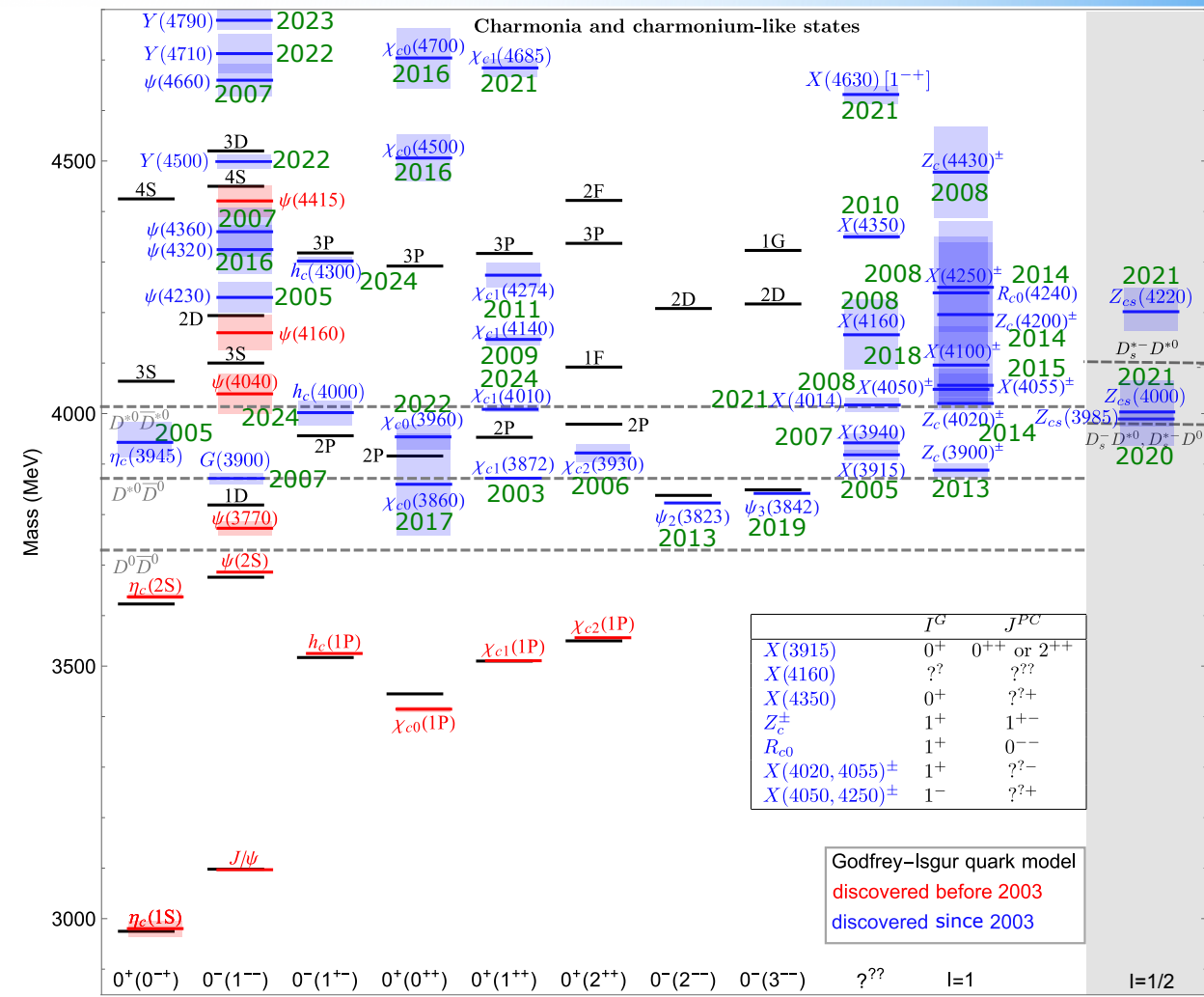


Physics of multiquark states

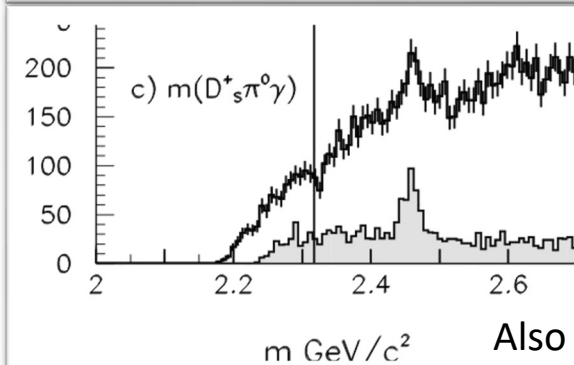
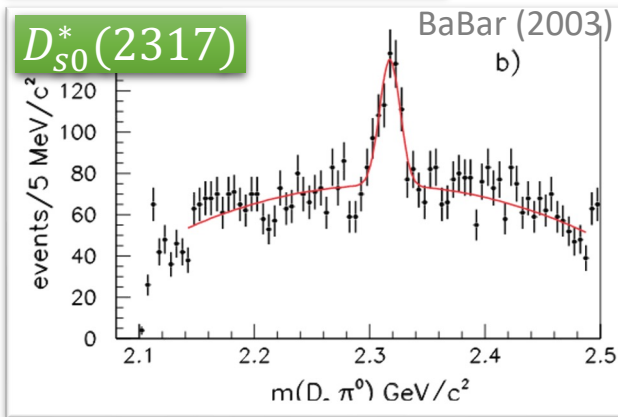
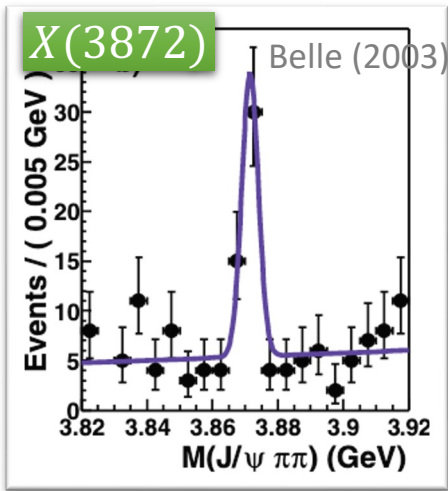
Feng-Kun Guo

Institute of Theoretical Physics,
Chinese Academy of Sciences

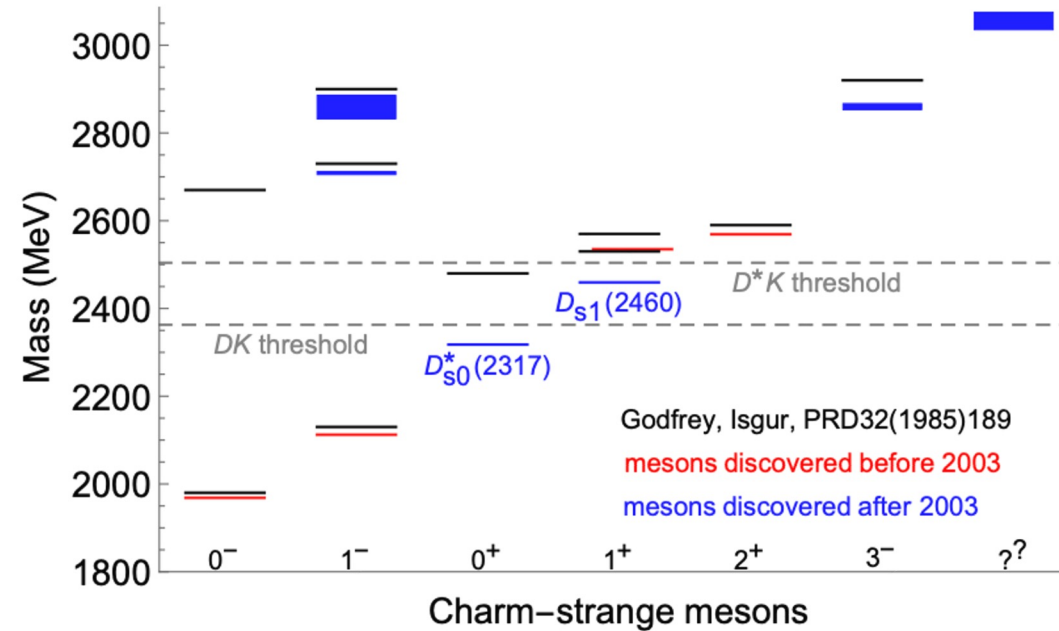
Sept. 18, 2015



Renaissance of hadron spectroscopy in 2003



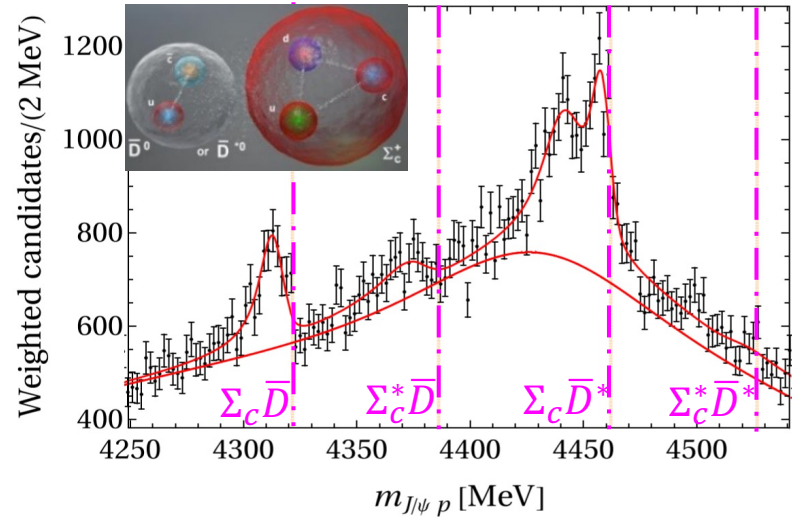
Also signal for $D_{s1}(2460)$ in BaBar (2003)



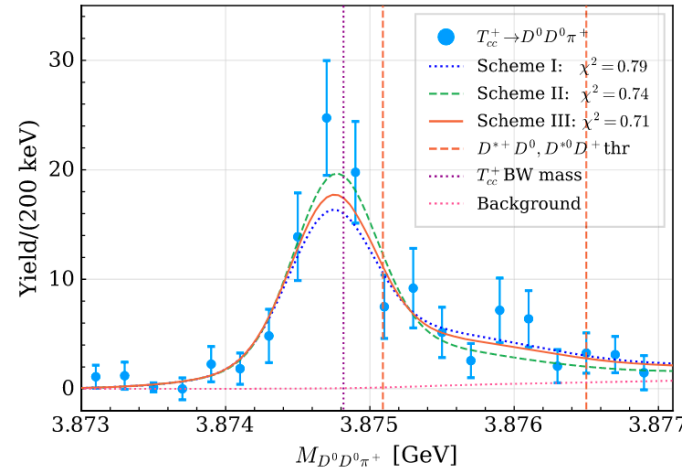
- $D_{s0}^*(2317)$ BaBar (2003)
 $J^P = 0^+$, $\Gamma < 3.8$ MeV
- $D_{s1}(2460)$ CLEO (2003)
 $J^P = 1^+$, $\Gamma < 3.5$ MeV
- no isospin partner observed, tiny widths
 $\Rightarrow I = 0$

- Mass problem: Why are $D_{s0}^*(2317)$ and $D_{s1}(2460)$ so light?
- Naturalness problem: Why $\underbrace{M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)}}_{(141.8 \pm 0.8) \text{ MeV}} \simeq \underbrace{M_{D^{*\pm}} - M_{D^\pm}}_{(140.67 \pm 0.08) \text{ MeV}}?$

Peaks and dips (some LHCb data)



Data: LHCb, PRL122 (2019) 222001;
Fit: M.-L. Du, et al., PRL 124 (2020) 072001



data from LHCb, Nat.Phys.18 (2022) 751;
fit from M.-L. Du et al., PRD 105 (2022) 014024

Neither all peaks correspond to resonances, nor all resonances appear as peaks

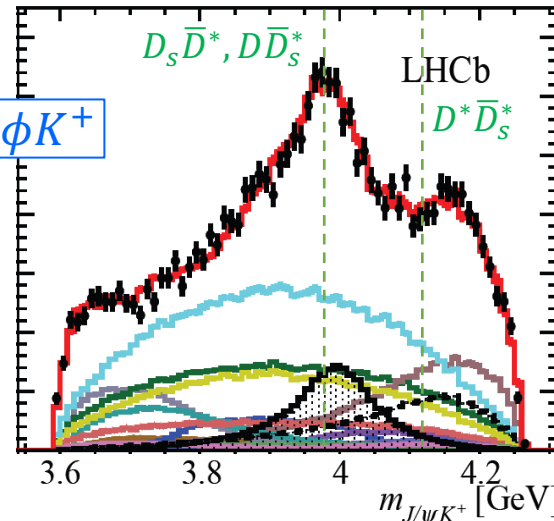
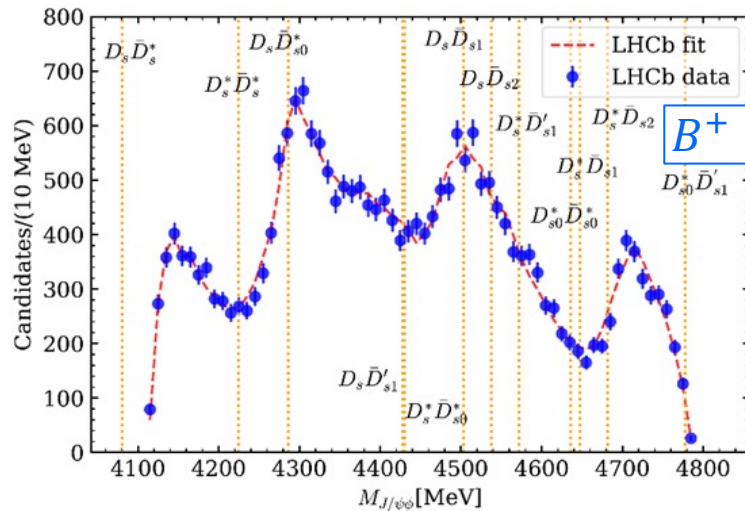
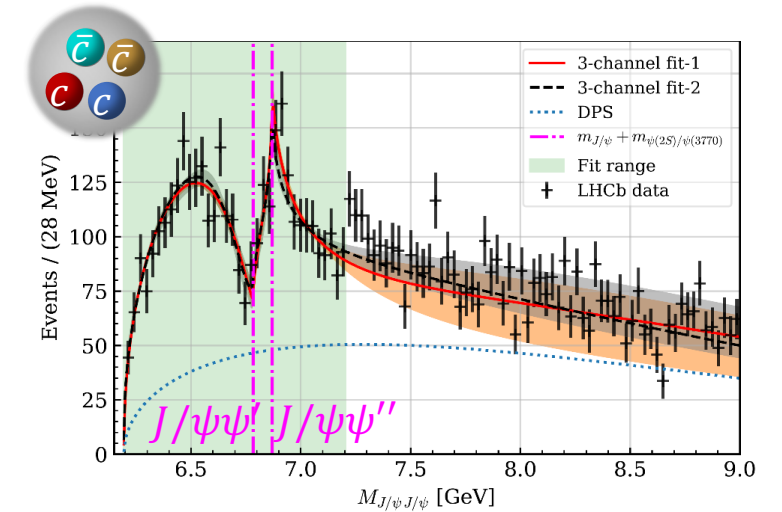


Fig. from X.-K. Dong, FKG, B.-S. Zou, arXiv:2101.01021; data from LHCb, PRL 127 (2021) 082001



data from LHCb, Sci.Bull.65 (2020) 1983;
fit from X.-K. Dong et al., PRL 126 (2021) 132001



- ## Reliable amplitude analysis lays the foundation to answer important questions

- See also talk by Q. Zhao

Multiquark states

- How is energy excited inside a hadron:

- Radial excitations?

- Excitation of light quark-antiquark pairs $\Rightarrow$ compact multiquarks?

- Hadron-hadron pairs? In the form of hadronic molecules

- Implication of **confinement** (large-size systems in favor of color-singlet clusters)?

- More and more molecular candidates have been observed (see below)

- If compact multiquarks exist too, why are the extended molecules so easily produced?

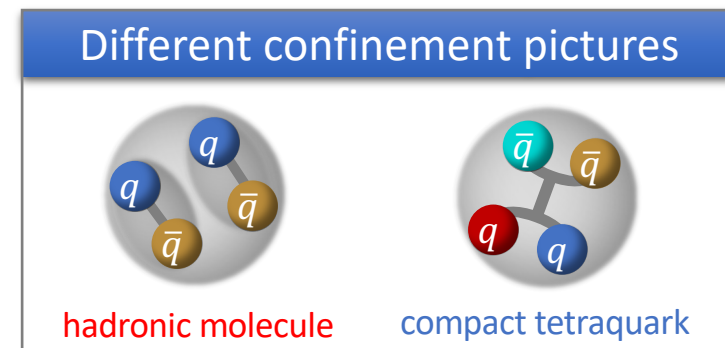
- Crucial quantity for near-threshold states: **compositeness** X , well-defined for S-wave loosely bound state; can be expressed in terms of low-energy observables

S. Weinberg (1965); V. Baru et al. (2004); T. Hyodo et al. (2012); F. Aceti, E. Oset (2012); Z.-H. Guo, J. Oller (2016); I. Matuschek et al. (2021); J. Song et al. (2022); M. Albaladejo, J. Nieves (2022) ; Y. Li, FKG, J.-Y. Pang, J.-J. Wu, PRD 105 (2022) L071502 ...

$$X \approx 1 - \exp \left(\frac{1}{\pi} \int_0^\infty dE \frac{\delta(E)}{E - E_B} \right) \in [0, 1]$$

Scattering phase shift

binding energy



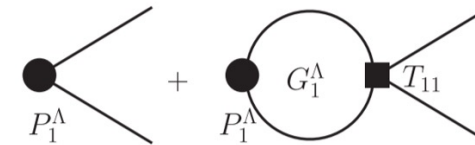
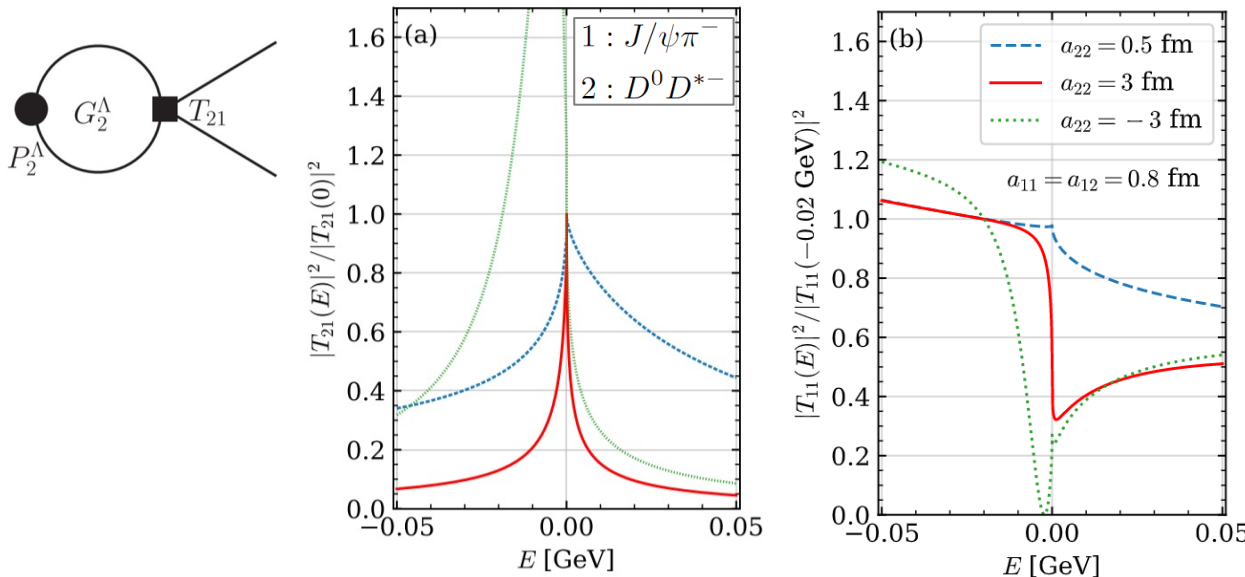
How thresholds affect line shapes

X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

Extension: classification of 2-channel near-threshold structures,
Z.-H. Zhang, FKG, PLB 863 (2025) 139387

- (Near-)threshold structures (S-wave)

- Nonrelativistic EFT at LO: nontrivial (near-)threshold structures for **attractive S-wave interaction**
- Either threshold cusp or below-threshold peak
- Peak more pronounced for heavier hadrons and stronger interaction ($\Rightarrow$ hadronic molecules)
- ✓ That's why many (near-)threshold structures were observed in hidden-charm spectra
- Structures are process (production-mechanism) dependent
- ✓ Universality of a dip of $|T_{11}|$ when the higher channel has a large scattering length

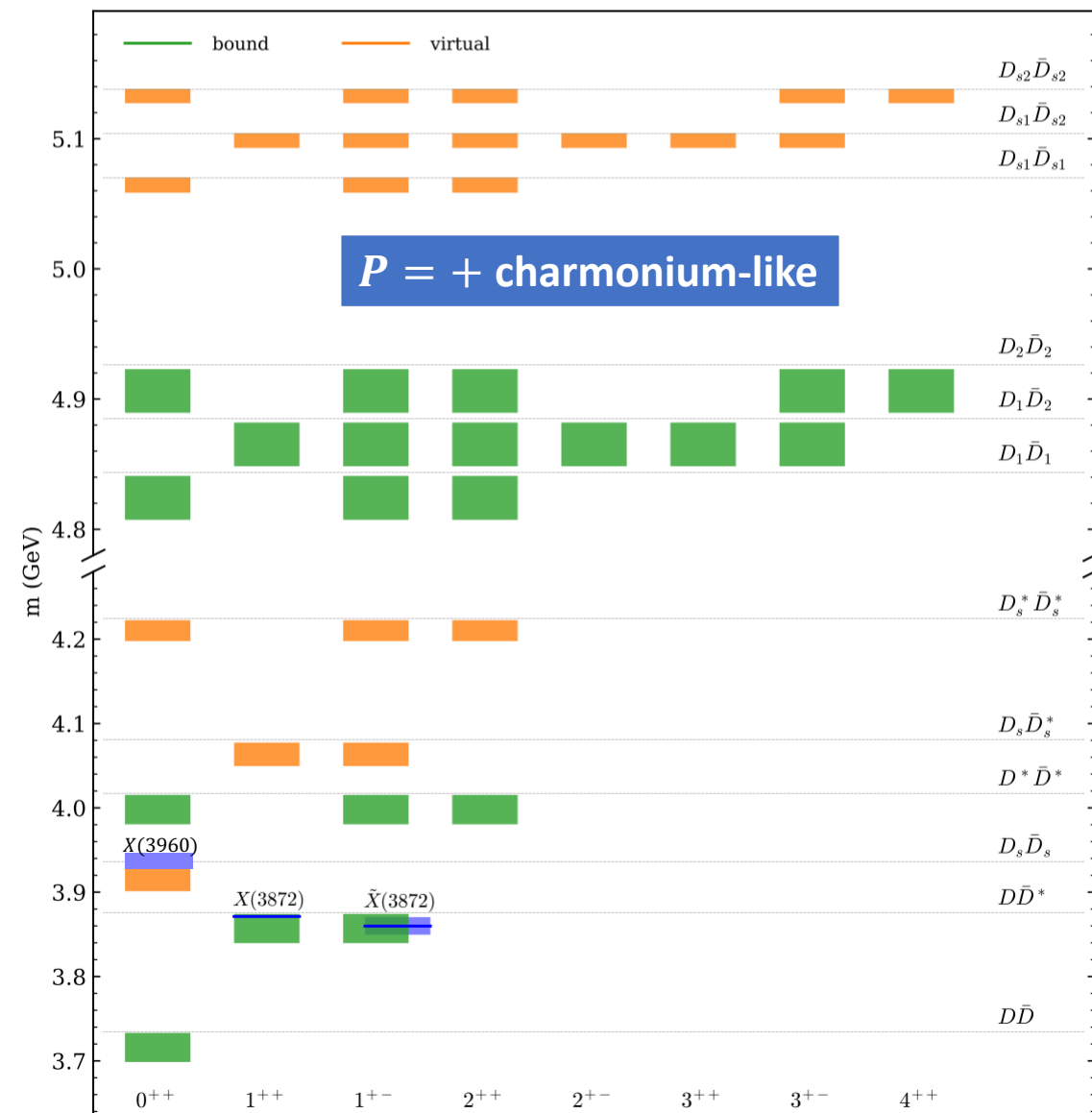


Process-dependent line shapes known since long;
see, e.g., J. Taylor, *Scattering Theory*

Distinct line shapes of amplitudes in the same coupled channels with the same poles

Survey of hadronic molecules

X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65 [arXiv:2101.01021];
Commun. Theor. Phys. 73 (2021) 125201



✓ Approximations: light-vector exchanges, single channel, no mixing

➤ >200 hidden-charm + >100 double-charm molecules

✓ $X(3872)$ as a $\bar{D}D^*$ bound state First predicted in Törnqvist (1993)

Some open questions (debates):

❑ Are radiative decays ($\rightarrow \psi\gamma$) sensitive to the structure?

E. Swanson, PLB 598 (2004) 297;

FKG et al., PLB 742 (2015) 394

❑ What can be learned from its production in heavy-ion collisions?

S. Cho et al., PRL 106 (2011) 212001;

H. Zhang et al., PRL 126 (2021) 012301;

B. Chen et al., PRC 105 (2022) 054901;

E. Braaten et al., PRL 134 (2025) 252301; ...

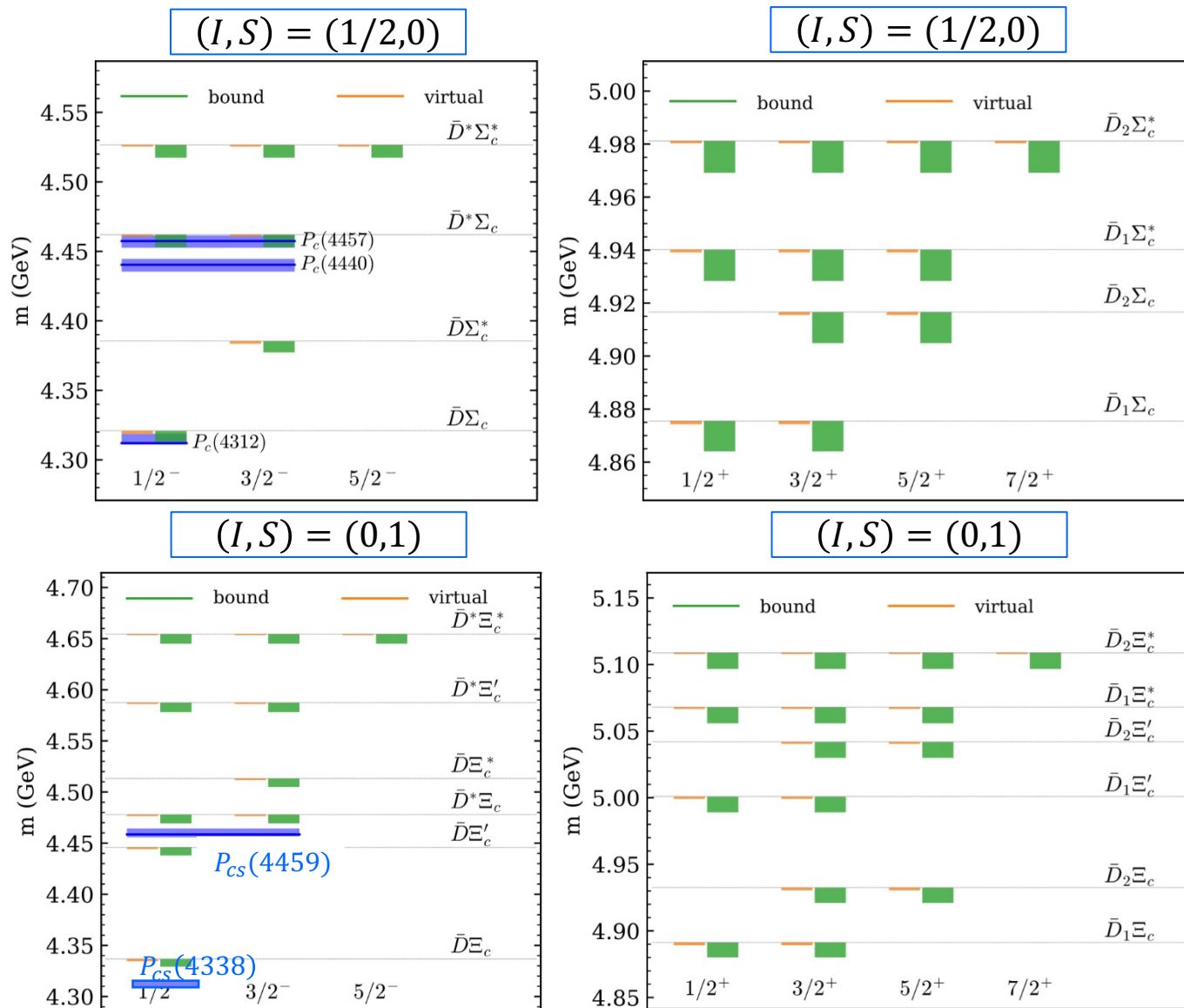
✓ $\bar{D}D$ bound state

Conflicting lattice QCD results, what is the reason?

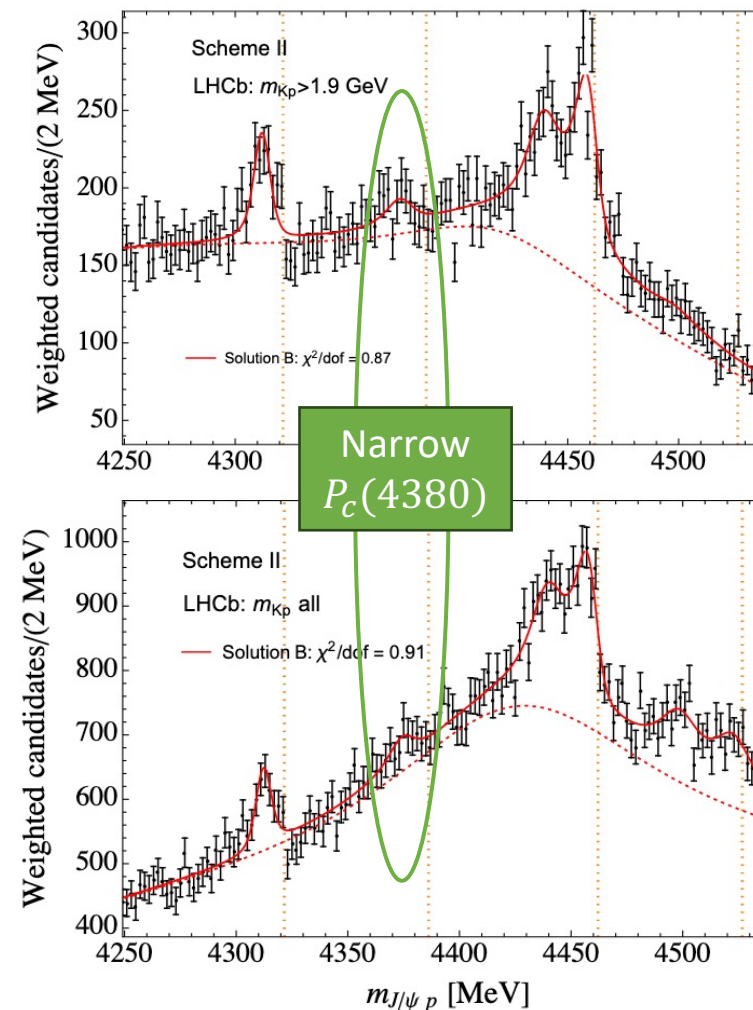
✓ Near-threshold bound state S.Prelovsek et al., JHEP 06 (2021) 035

✗ No near-threshold state D.Wilson et al., PRL 132 (2024) 241901

Survey of hadronic molecules: hidden-charm pentaquarks



✓ The LHCb data can be well described with
a **chiral EFT** M.-L. Du et al., JHEP 08 (2021) 157



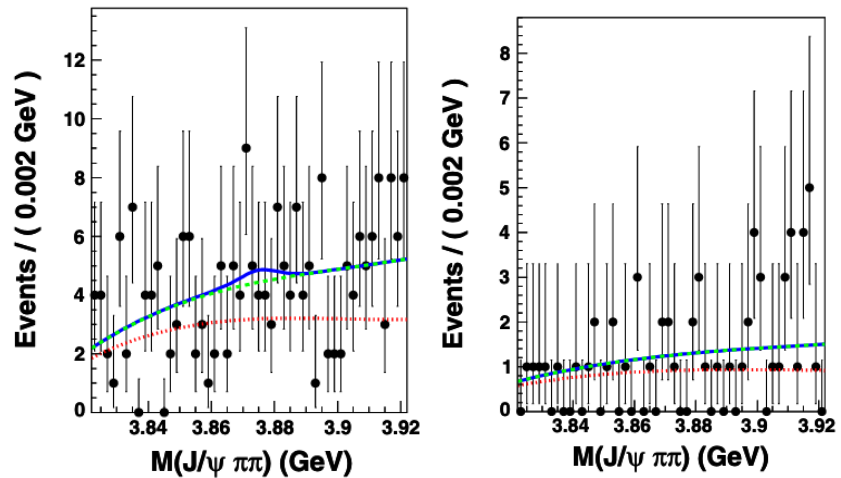
Closer look at $X(3872)$: Isospin-1 partner?

- Isospin-1 partner of $X(3872)$ was predicted in the compact tetraquark model

L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, PRD 71 (2004) 014028

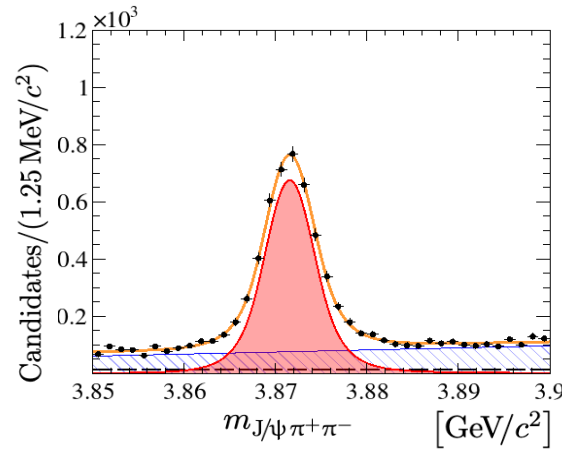
- No signal in the charged channel so far

- No signal around the $D^+ D^{*-}$ threshold



$$\bar{B}^0 \rightarrow K^- \pi^+ \pi^0 J/\psi \quad B^+ \rightarrow K^0 \pi^+ \pi^0 J/\psi$$

Belle, PRD 84 (2011) 052004

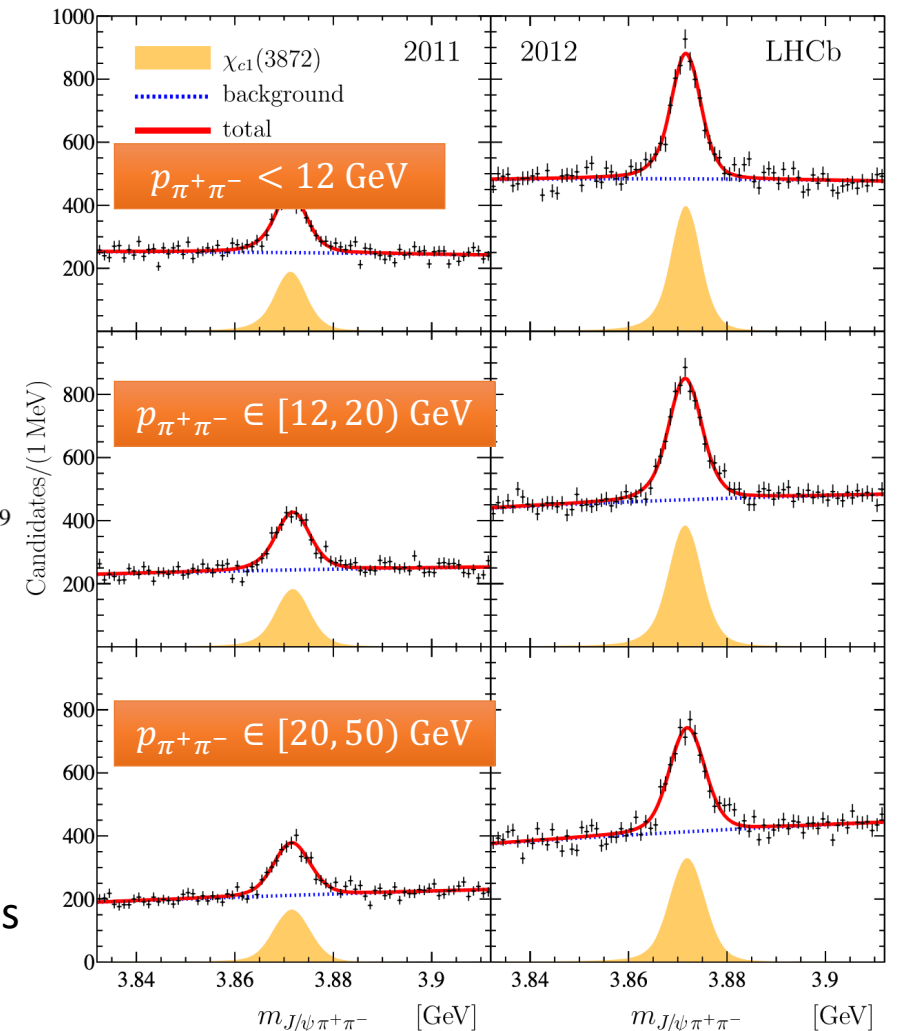


$$B^+ \rightarrow K^+ \pi^+ \pi^- J/\psi$$

LHCb, JHEP 08 (2020) 123

$$\pi^+ \pi^- J/\psi \text{ from } b\text{-hadrons}$$

LHCb, PRD 102 (2020) 092005



Closer look into $X(3872)$

Z.-H. Zhang, T. Ji, X.-K. Dong, FKG et al., JHEP 08 (2025) 130

- Chiral EFT for the $J^{PC} = 1^{++} D\bar{D}^*$ interaction with three-body effects. Two low-energy constants at LO
- Two inputs from $X(3872)$ properties :

➤ Mass

$$M_X = 3871.69_{-0.04-0.13}^{+0.00+0.05} \text{ MeV} \quad \text{LHCb, PRD 102 (2020) 092005}$$

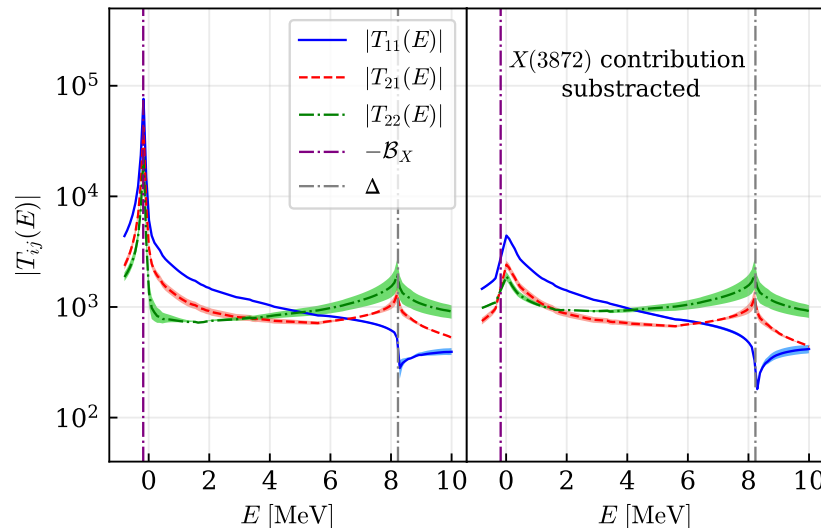
$$M_{D^0} + M_{D^{*0}} = 3871.69(7) \text{ MeV} \quad \text{PDG 2024}$$

➤ Isospin breaking in decays

LHCb, PRD 108 (2023) L011103

$$R_X = \left| \frac{\mathcal{M}_{X(3872) \rightarrow J/\psi \rho^0}}{\mathcal{M}_{X(3872) \rightarrow J/\psi \omega}} \right| = 0.29 \pm 0.04$$

- Prediction: there must exist an isovector $J^{PC} = 1^{++}$ state ($W_{c1}^{0,\pm}$)



➤ Virtual state

(like the $^1S_0 NN$)

➤ Threshold cusps

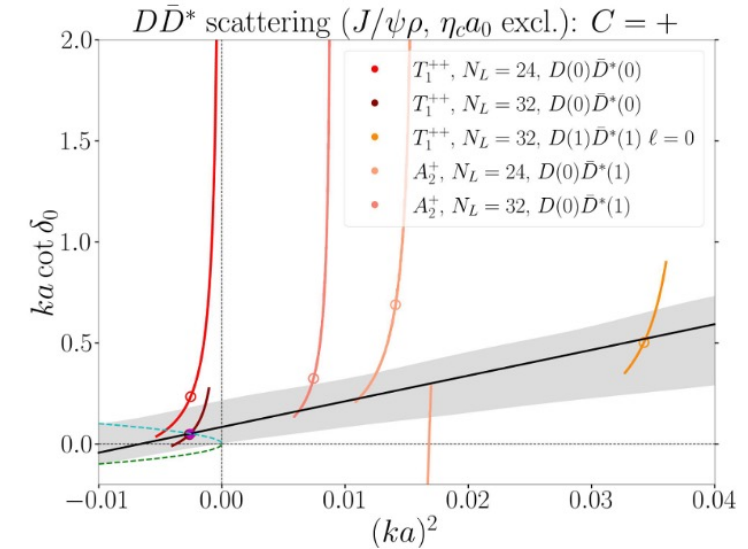
➤ $W_{c1}^{\pm}$:

-8_{-5}^{+8} MeV from

$D^0 D^{*-}$ threshold

✓ Support from lattice QCD

M. Sadl et al., PRD 111 (2025) 054513



J^{PC}	interpolators	$1/a_0$ [fm $^{-1}$]	r_0 [fm]	χ^2/N_{dof}	Δm_V [MeV]
1^{+-}	all	$0.46_{-0.45}^{+1.16}$	$0.96_{-0.73}^{+0.43}$	0.13	$-3.0_{-31.1}^{+3.0}$
	$\eta_c \rho$ excl.	$0.54_{-0.44}^{+1.07}$	$2.23_{-1.08}^{+0.95}$	0.24	$-2.8_{-17.1}^{+2.6}$
1^{++}	all	$0.62_{-0.51}^{+1.30}$	$1.78_{-2.44}^{+0.25}$	0.18	$-3.8_a^{+3.6}$
	$J/\psi \rho, \eta_c a_0$ excl.	$0.96_{-0.91}^{+1.42}$	$2.19_{-1.00}^{+0.36}$	0.15	$-6.7_{-19.5}^{+6.7}$

^a Uncertainty is so large that it is unbounded from below.

✓ Also in one-boson exchange model

X.-X. Chen, Z.-M. Ding, J. He, PRD 111 (2025) 114008

Combined analysis of BESIII + LHCb data w/ chiral EFT

Teng Ji, X.-K. Dong, FKG, C. Hanhart, U.-G. Meißner, arXiv:2502.04458

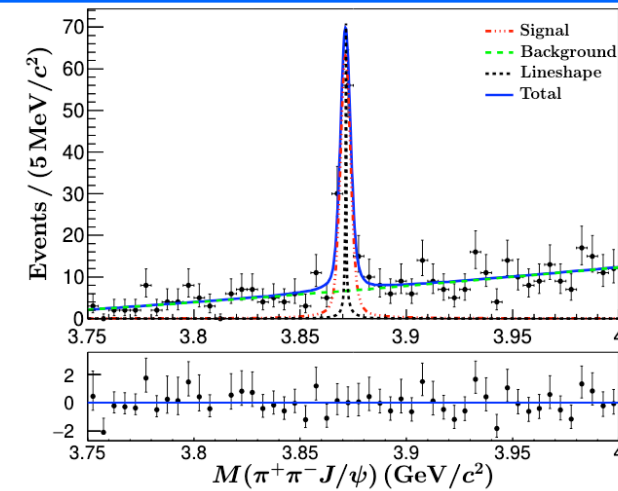
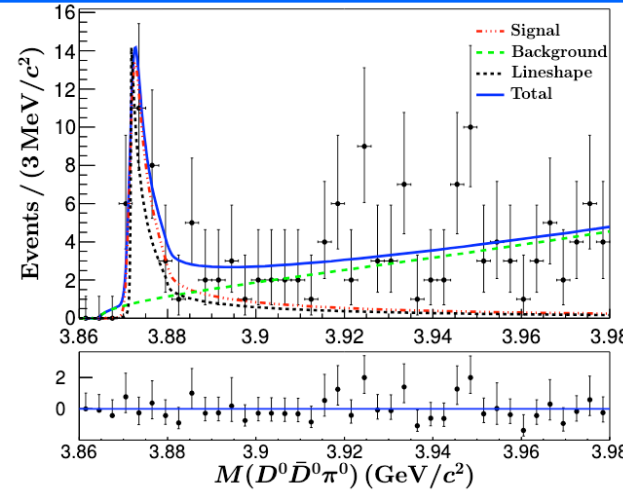
- $X(3872)$ line shapes $\Rightarrow X(3872)$ + possible $W_{c1}(3880)^0$
- $\pi^+\pi^-$ invariant mass distribution $\Rightarrow$ isospin breaking, information on $I = 1$

BESIII:

$$e^+e^- \rightarrow \gamma[D^0\bar{D}^0\pi^0]$$

$$e^+e^- \rightarrow \gamma[J/\psi\pi^+\pi^-]$$

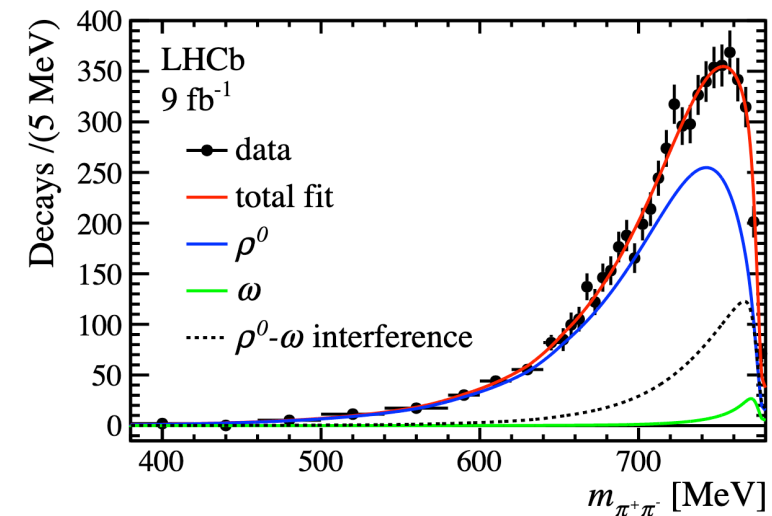
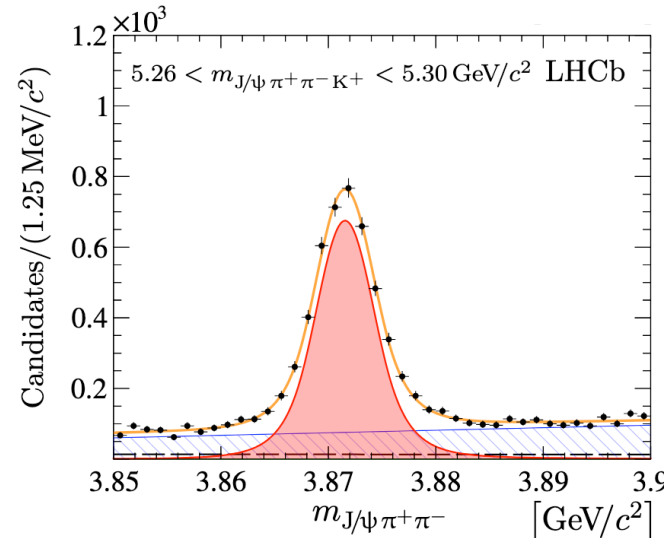
BESIII, PRL 132 (2024) 151903



LHCb:

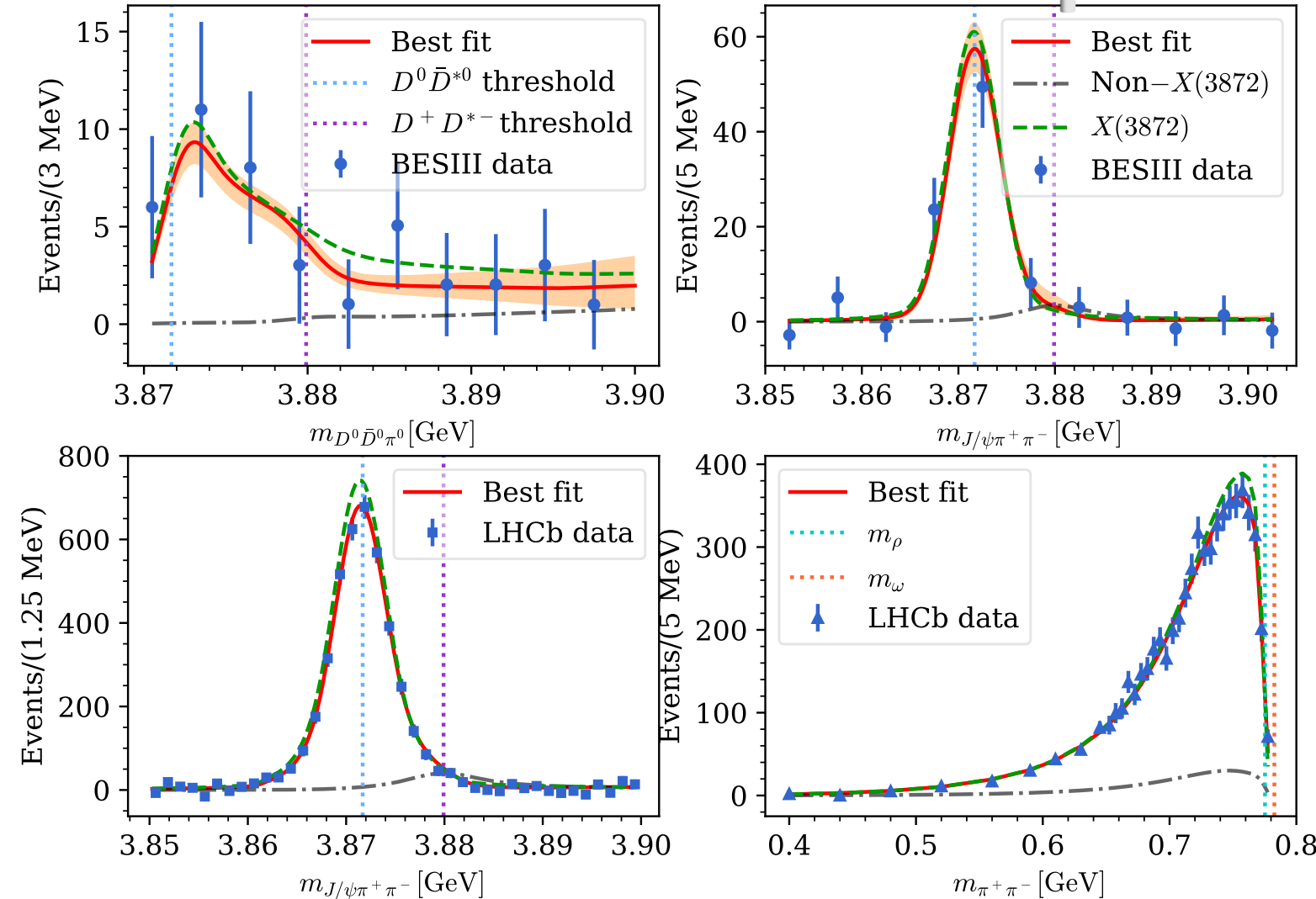
$$B^+ \rightarrow K^+[J/\psi\pi^+\pi^-]$$

LHCb, JHEP 08 (2020) 123;
PRD 108 (2023) L011103



Combined analysis of BESIII + LHCb data w/ chiral EFT

Results updated



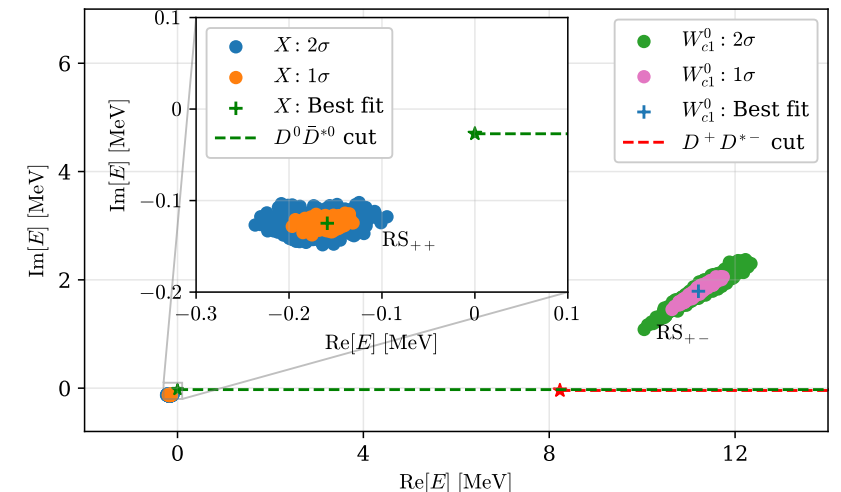
● Poles

□ $X(3872)$ as a bound state below $D^0 \bar{D}^{*0}$ threshold (2.8σ)

$$E_X = (-160_{-47-57}^{+43+38} - 125_{-25-28}^{+18+15}i) \text{ keV}$$

□ $W_{c1}(3880)^0$ pole on RS_{+-} , relative to the $D^+ D^{*-}$ threshold:

$$E_W = (3.1 \pm 0.7 + 1.3_{-0.6}^{+1.9}i) \text{ MeV}$$

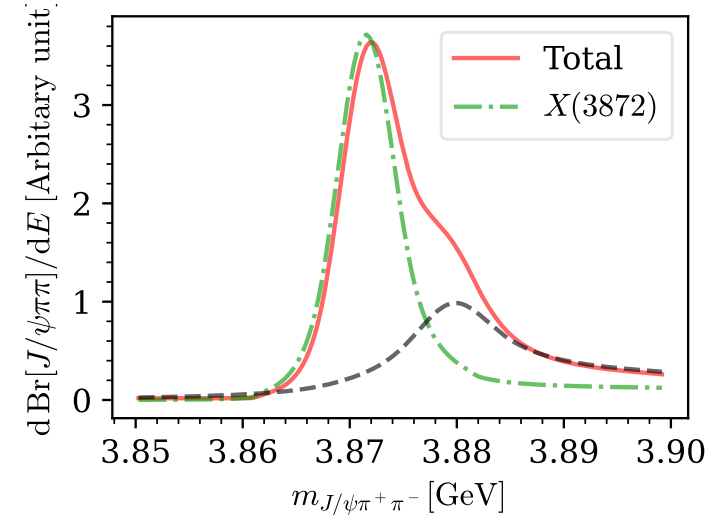
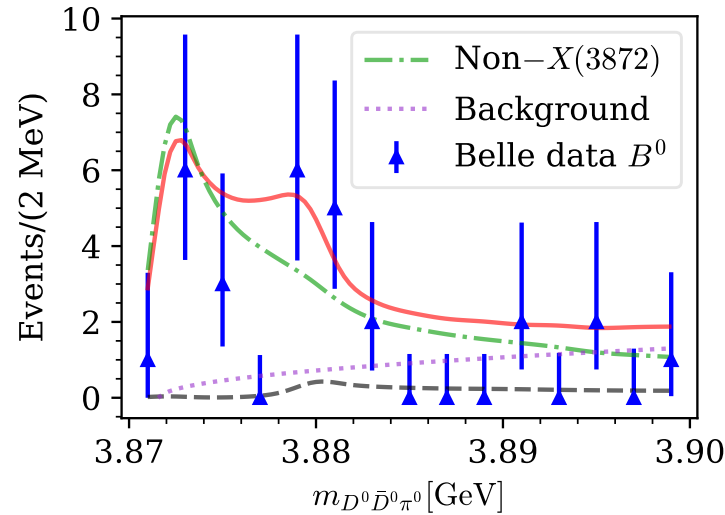
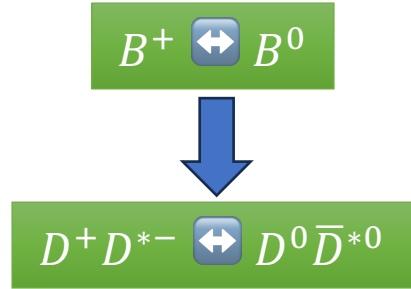


$$\chi^2/\text{dof} = 57/(96 - 10) = 0.66 \quad X = 1 - \exp\left(\frac{1}{\pi} \int_0^\infty dE \frac{\text{Re} \delta(E)}{E - \text{Re} E_X}\right) = 0.97(2)$$

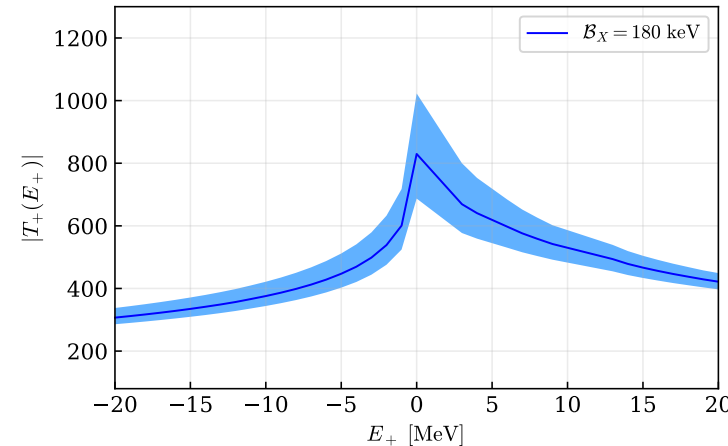
X is predominantly molecular!

Implications of the existence of $W_{c1}(3880)$

- $W_{c1}(3880)^0$ signal should be stronger in $B^0 \rightarrow K^0[D^0\bar{D}^0\pi^0, J/\psi\pi^+\pi^-]$ decays, to be checked @ LHCb, Belle II



- Cusp at $D^+\bar{D}^{*0}/D^{*+}\bar{D}^0$ threshold in $J/\psi\pi^\pm\pi^0$

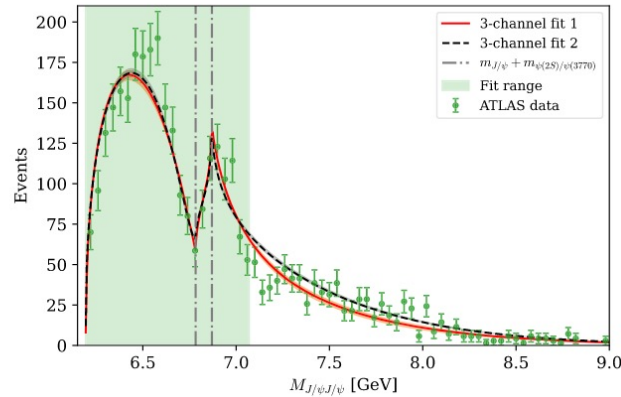
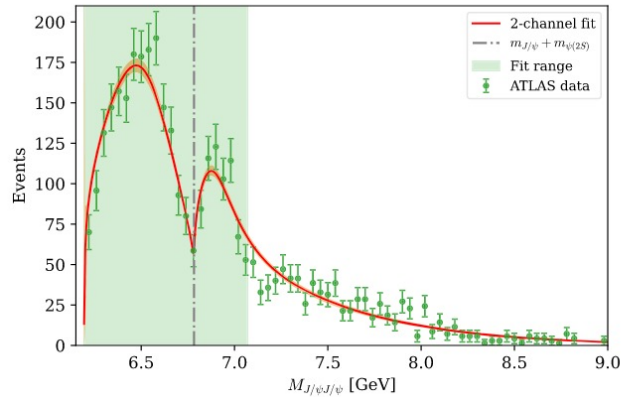
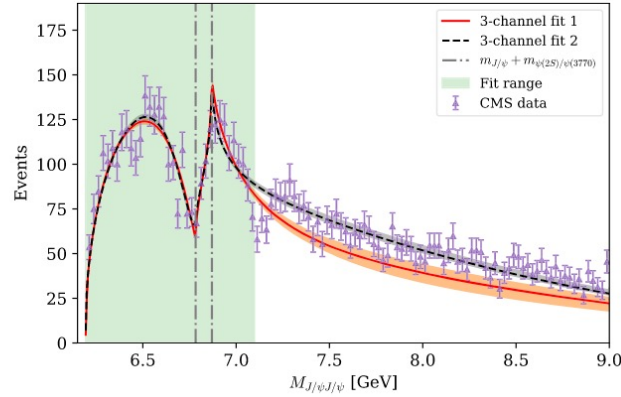
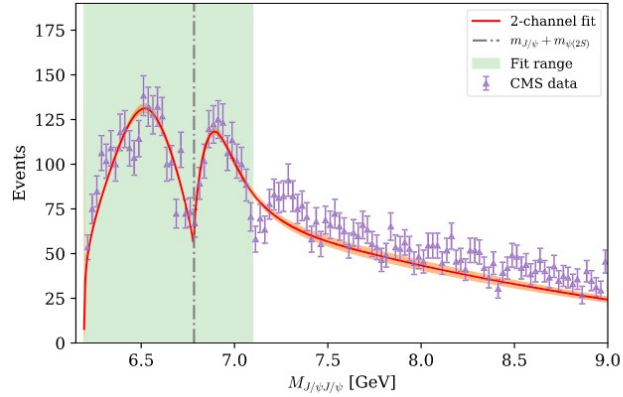
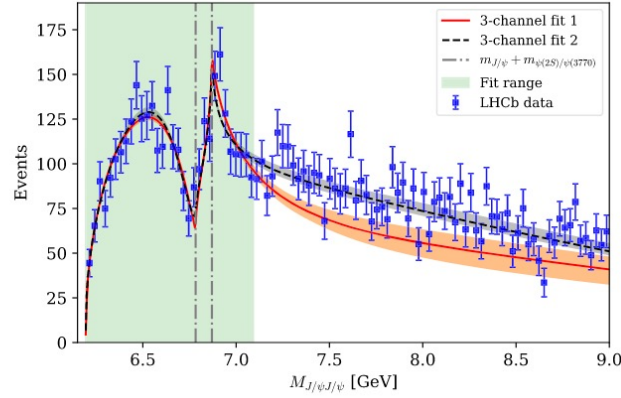
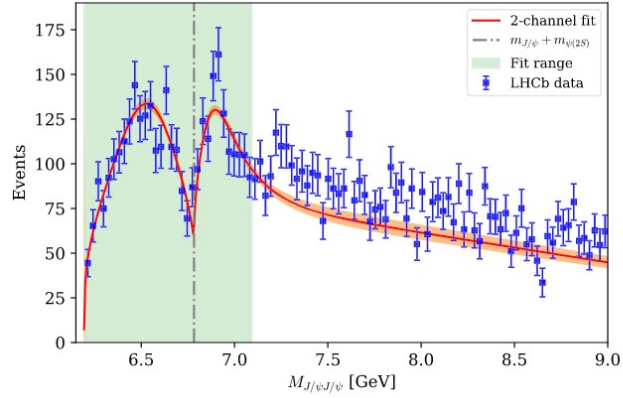


Z.-H. Zhang et al., JHEP 08 (2024) 130

- Compact tetraquarks (Maiani et al. (2005)) cannot be virtual states as they do not feel the thresholds

Structures in double- J/ψ distributions

Y.-L. Song et al., PRD 111 (2025) 034038 [updating analysis from X.-K. Dong et al., PRL 126 (2021) 132001]

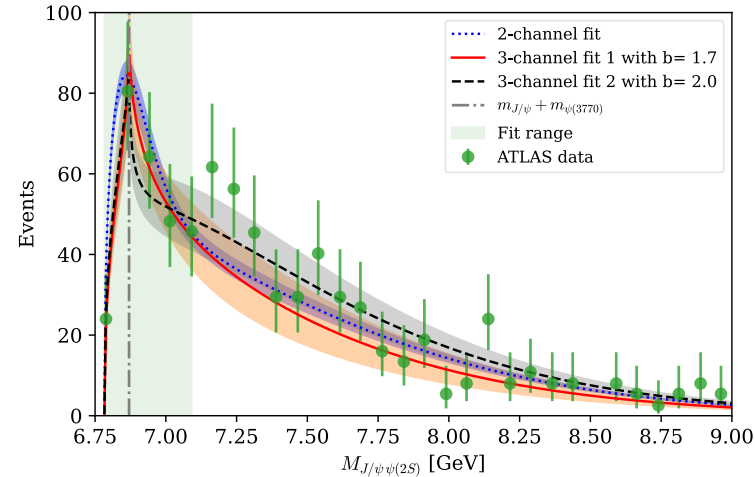


● Threshold effects could be important

● Two minimal coupled-channel models:

$$\square J/\psi J/\psi - J/\psi \psi'$$

$$\square J/\psi J/\psi - J/\psi \psi' - J/\psi \psi''$$



Data:

LHCb, Sci.Bull. 65 (2020) 1983;

CMS, PRL 132 (2024) 111901;

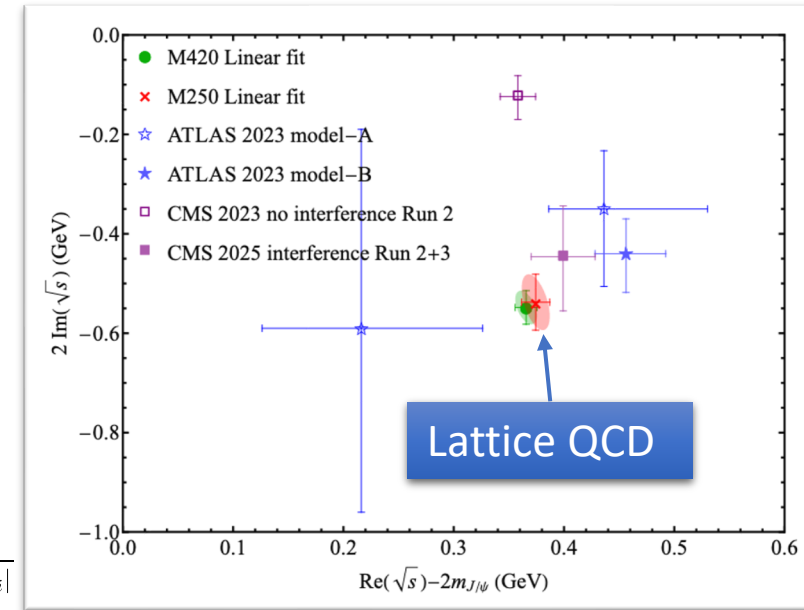
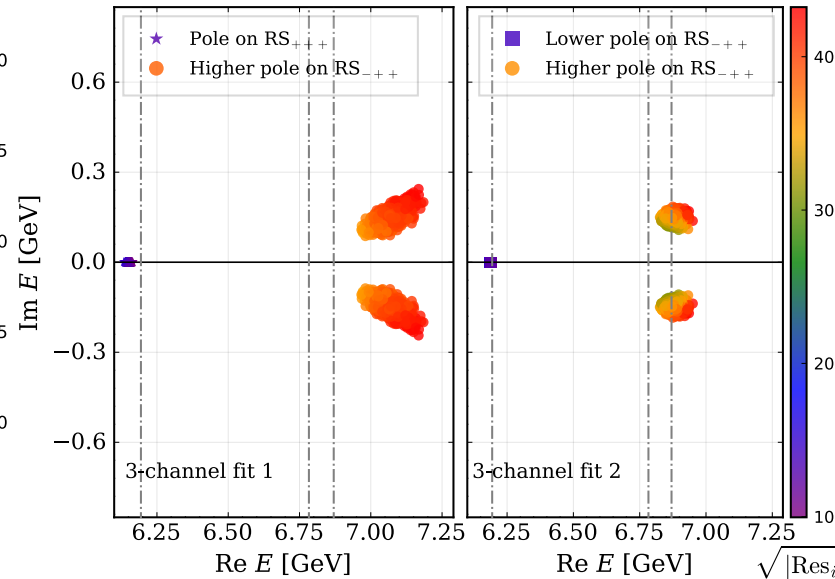
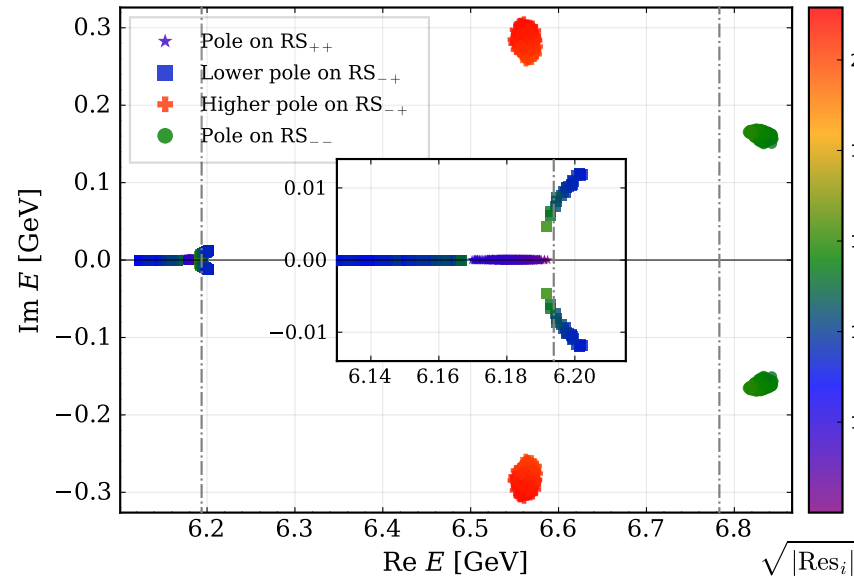
ATLAS, PRL 131 (2023) 151902

Structures in double- J/ψ distributions

- Pole content (resonances) depend on the amplitude model

Should go beyond summing BWs in amplitude analysis!

- A pole near 6.2 GeV
- A pole near 6.9 GeV (location depends on model)
- The one near 6.6 GeV exists only in the 2-channel model



- Lattice QCD results with $M_\pi \approx 250, 420$ MeV

G. Li, C. Shi, Y. Chen, W. Sun, arXiv:2505.23220; arXiv: 2505.24213

- A 2^{++} resonance around 6.54 GeV
- A 0^{++} double- J/ψ virtual state 30-40 MeV below threshold

Summary

- Multiquark candidates offer the opportunity to gain deep insights into confinement
- General rules for (near-)threshold structures
 - S-wave attraction, more prominent for heavier particles and stronger attraction
 - Pole behavior: distinct line shapes depending on reaction mechanism
 - Universality for the presence of a dip in certain line shapes
- Large number of hadronic molecular candidates
- Amplitude analyses with coupled-channel formalisms
 - ❑ Chiral EFT: prediction of an isovector partner of $X(3872)$: $W_{c1}^{\pm,0}$ as virtual states
 - ❑ Model: double- J/ψ distributions, full-charm tetraquarks

Thank you for your attention!

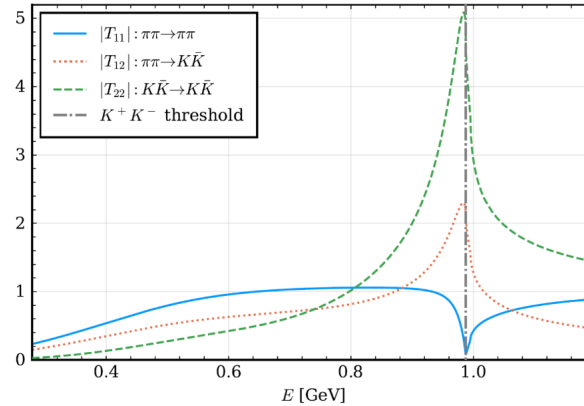
Distinct line shapes of the same pole

● Example-1: $f_0(980)$

□ T -matrix for $\pi\pi$ and $K\bar{K}$

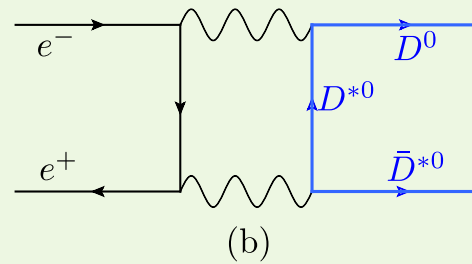
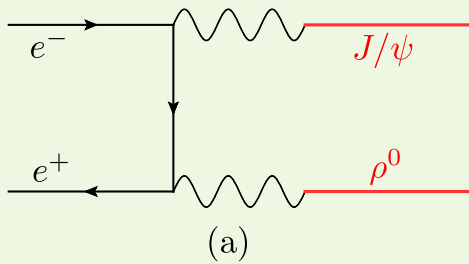
coupled channels

with the T-matrix from
L.-Y. Dai, M. R. Pennington,
PRD 90 (2014) 036004



● Example-2: direct production of $X(3872)$ in e^+e^-

V. Baru, FKG, C. Hanhart, A. Nefediev, PRD 109 (2024) L111501

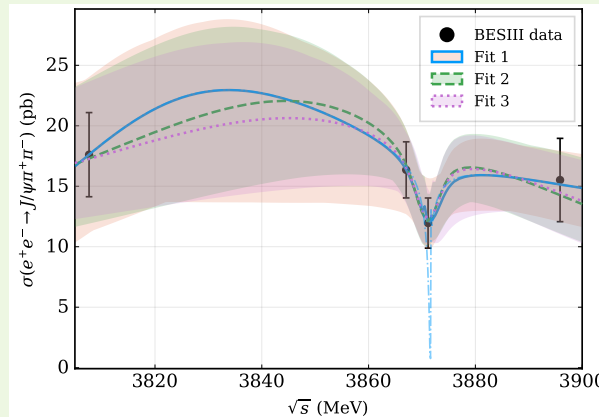


➤ Driving channel:

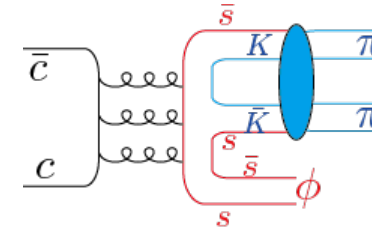
J/ψ + light vector

➤ Prediction: dip around

$D^*\bar{D}^*$ threshold

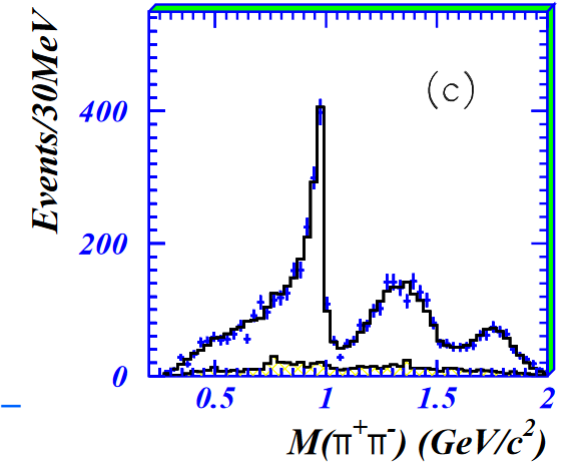


□ $J/\psi \rightarrow \phi \pi^+ \pi^-$



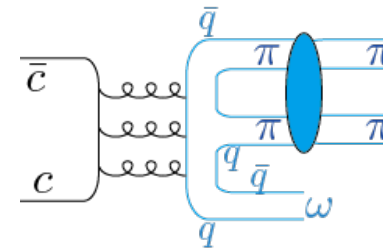
Driving channel: $K\bar{K}$

$J/\psi \rightarrow \phi K\bar{K} \rightarrow \phi \pi^+ \pi^-$



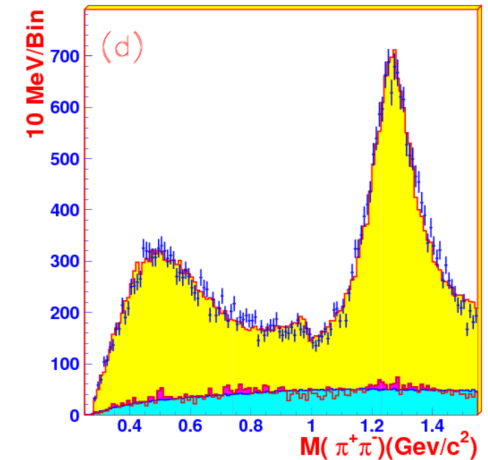
BES, PLB 607 (2005) 243

□ $J/\psi \rightarrow \omega \pi^+ \pi^-$



Driving channel: $\pi\pi$

$J/\psi \rightarrow \omega \pi\pi \rightarrow \omega \pi^+ \pi^-$



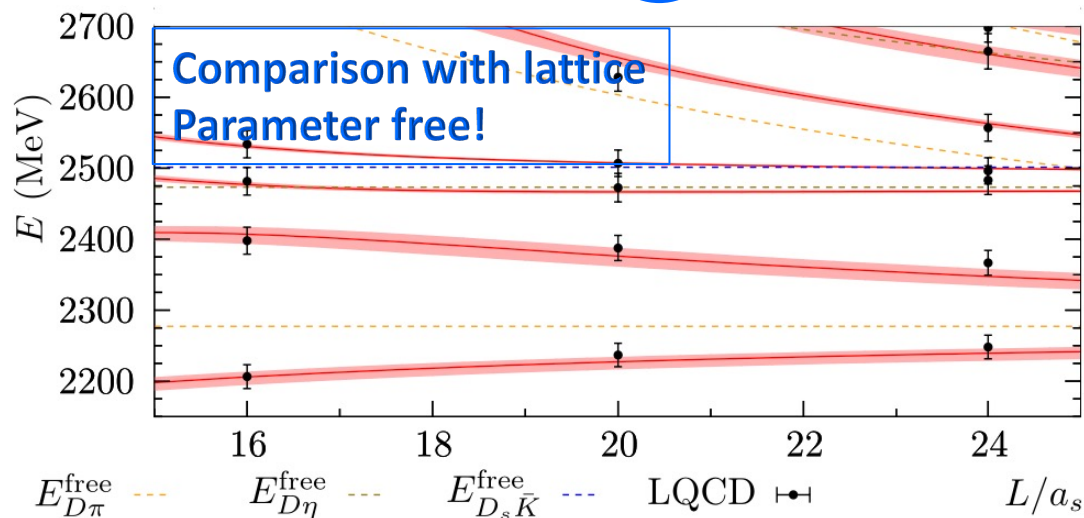
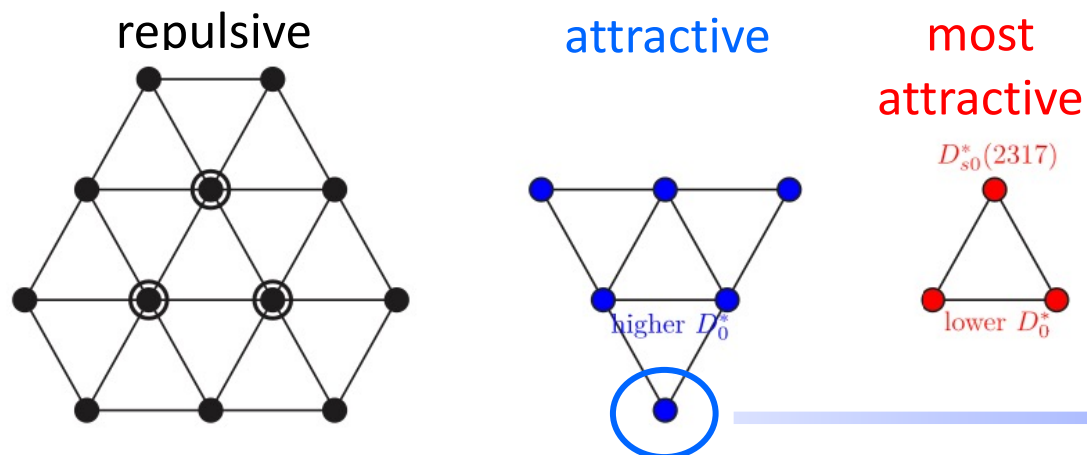
BES, PLB 598 (2004) 149

Charm-strange mesons from chiral EFT and lattice QCD

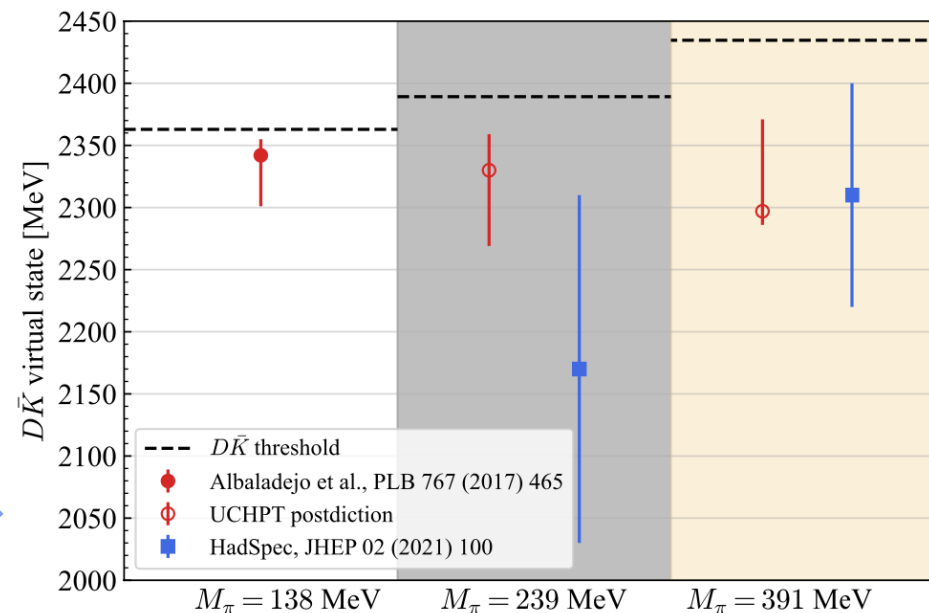
- **SU(3) structures differ from quark model!**

More exotic states: $\bar{3} \otimes 8 = \bar{15} \oplus 6 \oplus \bar{3}$

M.Albaladejo, P. Fernandez-Soler, FKG, J. Nieves, PLB 767 (2017) 465



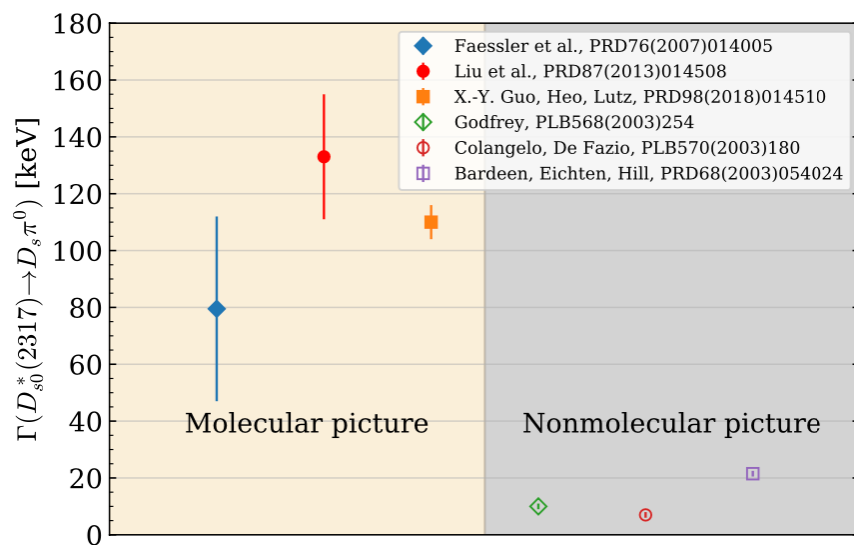
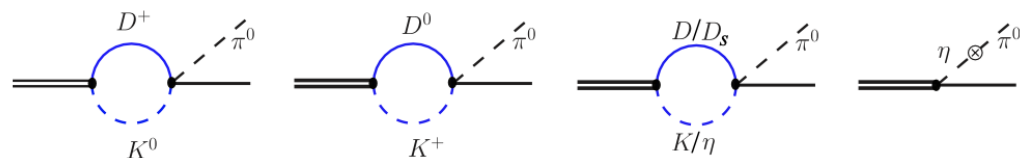
- Prediction of $I = 0, D\bar{K}$ virtual state confirmed by lattice QCD



- More supports from lattice & exp. not shown
M.-L. Du, FKG, Hanhart, Kubis, Meißner, PRL 126 (2021) 192001; ...
- Solutions to the two problems:
 - DK and D^*K molecular states
 - Consequence of heavy quark spin symmetry

Charm-strange mesons: smoking guns of molecular structure

- $D_{s0}^*(2317), D_{s1}(2460)$: total width ~ 100 keV



- BESIII measured $\text{Br}(D_{s0}^* \rightarrow D_s \pi^0) \approx 100\%$
width still not measured!

BESIII, PRL 97 (2018) 051103

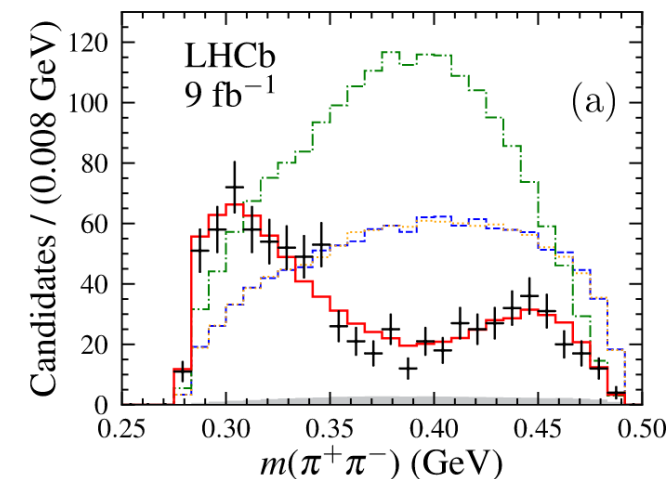
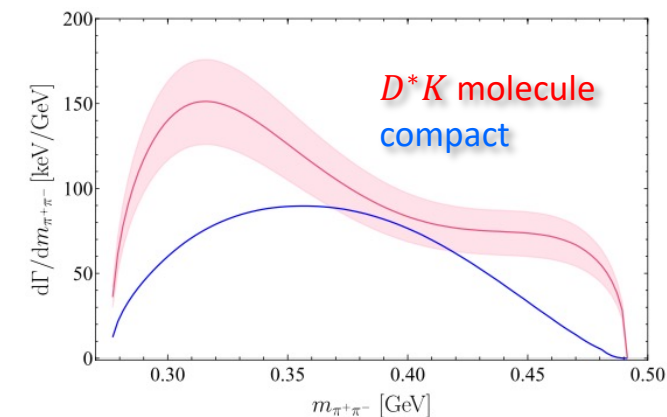
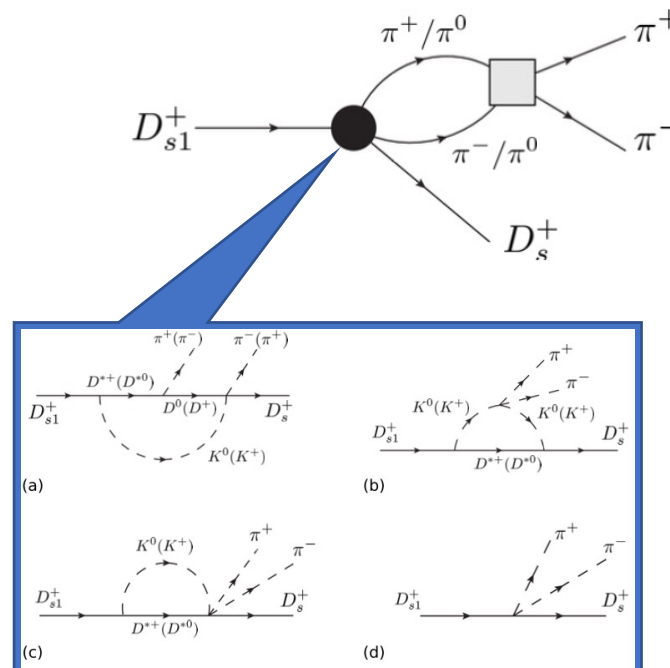
Universality of kaonic interaction with isospin-1/2 matter fields

⇒ a whole family of kaonic bound states!

✓ $\Lambda(1405), K$ -nucleus bound states, ...

- $D_{s1}(2460) \rightarrow D_s \pi^+ \pi^-$: double-bump

M.-N. Tang, Y.-H. Lin, FKG, U.-G. Meißner, CTP 75 (2023) 055203



double-bump seen by LHCb,

Sci.Bull. 70 (2025) 3219

Molecular line shapes at LO

- Scattering length approx.: $p \cot \delta = -\frac{1}{a} + \dots$

- Poles: **bound or virtual state** ($\kappa = 1/|a|$)

- Bound and virtual state can hardly be distinguished above threshold ($E > 0$)

$$|T_{\text{NR}}(E)|^2 \propto \left| \frac{1}{\pm\kappa + i\sqrt{2\mu E}} \right|^2 = \frac{1}{\kappa^2 + 2\mu E}$$

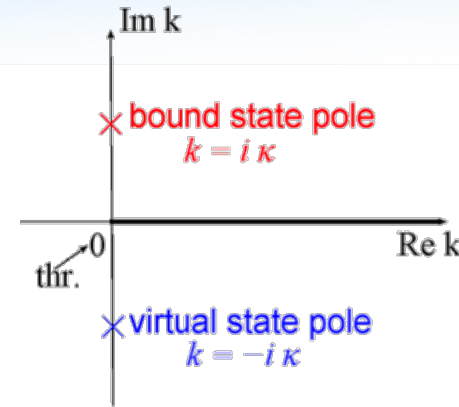
- Different below threshold ($E < 0$)

- bound state: peaked below threshold

$$|T_{\text{NR}}(E)|^2 \propto \frac{1}{(\kappa - \sqrt{-2\mu E})^2}$$

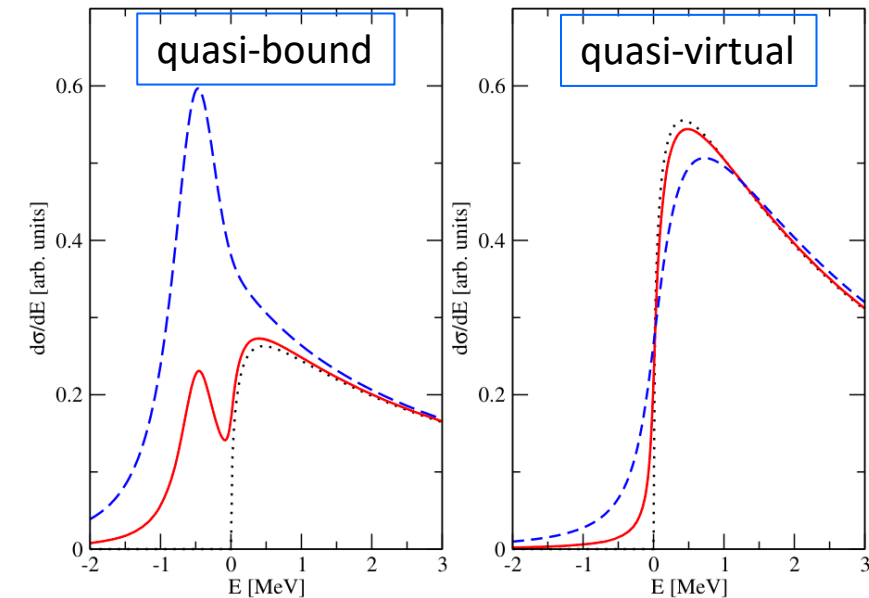
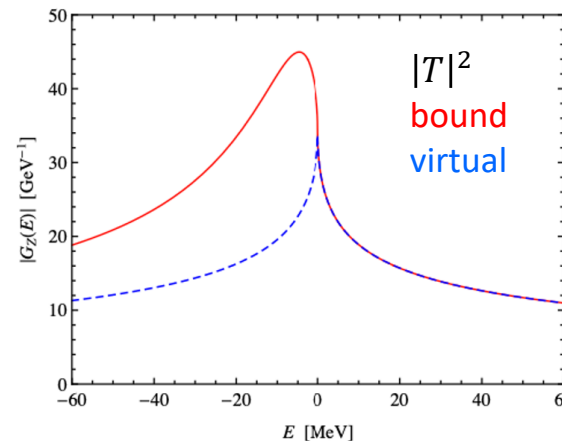
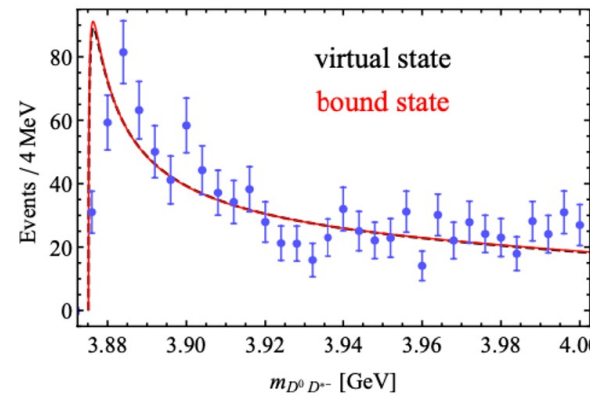
- virtual state: sharp cusp at threshold

$$|T_{\text{NR}}(E)|^2 \propto \frac{1}{(\kappa + \sqrt{-2\mu E})^2}$$



FKG, et al., RMP 90 (2018) 015004;
N. Brambilla et al., Phys.Rept. 873 (2020) 1

line shapes w/ phase space;
one unstable constituent:



$\Gamma = 0$	0.1 MeV	1 MeV
dotted	dashed	solid

Other properties of X(3872)

Teng Ji, X.-K. Dong, FKG, C. Hanhart, U.-G. Meißner, arXiv:2502.04458

- Width (twice of the imaginary part of the pole): 250^{+36+30}_{-50-56} keV

- Branching fractions computed using the method in L.A. Heuser, G. Chanturia, FKG, C. Hanhart, M. Hoferichter, B. Kubis, EPJC 84 (2024) 599

Mode	$D^0 \bar{D}^0 \pi^0$	$D^0 \bar{D}^0 \gamma$	$J/\psi \pi^+ \pi^-$	$J/\psi \pi^+ \pi^- \pi^0$	others
BR(%)	41^{+3}_{-4}	22 ± 2	5^{+2}_{-1}	16^{+4}_{-3}	16 ± 2

- Isospin breaking ratio $R_X \equiv \left| \frac{g_{XJ/\psi\rho}}{g_{XJ/\psi\omega}} \right| = 0.26(2)$

Results updated