

# CPV in Charmless B Meson Decays

on behalf of Belle, Belle II, and LHCb

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# Topics

## Belle II

$B^0 \rightarrow \pi^0 \pi^0$  — improved measurement,  $A_{CP}$  [arXiv:2412.14260, PRD111 L071102(2025)]

## Belle

$B^0 \rightarrow \omega \omega$  — first observation,  $f_L$ ,  $A_{CP}$  [arXiv:2401.04646, PRL133 081801(2024)]

## LHCb

$B^0 \rightarrow \phi \phi$  — new upper limit [arXiv:2507.20945]

$B^+ \rightarrow \rho^0 K^{*+}$  — improved measurement,  $f_L$ , non-zero  $A_{CP}$  [arXiv:2508.13563]

$B \rightarrow (K_S K \pi) K$  — two-body decays with light-resonances [arXiv:2501.06483, PRD111, 092009(2025)]

$B_{(s)}^0 \rightarrow K_S p \bar{p}$  — first observation for  $B_s$  [arXiv:2504.21269, JHEP07 (2025) 121]

$B^+ \rightarrow \bar{\Lambda} p \bar{p} p$  — first observation,  $A_{CP}$  [arXiv:2508.16259]

*4 charmless B decay modes with  $A_{CP}$  measurements (other charmless B decay modes are also relevant to CPV)*

# Roles of charmless B Meson Decays

## Rich sample for flavor physics

- Large number of interesting decay modes
- Interplays between  $b \rightarrow s$  or  $b \rightarrow d$  penguins and CKM-suppressed  $b \rightarrow u$  tree
- Sensitivity to new physics in penguin amplitudes
- **CPV** from CKM complex phase in the SM, and from new physics if any
- Branching fractions in  $10^{-5}$  to  $10^{-8}$  range

## Theory

- Individual branching fraction predictions are not so precise
- **Ratios / differences** ( $A_{CP}, \dots$ ) / **relations** (sum rules, ...) are more reliable
- Extraction of the CKM angle  $\phi_2/\alpha$  relies on the isospin analysis

## Yet unresolved issues

- $K$ - $\pi$  puzzle — simplest modes, but CPV difference between  $K^+\pi^-$  and  $K^+\pi^0$  **unexplained**
- Polarization puzzle — large transverse polarization in  $B \rightarrow (\rho/\omega/\phi)K^*$  modes **unexplained**

# Two-body charmless B decays

## Charmless two-body decays into **PP**, **PV**, **VV** (P:pseudoscalar V:vector)

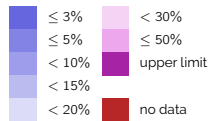
- PP modes are all measured except  $\eta^{(\prime)}\eta^{(\prime)}$
- Upper limit only for many PV and VV modes [only one mode ( $K^{*+}K_S$ ) not in PDG yet]

## Decays into **PS**, **VS** are also heavily involved (S:scalar)

- Narrow scalars (e.g.  $f_0(980)$ ) are easily distinguished
- Wide scalars (e.g.  $f_0(500)$ ) overlap with vectors (e.g.  $\rho(770)^0$ )

## Other combinations

- **Axial vectors**, tensors, Non-resonant 3- and 4-body decays, **baryonic** decays, ...



Two-body charmless  $B \rightarrow PP/PV/VV$  decay branching fraction errors (%) in PDG-live 2025

|                    | $\pi^+$ | $\pi^0$ | $K^+$ | $K_S$ | $\eta$ | $\eta'$ |
|--------------------|---------|---------|-------|-------|--------|---------|
| $\pi^-$ or $\pi^+$ | 4       | 5       | 2     | 3     | 7      | 33      |
| $\pi^0$            |         | 11      | 3     | 4     | 41     | 50      |
| $K^-$ or $K^+$     |         |         | 19    | 13    | 17     | 4       |
| $K_S$              |         |         |       | 13    | 22     | 6       |
| $\eta$             |         |         |       |       | UL     | UL      |
| $\eta'$            |         |         |       |       |        | UL      |

|                    | $\rho^+$ | $\rho^0$ | $K^{*+}$ | $K^{*0}$ | $\omega$ | $\phi$ |
|--------------------|----------|----------|----------|----------|----------|--------|
| $\pi^-$ or $\pi^+$ | 10       | 14       | 5        | 8        | 7        | 47     |
| $\pi^0$            | 12       | 25       | 13       | 18       | UL       | UL     |
| $K^-$ or $K^+$     | 13       | 14       | UL       | 14       | 6        | 8      |
| $K_S$              | 16       | 32       | NA       | UL       | 8        | 10     |
| $\eta$             | 41       | UL       | 8        | 6        | 43       | UL     |
| $\eta'$            | 23       | UL       | 38       | 21       | 50       | UL     |

|                                 | $\rho^+$ | $\rho^0$ | $K^{*+}$ | $K^{*0}$ | $\omega$ | $\phi$ |
|---------------------------------|----------|----------|----------|----------|----------|--------|
| $\rho^-$ or $\rho^+$            | 7        | 8        | 25       | 16       | 13       | UL     |
| $\rho^0$                        |          | 16       | 16       | 33       | UL       | UL     |
| $K^{*-}$ or $K^{*+}$            |          |          | UL       | 32       | UL       | 20     |
| $\overline{K}^{*0}$ or $K^{*0}$ |          |          |          | 29       | 25       | 5      |
| $\omega$                        |          |          |          |          | 18       | UL     |
| $\phi$                          |          |          |          |          |          | UL     |

# Belle II $B^0 \rightarrow \pi^0 \pi^0$

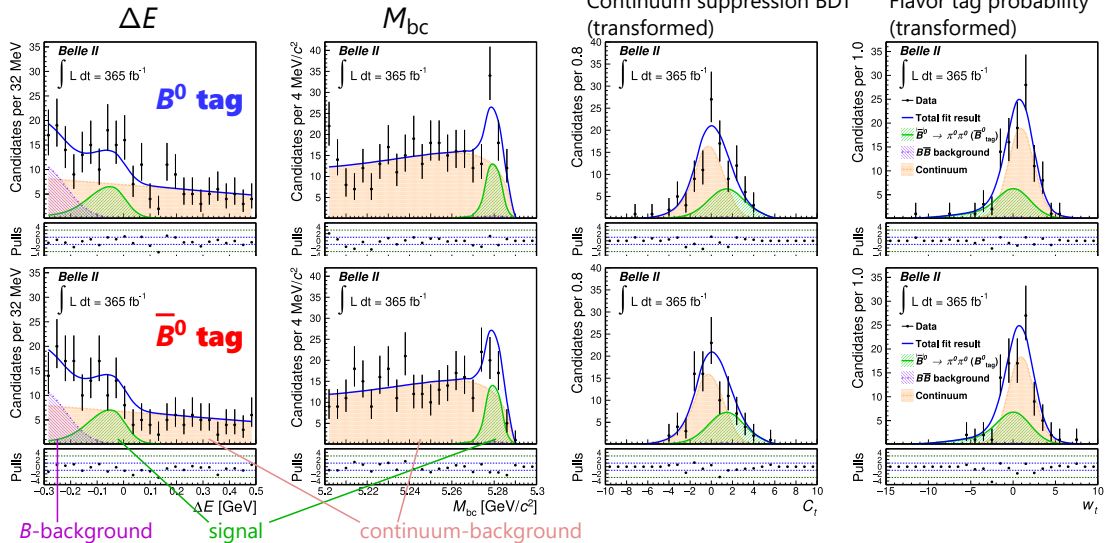
## Role of $B^0 \rightarrow \pi^0 \pi^0$

- Involves color-suppressed  $b \rightarrow u$  tree and  $b \rightarrow d$  penguin amplitudes
- Isospin analysis together with  $B^0 \rightarrow \pi^+ \pi^-$ ,  $B^+ \rightarrow \pi^+ \pi^0 \Rightarrow$  CKM UT angle  $\phi_2$
- Smallest  $\mathcal{B}$  among  $B \rightarrow \pi\pi$  (but larger than theory expectation)

## Analysis

- Belle II Run 1 dataset (387M  $B\bar{B}$ )
- Photon only final state, large beam-induced background  $\rightarrow$  new Photon-BDT  
(newly introduced to enhance the signal photon purity)
- Large background from  $e^+e^- \rightarrow q\bar{q} \rightarrow$  continuum-suppression-BDT  
( $B\bar{B}$ : spherical  $\Leftrightarrow$  continuum: jet-like)
- Signal: peak in  $M_{bc} = \sqrt{(E_{\text{beam}}^*)^2 - |p_B^*|^2}$  and  $\Delta E = E_B^* - E_{\text{beam}}^*$   
(Only peaking background from  $B \rightarrow (\pi\pi^0)\pi^0$  (including  $\rho\pi^0$ ) and well separated in  $\Delta E$ )
- Flavor from accompanying  $B \rightarrow$  Belle II's new BDT-based flavor tag

# Belle II $B^0 \rightarrow \pi^0 \pi^0$ 4-d fit



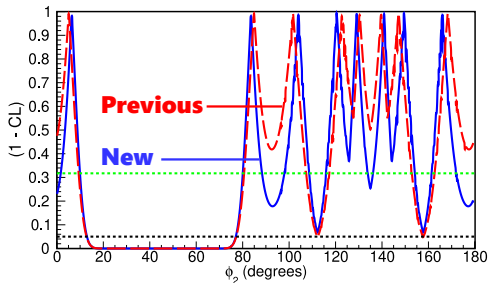
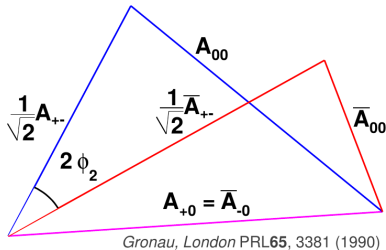
# Belle II $B^0 \rightarrow \pi^0 \pi^0$ and $\phi_2$

## Results (387M $B\bar{B}$ )

- $\mathcal{B} = (1.25 \pm 0.20 \pm 0.11) \times 10^{-6}$   
(best measurement)
- $A_{CP} = 0.03 \pm 0.30 \pm 0.04$   
(comparable with previous best)

## Isospin analysis

- Calculate new  $B^0 \rightarrow \pi^0 \pi^0$  average
- $B^0 \rightarrow \pi^+ \pi^-$  and  $B^+ \rightarrow \pi^+ \pi^0$  from PDG
- 10° reduction of 68% CL interval
- $B \rightarrow \pi\pi$  is now competitive with  $B \rightarrow \rho\rho$  in  $\phi_2$  extraction



# Belle $B^0 \rightarrow \omega\omega$

## Previous status

- One of the least measured charmless  $B \rightarrow VV$  mode
- Previous measurement:  $4.4\sigma$  signal by BaBar  
 $\mathcal{B} = (1.2 \pm 0.3^{+0.3}_{-0.2}) \times 10^{-6}$
- $f_L$  and  $A_{CP}$  were not reported
- First time to report from Belle

## Analysis

- Belle final dataset (772M  $B\bar{B}$ )
- Dominant background from continuum  $e^+e^- \rightarrow q\bar{q}$
- Background:  $\omega b_1(1235)^0 (\rightarrow \omega\pi^0)$ ,  $\omega K^{(*)0}$ ,  $\omega\eta^{(\prime)}$ ,  $\omega a_1(1260)^0$ , non-resonant components
- Belle's flavor tag algorithm for  $A_{CP}$
- 7-d fit:  $M_{bc}$ ,  $\Delta E$ , continuum suppression variable, masses and helicity angles of two  $\omega$ s  
 (non- $\omega$  background is small, no amplitude analysis is performed)

## Definitions

$B \rightarrow VV$  has three helicity amplitudes  $H_0$  (longitudinal),  $H_+$ ,  $H_-$  (transverse)  
 $f_L = |H_0|^2 / (|H_0|^2 + |H_+|^2 + |H_-|^2)$

## Polarization puzzle

$B \rightarrow VV$  is expected to be dominated by longitudinal amplitude, but it turns out  
 $f_L \sim 0.5$  for  $b \rightarrow s$  modes  
 $(f_L \sim 1$  for  $b \rightarrow u$  modes)



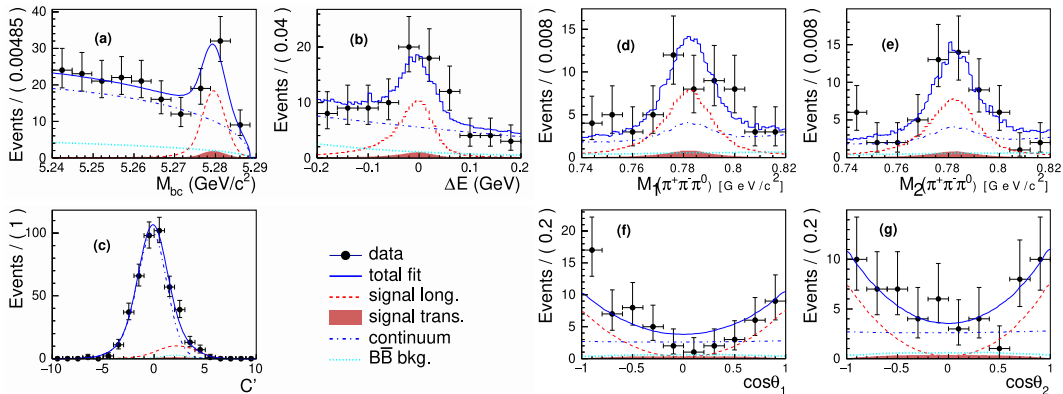
# Belle $B^0 \rightarrow \omega\omega$ results

## Results (Belle 772 M $B\bar{B}$ )

$$\mathcal{B} = (1.53 \pm 0.29 \pm 0.17) \times 10^{-6} \quad (7.9\sigma \text{ first observation})$$

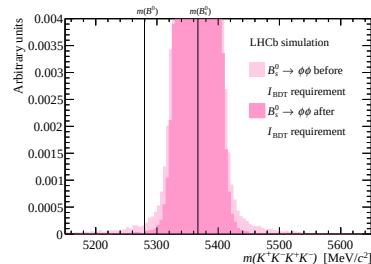
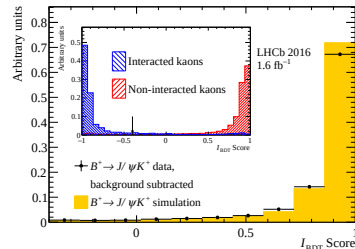
$$f_L = 0.87 \pm 0.13 \pm 0.13 \quad (\text{first measurement, consistent with } f_L \sim 1)$$

$$A_{CP} = -0.44 \pm 0.43 \pm 0.11 \quad (\text{first measurement})$$



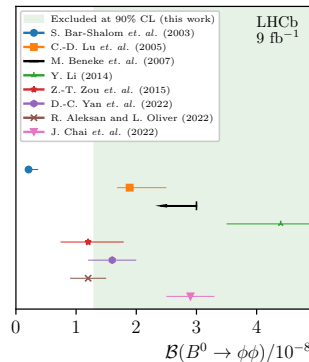
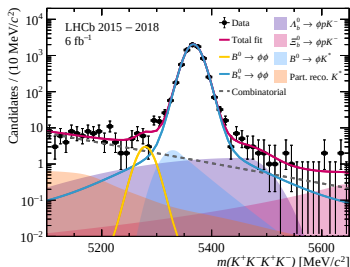
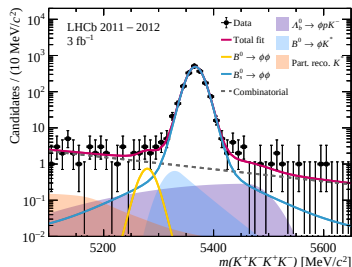
# LHCb $B^0 \rightarrow \phi\phi$

- Accessible only through annihilation / exchange amplitudes
- 4 charged-kaon final state
- Analysis strategy
  - Huge  $B_s \rightarrow \phi\phi$  control sample ( $b \rightarrow s$  penguin)  
also huge tail on top of  $B^0 \rightarrow \phi\phi$
  - **New kaon selection BDT**  $\rightarrow$  reduction of  $B_s \rightarrow \phi\phi$  tail  
(by suppressing kaons interacted inside the detector)
  - Huge random  $\phi$  from  $D_s^+ \rightarrow \phi\pi^+, \phi\mu^+\nu_\mu$  and  $\phi e^+\nu_e$ ,  
**tagged/removed by gradient-BDT** ( $\sim 50\%$  of  $D_s^+$ )
  - Still large untagged  $D_s^+ \rightarrow \phi X$ ,  $\rightarrow$  **multilayer perceptron**  
(99% background reduction while keeping 97.5% of signal)
  - Generic combinatorial background  
**reduction using a gradient-BDT (XGBOOST)**



# LHCb $B^0 \rightarrow \phi\phi$

- Yield:  $2.6 \pm 1.5$  (Run 1) +  $10.5 \pm 6.3$  (Run 2) ( $1.9 \sigma$  excess)
- Largest systematic uncertainty:  $B_s \rightarrow \phi\phi$  branching fraction (7.7%)
- Upper limit:  $\mathcal{B}(B^0 \rightarrow \phi\phi) < 1.3 \times 10^{-8}$  (90% CL) or  $< 1.4 \times 10^{-8}$  (95% CL)
- Already excluding many theory predictions, Signal would be seen with more LHCb data



# LHCb $B^+ \rightarrow \rho^0 K^{*+}$

## Large branching fraction

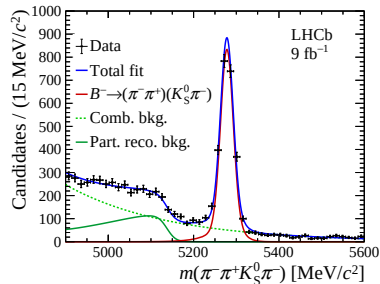
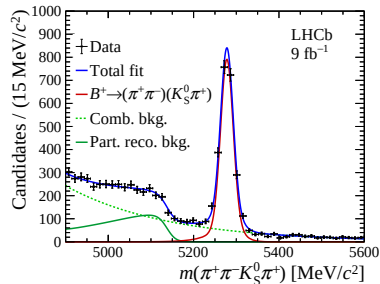
- $\mathcal{B}(B^+ \rightarrow \rho^0 K^{*+}) = (4.6 \pm 1.0 \pm 0.4) \times 10^{-6}$  by BaBar

## Huge signal of $B^+ \rightarrow (\pi^+ \pi^-)(K_S \pi^+)$

- Secondary production vertex for  $B$  meson decays
- $m_{\pi^+ \pi^-} \in [0.3, 1.1]$  GeV,  $m_{K_S \pi^+} \in [0.75, 1.2]$  GeV
- Veto  $\Lambda \rightarrow p \pi^-$  and charm meson decay combinations
- BDT to suppress combinatorials
- $2208 \pm 53(B^+) / 2333 \pm 55(B^-)$  events from Run 1+2
- 6.3% background fraction

## Multiple quasi-two-body modes

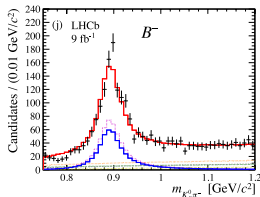
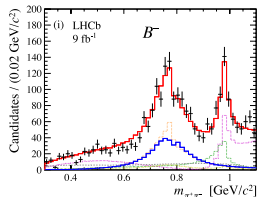
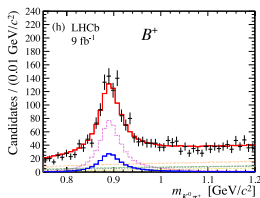
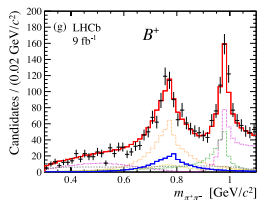
- $(\pi^+ \pi^-)$  includes  $\rho(770)$ ,  $\omega$ ,  $f_0(500)$ ,  $f_0(980)$ ,  $f_0(1370)$
- $(K_S \pi^+)$  includes  $K^*(892)$  and LASS-model scalars
- $(\pi^+ \pi^- \pi^+) K_S$  component with  $a_1(1260)$  and  $a_1(1640)$



# LHCb $B^+ \rightarrow \rho^0 K^{*+}$

## 5-d unbinned full-amplitude fit

- $m_{\pi^+\pi^-}$ ,  $m_{K_S^0\pi^\pm}$ ,  $\cos\theta_{\pi^+\pi^-}$ ,  $\cos\theta_{K_S^0\pi^\pm}$ ,  $\phi$   
to completely describe the kinematics



(k) Legend

⊕ Data

— Total fit

—  $VV = \rho(770)^0 K^{*0}(892)^+ + \omega(782) K^{*0}(892)^+ + \text{interf.}$

—  $VS = \rho(770)^0 (K_S^0 \pi^+)_S + \omega(782) (K_S^0 \pi^+)_S + \text{interf.}$

—  $SV = S_1 K^{*0}(892)^+ + S_2 K^{*0}(892)^+ + S_3 K^{*0}(892)^+ + \text{interf.}$

—  $SS = S_1 (K_S^0 \pi^+)_S + S_2 (K_S^0 \pi^+)_S + S_3 (K_S^0 \pi^+)_S + \text{interf.}$

—  $a_1 K_S^0 = a_1(1260)^+ K_S^0 + a_1(1640)^+ K_S^0 + \text{interf.}$

--- Background

where:  $S_1, S_2, S_3 = f_0(500), f_0(980), f_0(1370)$

## Results for $B^+ \rightarrow \rho^0 K^{*+}$

$$f_L = 0.720 \pm 0.028 \pm 0.009$$

(consistent with BaBar's  $0.78 \pm 0.12 \pm 0.03$ )

$$A_{CP} = 0.507 \pm 0.062 \pm 0.017 (> 9\sigma)$$

(consistent with BaBar's  $0.31 \pm 0.13 \pm 0.03$ )

(results for helicity amplitudes)

$$A_{CP}(A_0) = 0.664 \pm 0.083 \pm 0.025$$

$$A_{CP}(A_{||}) = -0.063 \pm 0.137 \pm 0.024$$

$$A_{CP}(A_{\perp}) = 0.284 \pm 0.140 \pm 0.022$$

(results for phase differences [rad])

$$\Delta_{CP}(A_0) = 0.720 \pm 0.177 \pm 0.048$$

$$\Delta_{CP}(A_{||}) = 0.477 \pm 0.187 \pm 0.040$$

$$\Delta_{CP}(A_{\perp}) = 0.412 \pm 0.180 \pm 0.048$$

**Longitudinal amplitude is the main source of CPV**

(triple product asymmetry results in backup slides)

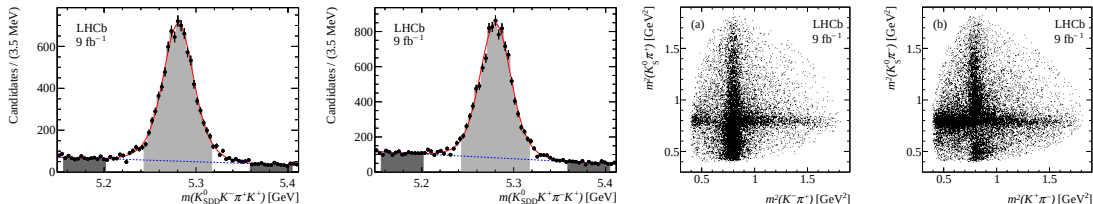
# LHCb $B^+ \rightarrow (K_S K \pi) K^+$

## Charmless B meson decays as a laboratory of light resonances ( $R^0$ )

- Many unresolved spectroscopy issues in the mass range around 1.4 GeV
- $(K_S K \pi)$  could be  $0^{-+}$  states ( $\eta$ ),  $1^{++}$  states ( $f_1$ ),  $1^{+-}$  states ( $h_1$ ), and  $2^{-+}$  states ( $\eta_2$ )
- $(K_S K \pi)$  could be  $(K_S K^+ \pi^-)$  or  $(K_S K^- \pi^+)$  — these are naively identical, but interference makes intermediate states look different
- Intermediate states of  $(K_S K \pi)$  include  $K^* \bar{K}$  (S-wave and D-wave),  $a_0 \pi$ , and phase space

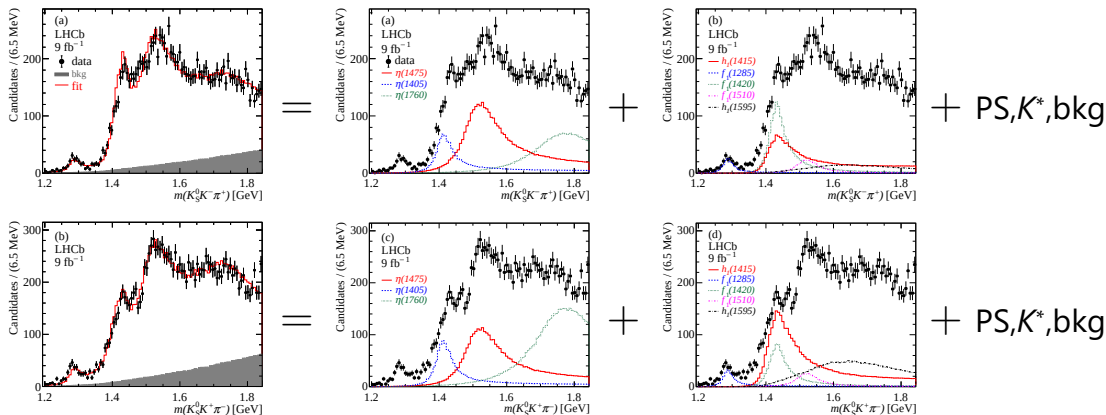
## Data sample

- Run 1+2 data, >10 k events, >80% purity for both combinations



# LHCb $B^+ \rightarrow (K_S K \pi) K^+$

- Dominated by  $0^{-+}$  ( $\eta(\dots)$ ),  $1^{++}$  ( $f_1(\dots)$ ), and  $1^{-+}$  ( $h_1(\dots)$ ) amplitudes
- Branching fractions for two-body  $B^+ \rightarrow R^0 K^+$  are reported ( $1.5 \times 10^{-6}$  to  $2.2 \times 10^{-5}$ , table in backup)
- New information on light mesons in the low-mass region



# LHCb $B^0 \rightarrow K_S p \bar{p}$ and $B_s^0 \rightarrow K_S p \bar{p}$

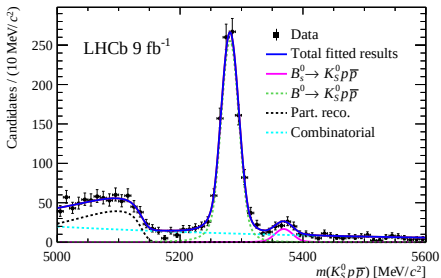
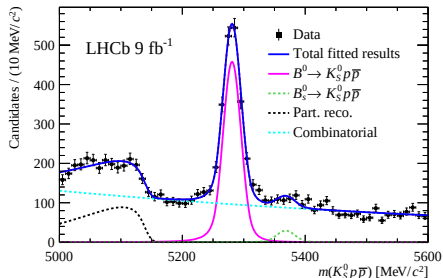
- $\Lambda_c^+ \rightarrow K^0 p, \eta_c, J/\psi, \psi(2S) \rightarrow p \bar{p}$  are vetoed
- normalization channel:  $B^0 \rightarrow K^0 \pi^+ \pi^-$
- Separate optimization for  $B^0$  and  $B_s^0$  on BDT requirement

## Results (Run 1+2 data)

- $\mathcal{B}(B^0 \rightarrow K^0 p \bar{p}) = (2.82 \pm 0.08_{\text{stat}} \pm 0.12_{\text{sys}} \pm 0.10_{\text{norm}}) \times 10^{-6}$ , much more precise than world average
- $\mathcal{B}(B_s^0 \rightarrow K^0 p \bar{p}) = (9.14 \pm 1.69_{\text{stat}} \pm 0.90_{\text{sys}} \pm 0.33_{\text{norm}} \pm 0.20_{f_s/f_d}) \times 10^{-7}$ ,  $5.6\sigma$ , **first observation**

Large enough data for time-dependent CPV study

$m_{p\bar{p}}$  information is not provided while  $B \rightarrow K^{(*)} p \bar{p}$  modes have  $p\bar{p}$  threshold enhancement

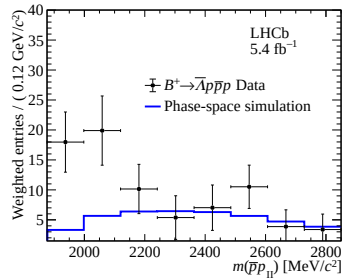
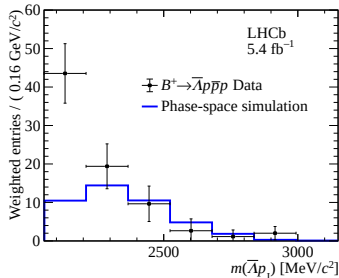
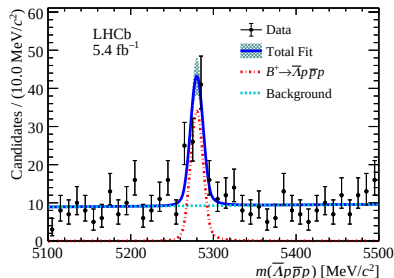




# LHCb $B^+ \rightarrow \bar{\Lambda} p \bar{p} p$

## Results (LHCb 5.4 fb<sup>-1</sup>)

- normalization channel:  $B^+ \rightarrow J/\psi(\rightarrow \Lambda p K^-) K^+$
- $\mathcal{B} = (2.08 \pm 0.34_{\text{stat}} \pm 0.12_{\text{sys}} \pm 0.26_{\text{norm}}) \times 10^{-7}$ ,  $>5\sigma$ , **first observation**
- $A_{CP} = (5.4 \pm 15.6 \pm 2.4)\%$   
(no non-zero  $A_{CP}$  yet in any baryonic charmless  $B$  decays)
- Threshold enhancements both in  $\bar{\Lambda} p$  and  $\bar{p} p$   
(like  $ppK$  and  $pp\pi$ , and in contrast to  $B^0 \rightarrow p \bar{p} p \bar{p}$  which shows no enhancement)



# Summary

**New ( $>5\sigma$ ):**  $B^0 \rightarrow \omega\omega$ ,  $A_{CP}(B^+ \rightarrow \rho^0 K^{*+})$ ,  $B \rightarrow R^0(\rightarrow K_S K\pi)K$ ,  $B_s \rightarrow K_S p\bar{p}$ ,  $B^+ \rightarrow \bar{\Lambda} p\bar{p}p$

**Update:**  $B^0 \rightarrow \pi^0\pi^0$ ,  $UL(B^0 \rightarrow \phi\phi)$ ,  $f_L(B^+ \rightarrow \rho^0 K^{*+})$ ,  $B^0 \rightarrow K_S p\bar{p}$

## Non-zero CP asymmetry is a common phenomenon in charmless B decays

- Whenever branching fraction is measured,  $A_{CP}$  can be usually measured
- CP final states are subject to time-dependent CPV measurement
- New non-zero  $A_{CP}$  is now added for  $B^+ \rightarrow \rho^+ K^{*0}$

## Spectroscopy techniques are more and more relevant in charmless B decays

- Amplitude analysis of quasi-two-body decays into multi-body final states
- $B \rightarrow VV$  modes are now reliably disentangled from  $B \rightarrow SV$ ,  $B \rightarrow SS$ , ...

## Many unknowns yet to tackle

- Polarization puzzle, threshold enhancement in baryonic decays,  $K\pi$  puzzle, ...
- Charmless decays with charged particle final states are extensively explored by LHCb
- Charmless decays with neutral particle final states are Belle II's expertise, still data-limited

**More results are expected from Belle II and LHCb,  
and charmless  $B$  decay is a very active topic!**

Thanks!

# Backup

# LHCb $B^+ \rightarrow \rho^0 K^{*+}$ triple product asymmetries

- Interference between CP-odd and -even amplitudes ( $A_{\perp} \Leftrightarrow A_0, A_{\perp} \Leftrightarrow A_{\parallel}$ )  
→ Triple product asymmetry (TPA)

## Definitions

$$\mathcal{A}_{\text{true,fake}}^{(1)} \equiv -\frac{2\sqrt{2}}{\pi} \frac{\Im(A_{\perp} A_0^* \mp \bar{A}_{\perp} \bar{A}_0^*)}{\sum_{\lambda} (|A_{\lambda}|^2 + |\bar{A}_{\lambda}|^2)}$$

$$\mathcal{A}_{\text{true,fake}}^{(2)} \equiv -\frac{2\sqrt{2}}{\pi} \frac{\Im(A_{\perp} A_{\parallel}^* \mp \bar{A}_{\perp} \bar{A}_{\parallel}^*)}{\sum_{\lambda} (|A_{\lambda}|^2 + |\bar{A}_{\lambda}|^2)}$$

- True TPA due to CP-violating weak phase  
Fake TPA due to CP-conserving strong phase

## Results

$$\mathcal{A}_{\text{true}}^{(1)} = -0.105 \pm 0.024 \pm 0.006 (\sim 4\sigma)$$

$$\mathcal{A}_{\text{fake}}^{(1)} = -0.157 \pm 0.024 \pm 0.008 (\sim 6\sigma)$$

$$\mathcal{A}_{\text{true}}^{(2)} = 0.007 \pm 0.019 \pm 0.003$$

$$\mathcal{A}_{\text{fake}}^{(2)} = 0.008 \pm 0.018 \pm 0.004$$

# LHCb $B^+ \rightarrow R^0 K^+$ branching fractions

| Contribution                                                   | $\mathcal{B}(B^+ \rightarrow R^0 K^+) \times 10^{-5}$ |
|----------------------------------------------------------------|-------------------------------------------------------|
| $B^+ \rightarrow \eta(1475)K^+ \rightarrow (K^* \bar{K})K^+$   | $1.45 \pm 0.11 \pm 0.12 \pm 0.09$                     |
| $B^+ \rightarrow \eta(1475)K^+ \rightarrow (a_0(980)\pi)K^+$   | $0.22 \pm 0.04 \pm 0.03 \pm 0.01$                     |
| $B^+ \rightarrow \eta(1475)K^+ \rightarrow (K^0 K \pi)K^+$     | $1.51 \pm 0.16 \pm 0.24 \pm 0.09$                     |
| $B^+ \rightarrow \eta(1760)K^+ \rightarrow (K^* \bar{K})K^+$   | $0.34 \pm 0.04 \pm 0.04 \pm 0.02$                     |
| $B^+ \rightarrow \eta(1760)K^+ \rightarrow (a_0(980)\pi)K^+$   | $0.26 \pm 0.04 \pm 0.02 \pm 0.02$                     |
| $B^+ \rightarrow \eta(1760)K^+ \rightarrow (K^0 K \pi)K^+$     | $2.21 \pm 0.20 \pm 0.33 \pm 0.13$                     |
| $B^+ \rightarrow \eta(1405)K^+ \rightarrow (K^* \bar{K})K^+$   | $0.38 \pm 0.05 \pm 0.09 \pm 0.02$                     |
| $B^+ \rightarrow \eta(1405)K^+ \rightarrow (K^0 K \pi)K^+$     | $0.80 \pm 0.05 \pm 0.09 \pm 0.05$                     |
| $B^+ \rightarrow f_1(1285)K^+ \rightarrow (a_0(980)\pi)K^+$    | $0.28 \pm 0.02 \pm 0.02 \pm 0.02$                     |
| $B^+ \rightarrow f_1(1420)K^+ \rightarrow (K^* \bar{K})K^+$    | $1.14 \pm 0.06 \pm 0.19 \pm 0.07$                     |
| $B^+ \rightarrow f_1(1510)K^+ \rightarrow (K^* \bar{K})K^+$    | $0.38 \pm 0.03 \pm 0.15 \pm 0.02$                     |
| $B^+ \rightarrow h_1(1415)K^+ \rightarrow (K^* \bar{K})K^+$    | $2.22 \pm 0.10 \pm 0.23 \pm 0.10$                     |
| $B^+ \rightarrow h_1(1595)K^+ \rightarrow (K^* \bar{K})K^+$    | $1.04 \pm 0.10 \pm 0.12 \pm 0.06$                     |
| $B^+ \rightarrow \eta_2(1645)K^+ \rightarrow (K^* \bar{K})K^+$ | $0.15 \pm 0.02 \pm 0.02 \pm 0.01$                     |

(inverse-variance-averages of  $B^+ \rightarrow (K^0 K^- \pi^+)K^+$  and  $B^+ \rightarrow (\bar{K}^0 K^+ \pi^-)K^+$ )