Multiple Top Quark Phenomenology and Higher-Order Corrections

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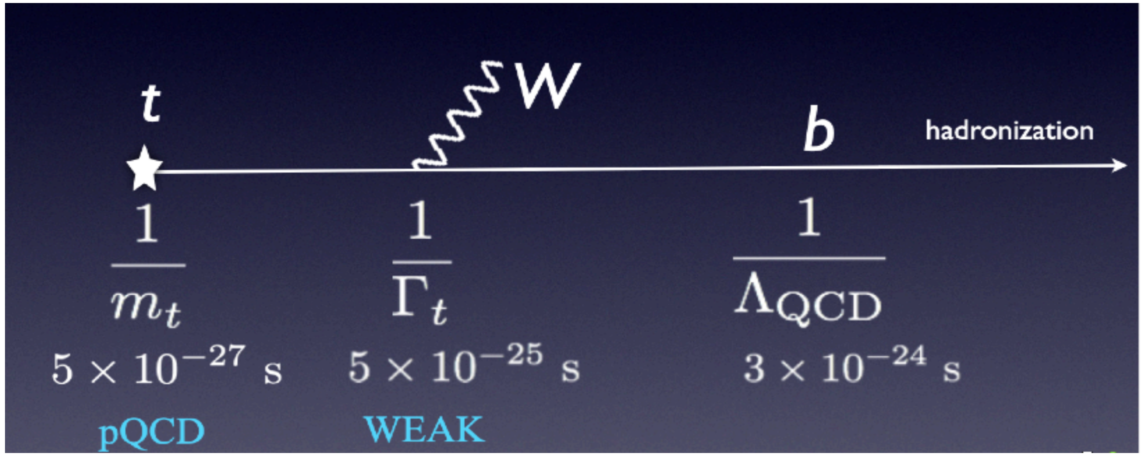
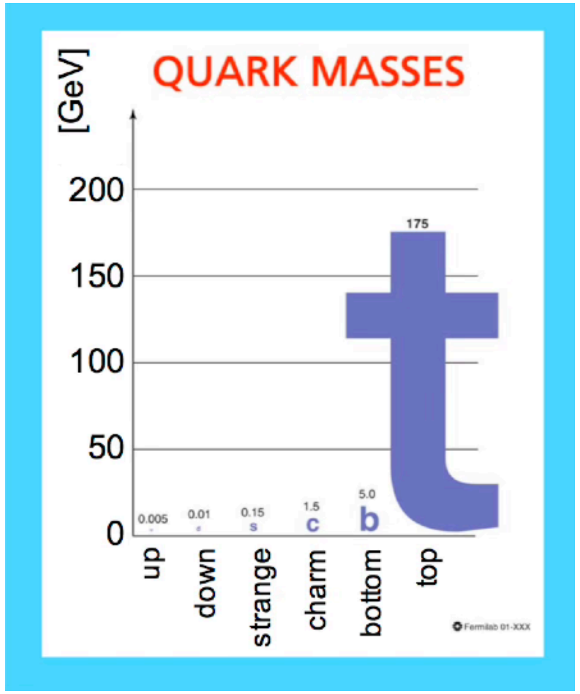
Top Quark: King of the Standard Model

Large mass: 173 GeV ($y_t \sim O(1)$)

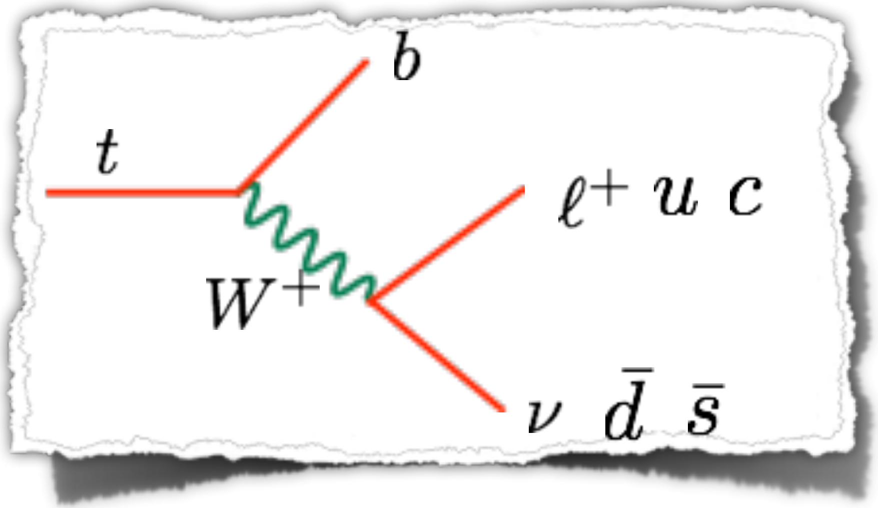
Short lifetime





"Bare" quark: information well kept

among its decay products



A unique position in the two frontiers:

Precision and Discovery

The key channel in the SM: $t\bar{t}$ pair production

Theory calculation has reached N^3LO.

TABLE III. The $t\bar{t}$ total cross sections (in pb, with central result for $\mu = m_t$, and uncertainties from scale variation and pdf) at different perturbative orders in pp collisions at the LHC with various values of $\sqrt{S}$, with $m_t = 172.5$ GeV and CT18 NNLO pdf.

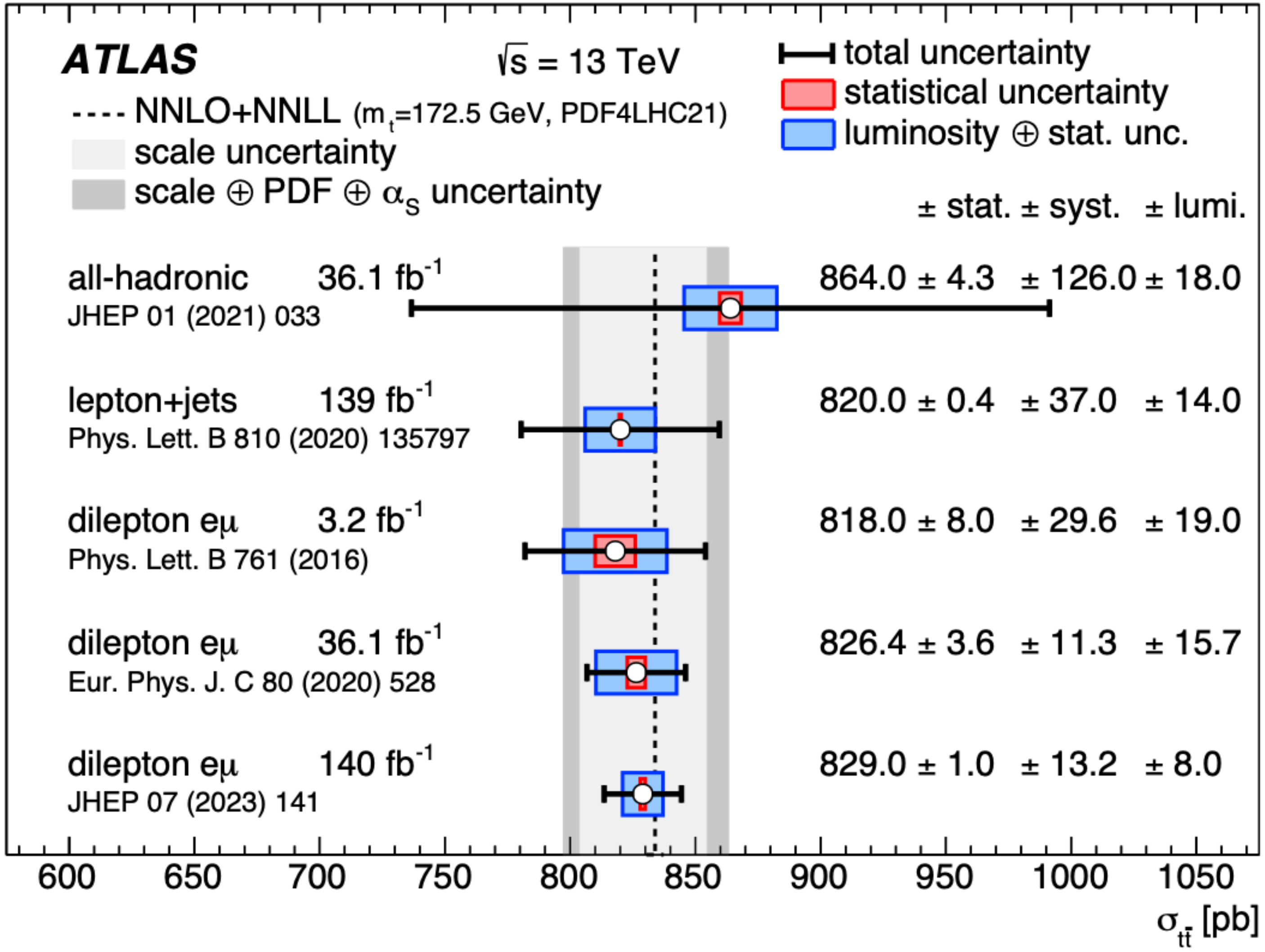
$t\bar{t}$ total cross sections at LHC energies with CT18 NNLO pdf						
σ in pb	5.02 TeV	7 TeV	8 TeV	13 TeV	13.6 TeV	14 TeV
LO QCD	$41.2^{+15.7+2.1}_{-10.5-1.3}$	106^{+37+4}_{-25-2}	151^{+51+5}_{-35-3}	$491^{+142+10}_{-104-10}$	$543^{+156+11}_{-113-10}$	$579^{+165+12}_{-120-11}$
NLO QCD	$60.3^{+7.2+3.1}_{-8.3-2.0}$	157^{+19+6}_{-20-4}	224^{+27+8}_{-28-5}	735^{+85+16}_{-86-15}	814^{+95+17}_{-95-16}	$869^{+101+18}_{-101-17}$
NLO QCD + EW	$60.2^{+7.2+3.1}_{-8.2-1.9}$	157^{+18+5}_{-21-4}	224^{+26+7}_{-29-6}	732^{+84+16}_{-85-14}	811^{+93+17}_{-94-16}	$866^{+99+17}_{-100-17}$
NNLO QCD	$67.9^{+3.0+3.5}_{-4.7-2.2}$	176^{+7+7}_{-11-4}	251^{+10+9}_{-15-6}	820^{+28+17}_{-46-16}	908^{+31+19}_{-50-18}	969^{+33+19}_{-53-18}
NNLO QCD + EW	$67.8^{+3.0+3.5}_{-4.7-2.2}$	176^{+7+7}_{-11-4}	251^{+10+9}_{-15-6}	817^{+28+17}_{-46-16}	905^{+31+19}_{-50-18}	966^{+33+19}_{-53-18}
aN^{3}LO QCD	$71.0^{+2.2+3.7}_{-3.3-2.3}$	183^{+5+7}_{-7-5}	261^{+7+8}_{-9-6}	845^{+23+18}_{-18-16}	935^{+25+20}_{-20-18}	997^{+27+21}_{-22-19}
aN^{3}LO QCD + EW	$70.9^{+2.2+3.7}_{-3.3-2.3}$	183^{+5+7}_{-7-5}	261^{+7+8}_{-9-6}	842^{+23+18}_{-18-16}	932^{+25+20}_{-20-18}	994^{+27+21}_{-22-19}

Nikolaos Kidonakis, Marco Guzzi, Alberto Tonero; Phys.Rev.D 108 (2023) 5, 054012

Scale uncertainty $\sim 2\,\%$

The precision measurement of $t\bar{t}$ pair production

Phys.Rept. 1116 (2025) 127-183



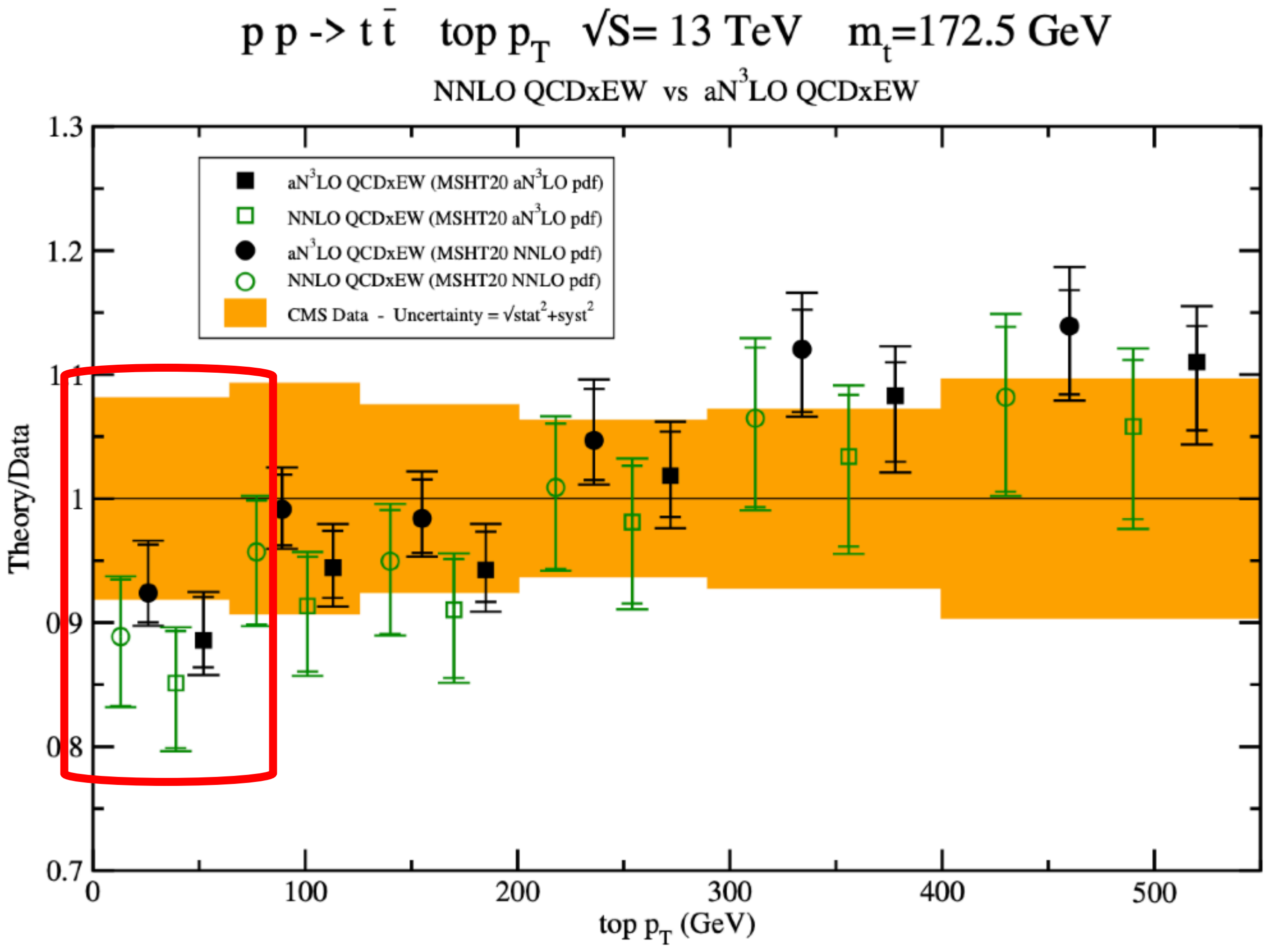
The experiment is steadily keeping pace.

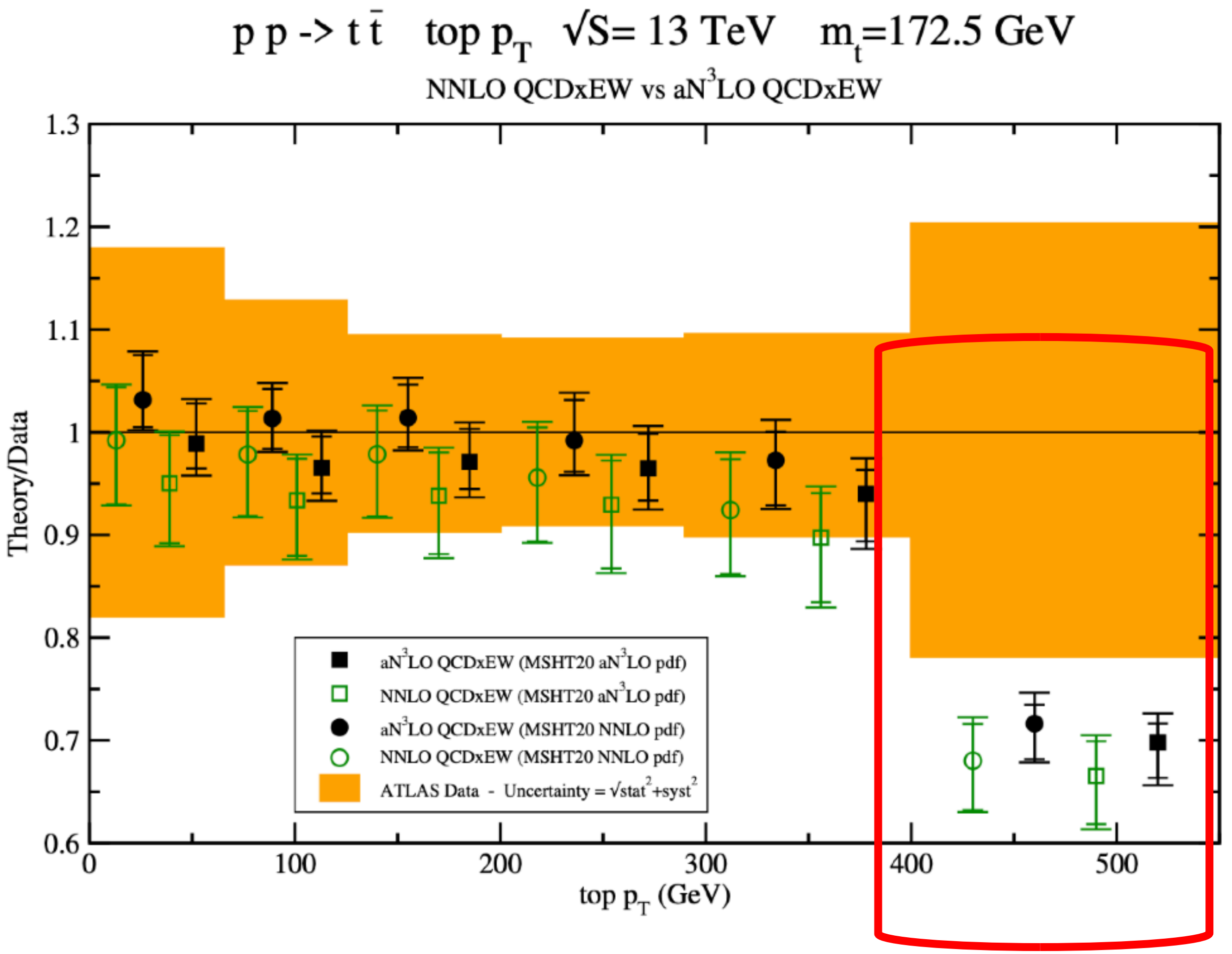
It is a true triumph of the SM.

It is now used to determine α_s and PDFs.

The Next Frontier: Differential Distributions

The real test is in the shapes of distributions, which are sensitive to the dynamics of QCD.

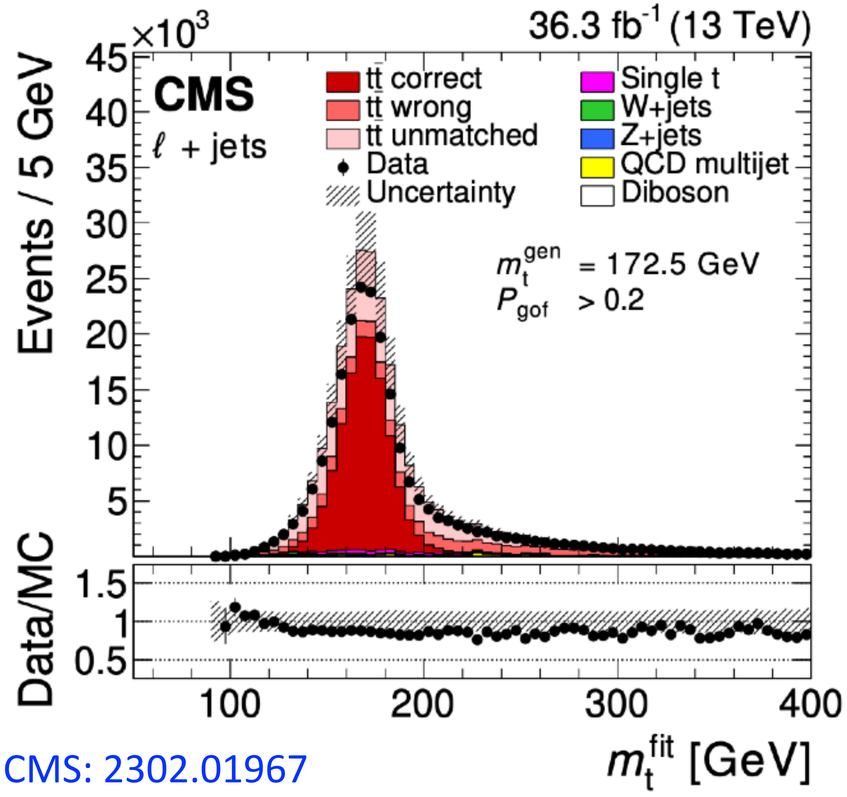




Tension requires high-order corrections and precision measurement.

Top quark mass measurement

Direct measurement

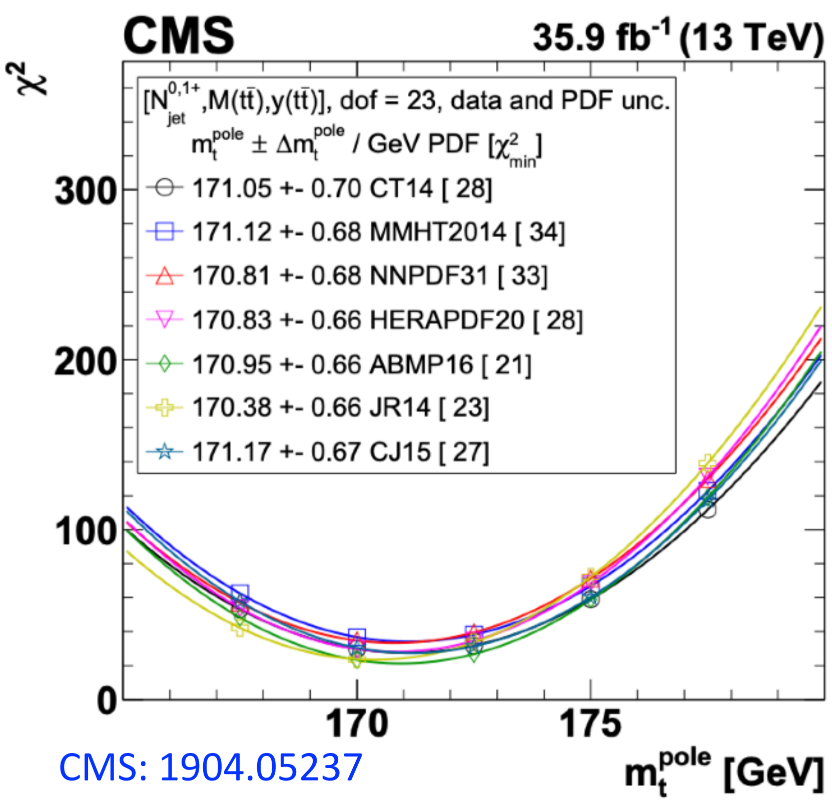


CMS: 2302.01967

Reconstruct the invariant mass of decay products

vs

Indirect measurement

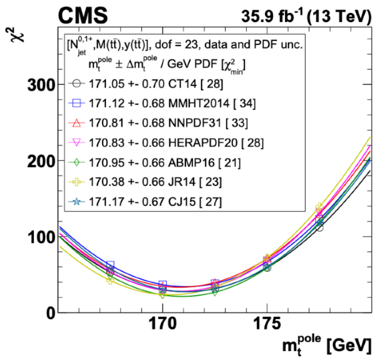


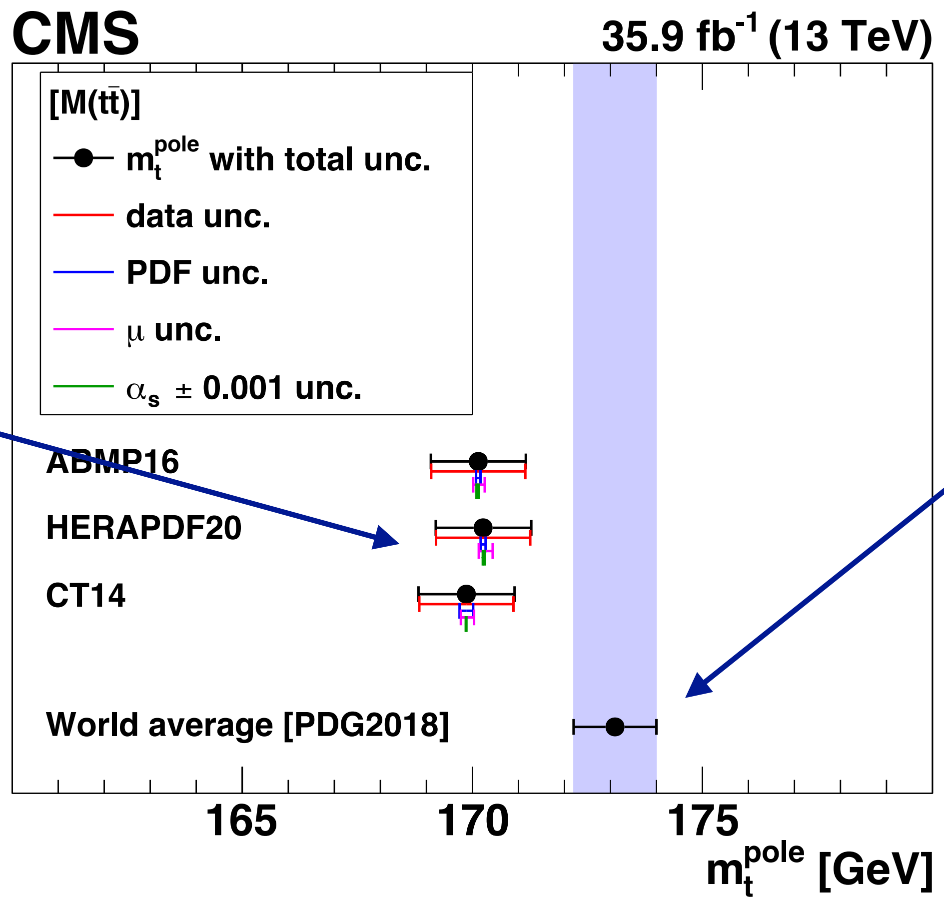
CMS: 1904.05237

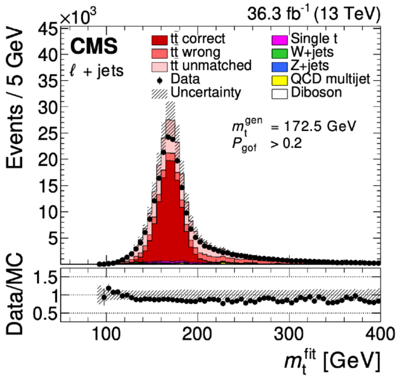
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}}^{\mathrm{TH}}(m_t) = \frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}}^{\mathrm{Exp}}$$

Top quark mass measurement

ATLAS: 1905.02302; CMS: 1904.05237

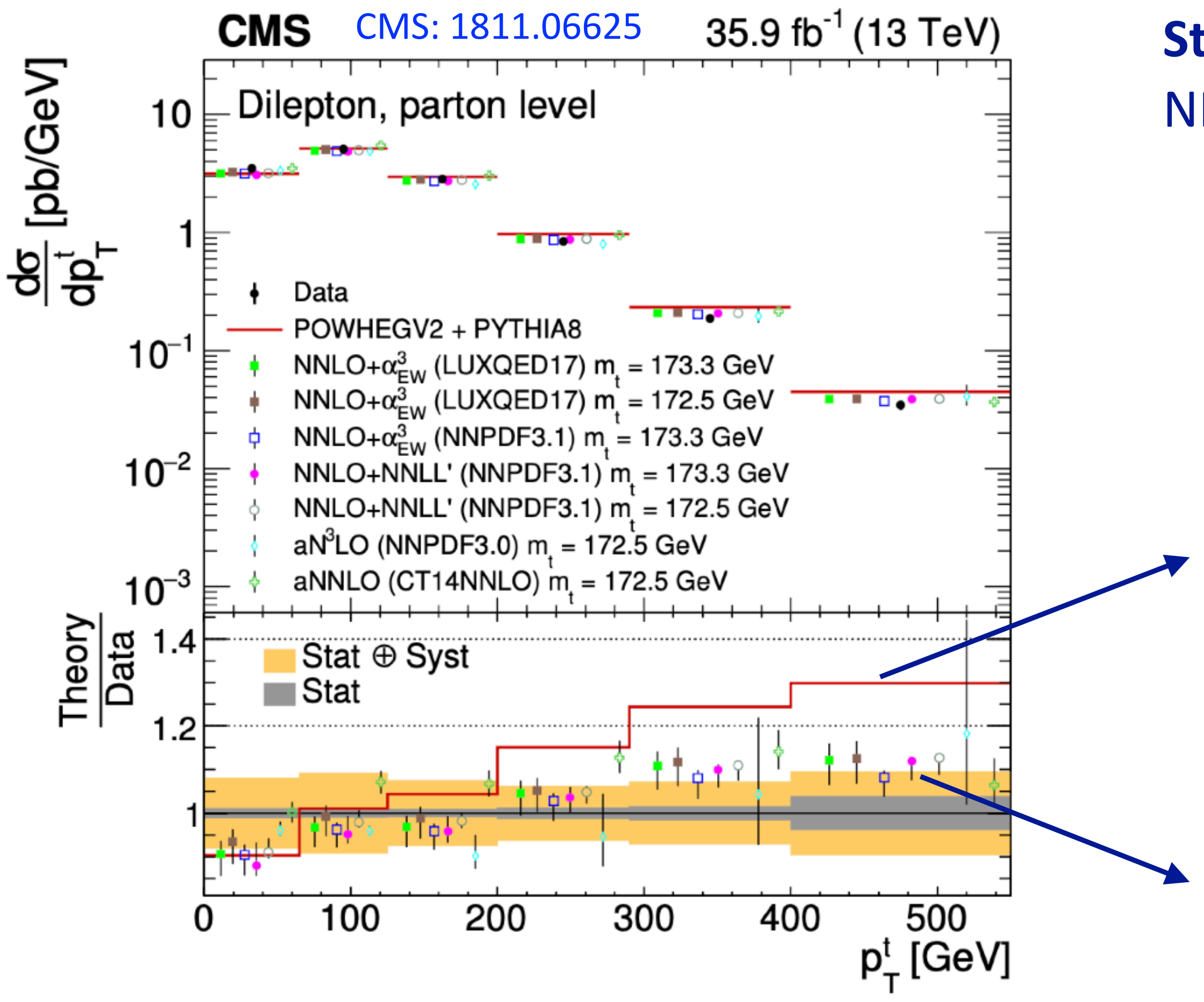






Tension between indirect and direct measurements!

Theoretical predictions vs experimental data



State-of-the-art theoretical prediction

NNLO+NNLL' in QCD + NLO in EW

Pecjak, Scott, Wang, Yang: 1601.07020
Czakon et al.: 1803.07623, 1901.08281

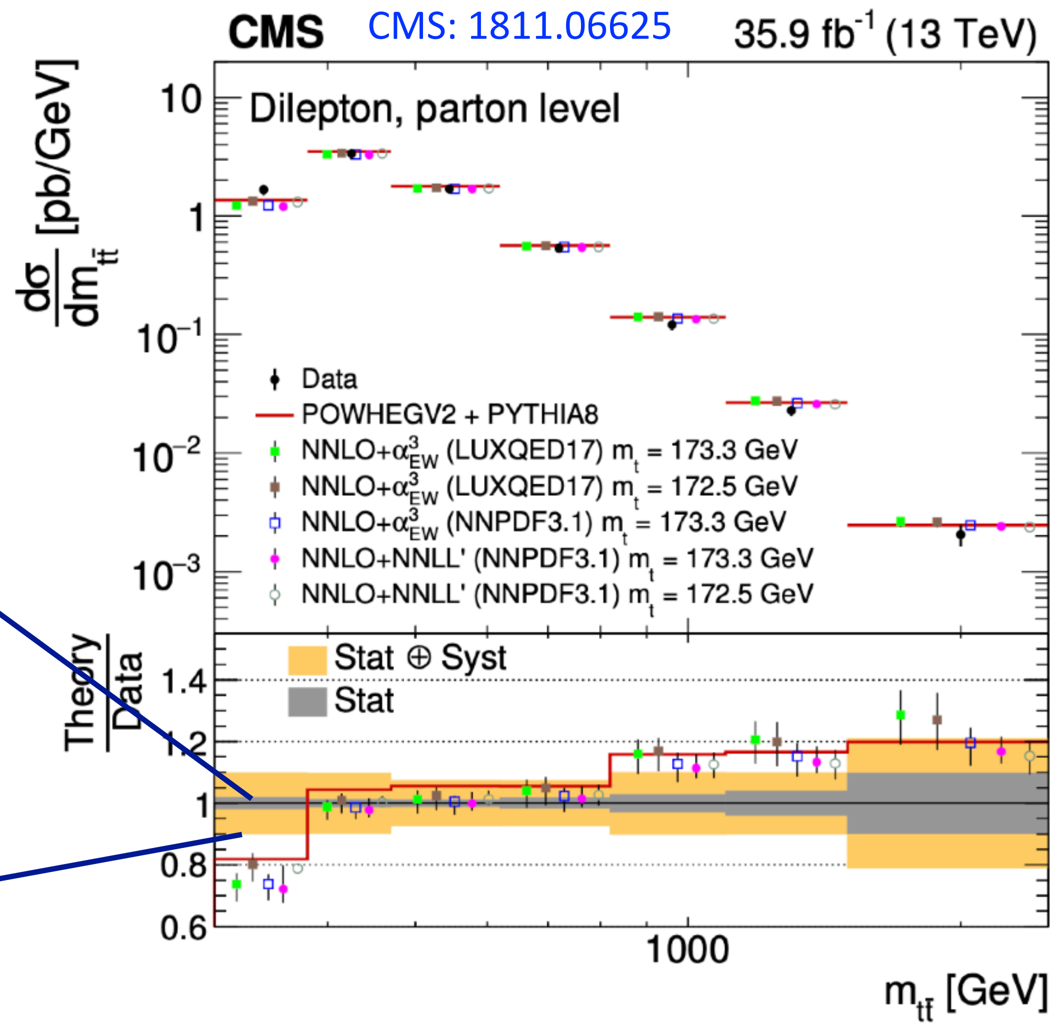
NLO alone cannot describe data

Including higher-order corrections

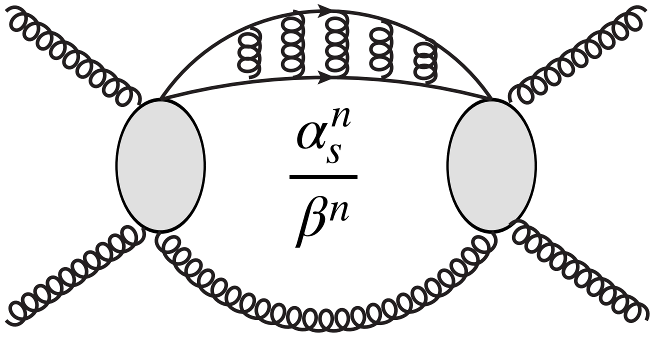
Theoretical predictions vs experimental data

Most advanced perturbative predictions cannot describe the threshold region

Most sensitive to top quark mass!

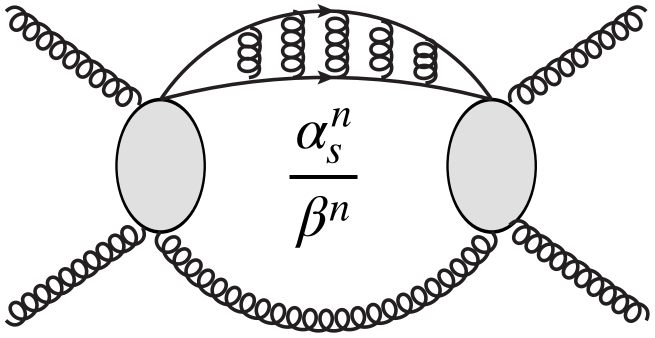


Bound-state effects near threshold



- ➤ Non-relativistic effects in $Q\bar{Q}$ system near the threshold
- ➤ Well-studied in quarkonium systems

Bound-state effects near threshold

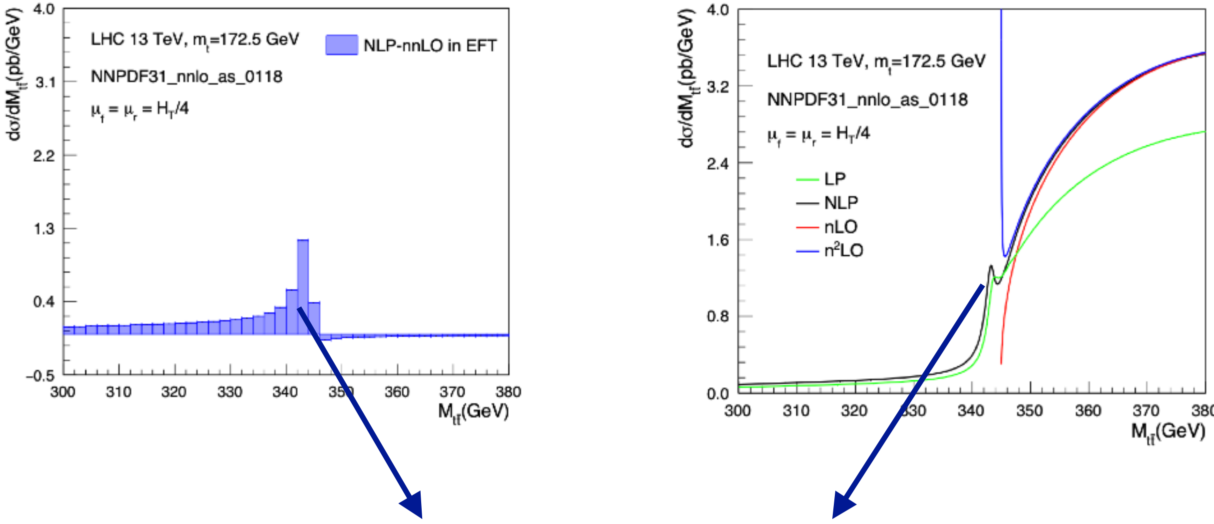


- Non-relativistic effects in $Q\bar{Q}$ system near the threshold
- Well-studied in quarkonium systems

$$\frac{\mathrm{d}\sigma}{\mathrm{d}M_{t\bar{t}}\mathrm{d}\Theta} \sim \int H \times J \times f \times f$$

Ju, Wang, Wang, Xu, Xu, Yang: 1908.02179, 2004.03088

NNLO+NLP resummation of bound-state effects

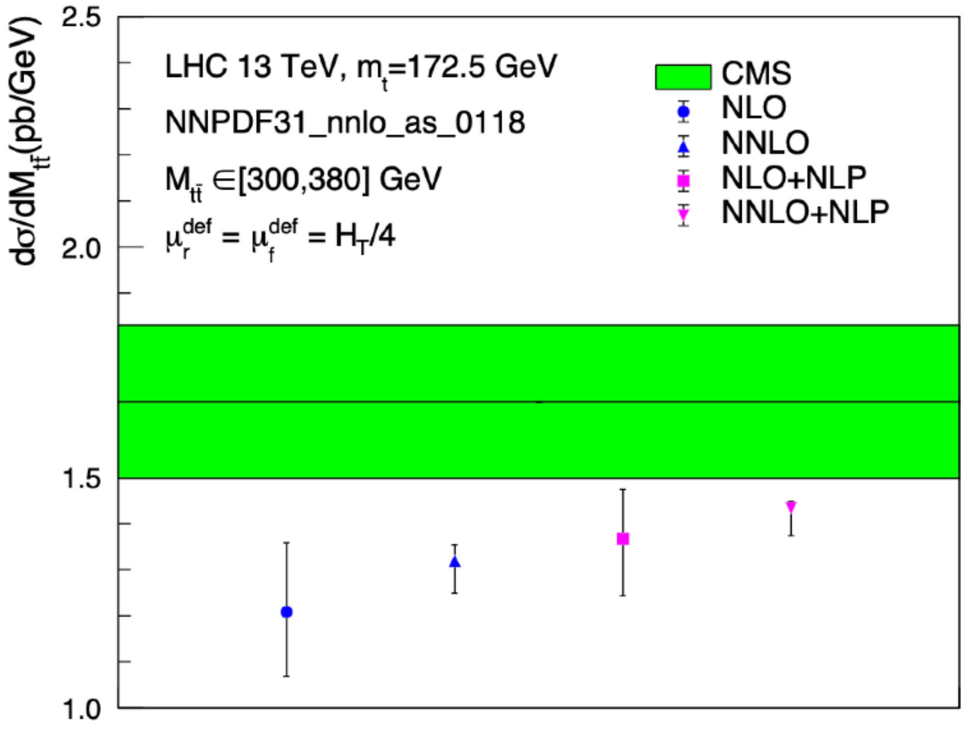


Enhancement near threshold

See also:
Kiyo et al.: 0812.0919
Sumino and Yokoya: 1007.0075
Fuks et al.: 2102.11281, 2411.18962
Garzelli et al.: 2412.16685

Bound-state effects near threshold

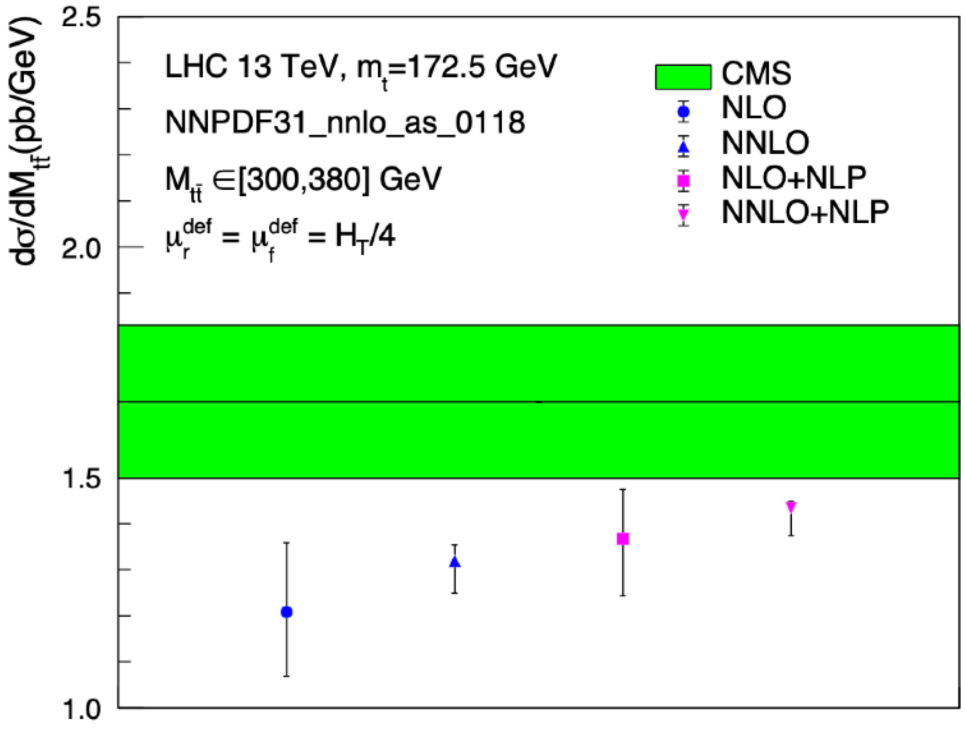
Ju, Wang, Wang, Xu, Xu, Yang:
1908.02179, 2004.03088

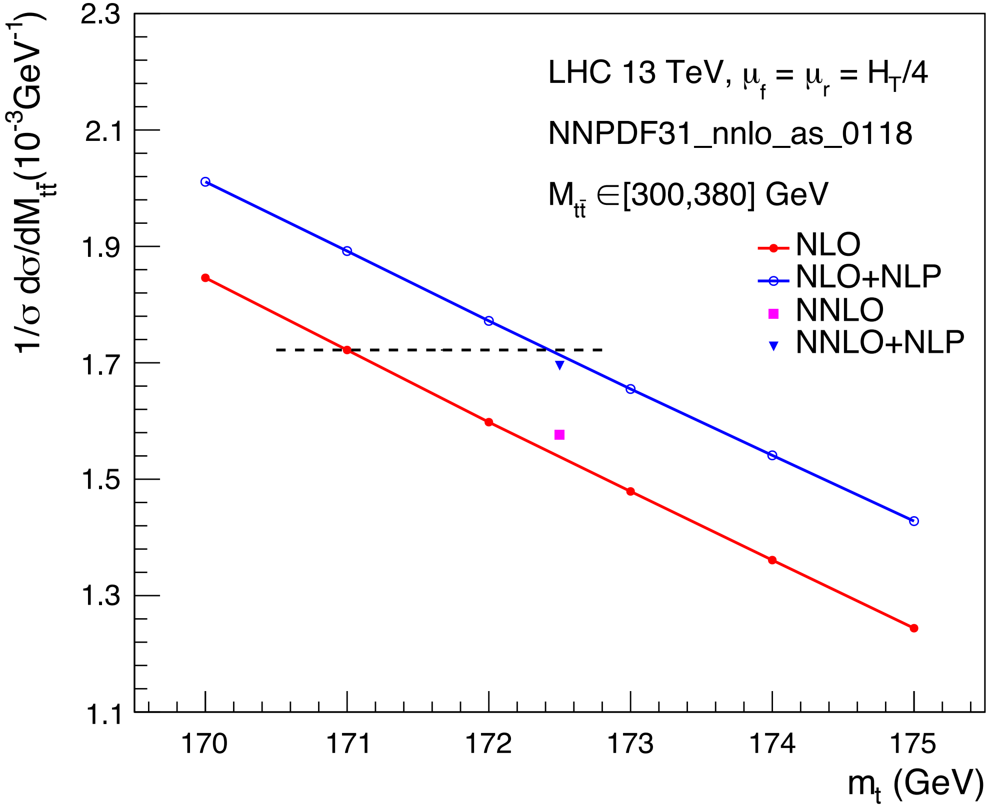


Enhancement ~ 10 pb

Bound-state effects near threshold

Ju, Wang, Wang, Xu, Xu, Yang: 1908.02179, 2004.03088





Enhancement ~ 10 pb

Bound-state effects near threshold

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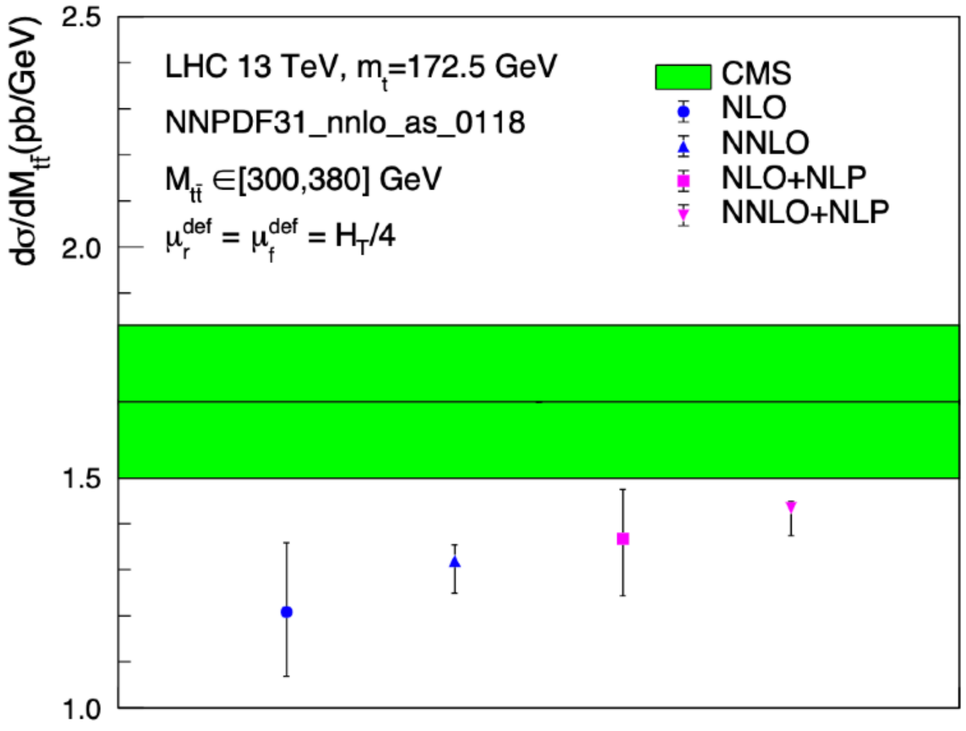
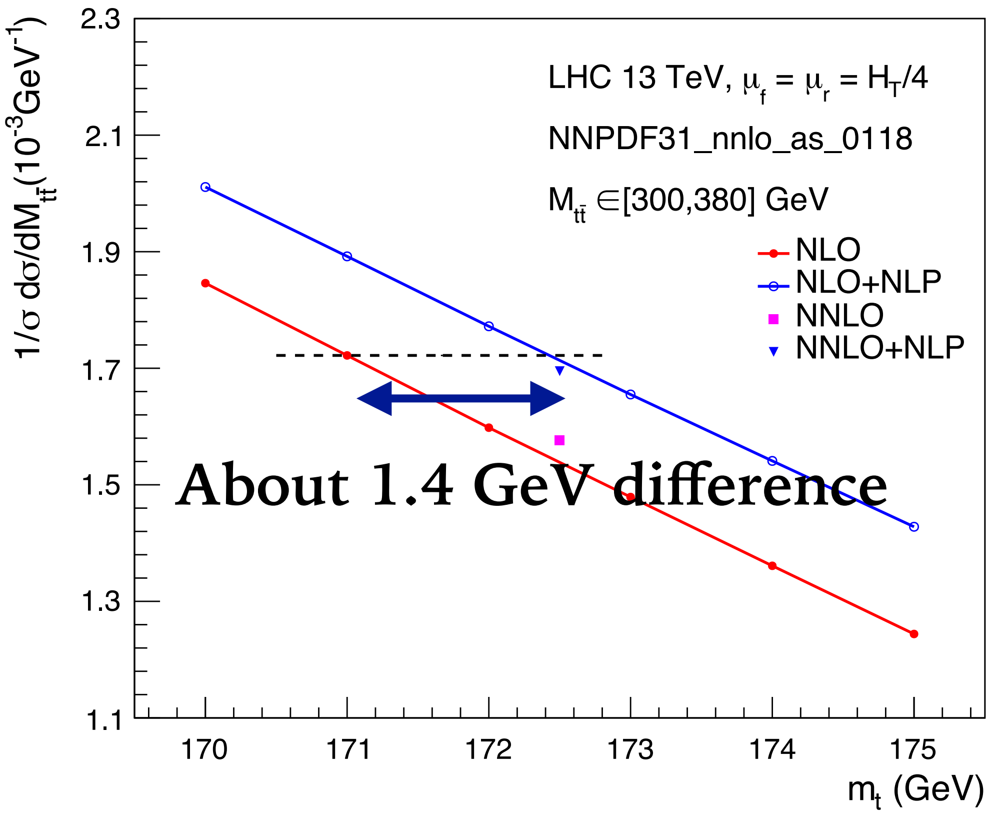




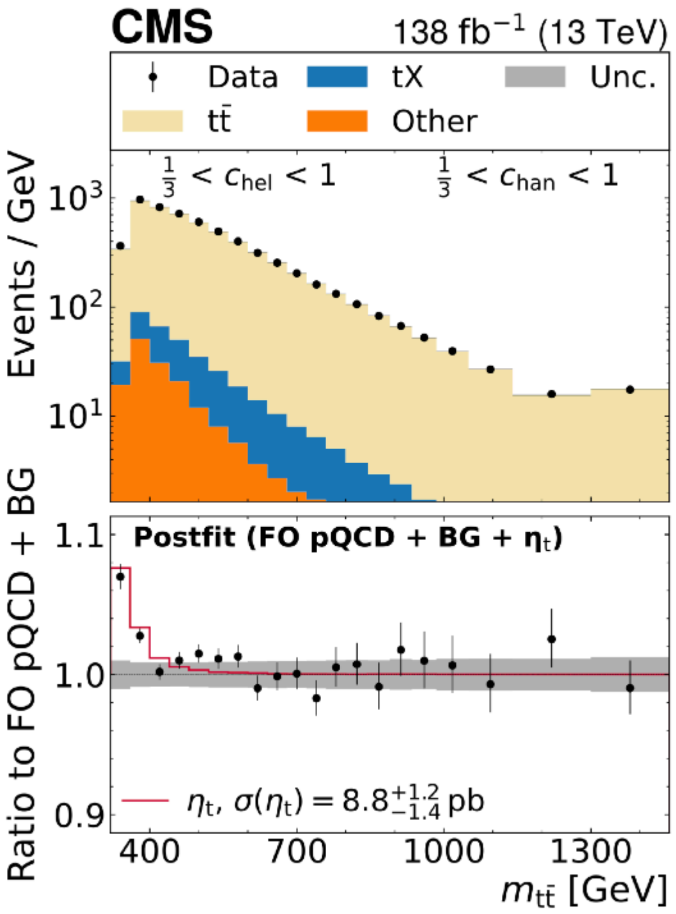
Enhancement ~ 10 pb

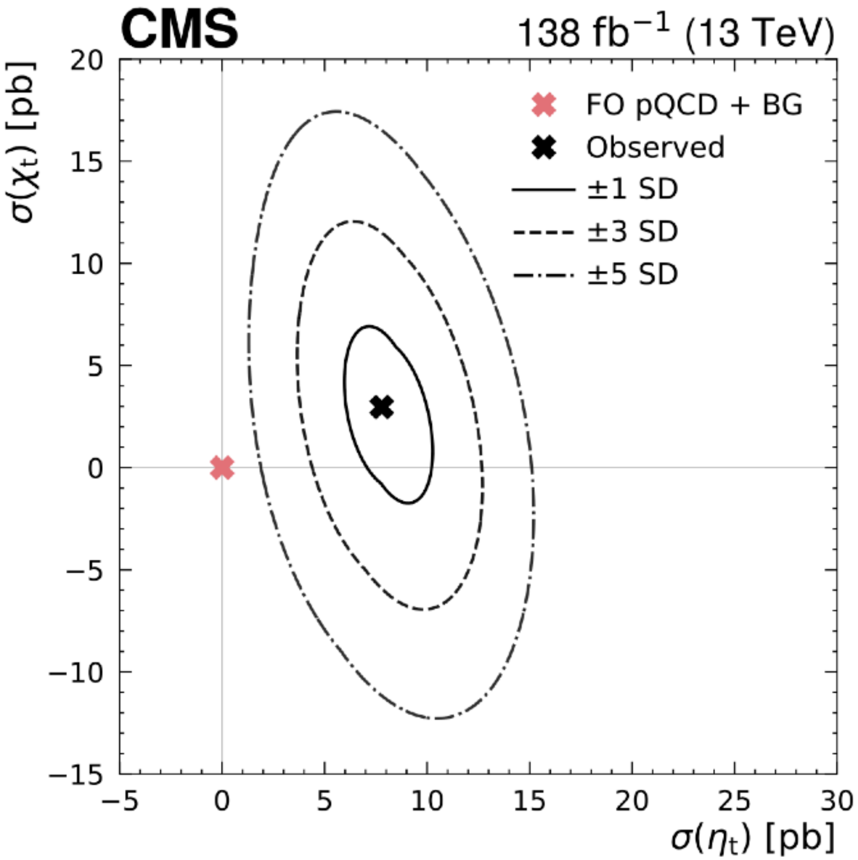
Account for most of the tension between direct and indirect measurements!

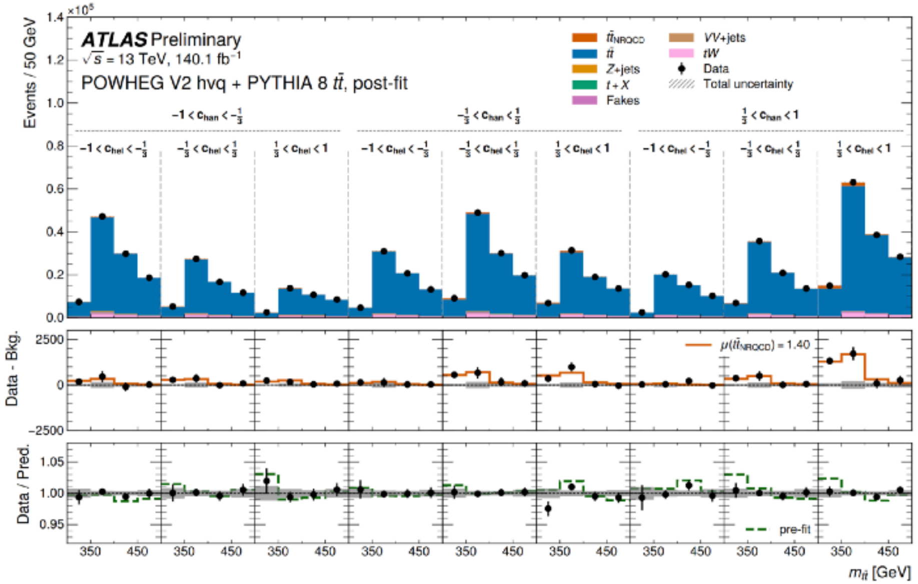
Recent CMS and ATLAS measurements

CMS: 2503.22382

ATLAS-CONF-2025-008

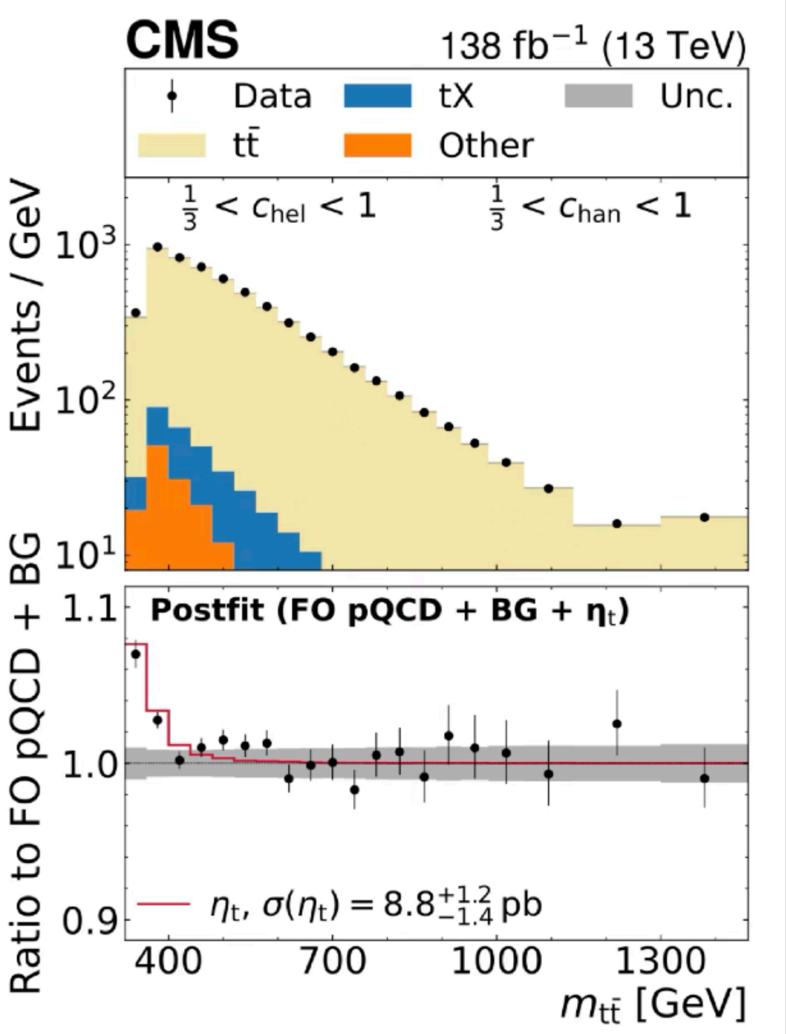






Observation of a pseudoscalar-like excess?

[CMS, RPP 88 (2025) 087801]



- A pseudo-scalar excess at $t\bar{t}$ threshold is observed with a selection cut on decay distribution

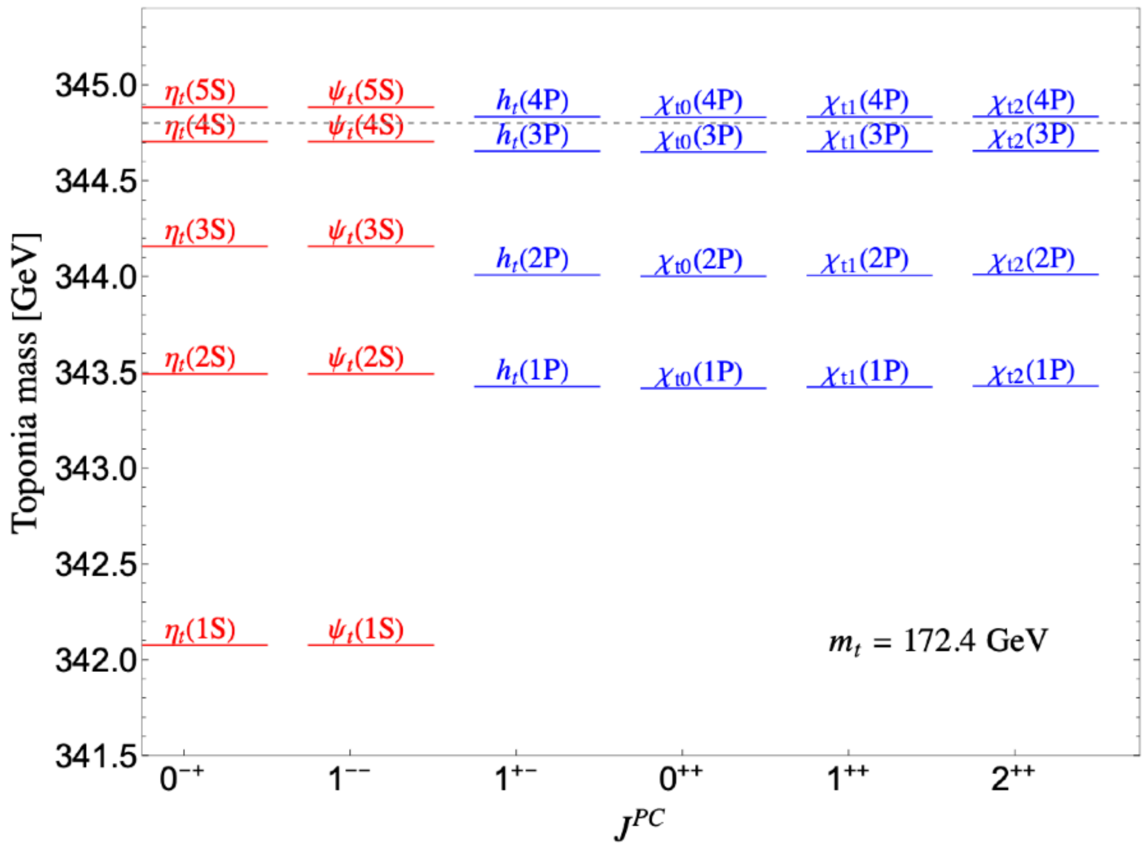
$$\sigma(\eta_t) = 8.8 \pm 0.5\,(\text{stat})\,^{+1.1}_{-1.3}\,(\text{syst})\,\text{pb} = 8.8\,^{+1.2}_{-1.4}\,\text{pb}.$$

- The cross section enhancement also confirmed by ATLAS [ATLAS-CONF-2025-008]
- Toponium? some theory modeling, e.g.

[Fuks, Hagiwara, Ma and Zheng, arXiv:2411.18962, arXiv:2509.03596]

Toponia

Bai, Chen, Yang, arXiv:2506.14552



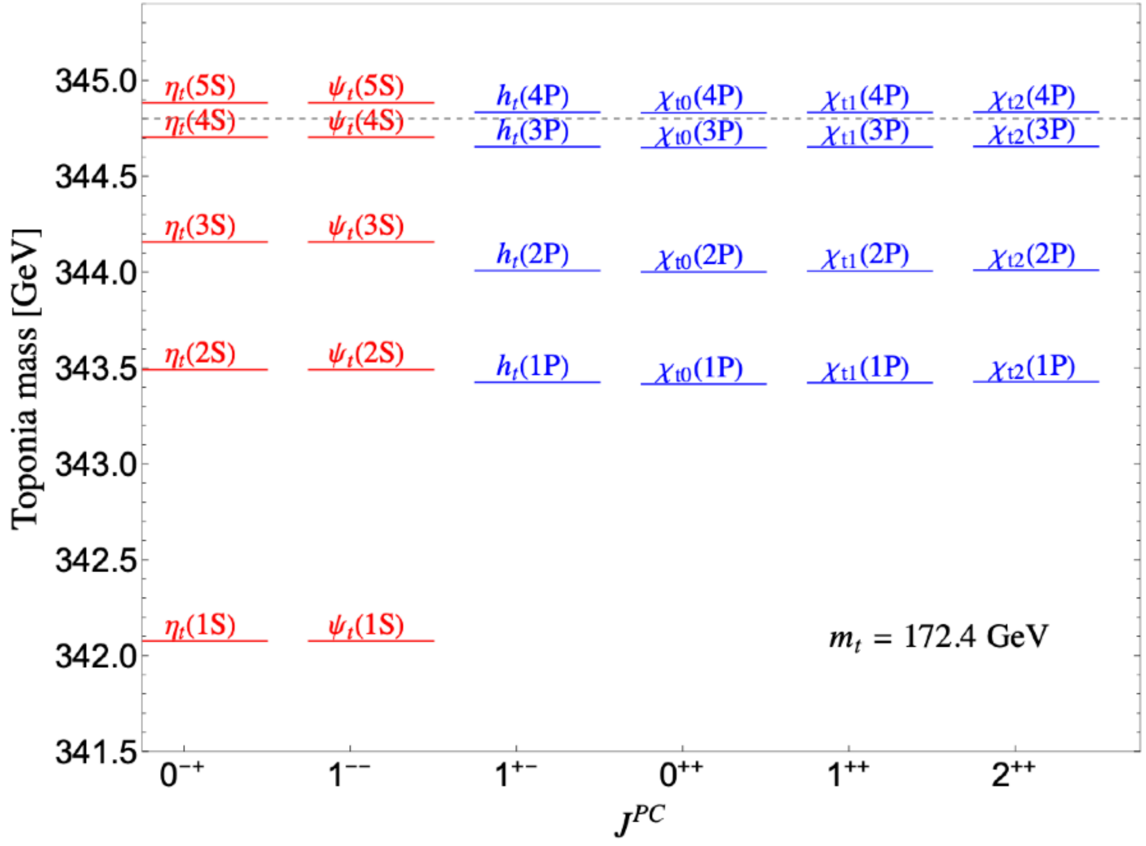
$$V_{\text{static}}(r) = \begin{cases} V_{\text{pQCD}}(r) & , \quad r \le r_0 \\ V_{\text{Cornell}}(r) & , \quad r > r_0 \end{cases}$$

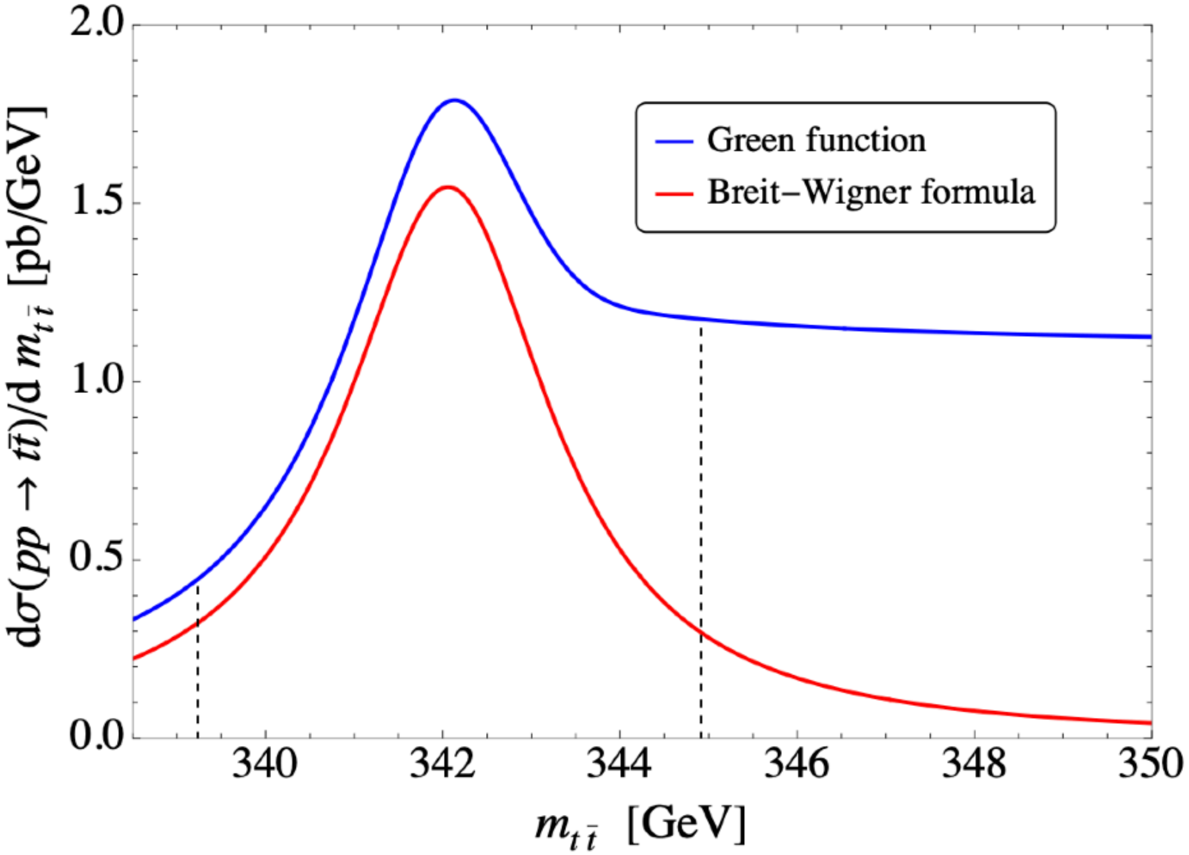
$$V_{\text{pQCD}}(r) = -\frac{C_F \alpha_s(\mu_R)}{r}\Big\{1 + \frac{\alpha_s(\mu_R)}{4\pi}\left[a_1 + 2\gamma_E \beta_0\right] + \left(\frac{\alpha_s(\mu_R)}{4\pi}\right)^2 \left[a_2 + \left(\frac{\pi^2}{3} + 4\gamma_E^2\right)\beta_0^2 + 2\gamma_E\left(2a_1\beta_0 + \beta_1\right)\right]\Big\}$$

$$V_{\text{Cornell}} = -\frac{C_F \alpha_s^{\text{Cornell}}}{r} + br$$

Toponia

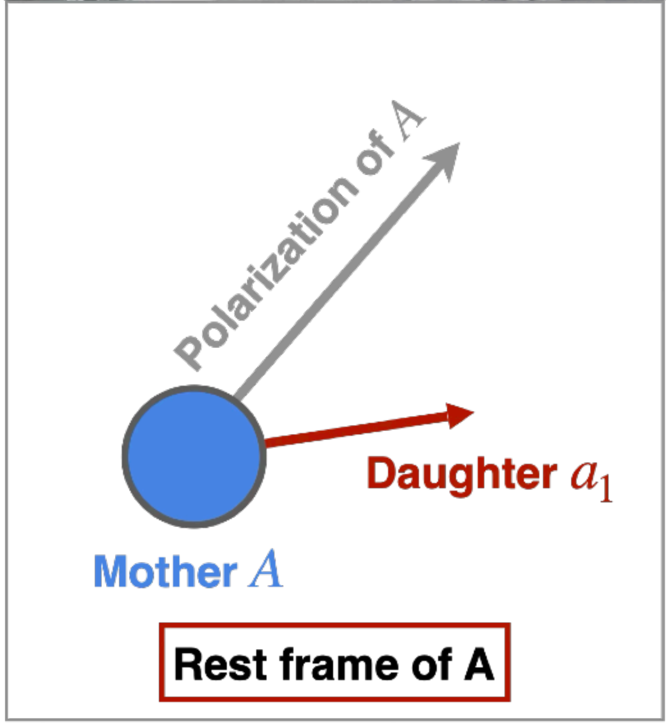
Bai, Chen, Yang, arXiv:2506.14552





Spin correlation and quantum information

- The decay of top quarks violates parity, allowing us to infer spin and spin correlation from decay distributions.
- E.g. $t \to \ell^+ \nu b, \bar{t} \to \ell^- \bar{\nu} \bar{b}$, spin vector σ_i can be measured from the mean value of the decay direction $\ell_i^{\pm}$ in $t/\bar{t}$ rest frame.



- Concurrence $\mathscr{C}$: a quantitative measure of entanglement

$$\mathscr{C} = \begin{cases} \frac{-\mathrm{Tr}(C) - 1}{2}, & \text{(spin singlet)} \\ \max_i \left[\frac{\mathrm{Tr}(C) - 2C_{ii} - 1}{2} \right], & \text{(spin triplet)} \end{cases}$$

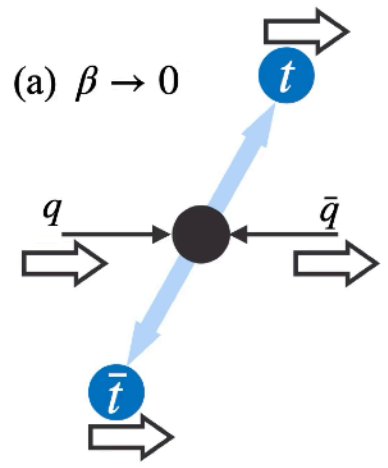
$\mathscr{C} > 0$: entangle
$\mathscr{C} \leq 0$: separable

$D = \mathrm{tr}(C)/3$ is directly related to the angle between ℓ^+ and ℓ^-

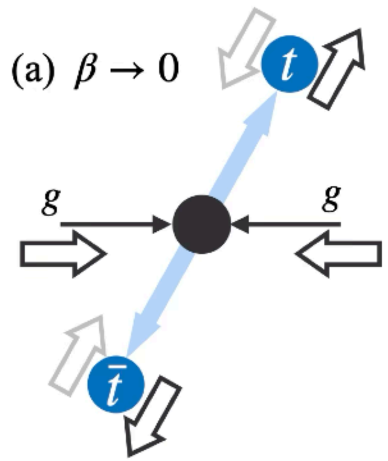
$D < -1/3$: entangle
$D \geq -1/3$: separable

Entangled $t\bar{t}$ at the LHC

[Fabbrichesi, Floreanini and Panizzo, arXiv:2102.11883]
[Afik and de Nova, arXiv:2203.05582]
[K. Cheng, T. Han and M. Low, arXiv:2311.09166] …



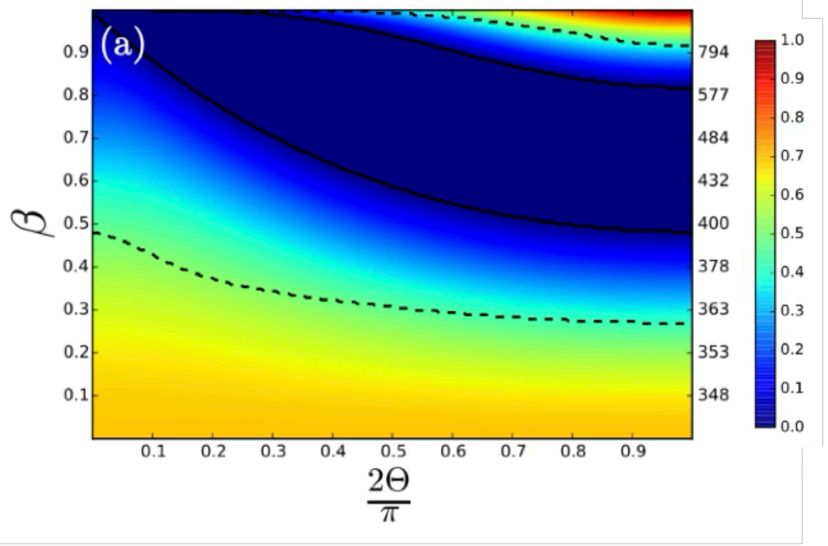
- Near threshold:
  - $q\bar{q}$ produces $|\uparrow\uparrow\rangle$ or $|\downarrow\downarrow\rangle$, —— **positive** spin correlation
  - gg produces $\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$ —— **negative** spin correlation
- The mixing of above two states cancels spin correlation, so entanglement is very sentitive to the percentage of contribution from spin singlet/triplet.



[K. Cheng, T. Han and M. Low, arXiv:2407.01672]

Near threshold, gg produces a Bell state, but a small mixing from $q\bar{q}$ process reduce the entanglement quite a bit.

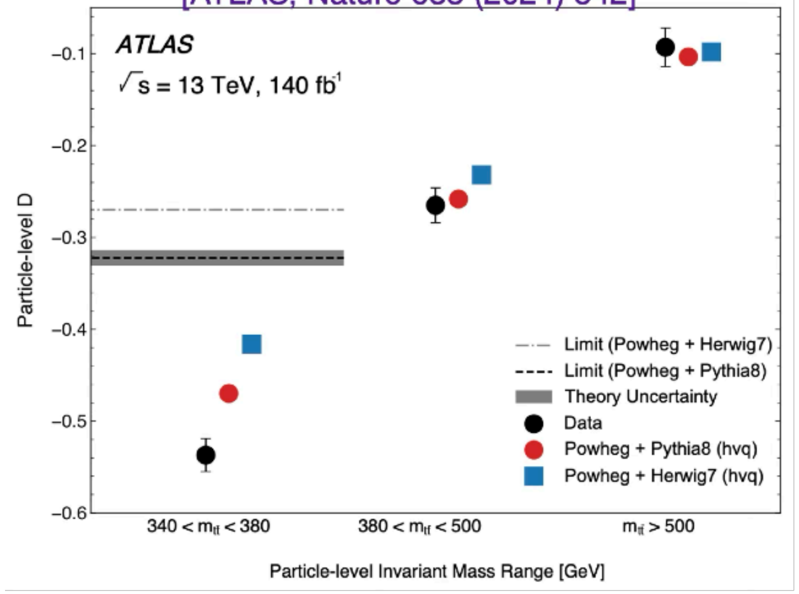
Concurrence



Entanglement of $t\bar{t}$ observed

more entangled than expected?

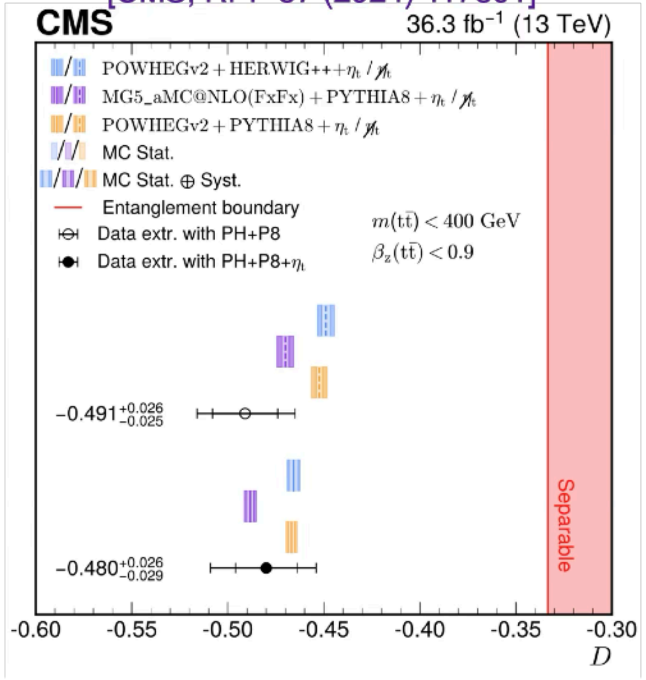
[ATLAS, Nature 633 (2024) 542]



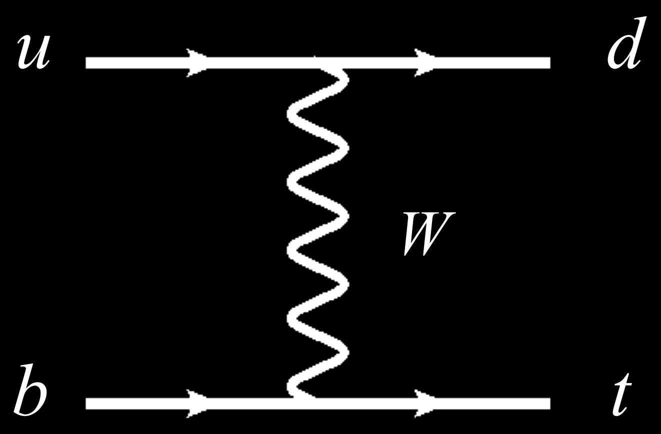
- Entanglement ($D < -1/3$) is observed near $t\bar{t}$ threshold
- First observation of entanglement in a pair of quarks.
- Entanglement is explored at the highest accesible energy scale

- Observation confirmed by CMS
- Prediction fits data better if there exists a pseudo-scalar η_t near $t\bar{t}$ threshold. (η_t couples to both gg and $t\bar{t}$)
- η_t produces a maximally entangled state

[CMS, RPP 87 (2024) 117801]

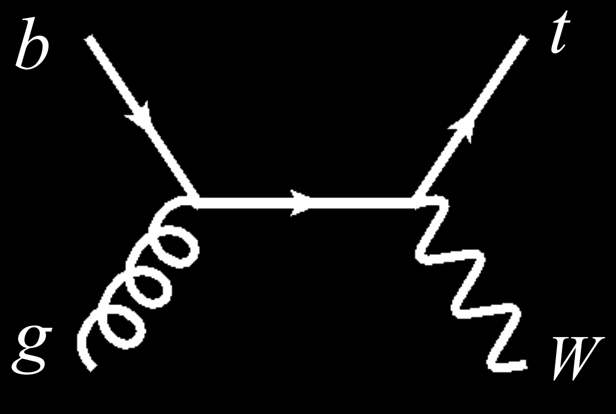


Single top-quark production



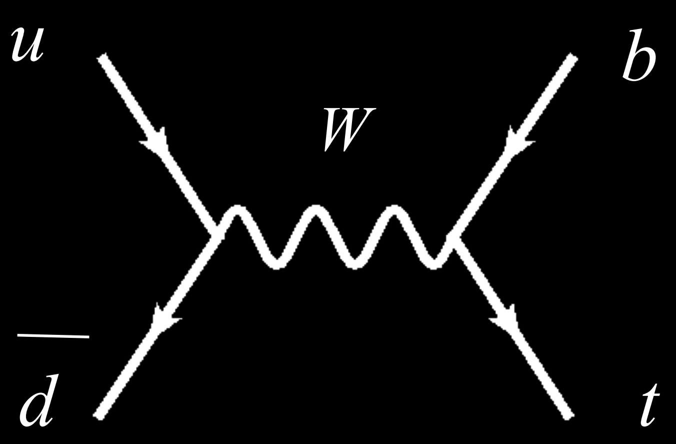
t-channel

$Q_W^2 < 0$



tW

$Q_W^2 = m_W^2$



s-channel

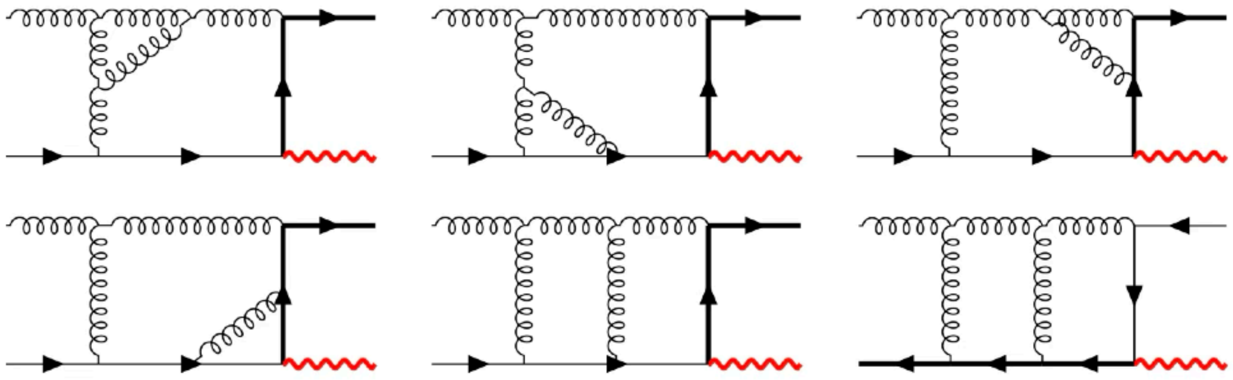
$Q_W^2 > 0$

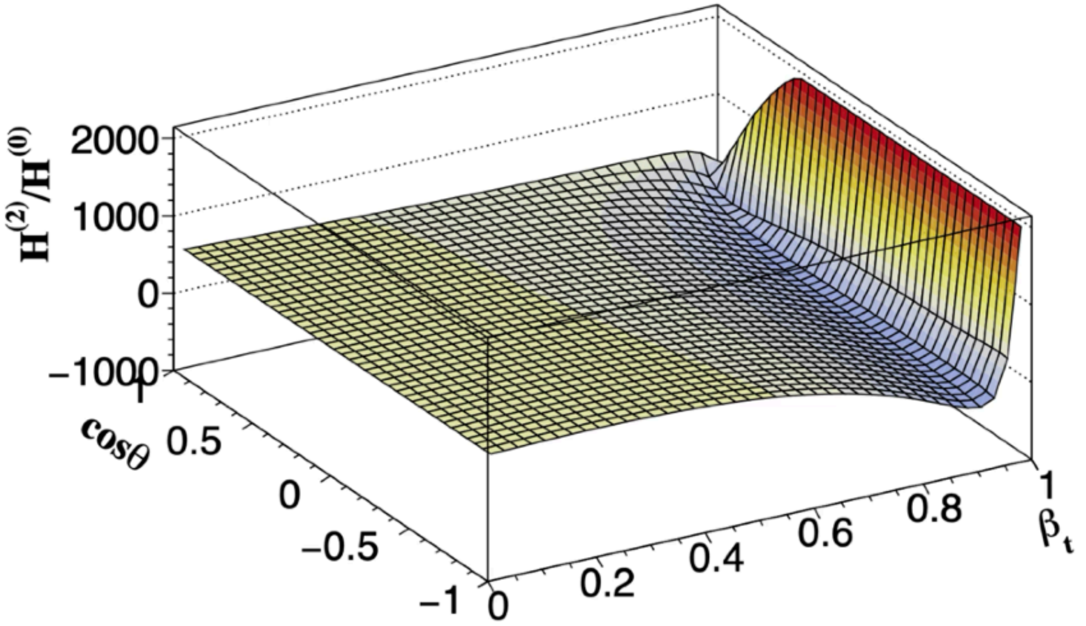
Single top production: tW production

Two-loop virtual corrections to tW production

Amplitude reads $\mathcal{M} = \mathcal{M}_0 + \frac{\alpha_s}{4\pi}\mathcal{M}_1 + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathcal{M}_2$

Tree   1-loop   2-loop





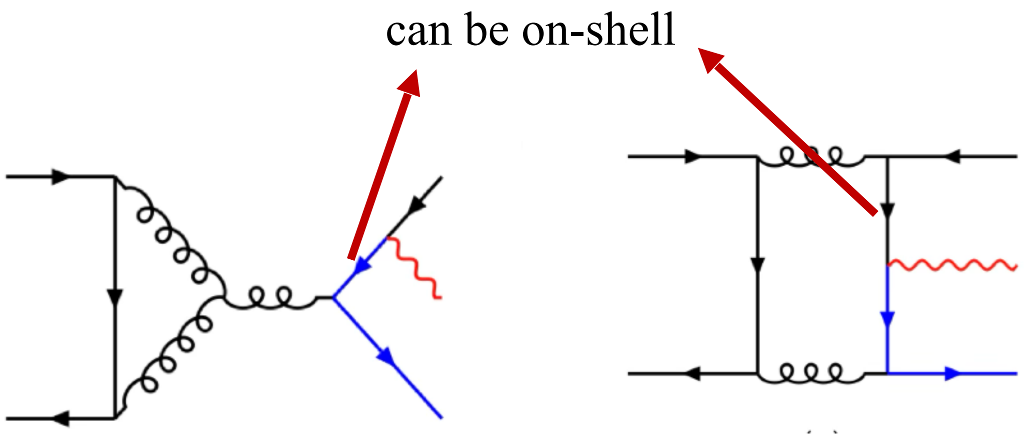
Chen, Dong, Li, Li, Wang, Wang, arXiv:2212.07190
Chen, Dong, Li, Li, Wang, Wang, arXiv:2208.08786

Single top production: tW production

Real-virtual for tW@NNLO

Investigate the resonance subtraction method at 1 loop level

Possible diagrams with resonance contribution



Matrix element square can be expanded in terms of

$$RV_{\text{Ren}} \equiv 2\,\mathbf{Re}\left[\mathscr{M}^{(0)*}\mathscr{M}^{(1)}_{\text{Ren}}\right] = \frac{C^{(2)}}{\Delta^2} + \frac{C^{(1)}}{\Delta} + C^{(0)} + \cdots$$

the leading divergent part can be subtracted

$$C^{(2)} = \frac{\alpha_s}{2\pi}B^{(2)}\Bigg\{-\frac{3C_F}{\epsilon^2} + \frac{1}{\epsilon}\Bigg[(C_A - 2C_F)\log\left(\frac{\mu^2}{s_{12}}\right) - 2\,(C_A - 2C_F)\log\left(\frac{\mu^2}{-s_{13}}\right)$$
$$+(C_A - 4C_F)\log\left(\frac{\mu^2}{-s_{23}}\right) + (C_A - 4C_F)\log\left(\frac{m_t\mu}{-s_{15} + m_t^2}\right)$$
$$-2\,(C_A - 2C_F)\log\left(\frac{m_t\mu}{-s_{25} + m_t^2}\right) + (C_A - 2C_F)\log\left(\frac{m_t\mu}{s_{35} - m_t^2}\right) - \frac{11}{2}C_F\Bigg]\Bigg\}$$
$$+\cdots$$

Dong, Li, Wang, arXiv:2411.07455

Top decay width Γ_t

First fully analytical NNLO QCD corrections

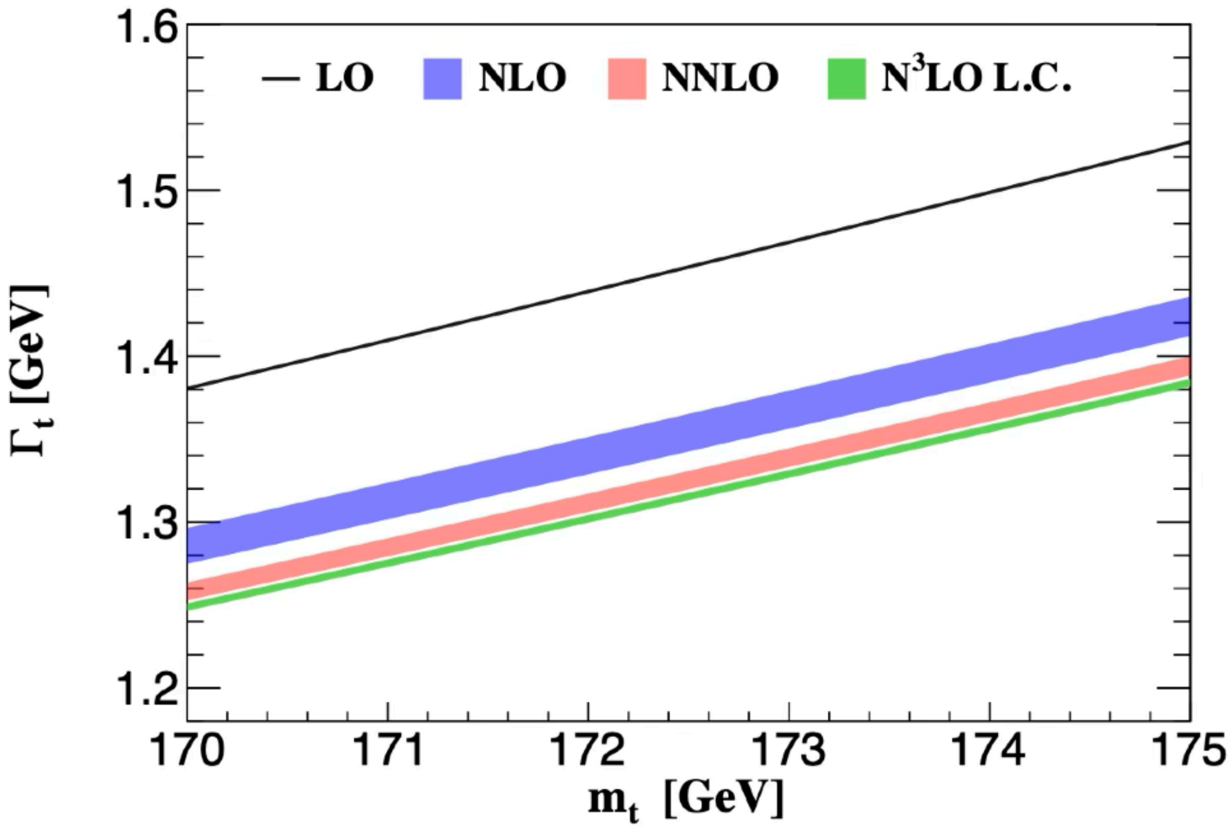
Chen, Li, Wang, Wang, arXiv:2212.06341

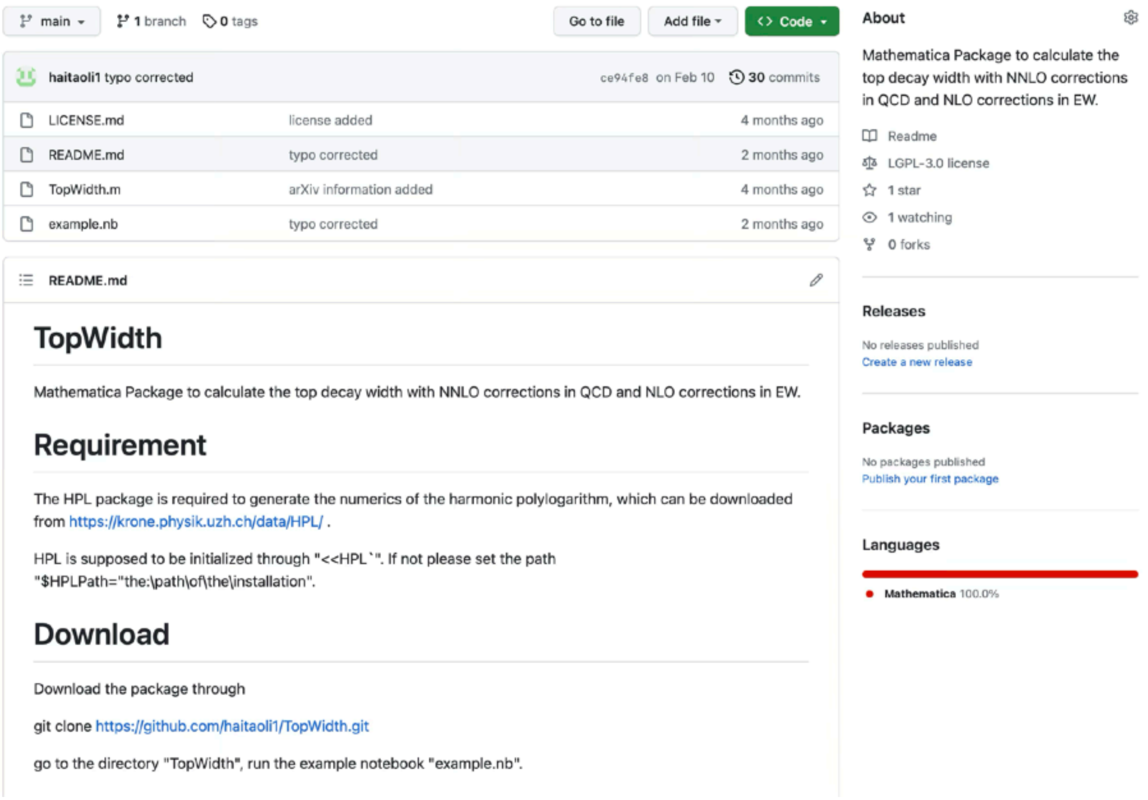
First analytical leading color N3LO QCD corrections

Chen, Li, Li, Wang, Wang, Wu, arXiv:2309.00762

Numerical N3LO QCD corrections

Chen, Chen, Guan, Ma,, arXiv: 2309.01937

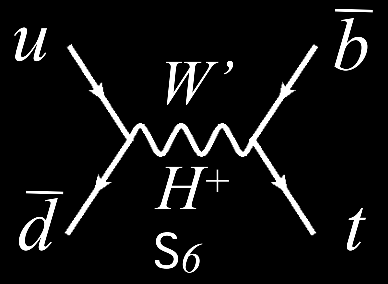


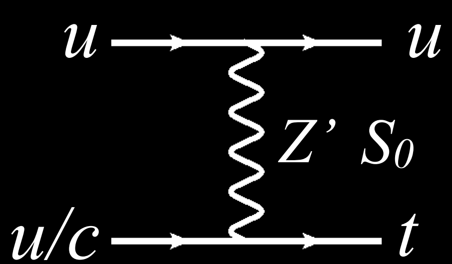


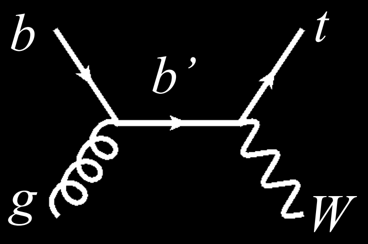
https://github.com/haitaoli1/TopWidth

Top Quark Physics : rich signatures of NP

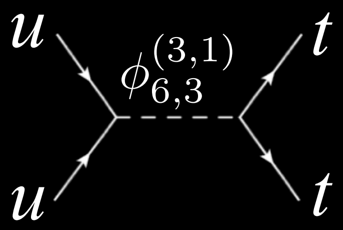
Single Top

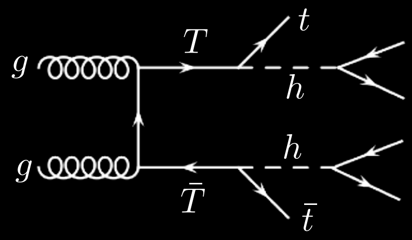


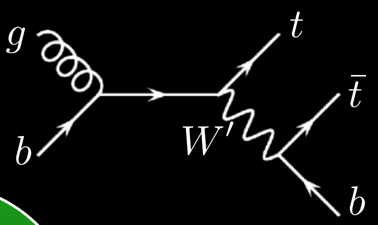


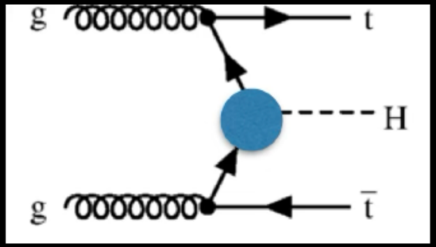


Top Pair



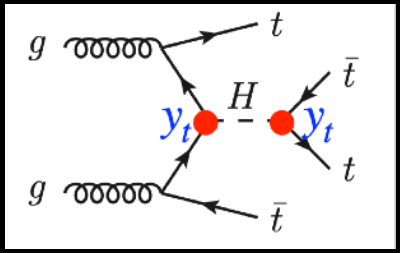


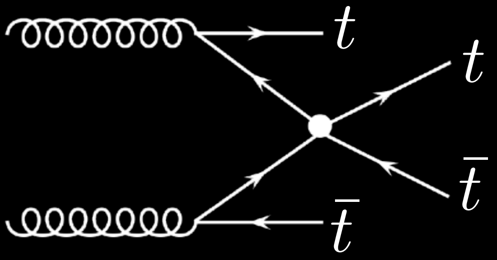


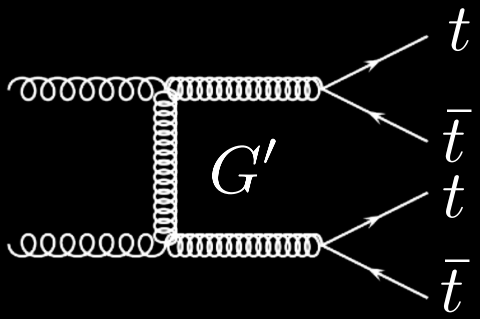


Yukawa y_t
CPV

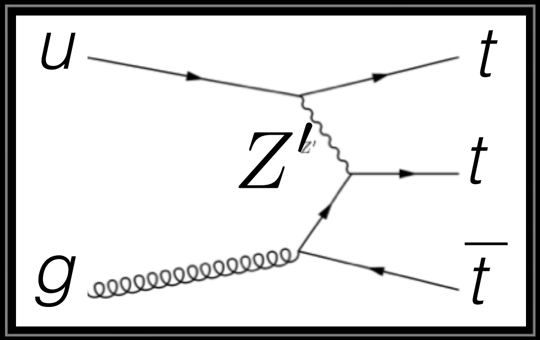
Four Tops







Triple Tops



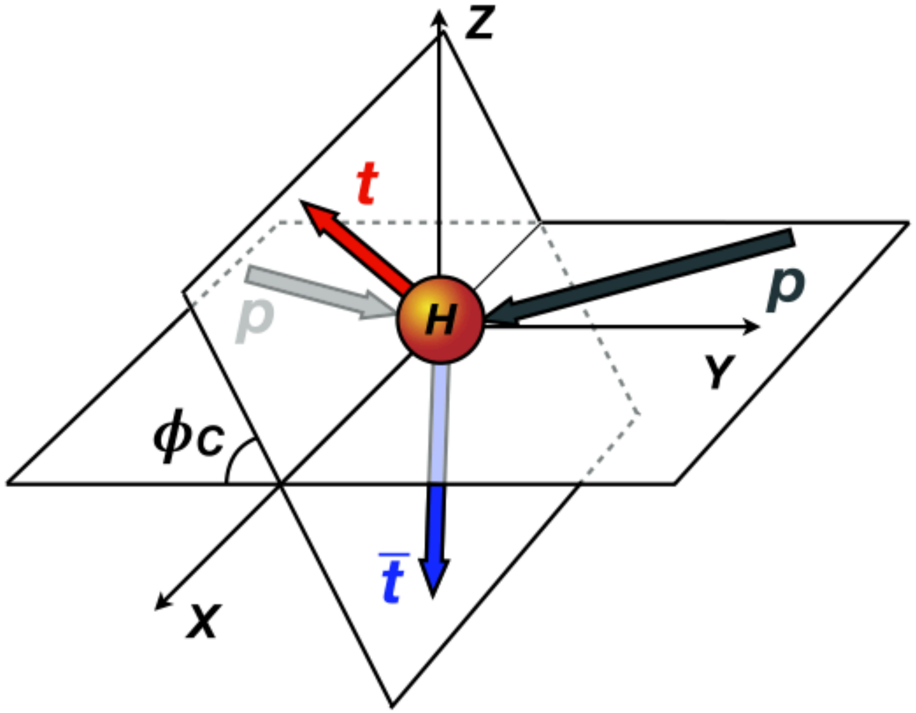
Flavor changing+Flavor conserving

1) CPV measurement in the $\bar{t}tH$ production

QHC, Xie, Zhang, Zhang, 2008.13442

CP property of $t\bar{t}H$ interaction

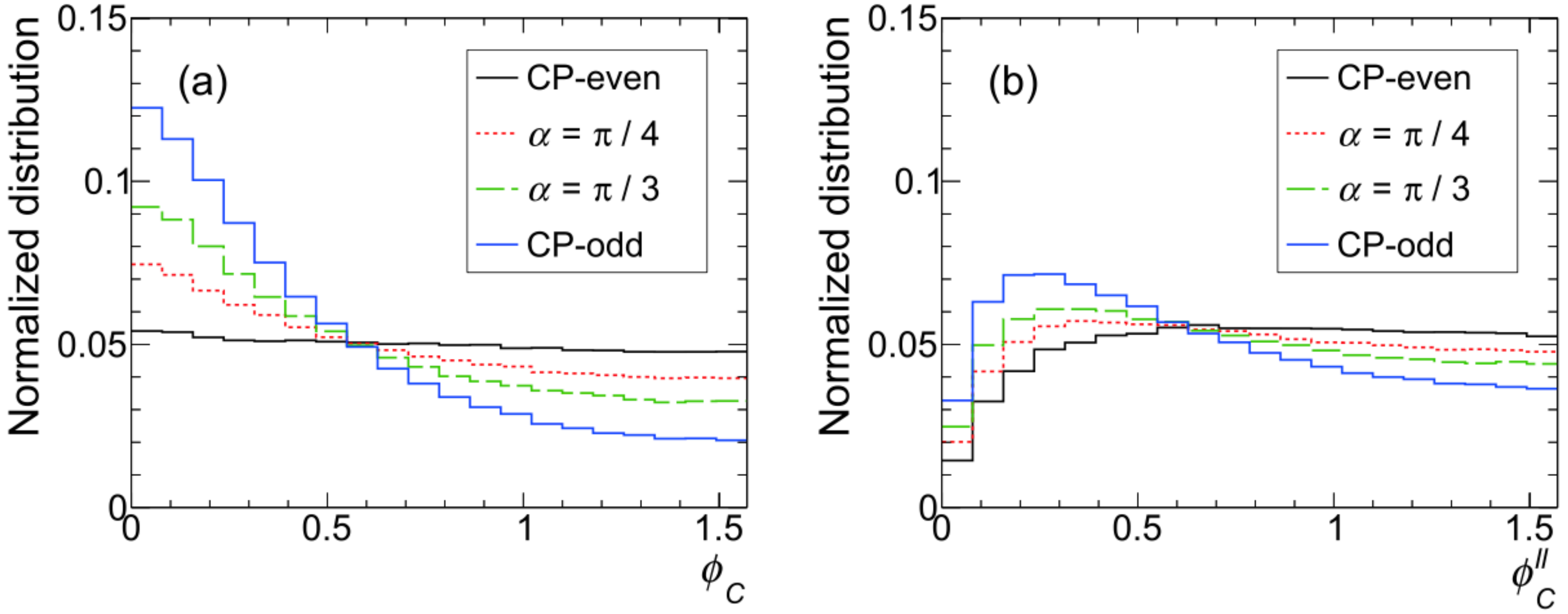
$$\mathscr{L} \supset -\, y_t\, \bar{t}\, e^{i\alpha\gamma^5} t\ H$$



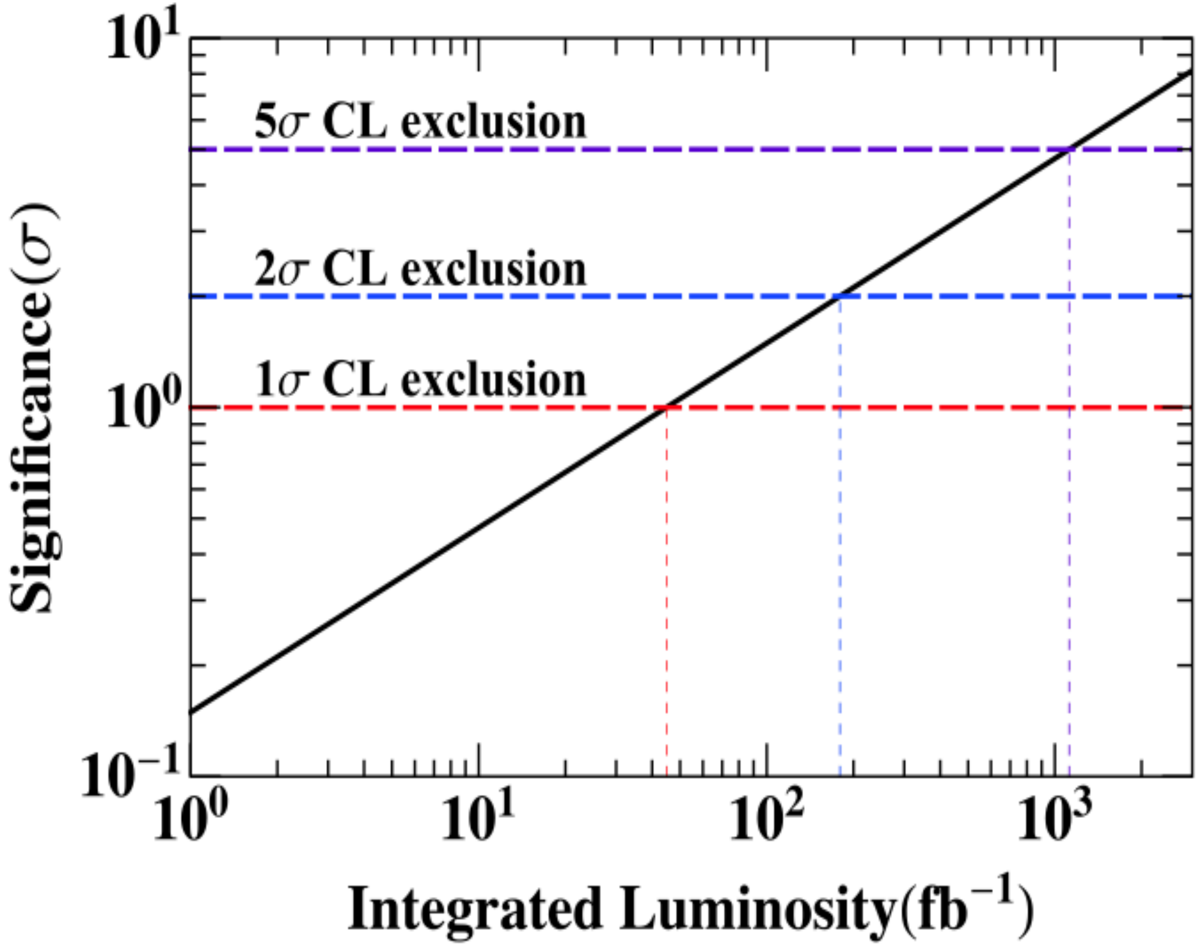
$$\cos\phi_C = \frac{\left|(\boldsymbol{n}_{p_1} \times \boldsymbol{n}_{p_2}) \cdot (\boldsymbol{n}_t \times \boldsymbol{n}_{\bar{t}})\right|}{\left|\boldsymbol{n}_{p_1} \times \boldsymbol{n}_{p_2}\right| \cdot \left|\boldsymbol{n}_t \times \boldsymbol{n}_{\bar{t}}\right|}.$$

$$\cos\phi_C^{\ell\ell} = \frac{\left|(\boldsymbol{n}_{p_1} \times \boldsymbol{n}_{p_2}) \cdot (\boldsymbol{n}_{\ell^+} \times \boldsymbol{n}_{\ell^-})\right|}{\left|\boldsymbol{n}_{p_1} \times \boldsymbol{n}_{p_2}\right| \cdot \left|\boldsymbol{n}_{\ell^+} \times \boldsymbol{n}_{\ell^-}\right|}.$$

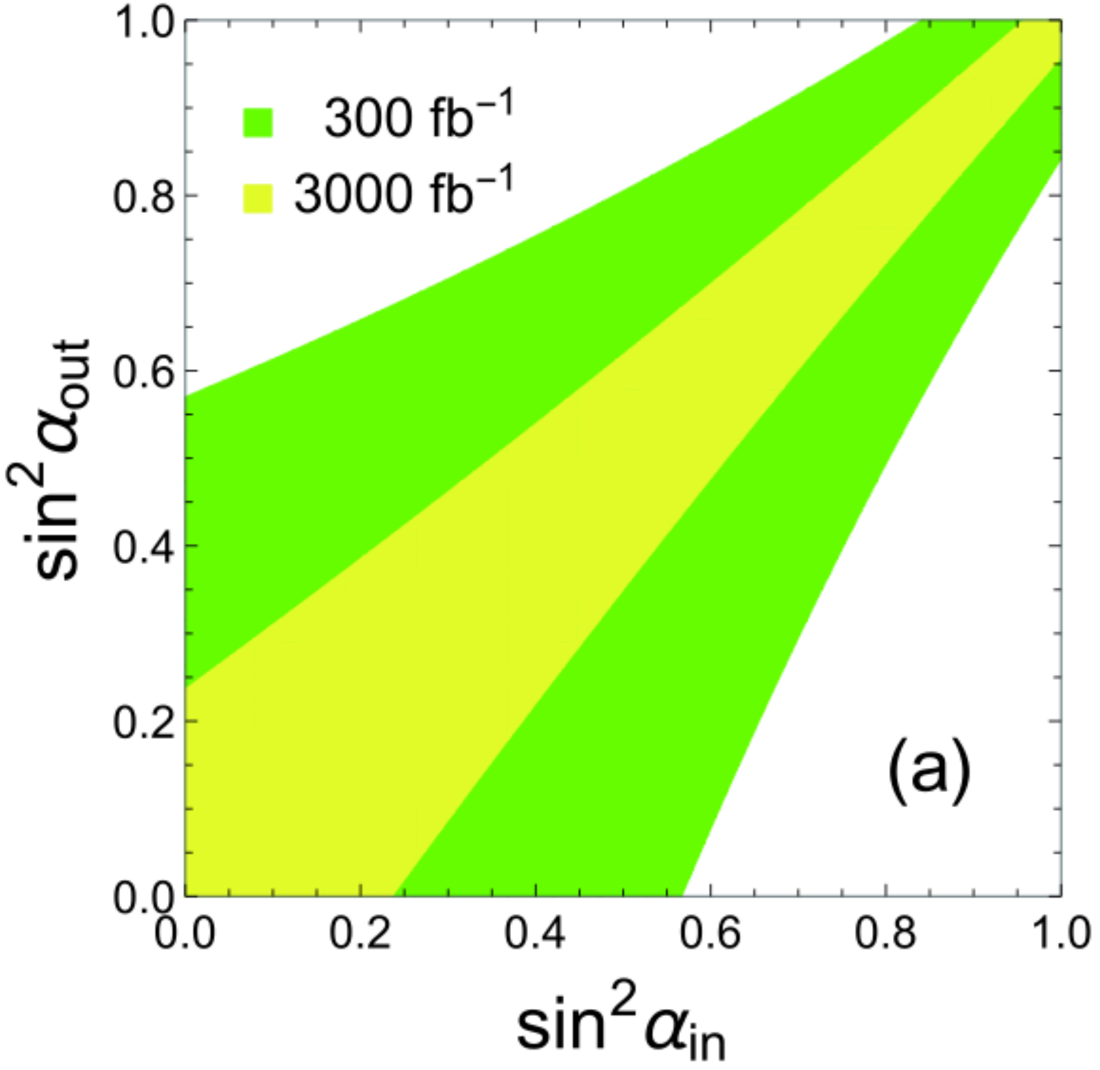
1) CPV measurement in the $\bar{t}tH$ production



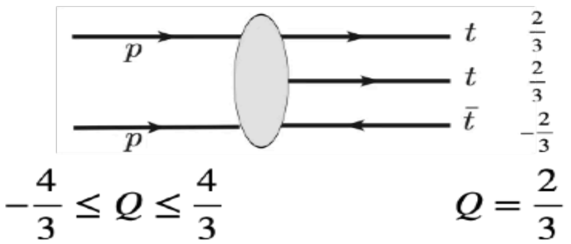
1) CPV measurement in the $\bar{t}tH$ production

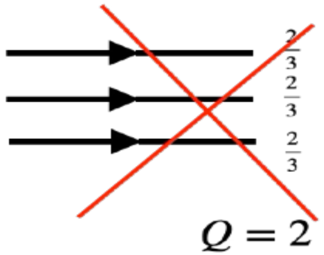


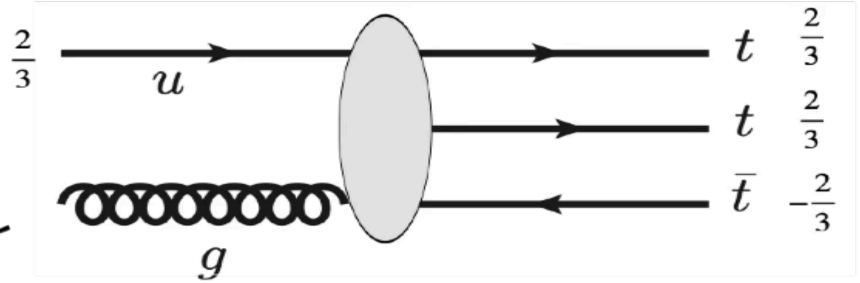
CP-odd vs CP-even

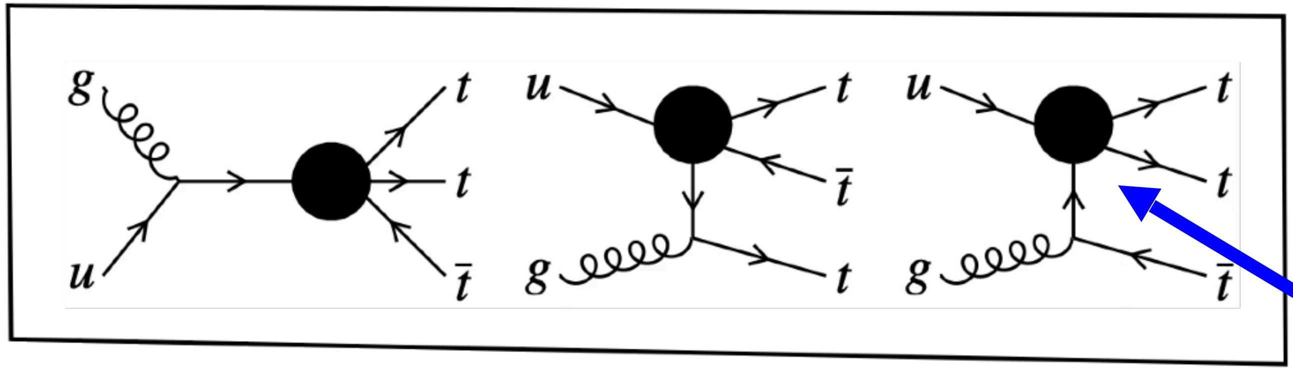


2) Triple Top-Quark Production at the LHC

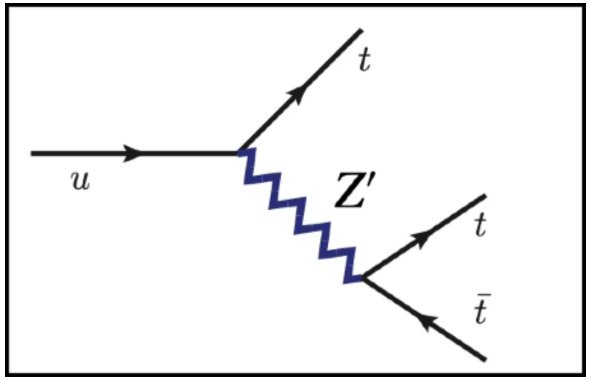








Top FCNC

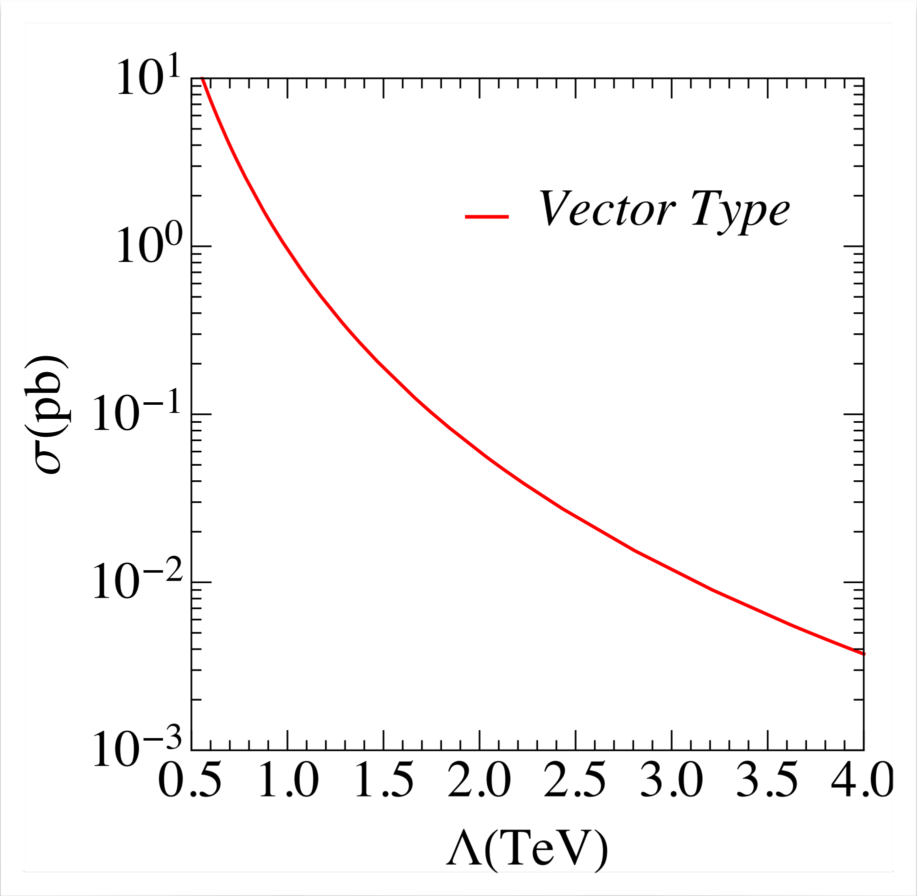


Coexistence
- Top FCNC
- Top flavor conserving

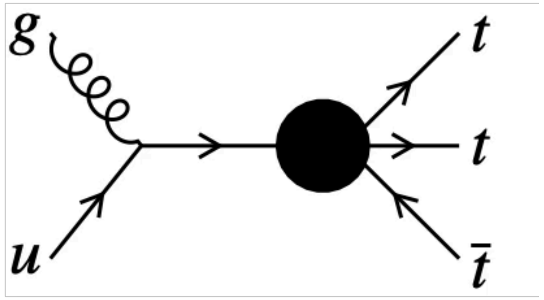
$$\mathcal{O}^{V}_{uttt} = \frac{f^{V}_{FVNI}\, f^{V}_{FCNI}}{\Lambda^2} (\bar{t}\gamma^{\mu} P_R t)(\,\bar{t}\,\gamma_{\mu} P_R u)$$

2) Triple Top-Quark Production at the LHC

QHC, Chen, Liu, Wang,1901.04643

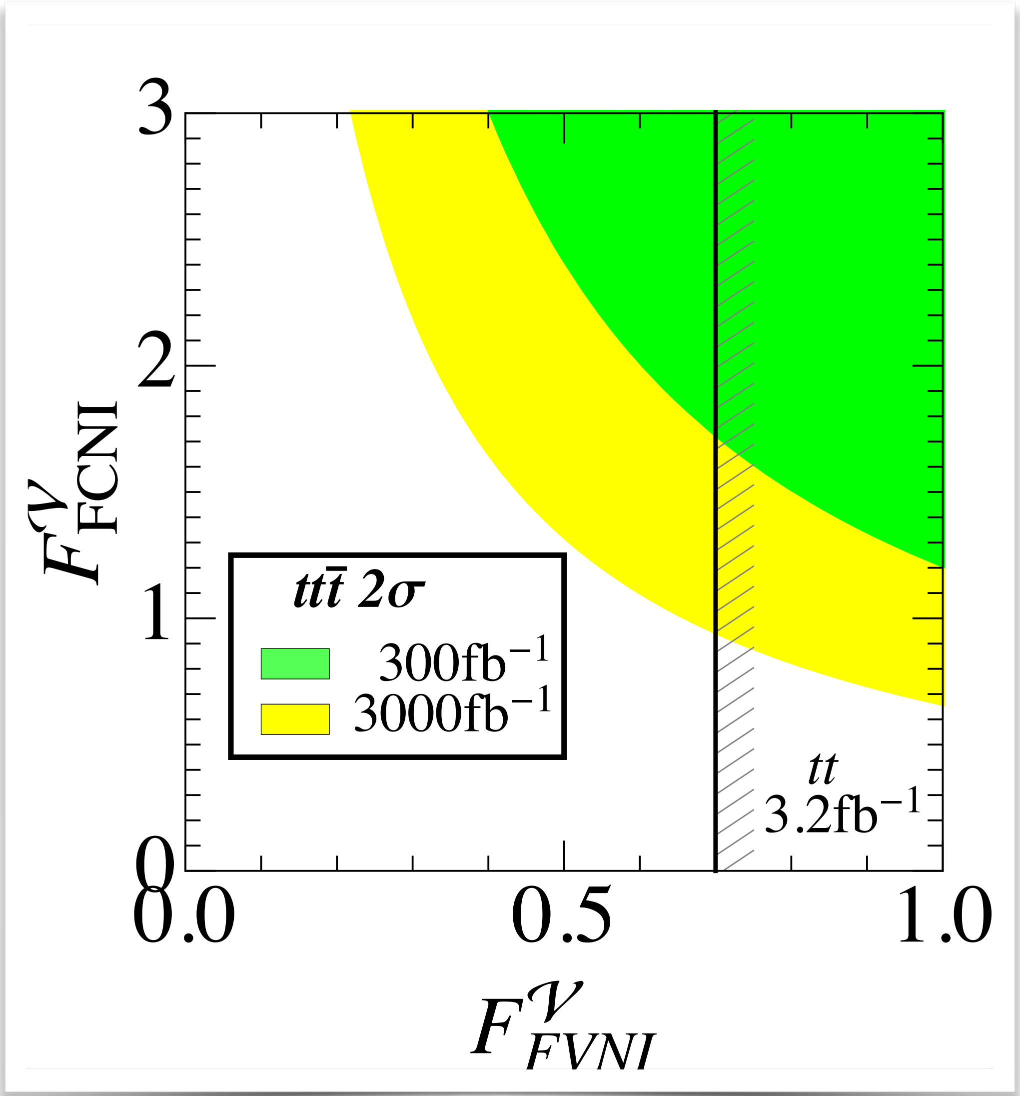


$$\mathcal{O}^V_{uttt} = \frac{f^V_{FVNI}\, f^V_{FCNI}}{\Lambda^2} (\bar{t}\gamma^\mu P_R t)(\bar{t}\,\gamma_\mu P_R u)$$

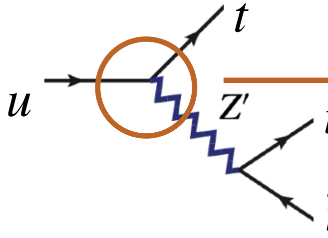


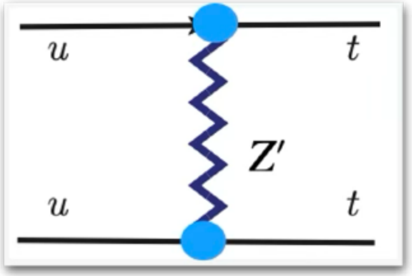
$$\sigma^V_{ttt} = 0.9582 \times \left(f^V_{FVNI}\, f^V_{FCNI}\right)^2 \left(\frac{TeV}{\Lambda}\right)^4 \text{pb}$$

$tt\bar{t}/\bar{t}\bar{t}t$ and $tt/\bar{t}\bar{t}$ Production



$$\mathcal{O}_{uutt} = \frac{1}{2}\left(\frac{f^{V}_{FVNI}}{\Lambda^2}\right)^2 \left(\bar{t}\gamma_\mu P_R u\right)\left(\bar{t}\gamma_\mu P_R u\right)$$





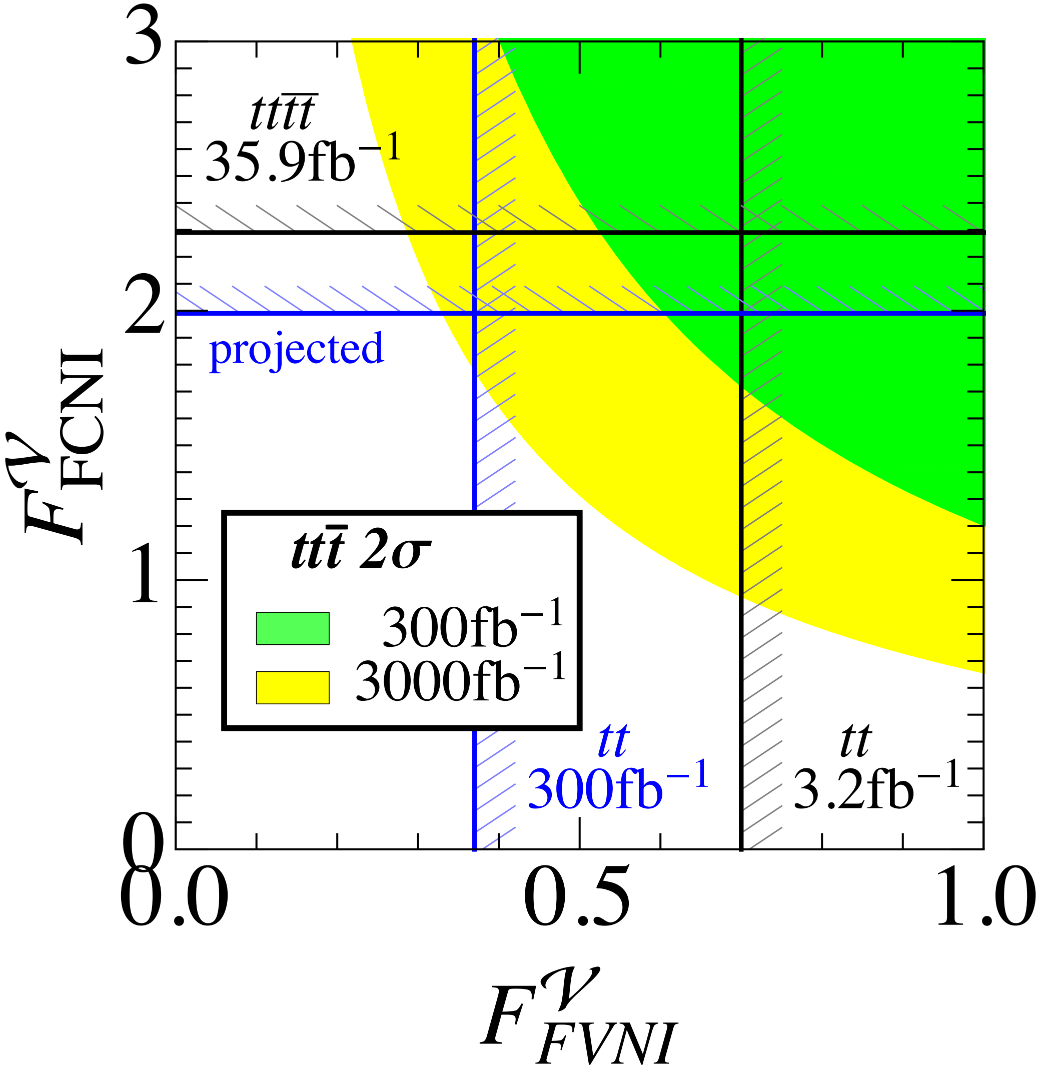
$$\sigma^{V}_{tt} = 52.28 \times \left(f^{V}_{FVNI}\right)^4 \left(\frac{TeV}{\Lambda}\right)^4 \text{ pb}$$

ATLAS, 13 TeV with $\mathcal{L} = 3.2\ fb^{-1}$ JHEP 1406 (2014) 035

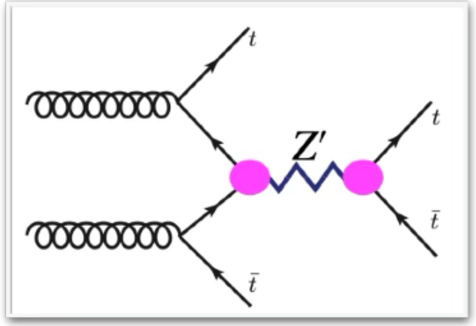
$$f^{V}_{FVNI} \leq 0.70\frac{\Lambda}{TeV}$$

Joint constraints from multiple top processes

triple top + four top quarks production



$$\mathcal{O}_{tttt} = \frac{1}{2}\left(\frac{f^V_{FCNI}}{\Lambda^2}\right)^2 (\bar{t}\,\gamma_\mu P_R t)\,(\bar{t}\,\gamma_\mu P_R t)$$



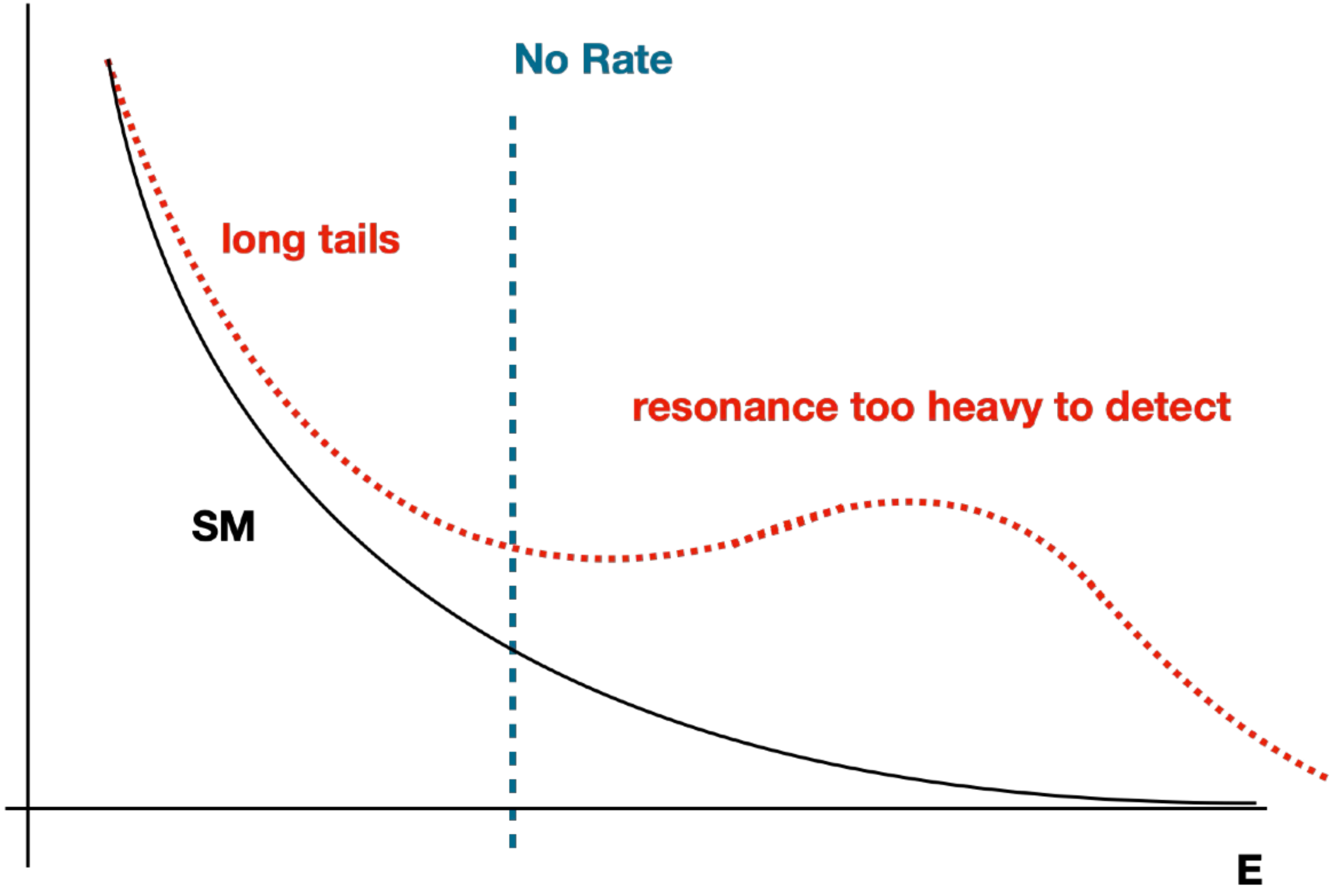
$$\sigma^V_{tttt} = 1.166 \times (f^V_{FCNI})^4 \left(\frac{TeV}{\Lambda}\right)^4 \text{ pb}$$

CMS, 13 TeV with $\mathscr{L} = 35.9\ \text{fb}^{-1}$

$$\Rightarrow\quad f^V_{FCNI} \leq 2.29\frac{\Lambda}{TeV}$$

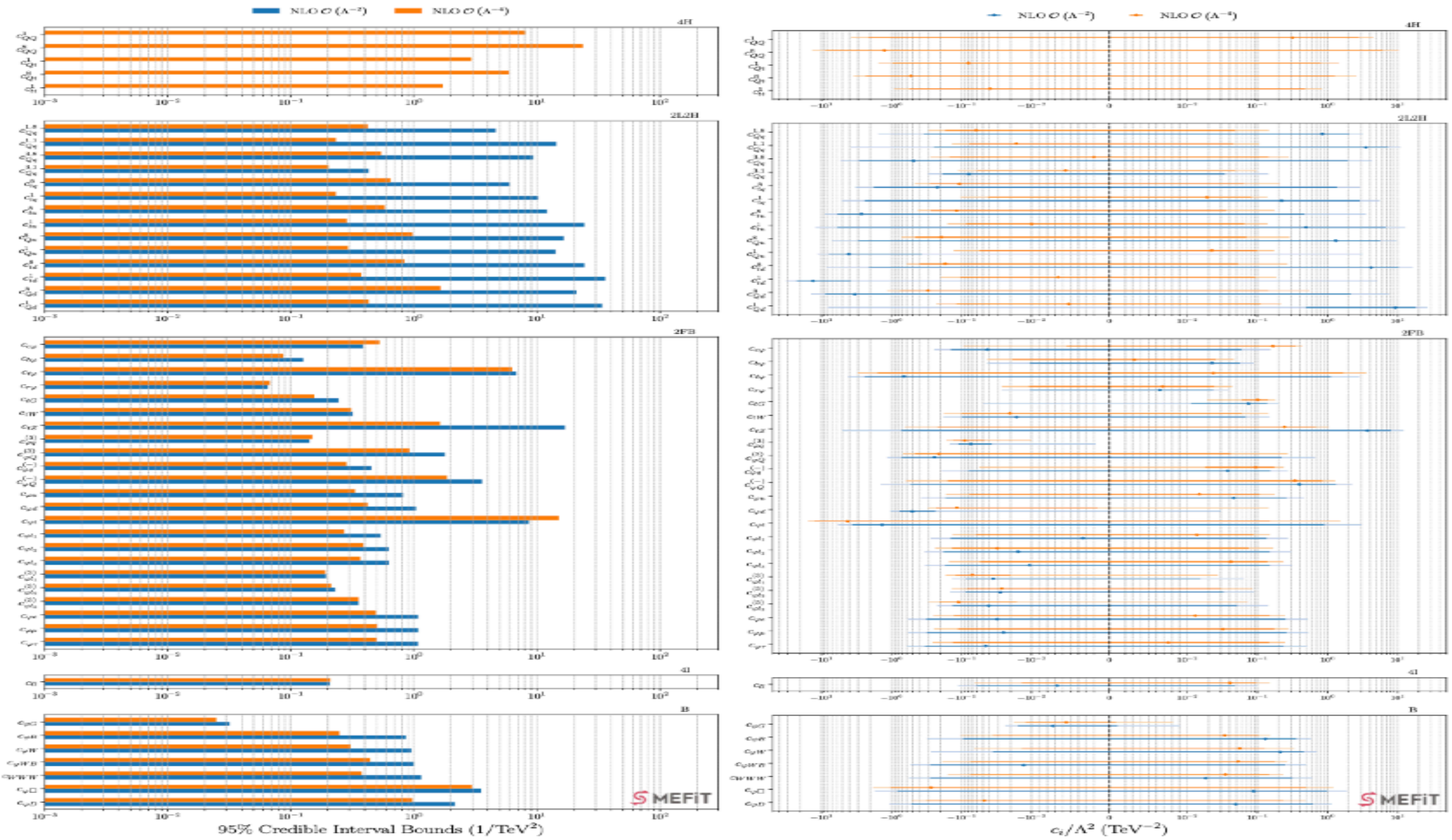
Top quark physics in the SMEFT

EFT provides a model-independent framework to search for NP.



Top quark physics in the SMEFT

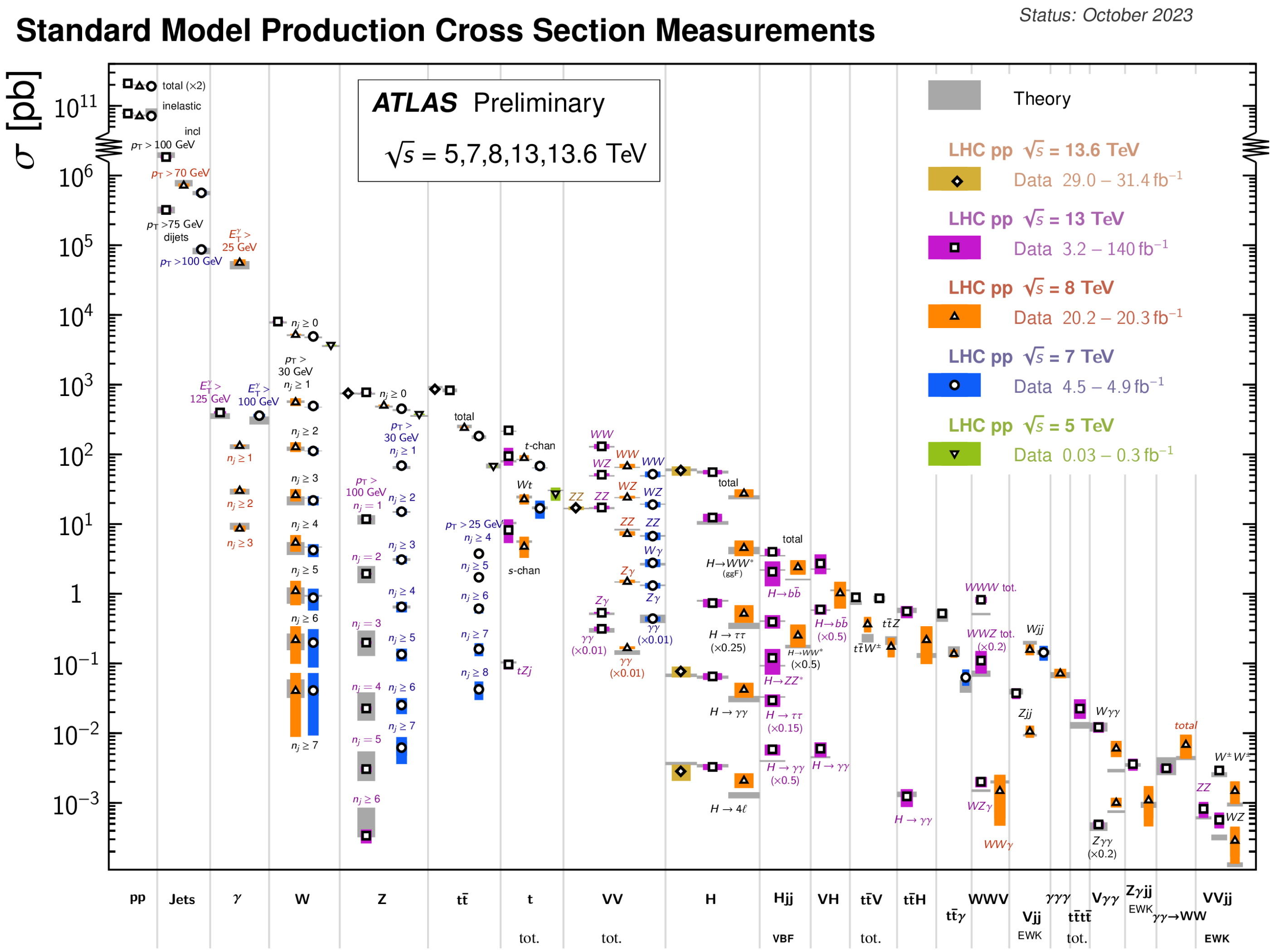
By combining top, Higgs, diboson, and electroweak precision data, one can constrain the parameter space from all directions.



E. Celada, T. Giani, J. Hoeve, L. Mantani, J. Rojo, A. N. Rossia, M. O. A. Thomas, E. Vryonidou
JHEP 09 (2024) 091

Summary

We are in an era of precision, and it is time to consider what we can learn from the data.



Thank You!