Measurements of the CKM angle γ/ϕ_3 at LHCb and Belle (II) with inputs from BES III

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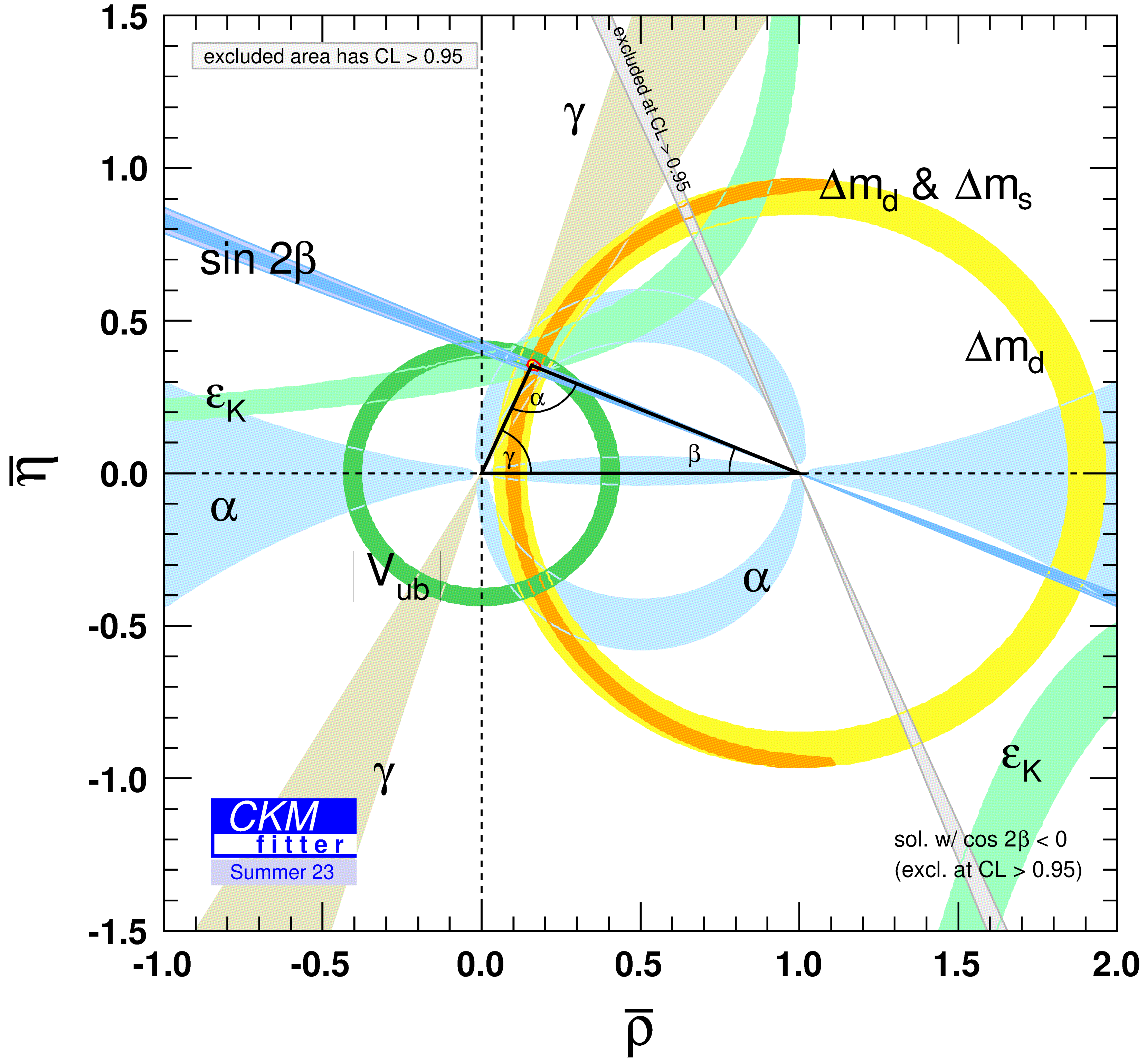
17th September 2025

CKM angle γ

- CKM matrix links quark mass and flavour eigenstates
- Unitary in the SM
  - $\implies$ triangles in the complex plane
- Use measurements to overconstrain them and search for new physics
- Also compare direct to indirect measurements
  - CKMfitter 2023 indirect world average $\gamma = (66.3^{+0.7}_{-1.9})^\circ$
  - HFLAV 2025 direct world average $\gamma = (66.4^{+2.7}_{-2.8})^\circ$
  - Global Beauty and Charm average $\gamma = (65.7 \pm 2.5)^\circ$ Phys. Rev. D 112, 013004
- Tree-level γ measurements have very low theory uncertainties - excellent benchmark parameter JHEP 01 (2014) 051
  - Next milestone for direct measurement is 1° uncertainty

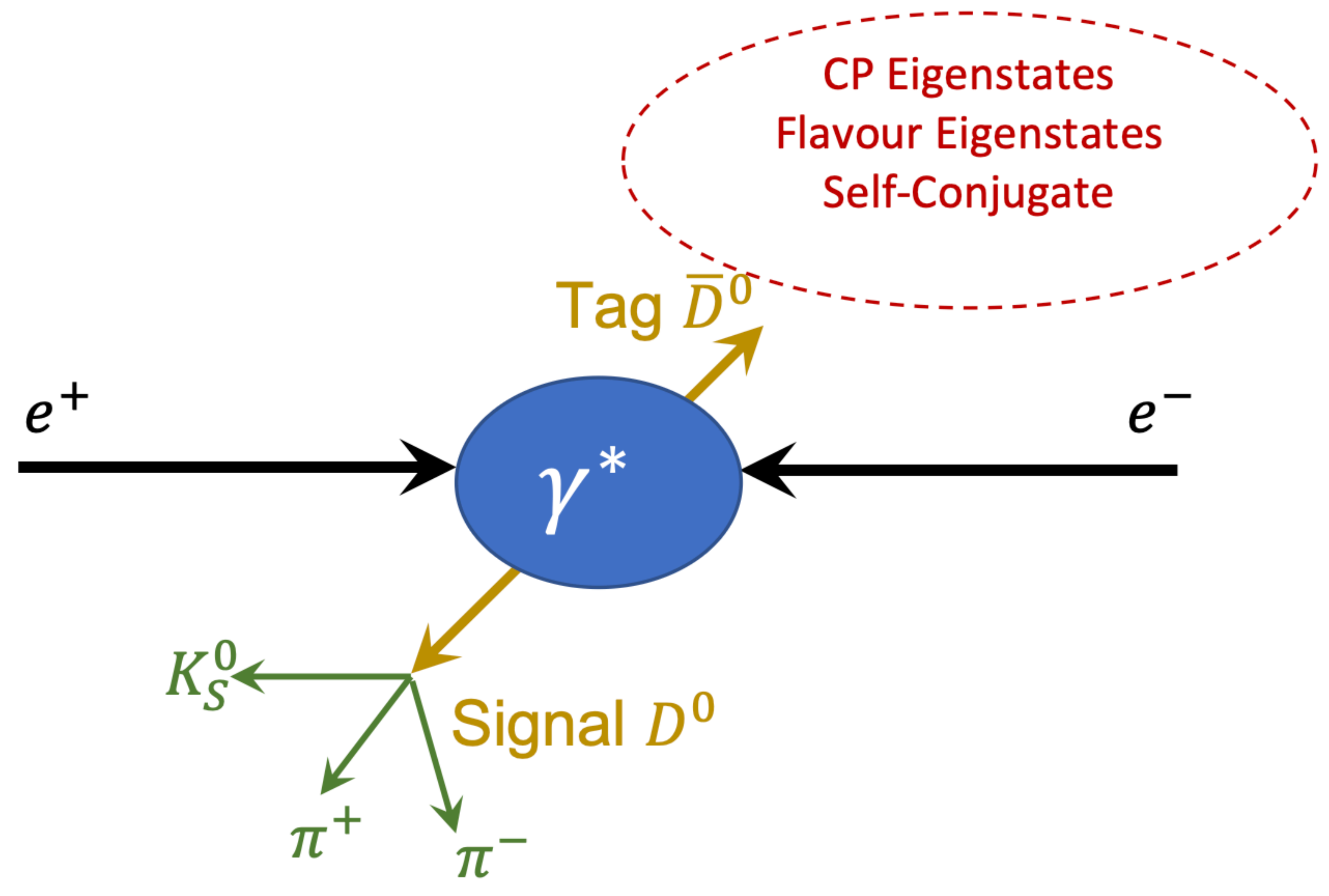
$$V_{\rm CKM} \sim \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix}$$



CKMfitter fit of the db unitarity triangle

BES III Inputs

- e^+e^- at the $D\bar{D}$ threshold ensures a clean environment
- CP-odd quantum correlated Charm pairs from a virtual photon
- $20.3\ \text{fb}^{-1}$ of $\psi(3770) \to D\bar{D}$
  - Some measurements don't use the full dataset yet
- $B \to DX(Y)$ measurements of γ rely on hadronic D^0 decay parameters
  - r_D - ratio of suppressed and favoured D decay
  - $\Delta\delta_D$ - strong-phase difference
  - κ_D - coherence factor, to account for multi-body decay resonance effects
  - Can also measure CP-even fractions F_+



Sketch of $D\bar{D}$ production at BES III, courtesy of Alex Gilman

Gronau-London-Wyler

Phys. Lett. B253 (1991) 483
Phys. Lett. B265 (1991) 172

- GLW method considers 2-body CP eigenstate modes, $D \to \pi^+\pi^-$, $D \to K^+K^-$, $D \to K_S^0\pi^0$
  - $R_{CP} = \dfrac{\Gamma(B^- \to D_{CP}X) + \Gamma(B^+ \to D_{CP}X)}{\Gamma(B^- \to D^0X) + \Gamma(B^+ \to \bar{D}^0X)} \propto 1 + r_B^2 \pm 2r_B\cos(\delta_B)\cos(\gamma)$
  - For any D final state can measure the charge asymmetry
  - $A_{CP} = \dfrac{\Gamma(B^- \to f^-) - \Gamma(B^+ \to f^+)}{\Gamma(B^- \to f^-) + \Gamma(B^+ \to f^+)} \propto \pm 2r_B\sin(\delta_B)\sin(\gamma)/R_{CP}$

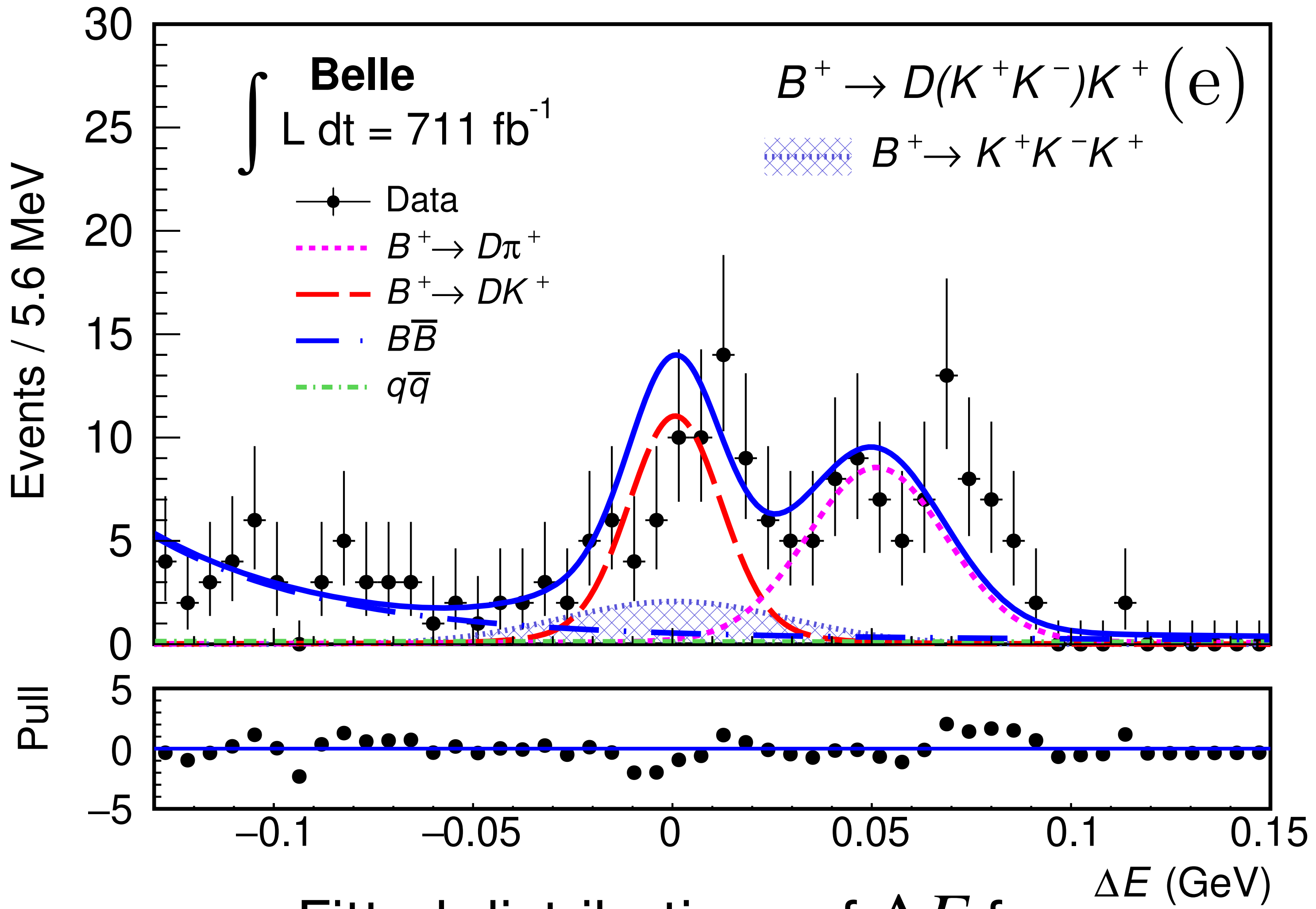
"Difference in peak heights"

- Doesn't require strong-phase input
- Extend fairly simply to 4-body modes such as $D \to \pi^+\pi^-\pi^+\pi^-$
  - Dilute interference by the CP-even fraction from BES III
- The same for B^0 decays

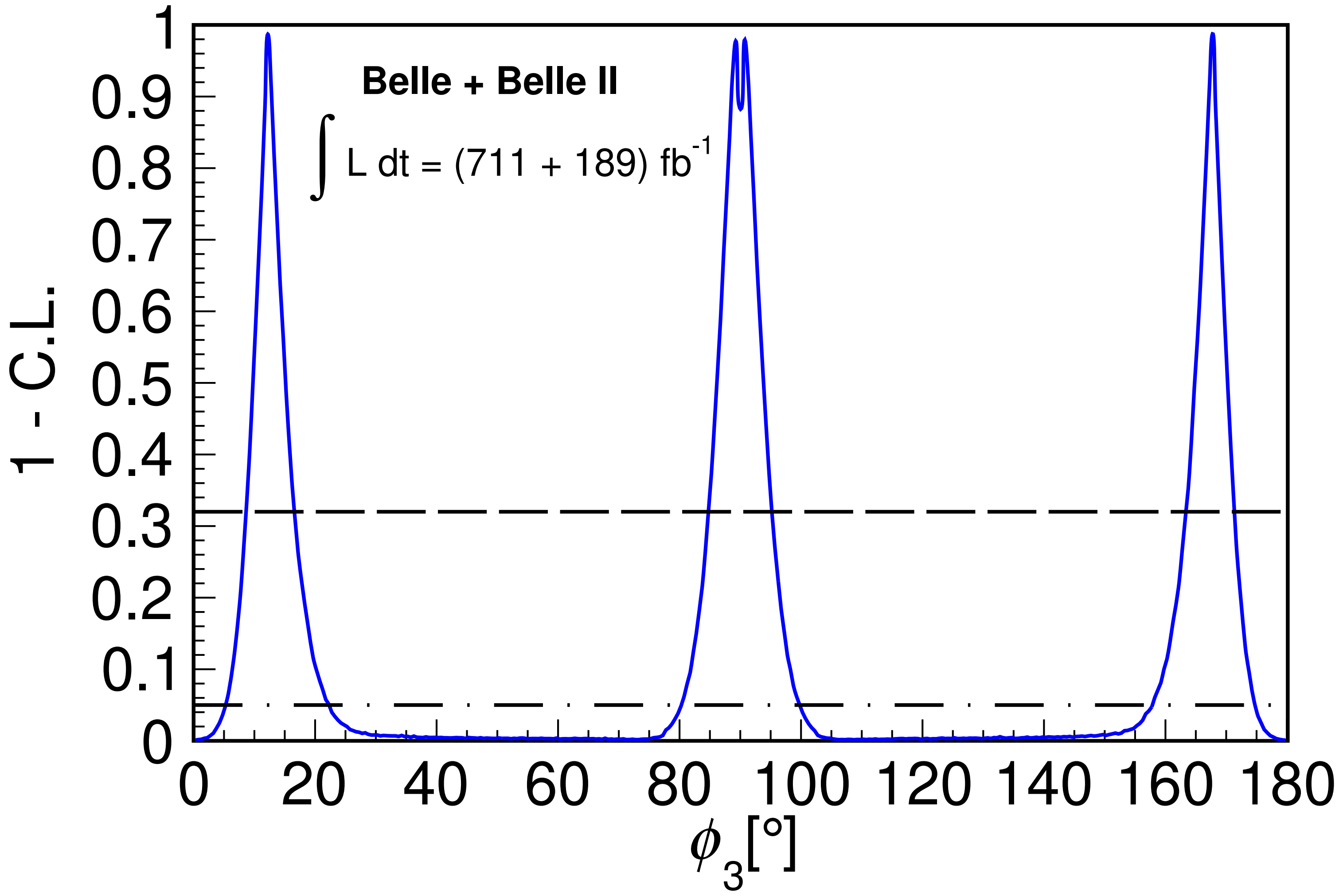
$B^+ \to DK^+, D \to (K^+K^-, K_S^0\pi^0)$ JHEP 05 (2024) 212

- $B^+ \to D\pi^+$ used to cancel efficiencies in CP observables (see backup)
- 772×10^6 $B\bar{B}$ pairs from Belle, 198×10^6 $B\bar{B}$ from Belle II
- Data split into 12 sets: B charges $\times$ B decays $\times$ D final states
- $\Delta E = E_B - E^*_{\text{beam}}$, difference of B candidate energy and beam energy
- Signal yields determined from a fit to ΔE and an ordered BDT output, C', “the fraction of signal events below C”.
  - Factorised due to negligible correlation

$$R_{CP+} = 1.164 \pm 0.081 \pm 0.036$$
$$R_{CP-} = 1.151 \pm 0.074 \pm 0.019$$
$$A_{CP+} = (+12.5 \pm 5.8 \pm 1.4)\,\%$$
$$A_{CP-} = (-16.7 \pm 5.7 \pm 0.6)\,\%$$



Fitted distributions of ΔE for $B^+ \to DK^+, D \to K^+K^-$ in Belle data

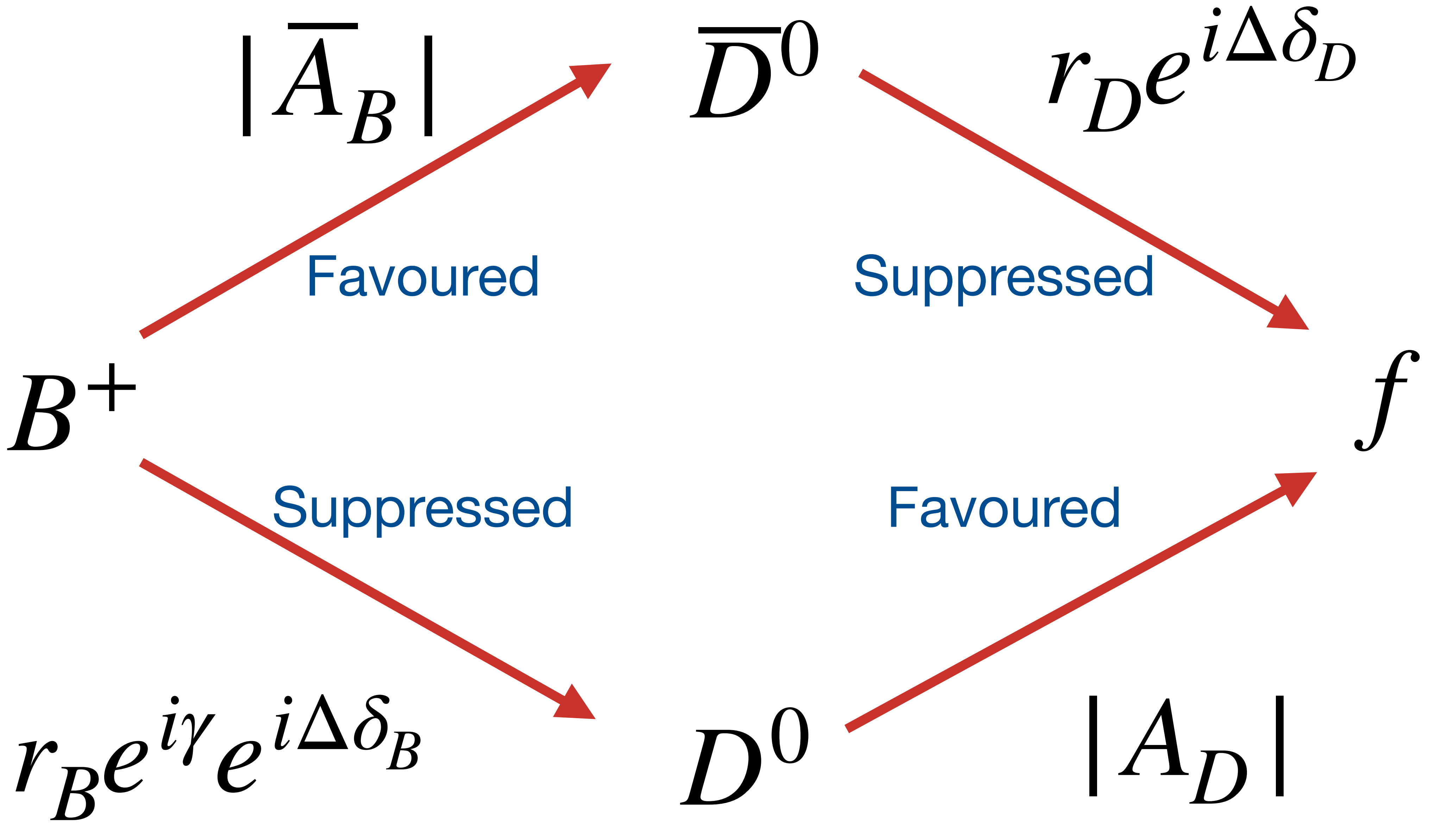


$1-$ CL as a function of ϕ_3, with dashed lines showing 68.3% CL, and 95.4% CL.

Atwood-Dunietz-Soni

Phys. Rev. Lett. 78 (1997) 3257

- ADS method considers 2-body Cabibbo favoured/suppressed modes $D \to K^{\pm}\pi^{\mp}$
  - $R_{CP} = \dfrac{\Gamma(B^- \to \overline{D}X, \overline{D} \to f) + \Gamma(B^+ \to DX, D \to f)}{\Gamma(B^- \to DX, D \to f) + \Gamma(B^+ \to \overline{D}X, \overline{D} \to f)}$ ~sup./fav.
    $\propto r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D)\cos(\gamma)$
- For any D final state can measure the charge asymmetry
  - $A_{CP} = \dfrac{\Gamma(B^- \to f) - \Gamma(B^+ \to f)}{\Gamma(B^- \to f) + \Gamma(B^+ \to f)} \propto 2r_B r_D \sin(\delta_B + \delta_D)\sin(\gamma)/R_{CP}$
- Fav/sup mode followed by sup/fav "balances" the size of terms in the amplitude $\implies$ relatively large interference
- These extend fairly simply to 4-body modes such as $D \to K^{\pm}\pi^{\mp}\pi^{+}\pi^{-}$
  - Dilute interference by a coherence factor, κ_D, to account for resonances
- Requires BES III inputs for r_D, $\Delta\delta_D$ and κ_D

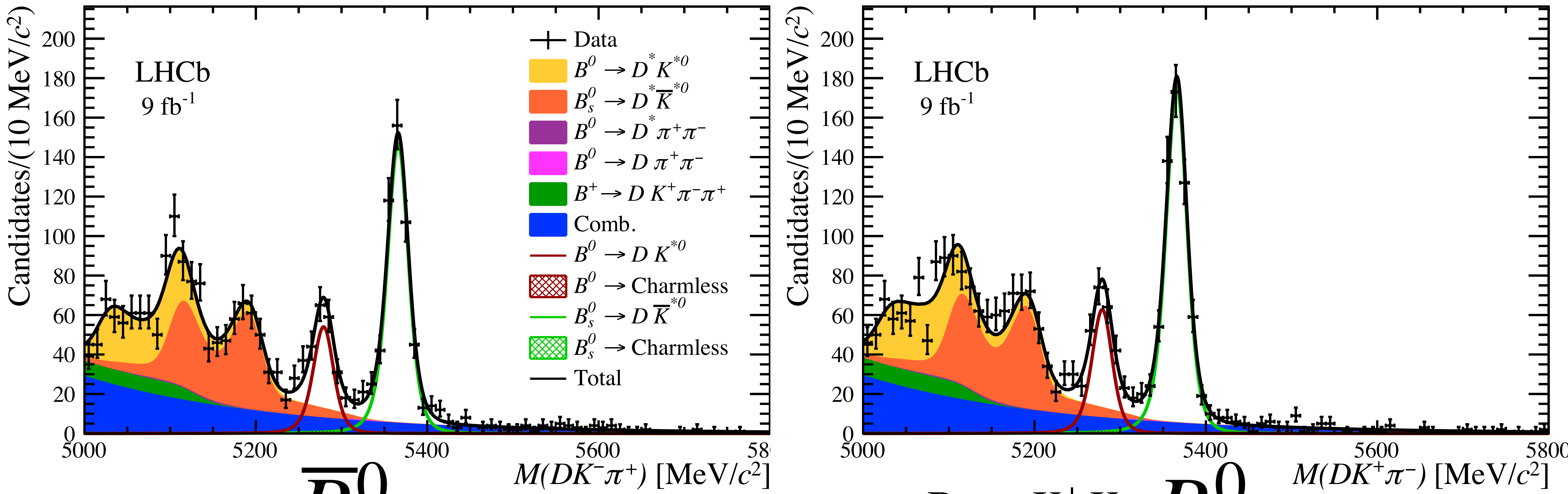


Sketch of the favoured and suppressed paths for a $B^+ \to DX, D \to f$ GLW decay

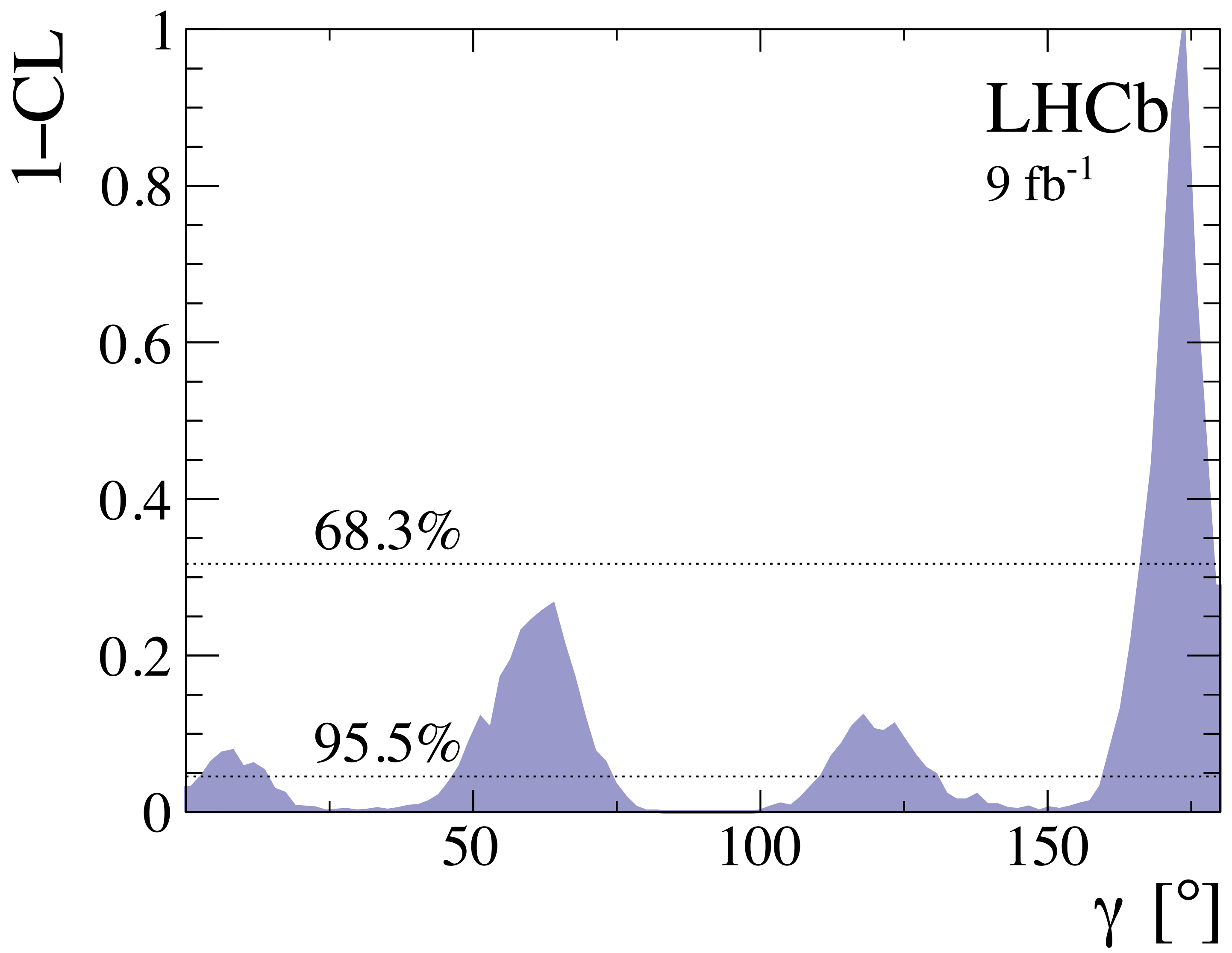
$B^0 \to DK^{*0}, D \to h^+h'^-(\pi^+\pi^-)$

JHEP 05 (2024) 025

- "Self-tagging", the charges of the K^{*0} children depend on the flavour of the B
- Same method as $B^+ \to DK^+$ but cut $B^0 \to DK^+\pi^-$ phase-space around the $K^*(892)$ resonance
    - Interference term gains a coherence parameter coefficient
- Simultaneous measurement of
    - $D \to K^\pm\pi^\mp(\pi^+\pi^-)$
    - $D \to \pi^+\pi^-(\pi^+\pi^-)$
    - $D \to K^+K^-$
- Fit interference effects to obtain 4 solutions of γ
    - Solution most compatible with existing measurements is $\gamma = (61.7 \pm 8.0)^\circ$
    - Require further input, such as $D \to K^0_S h^+h^-$, to resolve the ambiguity
- BES III input for $\kappa_D^{K\pi}$ JHEP 05 (2021) 164
- $F_+^{4\pi}$ is averaged from BES III and LHCB Phys Rev D 106 (2022) 092004, Phys. Lett. B 747 (2015) 9



$\overline{B}^0$ Invariant mass fit result for $D \to K^+K^-$ B^0



Confidence levels from the simultaneous interpretation in terms of $\gamma, r_{B^0}^{DK^*}, \delta_{B^0}^{DK^*}$

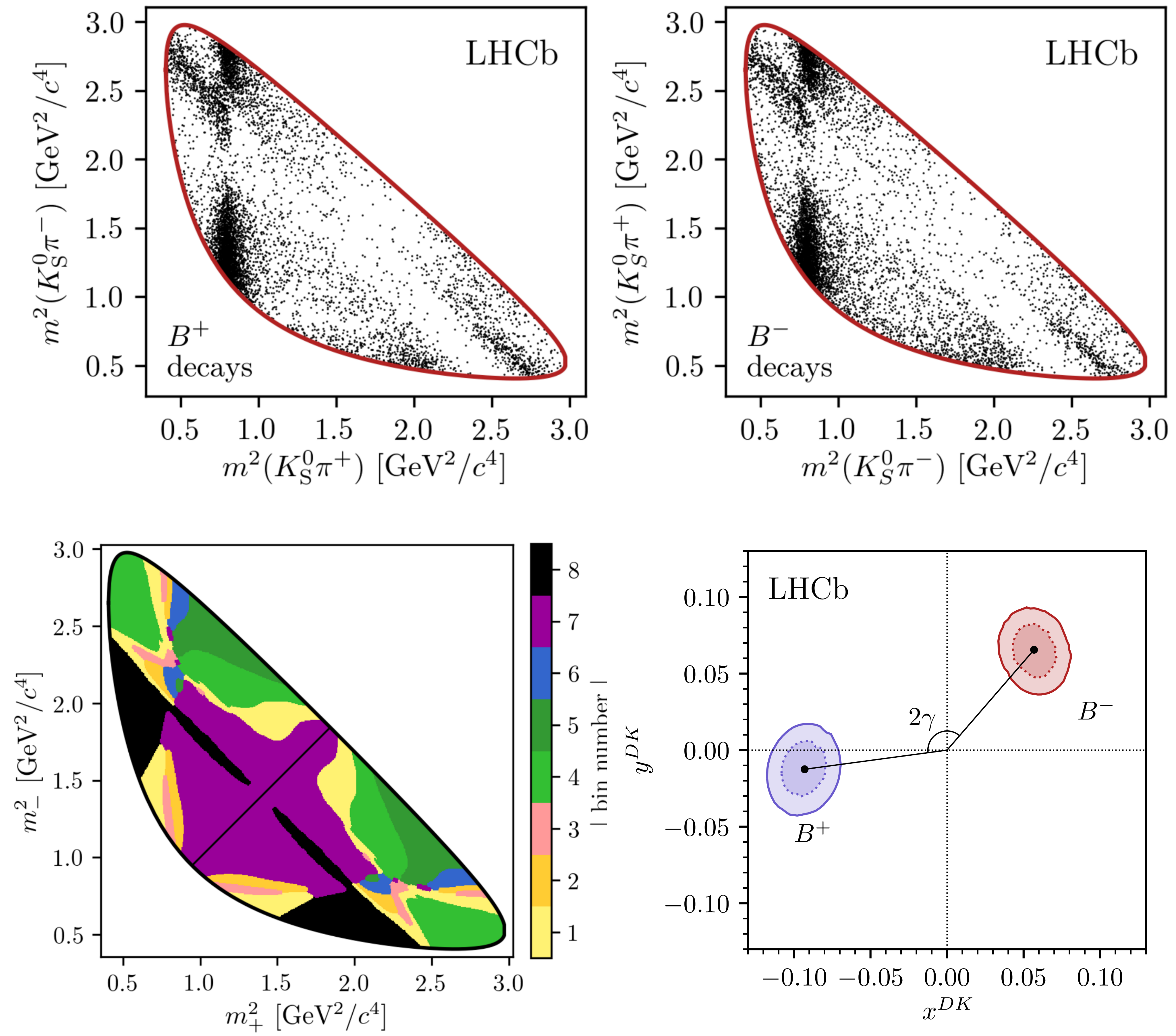
BPGGSZ (Bondar-Poluektov-Giri-Grossman-Soffer-Zupan)

- Most well known example is $B^+ \to DK^+, D \to K_S^0\pi^+\pi^-$
- $A = |\overline{A}_B||\overline{A}_D| + |A_B||A_D|e^{i\gamma}e^{i\Delta\delta_B}e^{i\Delta\delta_D}$ varies across the D Dalitz plane
  - Resonances overlap between favoured and suppressed $\Longrightarrow$ local asymmetries
- Binned Dalitz plane analysis $\Longrightarrow$ reduced $\sigma_{\text{syst.}}$ with a small increase to $\sigma_{\text{stat.}}$
- $\Longrightarrow$ bin populations $N_i^\pm = h^\pm[F_{\mp i} + (x^{\pm 2} + y^{\pm 2})F_{\pm i} + 2\sqrt{F_iF_{-i}}(c_ix^\pm \mp s_iy^\pm)]$

  fitted to obtain

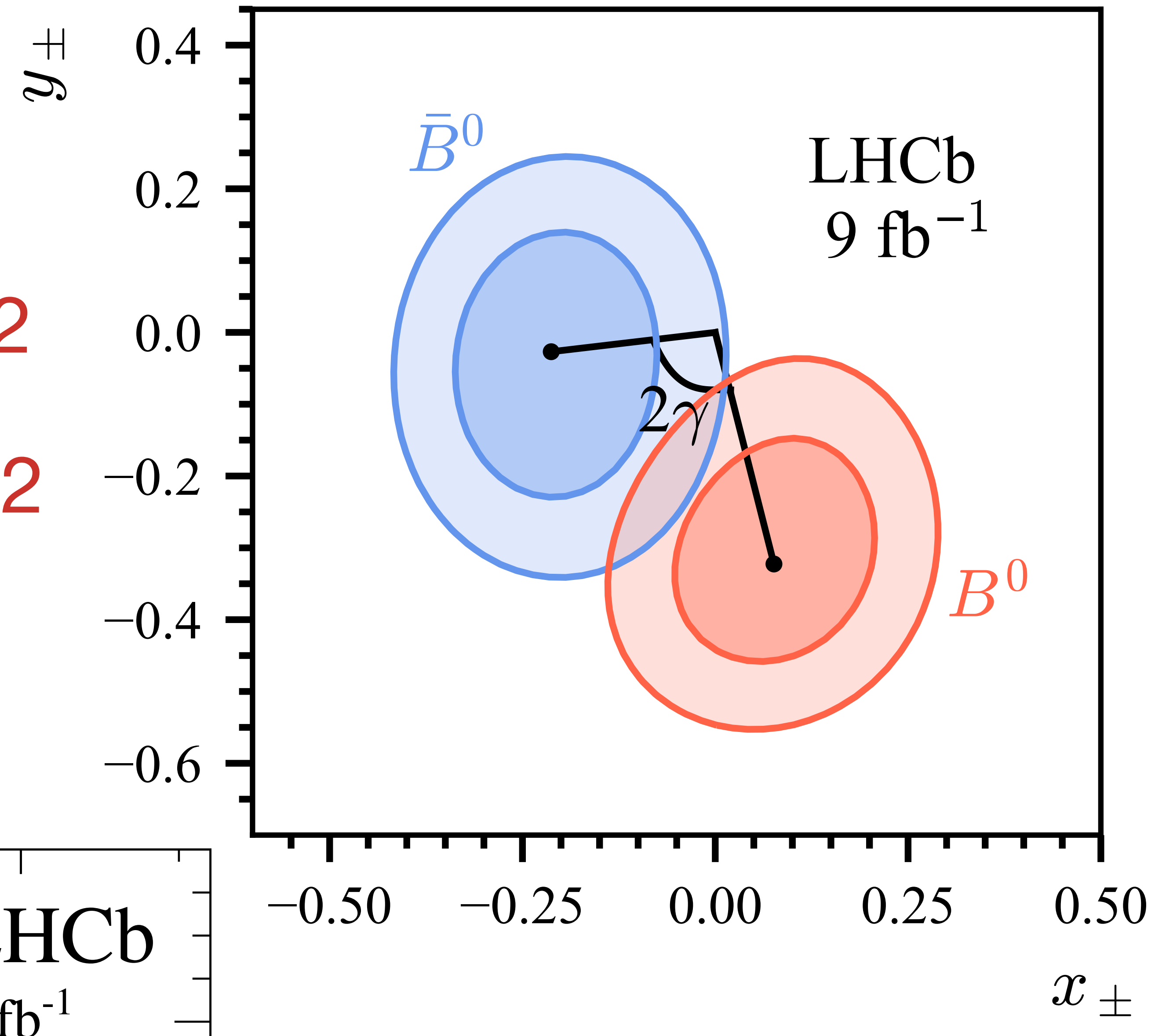
$$x^\pm = r_B\cos(\Delta\delta_B \pm \gamma)$$
$$y^\pm = r_B\sin(\Delta\delta_B \pm \gamma)$$

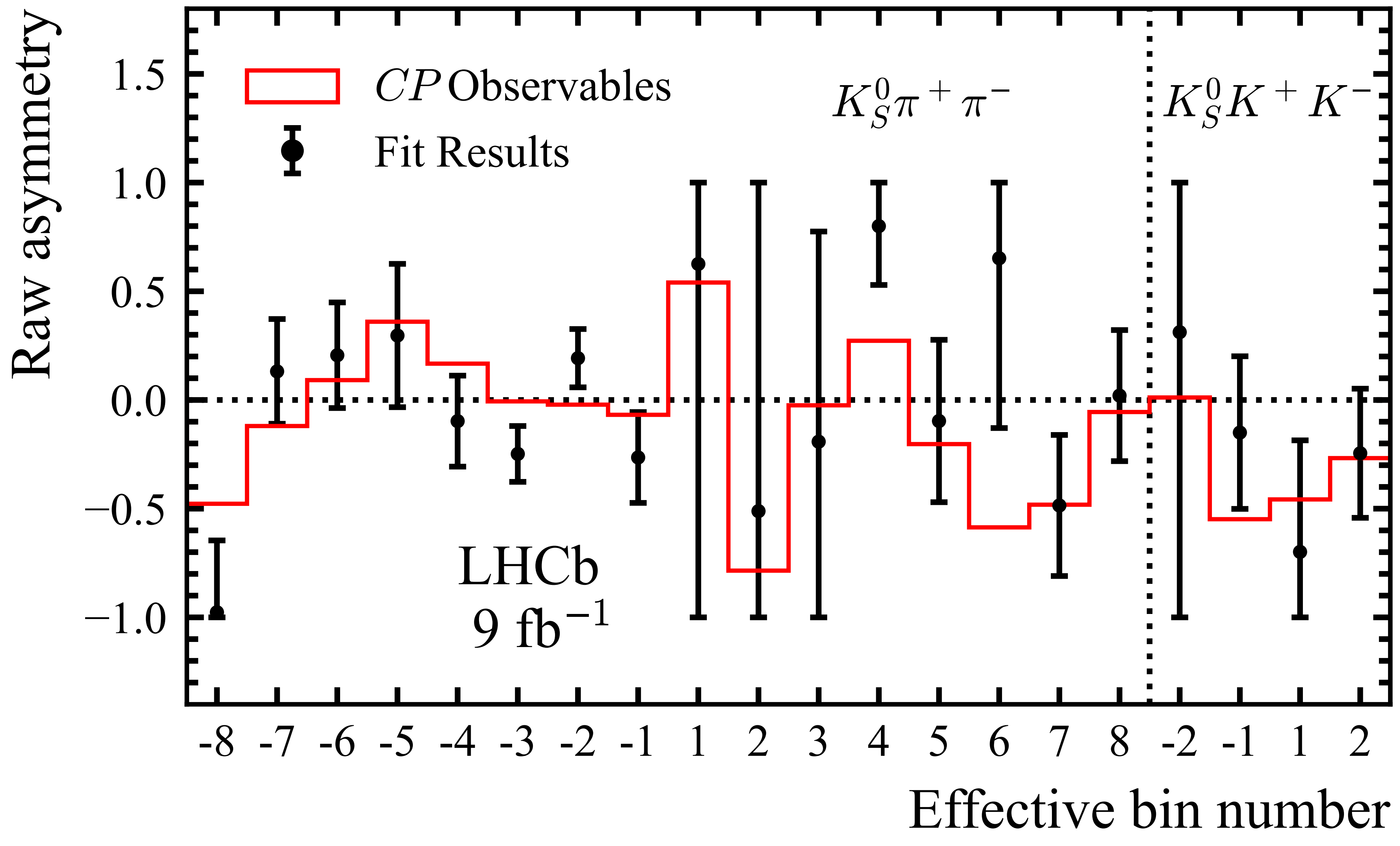
- c_i, s_i are fixed to CLEO-c + BES III values in the fit
- Finally γ can be extracted!



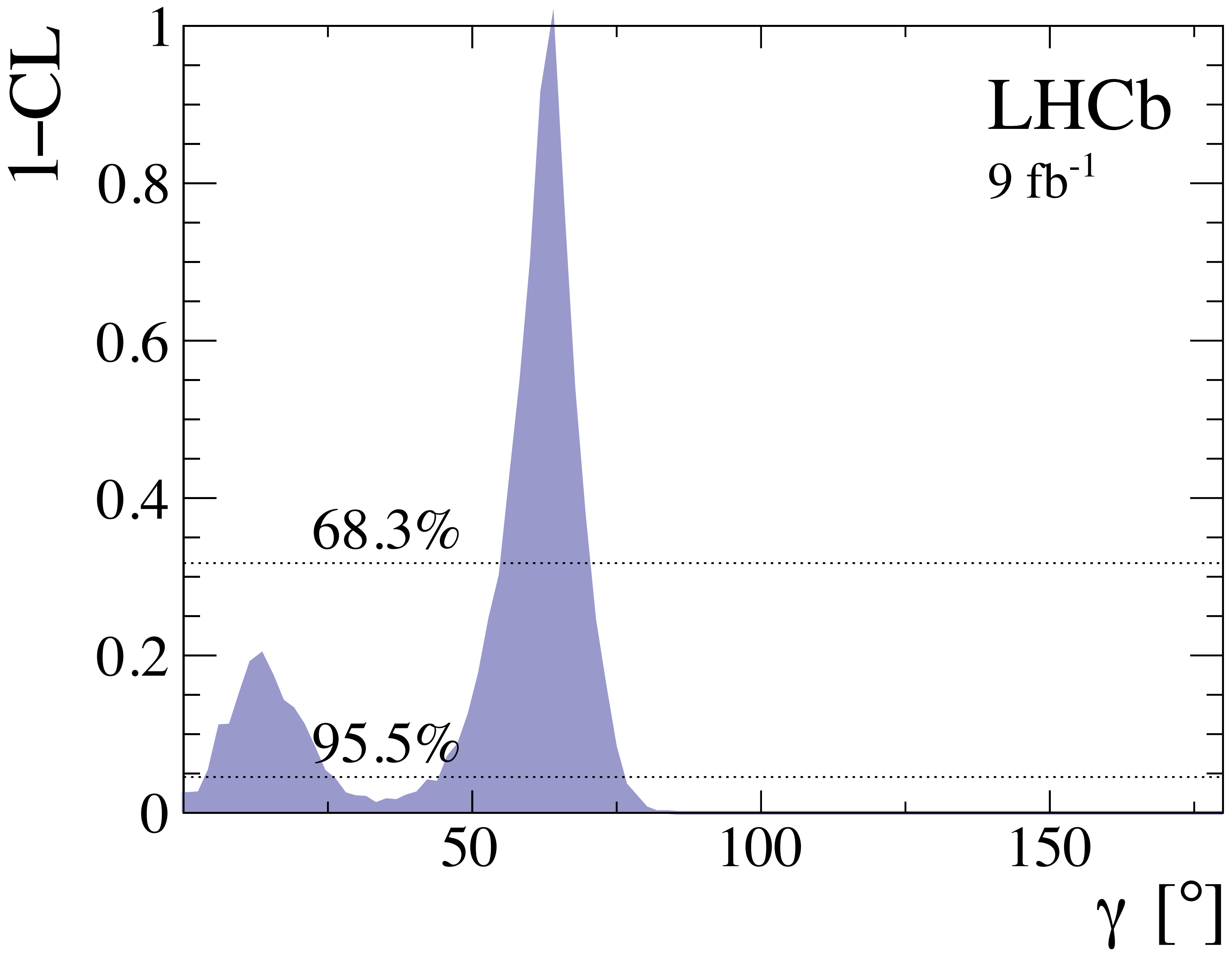
$D \to K_S^0\pi^+\pi^-$ Dalitz plane distribution (top) for B^+ (left) and B^- (right), the optimal binning (bottom left) and the extracted Cartesian parameters (bottom right). Figures from JHEP 02 (2021) 169

- Combination with $D \to h^+ h'^- (\pi^+\pi^-)$ yields $\gamma = (63.2^{+6.9}_{-8.1})^\circ$
- Much closer to where B^+ was in the previous LHCb combination! LHCb-CONF-2022-002
- Strong-phase parameters from BES III Phys. Rev. D 101, 112002 (2020), Phys. Rev. D 102 052008 (2020), Phys. Rev. Lett. 124 241802 (2020), combined with CLEO-c Phys. Rev. D 82, 112006 (2010)





Per-bin asymmetries determined by the CP fit parameters (red) and signal yields when allowed to float freely (black) with statistical uncertainties



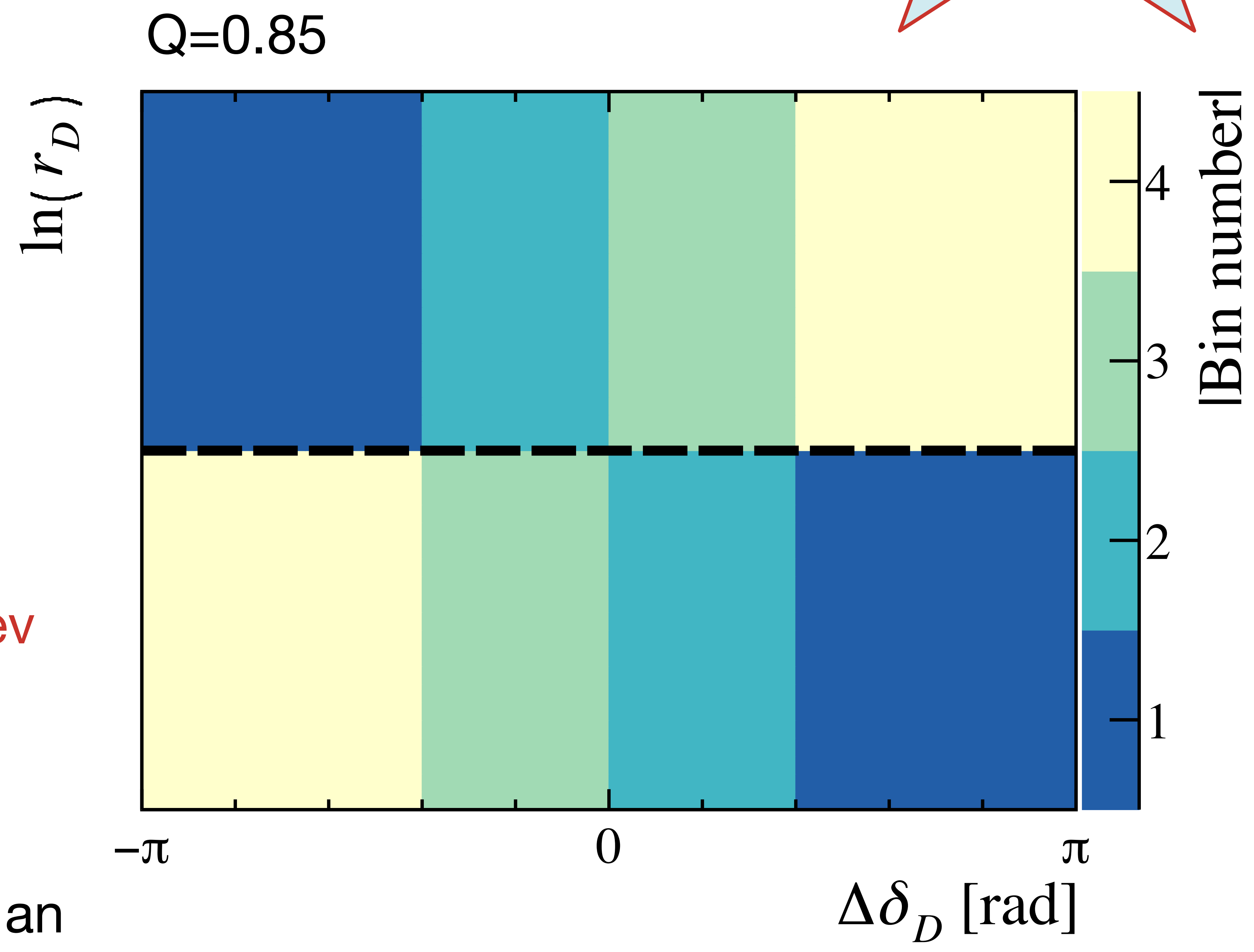
Statistical confidence regions for the measured $x_{\pm i}, y_{\pm i}$ values (right) and the confidence level profile for an extraction of γ from a combination of $D \to K_S^0 h^+ h^-$ and $D \to hh(hh)$ (left)

$B^+ \to Dh^+, D \to h'^+h'^-\pi^+\pi^-$

Preliminary
LHCb-PAPER-2025-019

NEW

- First phase-space binned model-independent γ measurement from
  - $D \to K^+K^-\pi^+\pi^-$, $D \to \pi^+\pi^-\pi^+\pi^-$
- 5 dimensional phase-space
- Binning schemes based on amplitude models
- $D \to K^+K^-\pi^+\pi^-$ binned as a projection onto $(\Delta\delta_D, ln(r_D))$
  - $+i$ for $ln(r_D) < 0$, $-i$ for $ln(r_D) > 0$,
  - $D \to \pi^+\pi^-\pi^+\pi^-$ hypercube model and binning from JHEP 01 (2018) 144, Phys Rev D 110 112008
    - Binning quality Q $\gtrsim 0.8$ (% sensitivity with respect to unbinned)
- Binning optimised according to a metric measuring the statistical sensitivity relative to an unbinned method
  - Effectively this corresponds to maximising the interference term $2\sqrt{F_iF_{-i}}(c_ix^\pm \mp s_iy^\pm)$
  - By construction, under CP $F_i \mapsto F_{-i}$, $c_i \mapsto c_i$, $s_i \mapsto -s_i \implies$ reduce free parameters



$D \to K^+K^-\pi^+\pi^-$ binning from LHCb Eur. Phys. J. C (2023) 83:547 based on the model from JHEP 02 (2019) 126

$B^+ \to Dh^+, D \to h'^+h'^-\pi^+\pi^-$

Preliminary
LHCb-PAPER-2025-019

NEW

- Previous analysis of $D \to K^+K^-\pi^+\pi^-$ Eur. Phys. J. C (2023) 83:547 determined c_i, s_i from the amplitude model
- New BES III result measured them $\Longrightarrow$ now fully model independent Phys Rev D 112 (2025) 012015
- BES III also provides them for $D \to \pi^+\pi^-\pi^+\pi^-$ Phys Rev D 110 (2024) 112008
- $F_{\pm i}$ are general to the given D decay, *assuming the same efficiency profile*, $\therefore$ can use $B^\pm \to D\pi^\pm$ as a "control" channel

Parameterise $B^\pm \to D\pi^\pm$ according to

$$x^\pm_{D\pi} = x_\xi x^\pm_{DK} - y_\xi y^\pm_{DK},$$
$$y^\pm_{D\pi} = x_\xi y^\pm_{DK} - y_\xi x^\pm_{DK},$$

where

$$x_\xi = \text{Re}(\xi_{D\pi}),\ y_\xi = \text{Im}(\xi_{D\pi})$$

$$\xi_{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}}\exp(i[\delta_B^{D\pi} - \delta_B^{DK}])$$

$$N^\pm_{Dh,i} = h^\pm_{Dh}[F_{\mp i} + ({x^\pm_{Dh}}^2 + {y^\pm_{Dh}}^2)F_{\pm i} + 2\sqrt{F_i F_{-i}}(c_i x^\pm_{Dh} \mp s_i y^\pm_{Dh})]$$

We need to extract $x^\pm_{DK}, y^\pm_{DK}, x_\xi, y_\xi$

$B^+ \to Dh^+, D \to h'^+h'^-\pi^+\pi^-$

Preliminary
LHCb-PAPER-2025-019

NEW

- Negligible correlation between binned LHCb-PAPER-2025-019, and integrated phase-space measurements in Eur. Phys. J. C (2023) 83:547

- Extract γ from these simultaneously (preliminary result)

$$\gamma = (52.6^{+8.5}_{-6.4})^\circ,$$
$$\delta_B^{DK} = (112.6^{+6.1}_{-7.8})^\circ \quad r_B^{DK} = (0.102^{+0.014}_{-0.017})$$
$$\delta_B^{D\pi} = (262^{+40}_{-52})^\circ \quad r_B^{D\pi} = (0.0043^{+0.0033}_{-0.0043})$$

- One of the most precise single measurements of γ to date!

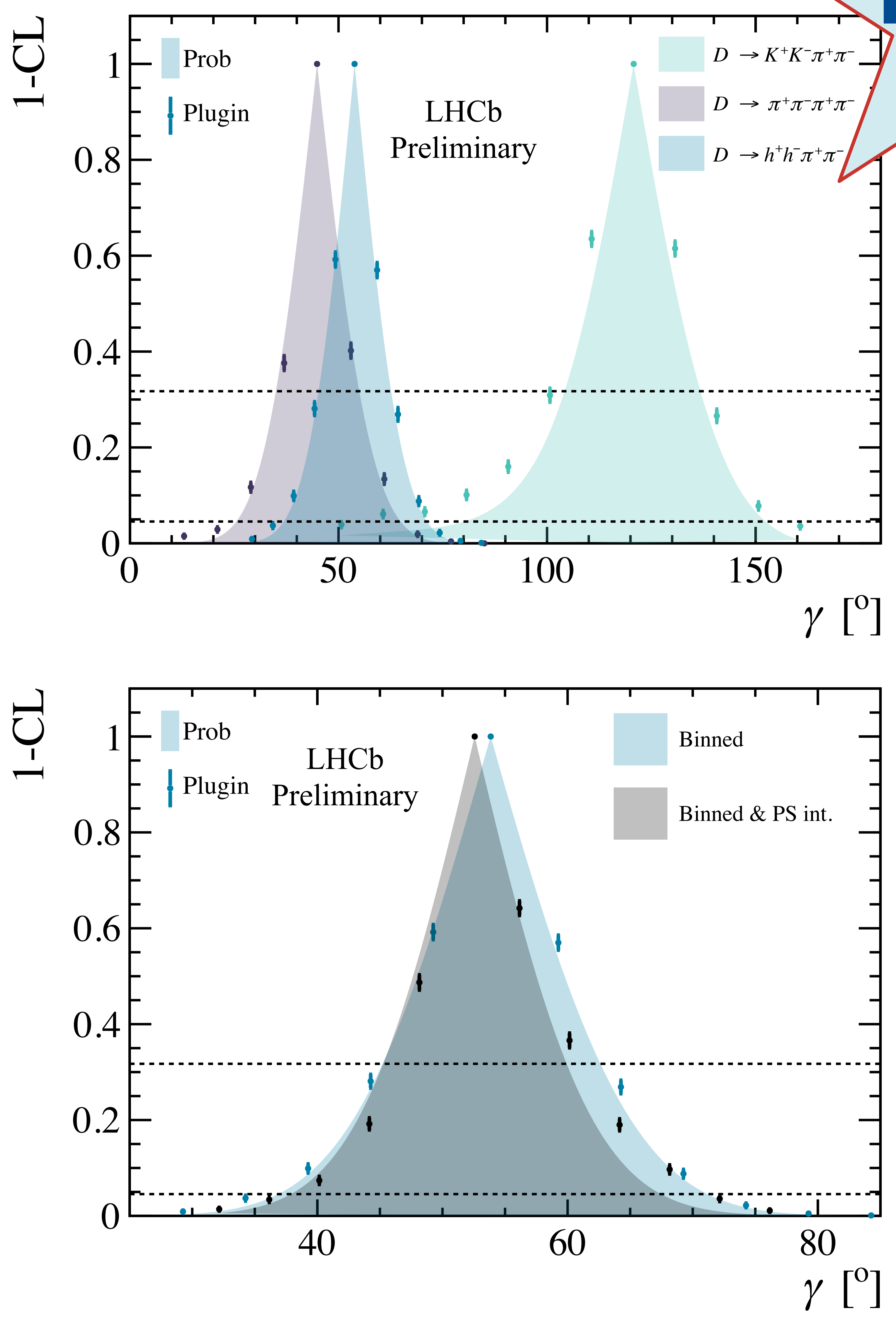
- Statistically limited $\implies$ Run 3 opportunities

- Leading systematics are strong-phase inputs (comparable to LHCb $\sigma_{\text{stat.}}$)

  - In future $D \to K^+K^-\pi^+\pi^-$ can exploit LHCb Charm mixing measurements

  - and $D \to \pi^+\pi^-\pi^+\pi^-$ the full BES III dataset $\implies$ factor of 2.5 reduction in statistical uncertainties of inputs

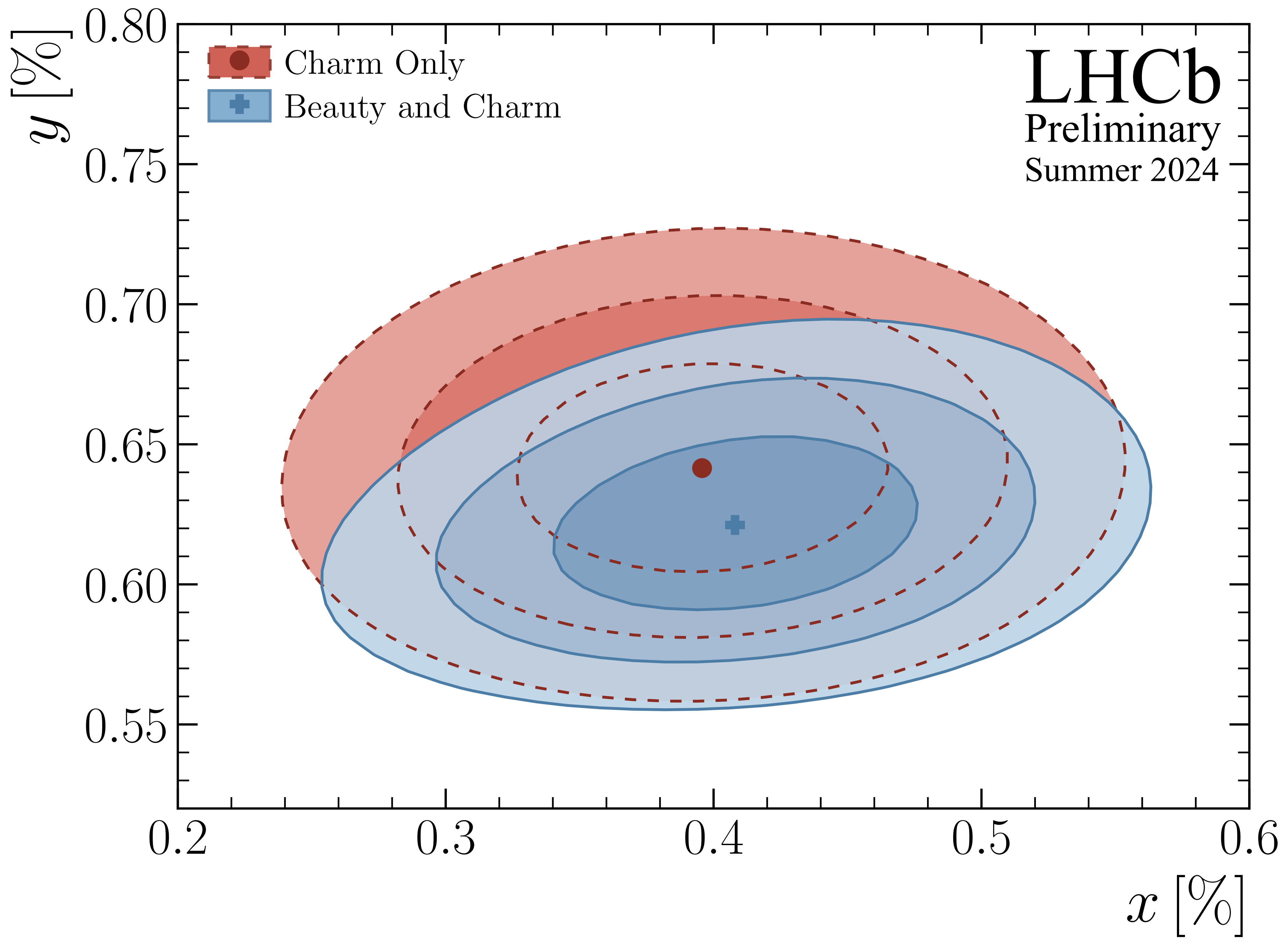
  - Other systematics are $\lesssim 20\,\%$ of LHCb σ_{stat}.



Combination of the BPGGSZ measurements (top) and the combination of these with the phase-space integrated measurement.

Beauty and Charm

- Combination of:
  - 19 LHCb B decay measurements (4 new, 3 superseded)*
  - 11 LHCb D decay measurements (1 new, 1 superseded)*
- 27 auxiliary inputs from LHCb, HFLAV, CLEO-c and BESIII (1 new, 2 updated)*
- Many Beauty and Charm measurements share parameters and provide complementary information
  - Detailed description of original method in 2013 Physics Letters B 726 (2013) 151–163
  - Added Charm in 2021 JHEP 12 (2021) 141
- Produces a single LHCb value for 29 physics parameters of interest (+ nuisance parameters)
- Latest update is LHCb-CONF-2024-004
  - Does not include the new $B^+ \to DK^+$ result shown today
  - Work on an update in progress

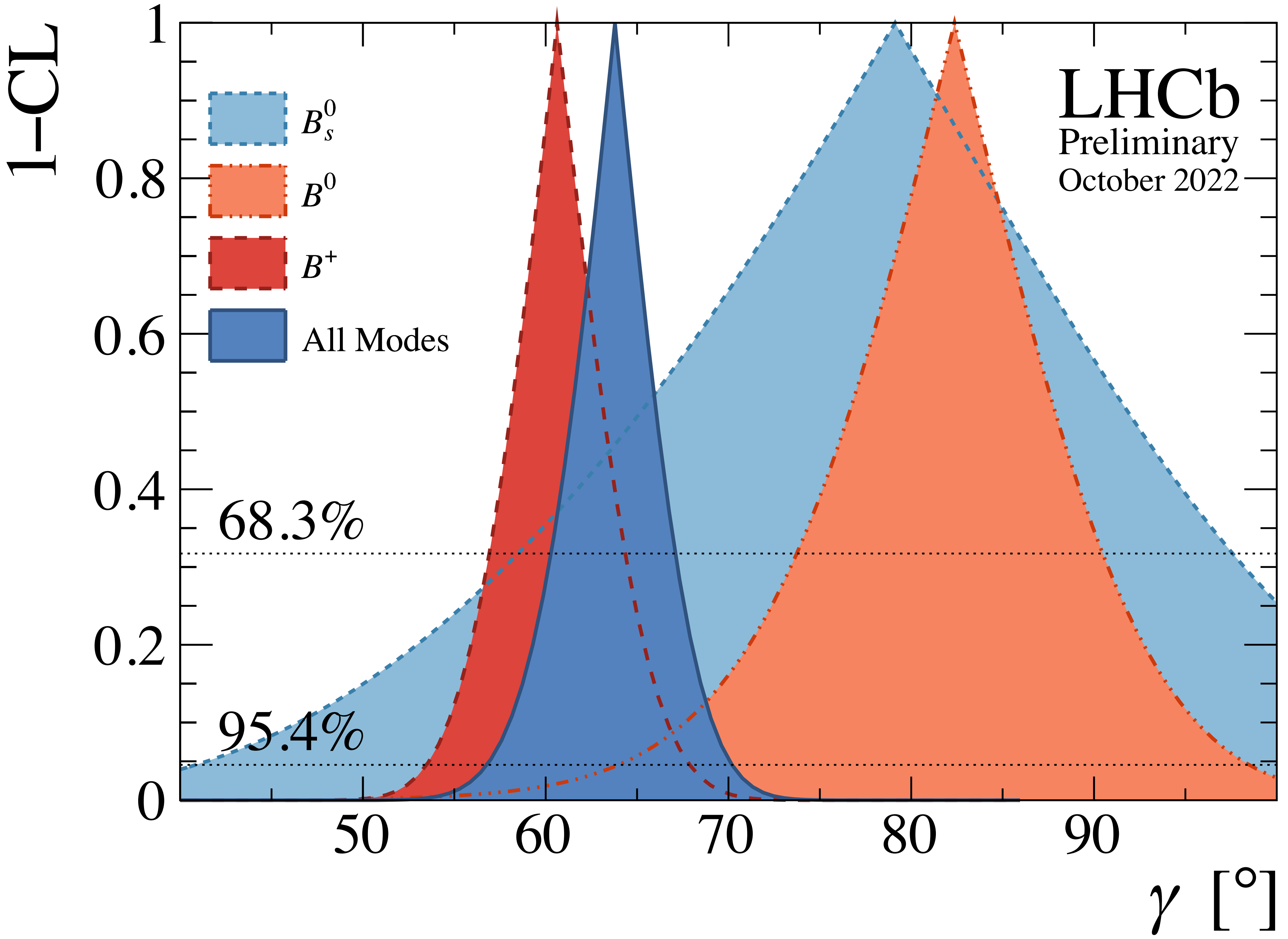


Charm mixing parameters when measured with and without complementary Beauty inputs

*See backup

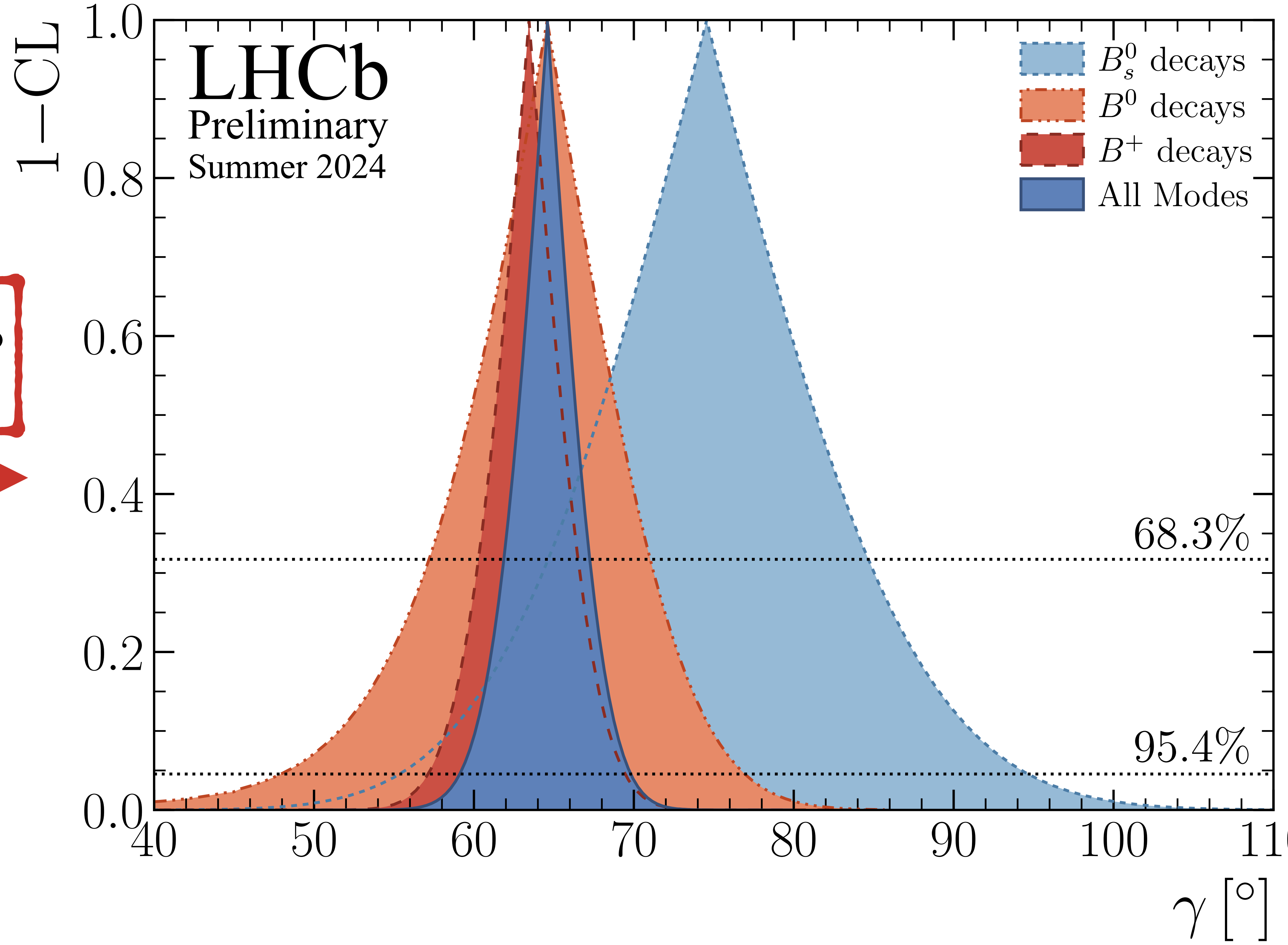
Per B species

LHCb-CONF-2022-002



$$\gamma = (64.6 \pm 2.8)^\circ$$

LHCb-CONF-2024-004



Species	Value [°]	68.3% CL Uncertainty [°]	95.4% CL Uncertainty [°]
B^+	60.6	$^{+4.0}_{-3.8}$	$^{+7.8}_{-7.5}$
B^0	82.0	$^{+8.1}_{-8.8}$	$^{+17}_{-18}$
B_s^0	79	$^{+21}_{-24}$	$^{+51}_{-47}$
All	63.8	$^{+3.5}_{-3.7}$	$^{+6.9}_{-7.5}$

Species	Value [°]	68.3% CL Uncertainty [°]	95.4% CL Uncertainty [°]
B^+	63.4	$^{+3.2}_{-3.3}$	$^{+6.4}_{-6.5}$
B^0	64.6	$^{+6.5}_{-7.5}$	$^{+12}_{-17}$
B_s^0	75	$^{+10}_{-11}$	± 20
All	64.6	± 2.8	$^{+5.5}_{-5.7}$

Belle (II) Combo inputs

JHEP 10 (2024) 143

- First Belle (II) combination of γ measurements
- Combination of:
  - $B^+ \to DK^+$, $B^+ \to D\pi^+$, $B^+ \to D^*K^+$ results
  - 7 measurements in total
- Much to be gained from future Belle II measurements

B decay	D decay	Method	Data set (Belle + Belle II)[fb^{-1}]	
$B^+ \to Dh^+$	$D \to K_S^0\pi^0, K^-K^+$	GLW	$711 + 189$	JHEP 05 (2024) 212
$B^+ \to Dh^+$	$D \to K^+\pi^-, K^+\pi^-\pi^0$	ADS	$711 + 0$	Phys Rev D 88 (2013) 091104, Phys Rev Lett 106 231803
$B^+ \to Dh^+$	$D \to K_S^0K^-\pi^+$	GLS	$711 + 362$	JHEP 09 (2023) 146
$B^+ \to Dh^+$	$D \to K_S^0h^-h^+$	BPGGSZ (m.i.)	$711 + 128$	JHEP 02 (2022) 063
$B^+ \to Dh^+$	$D \to K_S^0\pi^-\pi^+\pi^0$	BPGGSZ (m.i.)	$711 + 0$	JHEP 10 (2019) 178
$B^+ \to D^*K^+$	$D^* \to D\pi^0, D \to K_S^0\pi^0, K_S^0\phi, K_S^0\omega,$ $K^-K^+, \pi^-\pi^+$	GLW	210+0	Phys Rev D 73 (2006) 051106
$B^+ \to D^*K^+$	$D^* \to D\pi^0, D\gamma, D \to K_S^0\pi^-\pi^+$	BPGGSZ (m.d.)	$605 + 0$	Phys Rev D 81 (2010) 112002

Belle (II) γ Combination

JHEP 10 (2024) 143

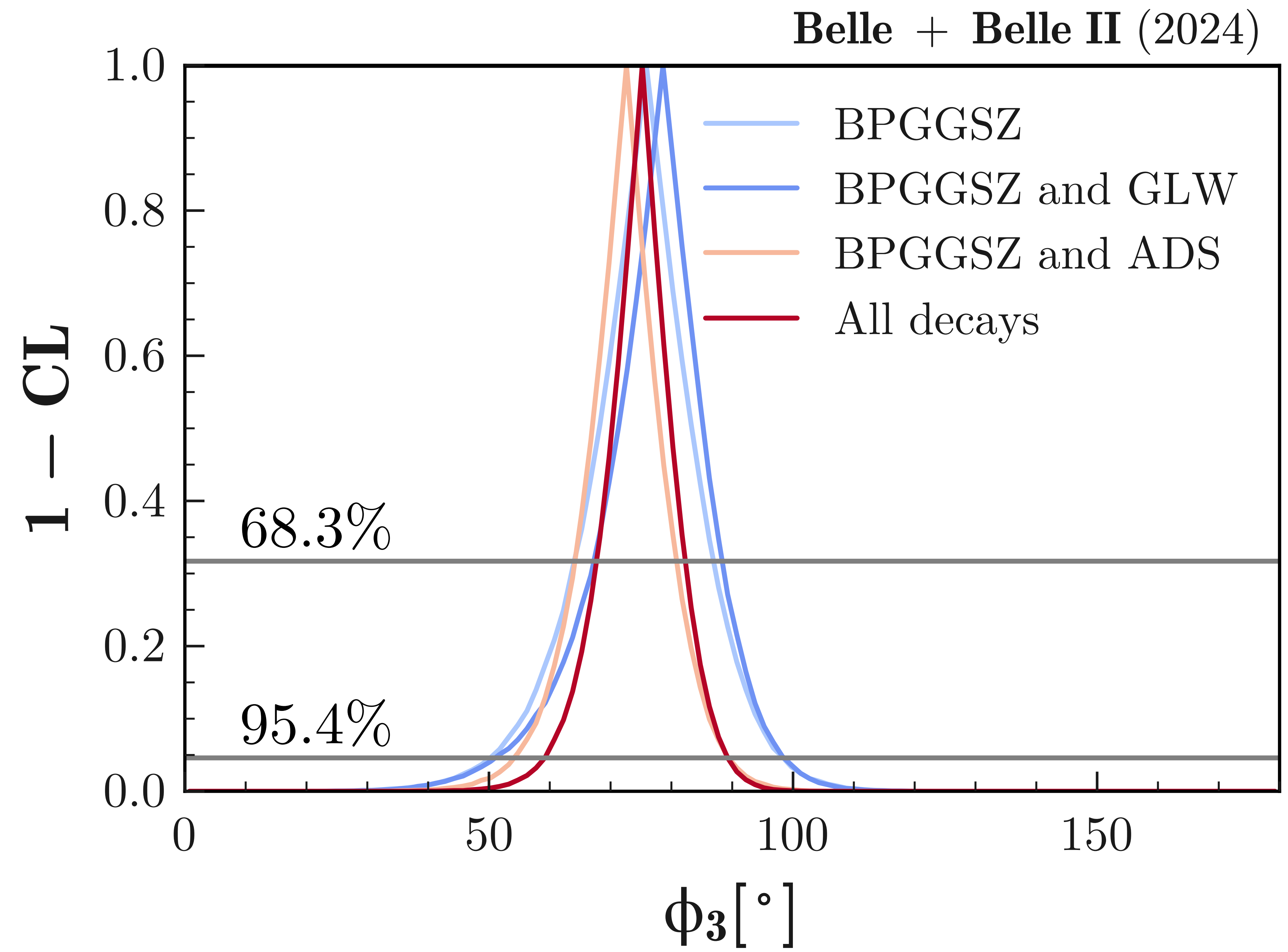
- 14 auxiliary input parameters from PDG, HFLAV, CLEO-c, BES III, LHCb
- Provides a single value of γ, r_B^{DK}, δ_B^{DK}, $r_B^{D\pi}$, $\delta_B^{D\pi}$, $r_B^{D^*K}$, $\delta_B^{D^*K}$ from Belle and Belle II
- Better sensitivity than expected arxiv:2207.06307, may be a statistical effect…

Parameters	$\phi_3(^\circ)$	r_B^{DK}	$\delta_B^{DK}(^\circ)$
Best-fit value	75.2	0.115	137.8
68.3% interval	[67.7, 82.3]	[0.102, 0.127]	[128.0, 146.3]
95.4% interval	[59, 89]	[0.089, 0.138]	[116, 154]

$r_B^{D\pi}$	$\delta_B^{D\pi}(^\circ)$	$r_B^{D^*K}$	$\delta_B^{D^*K}(^\circ)$
0.0165	347.0	0.229	342
[0.0113, 0.0220]	[337.4, 355.7]	[0.162, 0.297]	[326,356]
[0.006, 0.027]	[322, 366]	[0.10, 0.37]	[306, 371]

Combination results: best-fit values and 68.3% and 95.4% confidence intervals

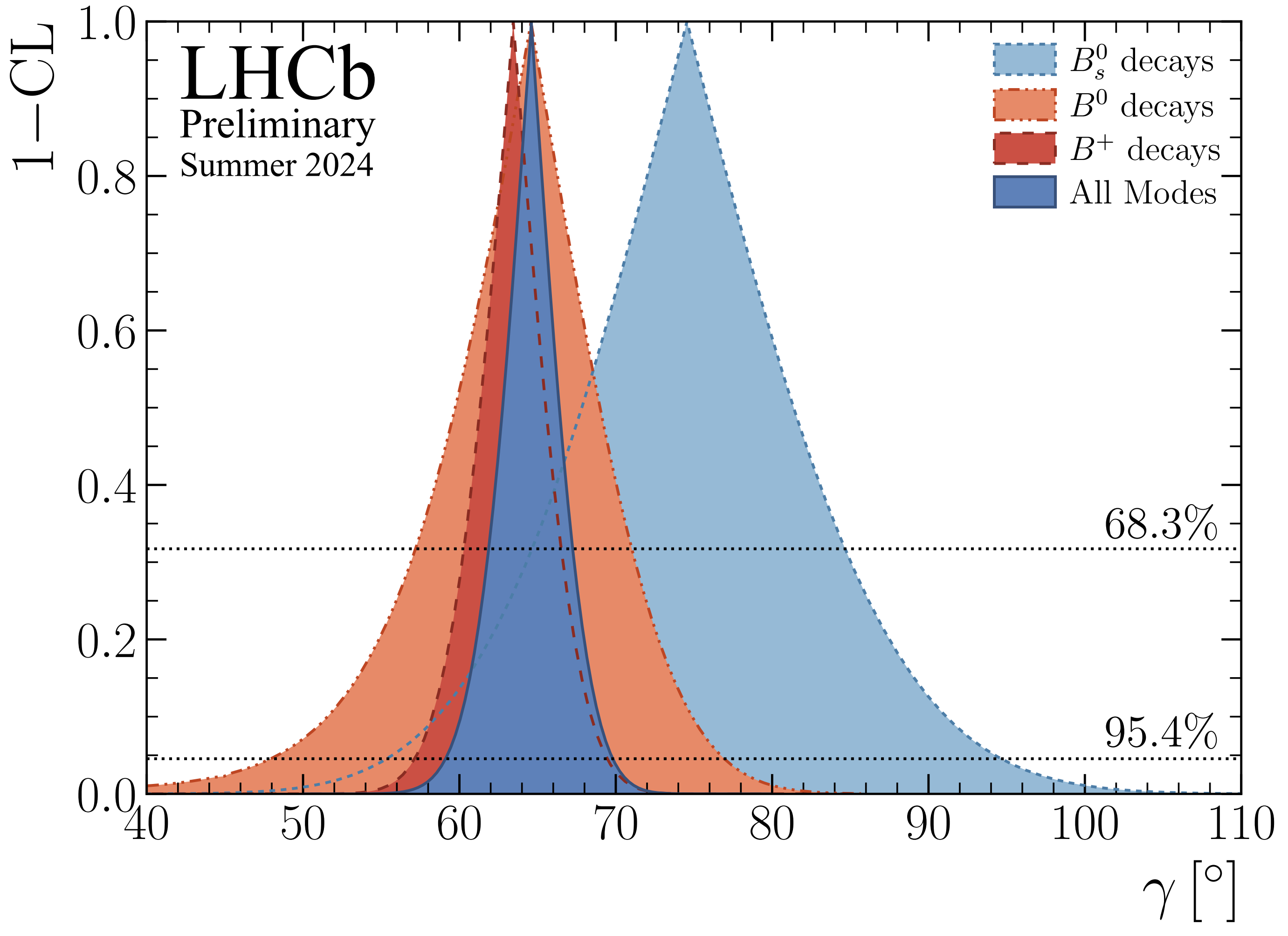
$$\gamma = (75.2^{+7.1}_{-7.5})^\circ$$

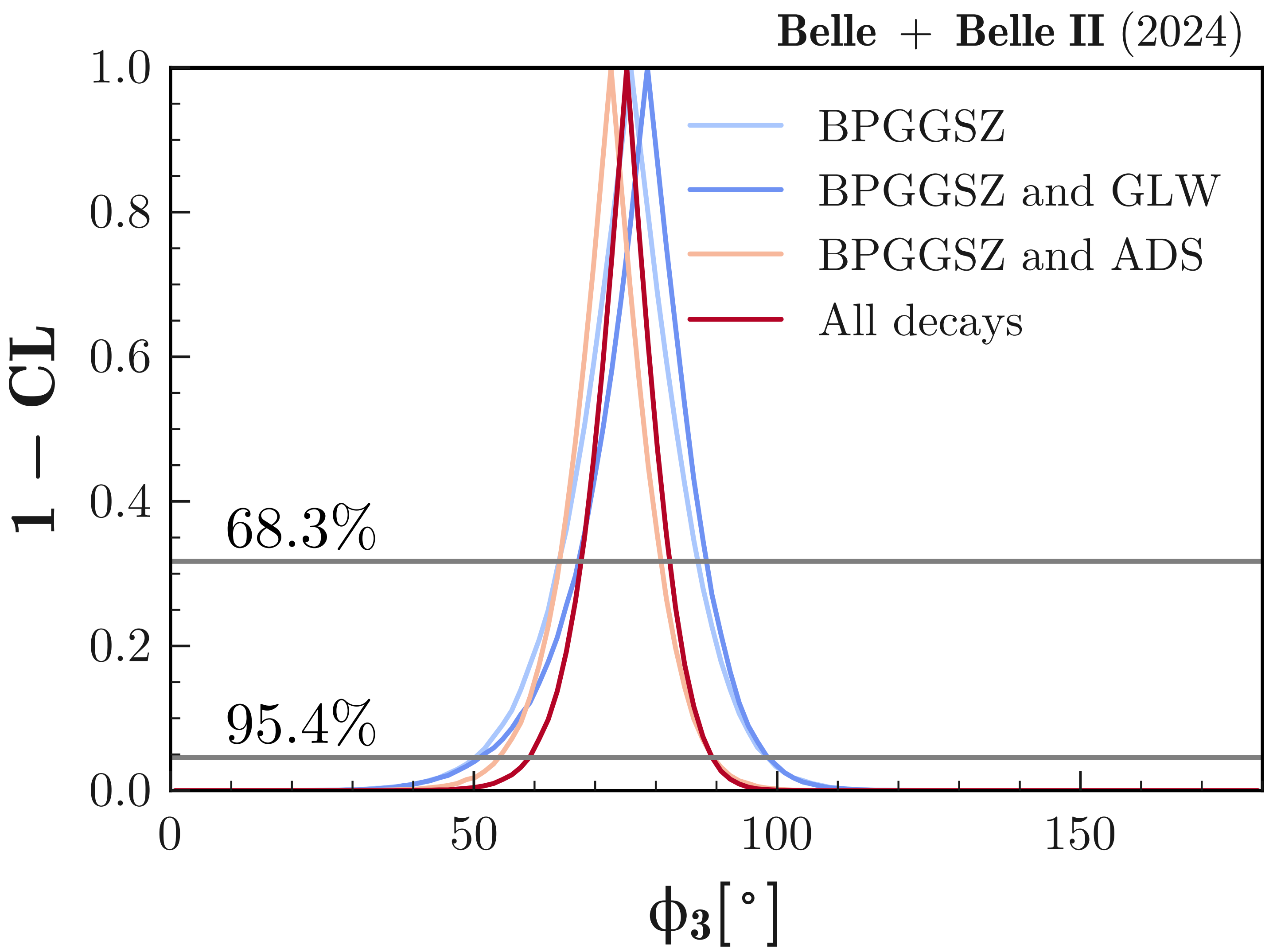


$1 - $ CL distributions as a function of ϕ_3 for various combinations of measurements

Summary

- LHCb and Belle II pushing down the direct uncertainty on γ in a complementary way
- Both experiments now perform our own dedicated combinations for γ
- Still pushing our datasets as far as we can
  - Statistically limited - future measurements wi be even better!
- BES III inputs are vital to achieve this performance
  - Uncertainty will be greatly reduced once the full dataset is exploited $\implies$ should not be a limiting factor
- Thanks to all the proponents of these analyses
- Thanks for listening!





CKMfitter indirect $\gamma = (66.3^{+0.7}_{-1.9})^\circ$

HFLAV direct $\gamma = (66.4^{+2.7}_{-2.8})^\circ$

Global Beauty and Charm $\gamma = (65.7 \pm 2.5)^\circ$

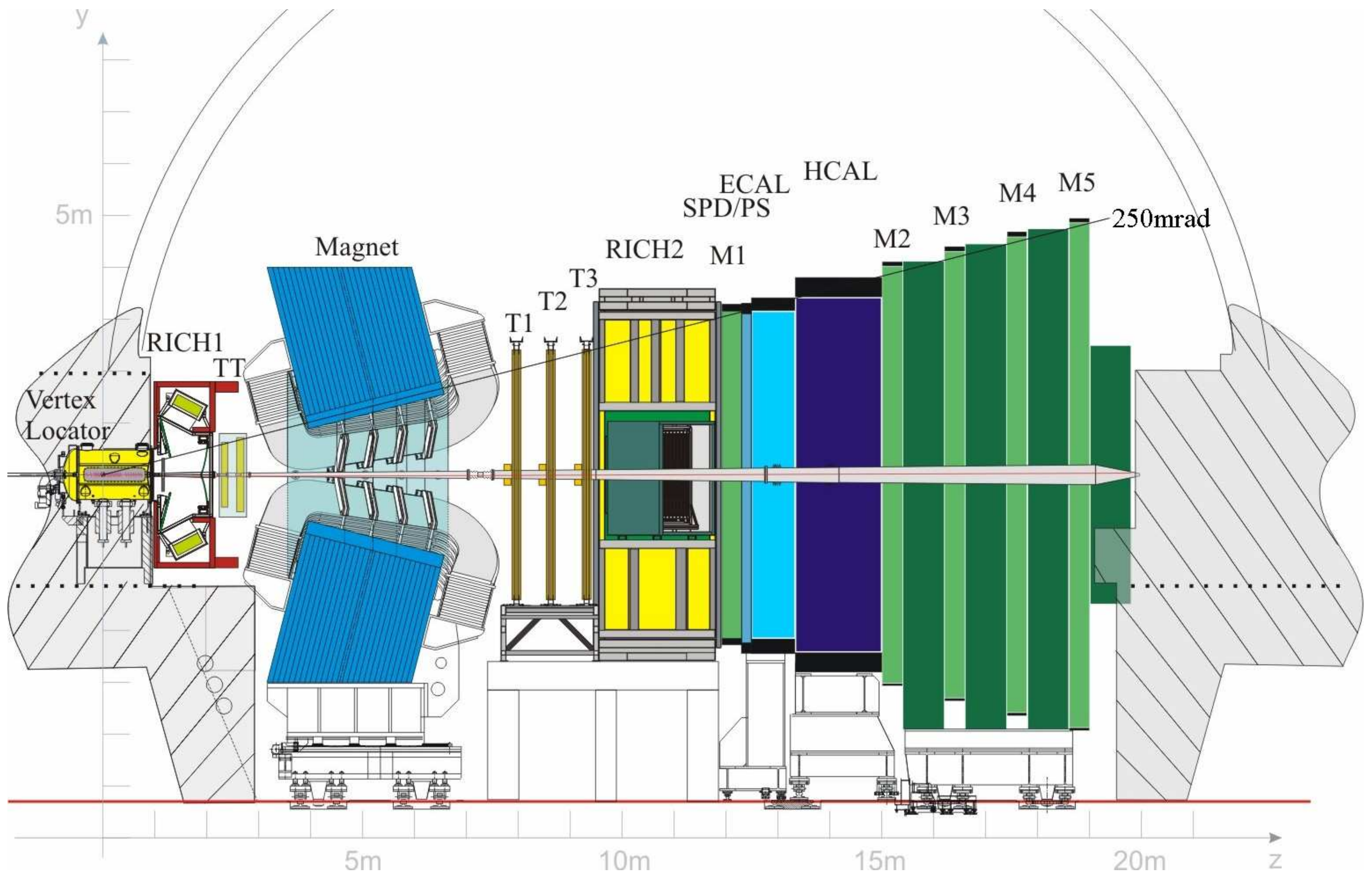
LHCb $\gamma = (64.6 \pm 2.8)^\circ$

Belle (II) $\gamma = (75.2^{+7.1}_{-7.5})^\circ$

Backup

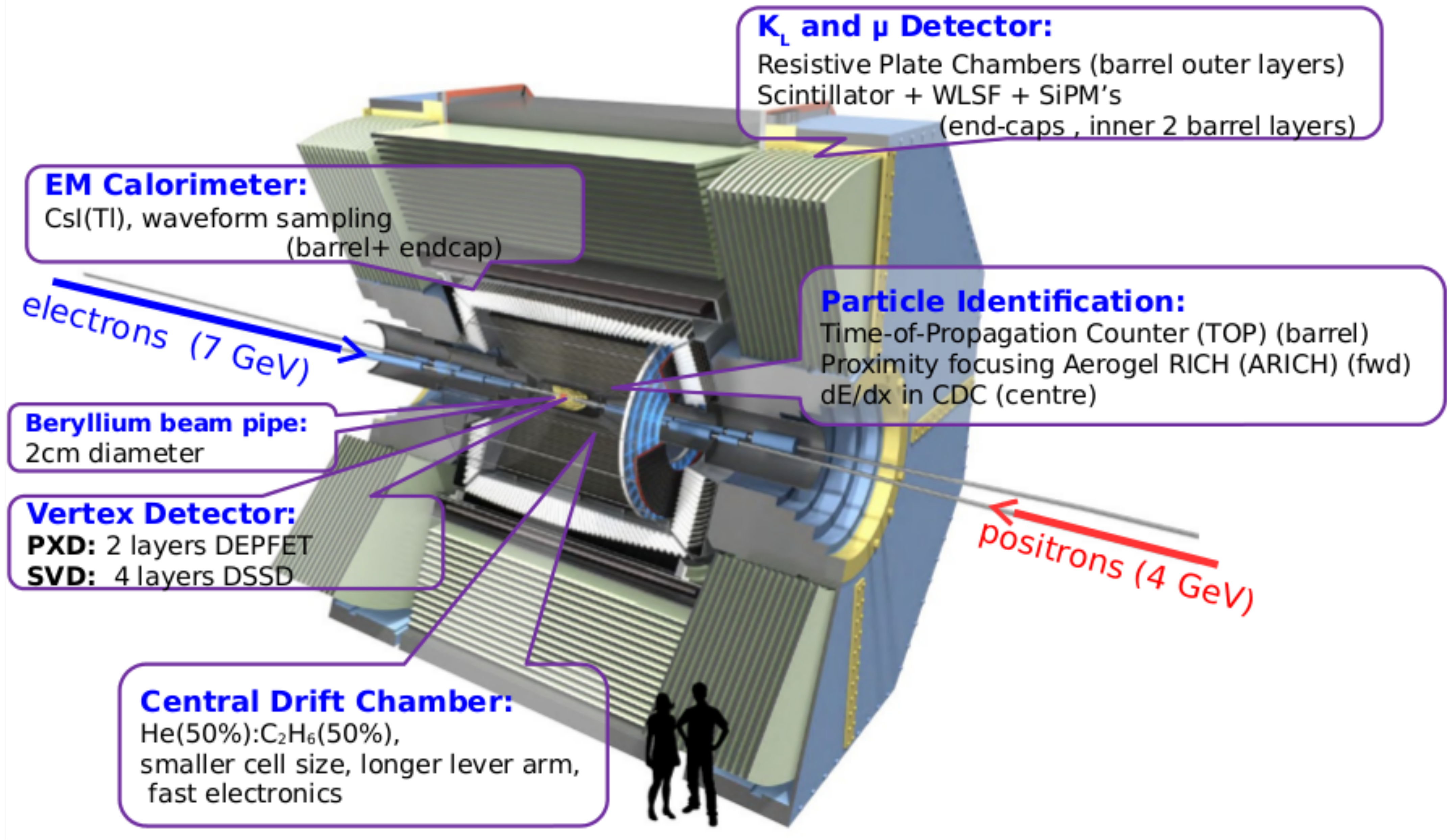
LHCb and Belle II

Complementary experiments for measurements of γ



Schematic of the Run 1/2 LHCb detector 2008 JINST 3 S08005, Int. J. Mod. Phys. A 30, 1530022 (2015)

- pp collisions, high production cross-section and boost
- Coverage only in the forward region
- Run 1+2: $9\ \text{fb}^{-1}$
- Run 3: higher integrated luminosity and hadron efficiency



Schematic of the Belle II detector

- $e^+e^- \to \Upsilon(4S)$ collisions, clean environment
- 4π coverage
- Good at reconstructing neutrals
- Belle: $711\ \text{fb}^{-1}$ at $\Upsilon(4S)$
- Belle II: $510\ \text{fb}^{-1}$at $\Upsilon(4S)$, more currently on the way

γ sensitive B decays

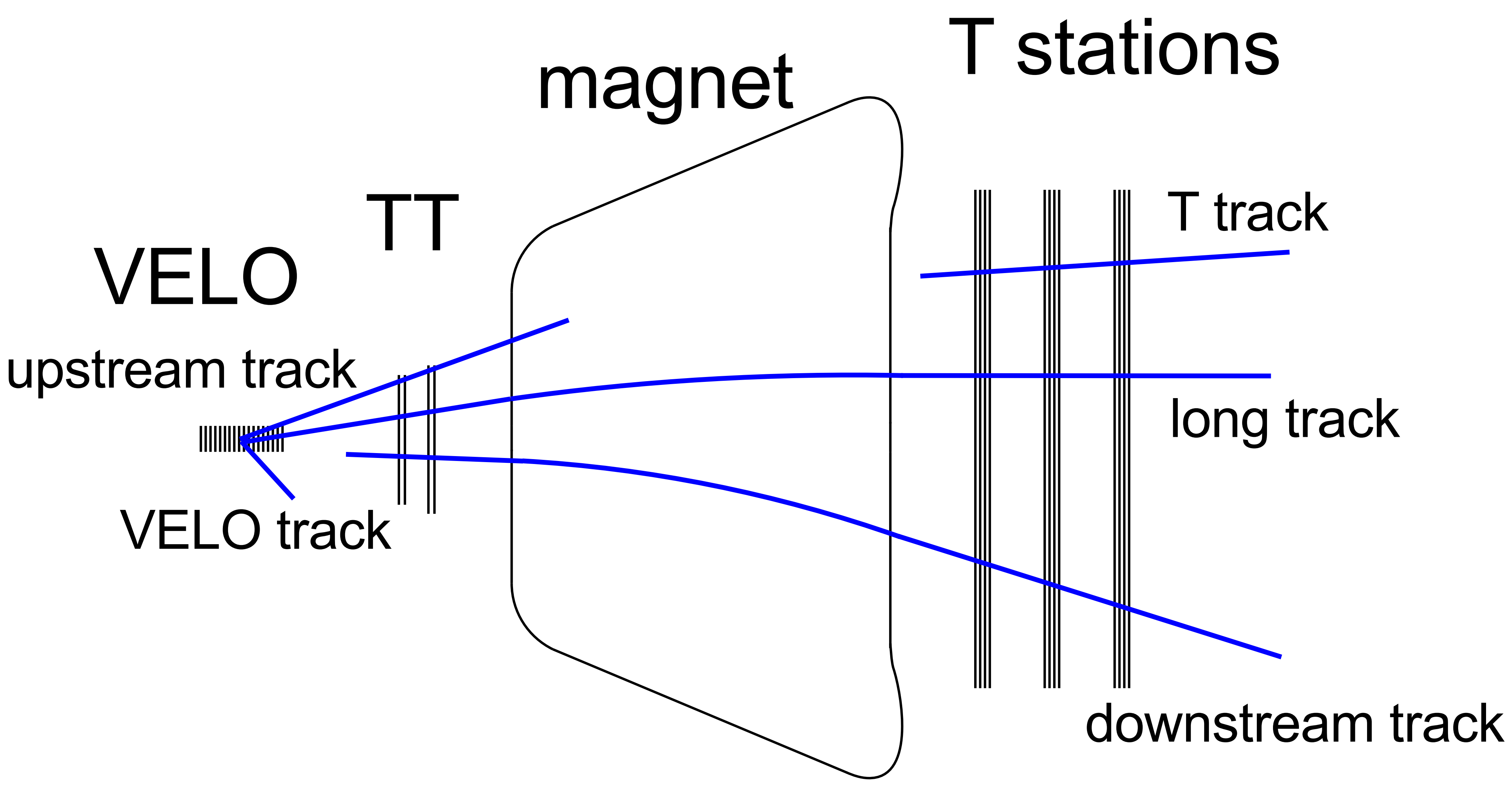
- $D \implies$ admixture of D^0 and $\bar{D}^0$
- $B^\pm \to Dh^\pm(\pi^+\pi^-)$ (New result today)
- $B^\pm \to D^*h^\pm$, where the D^* is partially reconstructed due to a missing π^0/γ
- $B^\pm \to DK^{*\pm}, K^{*\pm} \to K^0_S\pi^\pm \implies$ lower efficiency from extra K^0_S reconstruction
- $B^0 \to DK^{*0}$, "self tagging"
- $B^0 \to D^\mp\pi^\pm, D^- \to K^+\pi^-\pi^-$
- $B^0_s \to D^\mp_s K^\pm(h^+h^-), D^-_s \to h^-h^+\pi^-$

(last two:) Time dependent $\implies$ Flavour tagging

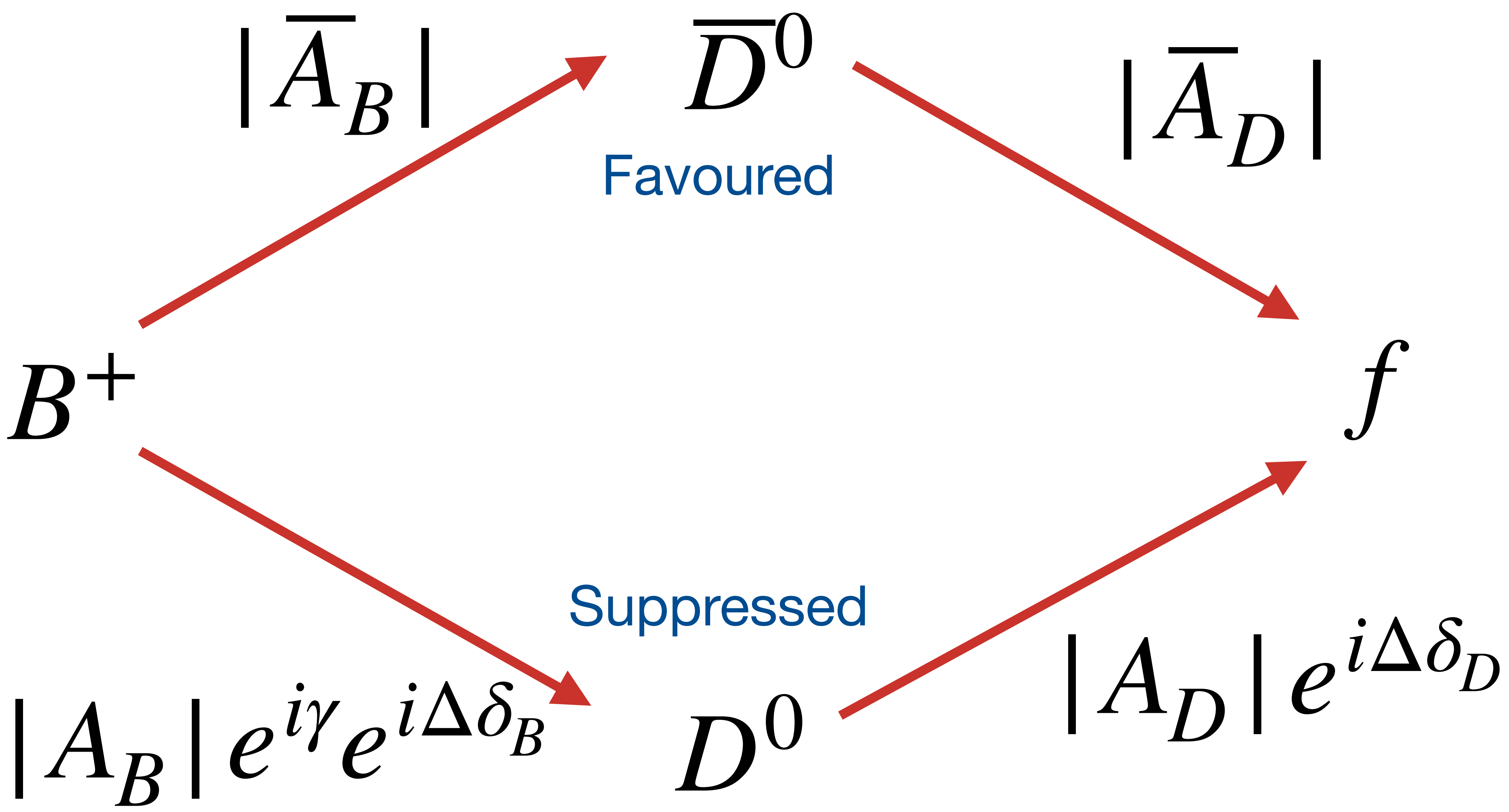
- Detailed summary available in the LHCb Beauty+Charm combination LHCb-CONF-2024-004

K_S^0 at LHCb

- For K_S^0 it's necessary to distinguish between those reconstructed with π tracks in different sub-detectors
  - Long-Long (LL) have hits in each tracking sub-detector
  - Down-Down (DD) are not seen in the VErtexLOcator (VELO) $\Longrightarrow$ slightly worse resolution



Time-integrated γ measurements



Sketch of the favoured and suppressed paths for a $B^+ \to DX, D \to f$ decay

- Can't tell which flavour D in each event
  - Interference between $b \to c$ and $b \to u$ transitions
- Squared amplitude depends on

  $\Delta\delta_B \pm \gamma$ for $B^\pm$

  $\implies$ asymmetries $\dfrac{\Gamma(B^- \to f) - \Gamma(B^+ \to f)}{\Gamma(B^- \to f) + \Gamma(B^+ \to f)}$
- Compare $B^\pm$ amplitudes to extract γ

γ from 2 and 4-body D decays

- For any D final state can measure the charge asymmetry
  - $\dfrac{\Gamma(B^- \to f) - \Gamma(B^+ \to f)}{\Gamma(B^- \to f) + \Gamma(B^+ \to f)}$ "Difference in peak heights"
- For 2-body modes can also measure ratios such as
  - $\dfrac{\Gamma(B^- \to \overline{D}X, \overline{D} \to f) + \Gamma(B^+ \to DX, D \to f)}{\Gamma(B^- \to DX, D \to f) + \Gamma(B^+ \to \overline{D}X, \overline{D} \to f)}$ for $D \to K^\pm\pi^\mp$ ~sup./fav.
  - $\dfrac{\Gamma(B^- \to D_{CP}X) + \Gamma(B^+ \to D_{CP}X)}{\Gamma(B^- \to DX) + \Gamma(B^+ \to DX)}$ for CP-even modes, $D \to \pi^+\pi^-$ and $D \to K^+K^-$
- These extend fairly simply to 4-body modes, multiply interference terms by
  - $D \to \pi^+\pi^-\pi^+\pi^-$: CP-even fraction
  - $D \to K^\pm\pi^\mp\pi^+\pi^-$: coherence factor - to account for resonances
- The same for B^0 decays

$B^+ \to DK^+, D \to (K^+K^-, K_S^0\pi^0)$

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$$Y_\pi(B^\pm \to D_X K^\pm) = \frac{1}{2}[1 \mp \mathcal{A}(B \to D_X K)]\, N(B \to D_X \pi)\, R_X\, \delta\, (1 - \varepsilon_\pm), \tag{4.1}$$

$$Y_K(B^\pm \to D_X K^\pm) = \frac{1}{2}[1 \mp \mathcal{A}(B \to D_X K)]\, N(B \to D_X \pi)\, R_X\, \delta\, \varepsilon_\pm, \tag{4.2}$$

$$Y_\pi(B^\pm \to D_X \pi^\pm) = \frac{1}{2}[1 \mp \mathcal{A}(B \to D_X \pi)]\, N(B \to D_X \pi)\, (1 - \kappa_\pm), \tag{4.3}$$

$$Y_K(B^\pm \to D_X \pi^\pm) = \frac{1}{2}[1 \mp \mathcal{A}(B \to D_X \pi)]\, N(B \to D_X \pi)\, \kappa_\pm, \tag{4.4}$$

We measure CP asymmetries,

$$\mathcal{A}_{CP\pm} \equiv \frac{\mathcal{B}(B^- \to D_{CP\pm}K^-) - \mathcal{B}(B^+ \to D_{CP\pm}K^+)}{\mathcal{B}(B^- \to D_{CP\pm}K^-) + \mathcal{B}(B^+ \to D_{CP\pm}K^+)}, \tag{1.1}$$

and the ratio of branching fractions for decays in which the D is reconstructed as a CP eigenstate and decays in which the D is reconstructed in a flavor-specific state:

$$\mathcal{R}_{CP\pm} \equiv \frac{\mathcal{B}(B^- \to D_{CP\pm}K^-) + \mathcal{B}(B^+ \to D_{CP\pm}K^+)}{(\mathcal{B}(B^- \to D_{\text{flav}}K^-) + \mathcal{B}(B^+ \to \overline{D}_{\text{flav}}K^+))/2}. \tag{1.2}$$

This ratio can be expressed as

$$\mathcal{R}_{CP\pm} \approx \frac{R_{CP\pm}}{R_{\text{flav}}}, \tag{1.3}$$

where

$$R_{CP\pm} \equiv \frac{\mathcal{B}(B^- \to D_{CP\pm}K^-) + \mathcal{B}(B^+ \to D_{CP\pm}K^+)}{\mathcal{B}(B^- \to D_{CP\pm}\pi^-) + \mathcal{B}(B^+ \to D_{CP\pm}\pi^+)}, \tag{1.4}$$

and

$$R_{\text{flav}} \equiv \frac{\mathcal{B}(B^- \to D_{\text{flav}}K^-) + \mathcal{B}(B^+ \to \overline{D}_{\text{flav}}K^+)}{\mathcal{B}(B^- \to D_{\text{flav}}\pi^-) + \mathcal{B}(B^+ \to \overline{D}_{\text{flav}}\pi^+)}. \tag{1.5}$$

Extracts from JHEP 05 (2024) 212

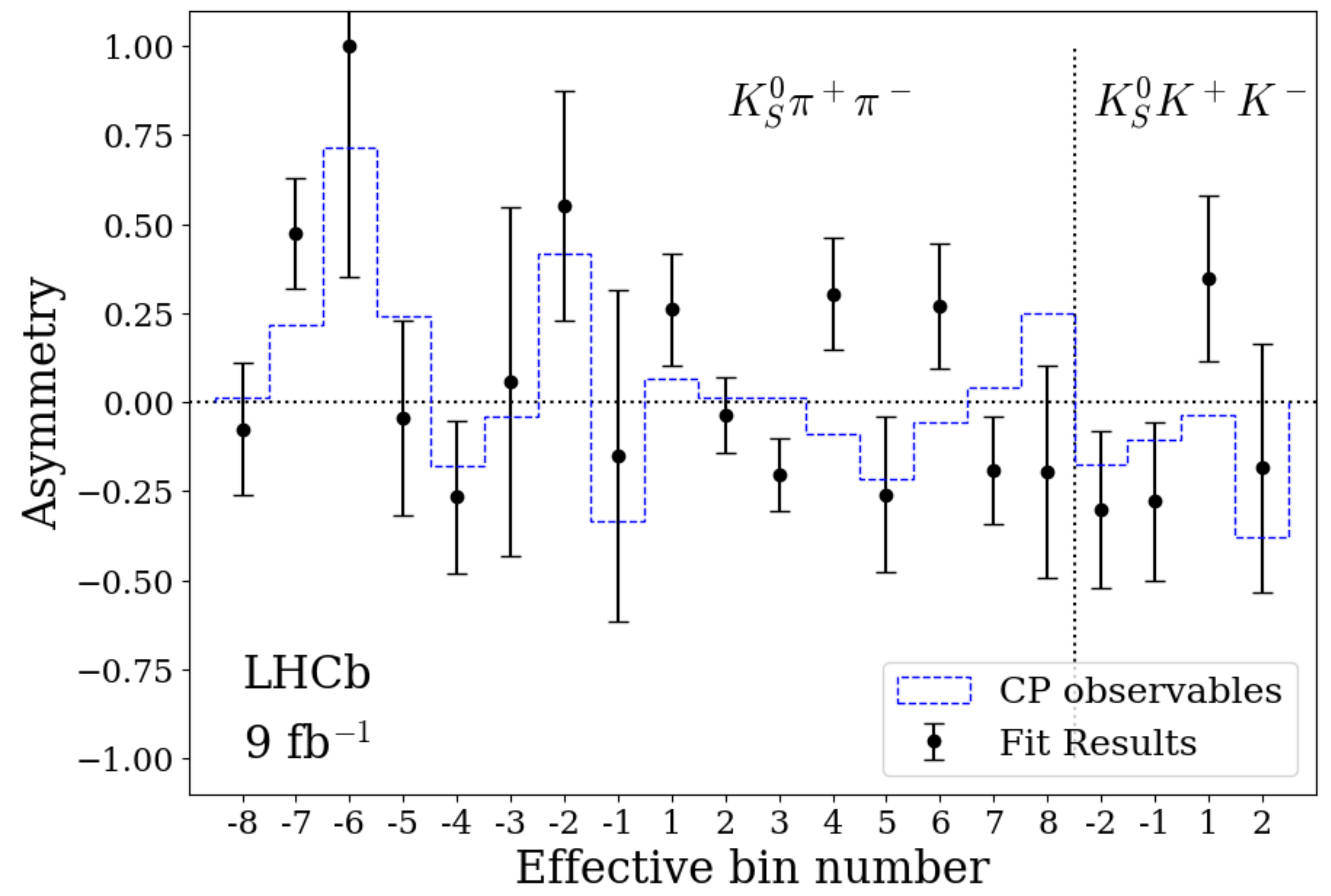
BPGGSZ Parameter Definitions

- $F_i = \frac{\int_i d\Phi \eta(\Phi) |A_{D^0}|^2}{\int d\Phi \eta(\Phi) |A_{D^0}|^2}$
- $c_i = \frac{\int_i d\Phi |A_{D^0}| |A_{\bar{D}^0}| \cos(\Delta\delta_D)}{\sqrt{\int_i d\Phi |A_{D^0}|^2 \int_i d\Phi |A_{\bar{D}^0}|^2}}$
- $s_i = \frac{\int_i d\Phi |A_{D^0}| |A_{\bar{D}^0}| \sin(\Delta\delta_D)}{\sqrt{\int_i d\Phi |A_{D^0}|^2 \int_i d\Phi |A_{\bar{D}^0}|^2}}$
- Where
  - Φ is the 5-dimensional phase-space coordinate
  - $\eta(\Phi)$ is the detection efficiency profile

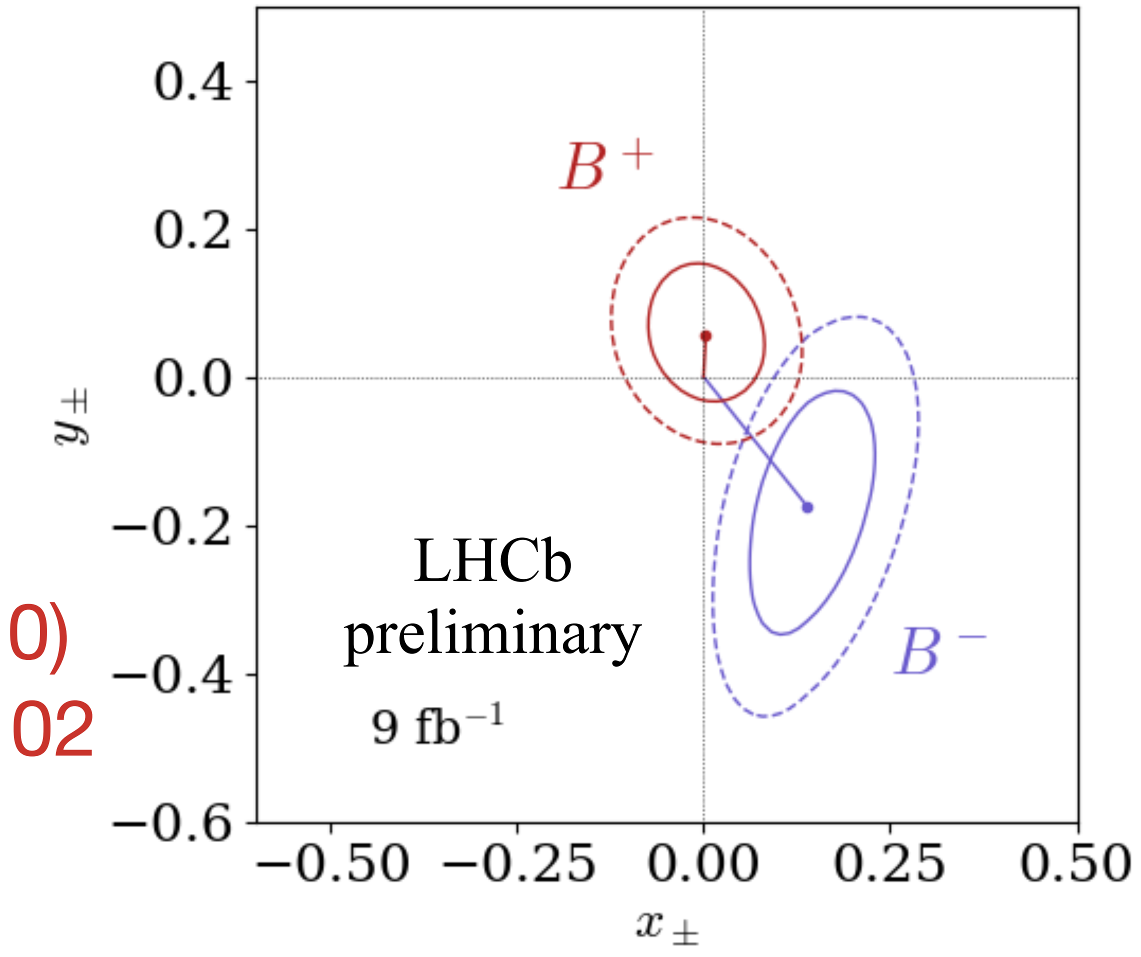
Separate γ results for $B^+ \to Dh^+, D \to h'^+h'^-\pi^+\pi^-$

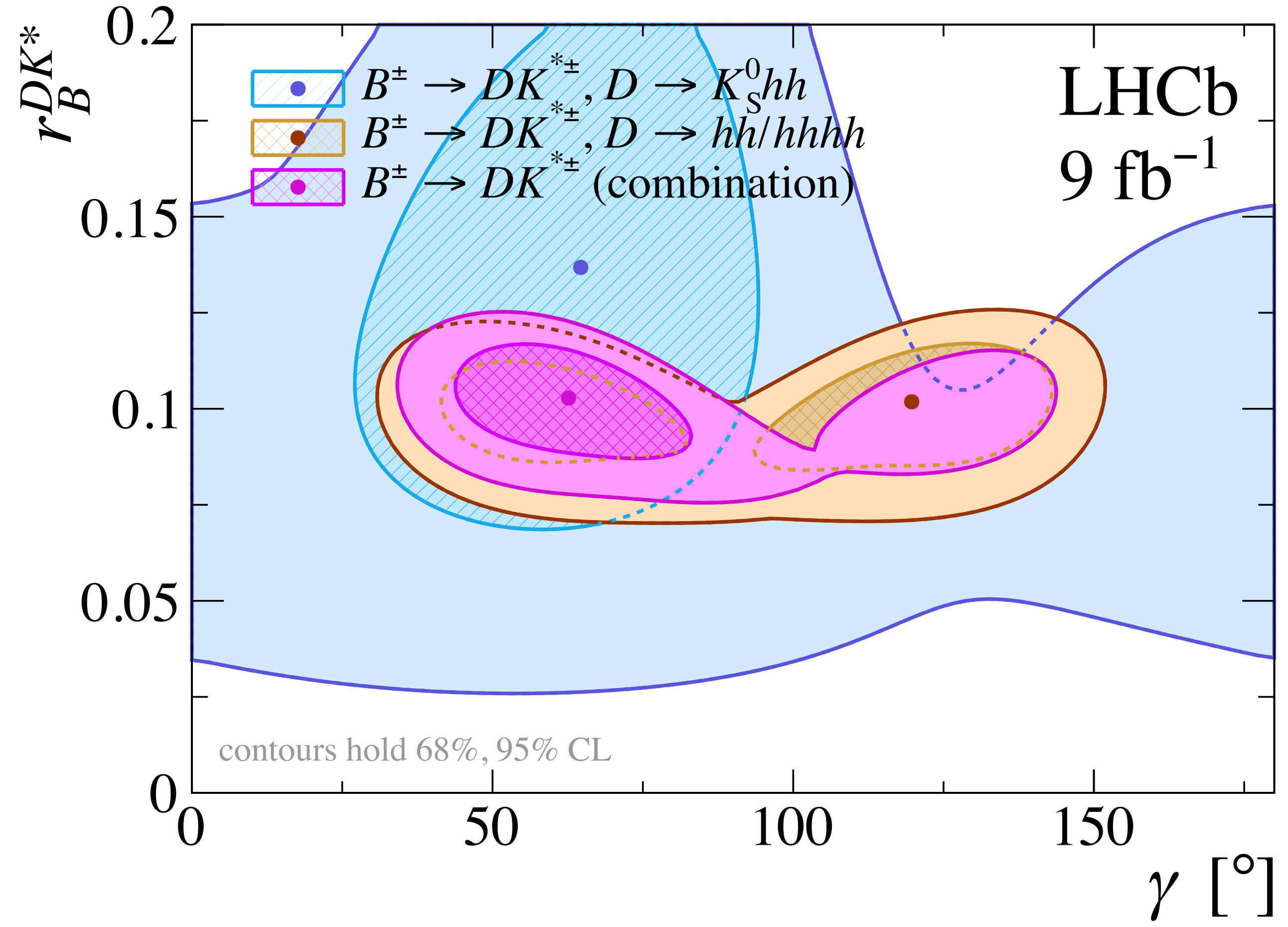
- Phase-space binned $\gamma = (53.9^{+9.5}_{-8.9})^\circ$ LHCb-PAPER-2025-019
- Phase-space integrated $\gamma = (116^{+12}_{-14})^\circ$ Eur. Phys. J. C (2023) 83:547

- Simultaneous measurement of γ using
  - $D \to K^\pm\pi^\mp(\pi^+\pi^-)$
  - $D \to \pi^+\pi^-(\pi^+\pi^-)$
  - $D \to K^+K^-$
  - $D \to K_S^0 h^+h^-$
- First time for $B^\pm \to DK^{*\pm}, D \to K_S^0 h^+h^-$
- First observation of the doubly Cabibbo suppressed $B^\pm \to DK^{*\pm}, D \to \pi^\pm K^\mp(\pi^+\pi^-)$
- Interpretation in terms of γ yields $\gamma = (63 \pm 13)^\circ$
- $F_+^{4\pi}$ input from BES III Phys Rev D 106 (2022) 092004
- c_i, s_i for $D \to K_S^0 h^+h^-$ combined from CLEO-c Phys Rev D 82 (2010) 112006 and BES III Phys. Rev. D 101 (2020) 112002, Phys. Rev. D 102 (2020) 052008
- $r_D^{K3\pi}, \delta_D^{K3\pi}, \kappa^{K3\pi}$ from a combination of LHCb, CLEO-c and BES III measurements JHEP 05 (2021) 164



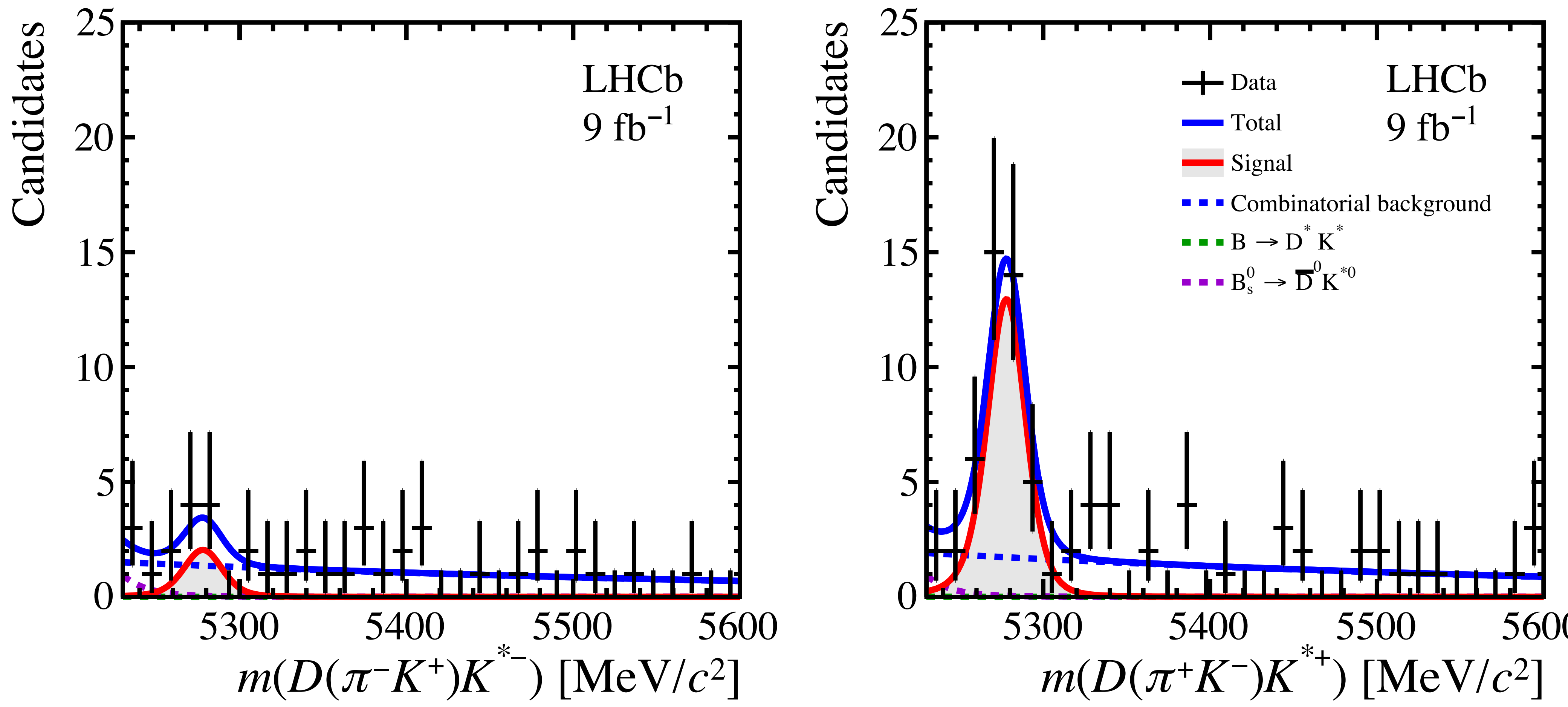
Per-bin asymmetries determined by the CP fit parameters (red) and signal yields when allowed to float freely (black) with statistical uncertainties



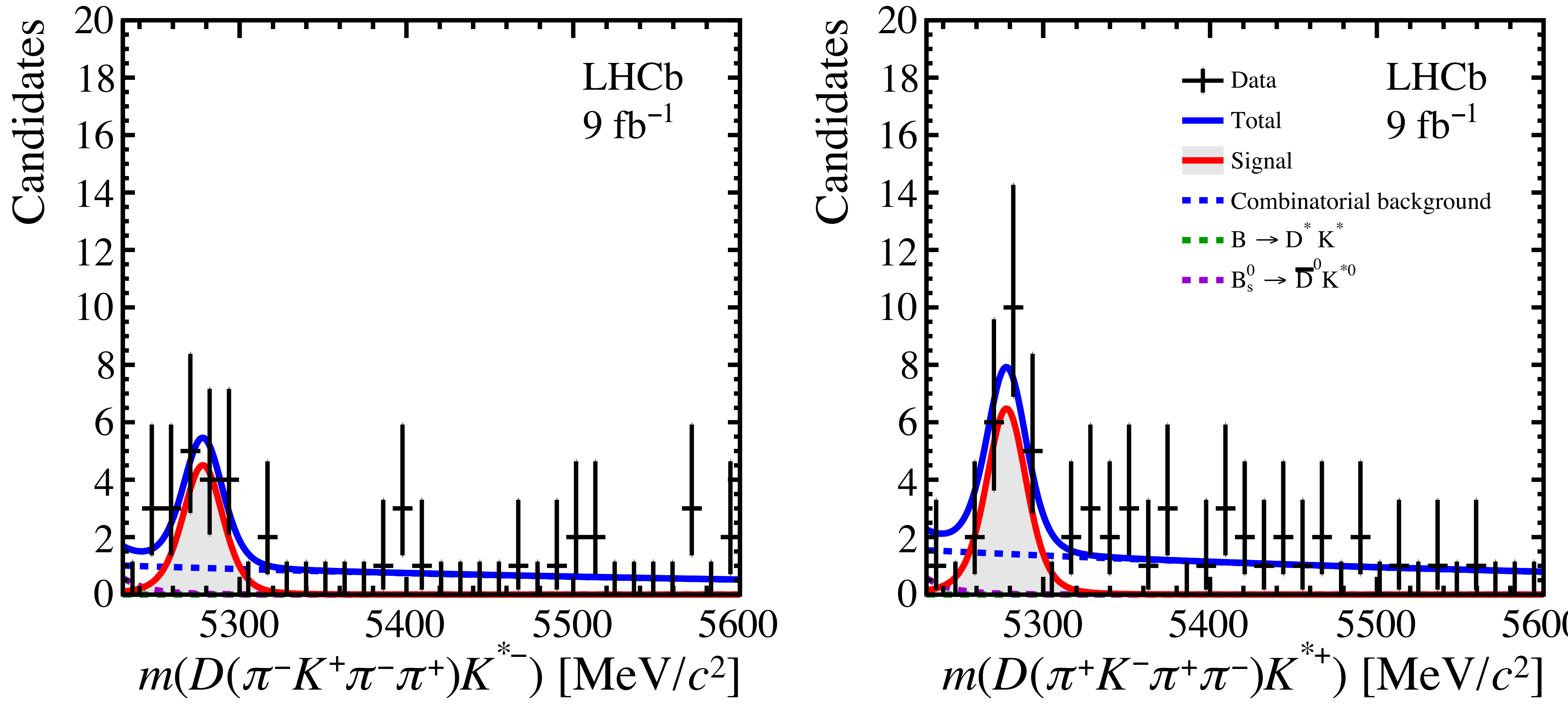


Statistical confidence regions for the measured $x_\pm, y_\pm$ values (left) and the contours for the extraction of $r_B^{DK^*}$ and γ (right)

- First observation of the doubly Cabibbo suppressed $B^\pm \to DK^{*\pm}, D \to \pi^\pm K^\mp(\pi^+\pi^-)$
- Amplitudes for favoured modes are of the form
  - $A^2 \propto 1 + r_B^2 r_D^2 + 2r_B r_D I$
  - $r_B, r_D < 1$
- Suppressed modes suffer from low statistics
  - But their amplitudes allow for large interference effects
  - $A^2 \propto r_D^2 + r_B^2 + 2r_B r_D I$
- We need more data!



CP-mass fit result for suppressed $B^\pm \to DK^{*\pm}, D \to \pi^\pm K^\mp$



CP-mass fit result for suppressed $B^\pm \to DK^{*\pm}, D \to \pi^\pm K^\mp \pi^+\pi^-$

Combination updates

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B decay	D decay	Ref.	Dataset	Status since Ref. [13]
$B^\pm \to Dh^\pm$	$D \to h^+h'^-$	[32]	Run 1&2	*As before*
$B^\pm \to Dh^\pm$	$D \to h^+h^-\pi^+\pi^-$	[19]	Run 1&2	**New**
$B^\pm \to Dh^\pm$	$D \to K^\pm\pi^\mp\pi^+\pi^-$	[33]	Run 1&2	*As before*
$B^\pm \to Dh^\pm$	$D \to h^+h'^-\pi^0$	[34]	Run 1&2	*As before*
$B^\pm \to Dh^\pm$	$D \to K^0_S h^+h^-$	[35]	Run 1&2	*As before*
$B^\pm \to Dh^\pm$	$D \to K^0_S K^\pm\pi^\mp$	[36]	Run 1&2	*As before*
$B^\pm \to D^*h^\pm$	$D \to h^+h'^-$ (PR)	[32]	Run 1&2	*As before*
$B^\pm \to D^*h^\pm$	$D \to K^0_S h^+h^-$ (PR)	[20]	Run 1&2	**New**
$B^\pm \to D^*h^\pm$	$D \to K^0_S h^+h^-$ (FR)	[21]	Run 1&2	**New**
$B^\pm \to DK^{*\pm}$	$D \to h^+h'^-$	[22]	Run 1&2	**Updated**
$B^\pm \to DK^{*\pm}$	$D \to h^+\pi^-\pi^+\pi^-$	[22]	Run 1&2	**Updated**
$B^\pm \to DK^{*\pm}$	$D \to K^0_S h^+h^-$	[22]	Run 1&2	**New**
$B^\pm \to Dh^\pm\pi^+\pi^-$	$D \to h^+h'^-$	[37]	Run 1	*As before*
$B^0 \to DK^{*0}$	$D \to h^+h'^-$	[23]	Run 1&2	**Updated**
$B^0 \to DK^{*0}$	$D \to h^+\pi^-\pi^+\pi^-$	[23]	Run 1&2	**Updated**
$B^0 \to DK^{*0}$	$D \to K^0_S h^+h^-$	[24]	Run 1&2	**Updated**
$B^0 \to D^\mp\pi^\pm$	$D^+ \to K^-\pi^+\pi^+$	[38]	Run 1	*As before*
$B^0_s \to D_s^\mp K^\pm$	$D_s^+ \to h^+h^-\pi^+$	[25,39]	Run 1&2	**Updated**
$B^0_s \to D_s^\mp K^\pm\pi^+\pi^-$	$D_s^+ \to h^+h^-\pi^+$	[40]	Run 1&2	*As before*

Beauty measurements in the combination

D decay	Observable(s)	Ref.	Dataset	Status since Ref. [13]
$D^0 \to h^+h^-$	ΔA_{CP}	[41–43]	Run 1&2	*As before*
$D^0 \to K^+K^-$	$A_{CP}(K^+K^-)$	[43–45]	Run 2	*As before*
$D^0 \to h^+h^-$	$y_{CP} - y_{CP}^{K^-\pi^+}$	[46,47]	Run 1&2	*As before*
$D^0 \to h^+h^-$	ΔY	[48–51]	Run 1&2	*As before*
$D^0 \to K^+\pi^-$ (double tag)	$R^\pm$, $(x'^\pm)^2$, $y'^\pm$	[52]	Run 1	*As before*
$D^0 \to K^+\pi^-$ (single tag)	$R_{K\pi}$, $A_{K\pi}$, $c_{K\pi}^{(\prime)}$, $\Delta c_{K\pi}^{(\prime)}$	[27,53]	Run 1&2	**Updated**
$D^0 \to K^\pm\pi^\mp\pi^+\pi^-$	$(x^2+y^2)/4$	[54]	Run 1	*As before*
$D^0 \to K^0_S\pi^+\pi^-$	x, y	[55]	Run 1	*As before*
$D^0 \to K^0_S\pi^+\pi^-$	x_{CP}, y_{CP}, Δx, Δy	[56]	Run 1	*As before*
$D^0 \to K^0_S\pi^+\pi^-$	x_{CP}, y_{CP}, Δx, Δy	[57,58]	Run 2	*As before*
$D^0 \to \pi^+\pi^-\pi^0$	ΔY^{eff}	[26]	Run 2	**New**

Charm measurements in the combination

Combination updates

LHCb-CONF-2024-004

Decay	Parameters	Source	Ref.	Status since Ref. [13]
$B^\pm \to DK^{*\pm}$	$\kappa_{B^\pm}^{DK^{*\pm}}$	LHCb	[59]	*As before*
$B^0 \to DK^{*0}$	$\kappa_{B^0}^{DK^{*0}}$	LHCb	[60]	*As before*
$B^0 \to D^\mp\pi^\pm$	β	HFLAV	[14]	**Updated**
$B_s^0 \to D_s^\mp K^\pm(\pi\pi)$	ϕ_s	LHCb	[61]	**Updated**
$D \to K^+\pi^-$	$\cos\delta_D^{K\pi}$, $\sin\delta_D^{K\pi}$, $(r_D^{K\pi})^2$, x^2, y	CLEO-c	[62]	*As before*
$D \to K^+\pi^-$	$A_{K\pi}$, $A_{K\pi}^{\pi\pi\pi^0}$, $r_D^{K\pi}\cos\delta_D^{K\pi}$, $r_D^{K\pi}\sin\delta_D^{K\pi}$	BESIII	[63]	*As before*
$D \to h^+h^-\pi^0$	$F_{\pi\pi\pi^0}^+$, $F_{KK\pi^0}^+$	CLEO-c	[64]	*As before*
$D \to \pi^+\pi^-\pi^+\pi^-$	$F_{4\pi}^+$	CLEO-c+BESIII	[64, 65]	*As before*
$D \to K^+K^-\pi^+\pi^-$	$F_{KK\pi\pi}^+$	BESIII	[66]	**New**
$D \to K^+\pi^-\pi^0$	$r_D^{K\pi\pi^0}$, $\delta_D^{K\pi\pi^0}$, $\kappa_D^{K\pi\pi^0}$	CLEO-c+LHCb+BESIII	[67–69]	*As before*
$D \to K^\pm\pi^\mp\pi^+\pi^-$	$r_D^{K3\pi}$, $\delta_D^{K3\pi}$, $\kappa_D^{K3\pi}$	CLEO-c+LHCb+BESIII	[54, 67–69]	*As before*
$D \to K_S^0K^\pm\pi^\mp$	$r_D^{K_S^0K\pi}$, $\delta_D^{K_S^0K\pi}$, $\kappa_D^{K_S^0K\pi}$	CLEO	[70]	*As before*
$D \to K_S^0K^\pm\pi^\mp$	$r_D^{K_S^0K\pi}$	LHCb	[71]	*As before*

Auxiliary inputs to the combination

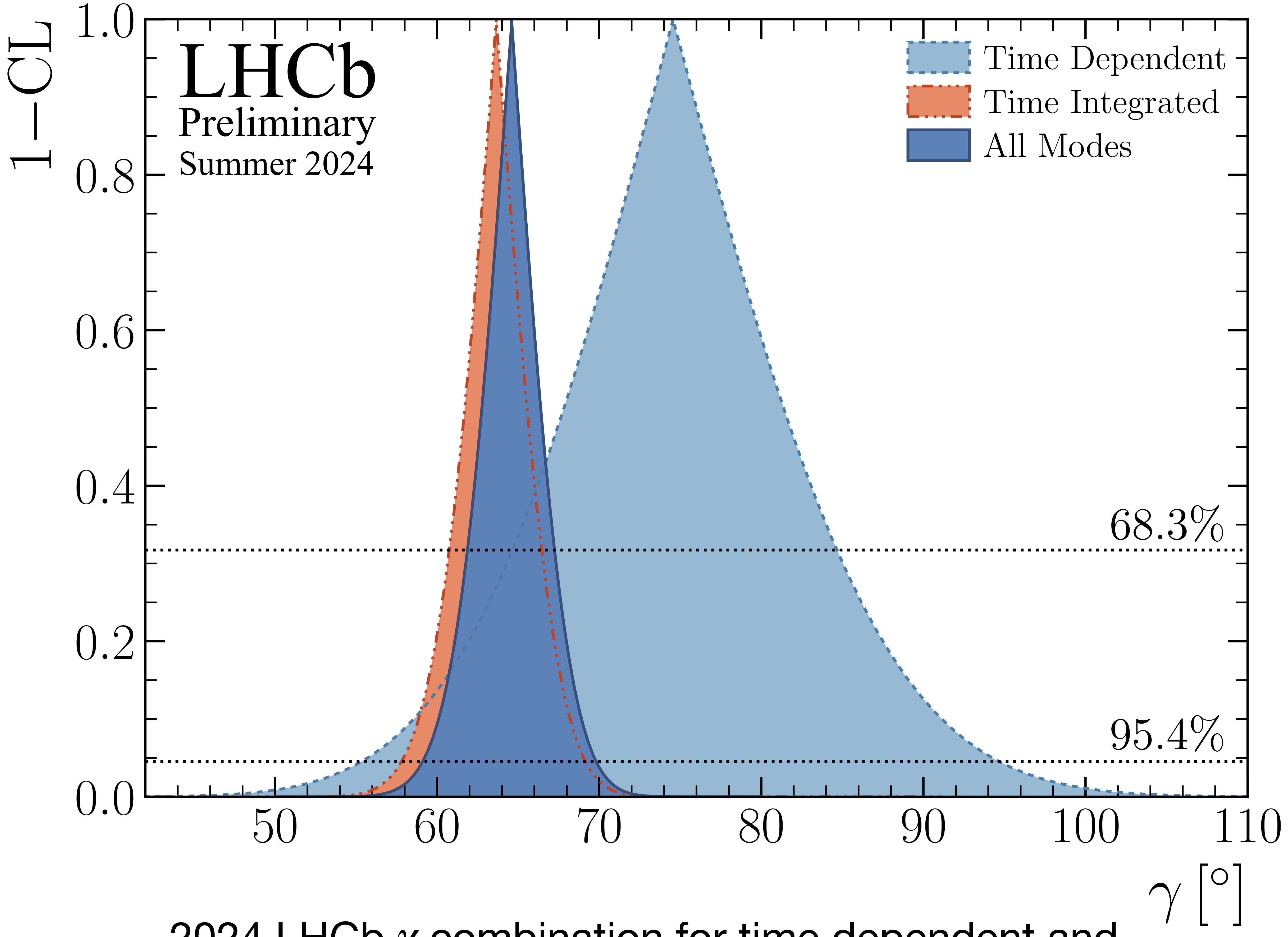
Quantity	Value	68.3% CL Uncertainty	68.3% CL Interval	95.4% CL Uncertainty	95.4% CL Interval
$\gamma[^\circ]$	64.6	±2.8	[61.8, 67.4]	$^{+5.5}_{-5.7}$	[58.9, 70.1]
$r_{B^\pm}^{DK^\pm}[\%]$	9.73	$^{+0.21}_{-0.20}$	[9.53, 9.94]	$^{+0.42}_{-0.40}$	[9.33, 10.15]
$\delta_{B^\pm}^{DK^\pm}[^\circ]$	127.4	$^{+2.8}_{-3.0}$	[124.4, 130.2]	$^{+5.6}_{-6.2}$	[121.2, 133.0]
$r_{B^\pm}^{D\pi^\pm}[\%]$	0.49	$^{+0.06}_{-0.05}$	[0.44, 0.55]	$^{+0.12}_{-0.10}$	[0.39, 0.61]
$\delta_{B^\pm}^{D\pi^\pm}[^\circ]$	292	$^{+10}_{-11}$	[281, 301]	$^{+19}_{-22}$	[269, 310]
$r_{B^\pm}^{D^*K^\pm}[\%]$	10.6	±1.0	[9.6, 11.6]	±2.0	[8.6, 12.6]
$\delta_{B^\pm}^{D^*K^\pm}[^\circ]$	312	$^{+6}_{-7}$	[304, 318]	$^{+12}_{-16}$	[296, 324]
$r_{B^\pm}^{D^*\pi^\pm}[\%]$	0.74	$^{+0.41}_{-0.32}$	[0.42, 1.15]	$^{+0.87}_{-0.62}$	[0.12, 1.61]
$\delta_{B^\pm}^{D^*\pi^\pm}[^\circ]$	37	$^{+39}_{-20}$	[17, 76]	$^{+94}_{-31}$	[6, 131]
$r_{B^\pm}^{DK^{*\pm}}[\%]$	10.6	$^{+0.9}_{-1.0}$	[9.6, 11.5]	$^{+1.7}_{-2.0}$	[8.6, 12.3]
$\delta_{B^\pm}^{DK^{*\pm}}[^\circ]$	49	$^{+14}_{-11}$	[38, 63]	$^{+30}_{-23}$	[26, 79]
$r_{B^0}^{DK^{*0}}[\%]$	23.4	$^{+1.5}_{-1.6}$	[21.8, 24.9]	$^{+2.9}_{-3.3}$	[20.1, 26.3]
$\delta_{B^0}^{DK^{*0}}[^\circ]$	192	±6	[186, 198]	$^{+13}_{-12}$	[180, 205]
$r_{B_s^0}^{D_s^\mp K^\pm}[\%]$	33.3	$^{+3.7}_{-3.5}$	[29.8, 37.0]	$^{+7.5}_{-7.1}$	[26.2, 40.8]
$\delta_{B_s^0}^{D_s^\mp K^\pm}[^\circ]$	349	±6	[343, 355]	±12	[337, 361]
$r_{B_s^0}^{D_s^\mp K^\pm\pi^+\pi^-}[\%]$	46	±8	[37, 54]	$^{+16}_{-17}$	[29, 62]
$\delta_{B_s^0}^{D_s^\mp K^\pm\pi^+\pi^-}[^\circ]$	345	$^{+13}_{-12}$	[333, 358]	$^{+26}_{-25}$	[320, 371]
$r_{B^0}^{D^\mp\pi^\pm}[\%]$	3.0	$^{+1.3}_{-1.2}$	[1.8, 4.3]	$^{+3.1}_{-2.7}$	[0.3, 6.1]
$\delta_{B^0}^{D^\mp\pi^\pm}[^\circ]$	30	$^{+25}_{-36}$	[−6, 55]	$^{+45}_{-77}$	[−47, 75]
$r_{B^\pm}^{DK^\pm\pi^+\pi^-}[\%]$	8.0	$^{+2.7}_{-3.3}$	[4.7, 10.7]	$^{+4.9}_{-8.0}$	[0.0, 12.9]*
$r_{B^\pm}^{D\pi^\pm\pi^+\pi^-}[\%]$	6.2	$^{+2.2}_{-3.0}$	[3.2, 8.4]	$^{+3.7}_{-6.2}$	[0.0, 9.9]*
$x[\%]$	0.41	±0.05	[0.36, 0.45]	±0.09	[0.31, 0.50]
$y[\%]$	0.621	$^{+0.022}_{-0.021}$	[0.600, 0.643]	$^{+0.044}_{-0.042}$	[0.579, 0.665]
$r_D^{K\pi}[\%]$	5.855	$^{+0.010}_{-0.009}$	[5.846, 5.865]	$^{+0.020}_{-0.019}$	[5.836, 5.875]
$\delta_D^{K\pi}[^\circ]$	191.6	$^{+2.5}_{-2.4}$	[189.2, 194.1]	$^{+4.9}_{-5.1}$	[186.5, 196.5]
$\lvert q/p\rvert$	0.989	±0.015	[0.974, 1.004]	$^{+0.031}_{-0.030}$	[0.959, 1.020]
$\phi[^\circ]$	−2.5	±1.2	[−3.7, −1.3]	±2.5	[−5.0, 0.0]
$a_{K^+K^-}^{\mathrm{d}}[\%]$	0.06	$^{+0.06}_{-0.05}$	[0.01, 0.12]	±0.11	[−0.05, 0.17]
$a_{\pi^+\pi^-}^{\mathrm{d}}[\%]$	0.22	±0.06	[0.16, 0.28]	±0.12	[0.10, 0.34]
$a_{K^+\pi^-}^{\mathrm{d}}[\%]$	−0.60	$^{+0.27}_{-0.26}$	[−0.86, −0.33]	$^{+0.53}_{-0.54}$	[−1.14, −0.07]

Combination results for Beauty and Charm parameters of interest

A matter of time

LHCb-CONF-2024-004

- Some small tension between time dependent and time integrated measurements
- Clearly need to push harder on time dependent analyses to get this uncertainty down



2024 LHCb γ combination for time dependent and time integrated analyses