

Bell inequality violation of light quarks in dihadron pair production at lepton colliders

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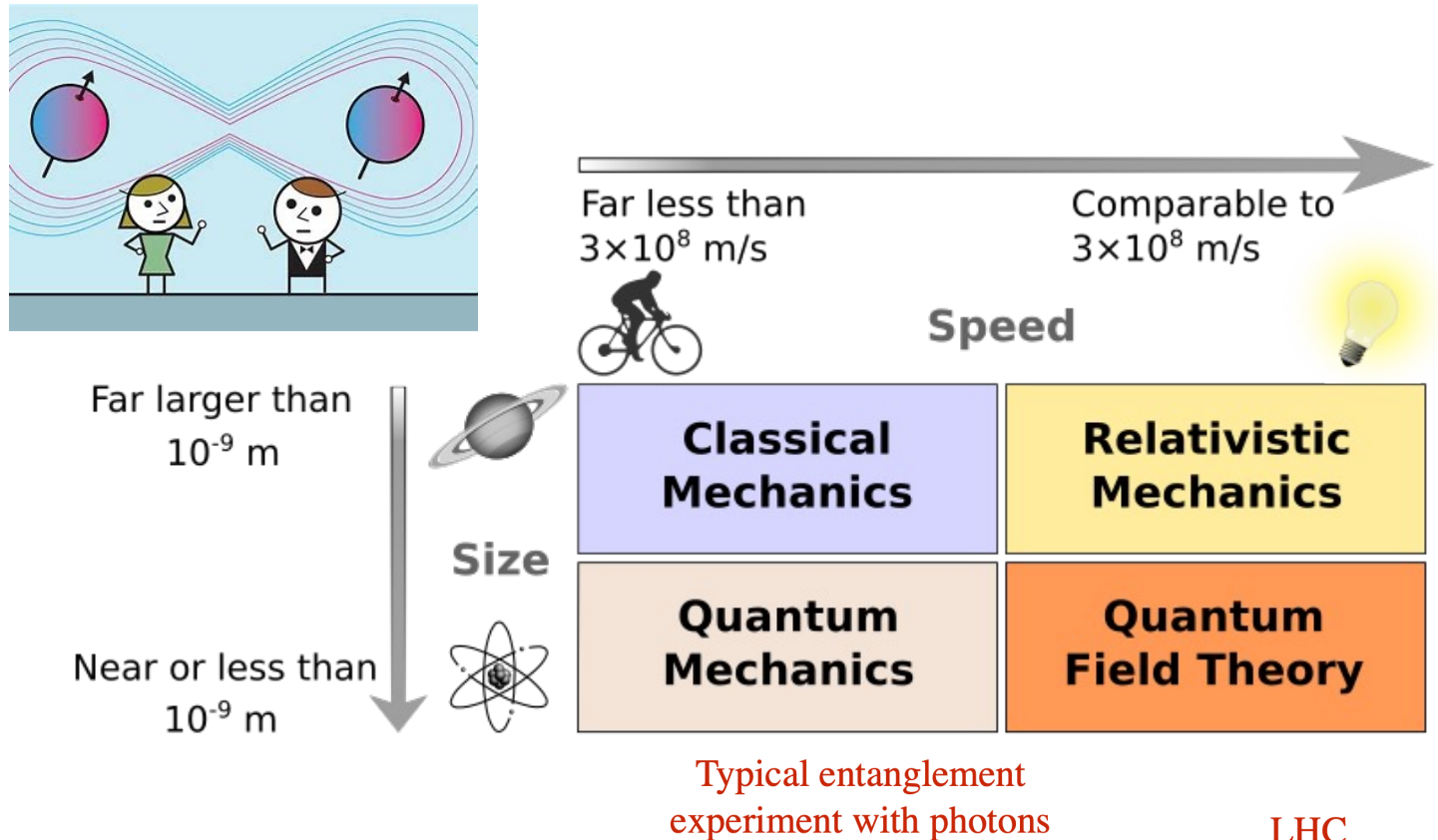
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Sep. 19-23, 2025

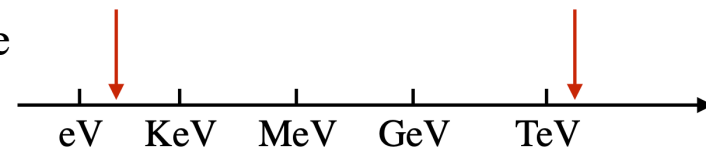
Based on PRL 135, 011902 in collaboration with Kun Cheng

Quantum information at collider

- Quantum entanglement and non-locality are distinctive features of quantum systems



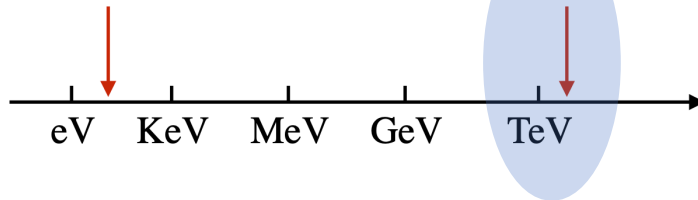
- Collider physics: high energy and small size



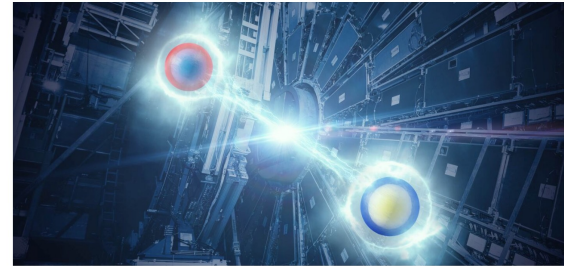
Quantum information at collider

➤ High energy: the scattering involves both the QCD and electroweak interactions

Typical entanglement
experiment with photons



The results open up a new perspective on the
complex world of quantum physics



ATLAS and CMS has observed the spin entanglement of top quark pair

➤ New features:

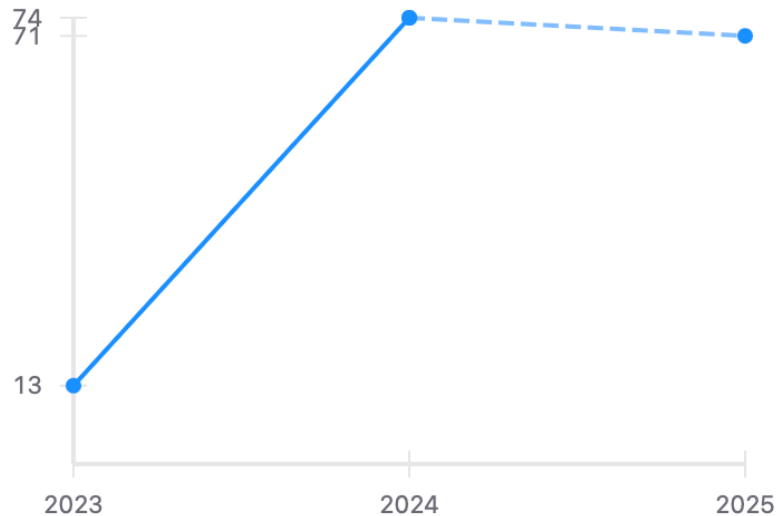
- ☐ Particle spin can not be measured directly
- ☐ New degree of freedom for spin-1 particle (Longitudinal mode)
- ☐ New features from interactions: parity violation, QCD confinement
- ☐ Entanglement beyond the spin space: flavor
- ☐

Quantum information in high-energy physics is an emerging and rapidly growing field at the intersection of particle physics and quantum theory

Quantum information in collider

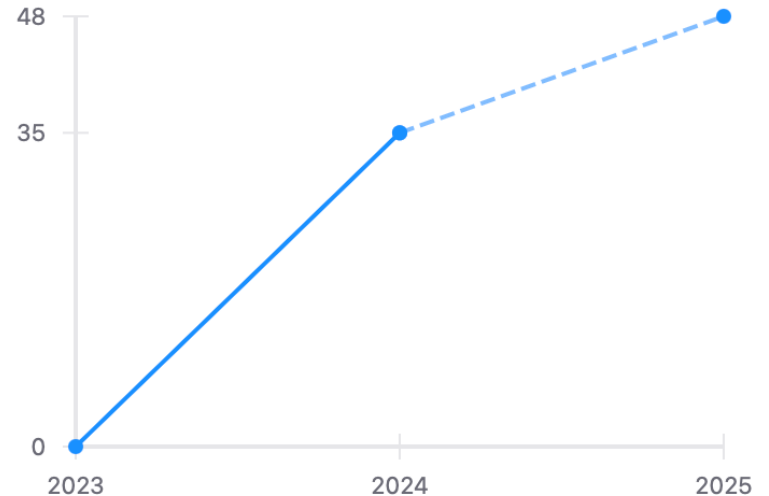
ATLAS and CMS has observed the spin entanglement of top quark pair

Citations per year



ATLAS: Nature 633 (2024) 8030, 542-547

Citations per year



CMS: Rept. Prog. Phys. 87 (2024) 117801

Quantum spin entanglement

The spin correlation of fermion pair can be described by the general density matrix

$$\rho = \frac{I_2 \otimes I_2 + B_i \sigma_i \otimes I_2 + \bar{B}_i I_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}$$

➤ $B_i, \bar{B}_i$: the polarization of each particle

➤ C_{ij} : the spin correlation of top quark pair

A separable state can: $\rho_{AB} = \sum_w p_w \rho_w^A \otimes \rho_w^B$, Entangled is non-separable

Entanglement can be measured by the **concurrence** observable

$$\mathcal{C}(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

λ_i : eigenvalues of matrix: $\sqrt{\sqrt{\rho} \tilde{\rho} \sqrt{\rho}}$ $\tilde{\rho} = (\sigma_2 \otimes \sigma_2) \rho^* (\sigma_2 \otimes \sigma_2)$

W. K. Wootters, PRL 80 (1998) 2245

Entanglement and Bell inequality

- Bell (CHSH) inequality: $\hat{A}_i = \pm 1$, $\hat{B}_i = \pm 1$

$$\left| \langle \hat{A}_1 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_2 \rangle + \langle \hat{A}_2 \hat{B}_1 \rangle - \langle \hat{A}_2 \hat{B}_2 \rangle \right| \leq 2 \quad [\text{Clauser et al, PRL 23, 880 (1969)}]$$

$$\hat{A} = \vec{a} \cdot \hat{\sigma}, \quad \hat{B} = \vec{b} \cdot \hat{\sigma}, \quad \langle \hat{A} \hat{B} \rangle = \vec{a} \cdot C \cdot \vec{b}$$

- Bell inequality is violated iff we can find four directions $\vec{a}_{1,2}, \vec{b}_{1,2}$ so that

$$\left| \vec{a}_1 \cdot C \cdot (\vec{b}_1 - \vec{b}_2) + \vec{a}_2 \cdot C \cdot (\vec{b}_1 + \vec{b}_2) \right| > 2$$

Fix some direction $\vec{a}_{1,2}, \vec{b}_{1,2}$

e.g. $\sqrt{2} |C_{xx} \pm C_{yy}| > 2$

Scan $\vec{a}_{1,2}, \vec{b}_{1,2}$ to maximize

$$\mathcal{B}[\rho] = 2\sqrt{c_1^2 + c_2^2} > 2$$

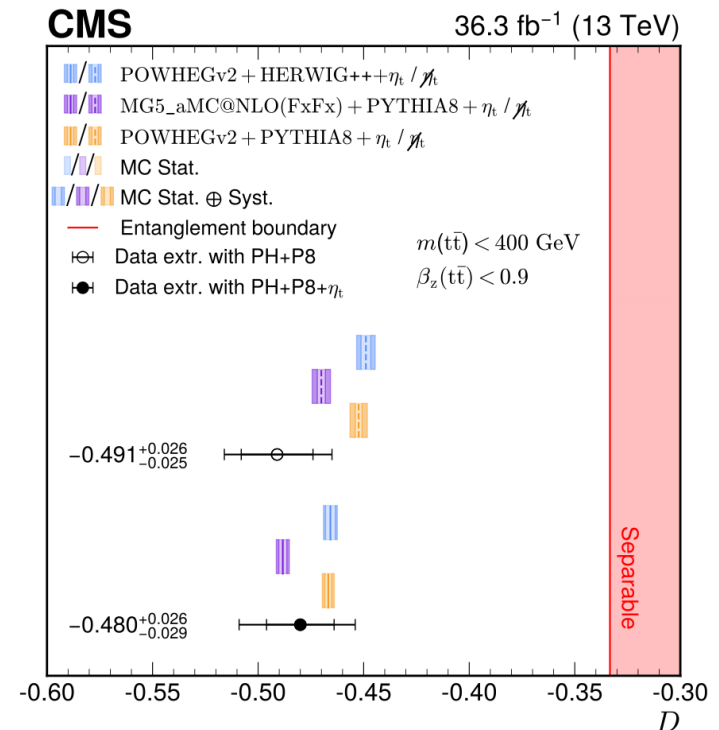
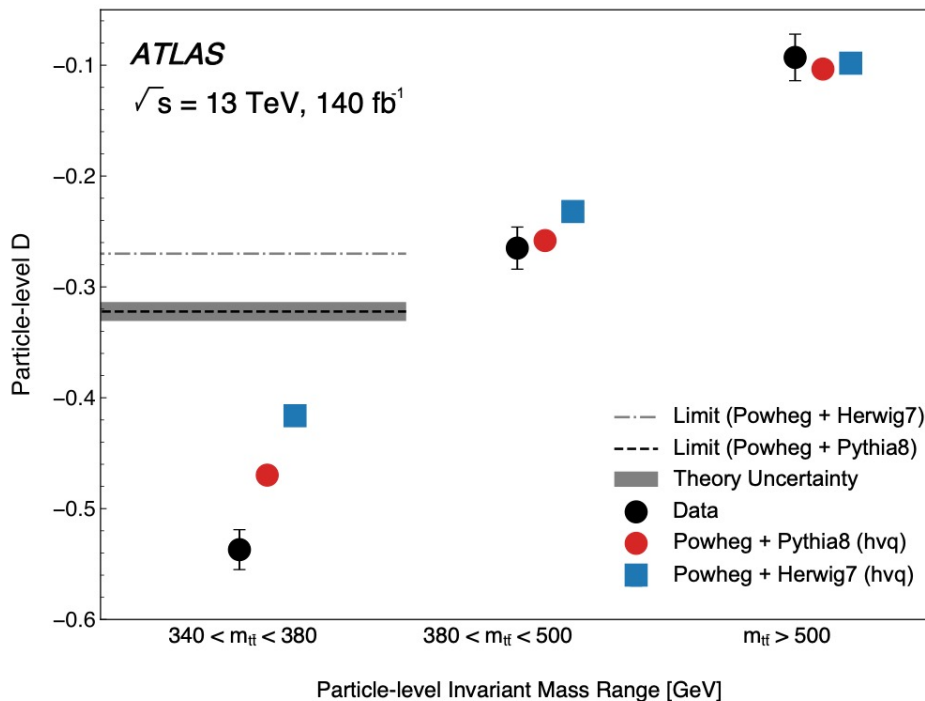
c_1^2, c_2^2 are the largest two eigenvalue of $C^T C$.

Quantum spin entanglement

Top quark pair: $\mathcal{C}[\rho] = \frac{-1 - 3D}{2} \quad \frac{1}{\sigma} \frac{d\sigma}{d \cos \varphi} = \frac{1}{2}(1 - D \cos \varphi)$

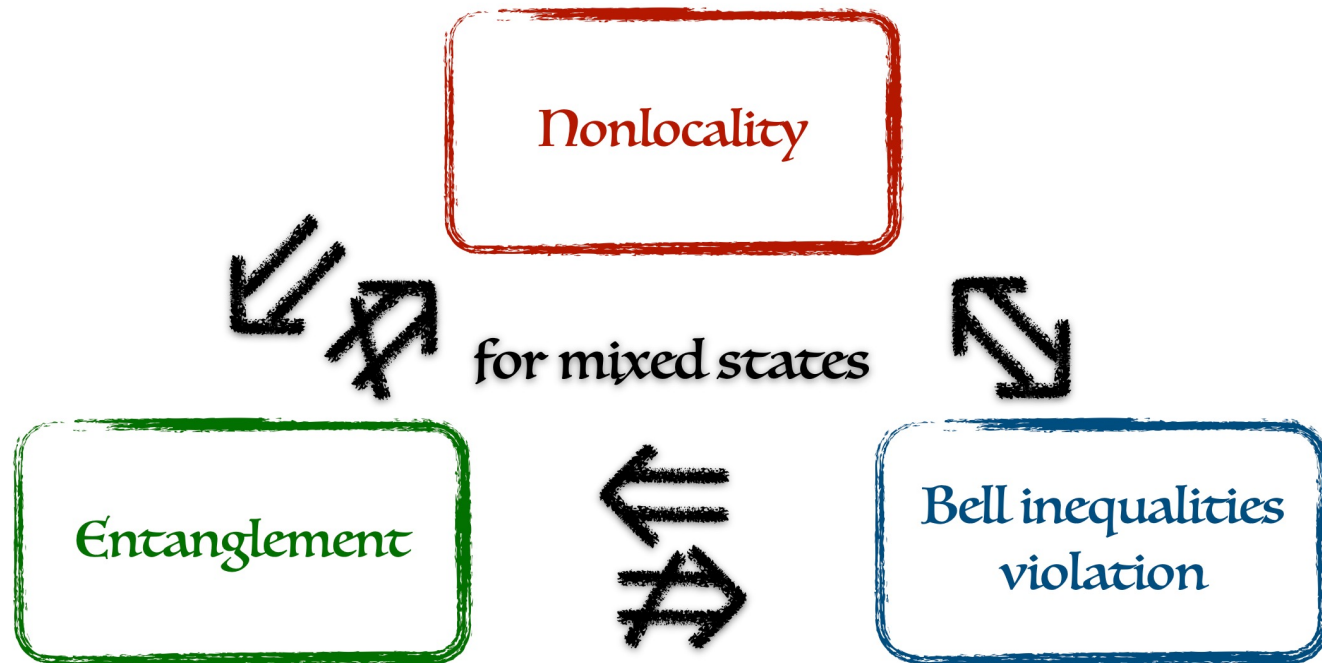
$$D = \min\left\{\frac{\text{tr}(C)}{3}, \frac{c_1 - c_2 - c_3}{3}, \frac{c_2 - c_1 - c_3}{3}, \frac{c_3 - c_1 - c_2}{3}\right\}, \quad c_i = \text{eig}(C)$$

Y. Afik, J. de Nova, EPJC 136 (2021) 907



Entangled : $D < -1/3$, separable : $D > -1/3$

Quantum entanglement and nonlocality



e.g. Werner states in quantum information

$$\rho_{\text{Werner}}(\alpha) = \alpha |\Psi_0\rangle\langle\Psi_0| + \frac{1-\alpha}{4} \hat{I}_4$$

Bell singlet

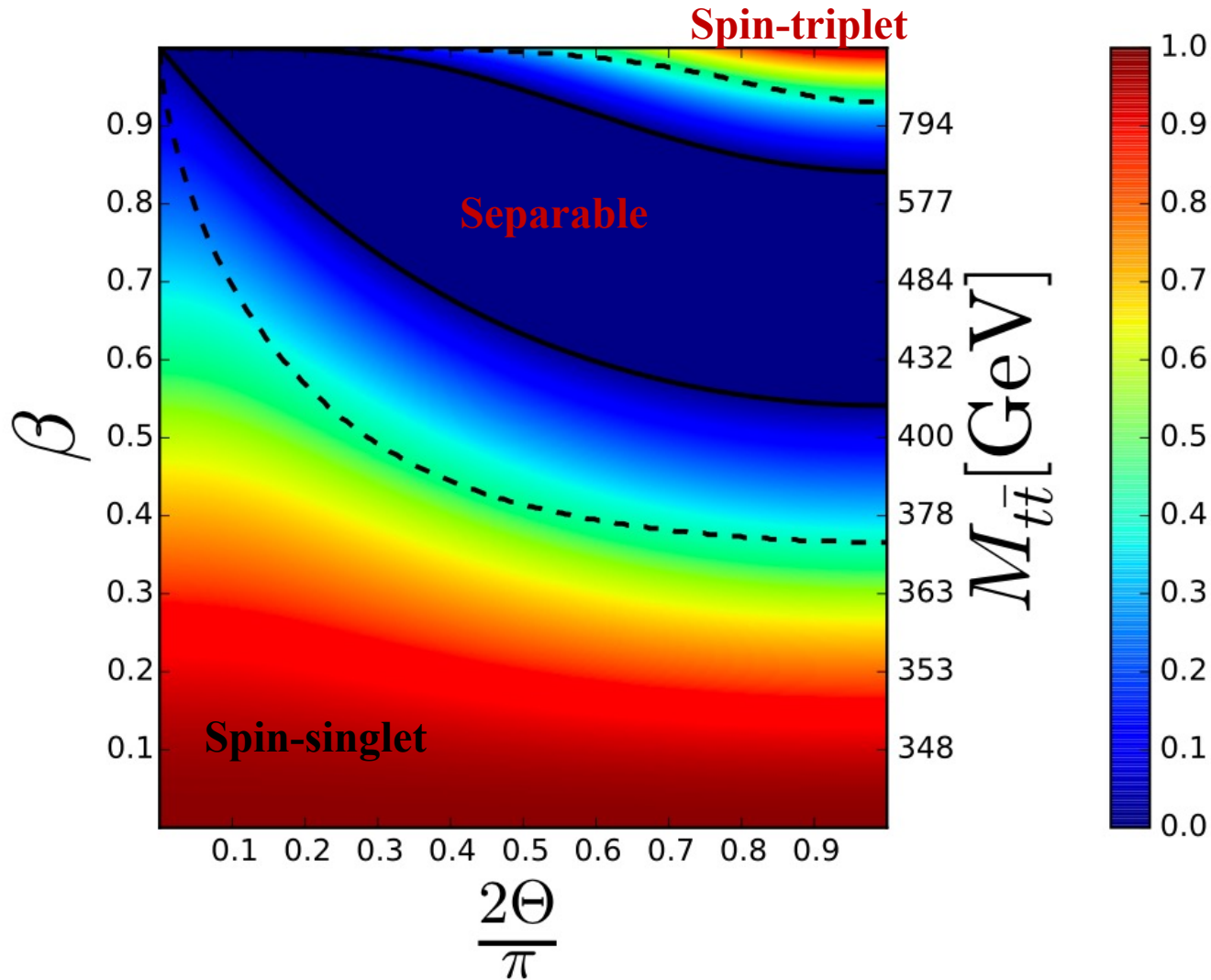
Testing Bell inequalities violation is much difficult than entanglement

Quantum entanglement of top quark

Solid: Entanglement

Dashed: Belle inequality violation

Y. Afik, J. de Nova, Quantum 6 (2022) 820



Quantum entanglement at colliders

- Top quark pair

Y. Afik, J. R. M. II. de Nova Eur. Phys. J. Plus 136, 907 (2021)
M. Fabbrichesi, R. Floreanini, G. Panizzo, PRL 127, 161801 (2021)
C. Severi, C. D. E. Boschi, F. Maltoni, and M. Sioli, EPJC 82, 285 (2022)
T. Han, M. Low, T. A. Wu, JHEP 07, 192 (2024)
T. Han, M. Low, N. McGinnis, and S. Su, 2412.21158
K. Cheng, T. Han and M. Low, 2410.08303,

- Tau lepton pair

M. M. Altakach et al, PRD 107, 093002 (2023)
K. Ehataht et al, PRD 109, 032005 (2024)
Y. Du, X.-G. He, C.-W. Liu and J.-P. Ma, 2409.15418
Y. Zhang et al, 2504.01496
T. Han, M. Low, Y. Su, 2501.04801

- Gauge boson pair

A. J. Barr et al, Quantum 7, 1070 (2023)
Q. Bi, Q.-H. Cao, K. Cheng, H. Zhang, PRD 109, 036022 (2024)
R. Ding et al, 2504.09832

- Flavor

K. Chen, Z. Xing, R. Zhu, 2407.19242
H. Feng, H. Tang, W. Guo Q. Qin, 2504.15798
K. Chen, T. Han, M. Low, T. Wu, 2507.12513

- ## ► Entanglement & NP

R. Aoude et al, PRD 106 (2022) 055007
M. Fabbrichesi et al, EPJC 83 (2023) 162, JHEP 09 (2023) 195
A. Bernal et al, EPJC 83 (2023) 11, 1050

See Talk
TianJun Li

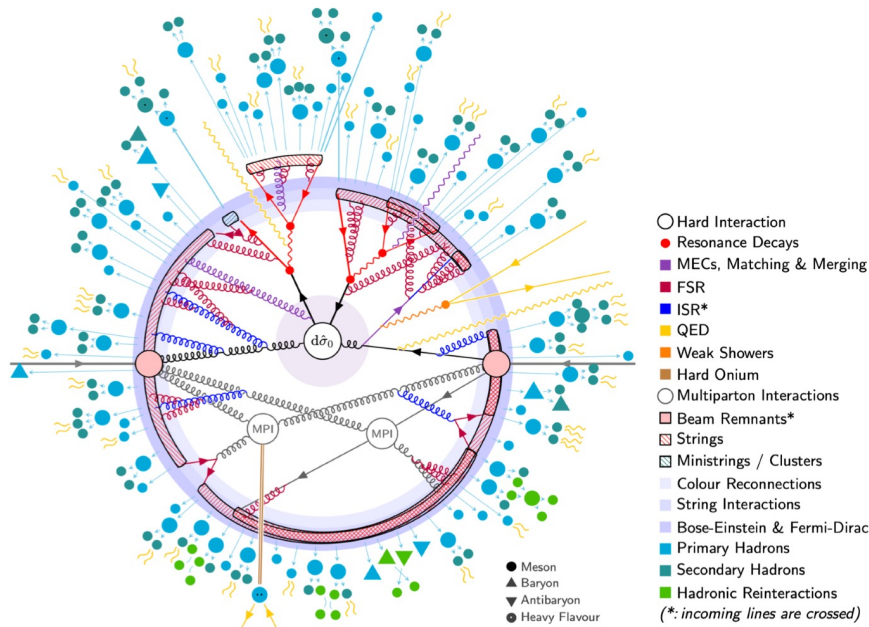
The spin correlation between particles can be measured from its decay products



How about the light quarks?

Quantum entanglement of light quarks

❑ The quark can not be a free particle due to the QCD confinement:

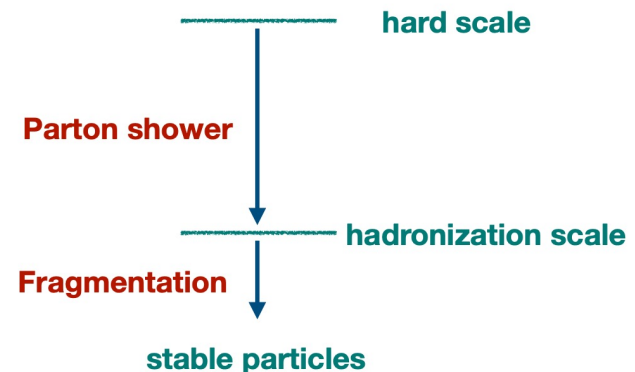


From PYTHIA 8.3

❑ Hard process in high energy

❑ Transition from high energy to low energy
—parton shower

❑ Low energy soft regime
—fragmentation

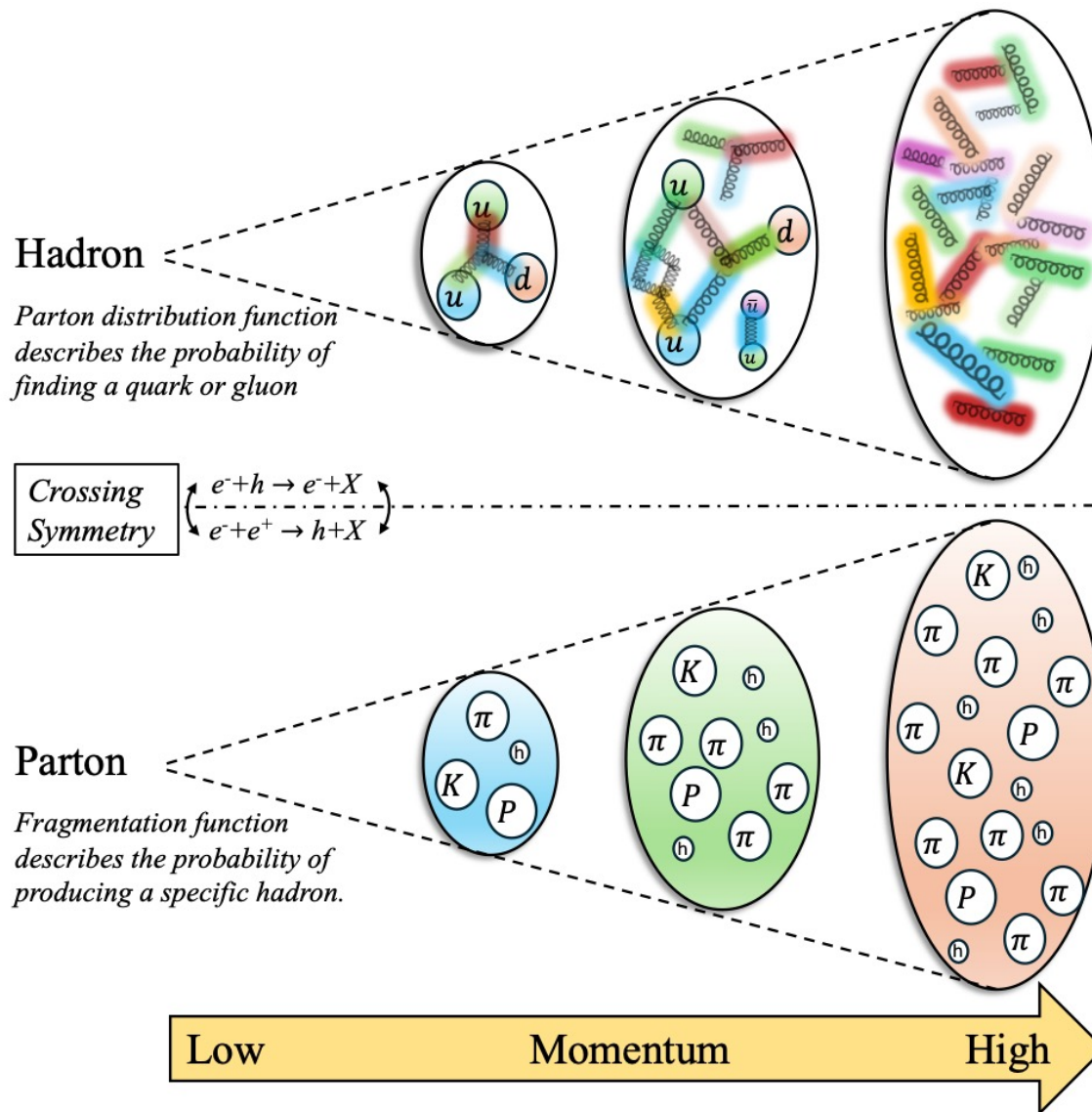


❑ The light quark does not decay but instead fragments into a jet of hadrons after produced from hard scattering

❑ How to probe the **spin information of light quarks?**

The non-perturbative functions: **the fragmentation functions**

PDF and FF



Spin information of light quarks

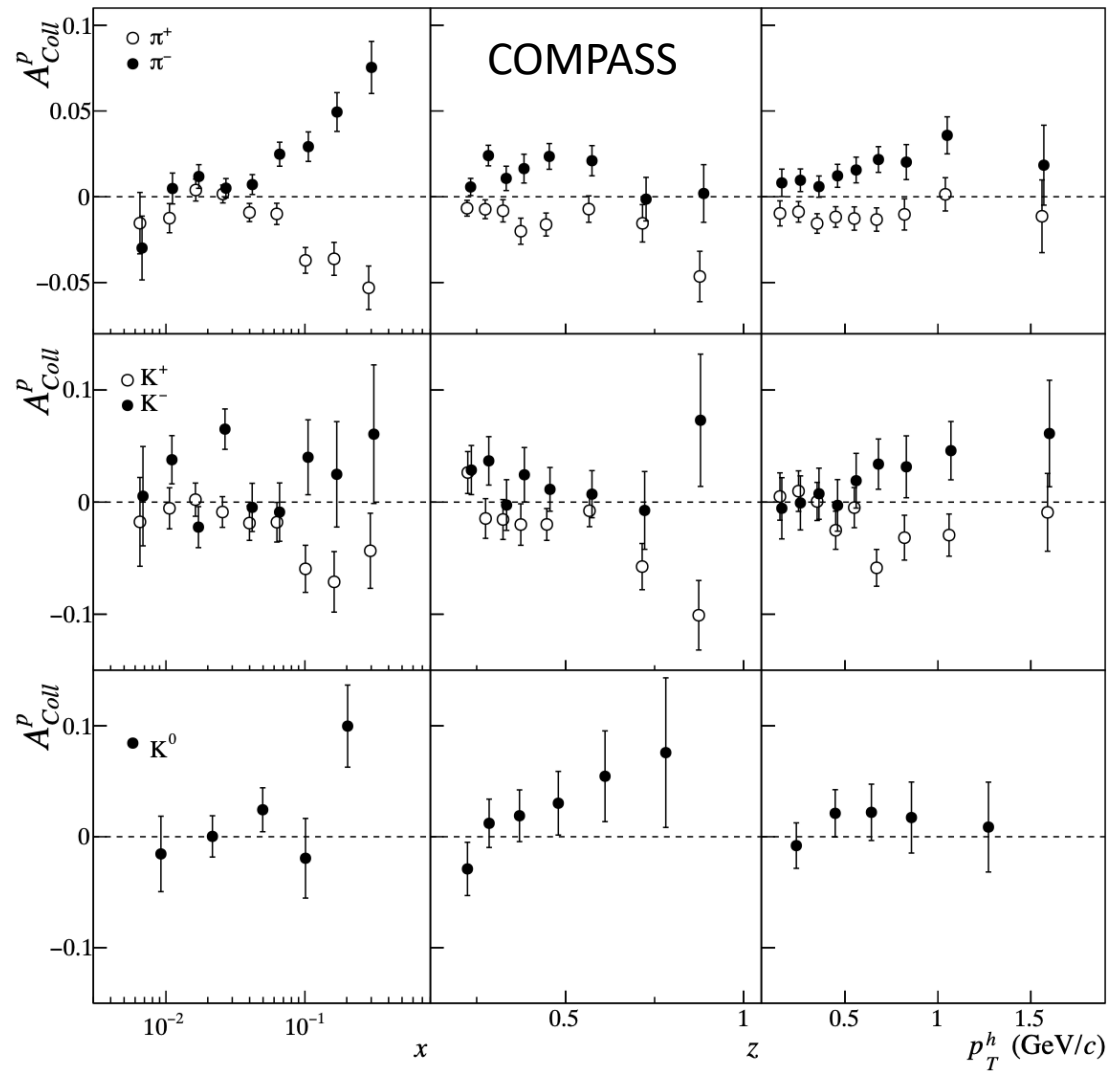
Leading Quark TMDFFs



		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		$D_1 = \text{circle with red dot}$ Unpolarized		$H_1^\perp = \text{circle with red dot and red arrow up} - \text{circle with red dot and red arrow down}$ Collins
			$G_1 = \text{circle with red dot and red arrow right} - \text{circle with red dot and red arrow left}$ Helicity	$H_{1L}^\perp = \text{circle with red dot and red arrow up-right} - \text{circle with red dot and red arrow down-left}$
Polarized Hadrons	L			
	T	$D_{1T}^\perp = \text{circle with red dot and red arrow up} - \text{circle with red dot and red arrow down}$ Polarizing FF	$G_{1T}^\perp = \text{circle with red dot and red arrow up} - \text{circle with red dot and red arrow down}$	$H_1 = \text{circle with red dot and red arrow up} - \text{circle with red dot and red arrow down}$ Transversity $H_{1T}^\perp = \text{circle with red dot and red arrow up-right} - \text{circle with red dot and red arrow down-left}$

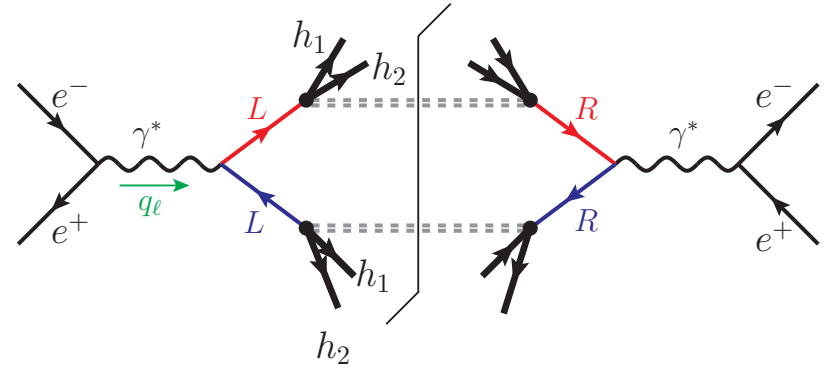
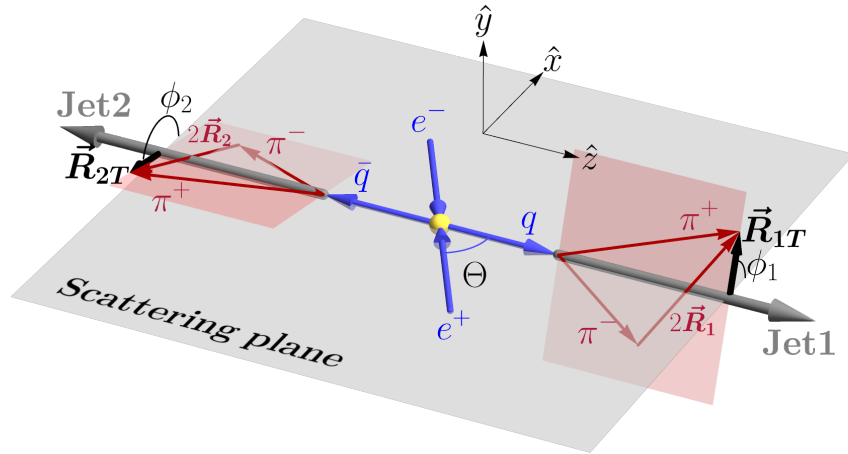
The spin information of light quarks can transfer to the hadrons: e.g. Collins functions

Example: Collins asymmetry



The spin transfer effects have been observed! (TMD)

Dihadron pair production at lepton colliders

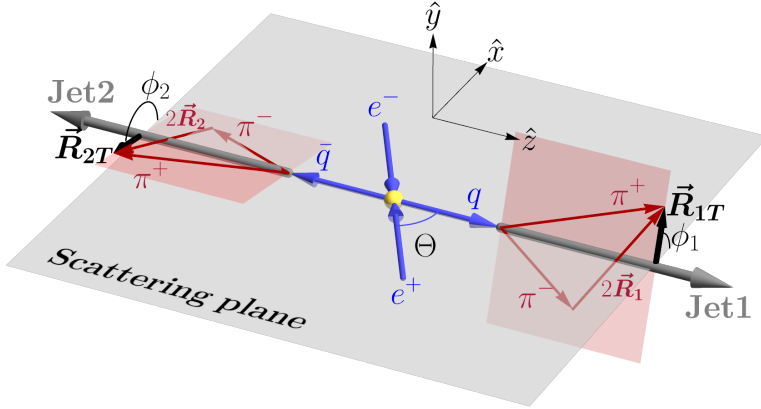


- The **transverse spin correlation** between light quarks: chiral-odd interference dihadron fragmentations (collinear factorization)
- Light quark pair are **100% correlated** in the central scattering region

$$C_{ij} = \text{diag} \left(\frac{\sin^2 \Theta}{1 + \cos^2 \Theta}, -\frac{\sin^2 \Theta}{1 + \cos^2 \Theta}, 1 \right)$$

- The **maximally entangled Bell state**: Bell inequality violation effects

Bell inequality of light quarks



J. C. Collins et al, NPB 420, 565 (1994)

$$\frac{d\sigma}{d\Omega_1 d\Omega_2} = \sigma_{\text{hard}} \sum_{ss' \bar{s}\bar{s}'} \bar{\rho}_{ss', \bar{s}\bar{s}'} \mathcal{D}_{\pi^+ \pi^- / q}^{ss'} \mathcal{D}_{\pi^+ \pi^- / \bar{q}}^{\bar{s}\bar{s}'}$$

$$\frac{1}{2} \text{Tr}(\mathcal{D}_{\pi^+ \pi^- / q}) = D_1^q(z_1, M_1),$$

$$\frac{1}{2} \text{Tr}(\sigma_z \mathcal{D}_{\pi^+ \pi^- / q}) = 0,$$

$$\frac{1}{2} \text{Tr}(\sigma_i \mathcal{D}_{\pi^+ \pi^- / q}) = -\frac{\varepsilon_T^{ij} R_{1,T}^j}{|\vec{R}_{1,T}|} H_1^{\triangleleft, q}(z_1, M_1),$$

$$\frac{d\sigma}{dz_1 dz_2 dM_1 dM_2 d\phi_1 d\phi_2} = \sigma_{\text{hard}} \left[\sum_q e_q^2 D_1^q(z_1, M_1) D_1^{\bar{q}}(z_2, M_2) + \frac{1}{2} \sum_q e_q^2 H_1^{\triangleleft, q}(z_1, M_1) H_1^{\triangleleft, \bar{q}}(z_2, M_2) \left(\mathcal{B}_- \cos(\phi_1 + \phi_2) - \mathcal{B}_+ \cos(\phi_1 - \phi_2) \right) \right]$$

Unpolarized diFF

Transverse polarized diFF

$$\mathcal{B}_{\pm} \equiv C_{xx} \pm C_{yy}$$

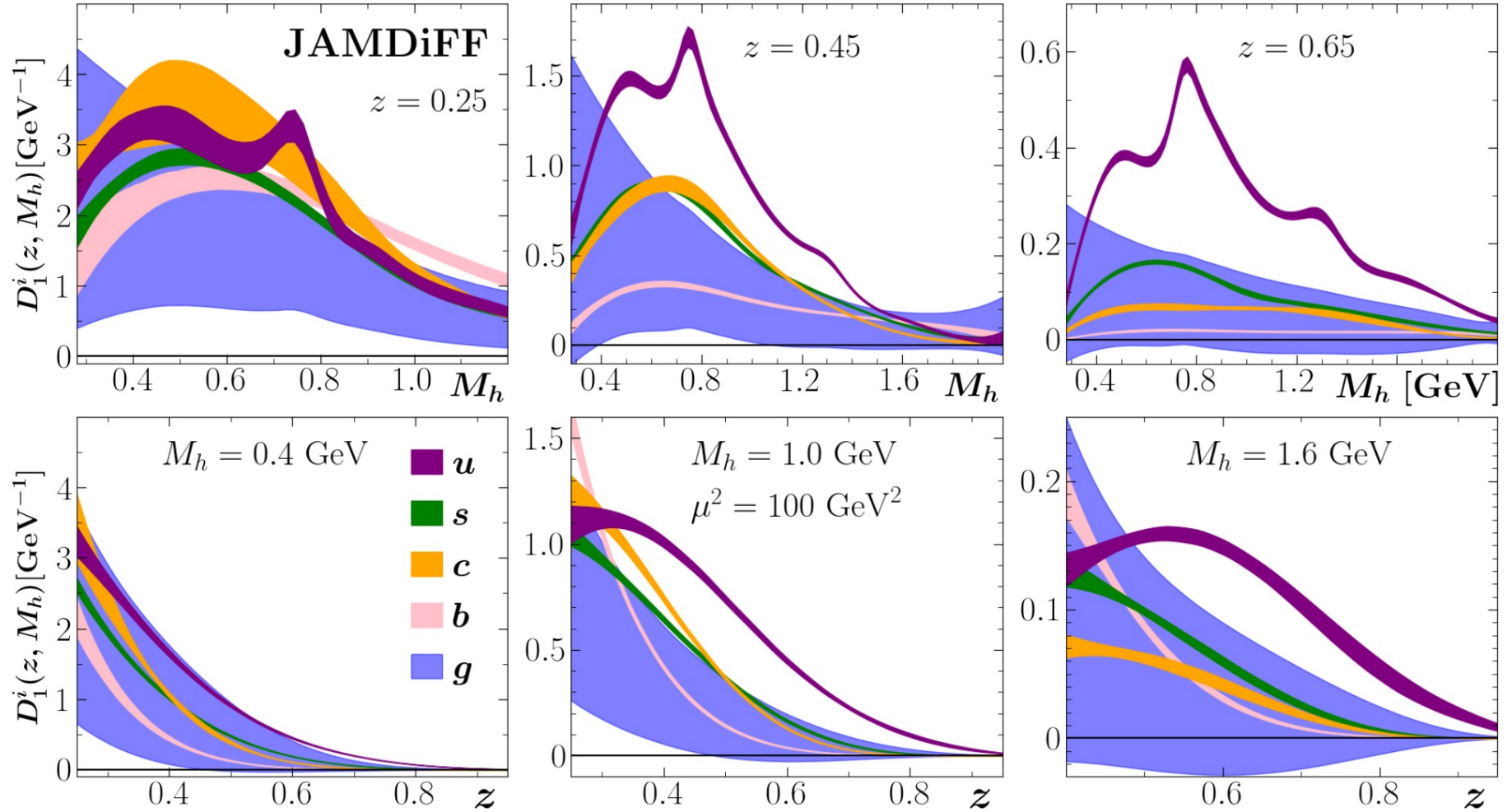
$$\mathcal{B}_+ = 0, \quad \mathcal{B}_- = \frac{2 \sin^2 \Theta}{1 + \cos^2 \Theta}.$$

$$\mathcal{B}_- = \frac{2 \langle \cos(\phi_1 + \phi_2) \rangle}{\alpha_{M_1, M_2}^{z_1, z_2}} = \frac{A_{12}}{\alpha_{M_1, M_2}^{z_1, z_2}}$$

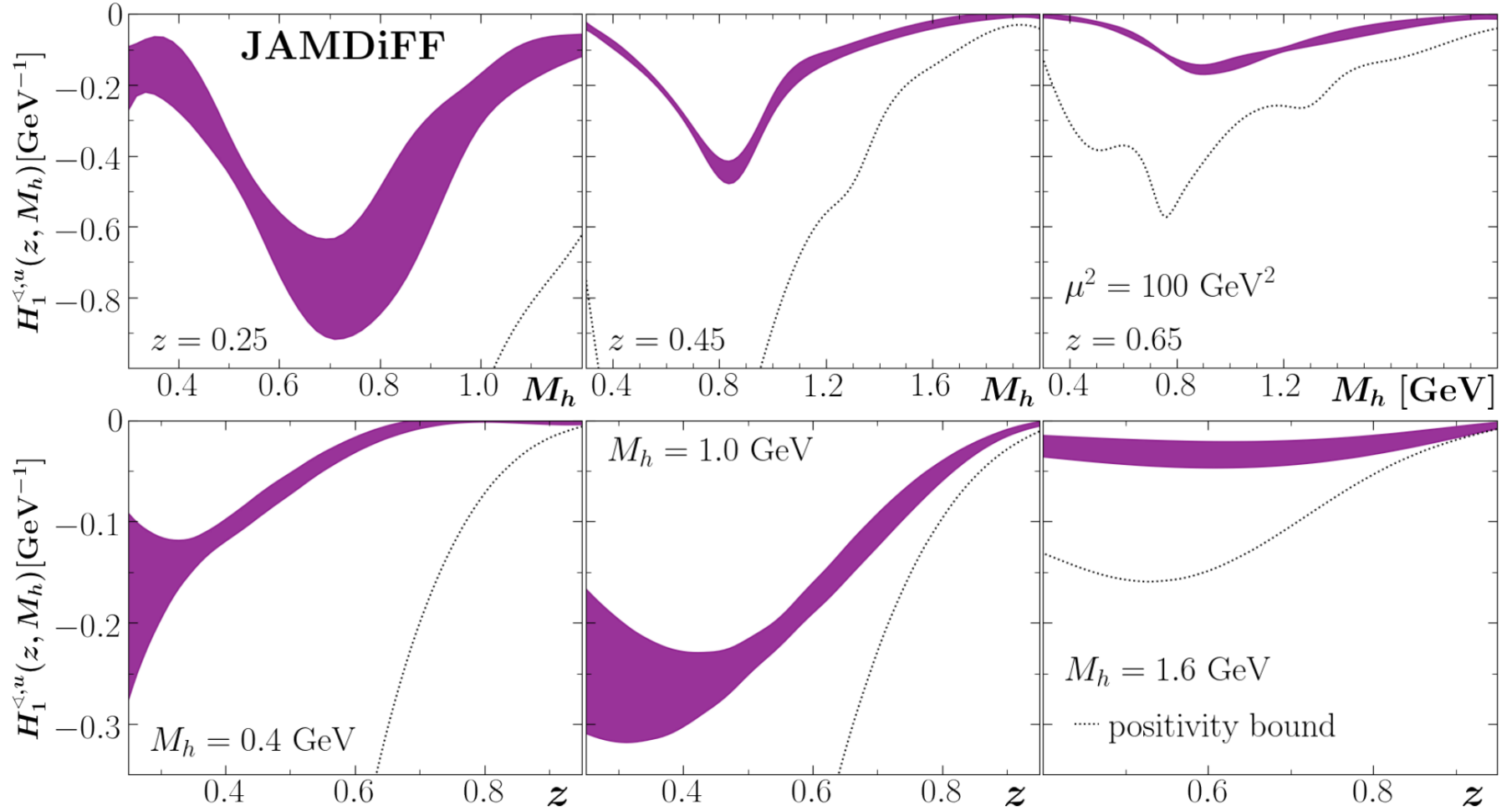
$$\alpha_{M_1, M_2}^{z_1, z_2} = \frac{1}{2} \frac{\sum_q e_q^2 H_1^{\triangleleft, q}(z_1, M_1) H_1^{\triangleleft, \bar{q}}(z_2, M_2)}{\sum_q e_q^2 D_1^q(z_1, M_1) D_1^{\bar{q}}(z_2, M_2)}.$$

$$\text{CHSH type Bell inequality} \quad |\mathcal{B}| > \sqrt{2}$$

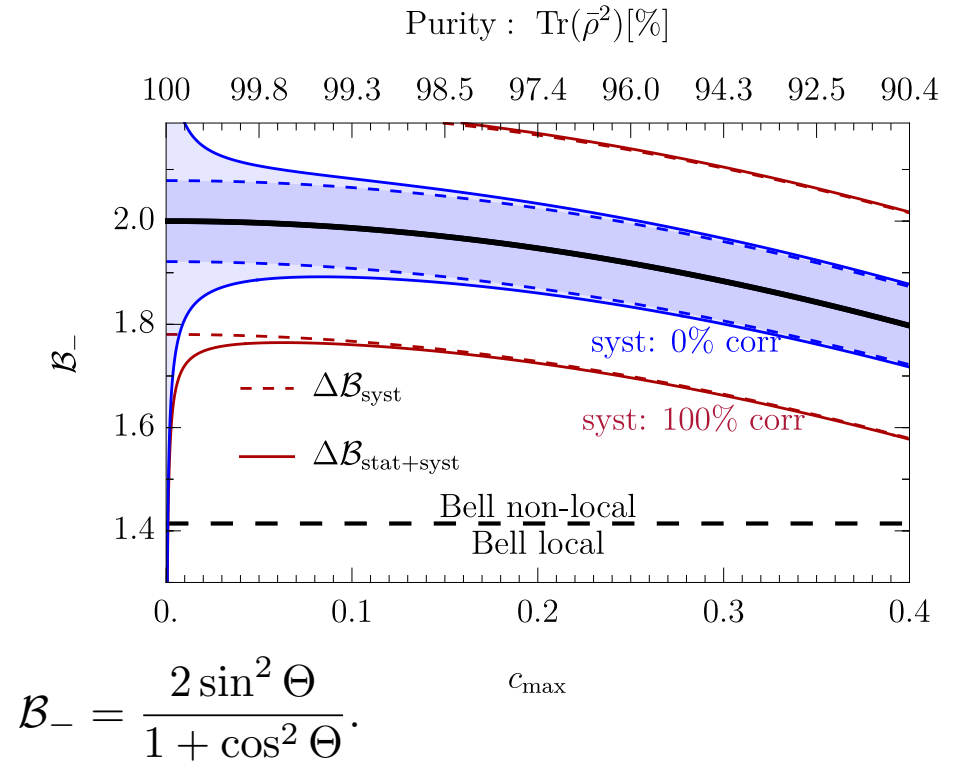
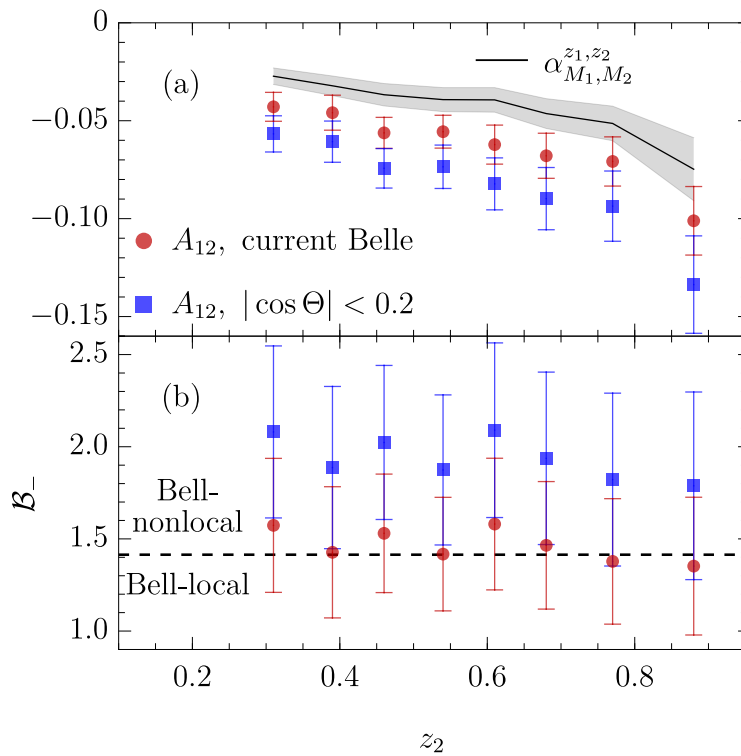
$\pi^+\pi^-$ Dihadron fragmentation functions



$\pi^+\pi^-$ Dihadron fragmentation functions



Dihadron pair production



$$B_- = \frac{2 \sin^2 \Theta}{1 + \cos^2 \Theta}.$$

- ❖ Current data exhibiting no significant evidence of Bell inequality violation
- ❖ The optimal cuts on scattering angle will significantly improve the results
- ❖ The light quark pair would be a **highly pure spin Bell state**
- ❖ Combined results: **2.5 σ for 100% correlated** systematic uncertainties **and 6.7 σ for the uncorrelated case**

Summary

- The era of quantum information study in colliders has just began
- Prior work focused only on massive particles with perturbative decays, leaving **massless particles unexplored**
- We proposed studying entanglement and Bell inequalities in massless quark pair via the **hadron final states by the fragmentation mechanism**
- The azimuthal correlations in Belle's $\pi^+\pi^-$ dihadron pair could probe **Bell inequality** for massless quarks, **with $> 5\sigma$ significance** when considering uncorrelated systematic uncertainties

Thank you