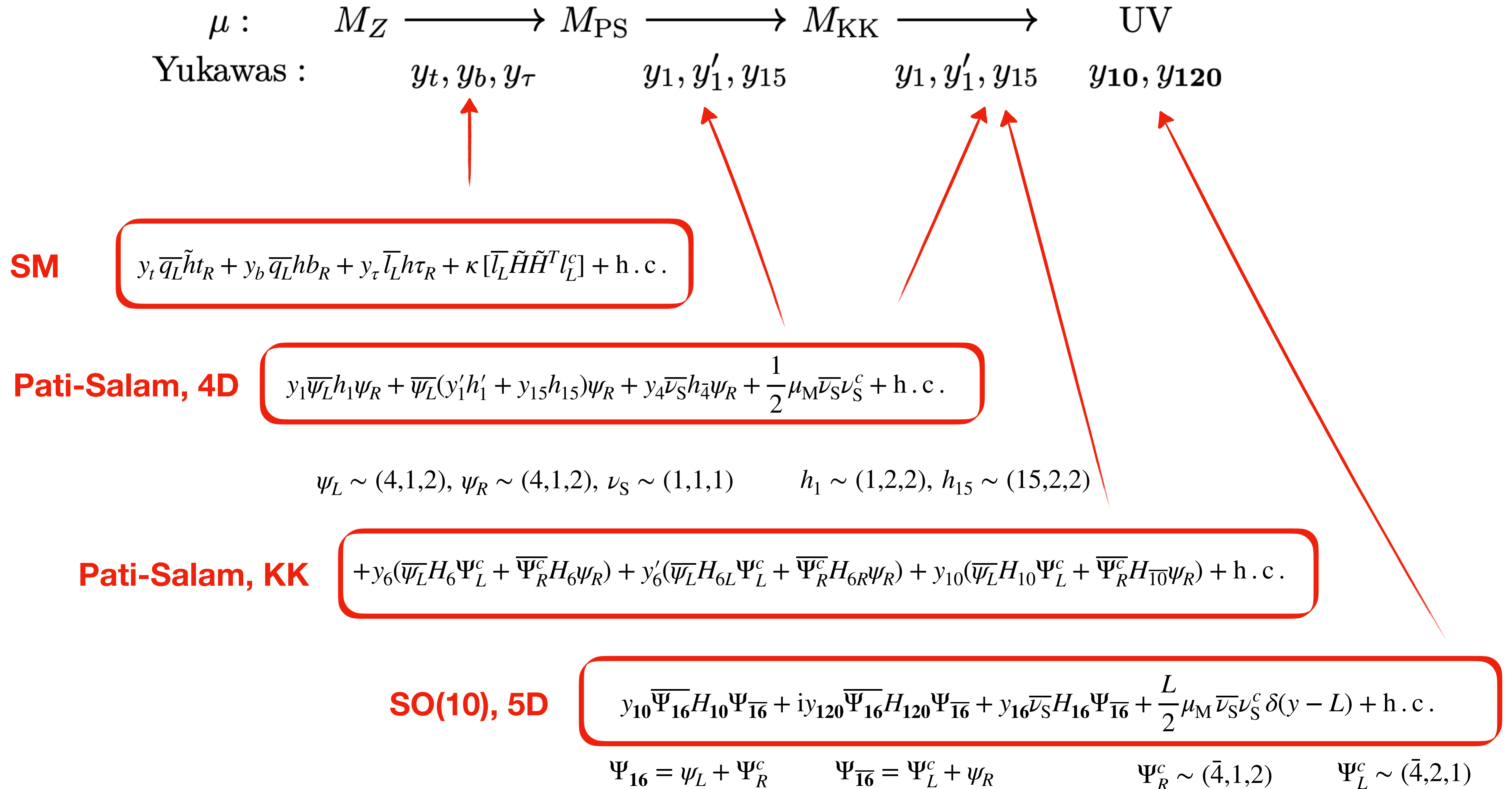
- RG flow of Yukawa couplings



$(\tilde{\alpha}_{y10}, \tilde{\alpha}_{y120})$	
Point	
$(0, 0)$	O
$(\frac{19\pi}{104}, 0)$	A
$(0, \frac{101\pi}{1824})$	B
$(\frac{65\pi}{456}, \frac{119\pi}{2736})$	C

Asymptotic freedom might be achieved for suitable size of gauge couplings

Explicitly running Yukawa couplings from EW scale to UV



Explicitly running Yukawa couplings from EW scale to UV

$$\begin{array}{ccccccc} \mu : & M_Z & \longrightarrow & M_{\text{PS}} & \longrightarrow & M_{\text{KK}} & \longrightarrow & \text{UV} \\ \text{Yukawas :} & y_t, y_b, y_\tau & & y_1, y'_1, y_{15} & & y_1, y'_1, y_{15} & & y_{10}, y_{120} \end{array}$$

$$\begin{aligned} 2\pi \frac{d\alpha_t}{dt} &= \left[\frac{9}{2}\alpha_t + \frac{3}{2}\alpha_b + \alpha_\tau - \frac{9}{4}\alpha_{2L} - \frac{17}{20}\alpha_1 - 8\alpha_3 \right] \alpha_t \\ 2\pi \frac{d\alpha_b}{dt} &= \left[\frac{3}{2}\alpha_t + \frac{9}{2}\alpha_b + \alpha_\tau - \frac{9}{4}\alpha_{2L} - \frac{1}{4}\alpha_1 - 8\alpha_3 \right] \alpha_b \\ 2\pi \frac{d\alpha_\tau}{dt} &= \left[3\alpha_t + 3\alpha_b + \frac{5}{2}\alpha_\tau - \frac{9}{4}\alpha_{2L} - \frac{9}{4}\alpha_1 \right] \alpha_\tau \end{aligned}$$

$$\begin{aligned} 2\pi \frac{d\tilde{\alpha}_{y10}}{dt} &= \left[2\pi + 26\tilde{\alpha}_{y10} + 24\tilde{\alpha}_{y120} - \frac{81}{8}\tilde{\alpha}_4 - \frac{45}{8}(\tilde{\alpha}_{2L} + \tilde{\alpha}_{2R}) \right] \tilde{\alpha}_{y10} \\ 2\pi \frac{d\tilde{\alpha}_{y120}}{dt} &= \left[2\pi + 10\tilde{\alpha}_{y10} + 120\tilde{\alpha}_{y120} - \frac{129}{8}\tilde{\alpha}_4 - \frac{45}{8}(\tilde{\alpha}_{2L} + \tilde{\alpha}_{2R}) \right] \tilde{\alpha}_{y120} \end{aligned}$$

$$\begin{aligned} 2\pi \frac{d\alpha_{y1}}{dt} &= \left[6\alpha_{y1} + 4\alpha_{y1'} - \frac{45}{4}\alpha_4 - \frac{9}{4}(\alpha_{2L} + \alpha_{2R}) \right] \alpha_{y1} \\ 2\pi \frac{d\alpha_{y1'}}{dt} &= \left[2\alpha_{y1} + 8\alpha_{y1'} - \frac{45}{4}\alpha_4 - \frac{9}{4}(\alpha_{2L} + \alpha_{2R}) \right] \alpha_{y1'} \\ 2\pi \frac{d\alpha_{y15}}{dt} &= \left[8\alpha_{y15} + 2\alpha_{y1} - \frac{45}{4}\alpha_4 - \frac{9}{4}(\alpha_{2L} + \alpha_{2R}) \right] \alpha_{y15} \end{aligned}$$

$$\begin{aligned} 2\pi \frac{d\alpha_{y1}}{dt} \Big|_{\text{KK}} &= \left[10\alpha_{y1} + \frac{3}{2}\alpha_{y6} + 4\alpha_{y1'} - \frac{5}{2}\alpha_{y10} + \frac{9}{4}\alpha_{y6'} - \frac{81}{8}\alpha_4 - \frac{45}{8}(\alpha_{2L} + \alpha_{2R}) \right] \alpha_{y1} \\ 2\pi \frac{d\alpha_{y1'}}{dt} \Big|_{\text{KK}} &= \left[2\alpha_{y1} + \frac{3}{2}\alpha_{y6} + 12\alpha_{y1'} + \frac{15}{2}\alpha_{y10} + \frac{9}{4}\alpha_{y6'} - \frac{129}{8}\alpha_4 - \frac{45}{8}(\alpha_{2L} + \alpha_{2R}) \right] \alpha_{y1'} \\ 2\pi \frac{d\alpha_{y15}}{dt} \Big|_{\text{KK}} &= \left[2\alpha_{y1} + \frac{3}{2}\alpha_{y6} + 9\alpha_{y15} + \frac{3}{2}\alpha_{y10} + \frac{9}{4}\alpha_{y6'} - \frac{129}{8}\alpha_4 - \frac{45}{8}(\alpha_{2L} + \alpha_{2R}) \right] \alpha_{y15} \end{aligned}$$

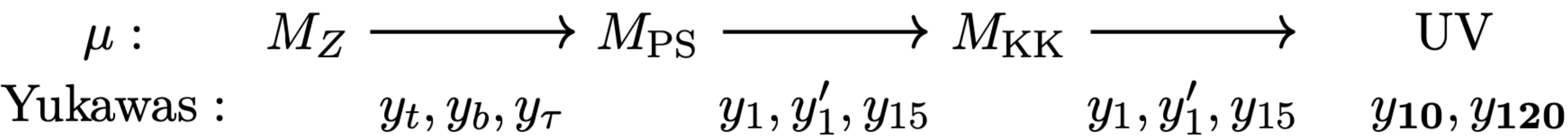
Explicitly running Yukawa couplings from EW scale to UV

$$\begin{array}{ccccccc}
 \mu : & M_Z & \longrightarrow & M_{\text{PS}} & \longrightarrow & M_{\text{KK}} & \longrightarrow & \text{UV} \\
 \text{Yukawas :} & y_t, y_b, y_\tau & & y_1, y'_1, y_{15} & & y_1, y'_1, y_{15} & & y_{10}, y_{120}
 \end{array}$$


Matching

$$\begin{aligned}
 y_t &= y_1 c_{10}^u + y'_1 c_{120}^{d'} + \frac{1}{2\sqrt{3}} y_{15} c_{120}^d, \\
 y_b &= y_1 c_{10}^d + y'_1 c_{120}^{d'} + \frac{1}{2\sqrt{3}} y_{15} c_{120}^d, \\
 y_\tau &= y_1 c_{10}^d + y'_1 c_{120}^{d'} - \frac{\sqrt{3}}{2} y_{15} c_{120}^d,
 \end{aligned}$$

Explicitly running Yukawa couplings from EW scale to UV



Matching

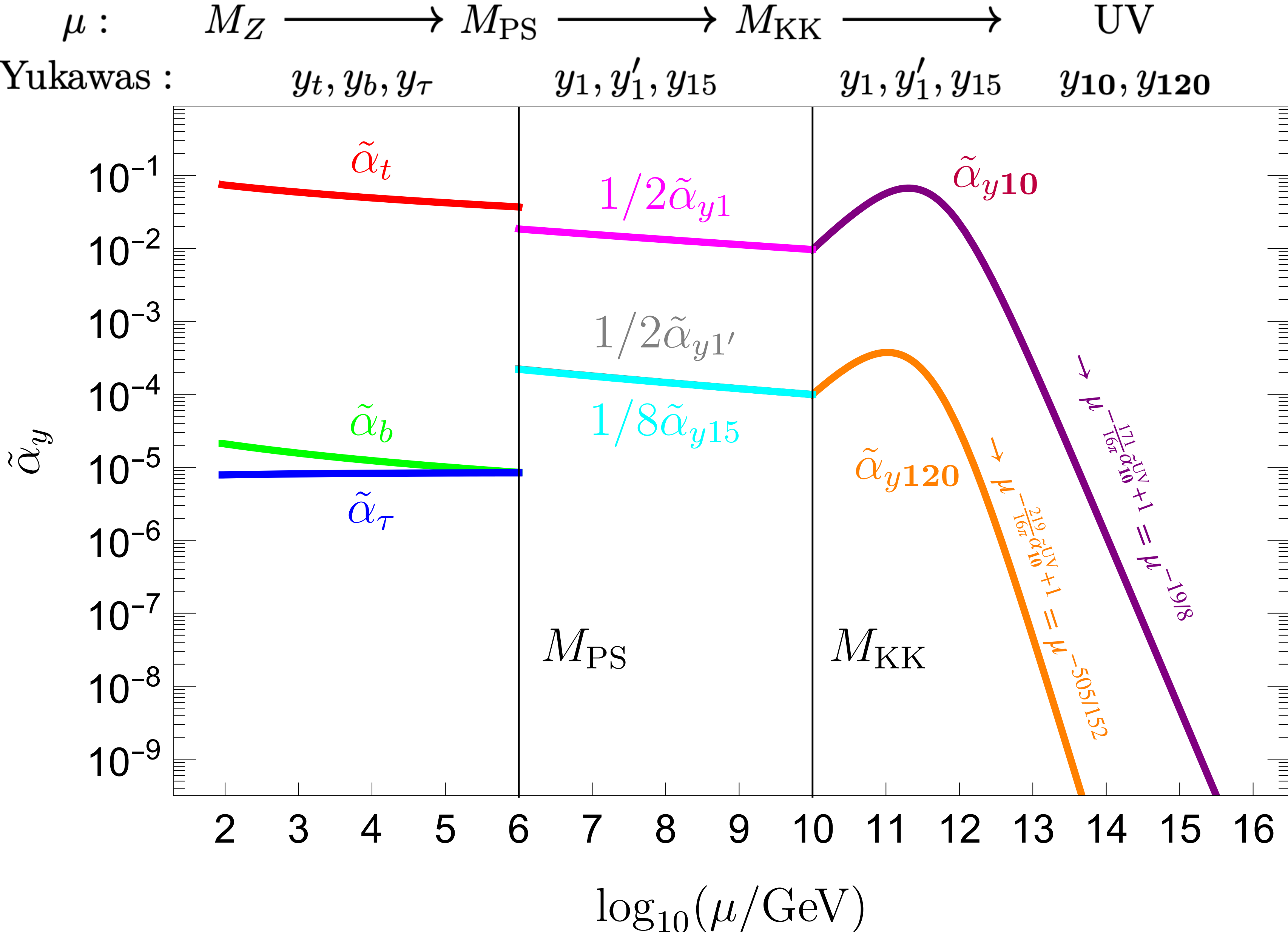
$$\begin{aligned} \frac{1}{2}\alpha_{y1}, \frac{1}{4}\alpha_{y6} &\rightarrow \alpha_{y10}, \\ \frac{1}{2}\alpha_{y1'}, \frac{1}{8}\alpha_{y15}, \frac{1}{8}\alpha_{y10}, \frac{1}{16}\alpha_{y6'} &\rightarrow \alpha_{y120}, \end{aligned} \quad \text{in the UV limit,}$$

Asymptotic unification?

$$\begin{aligned} \frac{1}{2}\alpha_{y1}, \frac{1}{4}\alpha_{y6} &= \alpha_{y10}, \\ \frac{1}{2}\alpha_{y1'}, \frac{1}{8}\alpha_{y15}, \frac{1}{8}\alpha_{y10}, \frac{1}{16}\alpha_{y6'} &= \alpha_{y120}, \end{aligned} \quad \text{at } \mu = M_{\text{KK}}.$$

Exact unification?

Explicitly running Yukawa couplings from EW scale to UV

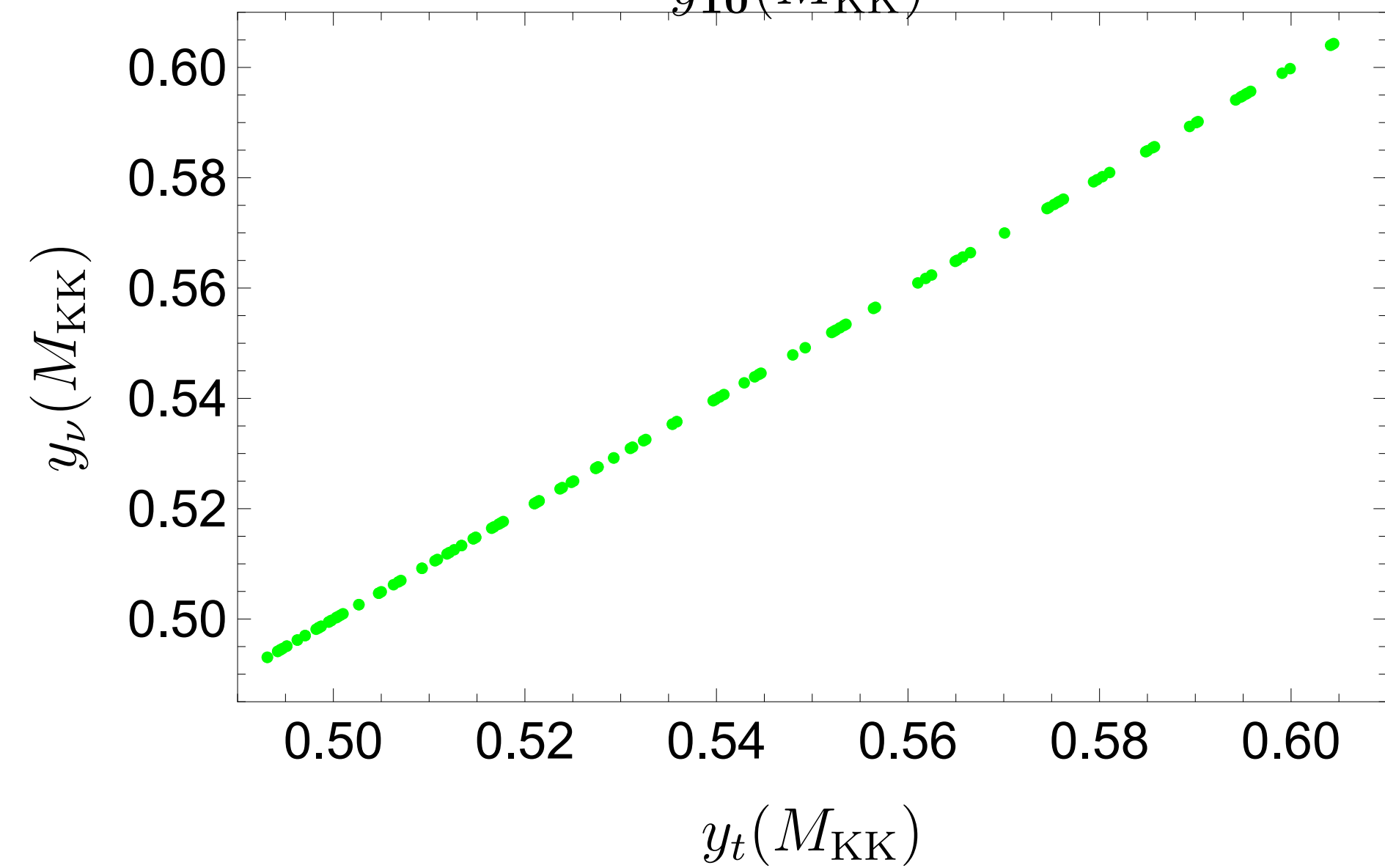
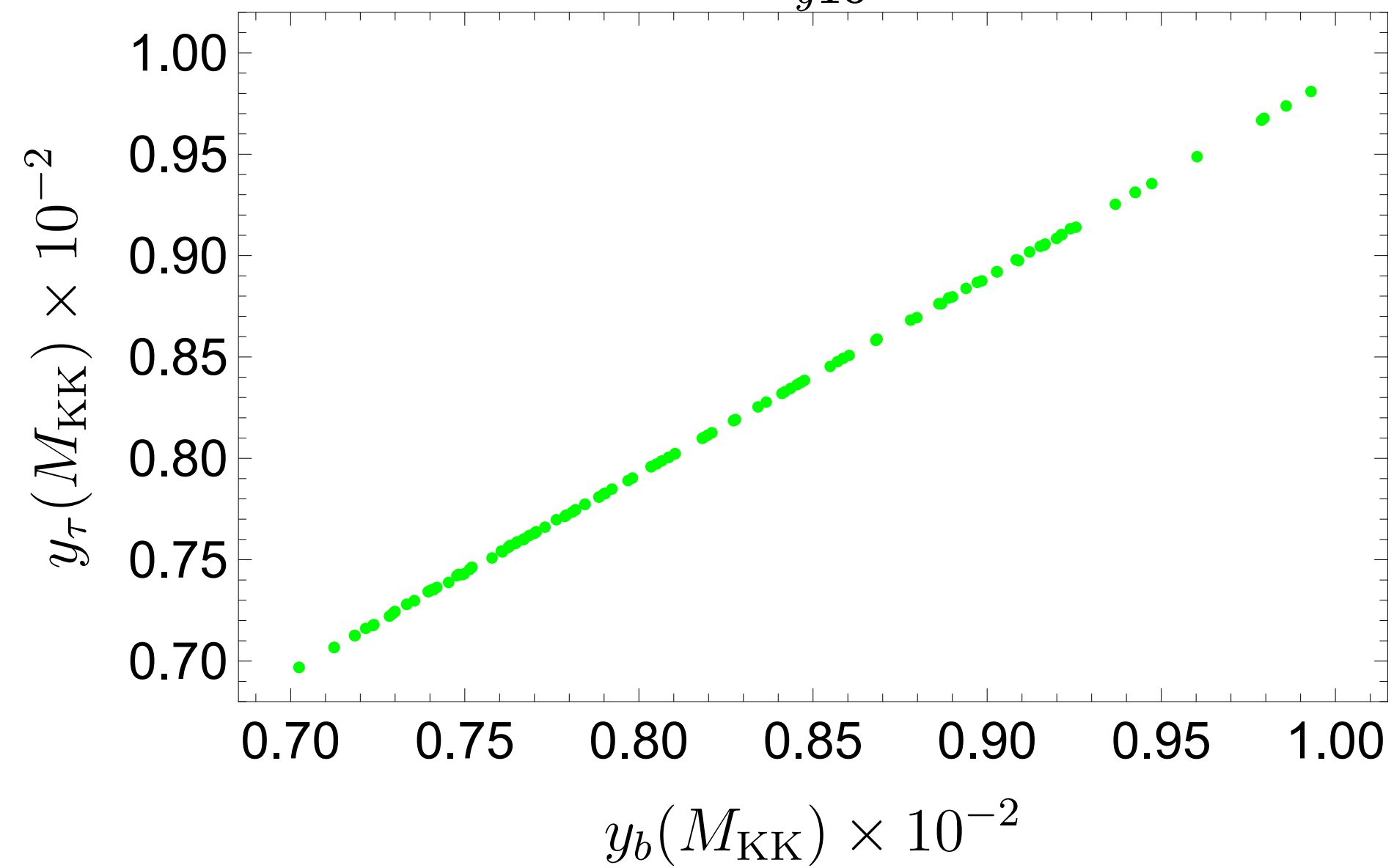
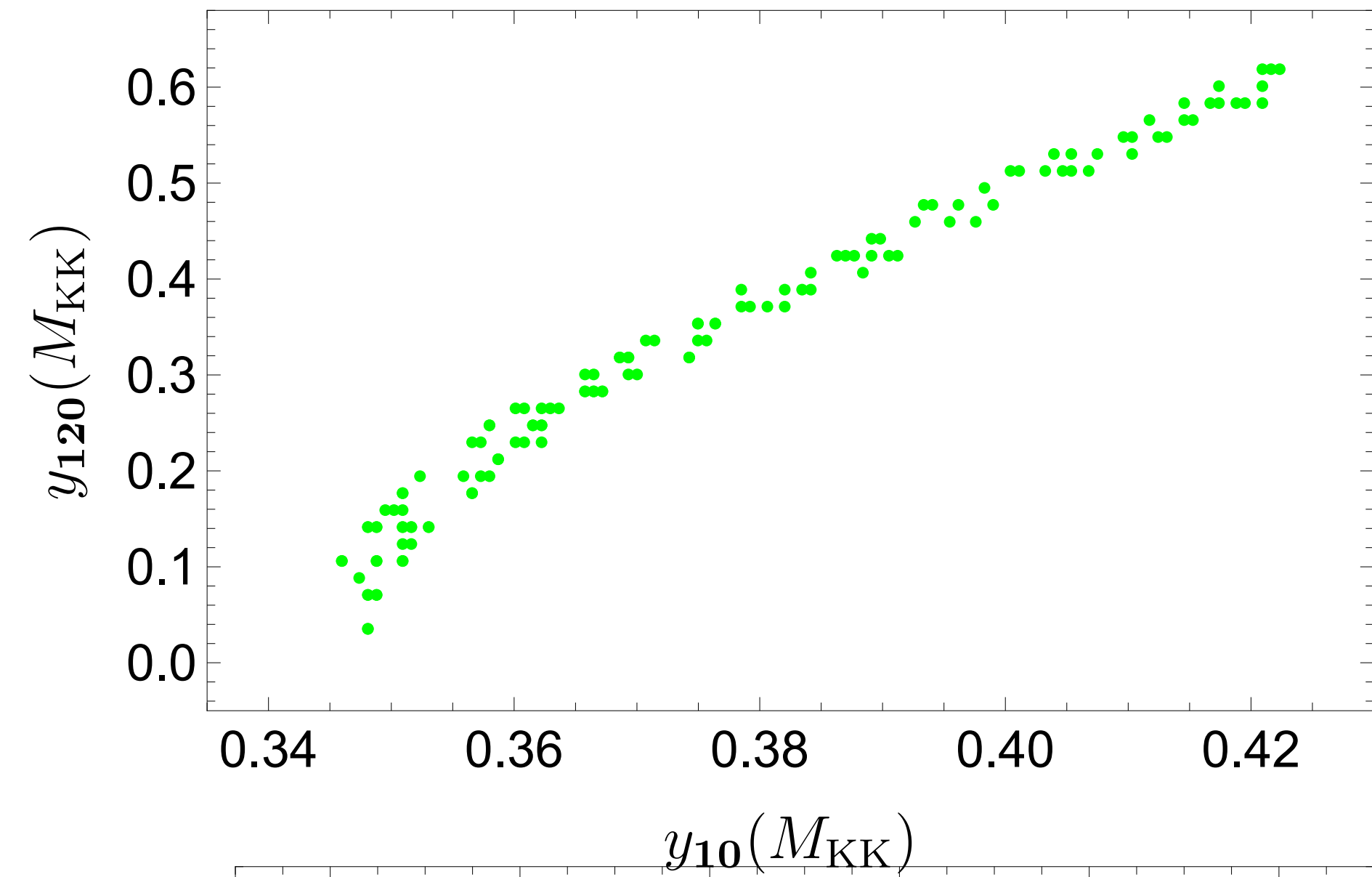
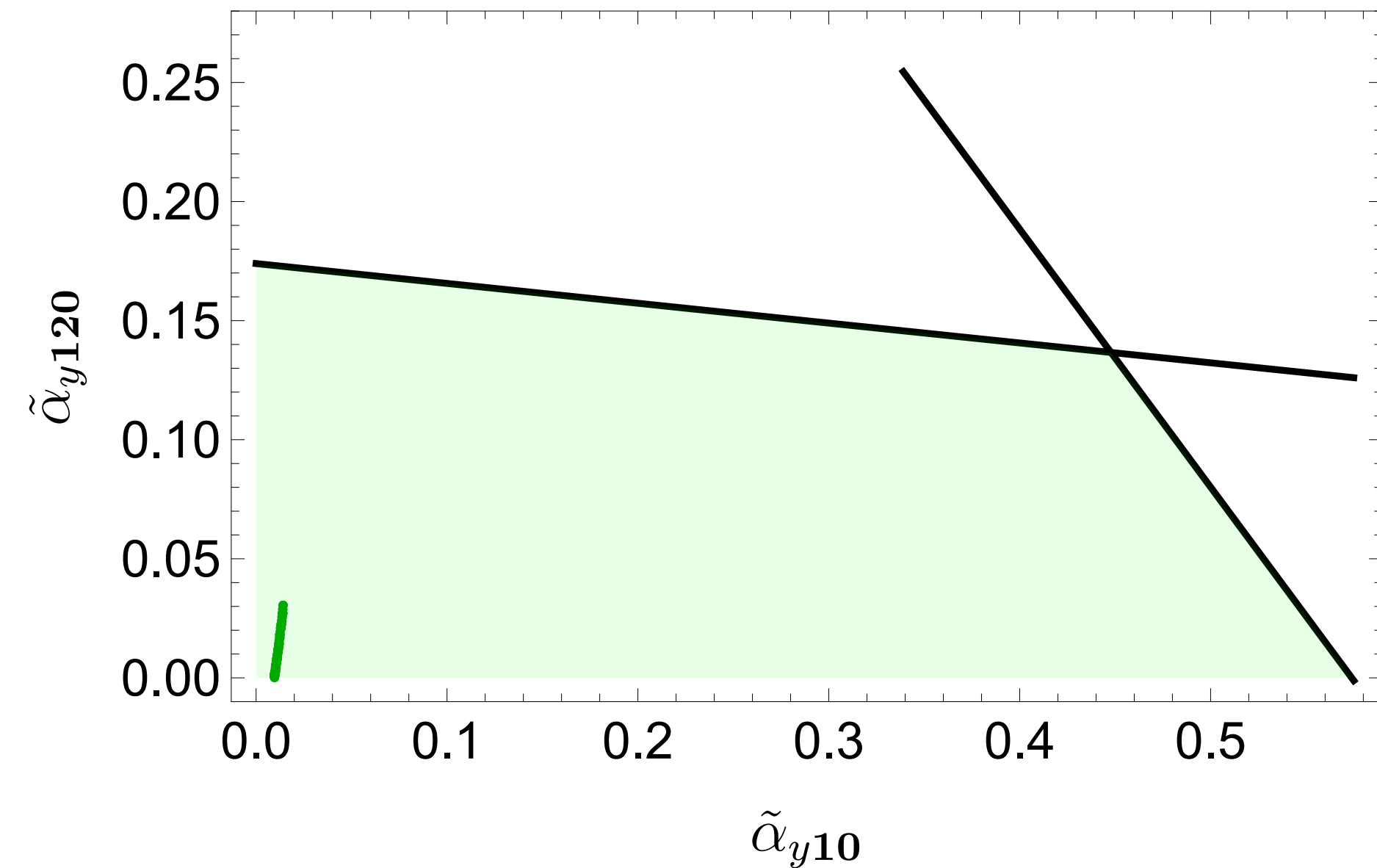


$$\tilde{\alpha}_{\text{yr}}(t) = \alpha_{\text{yr}}(t)S(t)$$

$$\alpha_{\text{yr}} = \frac{y_r^2}{4\pi}$$

$$\tilde{\beta}_y \sim (c_1\tilde{\alpha}_y - c_2\tilde{\alpha}_{10})\tilde{\alpha}_y$$

Parameter space for asymptotically free Yukawa couplings



Conclusion

- GUT is a hot topic for its rich phenos, in particular facing to the future neutrino and GW measurements. These studies focus at / below the GUT scale. Its UV behaviour has been paid less attention.
- GUT also provides a plateau to discuss fundamental properties of QFT, Asymptotic GUT (aGUT) is one of them.
- We demonstrate that aGUT is realised in a 5D SO(10) with minimal and realistic field content.

$$5\text{D SO}(10) \xrightarrow[\text{BC}]{M_{\text{KK}} \sim 10^{10} \text{ GeV}} 4\text{D Pati-Salam} \xrightarrow[\mathbf{16}]{M_{\text{PS}} \sim 10^6 \text{ GeV}} \text{SM}$$

- Gauge couplings are unified asymptotically in the deep UV regime.
- Yukawa couplings should be unified exactly at the KK scale. Their UV behaviour are very model-dependent, with a realistic content: complex **10**, real **120**, and **16**, asymptotic freedom for Yukawa couplings are achieved in some parameter space.
- The rep **$\overline{126}$** is not welcome in 5D SO(10) aGUT.

Thanks and questions ...

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- Theory of neutrino mass and flavor mixing
- Neutrino oscillation experiments and phenomenology
- Neutrino astronomy and cosmology
- Neutrinoless double beta decay
- Technology in neutrino experiments

Contact

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Yu-Feng Li IHEP, CAS
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Discussions led by

Liang-Jian Wen IHEP, CAS
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