# Charming lifetime, mixing and CPV

Puzzles and Opportunities in baryons

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HIAS

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### Exclusive (semi)leptonic decays

2-, 3-point cor. : 
$$\langle 0|J_{\mu}|h_{c}\rangle$$
,  $\langle h|J_{\mu}|h_{c}\rangle$ .

Inclusive decays and mixing

4-point cor. : 
$$\langle h_c | \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0) | h_c \rangle$$
 
$$m_c \gg \Lambda_{\rm QCD} \ \ (?)$$

3-point cor. : 
$$\sum m_c^n c_n \langle h \mid \mathcal{O}_n \mid h_c \rangle$$

Exclusive nonleptonic decays

4-point cor. : 
$$\langle h_1 h_2 | \mathcal{H}_{eff} | h_c \rangle$$

# • Inclusive decays; CKM leading c o uds

$$i\frac{d}{dt}\begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix} = \left( M - \frac{i}{2} \Gamma \right) \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix}$$

• i = j: lifetimes;  $i \neq j$ : mixing.

$$\Gamma_{ij} = \operatorname{Im}\left(\frac{i}{2m_D}\int T\left\langle \mathcal{H}_{eff}(x)\mathcal{H}_{eff}(0)\right\rangle d^4x\right) = \frac{m_c^5}{m_D}\sum_{n=0}^{\infty}\frac{1}{m_c^n}\left\langle D_i | C_nO_n | D_j \right\rangle$$

**Optical theorem** 

Separating energy scales  $M_W\gg m_c\gg \Lambda_{
m QCD}$   $\mathcal{H}_{eff}$   $C_n$   $\langle O_n 
angle$ 

 $\propto (4\pi)^2 m_c^2$ 

$$C_n O_n = \left( \underbrace{\begin{array}{c} \overline{d} \\ \overline{u} \\ \overline{s} \\ \overline$$

$$\propto m_c^5$$

## • Inclusive decays; CKM leading c o uds

$$(m_b, m_c, \Lambda_{QCD}) = (4.8, 1.5, 0.3) \text{ GeV}$$

Pole mass, non-perturbative input [2502.05901]

$$\left(16\pi^{2}\left(\frac{\Lambda_{QCD}}{m_{b}}\right)^{3}, 16\pi^{2}\left(\frac{\Lambda_{QCD}}{m_{c}}\right)^{3},\right) \approx \left(\frac{160}{4000}, \frac{160}{125}\right)$$

• The dim-6 operators are of order  $\mathcal{O}(10^{-2})$  and  $\mathcal{O}(1)$  relative to the dim-3 ones.

$$C_n O_n = \left( \underbrace{\begin{array}{c} \overline{d} \\ \overline{u} \\ \overline{s} \\ \end{array} \right) + \underbrace{\begin{array}{c} \overline{d} \\ \overline{u} \\ \overline{s} \\ \end{array} \right) + \underbrace{\begin{array}{c} \overline{d} \\ \overline{d} \\ \overline{s} \\ \end{array} \right) + \underbrace{\begin{array}{c} \overline{d} \\ \overline{d} \\ \overline{s} \\ \end{array} \right) + \underbrace{\begin{array}{c} \overline{d} \\ \overline{d} \\ \overline{s} \\ \end{array} \right) + \underbrace{\begin{array}{c} \overline{d} \\ \overline{d} \\ \overline{s} \\ \end{array} \right) + \underbrace{\begin{array}{c} \overline{d} \\ \overline{d} \\ \overline{s} \\ \end{array} \right) + \underbrace{\begin{array}{c} \overline{d} \\ \overline{d} \\ \overline{s} \\ \end{array} \right) + \underbrace{\begin{array}{c} \overline{d} \\ \overline{d} \\ \overline{s} \\ \end{array} \right) + \underbrace{\begin{array}{c} \overline{d} \\ \overline{s} \\ \overline{s} \\ \end{array} 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- Inclusive decays; CKM leading c o uds
  - Dim-6 contributions of  $D^0 \& D_s^+$  are proportional to the  $m_q$  in the internal loop:

$$(\Gamma_{D^0}, \Gamma_{D_s^+}) = \frac{\left(1.71^{+0.57}_{-0.59}, 1.71^{+0.66}_{-0.72}\right)_{\text{theory}}}{\left(2.44 \pm 0.01, 1.88 \pm 0.02\right)_{\text{exp}}}$$
[2204.11935]

• Nevertheless, it causes disaster in  $D^+$ :

$$\Gamma_{D^+} = (-0.07^{+0.82}_{-0.71})_{\text{theory}}$$
  $(0.96 \pm 0.01)_{\text{exp}}$  in units of ps<sup>-1</sup>

•  $\Gamma_{D^0}$  -  $\Gamma_{D^+}$  is consistent with HQE, suggesting long-distance effects are cancelled.

[1711.02100]

$$C_n O_n = \left( \begin{array}{c|c} \overline{d} & C & S & C \\ \hline u & S & \overline{d} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & C & S & C \\ \hline \overline{u} & \overline{d} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{u} & \overline{d} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & S & C \\ \hline \overline{d} & \overline{u} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & \overline{u} & \overline{u} \\ \hline \overline{d} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & \overline{u} & \overline{u} \\ \hline \overline{d} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & \overline{u} & \overline{u} \\ \hline \overline{d} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & \overline{u} \\ \overline{d} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & \overline{u} \\ \overline{d} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & \overline{u} \\ \overline{d} & \overline{u} \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \overline{d} & \overline{u} \\ \overline{d} & \overline{u}$$

# ullet Inclusive decays; $D-ar{D}$ mixing

Difficulties also encountered in the mixing :

$$\left(\frac{\Delta m}{\Gamma}, \frac{\Delta \Gamma}{2\Gamma}\right) = \left(0.405 \pm 0.043, 0.638 \pm 0.023\right) \%_{\text{HFlAV}}$$

Theory shows  $\propto V_{us}^2 (m_s/\Lambda_{OCD})^2$ , which is round  $10^{-6}$  in HQE due to GIM!

• Long distance approach has been considered. Master equation:

$$y = \sum_{n} (-1)^{ns} \eta_{\text{CP}} \cos \delta_n \sqrt{\mathscr{B}(D^0 \to n) \mathscr{B}(D^0 \to \bar{n})}.$$

D o PP and D o PV take up half of  $\Delta \Gamma/(2\Gamma)$ . [2401.06316]

Other nonperturbative methods: lattice QCD, inverse problem, sum rules...

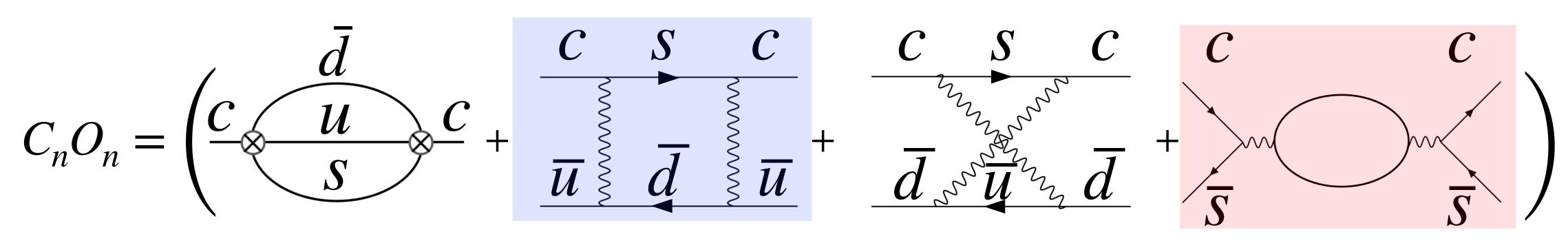
[1706.04622, 2504.16189]

[2001.04079]

[1711.02100]

## • Inclusive decays - Charmed baryons

Does the HQE work in charmed baryons?





• However, W-exchange and W-annihilation are no longer subject to helicity suppression!

#### Inclusive decays - Charmed baryons

Hai-Yang Cheng (LO + NRQM), March 19, 2018

	$\Gamma^{ m dec}$	$\Gamma^{\mathrm{ann}}$	$\Gamma^{ m int}_{-}$	$\Gamma_+^{ m int}$	$\Gamma_{ m SL}$	$\Gamma^{ m tot}$	$\tau(10^{-13}s)$	$ au_{ m expt}(10^{-13}s)$
$\overline{\Lambda_c^+}$	1.012	1.883	-0.209	0.021	0.308	3.015	2.18	$2.00\pm0.06$
$_{\scriptscriptstyle 5}\Xi_c^+$	1.012	0.115	-0.189	0.353	0.524	1.854	3.55	$4.42 \pm 0.26$
$\Xi_c^0$	1.012	2.160		0.351	0.524	4.083	1.61	$1.12^{+0.13}_{-0.10}$
$\Omega_c^0$	1.155	0.126		0.346	0.520	2.855	2.31	$0.69 \pm 0.12$

By the end of the work, I was very disappointed because […] the predicted  $\Omega_c$  lifetime […] opposite to the experiment.

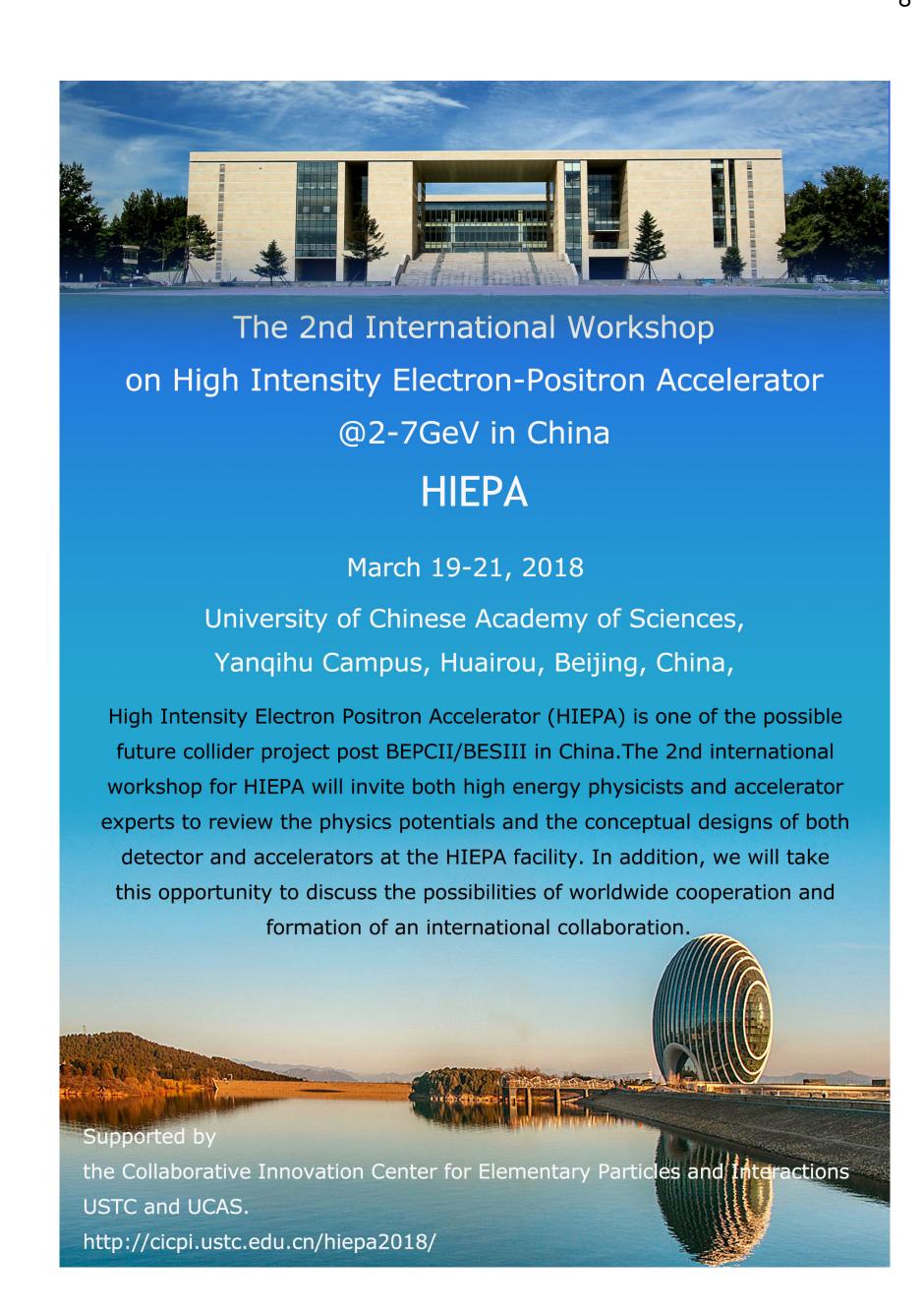
LHCb, June 8, 2018

$$\tau(\Omega_c^0) = (2.68 \pm 0.24 \pm 0.10) \times 10^{-13} s$$

Belle II, Aug 17, 2022

$$\tau(\Omega_c^0) = (2.43 \pm 0.58 \pm 0.11) \times 10^{-13} s$$

Shows predictive power of HQE in charm baryon!

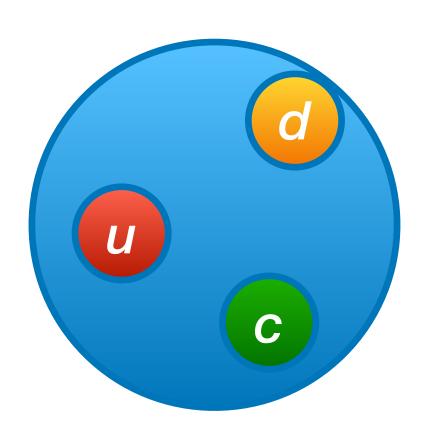


#### Inclusive decays - Charmed baryons

• From QCD and HQET sum rules  $L_{\Lambda_b}^{q_I} = -3.2 \pm 1.6 \& -2.38 \pm 0.11 \pm 0.34 \pm 0.22$  [9604425] [PLB 387, 371(1996)]

• The HQE expectation  $\langle O \rangle_c \sim \langle O \rangle_b$  works poorly in NRQM but well in BM.

Model	$(\mathcal{B}_Q,q)$	$(\Lambda_b,q_I)$	$(\Xi_b, q_I)$	$(\Xi_b, s)$	$(\Omega_b,s)$	$(\Lambda_c,q_I)$	$(\Xi_c,q_I)$	$(\Xi_c,s)$	$(\Omega_c,s)$
$\mathrm{BM}^{\;a}$	$L^q_{\mathcal{B}_Q}$	-5.44	-5.15	-5.88	-34.12	-4.83	-4.87	-5.34	-31.63
	$S^q_{\mathcal{B}_Q}$	2.44	2.32	2.74	-5.41	1.96	1.98	2.32	-4.65
	$P^q_{\mathcal{B}_Q}$	-0.27	-0.25	-0.20	-0.62	-0.44	-0.44	-0.34	-1.12
	$L^q_{\mathcal{B}_Q}$	-13(5)	-14(5)	-18(6) -	-126(60)	-5.1(15)	-5.4(16)	-7.4(22)	-46(14)
NRQM	$S^q_{\mathcal{B}_Q}$	7(2)	7(2)	9(3)	-21(10)	2.5(8)	2.7(8)	3.7(11)	-7.7(23)
	$P^q_{\mathcal{B}_Q}$	0	0	0	0	0	0	0	0



Bag is localized and cannot be 3-momentum eigenstate.
Underestimate a factor of 2.

[2305.00665]

#### Inclusive decays - Charmed baryons

Hai-Yang Cheng, Chia-Wei Liu (NLO + HBM), May 1, 2023

• The prediction of  $\Lambda_c^+ \to Xe^+$  is consistent with the data of  $(4.06 \pm 0.13)\,\%$  . BEST

•  $\mathscr{B}(\Xi_c^0 \to Xe^+)$  is consistent with the lattice result of  $\mathscr{B}(\Xi_c^0 \to \Xi^- e^+ \nu_e) \approx (3.58 \pm 0.12)\,\%$  together with the lowest bound-state saturation.

• For  $\Lambda_c^+, \Xi_c$  the HQE of  $\Gamma_3 > \Gamma_6 > \Gamma_7$  holds but not for  $\Omega_c$ .

• We are working on dim-7 NLO.

$\mathbf{B}_c$		$\Gamma_3^{ m SL}$	$\Gamma_6^{ m SL}$	$\Gamma_7^{ m SL}$	$\mathcal{B}_e^{\mathrm{SL}}(\%)$
$\Lambda_c^+$	LO	$0.40(13)_m$	0.01	0	$8.25(78)_m(44)_{\mu}(37)_4(37)_s$
	NLO	$0.35(11)_m$	0.01	-	$4.57(42)_m(24)_\mu(21)_4(13)_s$
$\Xi_c^0$	LO	$0.40(14)_m$	0.36	-0.15	$8.99(58)_m(29)_{\mu}(25)_4(43)_s$
	NLO	$0.35(12)_m$	0.18	-	$4.40(45)_m(22)_{\mu}(19)_4(30)_s$
$\Xi_c^+$	LO	$0.40(14)_m$	0.35	-0.15	$18.59(26)_m(22)_{\mu}(19)_4(39)_s$
	NLO	$0.35(12)_m$	0.18	-	$8.57(20)_m(5)_{\mu}(5)_4(44)_s$
$\Omega_c^0$	LO	$0.42(14)_m$	1.22	-0.83	$13.51(42)_m(10)_{\mu}(8)_4(23)_s$
		$0.37(12)_m$			$1.88(1.33)_m(47)_{\mu}(40)_4(85)_s$

## • Inclusive decays - Charmed baryons

	Hai-Yang Cheng,	Chia-Wei Liu	James Gratrex, Blaženka Melić, Ivan Nišandžić					
	HBI	<b>1</b> [2305.00665]	NR	QM [2204.11935]	Experiment			
	$\mathcal{BF}_e^{\mathrm{SL}}(\%)$	au	$\mathcal{BF}_e^{\mathrm{SL}}(\%)$	au	$\mathcal{BF}_e^{\mathrm{SL}}(\%)$	au		
$\Lambda_c^+$	4.57(54)	1.92(37)	$3.80^{+0.58}_{-0.57}$	$3.04^{+1.06}_{-0.80}$	$3.95 \pm 0.35$	2.029(11)		
$\Xi_c^0$	4.40(61)	1.66(32)	$4.31^{+0.87}_{-0.84}$	$2.31^{+0.84}_{-0.59}$	_	1.505(19)		
$\Xi_c^+$	8.57(49)	3.27(76)	$12.74^{+2.54}_{-2.45}$	$4.25_{-1.00}^{+1.22}$	_	4.53(5)		
$\Omega_c^0$	1.88(1.69)	2.30(58)	$7.59^{+2.49}_{-2.24}$	$2.59^{+1.03}_{-0.70}$	_	2.73(12)		

• Mostly consistent within uncertainties except for  $\mathscr{B}^{\mathrm{SL}}(\Omega_c)$  due to dim-7 operators.

First principle / reliable

Data driven / fruitful



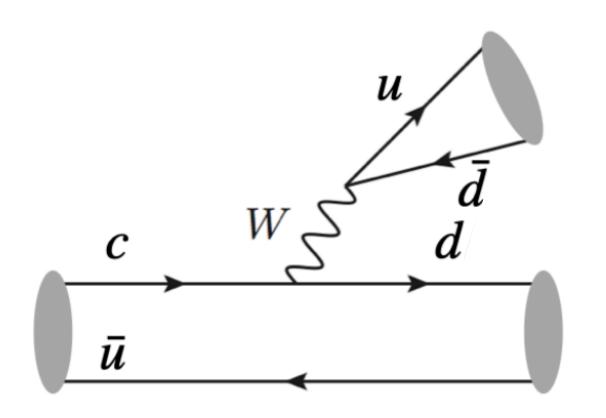
4-point cor. :  $\langle h_1 h_2 | \mathcal{H}_{eff} | h_c \rangle$ 

Lessons from lifetime and mixing:

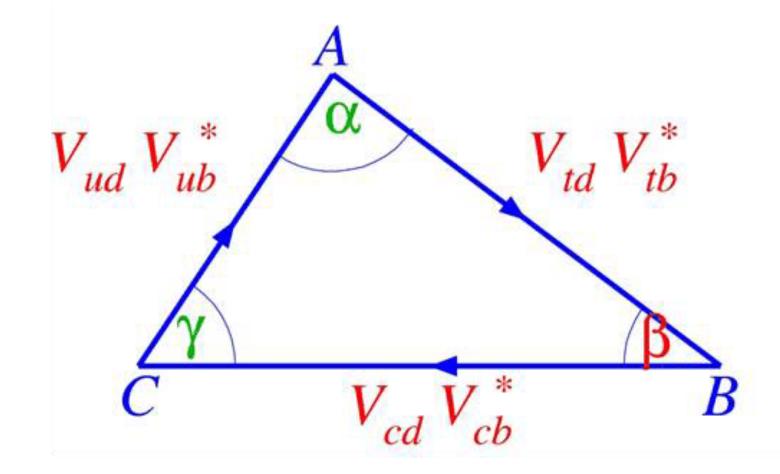
- 1.  $m_c$  converges slowly.
- 2. LD is important.
- 3. GIM cancellation does not exactly hold.

$$a_{CP} \approx 1.5 \times 10^{-3} \times \text{Im}(P/T)$$

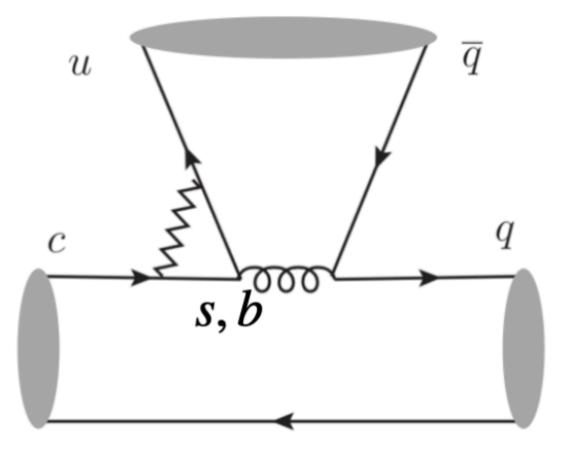
$$A(D^0 \to \pi^+ \pi^-) = V_{cd}^* V_{ud} T + V_{cb}^* V_{ub} P$$



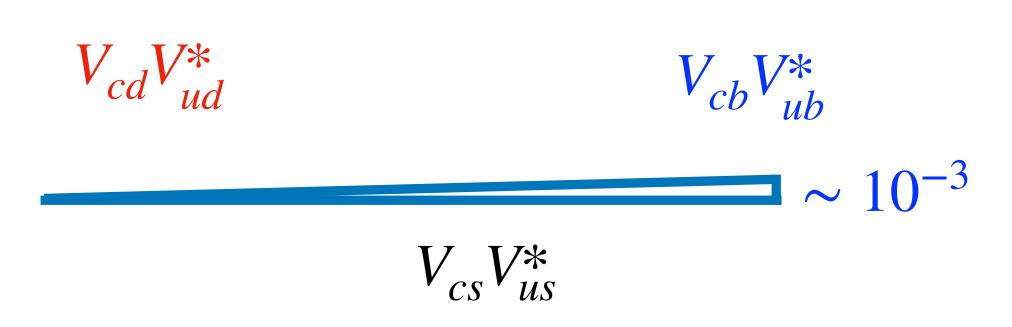
(T) Solvable from  $\mathscr{B}$ .



CKM triangle for  $b \rightarrow d$ 



(P) Indeterminate from  $\mathscr{B}$ .



CKM triangle for  $c \rightarrow u$ 

# • Charming physics - CP violation $a_{CP} \approx 1.5 \times 10^{-3} \times \text{Im}(P/T)$

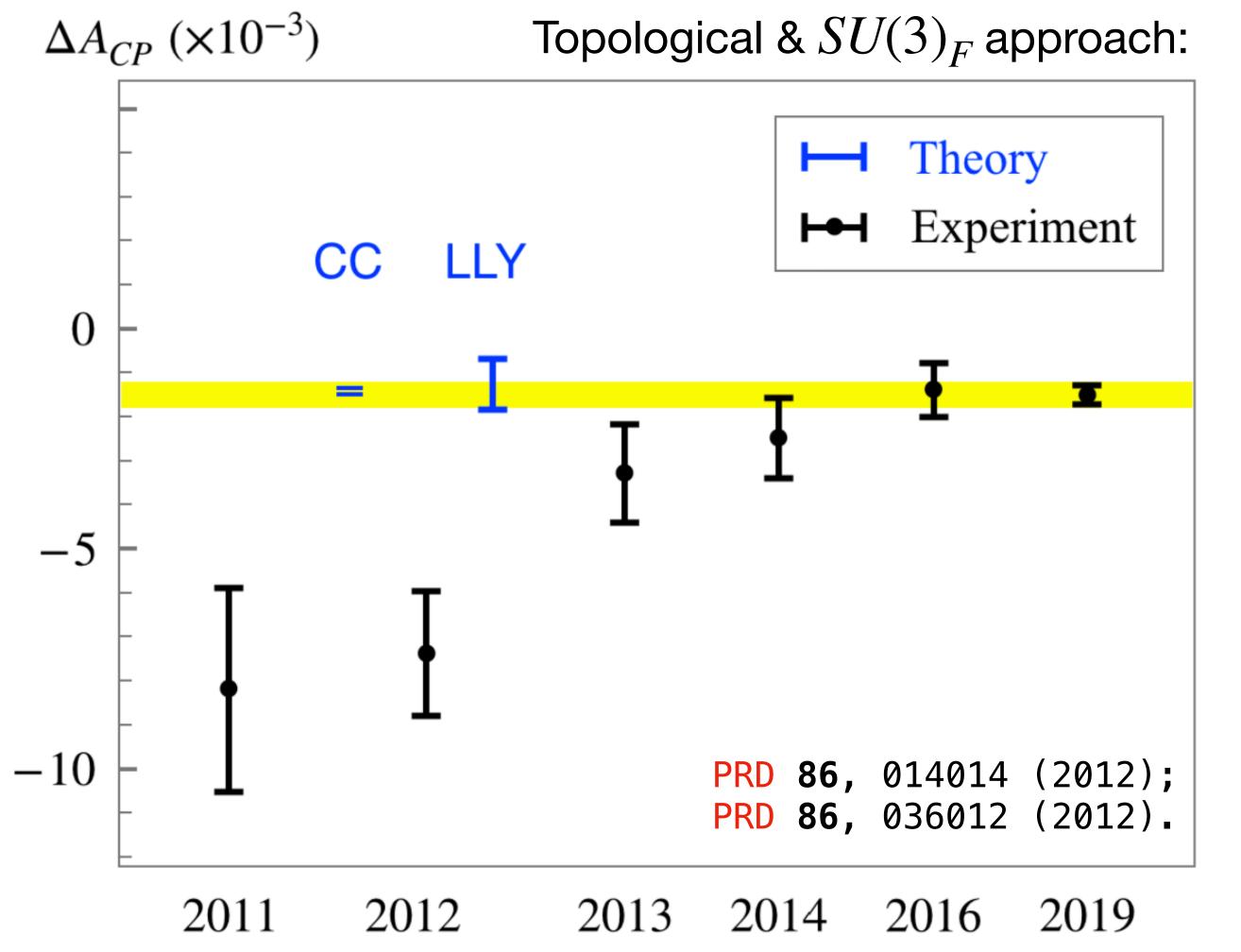
$$a_{CP} \approx 1.5 \times 10^{-3} \times \text{Im}(P/T)$$

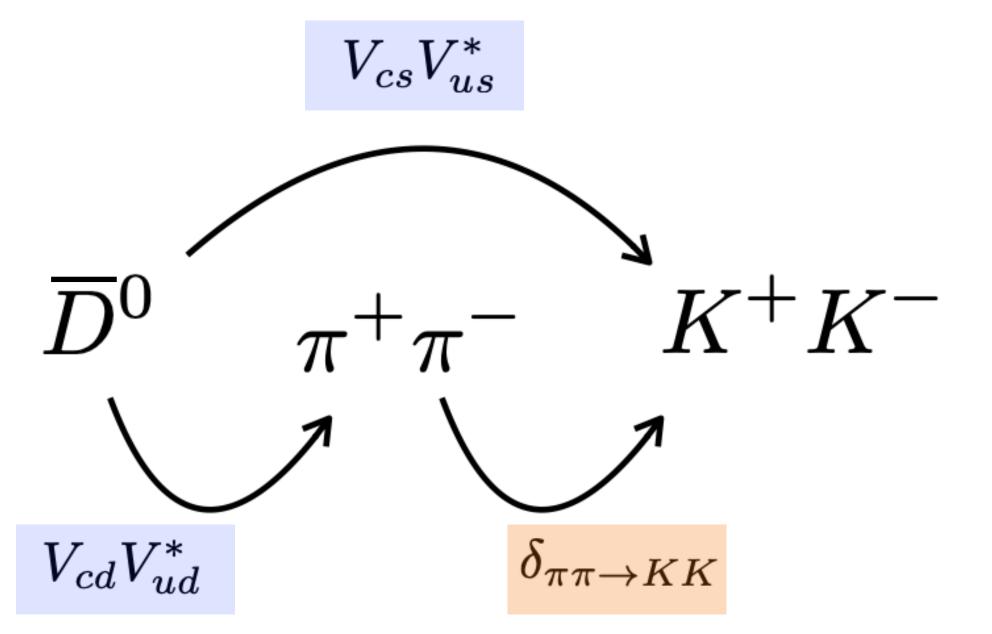
$$a_{CP}(D^0 \to K^+K^-) - a_{CP}(D^0 \to \pi^+\pi^-) = (-1.54 \pm 0.29) \times 10^{-3}$$



PRL **122**, 211803 (2019);

 $|P/T| \approx 1$ , an order larger than naive expectation!





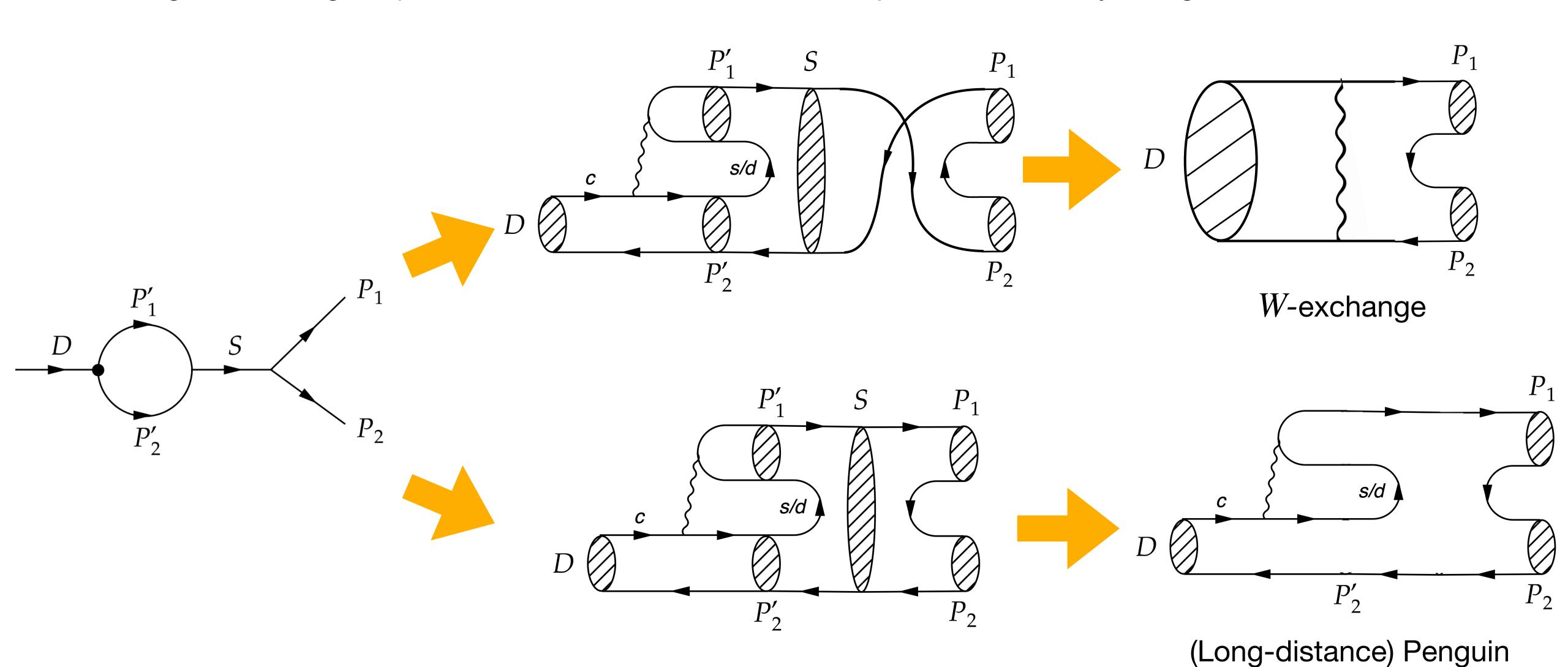
$$a_{CP} \propto \sin \delta_{\text{weak}} \sin \delta_{\text{strong}}$$

Two necessary and sufficient conditions for CPV:

**CKM** phases and strong phases.

PRL 131, 051802 (2023).

Cheng and Chiang conjectured P=E in 2012, which was proved in 2021 by Wang.



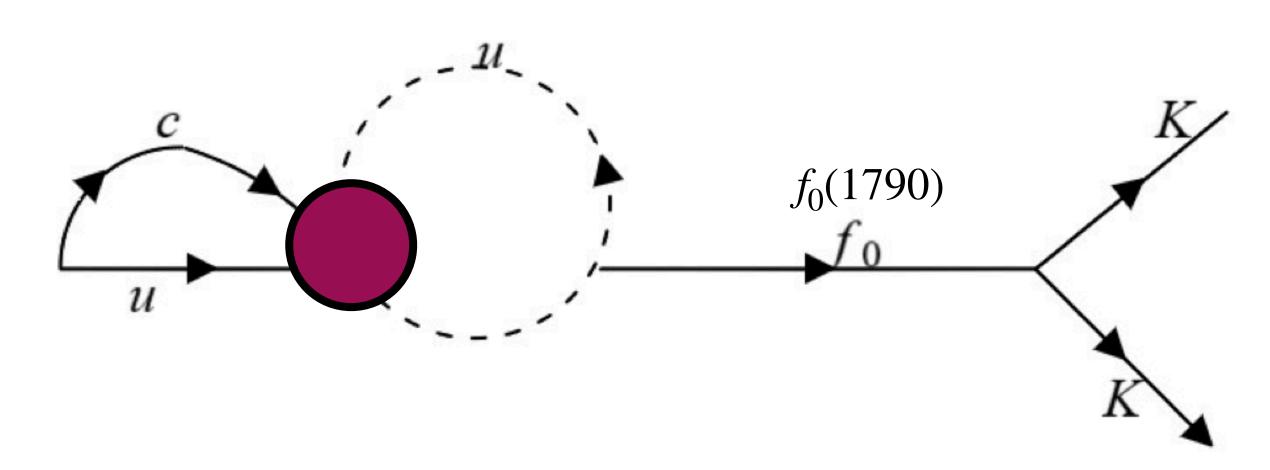
PRD 86, 014014 (2012), 2111.11201, 2505.07150.

Reasons to go beyond charmed mesons:

$$a_{CP}^{KK} = (7.7 \pm 5.7) \times 10^{-4}, \quad a_{CP}^{\pi\pi} = (23.2 \pm 6.1) \times 10^{-4}$$

PRL **131**, 091802 (2023)

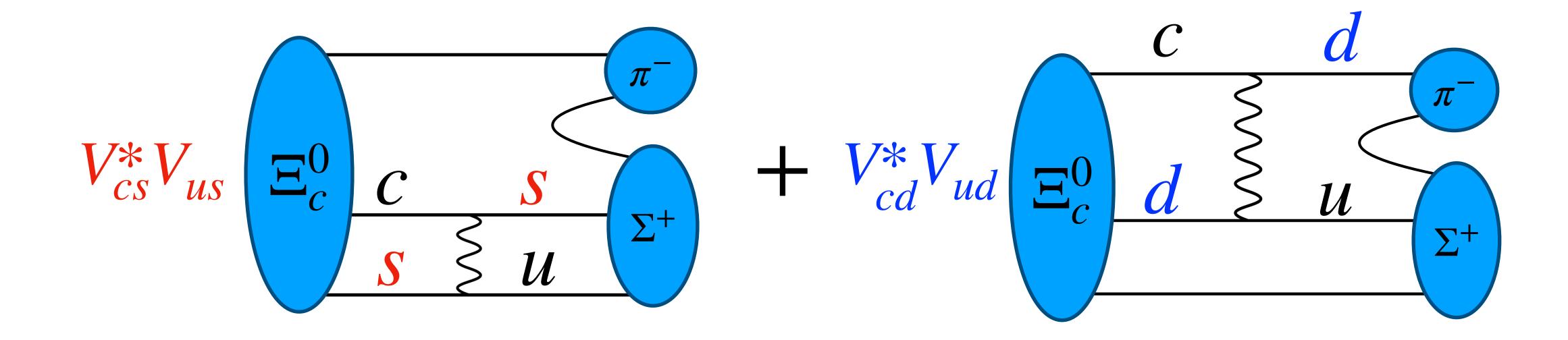




PRD 81, 074021 (2010), PLB 825, 136855 (2022).

- 1. Relative sign of  $a_{CP}^{KK}$  and  $a_{CP}^{\pi\pi}$  contradicts to the theoretical expectations.
- 2.  $f_0$  might be a glueball which mainly decays to kaons. Leading order amplitude  $\propto m_s$ .
- 3. Its mass is too close to D meson, enhancing SU(3) breaking effects from mass splitting.

Reasons to go beyond charmed mesons:

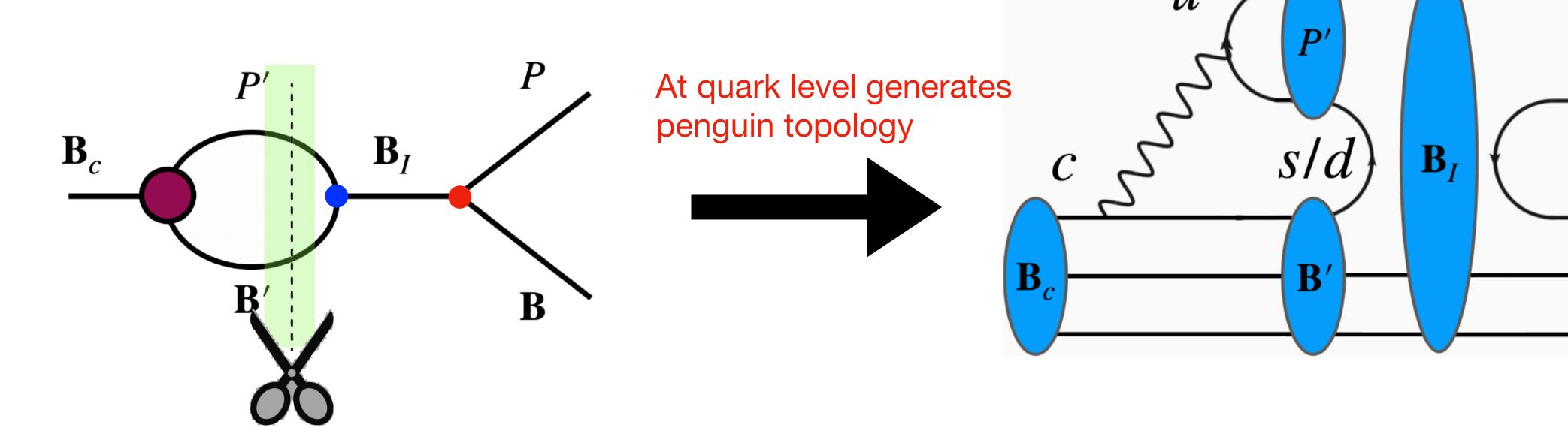


- 4. Quark structure provides CKM phase at tree level.
- 5. Unlike  $D^0 \to h^+h^-$ , CP-even **phase shifts** in baryon decays can be directly measured. Very important inputs and driven force in the study of charmed baryons!



X. G. He and **C. W. Liu** [Sci.Bull. **70** (2025) 2598]:

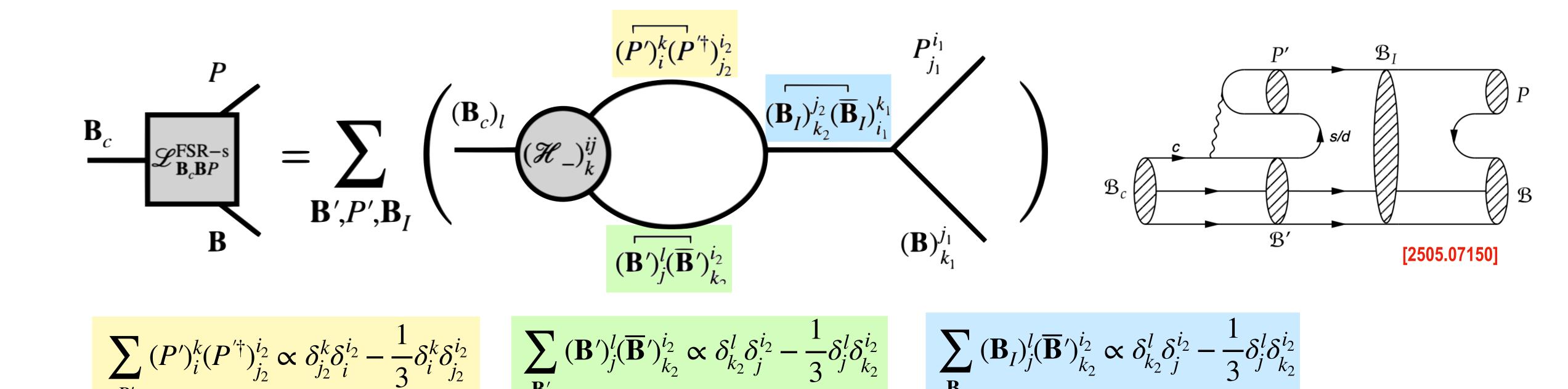
FIG from PRD **100**, 093002 (2019). CKM subleading  $\propto V_{ub}V_{cb}^* \sim 10^{-4}$ CKM leading  $\propto V_{us}V_{cs}^* \sim 10^{-1}$ Solved by experimental data. Unsolved!  $\mathbf{B}_c$  $\mathbf{B}_{I}$  $\mathbf{B}_c$  $\mathbf{B}'$ 



Generate necessary strong phase!

$$\langle \mathcal{L}_{\mathbf{B}_{c}\mathbf{B}P}^{\mathrm{FSR-s}} \rangle = \sum_{\mathbf{B}_{l},\mathbf{B}',P'} \overline{u}_{\mathbf{B}} \left( \int \frac{d^{4}q}{(2\pi)^{4}} \frac{g_{\mathbf{B}_{l}}p_{P}}{g_{\mathbf{B}_{c}}p_{P}} \frac{p_{\mathbf{B}_{c}}^{\mu}\gamma_{\mu} + m_{I}}{p_{\mathbf{B}_{c}}^{2} - m_{I}^{2}} \frac{q^{\mu}\gamma_{\mu} + m_{\mathbf{B}'}}{q^{2} - m_{\mathbf{B}'}^{2}} \frac{1}{(q - p_{\mathbf{B}_{c}})^{2} - m_{P'}^{2}} F_{\mathbf{B}_{c}\mathbf{B}'P'}^{\mathrm{Tree}} \right) u_{\mathbf{B}_{c}}$$

 $F_{\mathbf{B},\mathbf{B}'P'}^{\mathrm{Tree}}$  and  $g_{\mathbf{B}_{I}\mathbf{B}'P'}$  depend on  $q^2$  otherwise a cut-off has to be introduced.



#### **Approximations:**

- 1.  $\mathbf{B}_I \in \text{lowest-lying baryons of both parities.}$
- 2. The rescattering is closed, i.e.  $\mathbf{B}'P'$  belong to the same  $SU(3)_F$  group of  $\mathbf{B}P$ .

Amplitudes:  $\frac{\lambda_s - \lambda_d}{2} \tilde{f}^{b,c,d,e} + \lambda_b \tilde{f}^{b,c,d}_3$ 

CKM leading → rescattering parameters → CKM suppressed

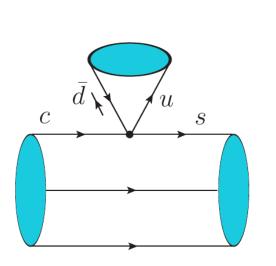
PRD 100, 093002 (2019)

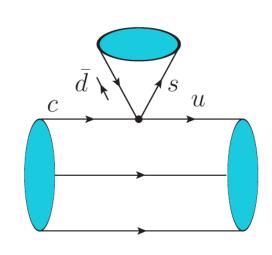
$$(\tilde{f}^b, \tilde{f}^c, \tilde{f}^d, \tilde{f}^e) \longrightarrow (\tilde{F}_V^+, \tilde{F}_V^-, \tilde{S}^-, \tilde{T}^-) \longrightarrow (\tilde{f}_3^b, \tilde{f}_3^c, \tilde{f}_3^d)$$

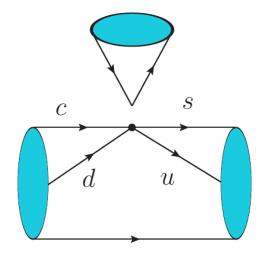
$$\tilde{f}^b = \tilde{F}_V^- - (r_- + 4)\tilde{S}^- + \sum_{\lambda = \pm} (2r_\lambda^2 - r_\lambda)\tilde{T}_\lambda^-,$$

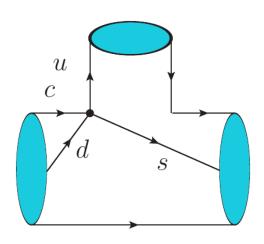
$$\tilde{f}^c = -r_-(r_- + 4)\tilde{S}^- + \sum_{\lambda = \pm} (r_\lambda^2 - 2r_\lambda + 3)\tilde{T}_\lambda^-,$$

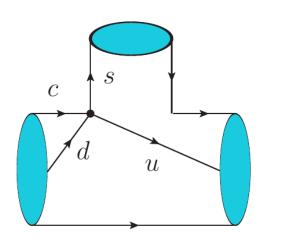
$$|\tilde{f}^d| = |\tilde{F}_V^-| + \sum_{\lambda} (2r_{\lambda}^2 - 2r_{\lambda} - 4)\tilde{T}_{\lambda}^-, \quad |\tilde{f}^e| = \tilde{F}_V^+$$

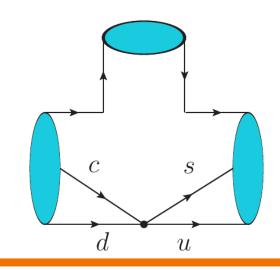


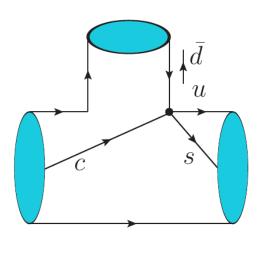








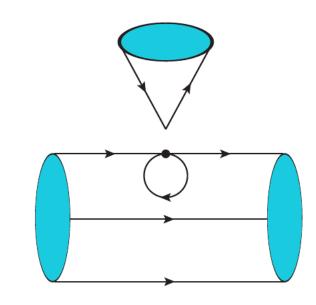


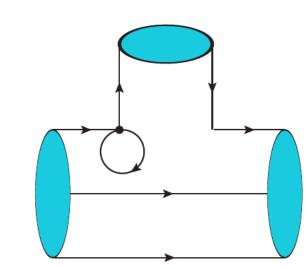


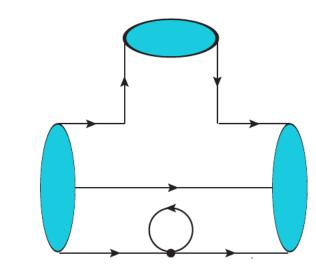
$$|\tilde{f}_{\mathbf{3}}^{b}| = (1 - \frac{7r_{-}}{2})\tilde{S}^{-} + \sum_{\lambda = \pm} (r_{\lambda}^{2} - 5r_{\lambda}/2 + 1)\tilde{T}_{\lambda}^{-},$$

$$\frac{\tilde{f}_{3}^{c}}{6} = \frac{(r_{-}+1)(7r_{-}-2)}{6}\tilde{S}^{-} - \sum_{\lambda=\pm} \frac{r_{\lambda}^{2}+11r_{\lambda}+1}{6}\tilde{T}_{\lambda}^{-},$$

$$|\tilde{f}_3^d| = \frac{2r_- - 7r_-^2}{2}\tilde{S}^- + \sum_{n=1}^{\infty} \frac{(r_{\lambda} + 1)^2}{2}\tilde{T}_{\lambda}^- - \frac{\tilde{F}_V^+ + 2\tilde{F}_V^-}{4}.$$







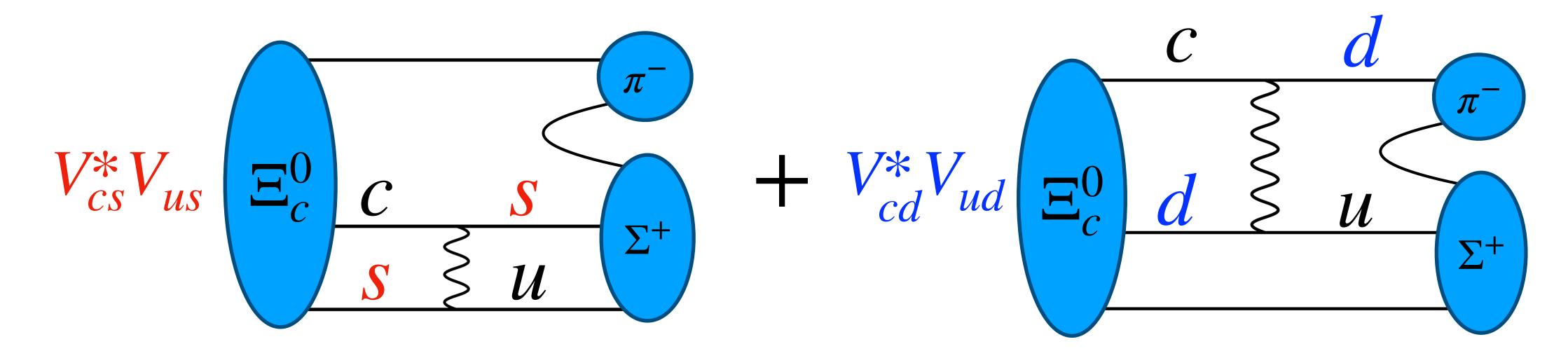
Much more complicated than D mesons!

•  $A_{CP}$  in the same size with the ones in D meson!

$$A_{CP} \left( \Xi_c^0 \to \Sigma^+ \pi^- \right) = (0.71 \pm 0.16) \times 10^{-3}$$
  
 $A_{CP} \left( \Xi_c^0 \to pK^- \right) = (-0.73 \pm 0.19) \times 10^{-3}$ 

In the U-spin limit, we have that

$$A_{CP}\left(\Xi_{c}^{0}\to\Sigma^{+}\pi^{-}\right)=-A_{CP}\left(\Xi_{c}^{0}\to pK^{-}
ight).$$
 EPJC 79, 429 (2019)



Two topological diagrams are in the same size, leads to  $A_{CP} \sim \left| 2 {\rm Im} ( V_{cs}^* V_{us} / V_{cd}^* V_{ud} ) \right| \sim 10^{-3}$  .

# $\bullet$ Final state rescattering Branching fractions and CP asymmetries in units of $10^{-3}$

Channels	$\mathcal{B}$	$A_{CP}$	$A^{lpha}_{CP}$	Channels	$\mathcal{B}$	$A_{CP}$	$A_{CP}^{lpha}$
$\Lambda_c^+  o p \pi^0$	0.18(2)	-0.01(7) $0.01(15)(45)$	-0.15(13) 0.55(20)(61)	$\Xi_c^0 \to \Sigma^+ \pi^-$	0.26(2)	$0 \\ 0.71(15)(6)$	0 $-1.83(10)(15)$
$\Lambda_c^+ \to n \pi^+$	0.68(6)	0.0(1) $-0.02(7)(28)$	0.03(2) $0.30(13)(41)$	$\Xi_c^0 \to \Sigma^0 \pi^0$	0.34(3)	-0.02(4) $0.44(24)(17)$	0.01(1) $-0.43(31)(16)$
$\Lambda_c^+ \to \Lambda K^+$	0.62(3)	0.00(2) $-0.15(13)(9)$	0.03(2) $0.50(9)(21)$	$\Xi_c^0 \to \Sigma^- \pi^+$	1.76(5)	0.01(1) $0.12(6)(2)$	-0.01(1) $-0.22(5)(21)$
$\Xi_c^+ \to \Sigma^+ \pi^0$	2.69(14)	-0.02(6) $0.05(7)(8)$	0.07(4) $-0.23(3)(15)$	$\Xi_c^0  o \Xi^0 K_{S/L}$	0.38(1)	$0 \\ 0.18(3)(5)$	0 -0.38(2)(11)
$\Xi_c^+  o \Sigma^0 \pi^+$	3.14(10)	0.00(1) $0.05(8)(7)$	-0.02(1) $-0.24(6)(13)$	$\Xi_c^0 \to \Xi^- K^+$	1.26(4)	0.00(1) $-0.12(5)(2)$	0.01(1) $0.21(4)(2)$
$\Xi_c^+ \to \Xi^0 K^+$	1.30(10)	0.00(0) $0.01(6)(17)$	-0.02(1) -0.23(9)(52)	$\Xi_c^0 \to pK^-$	0.31(2)	0 -0.73(18)(6)	0 $1.74(11)(14)$
$\Xi_c^+ \to \Lambda \pi^+$	0.18(3)	-0.01(2) $-0.31(21)(13)$	0.0(0) $0.96(25)(44)$	$\Xi_c^0  o n K_{S/L}$	0.86(3)	0 -0.14(3)(4)	$0 \\ 0.27(2)(7)$
$\Xi_c^+ \to pK_s$	1.55(7)	0 -0.13(3)(4)	$0 \\ 0.22(3)(7)$	$\Xi_c^0 \to \Lambda \pi^0$	0.06(2)	0.02(3) $-0.12(18)(10)$	0.0(1) $0.69(8)(43)$

# Charming puzzles and opportunities await!

## Inclusive decays:

Exp:  $\Omega_c^0 \to X\ell^+\nu_\ell$ .

Theory: LD physics and NLO of dim-7.







# Exclusive decays:

Exp: CPV and more data.

Theory: LD physics and CPV.

