Time-dependent results from ATLAS & CMS

Xin Chen， Tsinghua University
On behalf of ATLAS & CMS Collaborations

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High precision B_d^0 lifetime

[Eur. Phys. J. C 85 (2025) 736]

- Precise measurements on B-lifetimes and their ratios test our understanding of weak interactions
- In Heavy Quark Expansion (HQE) theory the decay width of B_q can be expressed as $\Gamma = \Gamma_3 + \delta\Gamma$, where Γ_3 is for free b-quark decay, and $\delta\Gamma$ contains non-perturbative contributions suppressed with at least two powers of $1/m_b$
- Theory prediction: $\Gamma_d = 0.63^{+0.11}_{-0.07}$ ps^{-1}, $\frac{\Gamma_d}{\Gamma_s} = 1.003 \pm 0.006$. Improvement in lifetimes measurements will constrain many BSM models
- The effective lifetime B^0 can be expressed as

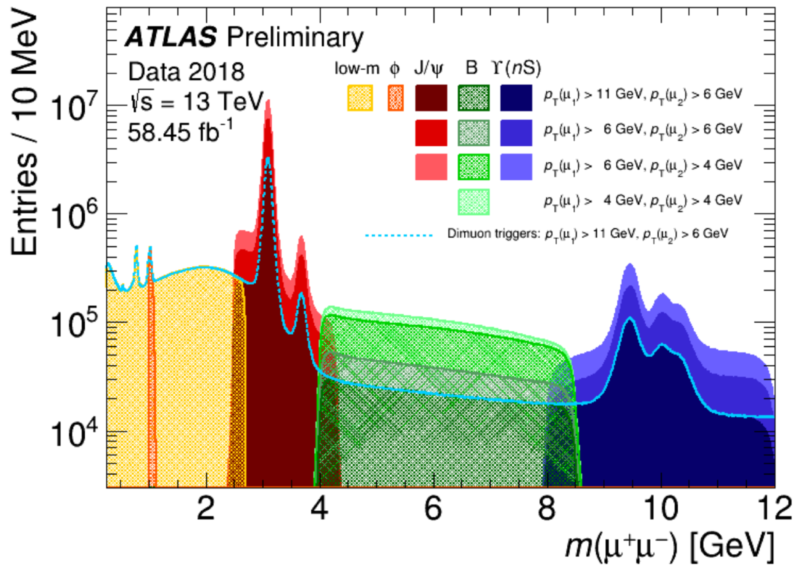
$$\tau_{B^0} = \frac{1}{\Gamma_d} \frac{1}{1-y^2} \left(\frac{1+2Ay+y^2}{1+Ay} \right)$$

where $\Gamma_d = \frac{\Gamma_L+\Gamma_H}{2}$, $y = \frac{\Gamma_L-\Gamma_H}{2\Gamma_d}$, and $A = \frac{R_H^f - R_L^f}{R_H^f + R_L^f}$, with R_H^f and R_L^f defined via the summed decay rate of the members of the $B_d^0 - \bar{B}_d^0$ system

$$\langle \Gamma(B^0(t)) \rangle = \Gamma(B^0(t)) + \Gamma(\bar{B}^0(t)) = R_H^f e^{-\Gamma_H t} + R_L^f e^{-\Gamma_L t}$$

High precision B_d^0 lifetime

- Use di-muon triggers with varying p_T thresholds of 4, 6 and 11 GeV. Low thresholds were activated in the end of fills



- $B^0 \to J/\psi\, K^{*0}$ selection:

  – At least one $J/\psi \to \mu^+\mu^-$ candidate with $\chi^2/N < 10$
  – For the two tracks from $K^{*0} \to K^+\pi^-$, both $K\pi$ and πK mass hypotheses are considered
  – B^0 vertex $\chi^2/N < 3$. If more than one B^0 candidates in an event (10% of the events), the one with least χ^2/N is chosen

- Primary vertex (PV) is recalculated after removing any tracks used in B^0. The PV candidate with the smallest 3D impact parameter, a_0 (minimum distance between PV and the line extrapolated from the B^0 vertex in B^0 momentum direction), is used.

- The proper decay time t is calculated as $t = \frac{L_{xy} m_B}{p_T}$, where p_T and m_B are the transverse momentum and PDG mass of B^0, and the transverse decay length L_{xy} is the distance in the transverse plane from PV to the B^0 decay vertex, projected onto p_T

High precision B_d^0 lifetime

- 2D unbinned maximum-likelihood fit applied simultaneously to mass and proper decay time:

$$\ln L = \sum_{i=1}^{N} w(t_i) \ln[f_{\rm sig}\mathcal{M}_{\rm sig}(m_i)\mathcal{T}_{\rm sig}(t_i, \sigma_{t_i}, p_{\mathrm{T}_i}) + (1 - f_{\rm sig})\mathcal{M}_{\rm bkg}(m_i)\mathcal{T}_{\rm bkg}(t_i, \sigma_{t_i}, p_{\mathrm{T}_i})],$$

where $\mathcal{M}$ and $\mathcal{T}$ are PDFs of mass and time, respectively, f_{sig} is the fraction of signal events

$\mathcal{M}_{\rm sig}(m_i)$ uses a Johnson S_U-distribution

$$\mathcal{M}_{\rm sig}(m_i) = \frac{\delta}{\lambda\sqrt{2\pi}\sqrt{1+\left(\frac{m_i-\mu}{\lambda}\right)^2}} \exp\left[-\frac{1}{2}\left(\gamma + \delta \sinh^{-1}\left(\frac{m_i-\mu}{\lambda}\right)\right)^2\right]$$

Background has two components: prompt and combinatorial (linear and sigmoid functions)

$$\mathcal{M}_{\rm bkg}(m_i) = f_{\rm poly}(1 + p_0 \cdot m_i) + (1 - f_{\rm poly})\left(1 - \frac{s(m_i - m_0)}{\sqrt{1 + (s(m_i - m_0))^2}}\right)$$

Signal time PDF: exponential convolved by R

$$P_{\rm sig}(t_i|\sigma_{t_i}, p_{\mathrm{T}_i}) = E(t', \tau_{B^0}) \otimes R(t' - t_i, \sigma_{t_i})$$

Background time PDF: prompt and combinatorial

$$P_{\rm bkg}(t_i|\sigma_{t_i}, p_{\mathrm{T}_i}) = \left(f_{\rm prompt} \cdot \delta_{\rm Dirac}(t') + (1 - f_{\rm prompt})\sum_{k=1}^{3} b_k \prod_{l=1}^{k-1}(1 - b_l)E(t', \tau_{{\rm bkg}_k})\right) \otimes R(t' - t_i, \sigma_{t_i})$$

Resolution R: sum of three Gaussians with $S^{(k)}$ as the free scaling parameters of fit

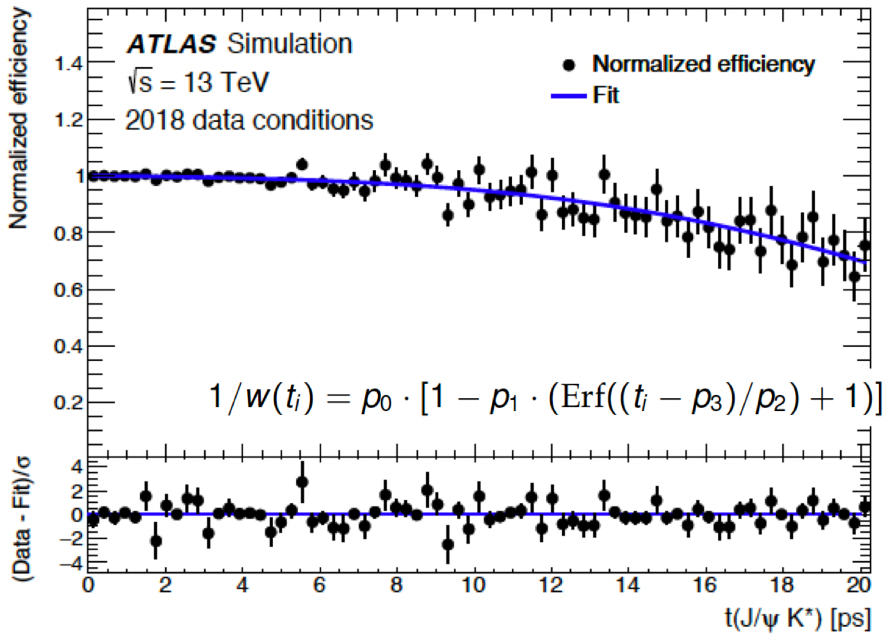
$$R(t' - t_i, \sigma_{t_i}) = \sum_{k=1}^{3} f_{\rm res}^{(k)} \frac{1}{\sqrt{2\pi}\, S^{(k)}\, \sigma_{t_i}} \exp\left(\frac{-(t' - t_i)^2}{2(S^{(k)}\, \sigma_{t_i})^2}\right)$$

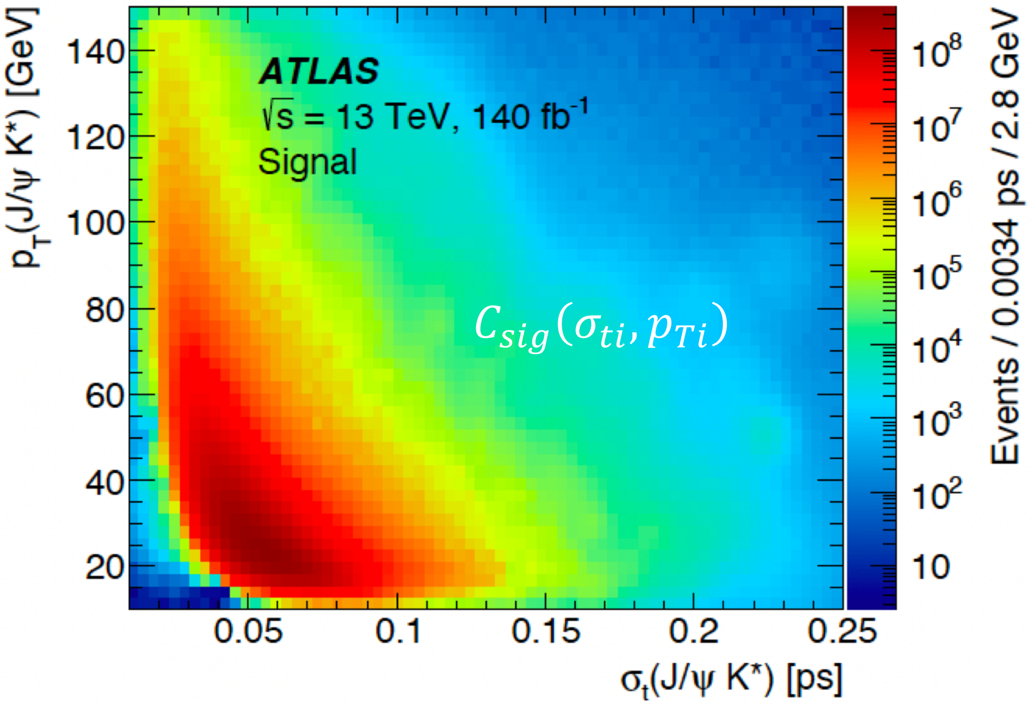
High precision B_d^0 lifetime

- Trigger, offline reconstruction and selections will bias the proper decay time distribution. For example, all four tracks in $B^0 \to \mu\mu KK$ have impact parameter cut $|d_0| < 10$ mm. As a result, B^0 reconstruction efficiency drops as the proper time increases – weight factor $w(t_i)$ is used to compensate for it

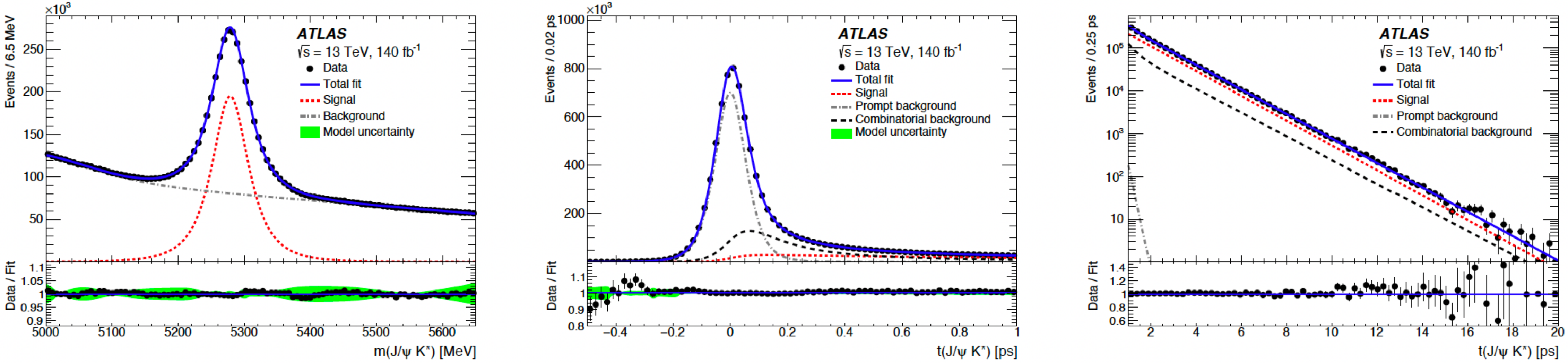
- The time uncertainty depends on σ_t and p_T per event, and they are also different for signal and background. It is not enough to treat them as parameters, but should have PDFs of their own

$$\mathcal{T}_j(\tau_i, \sigma_{\tau_i}, p_{T_i}) = P_j(\tau_i, \sigma_{\tau_i}, p_{T_i}) \cdot C_j(\sigma_{\tau_i}, p_{T_i})$$





High precision B_d^0 lifetime



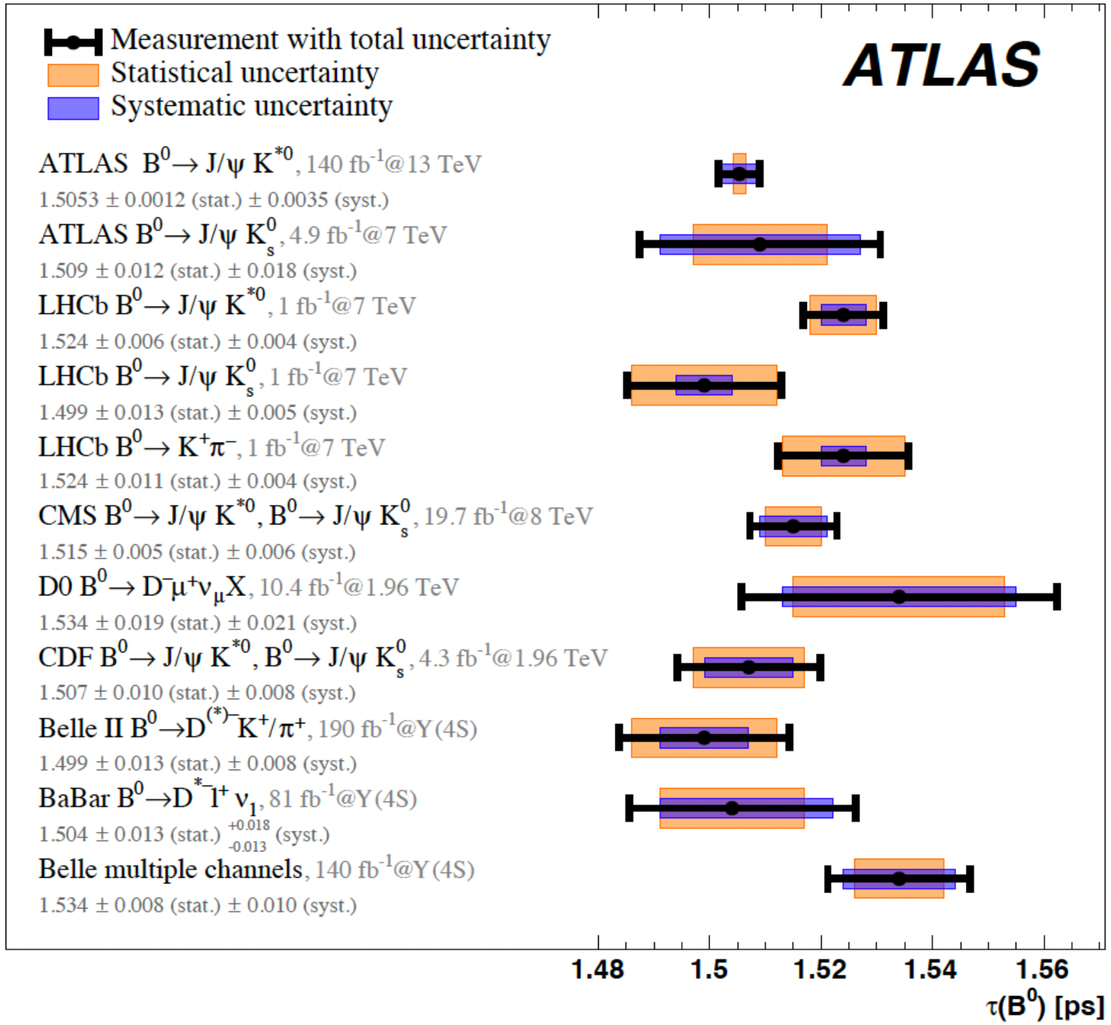
Source of uncertainty	Systematic uncertainty [ps]
ID alignment	0.00108
Choice of mass window	0.00104
Time efficiency	0.00135
Best-candidate selection	0.00041
Mass fit model	0.00152
Mass-time correlation	0.00229
Proper decay time fit model	0.00010
Conditional probability model	0.00070
Fit model test with pseudo-experiments	0.00002
Total	0.0035

A total of 2,450,500±2,400 $B^0 \to J/\psi\, K^{*0}$ events from the fit, and the extracted effective lifetime is

$$\tau = 1.5053 \pm 0.0012(\text{stat.}) \pm 0.0035\ (\text{syst.})\ \text{ps}$$

- Mass fit projection (left), proper decay time fit projections in two different ranges: (0.5, 1) ps (middle) and (1, 20) ps (right)
- Solid blue line - total fit; dashed red line - signal

High precision B_d^0 lifetime



- It provides the most precise result of the effective lifetime of B^0 to date
- Using $2y = 0.001 \pm 0.010$ and $A = -0.578 \pm 0.136$ from HFLAV group, the decay width is extracted:

$$\Gamma_d = 0.6639 \pm 0.0005(\text{stat.}) \pm 0.0016\,(\text{syst.}) \pm 0.0038(\text{ext.})\ \text{ps}^{-1}$$

where the last error originates from HFLAV

- With $\Gamma_s = 0.6703 \pm 0.0014(\text{stat.}) \pm 0.0018(\text{syst.})\ \text{ps}^{-1}$ from previous ATLAS result [EPJC 81 (2021) 342], decay width ratio is

$$\Gamma_d/\Gamma_s = 0.9905 \pm 0.0022(\text{stat.}) \pm 0.0036\,(\text{syst.}) \pm 0.0057(\text{ext.})$$

Time-dependent CPV in $B_s^0 \to J/\psi\ \phi$

$B_s^0 \to J/\psi\ \phi$ is a golden channel to measure the CP-violation phase ϕ_s (analogous to angle ϕ in $B^0 \to J/\psi\ K_S$)

$$\phi_s \approx -2\beta_s = -2\arg(-V_{ts}V_{tb}^* / V_{cs}V_{cb}^*)$$

CPV is induced by interference between direct decay and mixing

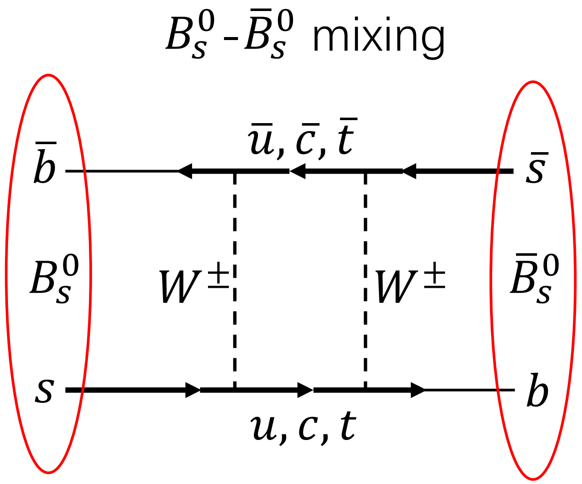
Need a simultaneous measurement of mass, decay angle, flavor and proper decay time. Differential decay rate of $B_s^0 \to J/\psi\ \phi \to J/\psi KK$:

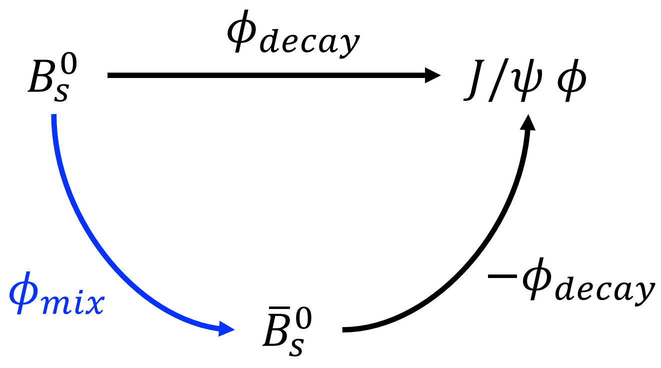
$$\frac{d^4\Gamma(B_s^0)}{d\Theta\, d(ct)} = \mathcal{F}(\Theta, ct, \alpha) \propto \sum_{i=1}^{10} O_i(ct, \alpha)\, g_i(\Theta)$$

Where Θ are the decay angles (next slide), ct is the proper decay length,

α are the interesting parameters such as Δm_s, $\Delta\Gamma_s$, $\lambda = \frac{q}{p}\frac{\bar{A}_f}{A_f}$, ϕ_s

$$B_s^{L,H} = p|B_s^0\rangle \pm q|\bar{B}_s^0\rangle$$

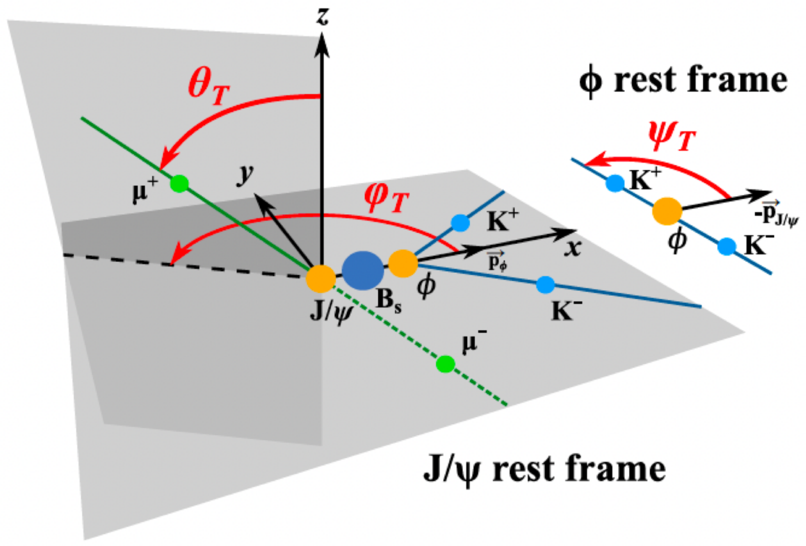


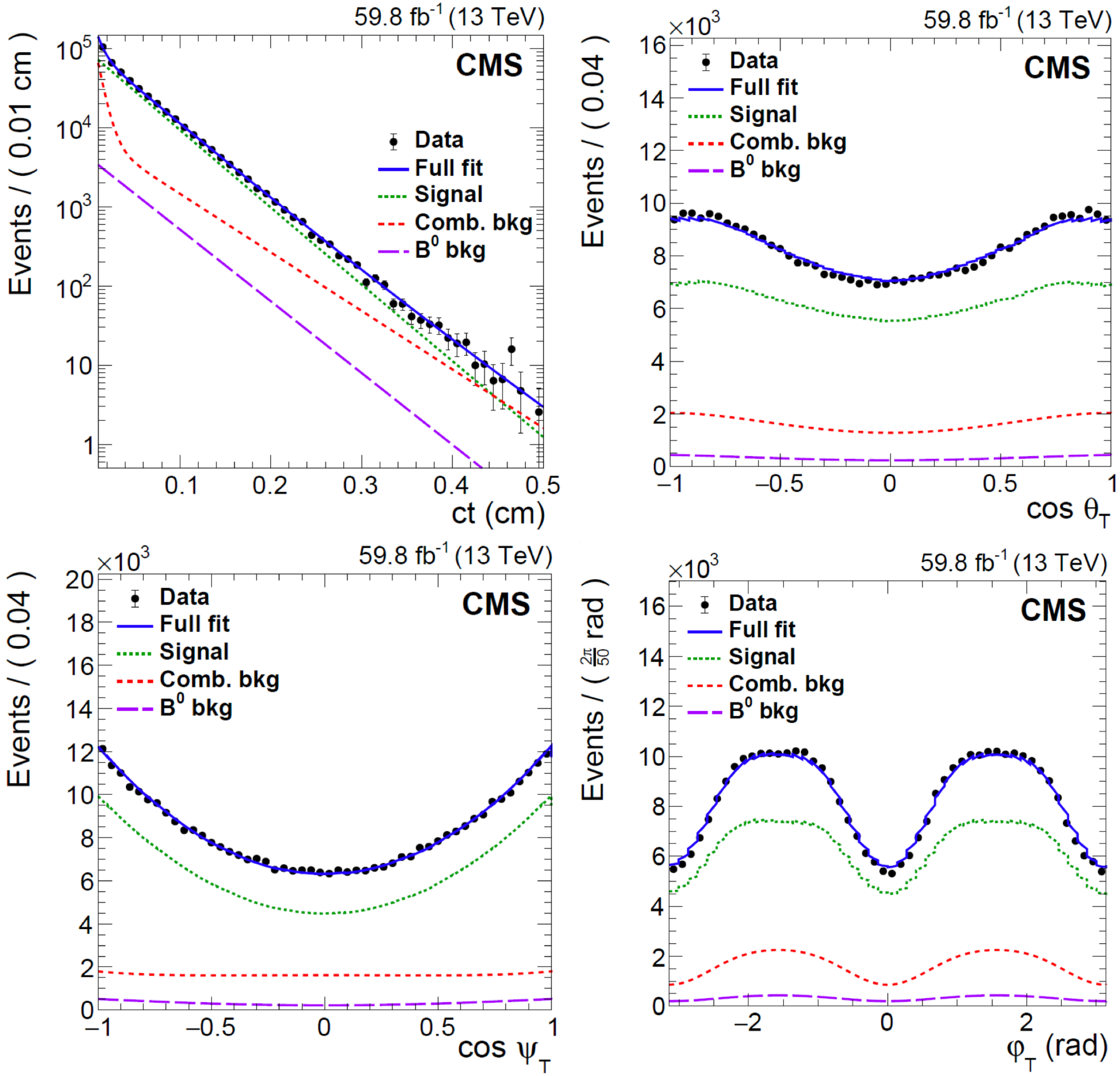


$\phi_s = \phi_{mix} - 2\phi_{decay}$

Time-dependent CPV in $B_s^0 \to J/\psi\ \phi$

[arXiv:2412.19952]





- Three transversity basis angles
- $\Theta = (\theta_T, \psi_T, \phi_T)$ are used to separate the amplitudes into CP even and odd parts
- An additional trigger path and a new flavour tagging algorithm are implemented

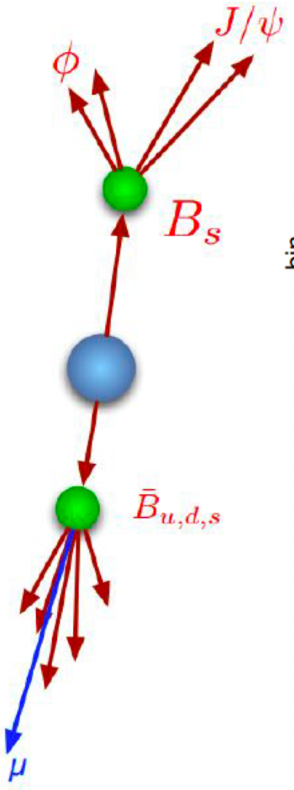
Time-dependent CPV in $B_s^0 \to J/\psi\, \phi$

- Efficiency: ε_{tag}. Fraction of tagged signals with a specific tagger.
- Dilution: $D = (1 - 2\omega)$, where ω is the mistag probability
- Tagging power: Fraction of correctly tagged signal events

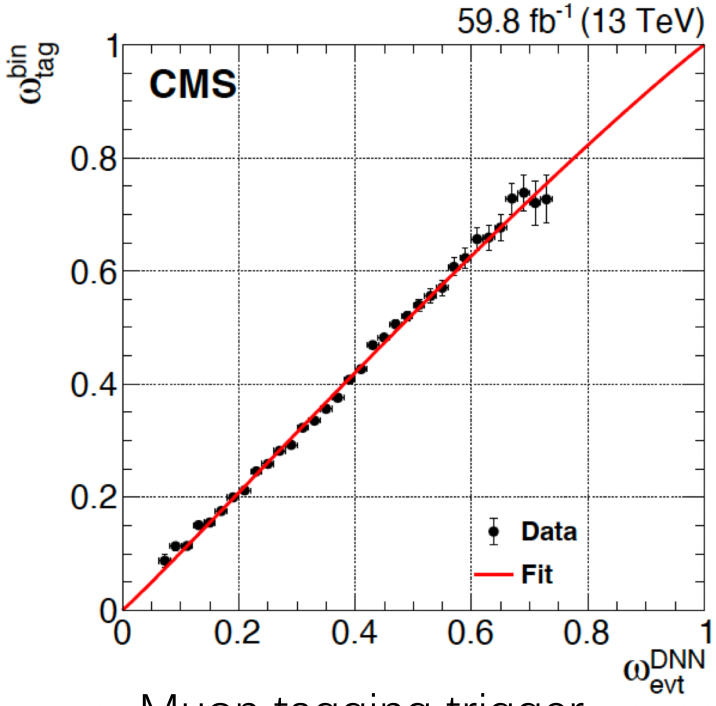
$$P_{tag} = \varepsilon_{tag} D^2$$

Category	ε_{tag} [%]	$\mathcal{D}^2_{tag,eff}$	P_{tag} [%]
Only OS muon	6.07 ± 0.05	0.212	1.29 ± 0.07
Only OS electron	2.72 ± 0.02	0.079	0.214 ± 0.004
Only OS jet	5.16 ± 0.03	0.045	0.235 ± 0.003
Only SS	33.12 ± 0.07	0.080	2.64 ± 0.01
SS + OS muon	0.62 ± 0.01	0.202	0.125 ± 0.003
SS + OS electron	2.77 ± 0.02	0.150	0.416 ± 0.005
SS + OS jet	5.40 ± 0.03	0.124	0.671 ± 0.006
Combined	55.9 ± 0.1	0.100	5.59 ± 0.02

Tagger is calibrated with self-tagging $B^+ \to J/\psi\, K^+$ data



Measured mistag probability versus the value predicted by the tagging algorithm:



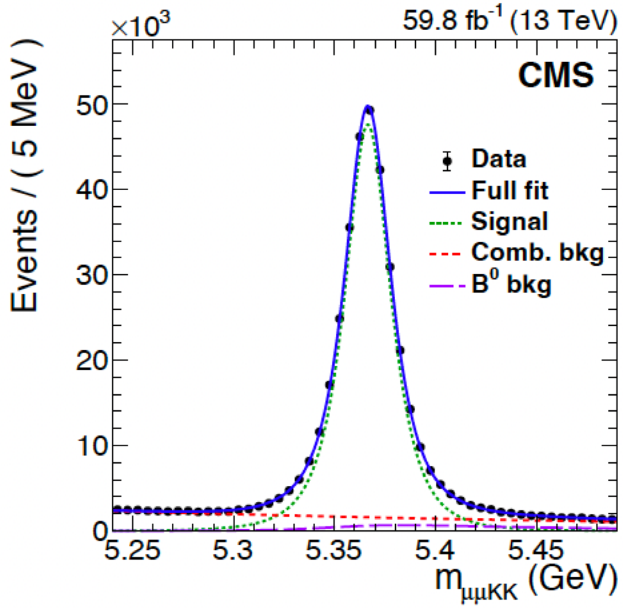
Muon tagging trigger

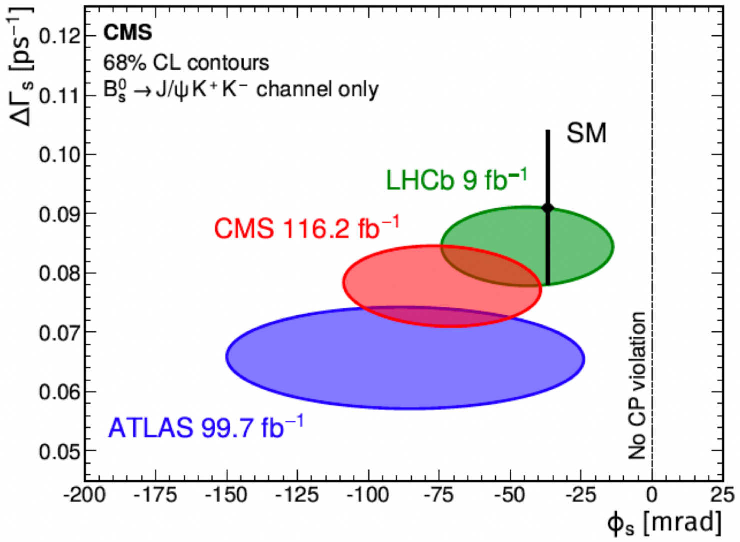
Time-dependent CPV in $B_s^0 \to J/\psi\, \phi$

Fitted parameters:

Parameter	Fit value	Stat. unc.	Syst. unc.
ϕ_s [mrad]	−73	± 23	± 7
$\Delta\Gamma_s$ [ps^{-1}]	0.0761	± 0.0043	± 0.0019
Γ_s [ps^{-1}]	0.6613	± 0.0015	± 0.0028
Δm_s [$\hbar\, ps^{-1}$]	17.757	± 0.035	± 0.017
$\lvert\lambda\rvert$	1.011	± 0.014	± 0.012
$\lvert A_0\rvert^2$	0.5300	$^{+0.0016}_{-0.0014}$	± 0.0044
$\lvert A_\perp\rvert^2$	0.2409	± 0.0021	± 0.0030
$\lvert A_S\rvert^2$	0.0067	± 0.0033	± 0.0009
$\delta_\parallel$ [rad]	3.145	± 0.089	± 0.025
$\delta_\perp$ [rad]	2.931	± 0.089	± 0.050
$\delta_{S\perp}$ [rad]	0.48	± 0.15	± 0.05

~491k signal events from fit, out of which ~27500 are tagged - the largest effective data sample of tagged signal events ever collected





- A_i and δ_i are the transversity amplitudes and their corresponding phases
- If combined with 8 TeV result, $\phi_s = -74 \pm 23$ mrad, which differs from zero by 3.2σ - first evidence for indirect CPV in this decay
- Still compatible with SM prediction of -37 ± 1 mrad

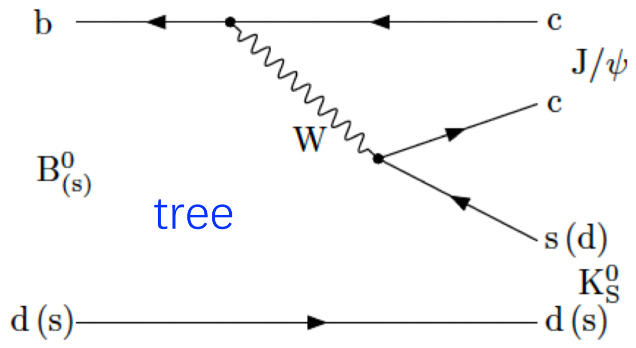
B_s^0 lifetime with $B_s^0 \to J/\psi\, K_S^0$ decay

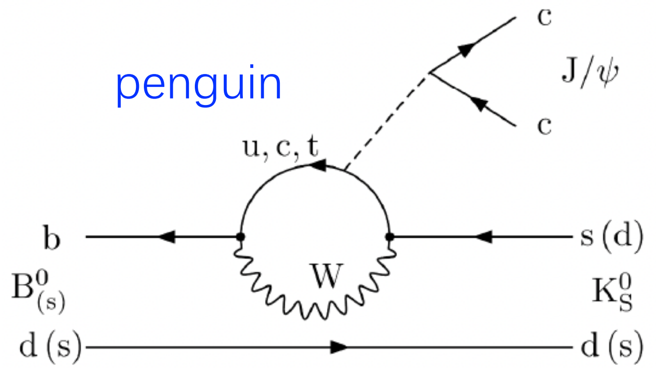
[JHEP10 (2024) 247]

- The phase ϕ_d from the interference between decay and mixing of $B^0 \to J/\psi\, K_S^0$ is related to angle β in the unitarity triangle: $\phi_d = 2\beta$. However, the equality is not exact due to a small contribution from doubly-Cabibbo-suppressed penguin diagram
- The penguin is not Cabibbo suppressed in the $B_s^0 \to J/\psi\, K_S^0$ decay. Measuring the effective B_s^0 lifetime in this decay can improve the measurement in $B^0 \to J/\psi\, K_S^0$
- The untagged effective lifetime of $B_s^0 \to J/\psi\, K_S^0$ is defined as

$$\tau(\mathrm{B_s^0} \to \mathrm{J/\psi K_S^0}) \equiv \frac{\int_0^\infty t\,\{\Gamma[\mathrm{B_s^0}(t) \to \mathrm{J/\psi K_S^0}] + \Gamma[\overline{\mathrm{B}}{}_\mathrm{s}^0(t) \to \mathrm{J/\psi K_S^0}]\}\,\mathrm{d}t}{\int_0^\infty \{\Gamma[\mathrm{B_s^0}(t) \to \mathrm{J/\psi K_S^0}] + \Gamma[\overline{\mathrm{B}}{}_\mathrm{s}^0(t) \to \mathrm{J/\psi K_S^0}]\}\,\mathrm{d}t}$$

- Since $J/\psi\, K_S^0$ is pure CP-odd state, and the CP violation in the kaon system can be neglected, this channel allows us to accurately measure the heavy B_S^0 state's lifetime
- Previous LHCb result $\tau(B_s^0 \to J/\psi\, K_S^0) = 1.75 \pm 0.12(\text{stat.}) \pm 0.07\,(\text{syst.})\,\text{ps}$ (Nucl. Phys. B 873 (2013) 275) is not as precise as the theory prediction of $\tau(B_s^0 \to J/\psi\, K_S^0) = 1.62 \pm 0.02\,\text{ps}$





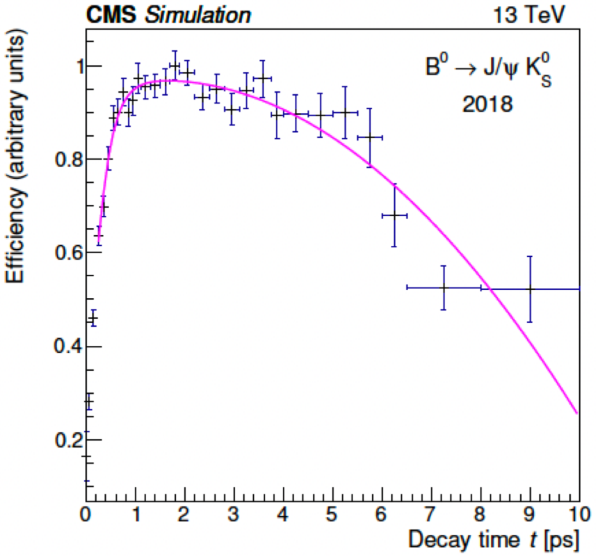
B_s^0 lifetime with $B_s^0 \to J/\psi\, K_S^0$ decay

- The $\mu^+\mu^-$ and $\pi^+\pi^-$pairs are each fitted to a common vertex. The K_S^0 vertex is required to be at least 15 times its uncertainty from the beam line. Their mass can within 2.5σ (σ=5.5/30 MeV for K_S^0 or J/ψ) of the nominal mass
- The $\Lambda \to p\pi^-$ background is removed by momentum asymmetry
- The $K_S^0 \to \pi^+\pi^-$ vertex is fitted with K_S^0 mass constrained. Similarly the $B_s^0 \to \mu^+\mu^- K_S^0$ vertex is fitted with the J/ψ mass constrained. Distance between B_s^0 and K_S^0 is greater than 5 times its uncertainty, and 4.9<m(B_s^0)<6.0 GeV
- BDT with 8 input variables (vertex probability, particle p_T, L_{xy} etc.) are used to reduce background. Signal MC and high mass sideband data with 5.6<m(B_s^0)<6.0 GeV are used for training
- One BDT in each data-taking year. The control channel $B^0 \to J/\psi\, K_S^0$ is used to validate inputs
- In the simultaneous mass and proper decay time 2D fit, the decay time and mass are further confined to 0.2<t<10 ps and 5.17<m(B_s^0)<5.57 GeV. The total likelihood function reads

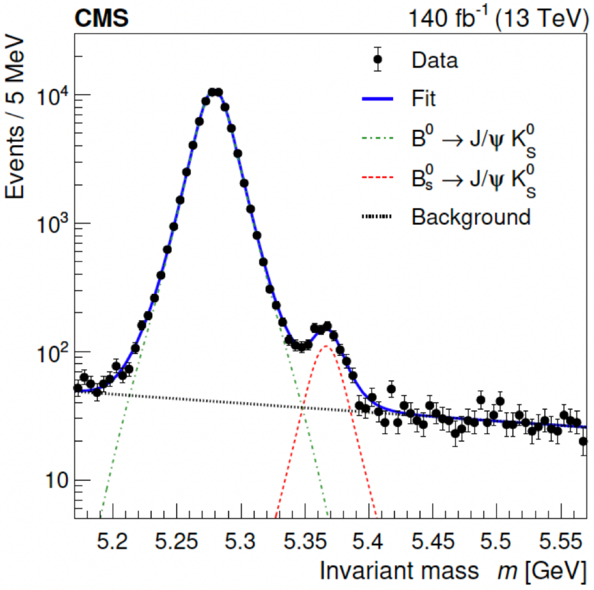
$$\mathcal{L}(m,t;\,\sigma_t) = N_{B_s^0}\, M_{B_s^0}(m)\, T_{B_s^0}(t;\,\sigma_t)\, \varepsilon_{B_s^0}(t) + N_{B^0}\, M_{B^0}(m)\, T_{B^0}(t;\,\sigma_t)\, \varepsilon_{B^0}(t) + N_{\rm bkg}\, M_{\rm bkg}(m)\, T_{\rm bkg}(t;\,\sigma_t)$$

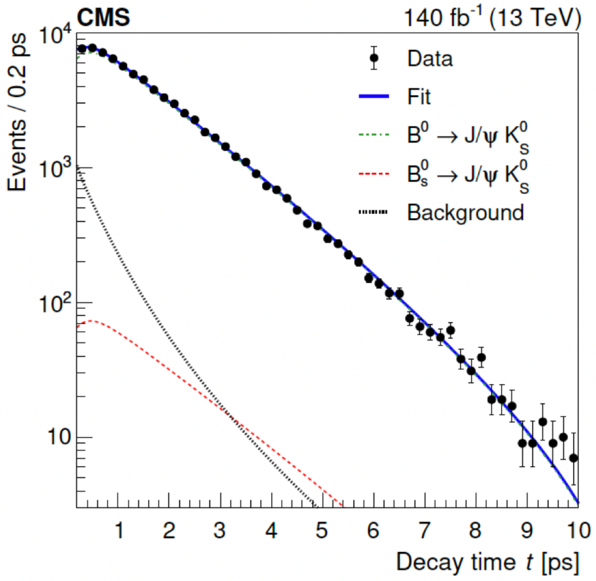
B_s^0 lifetime with $B_s^0 \to J/\psi\, K_S^0$ decay

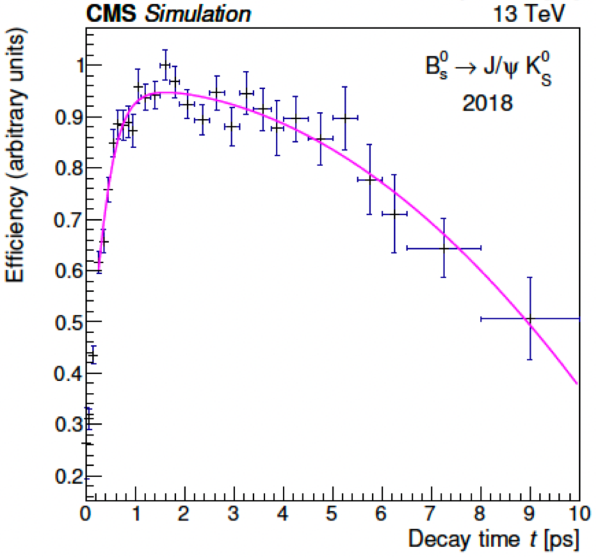
- The signal and control channel efficiencies, $\varepsilon_{B_s^0}(t)$ and $\varepsilon_{B^0}(t)$, are calculated from MC for each year. Their ratio is found to be flat (no dependence on time)
- Separate likelihood in each year (16-18) with independent parameters, except for the effective lifetimes of B^0 and B_s^0 which are common
- The means of B^0 and B_s^0 mass are floating, but their difference is constrained to 87.26±0.24 MeV according to PDG



Fit projections to 1D mass and decay time distributions:

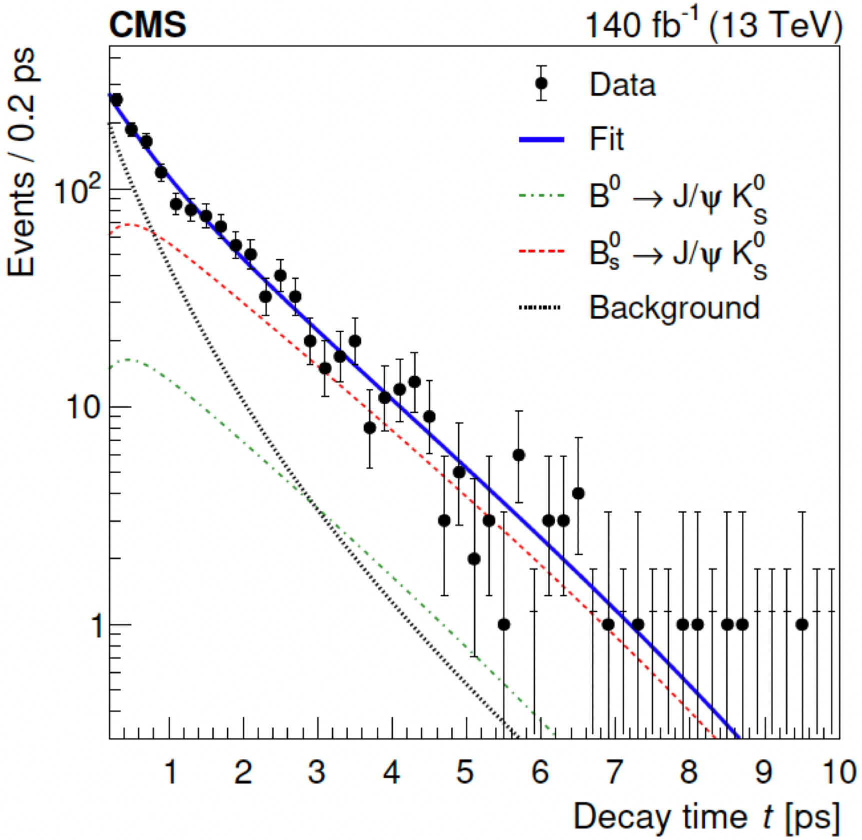






B_s^0 lifetime with $B_s^0 \to J/\psi\, K_S^0$ decay

Projection within B_s^0 mass window 5.34-5.42 GeV：



Systematic uncertainties

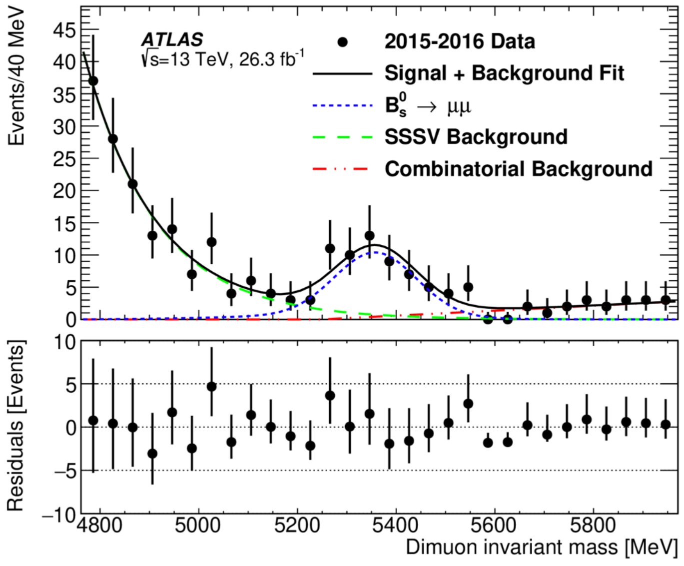
Source	Value (ps)
Limited MC event count	0.006
Efficiency modeling	0.002
Signal and background invariant mass model	0.022
Background decay time model	0.014
Invariant mass shape variation	0.004
Different fit strategy	0.006
Contribution of B_s^0 from B_c^+ decays	0.002
Control channel lifetime uncertainty	0.007
Total	0.029

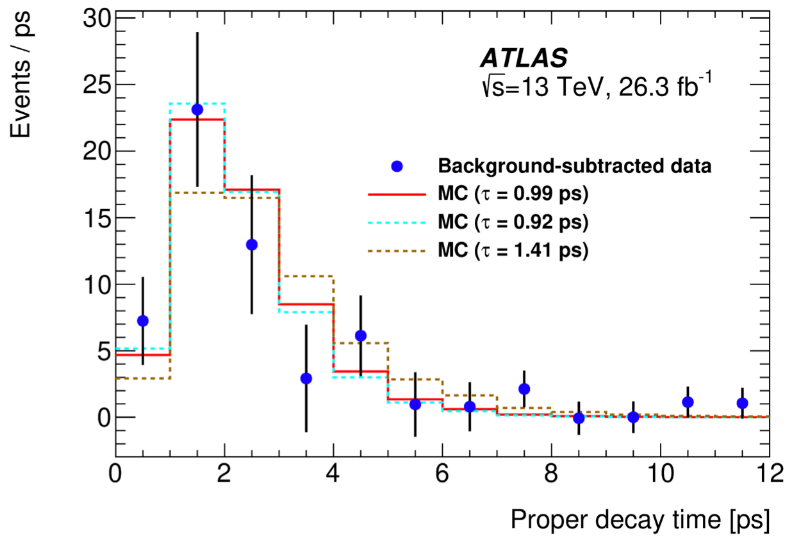
- A total of 727±35 and 68,460±270 signal events are obtained for B_s^0 and B^0, respectively
- Effective lifetime uncertainty has a factor of two improvement over previous result:

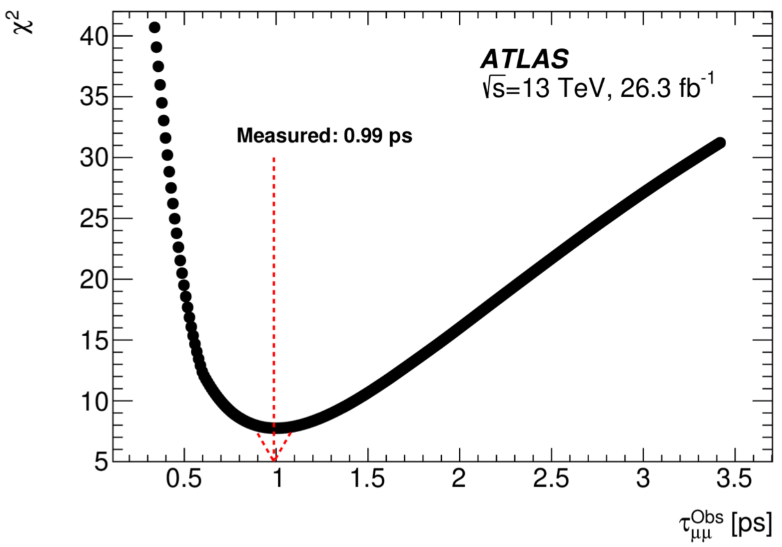
$$\tau(B_s^0 \to J/\psi\, K_S^0) = 1.59 \pm 0.07(\text{stat.}) \pm 0.03\,(\text{syst.})\ \text{ps}$$

$B_s^0 \to \mu\mu$ lifetime

[JHEP09 (2023) 199]

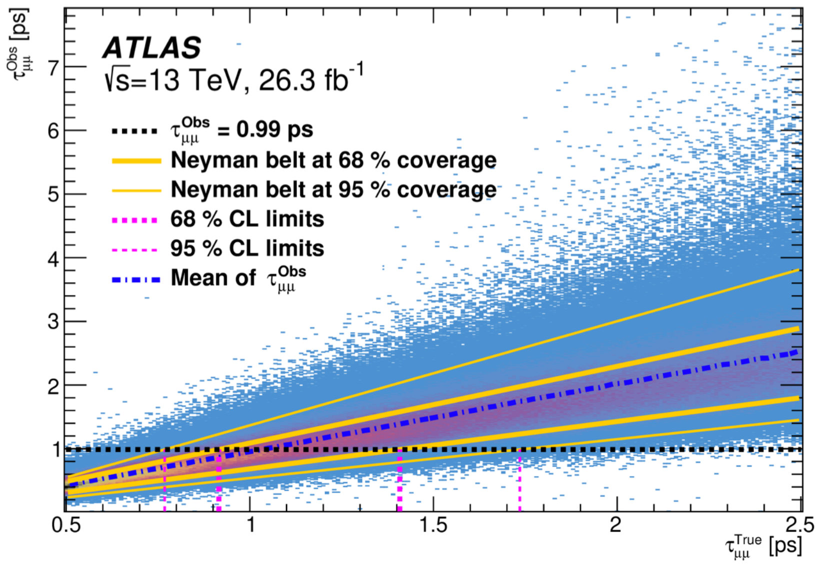


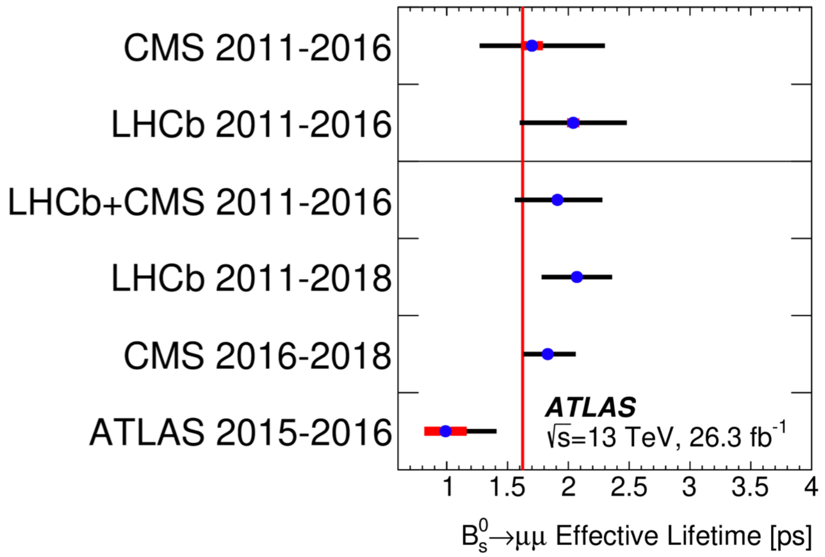




- $B_s \to \mu\mu$ is a FCNC process via loop diagrams, and can be used to measure CP-odd (heavy) B_s lifetime
- Unbinned Extended ML fit to m($\mu\mu$) distribution, with background parameters unconstrained and signal shape from MC. Signal yield is 58±13. Background is subtracted by *sPlot* technique to obtain the signal proper time distribution
- Signal templates with different proper decay times are generated from MC, fit to data with the smallest χ^2
- Full procedure is repeated with $B^\pm \to J/\psi K^\pm$ signal in data for L_{xy} resolution effect – found to be a 134 fs effect

$B_s^0 \to \mu\mu$ lifetime



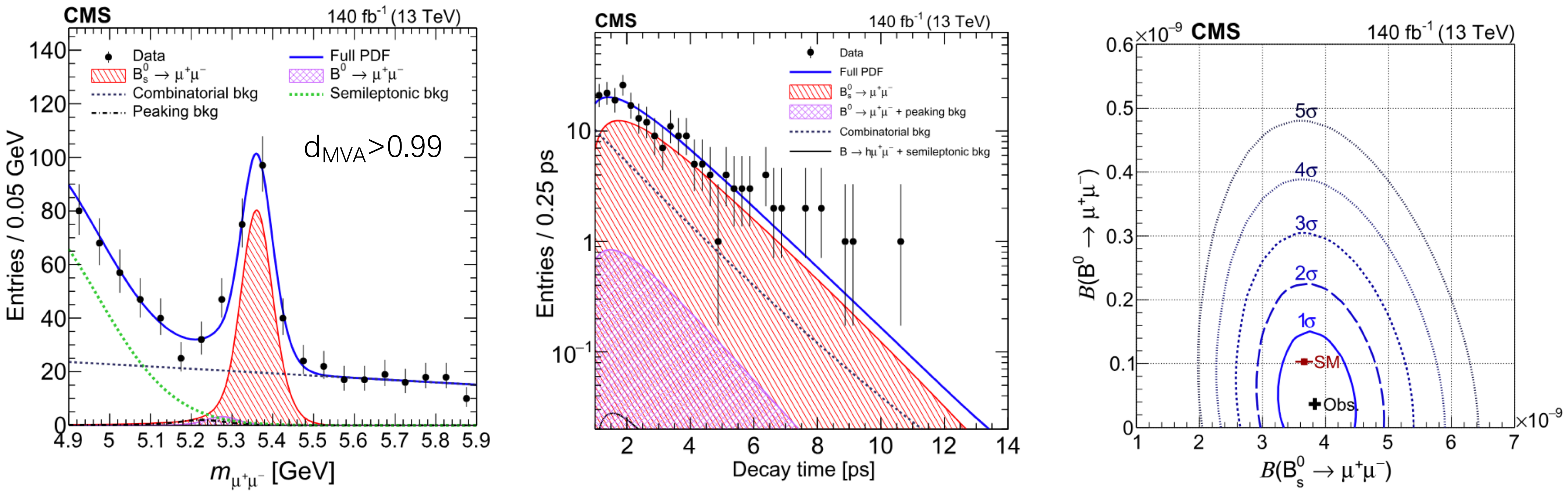


Uncertainty source	$\Delta\tau_{\mu\mu}^{\mathrm{Obs}}$ [fs]
Data - MC discrepancies	134
SSSV lifetime model	60
Combinatorial lifetime model	56
B kinematic reweighting	55
B isolation reweighting	32
SSSV mass model	22
B_d background	16
Fit bias lifetime dependency and B_s^0 eigenstates admixture	15
Combinatorial mass model	14
Pileup reweighting	13
B_c background	10
Muon Δ_η correction	6
$B \to hh'$ background	3
Muon reconstruction SF reweighting	2
Semileptonic background	2
Trigger reweighting	1
Total	174

- Neyman construction to go from observed to true proper time
- Largest systematics from Data-MC discrepancies
- Measured $\tau_{\mu\mu} = 0.99^{+0.42}_{-0.07}(stat.) \pm 0.17(sys.)$ ps, compared with SM prediction of 1.624±0.009 ps

$B_s^0 \to \mu\mu$ lifetime

[Phys. Lett. B 842 (2023) 137955]



- To extract the effective lifetime of $B_s^0 \to \mu\mu$, a 3D UML fit to the dimuon invariant mass, decay time, and decay time uncertainty. Data is divided by by data-taking period, d_{MVA} value, and rapidity of the most forward muon
- Use $B^{\pm} \longrightarrow J/\psi K^{\pm}$ as the normalization channel for branching fraction, and signal acceptance as a function of decay time is also corrected with it
- B($B_s^0 \to \mu\mu$)=$[3.83^{+0.38}_{-0.36}(stat)^{+0.19}_{-0.16}(sys)^{+0.14}_{-0.13}(f_s/f_d)]\times10^{-19}$, $\tau_{\mu\mu} = 1.83^{+0.23}_{-0.20}(stat.) \pm 0.04(sys.)$ ps

Heavy flavor physics projections

Projected CKM angle measurement sensitivities for different experiments and data sample sizes [ATL-PHYS-PUB-2025-020]:

Experiment Assumed data sample	**ATLAS** 20.3-99.7 fb^{-1}	**CMS** 116-140 fb^{-1}	**LHCb** 2-9 fb^{-1}	**Belle II** 364-1075 fb^{-1}
CKM angles				
β	—	—	0.57° [15]	1.2° [16]
α	—	—	—	6.6° [17]
γ	—	—	2.8° [18]	13° [17]
ϕ_s [mrad]	42 [19]	23 [20]	20 [21]	—

Experiment Assumed data sample	**ATLAS** 3000 fb^{-1}	**CMS** 3000 fb^{-1}	**LHCb** 300 fb^{-1}	**Belle II** 50 ab^{-1}
CKM angles				
β	—	—	0.08°	0.3°
α	—	—	—	0.6°
γ	—	—	0.3°	1.0°
ϕ_s [mrad]	(4 – 9)	3	3	—

Projected $\tau_{\mu\mu}$ sensitivities with 3000 fb^{-1} for different trigger working points [ATL-PHYS-PUB-2025-016]

Quantity	$\tau_{\mu\mu}$ [ps]		
SM value	1.624 ± 0.009 [7]		
ATLAS 2015–2016 measurements	$0.99^{+0.42}_{-0.07} \pm 0.17$ [8]		
Projected uncertainty	Stat.	Syst.	Total
$\mu10\mu10$	$^{+0.09}_{-0.06}$	0.06	$^{+0.11}_{-0.08}$
$\mu10\mu6$	$^{+0.06}_{-0.03}$	0.05	$^{+0.08}_{-0.06}$
$\mu6\mu6$	$^{+0.05}_{-0.03}$	0.05	$^{+0.07}_{-0.05}$

Summary

- Both ATLAS and CMS have a rich program for low energy hadron physics owing to their excellent muon and tracking systems
- Time-dependent analysis of b-hadrons are actively pursued in both, in an effort to unveil the mysteries of the electroweak interaction and CPV

  – Precise measurement of B^0 lifetime and decay width (ATLAS):

$$\tau(B^0 \to J/\psi\, K^{*0}) = 1.5053 \pm 0.0012(\text{stat.}) \pm 0.0035\,(\text{syst.})\ \text{ps}$$

$$\Gamma_d = 0.6639 \pm 0.0005(\text{stat.}) \pm 0.0016\,(\text{syst.}) \pm 0.0038(\text{ext.})\ \text{ps}^{-1}$$

$$\Gamma_d/\Gamma_s = 0.9905 \pm 0.0022(\text{stat.}) \pm 0.0036\,(\text{syst.}) \pm 0.0057(\text{ext.})$$

  – B_s^0 lifetime measurements in exclusive decays:

$$\tau\left(B_s^0 \to J/\psi\, K_S^0\right) = 1.59 \pm 0.07(\text{stat.}) \pm 0.03\,(\text{syst.})\ \text{ps}$$ (CMS)

$\tau(B_s^0 \to \mu\mu) = 0.99^{+0.42}_{-0.07}(\text{stat}) \pm 0.17(\text{sys})\ \text{ps}$ (ATLAS); $\tau(B_s^0 \to \mu\mu) = 1.83^{+0.23}_{-0.20}(\text{stat}) \pm 0.04(\text{sys})\ \text{ps}$ (CMS)

  – CPV phase in the decay $B_s^0 \to J/\psi\, \phi$: $\phi_s = -74 \pm 23$ mrad (CMS)

- List is not exhaustive, several analyses with run-2 ongoing, and look forward to run-3 results!

Backup Slides

Time-dependent CPV in $B_s^0 \to J/\psi\,\phi$

- The differential decay rate of $B_s^0 \to J/\psi\,\phi \to \mu^+\mu^-K^+K^-$ (PLB 816 (2021) 136188)

$$\frac{d^4\Gamma(B_s^0)}{d\Theta\, d(ct)} = \mathcal{F}(\Theta, ct, \alpha) \propto \sum_{i=1}^{10} O_i(ct,\alpha)\, g_i(\Theta)$$

$$O_i(ct,\alpha) = N_i e^{-\Gamma_s t}\left[a_i \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + b_i \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + c_i \cos(\Delta m_s t) + d_i \sin(\Delta m_s t)\right]$$

i	$g_i(\theta_T, \psi_T, \varphi_T)$	N_i	a_i	b_i	c_i	d_i
1	$2\cos^2\psi_T(1-\sin^2\theta_T\cos^2\varphi_T)$	$\|A_0(0)\|^2$	1	D	C	$-S$
2	$\sin^2\psi_T(1-\sin^2\theta_T\sin^2\varphi_T)$	$\|A_\parallel(0)\|^2$	1	D	C	$-S$
3	$\sin^2\psi_T\sin^2\theta_T$	$\|A_\perp(0)\|^2$	1	$-D$	C	S
4	$-\sin^2\psi_T\sin 2\theta_T\sin\varphi_T$	$\|A_\parallel(0)\|\|A_\perp(0)\|$	$C\sin(\delta_\perp-\delta_\parallel)$	$S\cos(\delta_\perp-\delta_\parallel)$	$\sin(\delta_\perp-\delta_\parallel)$	$D\cos(\delta_\perp-\delta_\parallel)$
5	$\frac{1}{\sqrt{2}}\sin 2\psi_T\sin^2\theta_T\sin 2\varphi_T$	$\|A_0(0)\|\|A_\parallel(0)\|$	$\cos(\delta_\parallel-\delta_0)$	$D\cos(\delta_\parallel-\delta_0)$	$C\cos(\delta_\parallel-\delta_0)$	$-S\cos(\delta_\parallel-\delta_0)$
6	$\frac{1}{\sqrt{2}}\sin 2\psi_T\sin 2\theta_T\cos\varphi_T$	$\|A_0(0)\|\|A_\perp(0)\|$	$C\sin(\delta_\perp-\delta_0)$	$S\cos(\delta_\perp-\delta_0)$	$\sin(\delta_\perp-\delta_0)$	$D\cos(\delta_\perp-\delta_0)$
7	$\frac{2}{3}(1-\sin^2\theta_T\cos^2\varphi_T)$	$\|A_S(0)\|^2$	1	$-D$	C	S
8	$\frac{1}{3}\sqrt{6}\sin\psi_T\sin^2\theta_T\sin 2\varphi_T$	$\|A_S(0)\|\|A_\parallel(0)\|$	$C\cos(\delta_\parallel-\delta_S)$	$S\sin(\delta_\parallel-\delta_S)$	$\cos(\delta_\parallel-\delta_S)$	$D\sin(\delta_\parallel-\delta_S)$
9	$\frac{1}{3}\sqrt{6}\sin\psi_T\sin 2\theta_T\cos\varphi_T$	$\|A_S(0)\|\|A_\perp(0)\|$	$\sin(\delta_\perp-\delta_S)$	$-D\sin(\delta_\perp-\delta_S)$	$C\sin(\delta_\perp-\delta_S)$	$S\sin(\delta_\perp-\delta_S)$
10	$\frac{4}{3}\sqrt{3}\cos\psi_T(1-\sin^2\theta_T\cos^2\varphi_T)$	$\|A_S(0)\|\|A_0(0)\|$	$C\cos(\delta_0-\delta_S)$	$S\sin(\delta_0-\delta_S)$	$\cos(\delta_0-\delta_S)$	$D\sin(\delta_0-\delta_S)$

$$C = \frac{1-|\lambda|^2}{1+|\lambda|^2}, \quad S = -\frac{2|\lambda|\sin\phi_s}{1+|\lambda|^2}, \quad D = -\frac{2|\lambda|\cos\phi_s}{1+|\lambda|^2}, \quad \lambda = (q/p)(\bar{A}_f/A_f)$$