Theory review of semi-leptonic decays

— selected topics

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Motivation

- Semileptonic decays play an essential role in the determintion of the CKM matrix elements

Discrepancy exists between inclusive and exclusive decays, the so called *Vcb*, *Vub* Puzzle

Theoretically only involve perturbative calculations, already in high precision

- $\sim 3\sigma$ difference between exclusive and inclusive (GGOU) measurement of $|V_{ub}|$:

$$|V_{ub}|^{\text{excl.}} = (3.75 \pm 0.20) \times 10^{-3}$$

$$|V_{ub}|^{\text{incl.}} = (4.06 \pm 0.12 \pm 0.11) \times 10^{-3}$$

- More than 3σ difference in $|V_{cb}|$:

$$|V_{cb}|^{\text{excl.}} = (39.62 \pm 0.47) \times 10^{-3}$$

$$|V_{cb}|^{\text{incl.}} = (41.97 \pm 0.48) \times 10^{-3}$$

Motivation

The exclusive $B \to \pi\, l\nu$ decay

$$\frac{d\Gamma(B \to \pi l \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2 m_B^3}{256\pi^3} \lambda^{\frac{3}{2}} \left(1 - \frac{m_l^2}{q^2}\right)^2 \left[\left(1 + \frac{2m_l^2}{q^2}\right) |f^+(q^2)|^2 + \frac{3m_l^2}{2\lambda q^2} \left(1 - \frac{m_M^2}{q^2}\right)^2 |f^0(q^2)|^2\right]$$

The decay rate depends on hadron transition form factors f^+ and f^0

- The accuracy of experimental data is far beyond theoretical predictions for most exclusive heavy flavor processes

Simileptonic decays are also sensitive to new physics, such as anomalies $R(D^{(*)})$, $R(K^{(*)})$ etc.

$B \to D^{(*)} l\nu$ decay is account for V_{cb} measurement

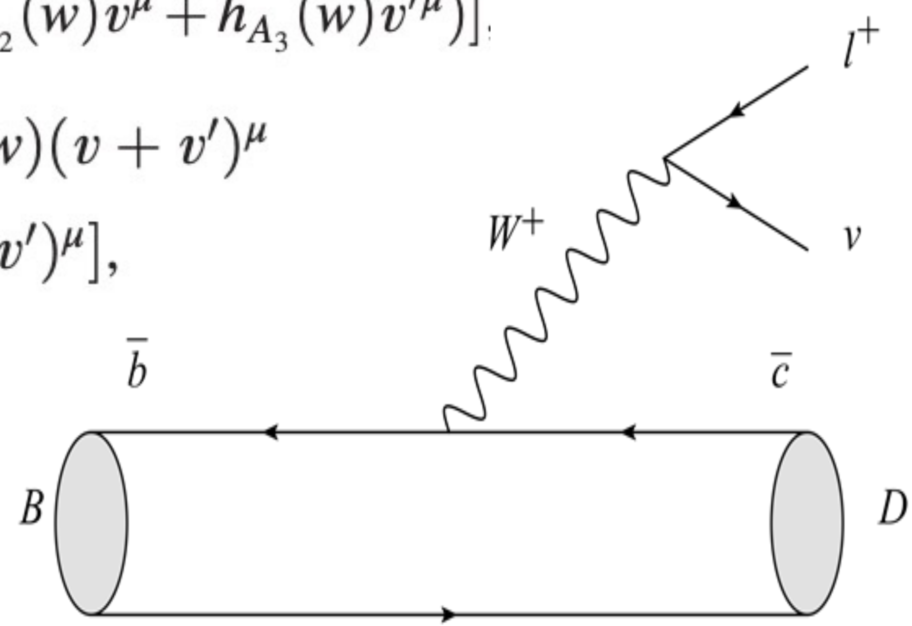
They are governed by $B \to D^{(*)}$ Form Factors, which are well defined in HQET

$$\langle D^*(k,\epsilon)|\bar{c}\gamma^\mu b|\bar{B}(p)\rangle = i\sqrt{m_B m_{D^*}} h_V(w)\varepsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu v'_\rho v_\sigma,$$

$$\langle D^*(k,\epsilon)|\bar{c}\gamma^\mu\gamma^5 b|\bar{B}(p)\rangle = \sqrt{m_B m_{D^*}}[h_{A_1}(w)(w+1)\epsilon^{*\mu} - (\epsilon^*\cdot v)(h_{A_2}(w)v^\mu + h_{A_3}(w)v'^\mu)],$$

$$\langle D(k)|\bar{c}\gamma^\mu b|\bar{B}(p)\rangle = \sqrt{m_B m_D}[h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu],$$

However, they are good only **for large q^2 and** leading power accuracy.



B decays: The multi-scale problem

NP scale	EW scale	Heavy quark scale	Intermediate scale	Hadronizati on scale
TeV or beyond	m_W	$m_b(m_c)$	$\sqrt{m_b\Lambda}, m_c$	Λ
$\mathcal{L}_{NP}$	$\mathcal{L}_{SM}+$ $\mathcal{L}_{D>4}$	$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\sum_i C_i O_i + \sum_i C_i' O_i'$	SCET-I	Low energy QCD HQET, SCET-II

- Factorization theorem need to be proved order by order
- Perturbation： matching, resummation, evolution

$B \to D^{(*)}$ Form Factors results

- **Lattice QCD.** **for large q²** **Next talk**

$B \to D$: f_+ and f_0 [Fermilab/MILC '15] [HPQCD '15 '16]

$B \to D^*$: $h_{A_1}(1)$ [Fermilab/MILC '14] [HPQCD '17]

$B \to D^*$ ffs at non-zero recoil [JLQCD '18][Fermilab/MILC '19]

- **QCD light-cone sum rule.** **for small q²**

LO in α_s, leading-twist B meson DAs [S. Faller et al., EPJC60, 603 (2009)]

NLO in α_s, leading-twist B meson DAs [YM. Wang et al, JHEP06, 062 (2017)]

LO in α_s, NL twist B meson DAs [N. Gubernari et al, JHEP01, 150 (2019)]

- **Parametrization of form factors** **q² dependence**

HQET parametrization including $O(\alpha_s)$ and part of $O(\Lambda_{QCD}^2)$ corrections

[M. Jung et al., JHEP 01 (2019) 009;F. Bernlochner et al., Phys.Rev.D 95 (2017) 11, 115008]

- **Strong unitarity bounds** on $0^-/0^+/1^-/1^+$ helicity amplitudes (with updated B_c masses)

Recent results of $B \to D(D^*)$ form factors

- **$B \to D$ form factors with power corrections**

  Gao, Huber, Ji, Wang, Wang, Wei, JHEP05 (2022) 024

- **An improved study on the $B \to D(D^*)$ form factors: NLO corrections + power corrections**

  Cui, Huang, Wang, Zhao, *PRD* 108 (2023) L071504

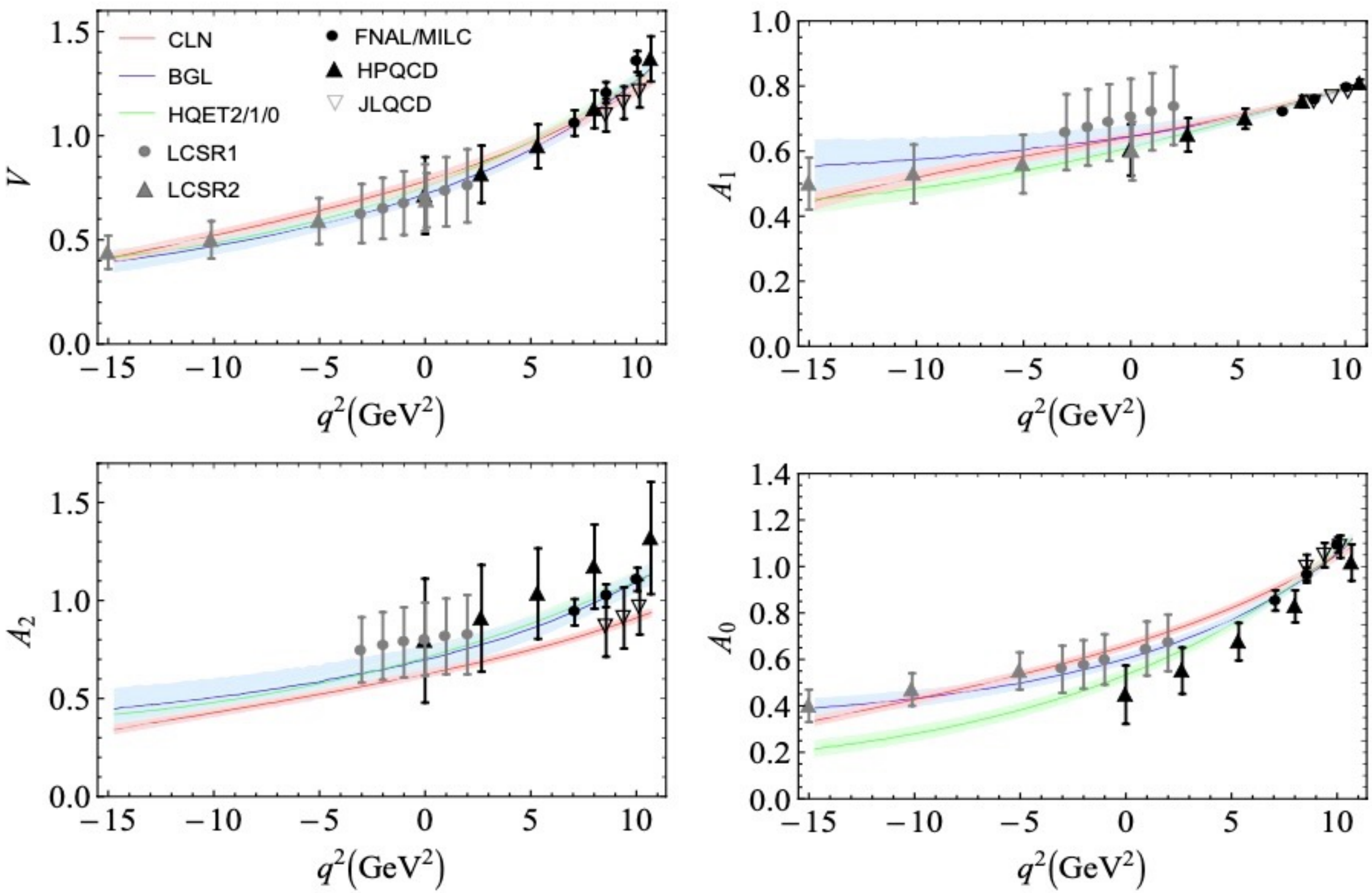
- **Combined analysis of Lattice simulation and LCSR**

Using Caprini-Lellouch-Neubert (CLN), Boyd-Grinstein-Lebed (BGL), and HQET parameterizations

  Li, Xu, Shi, Geng, Zhang. arXiv:2412.05989

Recent results of $B \to D^*$ form factors

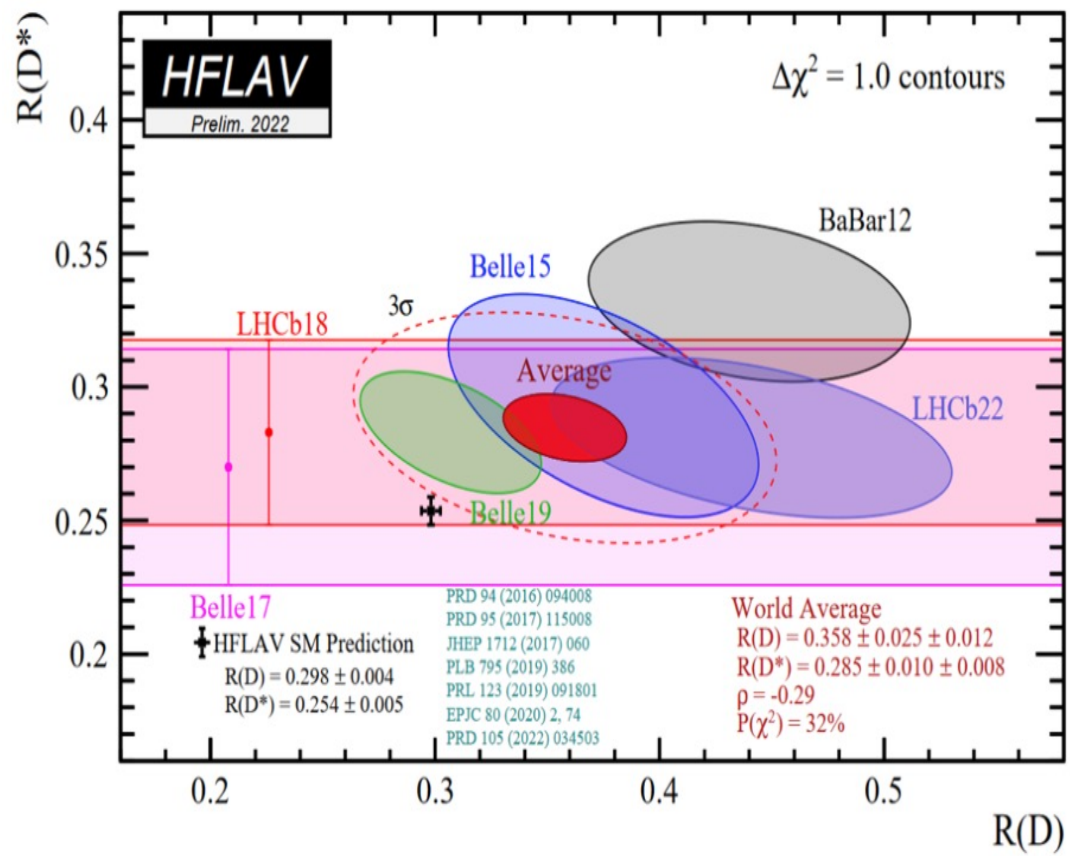
Li, Xu, Shi, Geng, Zhang. arXiv:2412.05989



$R(D^{(*)})$ Anomaly

$$R(D^{(*)}) = \frac{\mathfrak{B}(B \to D^{(*)}\tau\nu)}{\mathfrak{B}(B \to D^{(*)}\ell\nu)} \quad (\ell = e, \mu)$$

The combined results of $R(D^{(*)})$ indicate about 3σ deviation from the SM predictions



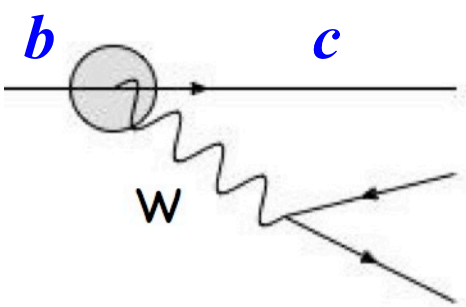
$$R(J/\psi) = \frac{\mathcal{B}(B_c \to J/\psi\tau\nu)}{\mathcal{B}(B_c \to J/\psi\mu\nu)}$$

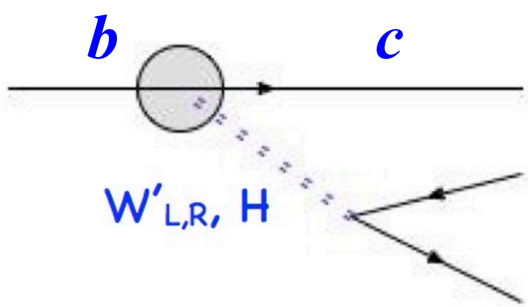
which deviate 2σ away from the SM prediction

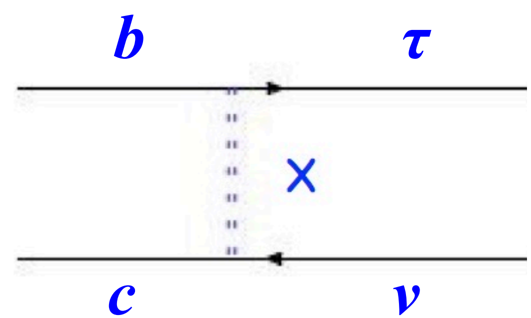
Taking the deviation seriously, apparently tau lepton has a stronger coupling

SM coupling

at tree level, several other possible couplings







Charged Higgs, seems a natural explanation but the simple models do not work

new W gauge boson with non-universal couplings (W_R)

leptoquark - need very specific flavour structure

A combined model independent analysis of the $R(D)$, $R(D*)$ and $R(J/\psi)$ anomalies

$$\mathcal{L}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{cb} [(1 + C_{V_1}) \mathcal{O}_{V_1} + C_{V_2} \mathcal{O}_{V_2} + C_{S_1} \mathcal{O}_{S_1} + C_{S_2} \mathcal{O}_{S_2} + C_T \mathcal{O}_T]$$

All possible Lorentz Invariant operators:

$$\mathcal{O}_{S_1} = (\bar{c}_L b_R)(\bar{\tau}_R \nu_L), \quad \mathcal{O}_{S_2} = (\bar{c}_R b_L)(\bar{\tau}_R \nu_L),$$

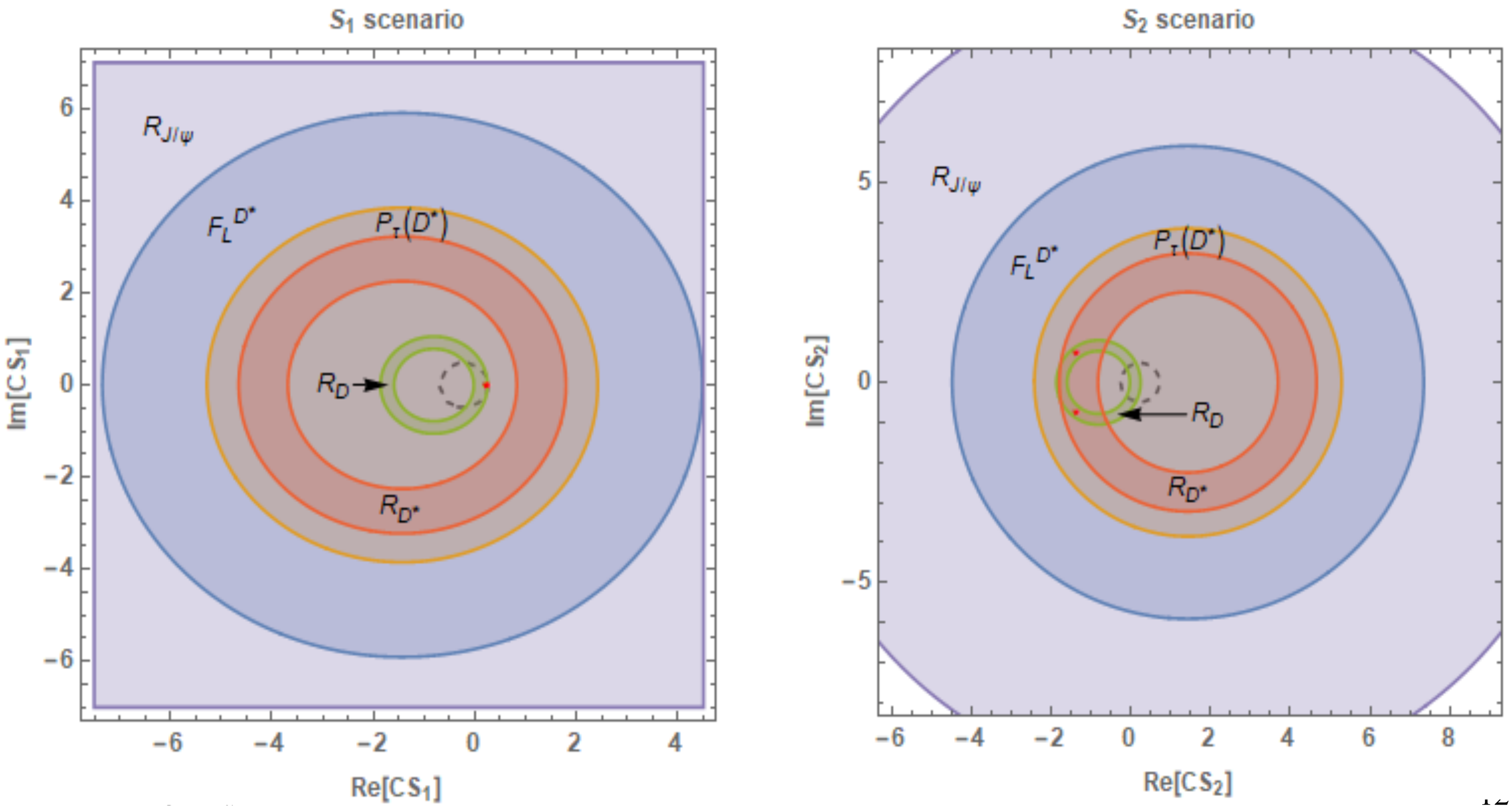
$$\mathcal{O}_{V_1} = (\bar{c}_L \gamma^\mu b_L)(\bar{\tau}_L \gamma_\mu \nu_L), \quad \mathcal{O}_{V_2} = (\bar{c}_R \gamma^\mu b_R)(\bar{\tau}_L \gamma_\mu \nu_L),$$

$$\mathcal{O}_T = (\bar{c}_R \sigma^{\mu\nu} b_L)(\bar{\tau}_R \sigma_{\mu\nu} \nu_L),$$

Huang, Li, Lu, Paracha, Wang, PRD98 (2018) no.9, 095018

It is found that none of the single operators can explain simultaneously the current experimental measurements of the ratios $R(D)$, $R(D*)$ and $R(J/\psi)$ at the confidence level of 1σ

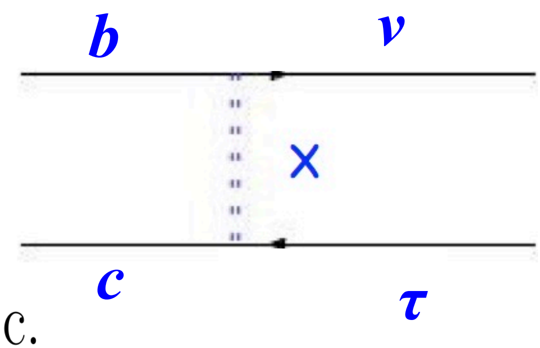
Even with 2σ Constraints, the NP scalar operators are also ruled out



Leptoquark model

Cheung, Huang, Li, Lu, Mao Tang, *NPB* 965 (2021) 115354

- **Lagrangian of Leptoquark**



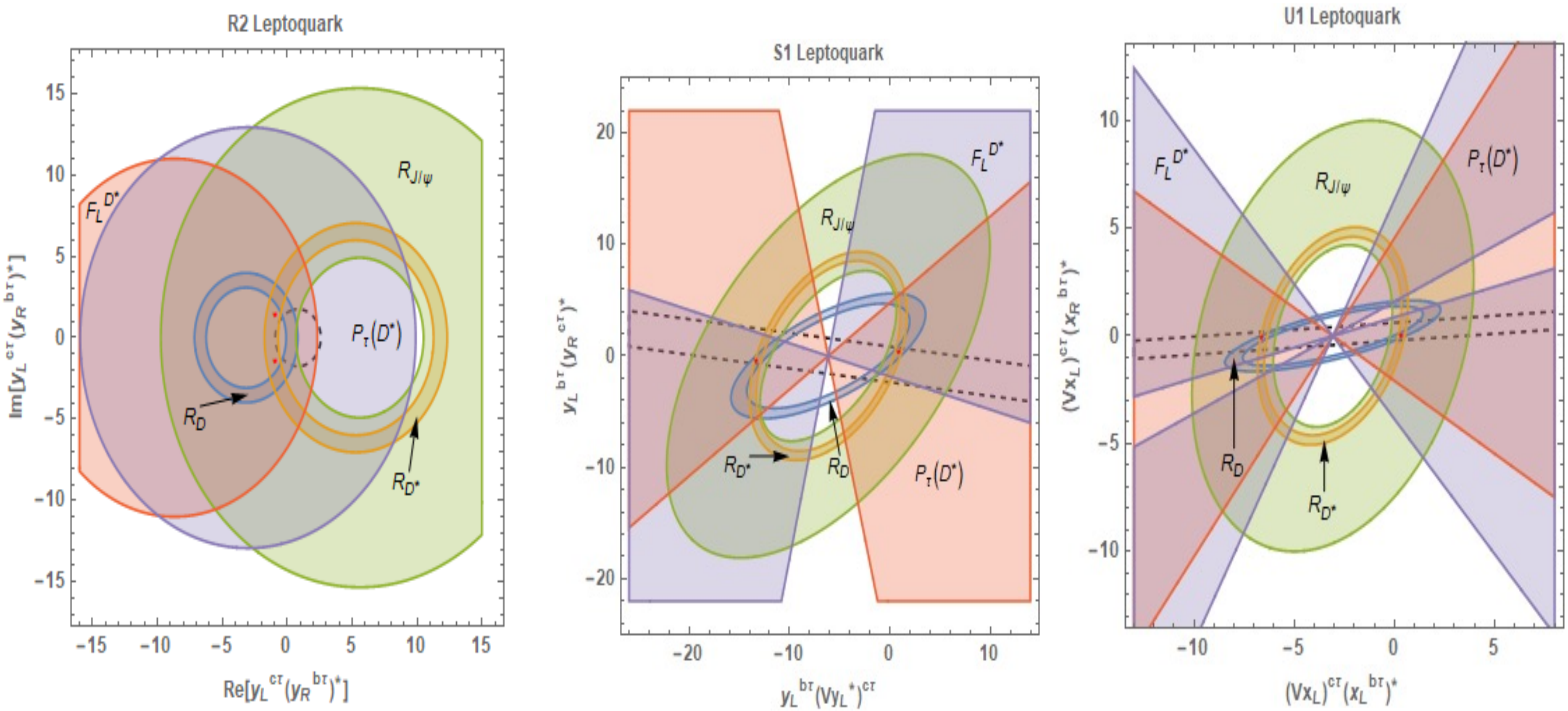
$$\mathcal{L}_{R_2} = \left(y_R^{b\tau} \bar{b}_L \tau_R + y_L^{c\tau} \bar{c}_R \nu_L \right) Y_{2/3} + \mathrm{H.c.}$$

$$\mathcal{L}_{S_1} = \left((V^*_{\mathrm{CKM}} y_L)^{c\tau} \bar{c}_L^c \tau_L - y_L^{b\tau} \bar{b}_L^c \nu_L + y_R^{c\tau} \bar{c}_R^c \tau_R \right) Y_{1/3} + \mathrm{H.c.}$$

$$\mathcal{L}_{U_1} = \left((V_{\mathrm{CKM}} x_L)^{c\tau} \bar{c}_L \gamma_\mu \nu_L + x_L^{b\tau} \bar{b}_L \gamma_\mu \tau_L + x_R^{b\tau} \bar{b}_R \gamma_\mu \tau_R \right) X^\mu_{2/3} + \mathrm{H.c.}$$

	SM quantum number [SU(3) × SU(2) × U(1)]	Spin	Fermions coupled to
R_2	$(3, 2, 7/6)$	0	$\bar{c}_R \nu_L, \bar{b}_L \tau_R$
S_1	$(\bar{3}, 1, 1/3)$	0	$\bar{b}_L^c \nu_L, \bar{c}_L^c \tau_L, \bar{c}_R^c \tau_R$
U_1	$(3, 1, 2/3)$	1	$\bar{c}_L \gamma_\mu \nu_L, \bar{b}_L \gamma_\mu \tau_L, \bar{b}_R \gamma_\mu \tau_R$

2σ Constraints on the Leptoquark couplings



Cheung, Huang, Li, Lu, Mao Tang, *NPB* 965 (2021) 115354

Nothing seen in other meson decay

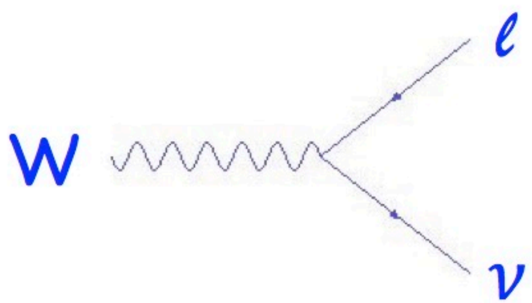
	Exp. (PDB)	SM
$\frac{B(K^+ \to \pi^0 \mu^+ \nu)}{B(K^+ \to \pi^0 e^+ \nu)}$	0.6608±0.0029	0.6631±0.0042 (Cirigliano et al)
$\frac{B(K^+ \to e^+ \nu)}{B(K^+ \to \mu^+ \nu)}$	2.488±0.009(10^{-5})	2.477±0.001 (10^{-5}) (Cirigliano et al)
$\frac{B(\pi^+ \to e^+ \nu(\gamma))}{B(\pi^+ \to \mu^+ \nu(\gamma))}$	1.2327±0.0023(10^{-4})	1.2352±0.0005(10^{-4}) (Marciano, Sirlin)

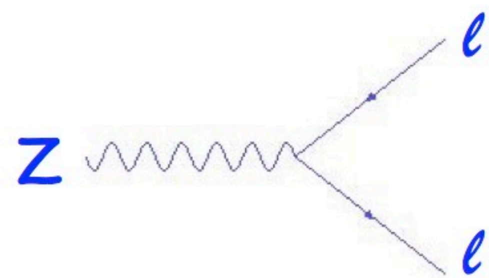
- no simple models
- need to arrange the flavour structure to single out this family: b, τ

Lepton universality

Lepton couplings to gauge bosons in the standard model are all the same

Very well tested, PDG averages:





$$\frac{B(W^+ \to \mu^+\nu)}{B(W^+ \to e^+\nu)} = 0.991 \pm 0.018$$

$$\frac{B(W^+ \to \tau^+\nu)}{B(W^+ \to e^+\nu)} = 1.043 \pm 0.024$$

$$\frac{B(W^+ \to \tau^+\nu)}{B(W^+ \to \mu^+\nu)} = 1.070 \pm 0.026$$

$$\frac{B(Z \to \mu^+\mu^-)}{B(Z \to e^+e^-)} = 1.0009 \pm 0.0028$$

$$\frac{B(Z \to \tau^+\tau^-)}{B(Z \to e^+e^-)} = 1.0019 \pm 0.0032$$

.9977 (SM)

Heavy-to-light form factors

$$F_{i,LP}^{B\to M}(E) = C_i^{(A0)}(E)\zeta_a(E) + \int_0^\infty \frac{d\omega}{\omega}\int_0^1 du\, C_i^{(B1)}(E,u)J_i(E,\omega)\phi_B^+(\omega)\phi_M(u)$$

- QCD/SCET factorization formulae for meson form factor [BBNS, BPRS, and many others].
- QCD/SCET factorization formulae for baryonic form factor [Wang, 2011].

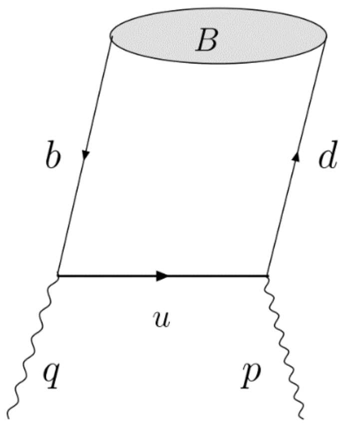
$$F_{LP}^{\Lambda_b\to\Lambda}(E) = \int_0^\infty d\omega_1 d\omega_2 \int_0^1 du\, dv C_i^{(A0)}(E,u,v)J_i(E,\omega_i)\psi_{\Lambda_b}^{(2)}(\omega_1,\omega_2)\psi_\Lambda^{(3)}(u,v)$$

Leading power contribution; but numerically suppressed

Lattice calculation Works well at small recoil region, and needs to be extrapolated to the whole physical region

Light cone sum rule for the heavy-to-light form factors

- LCSR is a QCD inspired method for the large recoil region of the heavy-to-light form factors.



The correlation function

$\langle 0|T\{J_{\text{weak}}(0), J_B(x)\}|\bar{B}(P)\rangle$

- B-meson LCSR (SCET sum rules).

or $\langle \bar{M}(P)|T\{J_{\text{weak}}(0), J_M(x)\}|0\rangle$

- Light meson LCSR

Quark level OPE ⟺ Quark hadron duality ⟺ Hardon level quark hadron duality

- The correlation function can be calculated with standard QCD factorization technique.
  - $\mathrm{B} \to \pi K$ form factors [Cui, Huang, Shen, Wang, Wang, 2022]

LCSR results of heavy-to-light form factors

Model dependence

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

$$f^0_{B\to\pi}(0) = 0.122 \times \Big[1 \pm 0.07\Big|_{S_0^\pi} \pm 0.11\Big|_{\Lambda_q} \pm 0.02\Big|_{\lambda_E^2/\lambda_H^2}{}^{+0.05}_{-0.06}\Big|_{M^2} \pm 0.05\Big|_{2\lambda_E^2+\lambda_H^2}{}^{+0.06}_{-0.10}\Big|_{\mu_h} \pm 0.04\Big|_{\mu}{}^{+1.36}_{-0.56}\Big|_{\lambda_B}{}^{+0.25}_{-0.43}\Big|_{\sigma_1,\sigma_2}\Big]$$

Inverse moment and log moment

Gao, Lu, Shen, Wang, Wei, 2020; Cui, Huang, Shen, Wang, 2023

The largest theoretical uncertainty is from light cone distribution amplitudes (LCDAs) of hadrons.

Limited understanding of the nonperturbative heavy meson LCDAs

Hadron LCDAs are also essential inputs in any of the factorization method of non-leptonic hadron decays

$$\langle \pi(p')\,\pi(q)\,|Q_i|\,\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx\,T_i^{\mathrm{I}}(x)\,\phi_\pi(x) + \int_0^1 d\xi\, dx\, dy\, T_i^{\mathrm{II}}(\xi,x,y)\,\phi_B(\xi)\,\phi_\pi(x)\,\phi_\pi(y)$$

Form factor

Heavy meson LCDA

Hard kernel ⊗ LCDA

Hard kernel: Perturbative

Meson LCDAs: Nonperturbative

Matrix elements = Hard kernel ⊗ Form factor ⊗ LCDAs

The Distribution amplitudes of B meson

The definition of leading twist B meson DA **[Grozin, Nuebert, 1997]**

$$\langle 0|\bar{q}_\beta(z)[z,0]h_{v\alpha}(0)|\bar{B}(v)\rangle = -\frac{i}{4}F_B m_B\left[\frac{1+v\cdot\gamma}{2}\left\{2\tilde{\phi}^+(t,\mu)+\frac{\tilde{\phi}^-(t,\mu)-\tilde{\phi}^+(t,\mu)}{2}z\cdot\gamma\right\}\gamma_5\right]_{\alpha\beta}$$

The Evolution: Lange-Neubert equation **[Lange, Nuebert, 2003]**

$$\mu\frac{d}{d\mu}\phi^+(\omega,\mu) = -\left[\Gamma_{\rm cusp}(\alpha_s)ln\frac{\omega}{\mu}+\gamma_+(\alpha_s)\right]\phi^+(\omega,\mu)-\omega\int_0^\infty d\eta\Gamma_+(\omega,\eta,\alpha_s)\phi^+(\eta,\mu)$$

The evolution equation at two loops: [Braun, Ji, Manashov, 2019]

Solution to the LN equation: dual space: [Bell, Feldmann, Wang, Yip, 2013]

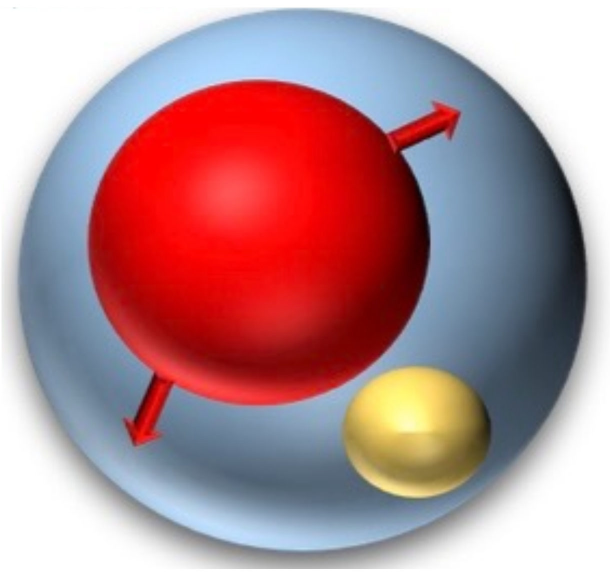
Solution to the evolution equation @ two loop level [Galda, Neubert, 2020]

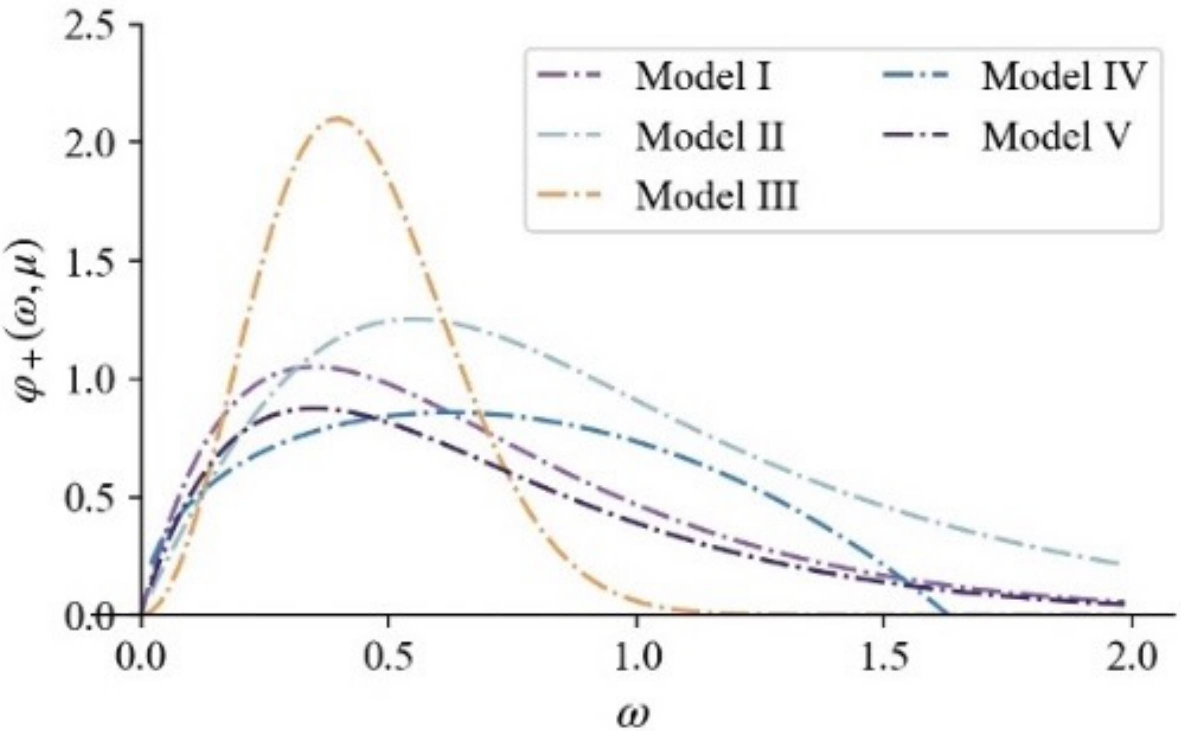
The higher twist DAs of B meson[Braun et al, 2017]

Modelling the B meson LCDA: Expernential[Grozin, Nuebert, 1997], Free parton[KKQT 2001], Local duality[Braun etc. 2003]

Limited understanding of the nonperturbative heavy meson LCDAs

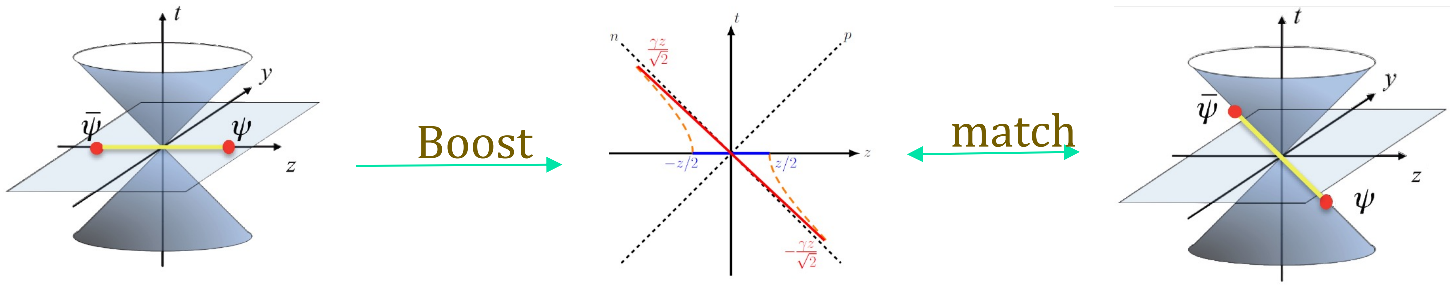
$$\varphi^+(\omega,\mu) = \frac{1}{i\tilde{f}_{H_Q} m_{H_Q} n_+ \cdot v} \int \frac{dt}{2\pi} e^{-i\omega t n_+ \cdot v} \langle 0 | \bar{q}(tn_+) \not{n}_+ \gamma_5 W_c(tn_+, 0) h_v(0) | H_Q(v) \rangle$$





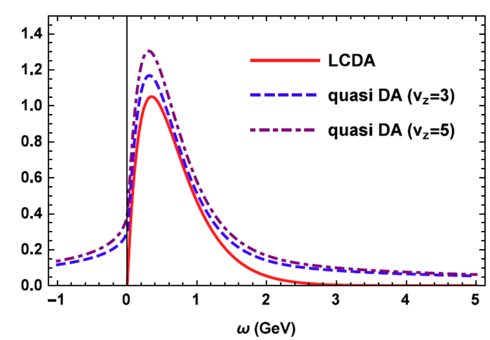
The Quasi-DA of B meson

- The Large momentum effective theory [Ji, 2013]



- The Quasi-DA of leading twist B meson [Wang, Wang, Xu, Zhao 2019]

$$iF_B m_B\, \varphi_B^+(\xi,\mu) = \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} e^{in_z\cdot v\xi\tau} \langle 0|(\bar{q}_s Y_s)(\tau n_z) n_z\cdot\, \gamma\gamma_5 (Y_s^\dagger\, h)_v(0)|\bar{B}(v)\rangle$$



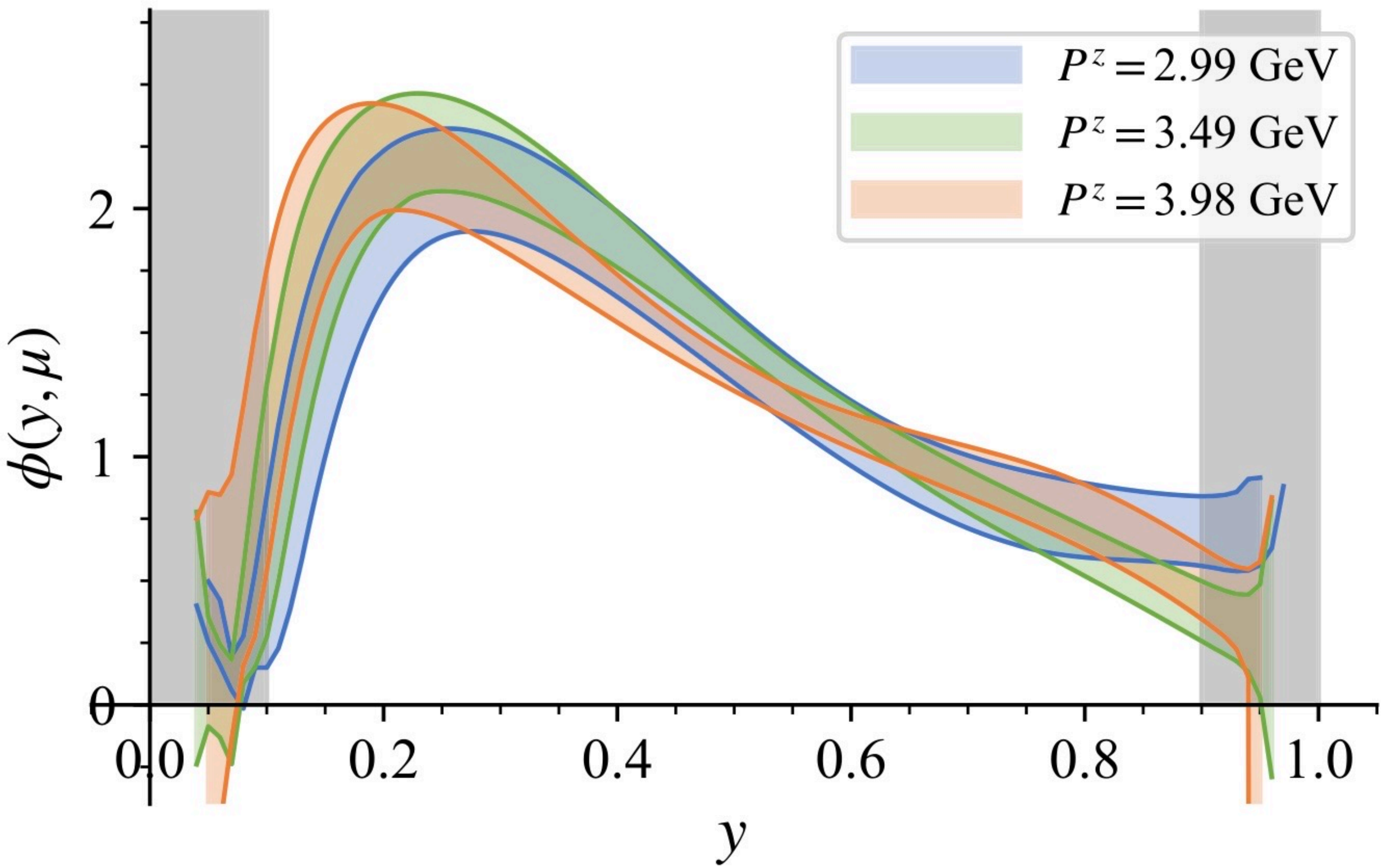
- The **matching**

$$\varphi_B^+(\xi,\mu) = \int_0^{+\infty} d\omega H(\xi,\omega,\mathrm{n}_z\cdot v,\mu)\, \phi_B^+(\omega,\mu) + O\left(\frac{\Lambda}{n_z\cdot v\xi}\right)$$

- The Quasi-DA of subleading twist B meson [Hu, Wang, Xu, Zhao 2023]

- Problem: Calculation of Quasi-DA of B meson on Lattice

QCD LCDA of D meson

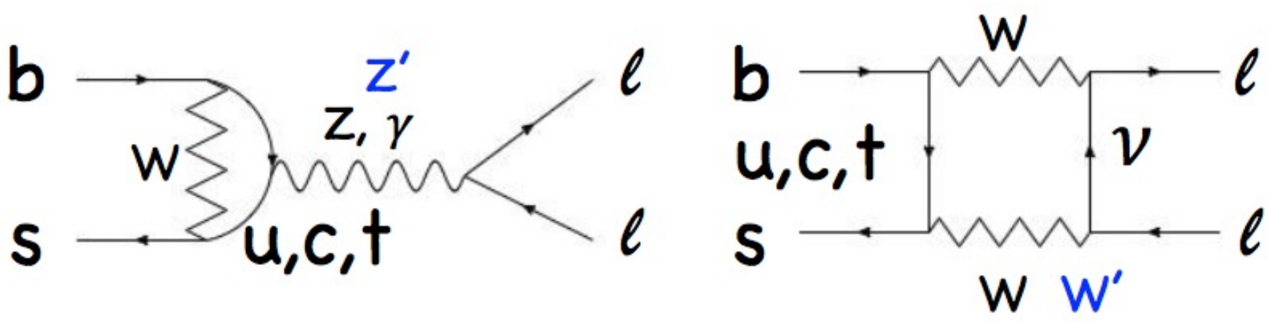


Han et. al, PRD 111, 034503 (2025)

End-point region (grey):
LaMET matching kernel suffer large power corrections

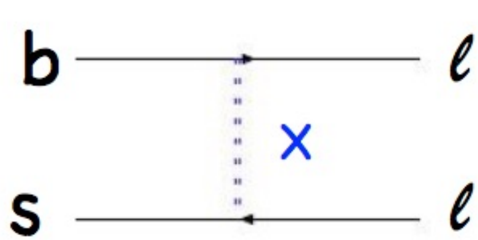
b → s l⁺l⁻

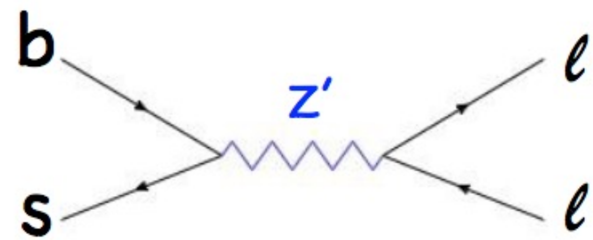
- **Rare decays, sensitive to new physics**



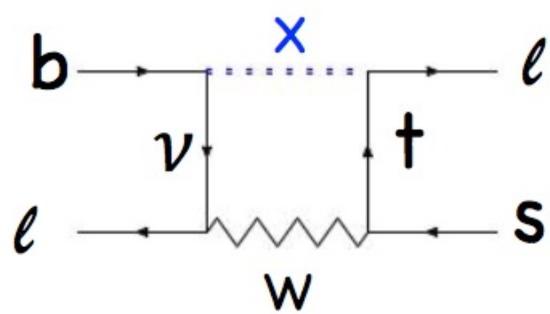
SM

Lepto-quark





supersymmetry



$B \to K(K^*)\ell^+\ell^-$ decays

- **The decay amplitudes**

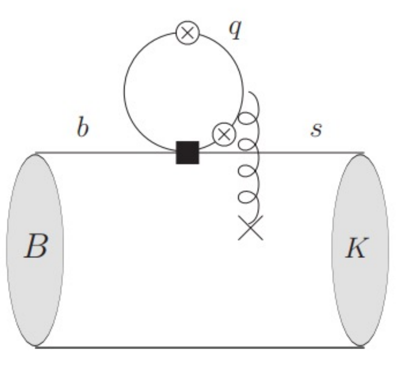
Yuelong Shen' talk

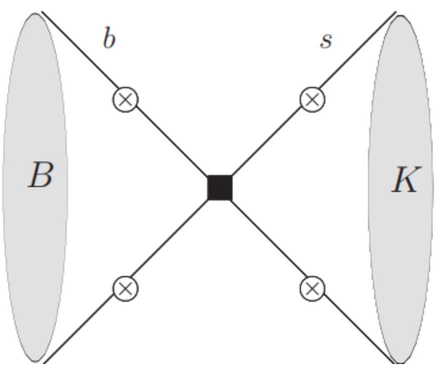
$$\mathcal{A}^{M\ell\ell} \equiv \frac{G_F\,\alpha_e\,V_{tb}V_{ts}^*}{\sqrt{2}\pi}\left\{(C_9\,L_V^\mu + C_{10}\,L_A^\mu)\,\mathcal{F}_\mu^{B\to M} - \frac{L_V^\mu}{q^2}\left[2im_bC_7\,\mathcal{F}_{T,\mu}^{B\to M} + 16\pi^2\mathcal{H}_\mu^{B\to M}\right]\right\}$$

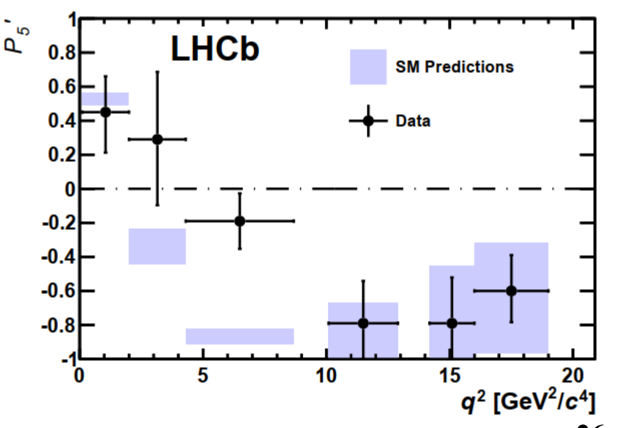
- **The clean observables from angular distribution analysis**

$$P_5' = S_5/\sqrt{F_L(1-F_L)}$$

- **long distance charm loop and annihilation diagrams**







Semi-leptonic decays of Baryon and transition form factors

$\Lambda_b \to \Lambda_c l^- \nu$ decay and $\Lambda_b \to \Lambda_c$ transition form factor

HQET, Bernlochner, Ligeti, Robinson, and Sutcliffe, PRD 99 (2019) 055008

Lattice QCD, Detmold, Lehner, and Meinel, PRD 92 (2015) 034503

Light-cone sum Rules, Duan, Liu, and Huang, EPJC 82 (2022) 951

Perturbative QCD, Li, Chen, Wang and Zou, arXiv:2509.02257

More complicated than mesons, with more degrees of freedom and more important contributions from power corrections

Semi-leptonic decays of Baryon and transition form factors

$\Lambda_b \to \Lambda\, l^+ l^-$ decay and $\Lambda_b \to \Lambda$ transition form factor

Soft-collinear effective theory, Feldmann and Yip, PRD 85 (2012) 014035

Lattice QCD, Detmold and Meinel, PRD 93 (2016) 074501

Light-cone sum Rules, Wang and Shen, JHEP 02 (2016) 179

SCET sum rules: Lu, Lü, Shen and Wei, arXiv:2506.21419

Perturbative QCD, Yang, Han, Chang, Yu, arXiv:2508.18069

Summary

- **High precision theoretical study of semi-leptonic decays are in progress.**
- **Hadron LCDAs study are urgently needed.**
- **Some flavor anomalies have been discussed**
- **We are still waiting for a clear New physics signal in the heavy flavor sector**

Thanks !