Study of the $D_s \to \phi \ell \nu_\ell$ semileptonic decay with (2+1)-flavor lattice QCD

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Introduction

The semileptonic decays of heavy mesons provide an excellent opportunity to determine the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and a strong test of QCD. The semileptonic decay rates involve the hadronic form factors, which describe the non-perturbative strong interactions. Precise calculations of form factors can not only rigorously test the Standard Model, but also relate to potential new physics. The semileptonic decays also provide support for testing the lepton flavor universality.

Methodology

The Euclidean hadronic matrix element can be parametrized to be [1]

$$
\begin{aligned}
&\langle \phi(\vec{p},\varepsilon)| J^W_\mu |D_s(p')\rangle \\
&= \varepsilon^*_\nu \epsilon_{\mu\nu\alpha\beta} p'_\alpha p_\beta \frac{2V}{m+M} + (M+m)\,\varepsilon^*_\mu A_1 \\
&+ \frac{\varepsilon^* \cdot q}{M+m}(p+p')_\mu A_2 + 2m\frac{\varepsilon^*\cdot q}{q^2} q_\mu (A_0 - A_3)\,, \qquad (1)
\end{aligned}
$$

where the weak current is $J^W_\mu = \bar{s}\gamma_\mu(1-\gamma_5)c$. It is related to the hadronic function $H_{\mu\nu}(x) \equiv V_{\mu\nu}(x) - A_{\mu\nu}(x) = \langle \phi_\nu(\vec{x},t) J^W_\mu(0)|D_s(p')\rangle$ by inserting the complete basis. To extract the form factors V and A_i $(i=0,1,2)$, we construct the following scalar functions [2]

$$
\begin{aligned}
\mathcal{I}_0 &= \frac{1}{M|\vec{p}|^2}\epsilon_{\mu\nu\alpha'\beta'} p'_{\alpha'} p_{\beta'} \int \mathrm{d}^3\vec{x}\, e^{-i\vec{p}\cdot\vec{x}} V_{\mu\nu}(\vec{x},t)\,, \\
\mathcal{I}_1 &= \delta_{\mu\nu} \int \mathrm{d}^3\vec{x}\, e^{-i\vec{p}\cdot\vec{x}} A_{\mu\nu}(\vec{x},t)\,, \\
\mathcal{I}_2 &= \frac{E}{M}\frac{p_\mu p'_\nu}{|\vec{p}|^2} \int \mathrm{d}^3\vec{x}\, e^{-i\vec{p}\cdot\vec{x}} A_{\mu\nu}(\vec{x},t)\,, \\
\mathcal{I}_3 &= \frac{p'_\mu p'_\nu}{|\vec{p}|^2} \int \mathrm{d}^3\vec{x}\, e^{-i\vec{p}\cdot\vec{x}} A_{\mu\nu}(\vec{x},t)\,. \qquad (2)
\end{aligned}
$$

The form factors then can be accessed via solving the equations. The three point function is

$$
C_{\mu\nu}(\vec{x},t,t_s) = \langle \mathcal{O}_{\phi_\nu}(t) J^W_\mu(0) \mathcal{O}^\dagger_{D_s}(-t_s)\rangle = \frac{Z_{D_s}}{2M} e^{-Mt_s} H_{\mu\nu}(\vec{x},t)\,, \qquad (3)
$$

where M, m, Z_{D_s}, and Z_ϕ are extracted from the two point function. The differential decay distribution over q^2 is

$$
\frac{\mathrm{d}\Gamma(D_s\to\phi\ell\nu)}{\mathrm{d}q^2} = \frac{G_F^2|V_{cs}|^2|\vec{p}|q^2 v^2}{96\pi^3 M^2}\left[(1+\delta)\sum_{i=+,-,0}|H_i|^2 + 3\delta\,|H_t|^2\right], \qquad (4)
$$

where $v = 1 - m_\ell^2/q^2$, $\delta = m_\ell^2/(2q^2)$, and $H_{\pm,0,t}$ are the helicity amplitudes constructed by using V and A_i $(i=0,1,2)$.

Simulation Results

We employ seven $(2+1)$-flavor Wilson-clover gauge ensembles generated by the CLQCD collaboration [3], the parameters of which are shown in Table 1. The valence strange and charm quark masses are tuned using the "fictitious" meson η_s and the D_s meson masses.

Ensemble	C24P29	C32P23	C32P29	F32P30	F48P21	G36P29	H48P32
a (fm)		0.10524(05)(62)		0.07753(03)(45)		0.06887(12)(41)	0.05199(08)(31)
am_l	−0.2770	−0.2790	−0.2770	−0.2295	−0.2320	−0.2150	−0.1850
am_s	−0.2400	−0.2400	−0.2400	−0.2050	−0.2050	−0.1926	−0.1700
am_s^V	−0.2356(1)	−0.2337(1)	−0.2358(1)	−0.2038(1)	−0.2025(1)	−0.1928(1)	−0.1701(1)
am_c^V	0.4159(07)	0.4190(07)	0.4150(06)	0.1974(05)	0.1997(04)	0.1433(12)	0.0551(07)
$L^3 \times T$	$24^3\times 72$	$32^3\times 64$	$32^3\times 64$	$32^3\times 96$	$48^3\times 96$	$36^3\times 108$	$48^3\times 144$
$N_{\rm cfg}\times N_{\rm src}$	450 × 288	200 × 192	397 × 128	180 × 192	150 × 192	200 × 108	150 × 144
m_π (MeV/c^2)	292.3(1.0)	227.9(1.2)	293.1(0.8)	300.4(1.2)	207.5(1.1)	297.2(0.9)	316.6(1.0)
t	2 − 17	2 − 20	2 − 20	4 − 22	4 − 26	2 − 32	8 − 30
Z_V^s	0.85184(06)	0.85350(04)	0.85167(04)	0.86900(03)	0.86880(02)	0.87473(05)	0.88780(01)
Z_V^c	1.57353(18)	1.57644(12)	1.57163(14)	1.30566(07)	1.30673(04)	1.23990(13)	1.12882(11)
Z_A/Z_V	1.07244(70)	1.07375(40)	1.07648(63)	1.05549(54)	1.05434(88)	1.04500(22)	1.03802(28)

Table 1: Parameters of gauge ensembles used in this work.

After obtaining the form factors on each ensemble, the z-expansion extrapolation to the continuum limit and physical pion mass point is performed, where

$$
\begin{aligned}
V &= \frac{1}{1-q^2/m^2_{D_s^*}}\sum_{i=0}^{2}\left(c_i + d_i a^2\right)\left[1 + f_i\left(m_\pi^2 - m^2_{\pi,\rm phys}\right)\right] z^i, \\
A_{0,1,2} &= \frac{1}{1-q^2/m^2_{D_{s1}}}\sum_{i=0}^{2}\left(c_i + d_i a^2\right)\left[1 + f_i\left(m_\pi^2 - m^2_{\pi,\rm phys}\right)\right] z^i. \qquad (5)
\end{aligned}
$$

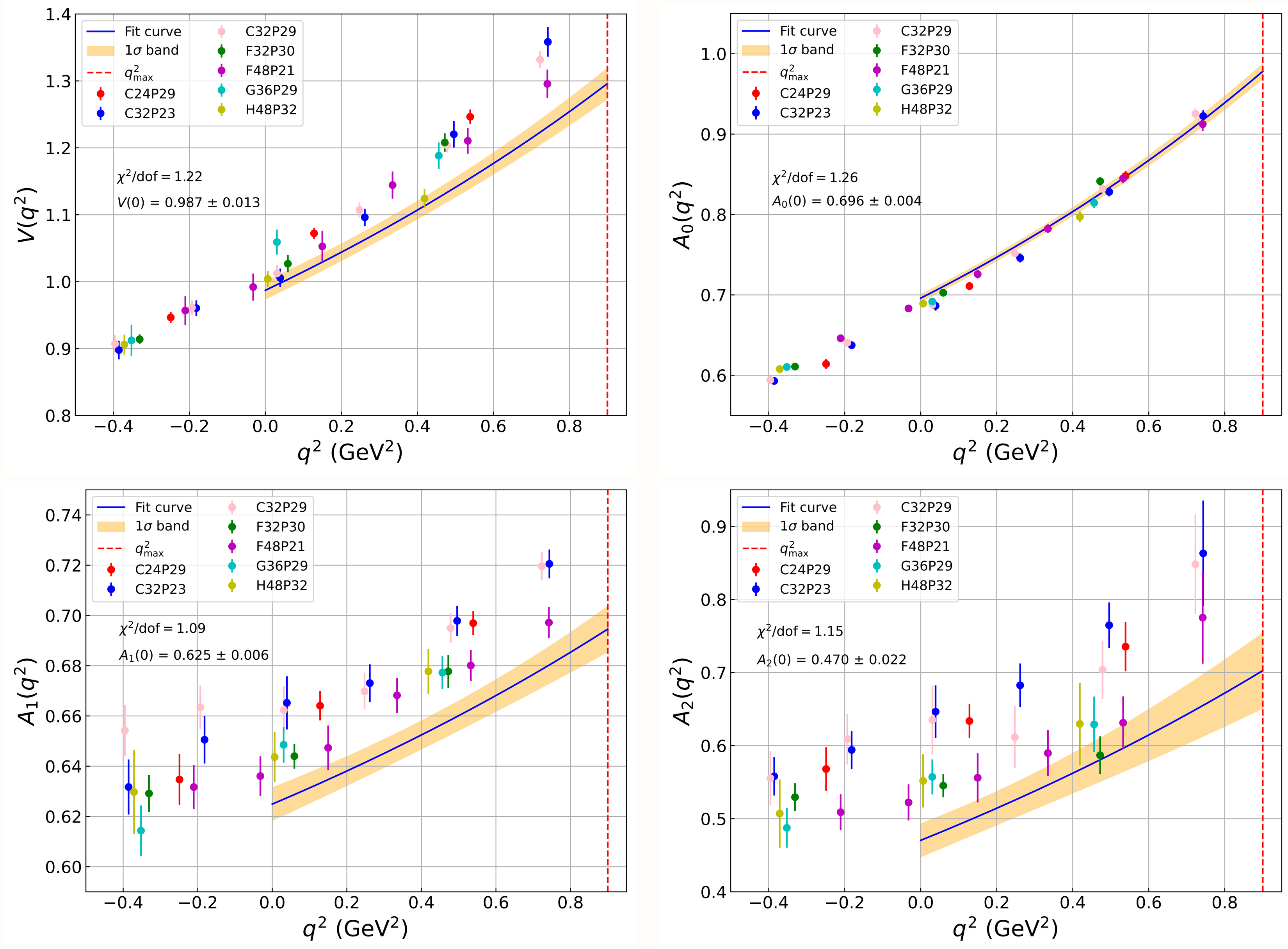

Figure 1: The lattice results and the extrapolation results.

The ratio $r_V \equiv V(0)/A_1(0)$, $r_2 \equiv A_2(0)/A_1(0)$, and the CKM matrix $|V_{cs}|$ are calculated, as shown in Fig. 2.

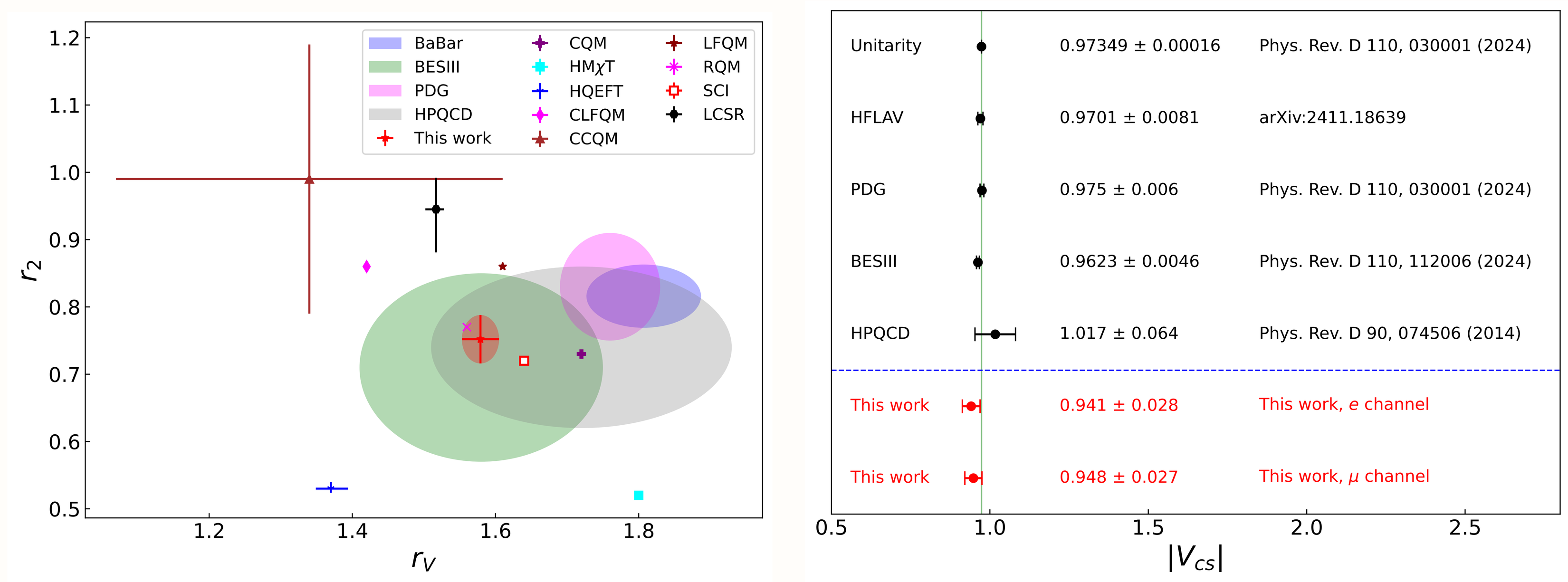

Figure 2: The comparisons of r_V and r_2 obtained in this work (left), and $|V_{cs}|$ extracted by comparing the lattice calculations and PDG results (right).

Summary

We present the full lattice QCD calculations of the vector and axial vector form factors V and A_i $(i=0,1,2)$. The form factor ratios are obtained to be $r_V = 1.579(26)$ and $r_2 = 0.752(36)$. For the ϕ meson, we obtain the physical mass $m = 1.0138(80)$ GeV/c^2 and the decay constant $f_\phi = 0.2435(50)$ GeV, which are in agreement with the PDG results. We finally obtain the branching fractions $\mathcal{B}(D_s\to\phi\mu\nu_\mu) = 2.372(79)\times 10^{-2}$ and $\mathcal{B}(D_s\to\phi e\nu_e) = 2.514(86)\times 10^{-2}$. The corresponding ratio of the branching fractions between the μ and e lepton final states is $\mathcal{R}_{\mu/e} = 0.9433(14)$, which greatly improves the accuracy and can be tested in future experiments.

References

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17th International Conference on Heavy Quarks and Leptons (HQL 2025), 15-19 Sept. 2025, Beijing, China