Study of the $D_s \to \phi \ell \nu_\ell$ semi-leptonic decay with (2+1)-flavor lattice QCD

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- Lattice set up
- Methods
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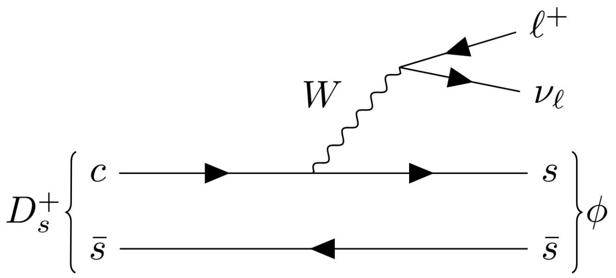
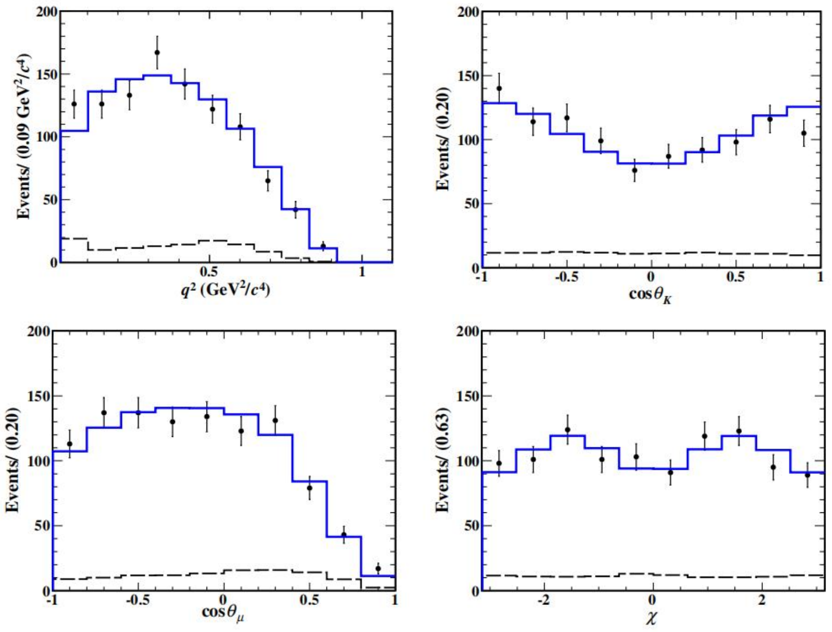
Motivations



- Semi-leptonic decays offer an ideal place to deeply understand hadronic transitions in the nonperturbative region of QCD, and can help to explore the weak and strong interactions in charm sector
- Vector meson decay makes this transition notoriously difficult to model due to theoretical complexity
- Combining with the experimental data, the CKM matrix element can be extracted, and it helps to test unitarity of CKM matrix and search for new physics beyond SM
- Calculating branching fractions helps to test $\mu - e$ lepton flavor universality



[BESIII, JHEP 12, 072 (2023)]

SM parameter

$$\frac{d\Gamma(D_s \to \phi\ell\nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2 |\boldsymbol{p}_\phi| q^2}{96\pi^3 M_{D_s}^2}\left(1 - \frac{m_\ell^2}{q^2}\right)^2 \left[\left(1 + \frac{m_\ell^2}{2q^2}\right)\left(|H_+|^2 + |H_-|^2 + |H_0|^2\right) + \frac{3m_\ell^2}{2q^2}|H_t|^2\right]$$

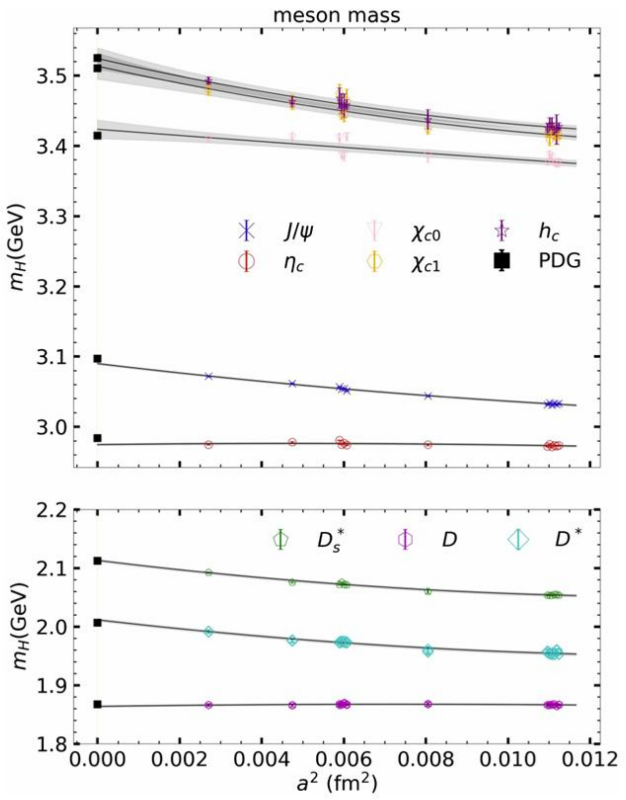
Experimental data

non-perturbative

Lattice set up

- (2+1)-flavor Wilson-clover gauge ensembles [CLQCD, PRD 111, 054504 (2025)]
- Computer resources: "SongShan" supercomputer at Zhengzhou University

Ensemble	C24P29	C32P23	C32P29	F32P30	F48P21	G36P29	H48P32
a (fm)		0.10524(05)(62)		0.07753(03)(45)		0.06887(12)(41)	0.05199(08)(31)
am_l	−0.2770	−0.2790	−0.2770	−0.2295	−0.2320	−0.2150	−0.1850
am_s	−0.2400	−0.2400	−0.2400	−0.2050	−0.2050	−0.1926	−0.1700
am_s^{V}	−0.2356(1)	−0.2337(1)	−0.2358(1)	−0.2038(1)	−0.2025(1)	−0.1928(1)	−0.1701(1)
am_c^{V}	0.4159(07)	0.4190(07)	0.4150(06)	0.1974(05)	0.1997(04)	0.1433(12)	0.0551(07)
$L^3 \times T$	$24^3 \times 72$	$32^3 \times 64$	$32^3 \times 64$	$32^3 \times 96$	$48^3 \times 96$	$36^3 \times 108$	$48^3 \times 144$
N_{mea}	$450 \times 72 \times 4$	$200 \times 64 \times 3$	$397 \times 64 \times 2$	$180 \times 96 \times 2$	$150 \times 48 \times 4$	$200 \times 54 \times 2$	$150 \times 72 \times 2$
m_π (MeV/c^2)	292.3(1.0)	227.9(1.2)	293.1(0.8)	300.4(1.2)	207.5(1.1)	297.2(0.9)	316.6(1.0)
t	2 − 17	2 − 20	2 − 20	4 − 22	4 − 26	2 − 32	8 − 30
Z_V^s	0.85184(06)	0.85350(04)	0.85167(04)	0.86900(03)	0.86880(02)	0.87473(05)	0.88780(01)
Z_V^c	1.57353(18)	1.57644(12)	1.57163(14)	1.30566(07)	1.30673(04)	1.23990(13)	1.12882(11)
Z_A/Z_V	1.07244(70)	1.07375(40)	1.07648(63)	1.05549(54)	1.05434(88)	1.04500(22)	1.03802(28)



Methods

- The parameterization for $P \to V$ semileptonic matrix element

$$\langle \phi_\sigma(\vec{p})|J_\mu^W(0)|D_s(p')\rangle = \frac{F_0(q^2)}{Mm}\epsilon_{\mu\sigma\alpha\beta}p'^\alpha p^\beta + F_1(q^2)\delta_{\mu\sigma} + \frac{F_2(q^2)}{Mm}p_\mu p'_\sigma + \frac{F_3(q^2)}{M^2}p'_\mu p'_\sigma$$

$$\langle \phi(\varepsilon,\vec{p})|J_\mu^W(0)|D_s(p')\rangle = \varepsilon_\nu^* \varepsilon_{\mu\nu\alpha\beta}p'_\alpha p_\beta \frac{2V}{m+M} + (M+m)\varepsilon_\mu^* A_1 + \frac{\varepsilon^*\cdot q}{M+m}(p+p')_\mu A_2 - 2m\frac{\varepsilon^*\cdot q}{Q^2}q_\mu(A_0 - A_3)$$

- Correlation functions ⟶ Scalar functions ⟶ Form factors

$\langle \phi_\sigma(\vec{p})|J_\mu^W(0)|D_s(p')\rangle$ $\tilde{\mathcal{I}}_j$ V, A_0, A_1, A_2

- Relationship with the form factor

$$V = \frac{(m+M)}{2mM}F_0,$$
$$A_1 = \frac{F_1}{M+m},$$
$$A_2 = \frac{M+m}{2mM^2}(MF_2 + mF_3),$$
$$A_0 - A_3 = Q^2\left(\frac{F_2}{4m^2M} - \frac{F_3}{4mM^2}\right).$$

A_3 is not an independent form factor

$$A_3(q^2) = \frac{M+m}{2m}A_1(q^2) - \frac{M-m}{2m}A_2(q^2)$$

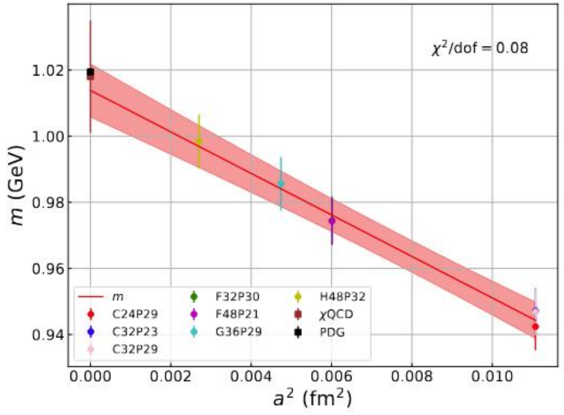
A_0 is then

$$A_0(q^2) = \frac{F_1}{2m} + \frac{m^2 - M^2 + Q^2}{4m^2M}F_2 + \frac{m^2 - M^2 - Q^2}{4mM^2}F_3$$

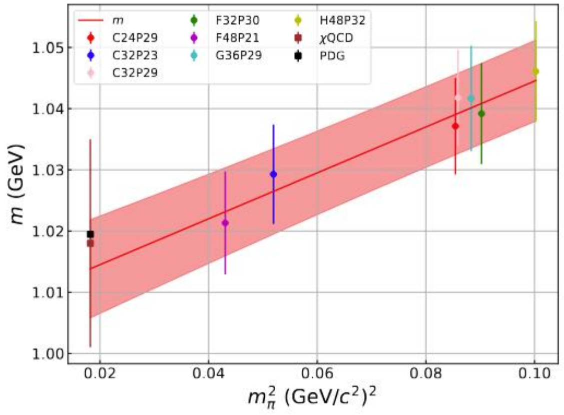
$A_0(0) = A_3(0)$ is automatically perserved

Results

- We extrapolate masses and decay constants of the ϕ meson to get physical results



(i) ϕ mass at $m_\pi = 0.135$ GeV$/c^2$.

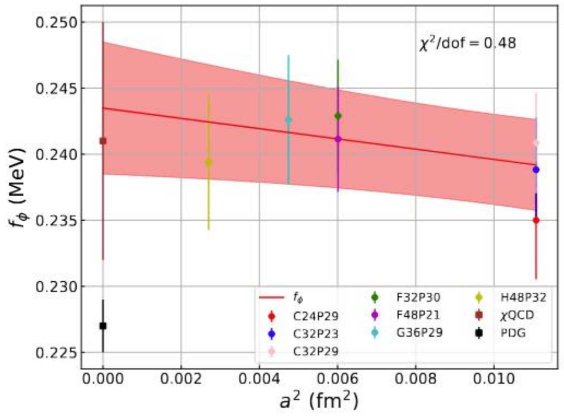


(ii) ϕ mass at $a = 0.0$ fm.

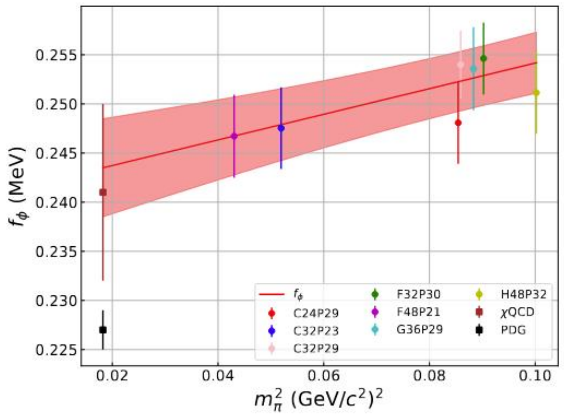
$$m/f_\phi = c + da^2 + f\left(m_\pi^2 - m_{\pi,\mathrm{phys}}^2\right)$$

$$m \;=\; 1.0138(80)\;\; \mathrm{GeV}/c^2$$

$$f_\phi = 0.2435(50)\;\mathrm{GeV}$$



(iii) ϕ decay constant at $m_\pi = 0.135$ GeV$/c^2$.



(iv) ϕ decay constant at $a = 0.0$ fm.

Results

- Extrapolate results to the continuum limit and the physical pion mass using z-expansion

$$z\left(q^2, t_0\right) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

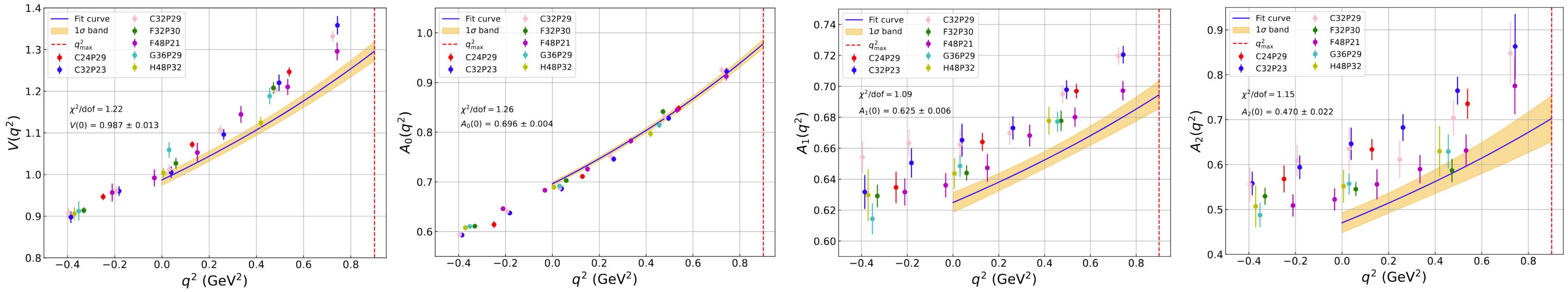
where $t_+ = \left(m_{D_s} + m_\phi\right)^2,\ t_0 = 0$

$$V\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/m_{D_s^*}^2} \sum_{i=0}^{2} \left(c_i + d_i a^2\right)\left[1 + f_i\left(m_\pi^2 - m_{\pi,\text{phys}}^2\right)\right] z^i$$

$$A_{0,1,2}\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/m_{D_{s1}}^2} \sum_{i=0}^{2} \left(c_i + d_i a^2\right)\left[1 + f_i\left(m_\pi^2 - m_{\pi,\text{phys}}^2\right)\right] z^i$$

$$m_{\pi,\text{phys}}^2 = 135.0\ \text{MeV}/c^2,\ m_{D_s^*} = 2112.2\ \text{MeV}/c^2,\ m_{D_{s1}} = 2459.5\ \text{MeV}/c^2$$

- $A_3(0) - A_0(0) = 0.001(14)$, consistent with zero



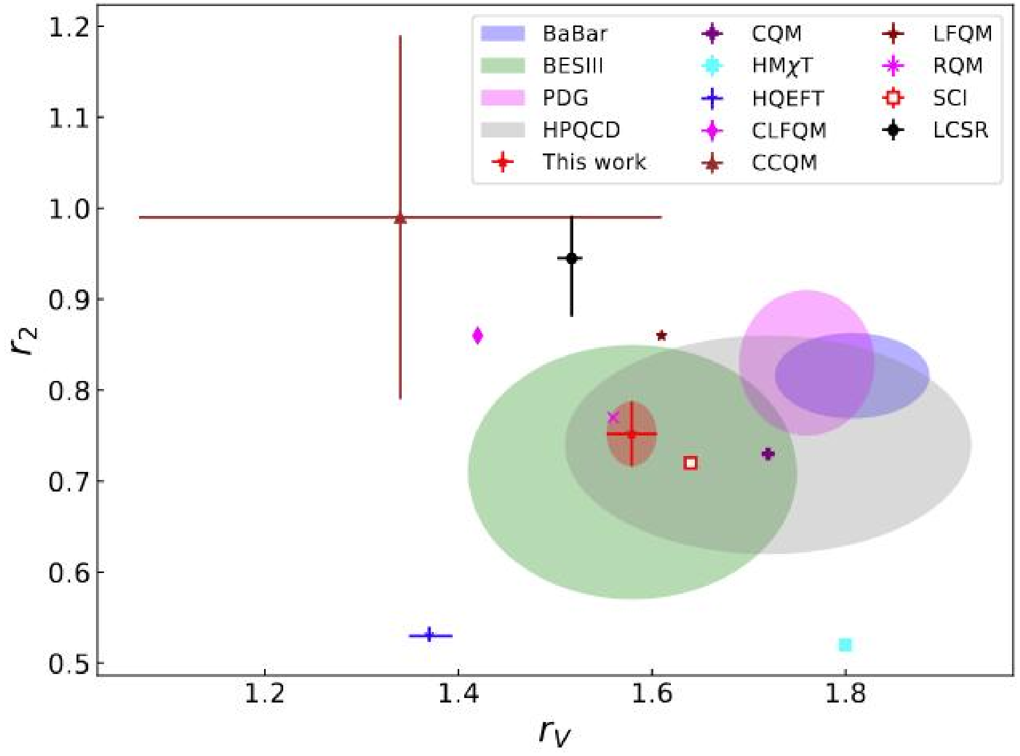
Results

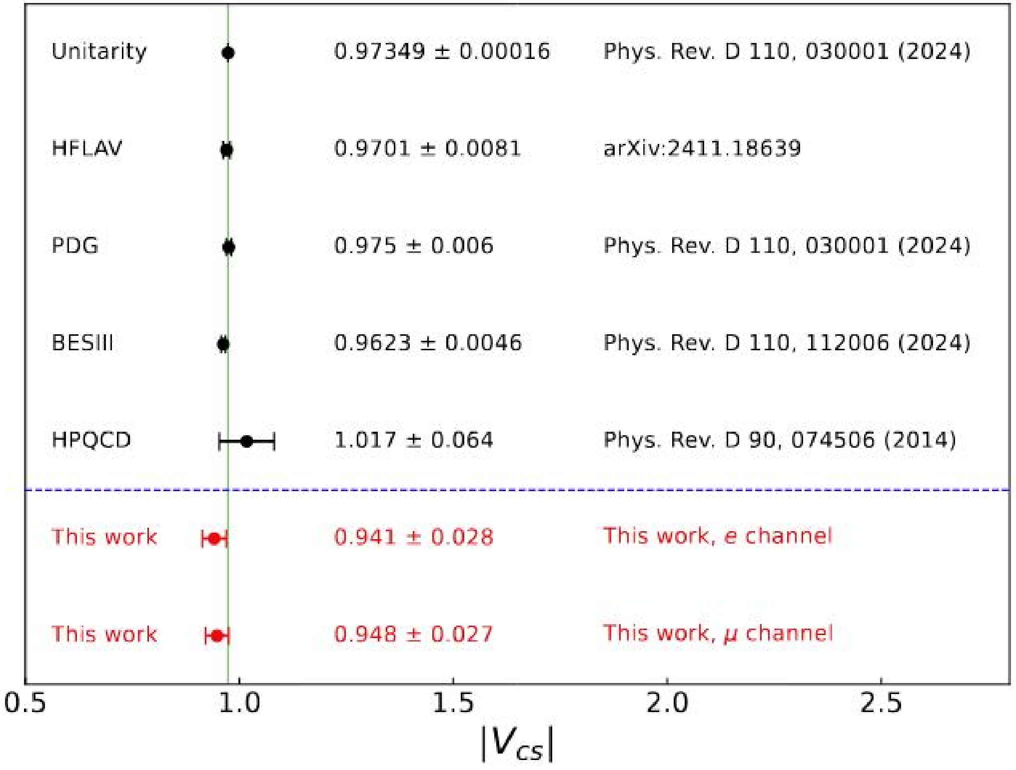
$$r_V \equiv \frac{V(0)}{A_1(0)} = \frac{(m+M)^2}{2mM}\frac{F_0(0)}{F_1(0)}$$

$$r_2 \equiv \frac{A_2(0)}{A_1(0)} = \frac{(m+M)^2}{2mM^2}\left[M\frac{F_2(0)}{F_1(0)} + m\frac{F_3(0)}{F_1(0)}\right]$$

$$|V_{cs}| = \sqrt{\frac{1}{\Gamma_{\mathrm{latt}}} \times \frac{\mathcal{B}_{\mathrm{PDG}} \times \hbar}{\tau_{D_s}}}$$

- Summary of preliminary results (form factor and $|V_{cs}|$)





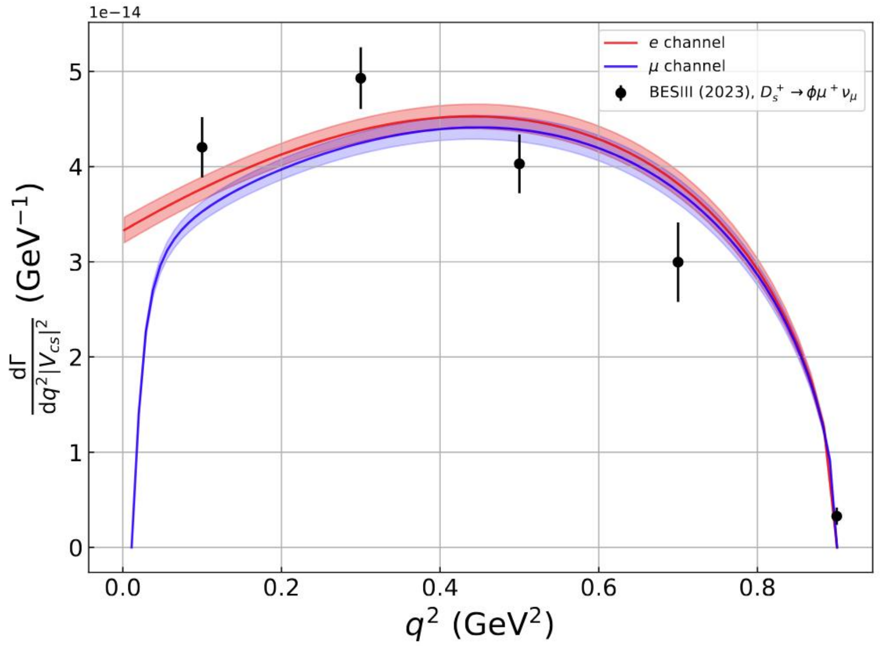
Results

- Summary of preliminary results (branching fraction)

PDG $|V_{cs}|$ as input

$$\frac{d\Gamma(D_s \to \phi \ell \nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2 |\boldsymbol{p}_\phi| q^2}{96\pi^3 M_{D_s}^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \left[\left(1 + \frac{m_\ell^2}{2q^2}\right)(|H_+|^2 + |H_-|^2 + |H_0|^2) + \frac{3m_\ell^2}{2q^2}|H_t|^2\right]$$

$\mathcal{B}(D_s \to \phi \ell \nu_\ell) \times 10^{-2}$	μ channel	e channel	$\mathcal{R}_{\mu/e}$
This work	2.372(79)	2.514(86)	0.9433(14)
BaBar	–	2.61(17)	–
CLEO	–	2.14(19)	–
BESIII (2018)	1.94(54)	2.26(46)	0.86(29)
BESIII (2023)	2.25(11)	–	–
PDG	2.24(11)	2.34(12)	0.957(68)



Summary

- Mass and decay constant of the ϕ meson are calculated
- Form factors on seven lattice sets with different q^2 are calculated
- Extrapolate form factors to the continuum limit and the physical pion mass
- Differential decay width and branching fraction results are calculated
- Preliminary work on $D \to K^* l\nu$ form factors are ongoing

Thank you for your attention!