第四届高能物理理论与实验融合发展研讨会

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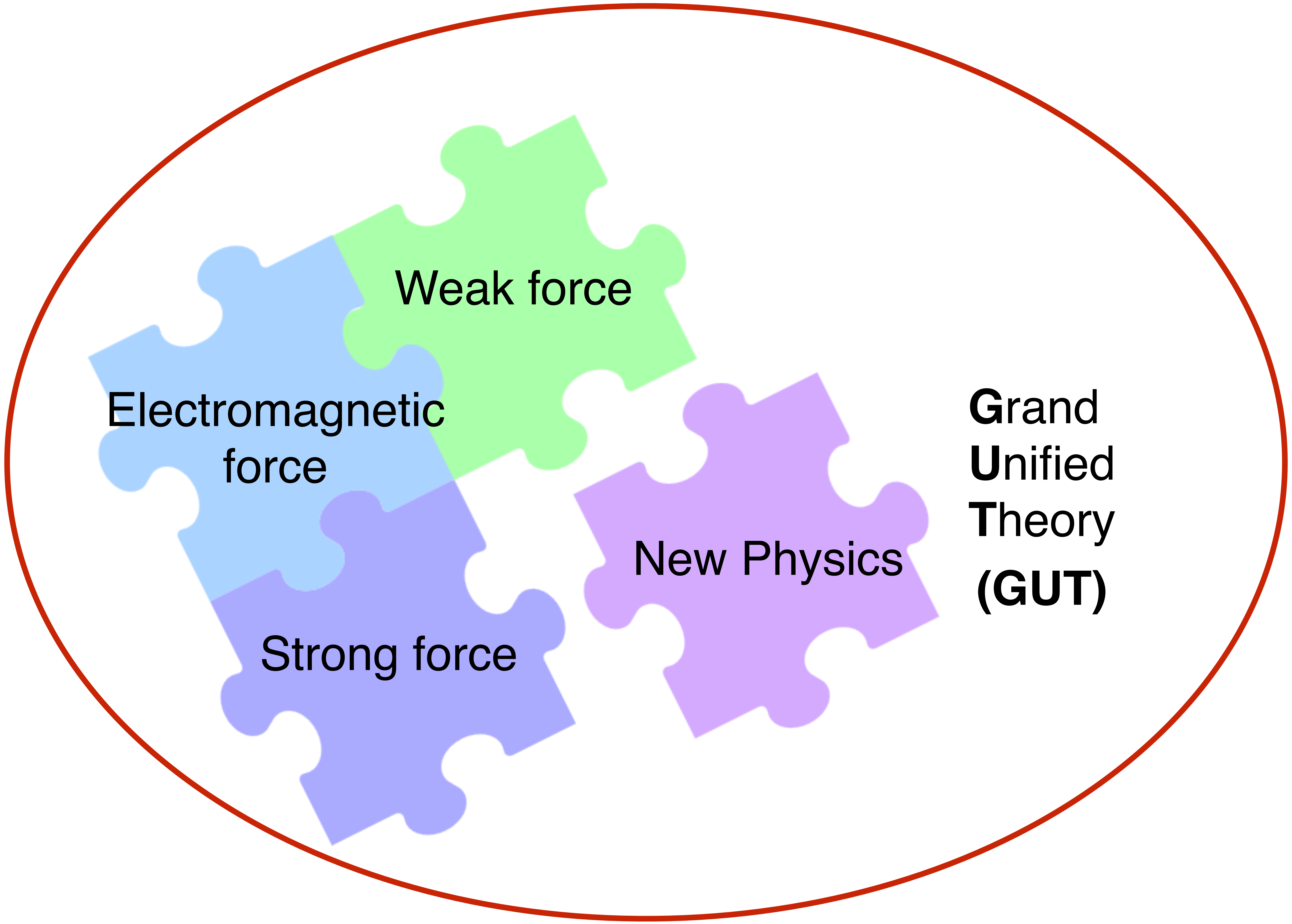
Asymptotic GUT in extra dimension

周 也 铃 2025-09-20

國科大杭州高等研究院
Hangzhou Institute for Advanced Study, UCAS

基础物理与数学科学学院
School of Fundamental Physics and Mathematical Sciences

Introduction



Gravity… not included

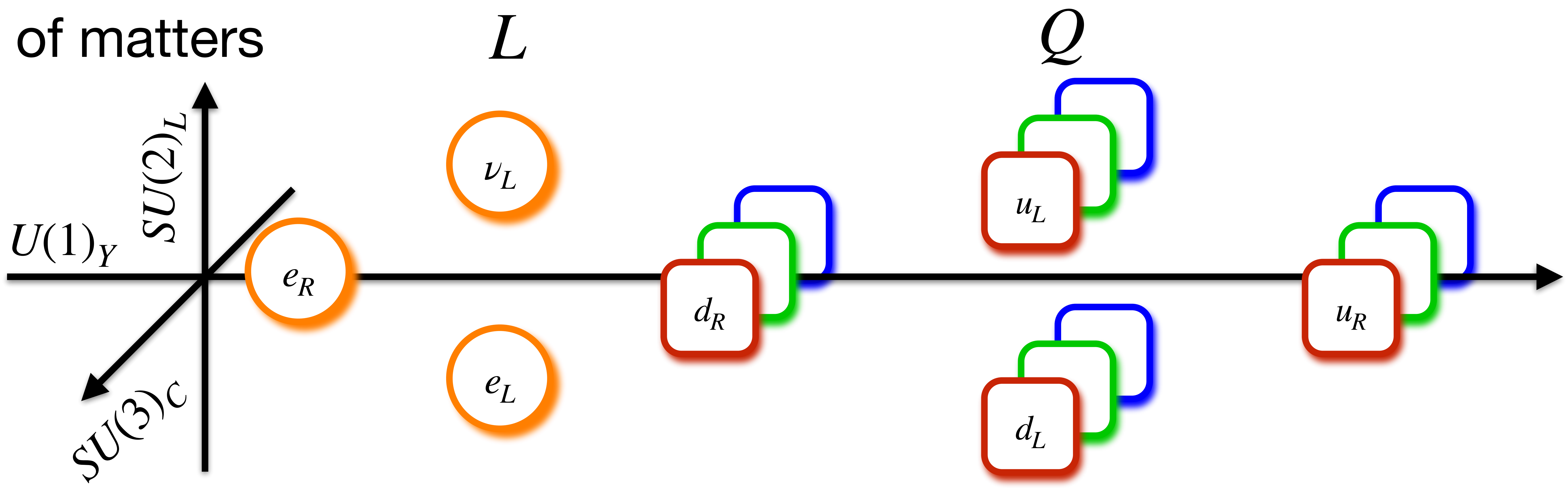
Basics of GUTs

- Unification of symmetries $G_{\rm GUT} \supset G_{\rm SM} = SU(3)_C \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{EW}}$

- Unification of couplings $g_3 = g_2 = g_1$ up to a loop factor for a simple Lee group

The scale where three gauge couplings are unified, denoted as $M_{\rm GUT}$ in this talk

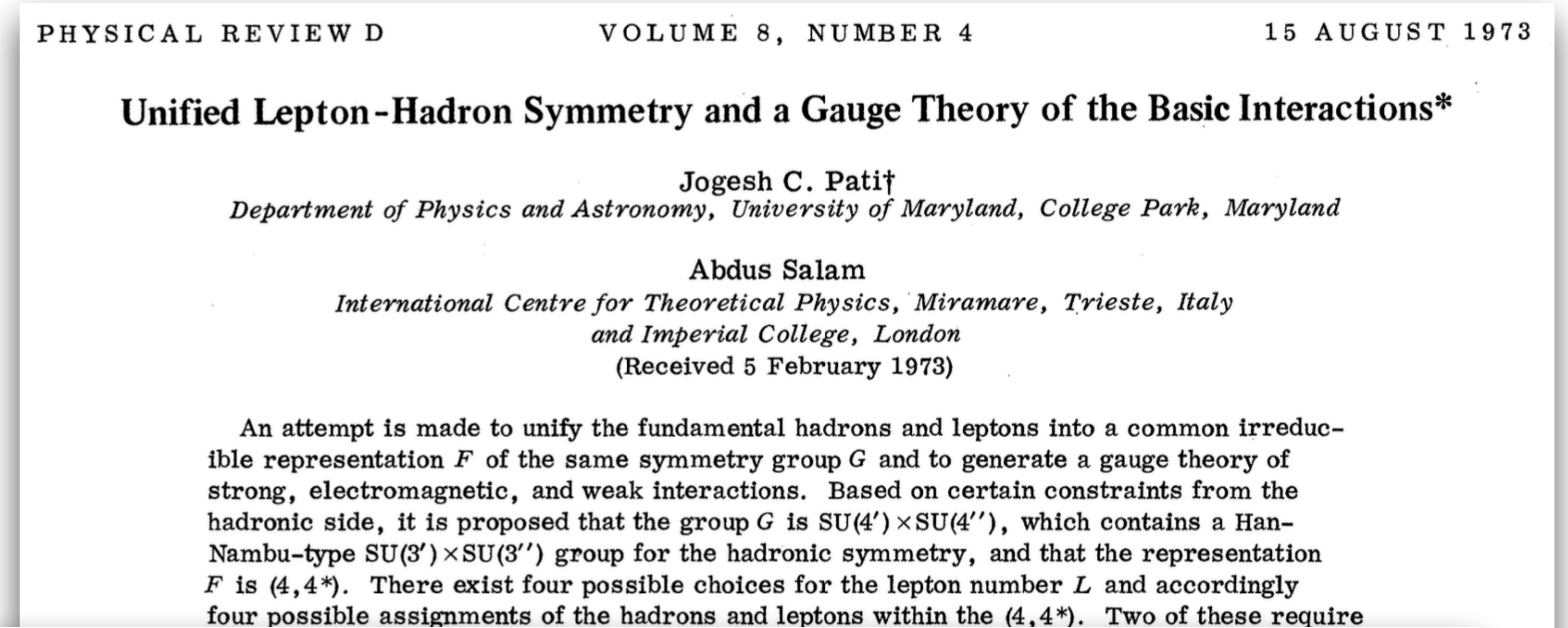
- Unification of matters



Weak hypercharge: $Y = -1$ $\quad Y = -\frac{1}{2}$ $\quad Y = -\frac{1}{3}$ $\quad Y = \frac{1}{6}$ $\quad Y = \frac{2}{3}$

GUT models

Pati-Salam (1973, 1974) $SU(4)_c \times SU(2)_L \times SU(2)_R := G_{422}$

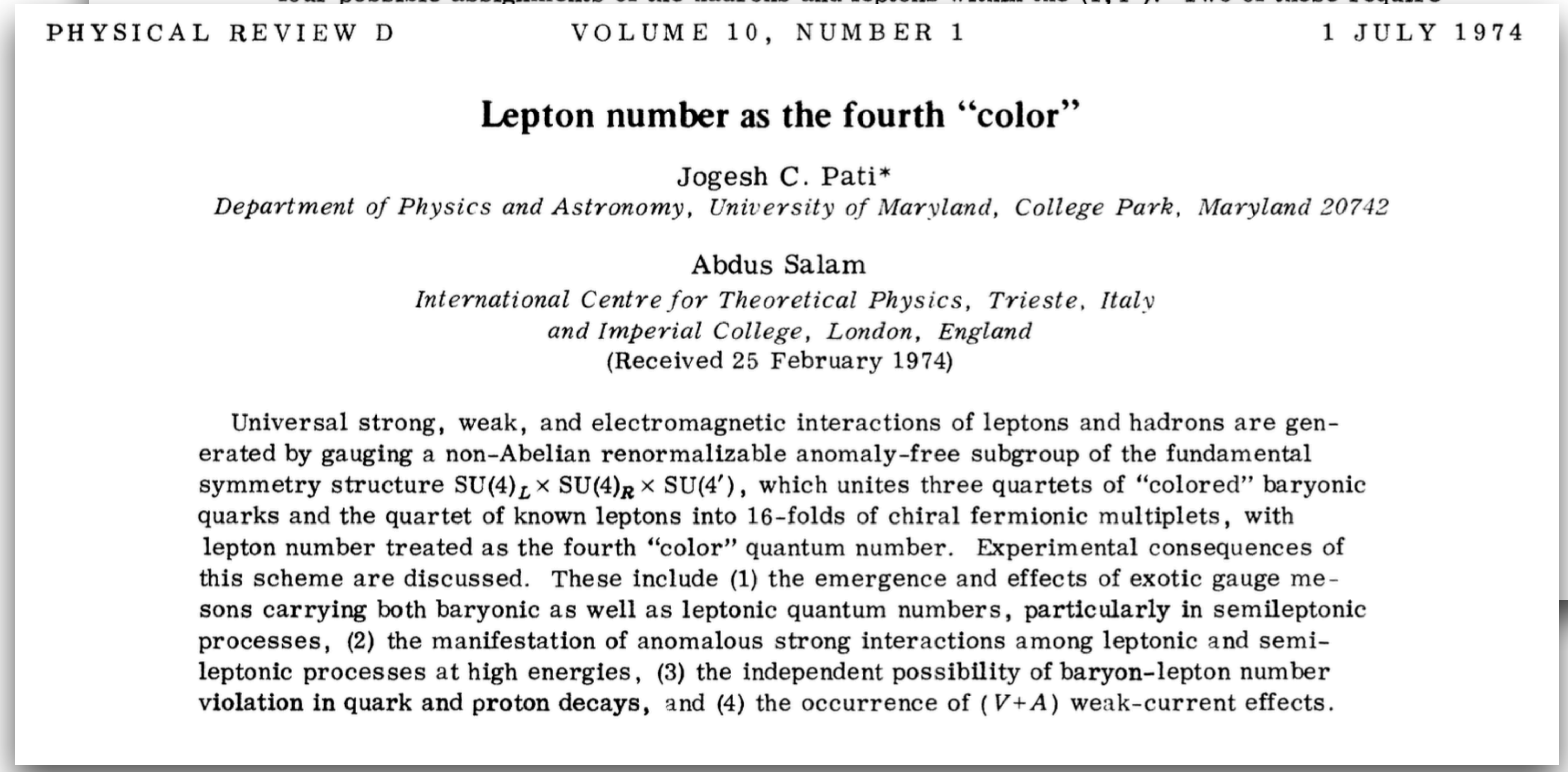

PHYSICAL REVIEW D VOLUME 8, NUMBER 4 15 AUGUST 1973

Unified Lepton-Hadron Symmetry and a Gauge Theory of the Basic Interactions*

Jogesh C. Pati†
Department of Physics and Astronomy, University of Maryland, College Park, Maryland

Abdus Salam
International Centre for Theoretical Physics, Miramare, Trieste, Italy and Imperial College, London
(Received 5 February 1973)

An attempt is made to unify the fundamental hadrons and leptons into a common irreducible representation F of the same symmetry group G and to generate a gauge theory of strong, electromagnetic, and weak interactions. Based on certain constraints from the hadronic side, it is proposed that the group G is SU(4′)×SU(4″), which contains a Han-Nambu-type SU(3′)×SU(3″) group for the hadronic symmetry, and that the representation F is (4,4*). There exist four possible choices for the lepton number L and accordingly four possible assignments of the hadrons and leptons within the (4,4*). Two of these require

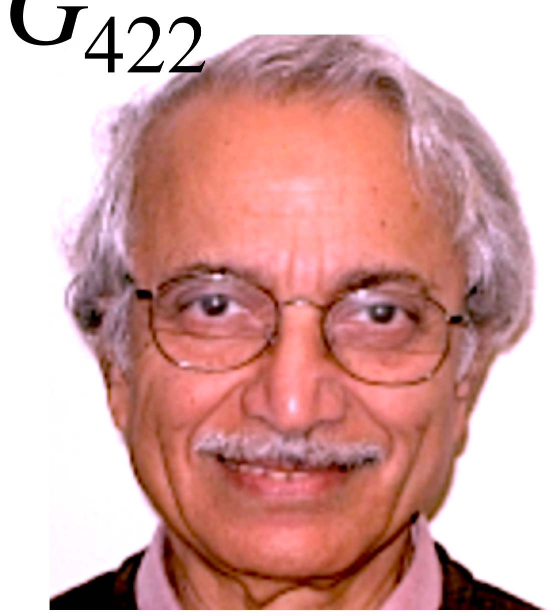

PHYSICAL REVIEW D VOLUME 10, NUMBER 1 1 JULY 1974

Lepton number as the fourth "color"

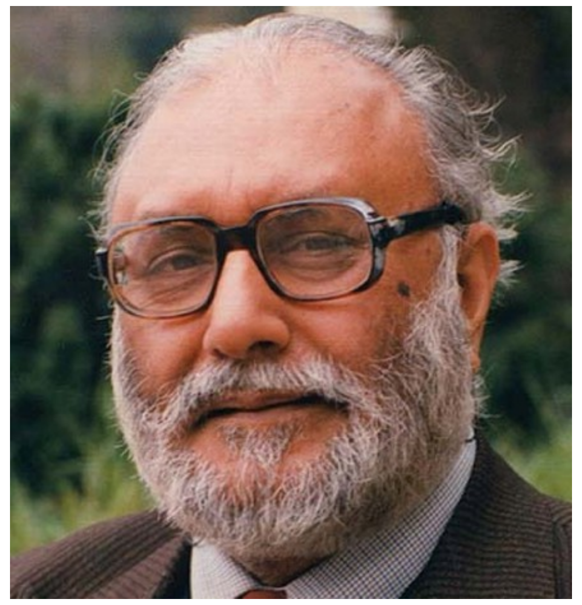
Jogesh C. Pati*
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

Abdus Salam
International Centre for Theoretical Physics, Trieste, Italy and Imperial College, London, England
(Received 25 February 1974)

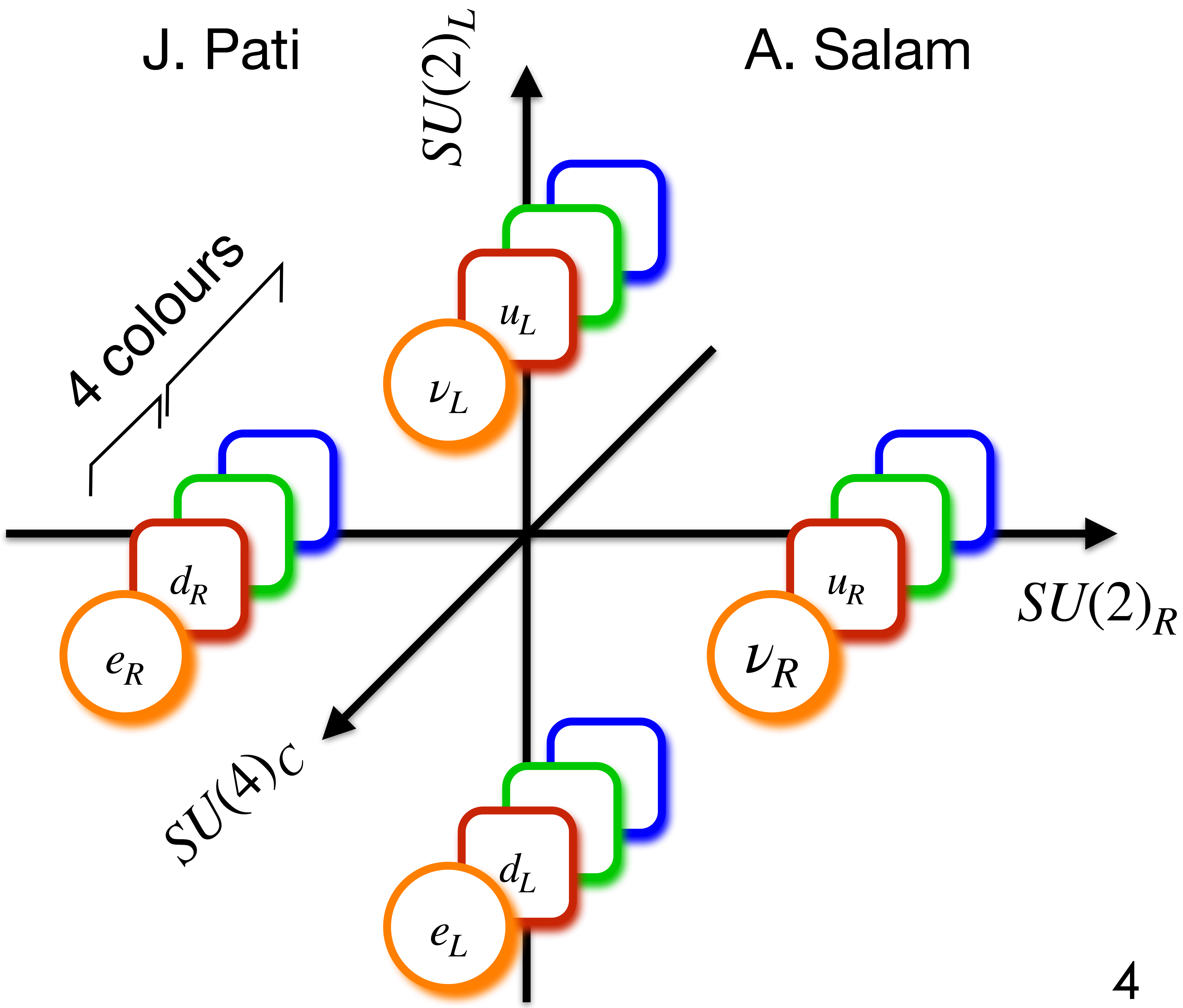
Universal strong, weak, and electromagnetic interactions of leptons and hadrons are generated by gauging a non-Abelian renormalizable anomaly-free subgroup of the fundamental symmetry structure $SU(4)_L \times SU(4)_R \times SU(4')$, which unites three quartets of "colored" baryonic quarks and the quartet of known leptons into 16-folds of chiral fermionic multiplets, with lepton number treated as the fourth "color" quantum number. Experimental consequences of this scheme are discussed. These include (1) the emergence and effects of exotic gauge mesons carrying both baryonic as well as leptonic quantum numbers, particularly in semileptonic processes, (2) the manifestation of anomalous strong interactions among leptonic and semileptonic processes at high energies, (3) the independent possibility of baryon-lepton number violation in quark and proton decays, and (4) the occurrence of ($V+A$) weak-current effects.



J. Pati

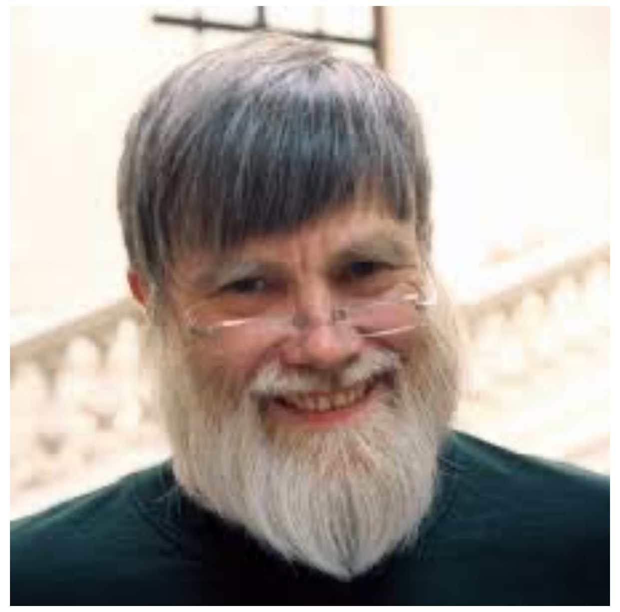


A. Salam

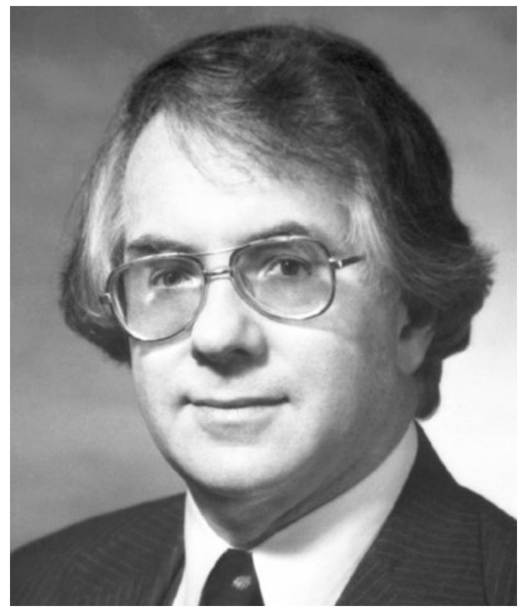


GUT models

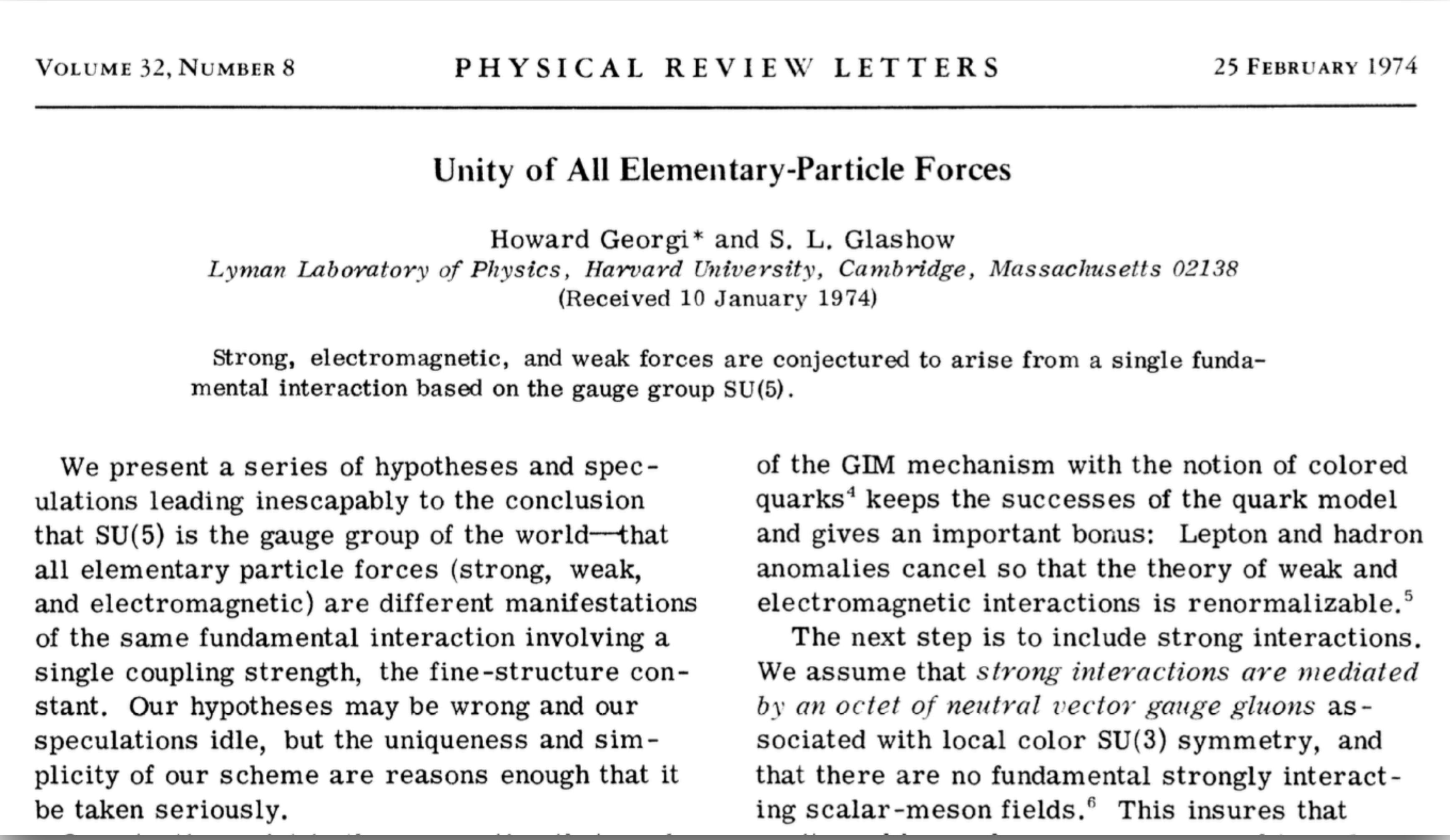
Georgi-Glashow (1974), $SU(5)$ GUT



H. Georgi



S. Glashow



VOLUME 32, NUMBER 8 PHYSICAL REVIEW LETTERS 25 FEBRUARY 1974

Unity of All Elementary-Particle Forces

Howard Georgi* and S. L. Glashow
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 10 January 1974)

Strong, electromagnetic, and weak forces are conjectured to arise from a single fundamental interaction based on the gauge group SU(5).

We present a series of hypotheses and speculations leading inescapably to the conclusion that SU(5) is the gauge group of the world—that all elementary particle forces (strong, weak, and electromagnetic) are different manifestations of the same fundamental interaction involving a single coupling strength, the fine-structure constant. Our hypotheses may be wrong and our speculations idle, but the uniqueness and simplicity of our scheme are reasons enough that it be taken seriously.

of the GIM mechanism with the notion of colored quarks[4] keeps the successes of the quark model and gives an important bonus: Lepton and hadron anomalies cancel so that the theory of weak and electromagnetic interactions is renormalizable.[5]

The next step is to include strong interactions. We assume that *strong interactions are mediated by an octet of neutral vector gauge gluons* associated with local color SU(3) symmetry, and that there are no fundamental strongly interacting scalar-meson fields.[6] This insures that

$$\bar{\mathbf{5}} \Rightarrow \begin{pmatrix} (d_R^1)^c \\ (d_R^2)^c \\ (d_R^3)^c \\ e_L \\ -\nu_L \end{pmatrix}$$

$$\mathbf{10} \Downarrow$$

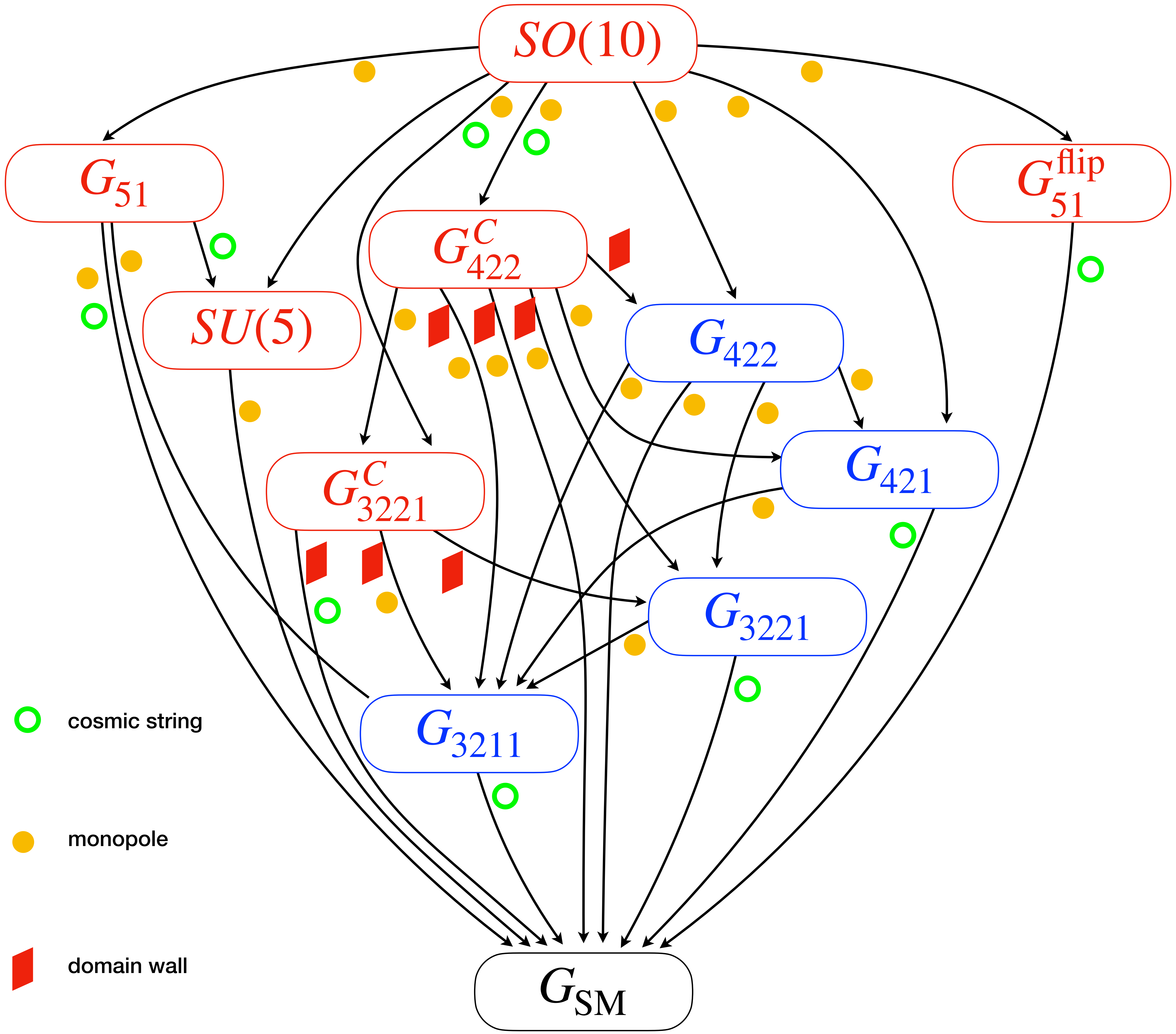
$$\begin{pmatrix} 0 & (u_R^3)^c & -(u_R^2)^c & -u_L^1 & -d_L^1 \\ -(u_R^3)^c & 0 & (u_R^1)^c & -u_L^2 & -d_L^2 \\ (d_R^2)^c & -(d_R^1)^c & 0 & -u_L^3 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 & 0 & (e_R)^c \\ d_L^1 & d_L^2 & d_L^3 & -(e_R)^c & 0 \end{pmatrix}$$

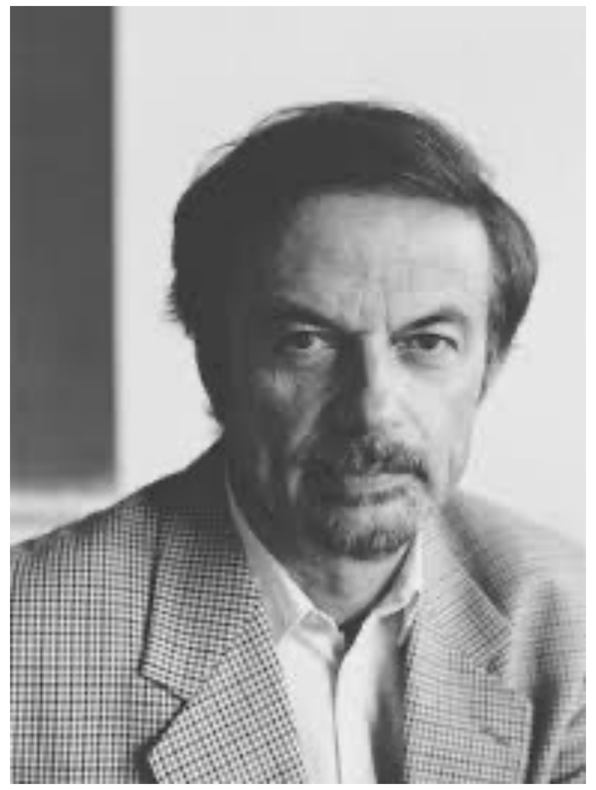
And Higgses $\mathbf{5}, \mathbf{45}, \mathbf{24}$.

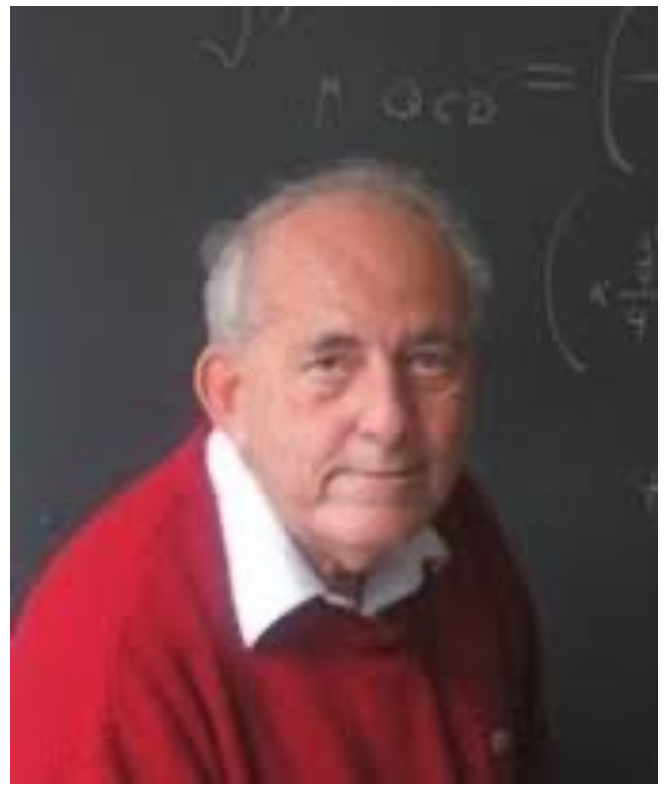
GUT models

SO(10) GUT

Fritzsch, Minkowski (1975)







$$G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$$

$$G_{51} = SU(5) \times U(1)$$

$$G_{421} = SU(4)_c \times SU(2)_L \times U(1)_R$$

$$G_{3221} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$G_{3211} = SU(3)_c \times SU(2)_L \times SU(1)_Y \times U(1)_{B-L}$$

C: parity $\psi_L \leftrightarrow \psi_R^C$

flip: isospin flipping $u \leftrightarrow d, \nu \leftrightarrow e$

RG running of gauge couplings

Given $\alpha_i = \dfrac{g_i^2}{4\pi}$ for gauge coupling g_i of group G_i $G_i = SU(3)_c,\ SU(2)_L,\ U(1)_Y, \cdots$

$$\frac{\mathrm{d}\alpha_i}{\mathrm{d}t} = \beta_i$$

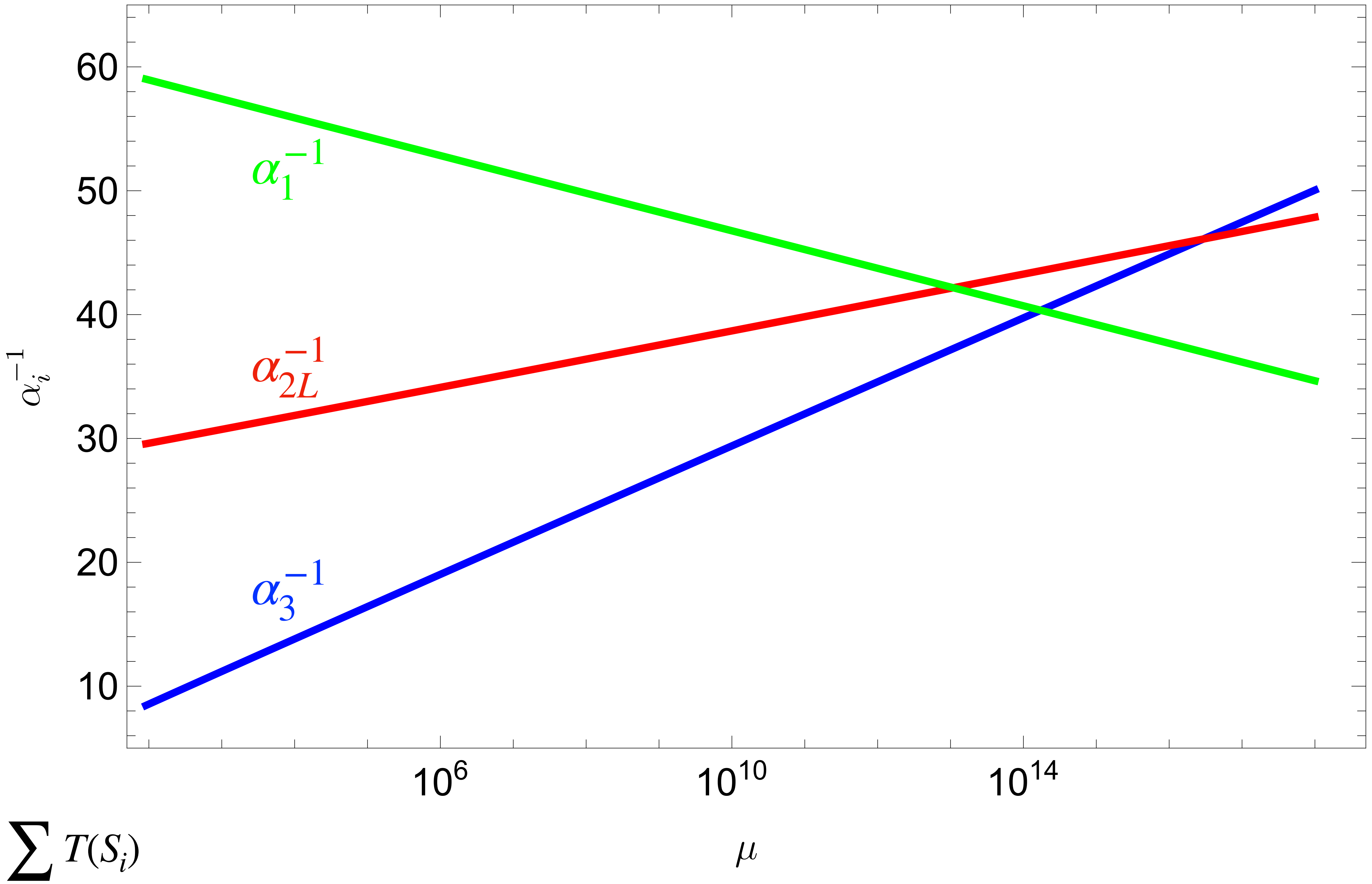
$$t = \log\frac{\mu}{\mu_0}$$

β function at leading order

$$\beta_i = b_i\frac{\alpha_i^2}{2\pi} + \cdots$$

$$b_i = -\frac{11}{3}C_2(G_i) + \frac{4}{3}\sum_F T(F_i) + \frac{1}{6}\sum_S T(S_i)$$

$M_Z \xrightarrow{\ G_{\rm SM}\ }$ Unification scale?



$$b_3 = -7$$

$$b_{2L} = -\frac{19}{6}$$

$$b_1 = \frac{41}{10}$$

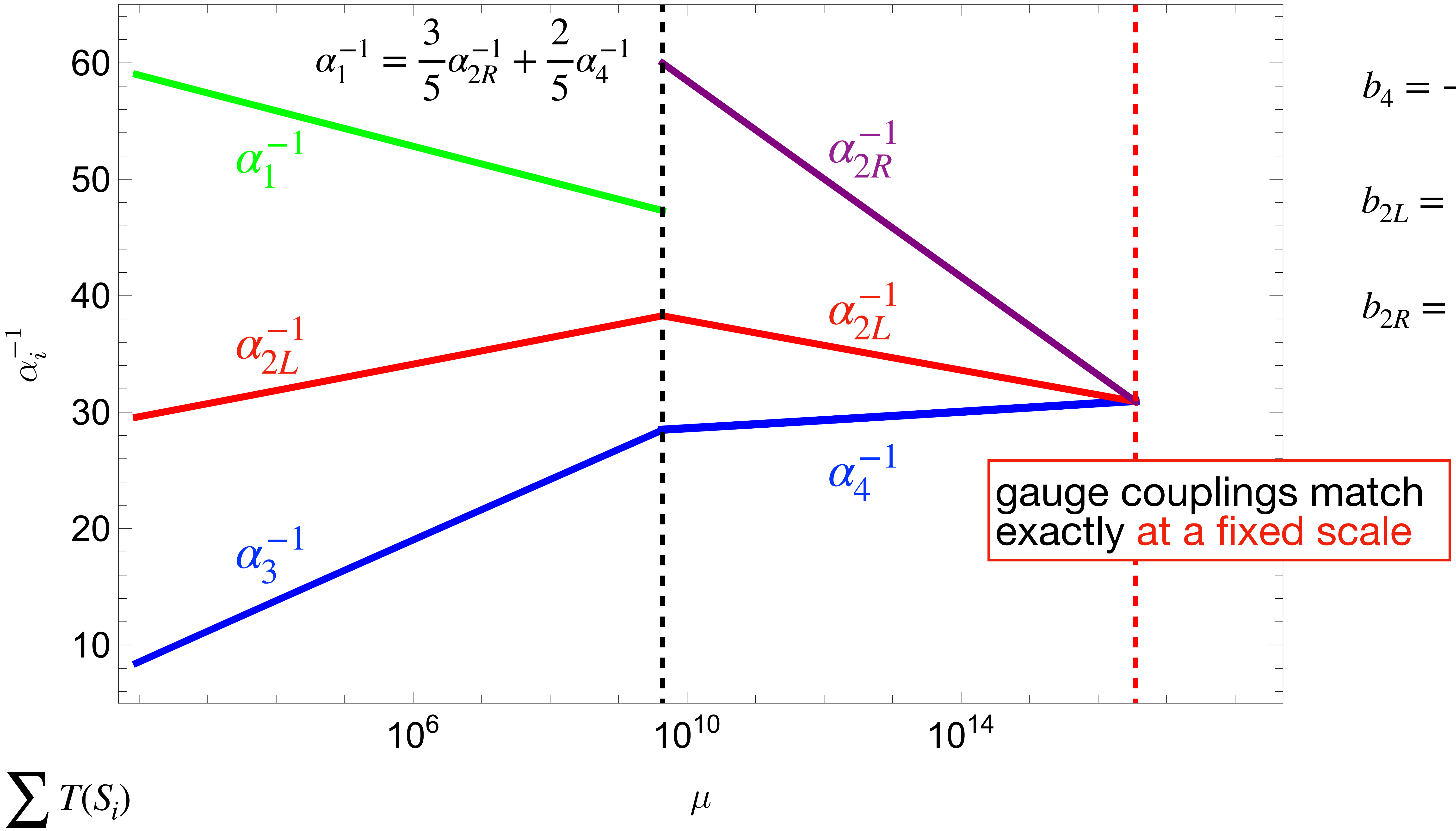
RG running of gauge couplings

Given $\alpha_i = \dfrac{g_i^2}{4\pi}$ for gauge coupling g_i of group G_i $\qquad G_i = SU(4)_c,\, SU(2)_L,\, SU(2)_R, \cdots$

$$M_Z \xrightarrow{\;G_{\rm SM}\;} M_{\rm PS} \xrightarrow{\;G_{422}\;} M_X \text{ (GUT scale)}$$

$$G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$$

$$G_{\rm SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$



$b_4 = -\dfrac{7}{3}$

$b_{2L} = 2$

$b_{2R} = \dfrac{28}{3}$

$$b_i = -\frac{11}{3}C_2(G_i) + \frac{4}{3}\sum_F T(F_i) + \frac{1}{6}\sum_S T(S_i)$$

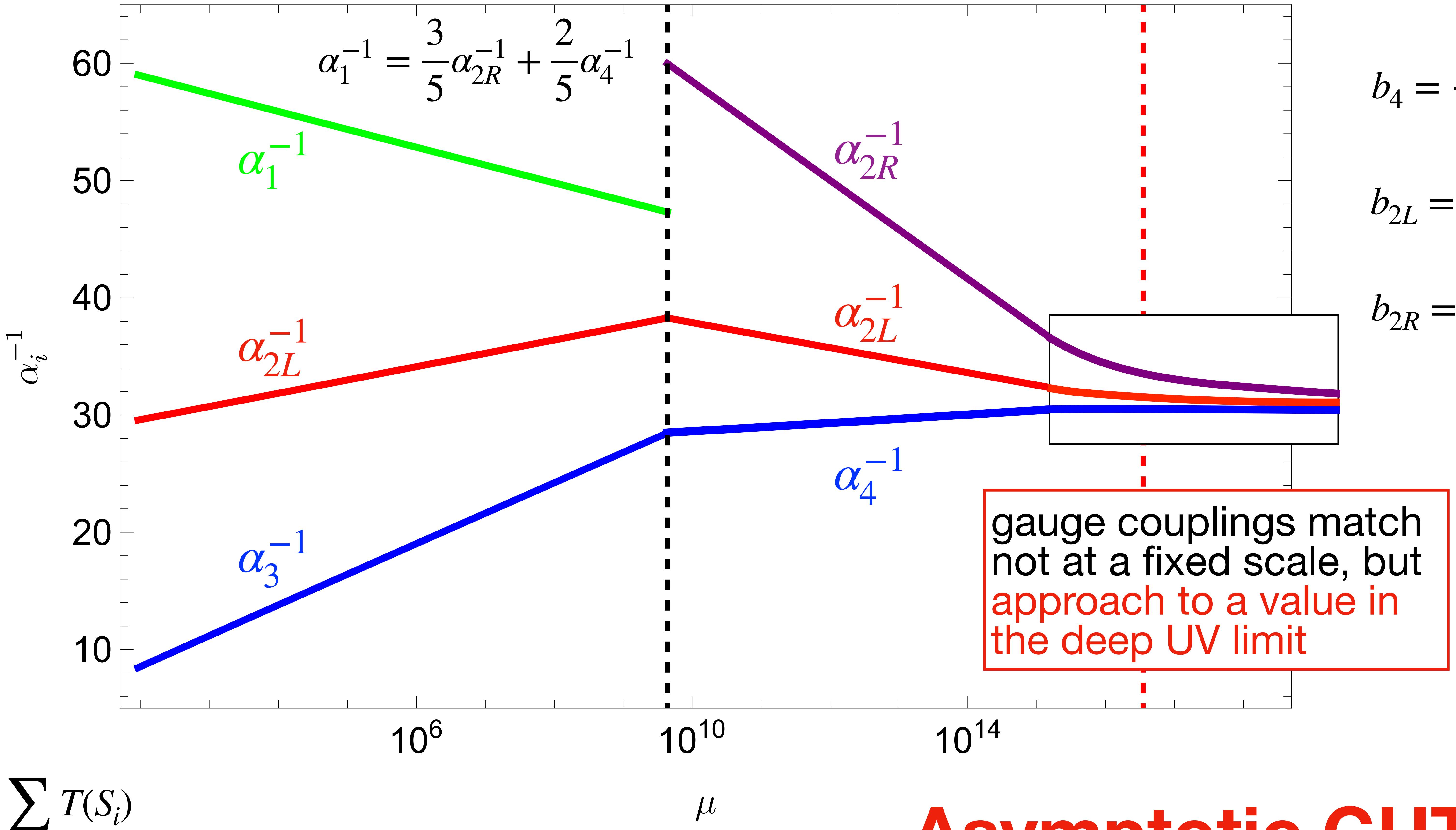
RG running of gauge couplings

Given $\alpha_i = \dfrac{g_i^2}{4\pi}$ for gauge coupling g_i of group G_i $\quad G_i = SU(4)_c, SU(2)_L, SU(2)_R, \cdots$

$$M_Z \xrightarrow{G_{\rm SM}} M_{\rm PS} \xrightarrow{G_{422}} M_X \text{ (GUT scale)}$$

$G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$

$G_{\rm SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$



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Asymptotic GUT

UV behaviour of GUT

- RGE of gauge coupling above the GUT scale

take SO(10) as an example, $\alpha_{10} = \dfrac{g_{10}^2}{4\pi}$

$$2\pi \frac{\mathrm{d}\alpha_{10}}{\mathrm{d}t} = b_{10}\alpha_{10}^2 \qquad t = \log(\mu/\mu_0)$$

$$\boxed{\alpha_{10}(\mu > M_X) = \frac{\alpha_{10}(M_X)}{1 - \alpha_{10}(M_X)\frac{b_{10}}{2\pi}\log(\frac{\mu}{M_X})}}$$

b_{10}: depending on fermion and Higgs particle contents of the model

Higgs contents	b_{10}
$(\mathbf{10}_\mathrm{c}, \overline{\mathbf{126}}, \mathbf{45}_\mathrm{c})$	$-\frac{32}{3}$
$(\mathbf{10}_\mathrm{r}, \mathbf{120}_\mathrm{r}, \overline{\mathbf{126}}, \mathbf{45}_\mathrm{r})$	$-\frac{15}{2}$
$(\mathbf{10}_\mathrm{c}, \mathbf{120}_\mathrm{c}, \overline{\mathbf{126}}, \mathbf{45}_\mathrm{c})$	$-\frac{4}{3}$
$(\mathbf{10}_\mathrm{c}, \mathbf{120}_\mathrm{c}, \overline{\mathbf{126}}, \mathbf{45}_\mathrm{c}, \mathbf{54}_\mathrm{c})$	$\frac{8}{3}$
$(\mathbf{10}_\mathrm{c}, \mathbf{120}_\mathrm{c}, \overline{\mathbf{126}}, \mathbf{210}_\mathrm{c})$	$\frac{44}{3}$

→ $\alpha_{10} \to \infty$ for $b_{10} > 0$ Landau pole at $\mu = M_X \exp\left(\dfrac{2\pi}{\alpha_{10} b_{10}}\right)$

→ $\alpha_{10} \to 0$ for $b_{10} < 0$ asymptotically approach to $\dfrac{2\pi}{-b_{10}\log(\mu/M_X)}$

UV behaviour of GUT: another possibility?

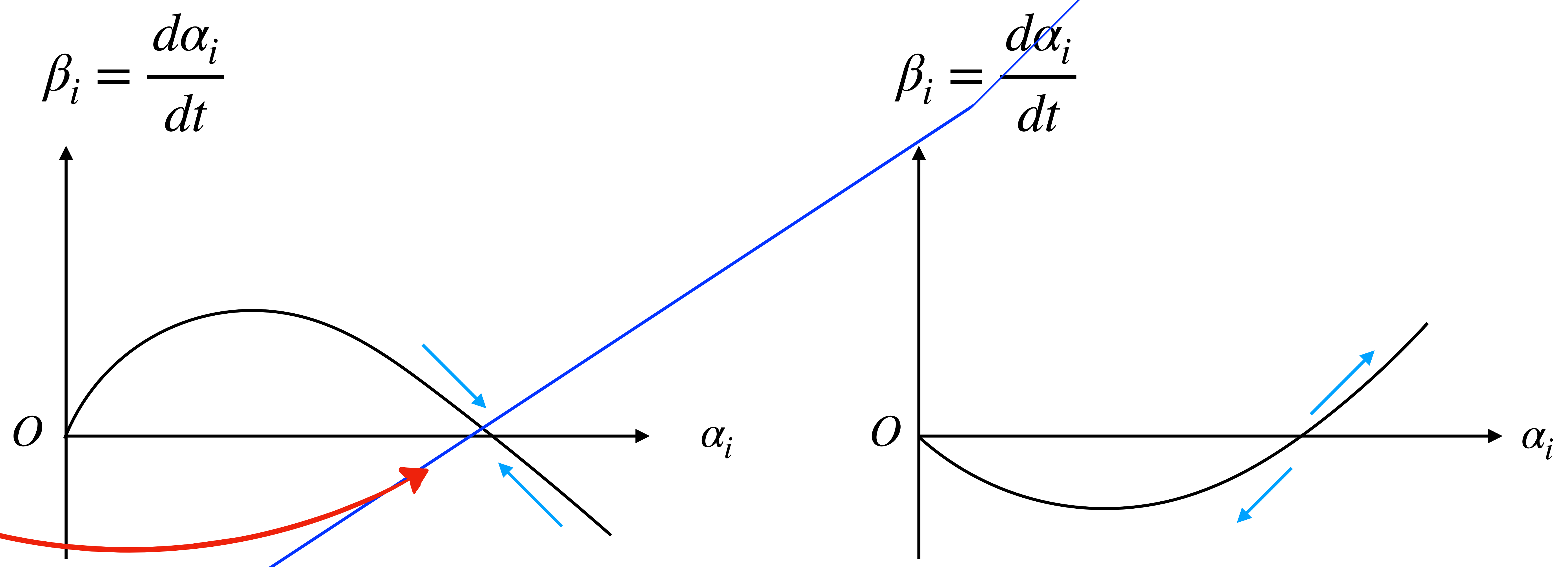
- *A theory is said to be asymptotically safe if the 'essential' coupling parameters approach a fixed point as the momentum scale of their renormalization point goes to infinity.*

  — —S. Weinberg in *Ultraviolet divergences in quantum theories of gravitation*, 1979

- Fixed points in β function

Asymptotic safety in the UV limit

α_{10} might approach to a fixed, non-vanishing and finite value.
⇒The third phase for UV behaviour of gauge theories



stable, $\frac{d^2\alpha_i}{dt^2} = \frac{d\beta_i}{dt} < 0$ unstable, $\frac{d^2\alpha_i}{dt^2} = \frac{d\beta_i}{dt} > 0$

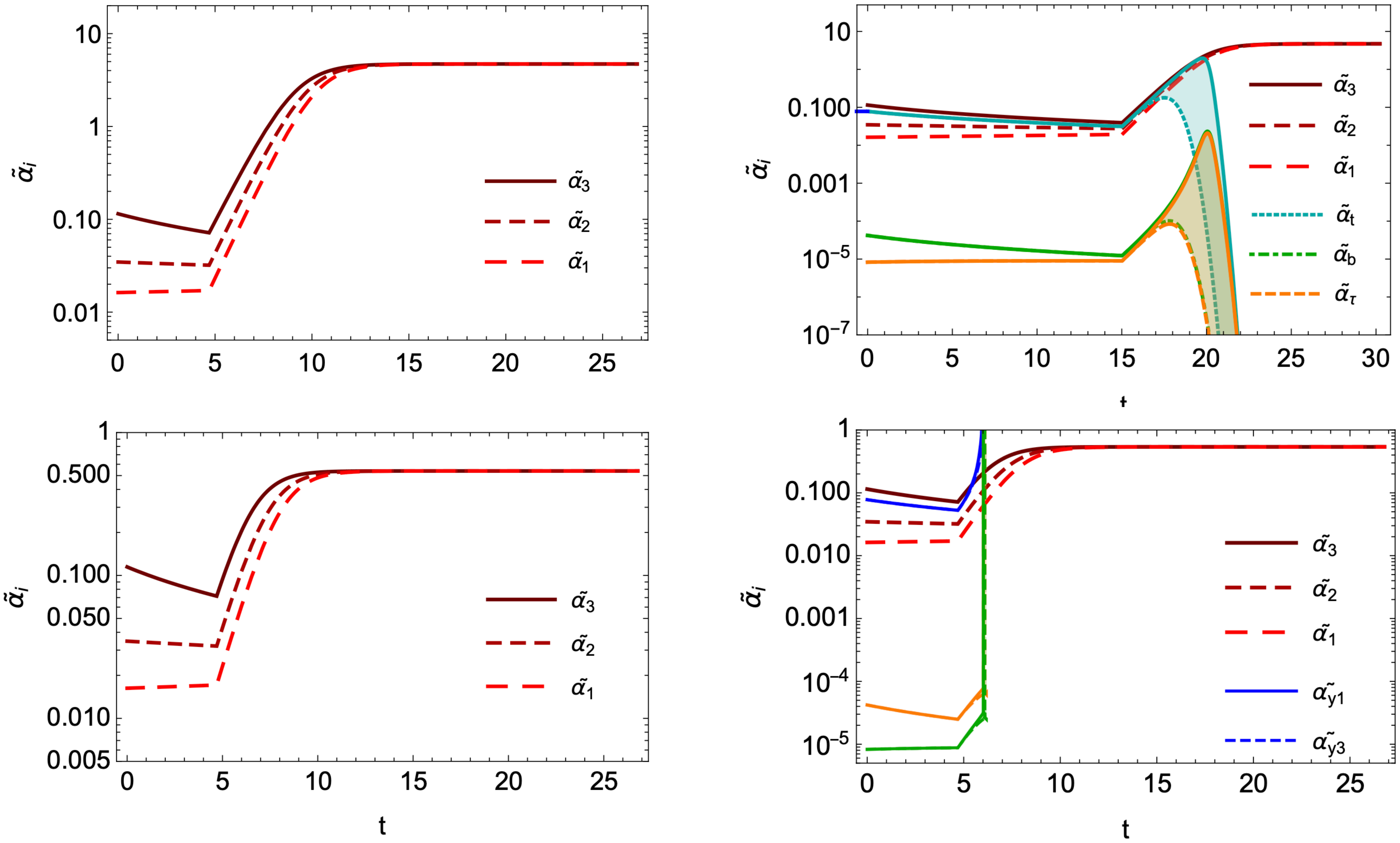
- RG running of Yukawa couplings above the GUT scale

  (in particular, the top quark Yukawa coupling)

→ asymptotically free?
→ asymptotically safe?
→ Landau pole?

Asymptotic GUT in 5D

Cacciapaglia, and Cornell, Cot, Deandrea, 2012.14732; 2210.03596

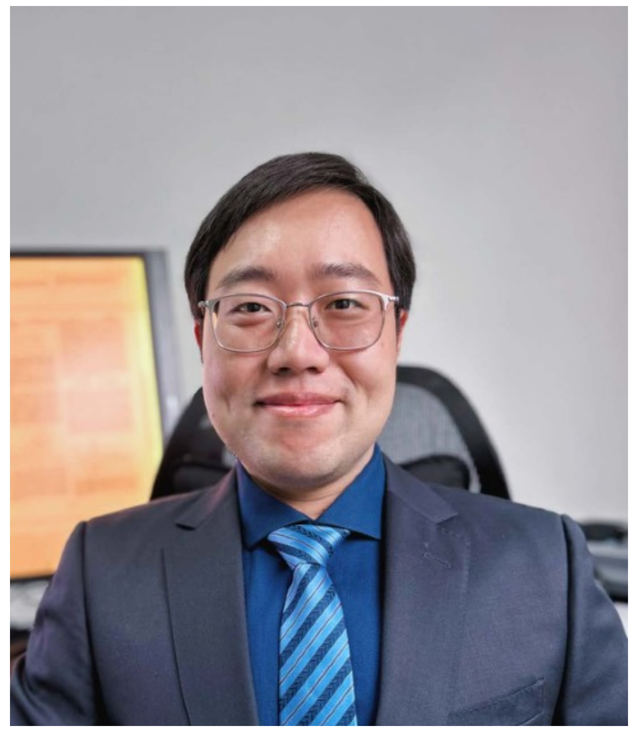


5D SU(5) ✔

5D SO(10) ✘

A successful and realistic SO(10) aGUT

Gao-Xiang Fang



Zhi-Wei Wang

Asymptotic grand unification in SO(10) with one extra dimension

Fang, Wang, YLZ, arXiv:2505.08068

- Breaking chain 5D SO(10) $\xrightarrow[\text{BC}]{M_{\rm KK}\sim 10^{10}\ \rm GeV}$ 4D Pati-Salam $\xrightarrow[\mathbf{16}]{M_{\rm PS}\sim 10^{6}\ \rm GeV}$ SM

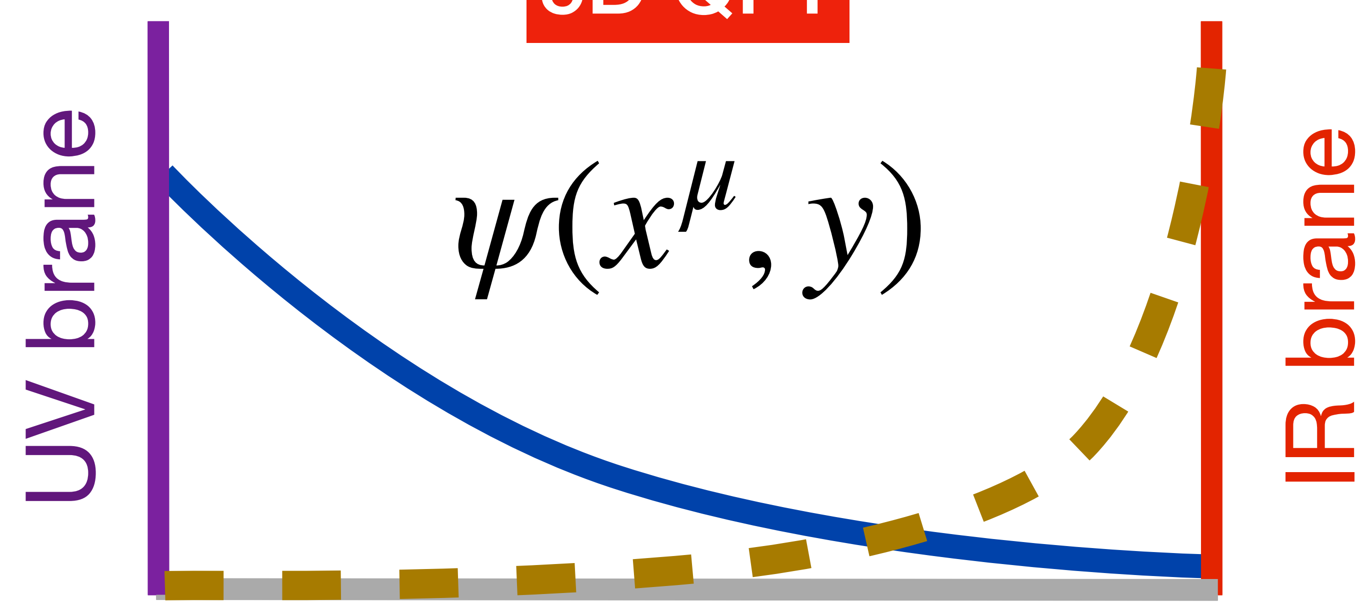
- Particle contents

Energy scale	Symmetry	Fermion	Higgs
$\mu > M_{\rm KK}$	$SO(10)$	$\Psi_{\mathbf{16}} \sim \mathbf{16}$ $\Psi_{\overline{\mathbf{16}}} \sim \overline{\mathbf{16}}$ $\nu_{\rm S} \sim \mathbf{1}$	$H_{\mathbf{10}} \sim \mathbf{10}$, complex $H_{\mathbf{120}} \sim \mathbf{120}$, real $H_{\mathbf{16}} \sim \mathbf{16}$
$M_{\rm PS} < \mu < M_{\rm KK}$	G_{422}	$\psi_L \sim (4,2,1)$ $\psi_R \sim (4,1,2)$ $\nu_{\rm S} \sim (1,1,1)$	$h_1, h_1' \sim (1,2,2), h_{15} \sim (15,2,2)$ $h_{\bar{4}} \sim (\bar{4},1,2)$
$M_Z \ll \mu < M_{\rm PS}$	$G_{\rm SM}$	q_L, d_R, u_R $l_L, \nu_R, e_R, \nu_{\rm S}$	$h_{\rm SM}$

No $\overline{\mathbf{126}}$ Higgs

5D GUTs

5D QFT



One extra dimension $S_1/(Z_2 \times Z_2)$

$\simeq$

4D EFT

SM particles KK modes

$$\psi_0(x^\mu) + \sum_{\mathrm{KK}=1}^{\mathrm{dof}} \psi_{\mathrm{KK}}(x^\mu)$$

$$\mathrm{dof} = \begin{cases} 1, & \mu < M_{\mathrm{KK}} \\ [\mu/M_{\mathrm{KK}}], & \mu > M_{\mathrm{KK}} \end{cases}$$

$$S = \int \mathrm{d}^4x\mathrm{d}y\, \mathscr{L}_{5\mathrm{D}} = \int \mathrm{d}^4x \mathscr{L}_{4\mathrm{D}} + \sum_{\mathrm{KK}=1}^{\mathrm{dof}} \int \mathrm{d}^4x\, \mathscr{L}_{\mathrm{KK}}$$

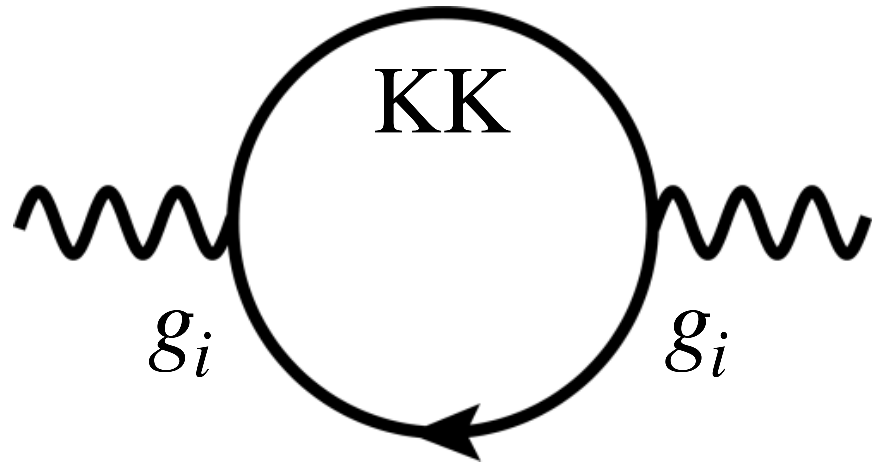
$$M_{\mathrm{KK}} = 1/R$$

In 5D, it is useful to introduce 't Hooft coupling for the gauge coupling $\tilde{\alpha}_i = \mathrm{dof} \times \alpha_i$, for $\mu \gg M_{\mathrm{KK}}$

$$\tilde{g} = \sqrt{k} g_{\mathrm{b}}$$

$$\tilde{\alpha}_i(t) = \alpha_i(t) S(t)$$

$$S(t) = \begin{cases} 1 & \text{for } \mu < 1/R\,, \\ \mu R = M_Z R e^t & \text{for } \mu > 1/R\,. \end{cases}$$



Holger Gies, hep-th/0305208; Tim Morris, hep-ph/0410142

Symmetry breaking

$$\text{5D SO(10)} \xrightarrow[\text{BC}]{M_{\rm KK} \sim 10^{10}\ \text{GeV}} \text{4D Pati-Salam} \xrightarrow[\mathbf{16}]{M_{\rm PS} \sim 10^{6}\ \text{GeV}} \text{SM}$$

- Boundary Conditions (BCs) to achieve the first breaking: $SO(10) \to SO(6) \times SO(4)$

$$SO(6) \simeq SU(4)_c,\ SO(4) \simeq SU(2)_L \times SU(2)_R$$

BC on UV brane
$$\begin{pmatrix} + & + & + & + & + & + & - & - & - & - \\ + & + & + & + & + & + & - & - & - & - \\ + & + & + & + & + & + & - & - & - & - \\ + & + & + & + & + & + & - & - & - & - \\ + & + & + & + & + & + & - & - & - & - \\ + & + & + & + & + & + & - & - & - & - \\ - & - & - & - & - & - & + & + & + & + \\ - & - & - & - & - & - & + & + & + & + \\ - & - & - & - & - & - & + & + & + & + \\ - & - & - & - & - & - & + & + & + & + \end{pmatrix}$$

BC on IR brane
$$\begin{pmatrix} + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + & + & + \end{pmatrix}$$

Fermions and Higgses BCs are arranged accordingly, such that those appearing in 4D PS model have zero mode, and

- $\mathbf{16}$ Higgs to achieve the second breaking: $G_{422} \to G_{\rm SM}$

$$\mathbf{16} \quad = \underbrace{(4,2,1) + (\bar{4},2,1)}_{G_{422}} \quad = \underbrace{(1,1,0) + (1,1,1) + (1,2,-\frac{1}{2}) + (\bar{3},1,-\frac{2}{3}) + (\bar{3},1,\frac{1}{3}) + (\bar{3},2,\frac{1}{3})}_{G_{321}}$$

Yukawa coupling in 5D SO(10)

$$S \supset -\int \mathrm{d}^4x\mathrm{d}y \quad y_{10}\overline{\Psi_{\mathbf{16}}}H_{10}\Psi_{\overline{\mathbf{16}}} + \mathrm{i}y_{120}\overline{\Psi_{\mathbf{16}}}H_{120}\Psi_{\overline{\mathbf{16}}} + y_{16}\overline{\nu_{\mathrm{S}}}H_{16}\Psi_{\overline{\mathbf{16}}} + \frac{L}{2}\mu_{\mathrm{M}}\,\overline{\nu_{\mathrm{S}}}\nu^c_{\mathrm{S}}\,\delta(y-L) + \mathrm{h.c.}$$

$$S \supset -\int \mathrm{d}^4x \quad y_1\overline{\psi_L}h_1\psi_R + \overline{\psi_L}(y_1'h_1' + y_{15}h_{15})\psi_R + y_4\overline{\nu_{\mathrm{S}}}h_{\bar{4}}\psi_R + \frac{1}{2}\mu_{\mathrm{M}}\overline{\nu_{\mathrm{S}}}\nu^c_{\mathrm{S}} + \mathrm{h.c.}$$

$$\Psi_{\mathbf{16}} = \psi_L + \Psi^c_R \qquad \Psi_{\overline{\mathbf{16}}} = \Psi^c_L + \psi_R \qquad H_{\mathbf{10}} \subset h_1, \quad H_{\mathbf{120}} \supset h_{1'} + h_{15}, \quad H_{\mathbf{16}} \supset h_{\bar{4}}$$

$$y_t = \sqrt{2}\,y_{10}c^u_{10} + \sqrt{2}\,y_{120}(c^{d'}_{120} + \frac{1}{\sqrt{3}}c^d_{120})$$

$$y_b = \sqrt{2}\,y_{10}c^d_{10} + \sqrt{2}\,y_{120}(c^{d'}_{120} + \frac{1}{\sqrt{3}}c^d_{120})$$

$$y_\tau = \sqrt{2}\,y_{10}c^d_{10} + \sqrt{2}\,y_{120}(c^{d'}_{120} - \sqrt{3}c^d_{120})$$

$$y_\nu = \sqrt{2}\,y_{10}c^u_{10} + \sqrt{2}\,y_{120}(c^{d'}_{120} - \sqrt{3}c^d_{120})$$

Inverse seesaw

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_{\mathrm{D}} & 0 \\ m_{\mathrm{D}} & 0 & m_{\mathrm{S}} \\ 0 & m_{\mathrm{S}} & \mu_{\mathrm{M}} \end{pmatrix}$$

$$m_{\mathrm{D}} = y_\nu\langle H_{\mathrm{SM}}\rangle$$

$$m_{\mathrm{S}} = y_{16}\langle H_{16}\rangle$$

$$m_\nu = \frac{m_D^2}{m_S^2}\mu_{\mathrm{M}}$$

UV behaviour of gauge coupling

- RGEs for gauge couplings in 4D

$$\boxed{\beta_i = \frac{b_i}{2\pi}\alpha_i^2}$$

Standard Model, $(b_3, b_{2L}, b_1) = \left(-7,\ -\frac{19}{6}, \frac{41}{10}\right)$

Pati-Salam Model, $(b_4, b_{2L}, b_{2R}) = \left(-5,\ \frac{7}{3},\ 3\right)$

- RGEs for gauge couplings in 5D

$$\boxed{\beta_i = \frac{b_i}{2\pi}\alpha_i^2 + \left(S(t) - 1\right)\frac{b_{\mathbf{10}}}{2\pi}\tilde{\alpha}_i^2}$$

't Hooft coupling: $\tilde{\alpha}_i(t) = \alpha_i(t)S(t)$

$$\boxed{\frac{\mathrm{d}\tilde{\alpha}_i}{\mathrm{d}t} = \tilde{\beta}_i = \tilde{\alpha}_i + \frac{b_{\mathbf{10}}}{2\pi}\tilde{\alpha}_i^2}$$

β coefficient including KK contributions

$$b_{\mathbf{10}} = \left(-\frac{11}{3} + \frac{1}{6}\right) C_2(SO(10)) + \frac{4}{3}\sum_F T(F) + \frac{1}{6}\sum_S T(S)$$

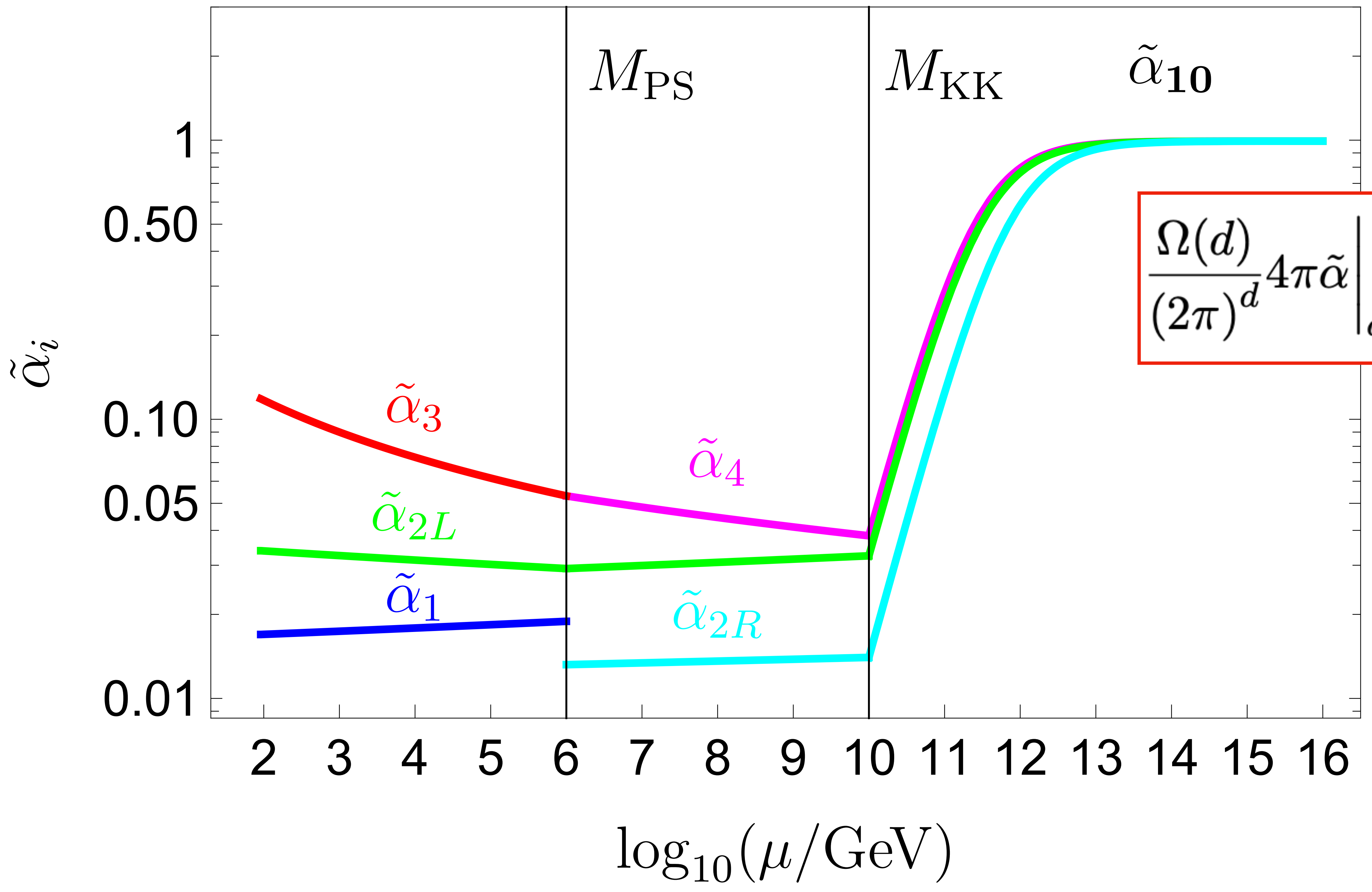
V_5 appears as scalar

$$b_{\mathbf{10}} < 0 \qquad \Longrightarrow \qquad \tilde{\alpha}_i = \frac{2\pi}{e^{-t+c_i} - b_{\mathbf{10}}} \to \frac{2\pi}{-b_{\mathbf{10}}}$$

UV fixed point

UV behaviour of gauge coupling

Gauge couplings asymptotically safe



$$\frac{\Omega(d)}{(2\pi)^d} 4\pi\tilde{\alpha}\Big|_{d=5} = \frac{2}{19\pi} \sim 0.033 \ll 1.$$

Higgs contents	$\tilde{\alpha}_{\mathbf{10}}^{\rm UV}$
$(\mathbf{10}_{\rm r}, \mathbf{120}_{\rm r}, \mathbf{16})$	$\frac{4\pi}{13}$
$(\mathbf{10}_{\rm c}, \mathbf{120}_{\rm r}, \mathbf{16})$	$\frac{6\pi}{19}$
$(\mathbf{10}_{\rm r}, \mathbf{120}_{\rm c}, \mathbf{16})$	$\frac{12\pi}{11}$
$(\mathbf{10}_{\rm c}, \mathbf{120}_{\rm c}, \mathbf{16})$	$\frac{6\pi}{5}$
$(\mathbf{10}_{\rm r}, \mathbf{120}_{\rm r}, \mathbf{45}_{\rm r}, \mathbf{16})$	$\frac{12\pi}{31}$
$(\mathbf{10}_{\rm r}, \mathbf{120}_{\rm r}, \mathbf{45}_{\rm r}, \mathbf{16})$	$\frac{12\pi}{31}$
$(\mathbf{10}_{\rm c}, \mathbf{120}_{\rm r}, \mathbf{45}_{\rm r}, \mathbf{16})$	$\frac{2\pi}{5}$
$(\mathbf{10}_{\rm r}, \mathbf{120}_{\rm r}, \mathbf{54}_{\rm r}, \mathbf{16})$	$\frac{4\pi}{9}$
$(\mathbf{10}_{\rm r}, \overline{\mathbf{126}})$	12π
$(\mathbf{10}_{\rm c}, \overline{\mathbf{126}})$	∞

$\overline{\mathbf{126}}$ Higgs is not recommended

$$b_{10} = -28 + \frac{16}{3} n_{\Psi_{16},{\rm c}} + \frac{1}{6} n_{H_{10},{\rm r}} + \frac{14}{3} n_{H_{120},{\rm r}} + \frac{4}{3} n_{H_{45},{\rm r}} + 2 n_{H_{54},{\rm r}} + \frac{35}{3} n_{H_{\overline{126}},{\rm c}} + \frac{2}{3} n_{H_{16},{\rm c}} + \cdots$$

Deriving Yukawa RGEs

- Gamma matrices (32×32) and chiral representation $\mathbf{16}$ in the gauge space of $SO(10)$

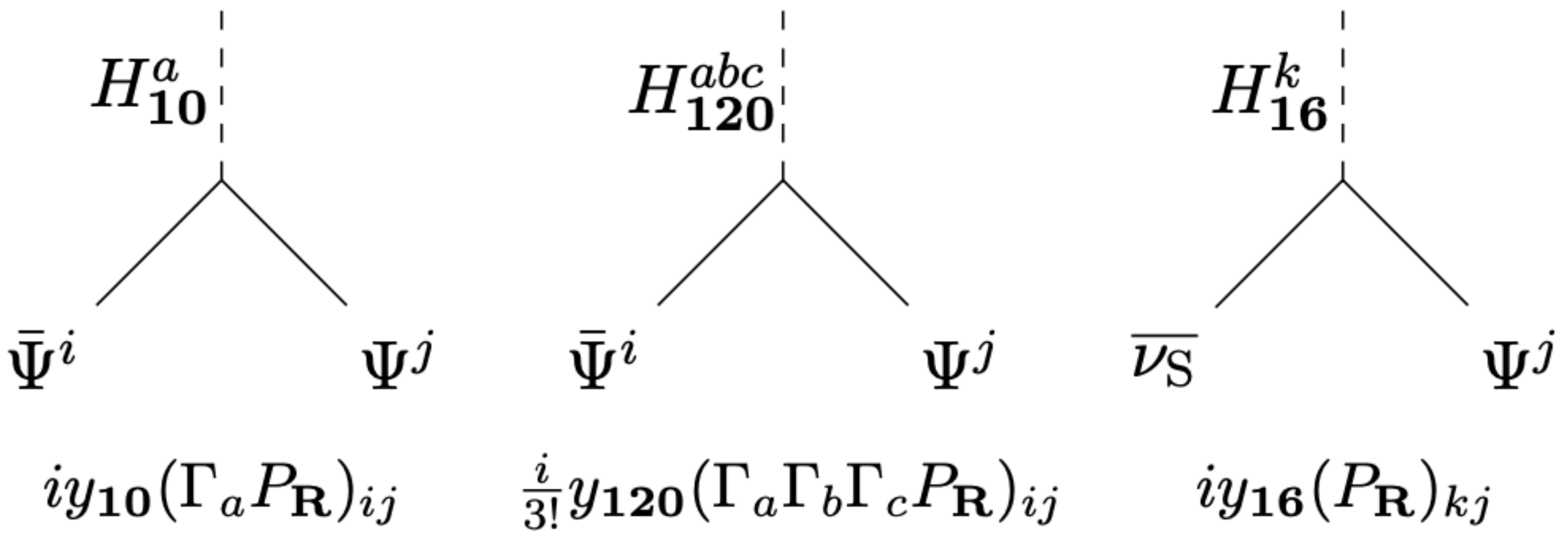
$$\{\Gamma_a, \Gamma_b\} = 2\delta_{ab} \qquad \Gamma_\chi = i\Gamma_1\Gamma_2\cdots\Gamma_{10} = \begin{pmatrix} -I_{16} & 0 \\ 0 & I_{16} \end{pmatrix} \qquad P_{\mathbf{L},\mathbf{R}} = \frac{1}{2}(I_{32} \mp \Gamma_\chi)$$

$$\Psi_{\mathbf{L}} = P_{\mathrm{L}}\Psi = \begin{pmatrix} \Psi_{\mathbf{16}} \\ 0 \end{pmatrix} \qquad \Psi_{\mathbf{R}} = P_{\mathrm{L}}\Psi = \begin{pmatrix} 0 \\ \Psi_{\overline{\mathbf{16}}} \end{pmatrix} \qquad \mathbf{32} = \mathbf{16} + \overline{\mathbf{16}}$$

- Field arrangements: Fermion $\sim \mathbf{16} + \overline{\mathbf{16}}$, Scalars $\sim \mathbf{10} + \mathbf{120} + \mathbf{16}$ ($\overline{\mathbf{126}}$ is not included)

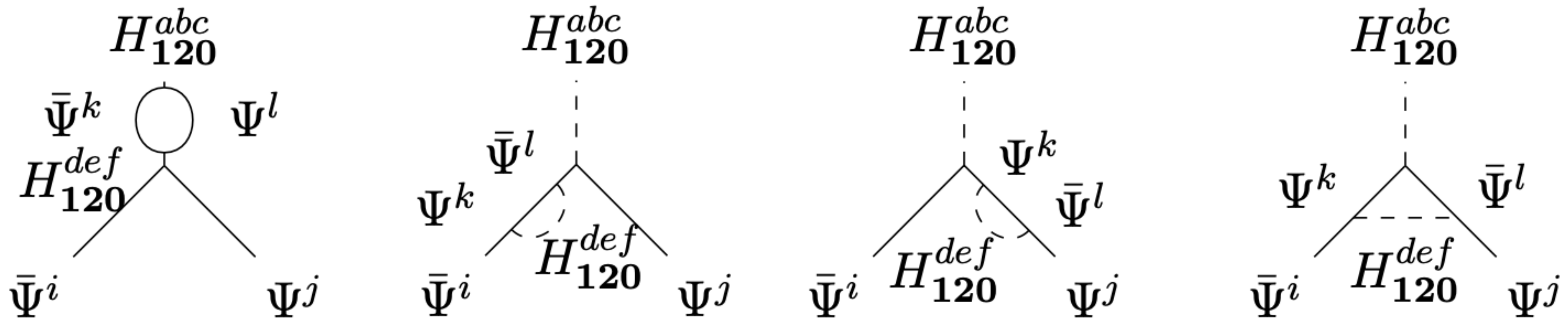
$$-\mathscr{L}_Y = \bar{\Psi}^i \left[\, y_{\mathbf{10}}(\Gamma_a P_{\mathbf{R}})_{ij} H^a_{\mathbf{10}} + \frac{1}{3!} y_{\mathbf{120}}(\Gamma_a\Gamma_b\Gamma_c P_{\mathbf{R}})_{ij} H^{abc}_{\mathbf{120}} \,\right] \Psi^j + y_{\mathbf{16}}\overline{\nu_{\mathrm{S}}} H^k_{\mathbf{16}}(P_{\mathbf{R}})_{kj}\Psi^j + \mathrm{h.c.}$$

- Feynman rules



Deriving Yukawa RGEs

- Calculate loop diagrams one by one



- RGE for Yukawa couplings

$$16\pi^2\frac{\mathrm{d}y_{\mathrm{r}}}{\mathrm{d}t} = 16\pi^2\frac{\mathrm{d}y_{\mathrm{r}}}{\mathrm{d}t}\bigg|_{4\mathrm{D}} + (S(t)-1)16\pi^2\frac{\mathrm{d}y_{\mathrm{r}}}{\mathrm{d}t}\bigg|_{\mathrm{KK}}$$

- 0-mode contribution

Assuming $y_{\mathbf{16}}$ small enough, thus its contribution can be ignored.

$$16\pi^2\frac{\mathrm{d}y_{\mathbf{10}}}{\mathrm{d}t}\bigg|_{4\mathrm{D}} = \eta_{\mathbf{10}}\Big|_{4\mathrm{D}} y_{\mathbf{10}} - 96Y_{\mathbf{120}}y_{\mathbf{10}}^{\dagger}y_{\mathbf{120}} + 10Y_{\mathbf{10}}y_{\mathbf{10}}^{\dagger}y_{\mathbf{10}} + +60(y_{\mathbf{10}}y_{\mathbf{120}}^{\dagger}y_{\mathbf{120}} + y_{\mathbf{120}}y_{\mathbf{120}}^{\dagger}y_{\mathbf{10}}) \qquad \eta_{\mathbf{10}}\Big|_{4\mathrm{D}} = -\frac{270}{8}g_{\mathbf{10}}^2 + 8\ \mathrm{Tr}(y_{\mathbf{10}}y_{\mathbf{10}}^{\dagger})$$

$$16\pi^2\frac{\mathrm{d}y_{\mathbf{120}}}{\mathrm{d}t}\bigg|_{4\mathrm{D}} = \eta_{\mathbf{120}}\Big|_{4\mathrm{D}} y_{\mathbf{120}} + 104y_{\mathbf{120}}y_{\mathbf{120}}^{\dagger}y_{\mathbf{120}} + 5(y_{\mathbf{120}}y_{\mathbf{10}}^{\dagger}y_{\mathbf{10}} + y_{\mathbf{10}}y_{\mathbf{10}}^{\dagger}y_{\mathbf{120}}) \qquad \eta_{\mathbf{120}}\Big|_{4\mathrm{D}} = -\frac{270}{8}g_{\mathbf{10}}^2 + 8\ \mathrm{Tr}(y_{\mathbf{120}}y_{\mathbf{120}}^{\dagger})$$

- KK-mode contribution

$$16\pi^2\frac{\mathrm{d}y_{\mathbf{10}}}{\mathrm{d}t}\bigg|_{\mathrm{KK}} = \eta_{\mathbf{10}}\Big|_{\mathrm{KK}} y_{\mathbf{10}} + 10y_{\mathbf{10}}y_{\mathbf{10}}^{\dagger}y_{\mathbf{10}} + 60(y_{\mathbf{10}}y_{\mathbf{120}}^{\dagger}y_{\mathbf{120}} + y_{\mathbf{120}}y_{\mathbf{120}}^{\dagger}y_{\mathbf{10}}) + +60(y_{\mathbf{10}}y_{\mathbf{120}}^{\dagger}y_{\mathbf{120}} + y_{\mathbf{120}}y_{\mathbf{120}}^{\dagger}y_{\mathbf{10}}) \qquad \eta_{\mathbf{10}}\Big|_{\mathrm{KK}} = -\frac{171}{8}g_{\mathbf{10}}^2 + 16\ \mathrm{Tr}(y_{\mathbf{10}}y_{\mathbf{10}}^{\dagger})$$

$$16\pi^2\frac{\mathrm{d}y_{\mathbf{120}}}{\mathrm{d}t}\bigg|_{\mathrm{KK}} = \eta_{\mathbf{120}}\Big|_{\mathrm{KK}} y_{\mathbf{120}} + 104y_{\mathbf{120}}y_{\mathbf{120}}^{\dagger}y_{\mathbf{120}} + 5(y_{\mathbf{120}}y_{\mathbf{10}}^{\dagger}y_{\mathbf{10}} + y_{\mathbf{10}}y_{\mathbf{10}}^{\dagger}y_{\mathbf{120}}) \qquad \eta_{\mathbf{120}}\Big|_{\mathrm{KK}} = -\frac{219}{8}g_{\mathbf{10}}^2 + 16\ \mathrm{Tr}(y_{\mathbf{120}}y_{\mathbf{120}}^{\dagger})$$

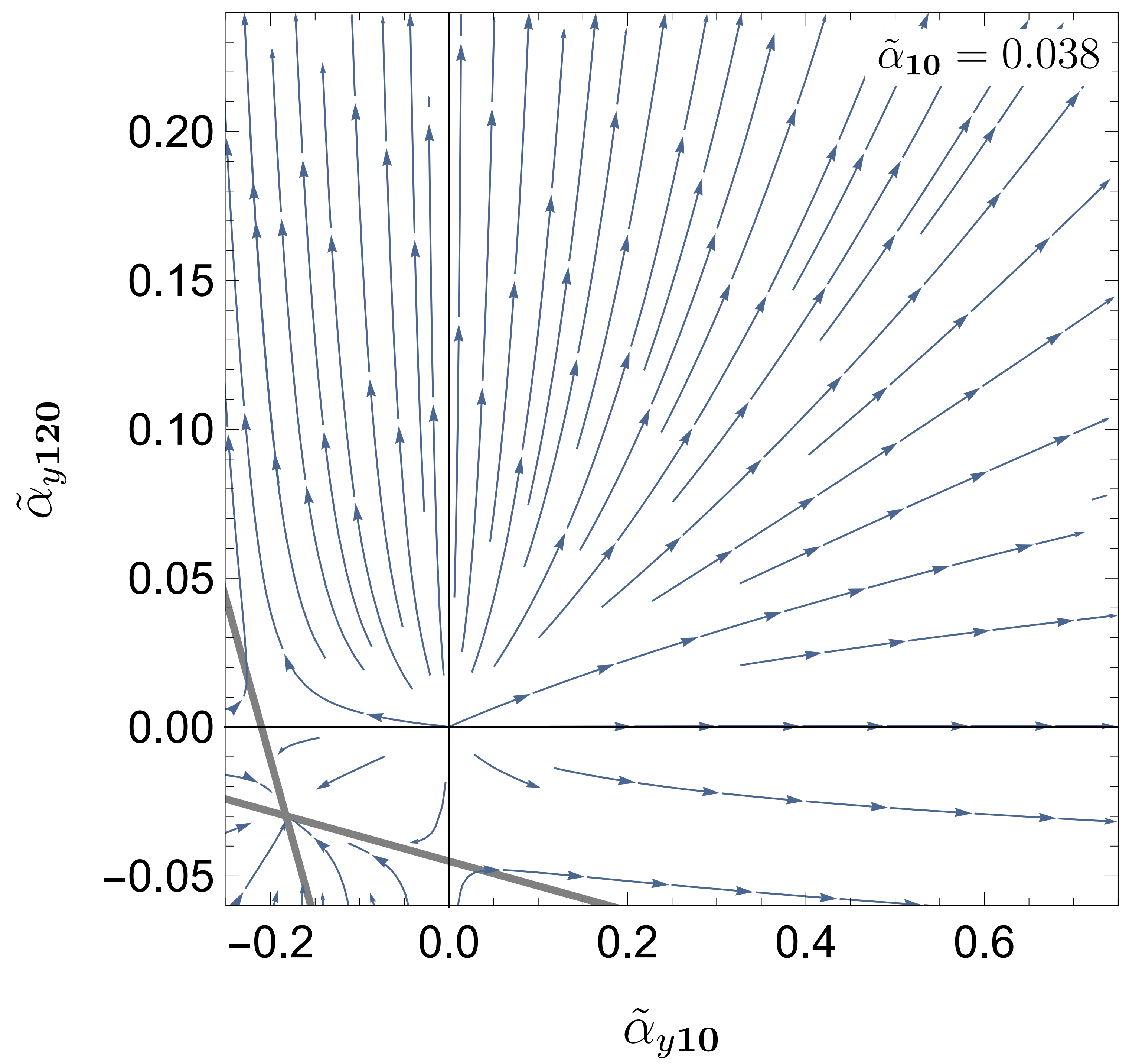
Deriving Yukawa RGEs

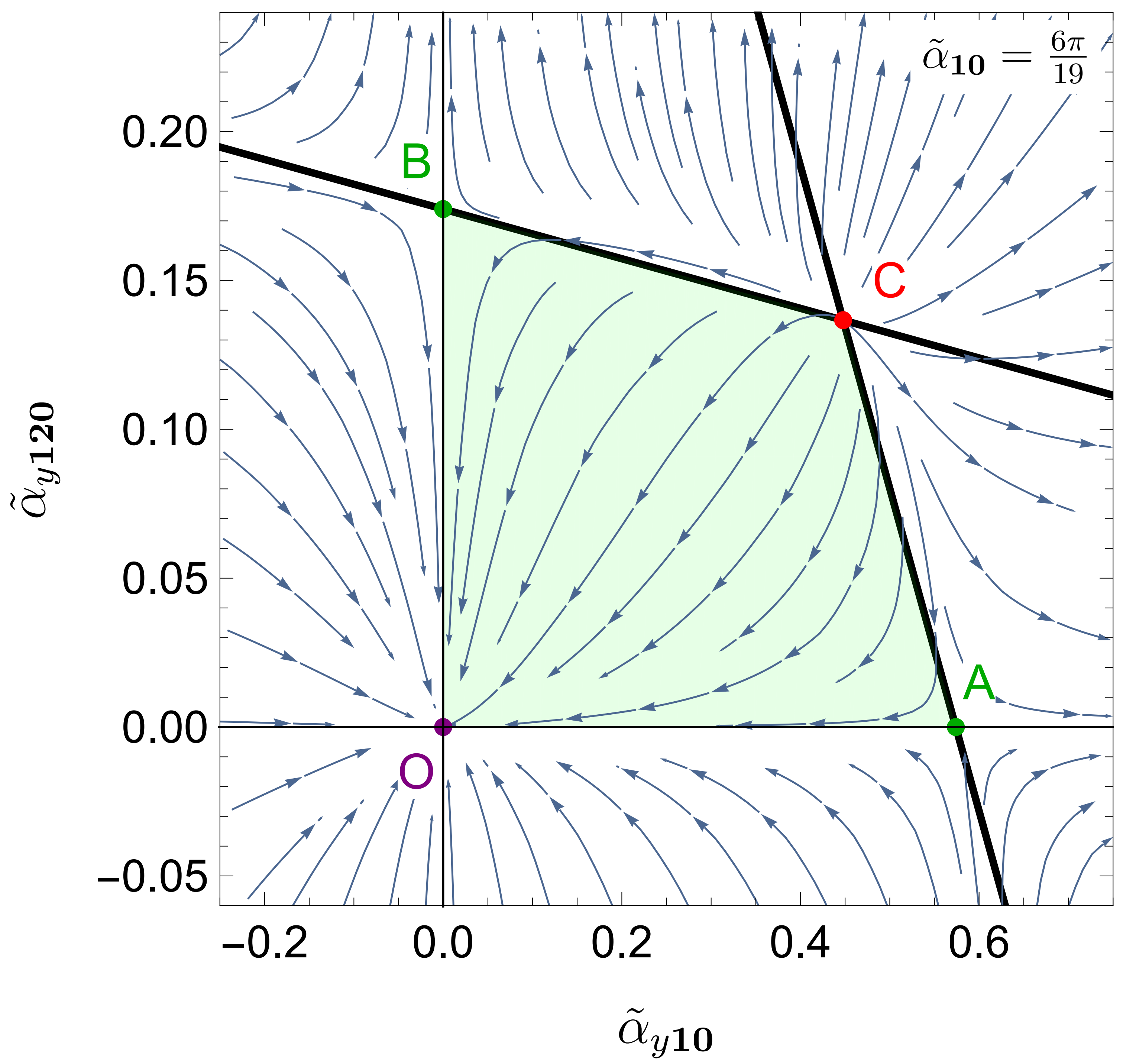
- RGEs for 't Hooft couplings $\quad \tilde{\alpha}_{\rm yr}(t) = \alpha_{\rm yr}(t) S(t), \quad \alpha_{yr} = \dfrac{y_r^2}{4\pi}, \quad r = \mathbf{10}, \mathbf{120}$

$$2\pi \frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{120}}}{\mathrm{d}t} = \left[2\pi + 10\tilde{\alpha}_{y\mathbf{10}} + 120\tilde{\alpha}_{y\mathbf{120}} - \frac{219}{8}\tilde{\alpha}_{\mathbf{10}}\right]\tilde{\alpha}_{y\mathbf{120}}$$

$$2\pi \frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{10}}}{\mathrm{d}t} = \left[2\pi + 26\tilde{\alpha}_{y\mathbf{10}} + 24\tilde{\alpha}_{y\mathbf{120}} - \frac{171}{8}\tilde{\alpha}_{\mathbf{10}}\right]\tilde{\alpha}_{y\mathbf{10}}$$

- RG flow of Yukawa couplings





$(\tilde{\alpha}_{y\mathbf{10}}, \tilde{\alpha}_{y\mathbf{120}})$

Point

$$\underset{O}{(0,0)},\ \underset{A}{(\tfrac{19\pi}{104}, 0)},\ \underset{B}{(0, \tfrac{101\pi}{1824})},\ \underset{C}{(\tfrac{65\pi}{456}, \tfrac{119\pi}{2736})}.$$

Asymptotic freedom might be achieved for suitable size of gauge couplings

Explicitly running Yukawa couplings from EW scale to UV

$$\mu: \quad M_Z \longrightarrow M_{\rm PS} \longrightarrow M_{\rm KK} \longrightarrow \text{UV}$$

$$\text{Yukawas}: \quad y_t, y_b, y_\tau \qquad y_1, y_1', y_{15} \qquad y_1, y_1', y_{15} \qquad \boldsymbol{y_{10}, y_{120}}$$

SM $\boxed{y_t \overline{q_L}\tilde{h}t_R + y_b \overline{q_L} h b_R + y_\tau \overline{l_L} h \tau_R + \kappa\,[\overline{l_L}\tilde{H}\tilde{H}^T l_L^c] + \text{h.c.}}$

Pati-Salam, 4D $\boxed{y_1 \overline{\psi_L} h_1 \psi_R + \overline{\psi_L}(y_1' h_1' + y_{15} h_{15})\psi_R + y_4 \overline{\nu_S} h_{\bar 4} \psi_R + \frac{1}{2}\mu_M \overline{\nu_S}\nu_S^c + \text{h.c.}}$

$\psi_L \sim (4,1,2),\ \psi_R \sim (4,1,2),\ \nu_S \sim (1,1,1) \qquad h_1 \sim (1,2,2),\ h_{15} \sim (15,2,2)$

Pati-Salam, KK $+y_6(\overline{\psi_L} H_6 \Psi_L^c + \overline{\Psi_R^c} H_6 \psi_R) + y_6'(\overline{\psi_L} H_{6L} \Psi_L^c + \overline{\Psi_R^c} H_{6R} \psi_R) + y_{10}(\overline{\psi_L} H_{10} \Psi_L^c + \overline{\Psi_R^c} H_{\overline{10}} \psi_R) + \text{h.c.}$

SO(10), 5D $y_{10}\overline{\Psi_{16}} H_{10} \Psi_{\overline{16}} + \mathrm{i}y_{120}\overline{\Psi_{16}} H_{120} \Psi_{\overline{16}} + y_{16}\overline{\nu_S} H_{16} \Psi_{\overline{16}} + \frac{L}{2}\mu_M \overline{\nu_S}\nu_S^c\,\delta(y-L) + \text{h.c.}$

$\Psi_{16} = \psi_L + \Psi_R^c \qquad \Psi_{\overline{16}} = \Psi_L^c + \psi_R \qquad \Psi_R^c \sim (\bar 4,1,2) \qquad \Psi_L^c \sim (\bar 4,2,1)$

Explicitly running Yukawa couplings from EW scale to UV

$$\begin{array}{lcccccccc} \mu: & M_Z & \longrightarrow & M_{\rm PS} & \longrightarrow & M_{\rm KK} & \longrightarrow & \text{UV} \\ \text{Yukawas}: & & y_t, y_b, y_\tau & & y_1, y_1', y_{15} & & y_1, y_1', y_{15} & \boldsymbol{y_{10}, y_{120}} \end{array}$$

$$2\pi\frac{\mathrm{d}\alpha_t}{\mathrm{d}t} = \left[\frac{9}{2}\alpha_t + \frac{3}{2}\alpha_b + \alpha_\tau - \frac{9}{4}\alpha_{2L} - \frac{17}{20}\alpha_1 - 8\alpha_3\right]\alpha_t$$

$$2\pi\frac{\mathrm{d}\alpha_b}{\mathrm{d}t} = \left[\frac{3}{2}\alpha_t + \frac{9}{2}\alpha_b + \alpha_\tau - \frac{9}{4}\alpha_{2L} - \frac{1}{4}\alpha_1 - 8\alpha_3\right]\alpha_b$$

$$2\pi\frac{\mathrm{d}\alpha_\tau}{\mathrm{d}t} = \left[3\alpha_t + 3\alpha_b + \frac{5}{2}\alpha_\tau - \frac{9}{4}\alpha_{2L} - \frac{9}{4}\alpha_1\right]\alpha_\tau$$

$$2\pi\frac{\mathrm{d}\tilde{\alpha}_{\mathbf{y10}}}{\mathrm{d}t} = \left[2\pi + 26\tilde{\alpha}_{\mathbf{y10}} + 24\tilde{\alpha}_{\mathbf{y120}} - \frac{81}{8}\tilde{\alpha}_4 - \frac{45}{8}(\tilde{\alpha}_{2L} + \tilde{\alpha}_{2R})\right]\tilde{\alpha}_{\mathbf{y10}}$$

$$2\pi\frac{\mathrm{d}\tilde{\alpha}_{\mathbf{y120}}}{\mathrm{d}t} = \left[2\pi + 10\tilde{\alpha}_{\mathbf{y10}} + 120\tilde{\alpha}_{\mathbf{y120}} - \frac{129}{8}\tilde{\alpha}_4 - \frac{45}{8}(\tilde{\alpha}_{2L} + \tilde{\alpha}_{2R})\right]\tilde{\alpha}_{\mathbf{y120}}$$

$$2\pi\frac{\mathrm{d}\alpha_{y1}}{\mathrm{d}t} = \left[6\alpha_{y1} + 4\alpha_{y1'} - \frac{45}{4}\alpha_4 - \frac{9}{4}(\alpha_{2L} + \alpha_{2R})\right]\alpha_{y1}$$

$$2\pi\frac{\mathrm{d}\alpha_{y1'}}{\mathrm{d}t} = \left[2\alpha_{y1} + 8\alpha_{y1'} - \frac{45}{4}\alpha_4 - \frac{9}{4}(\alpha_{2L} + \alpha_{2R})\right]\alpha_{y1'}$$

$$2\pi\frac{\mathrm{d}\alpha_{y15}}{\mathrm{d}t} = \left[8\alpha_{y15} + 2\alpha_{y1} - \frac{45}{4}\alpha_4 - \frac{9}{4}(\alpha_{2L} + \alpha_{2R})\right]\alpha_{y15}$$

$$2\pi\frac{\mathrm{d}\alpha_{y1}}{\mathrm{d}t}\bigg|_{\rm KK} = \left[10\alpha_{y1} + \frac{3}{2}\alpha_{y6} + 4\alpha_{y1'} - \frac{5}{2}\alpha_{y10} + \frac{9}{4}\alpha_{y6'} - \frac{81}{8}\alpha_4 - \frac{45}{8}(\alpha_{2L} + \alpha_{2R})\right]\alpha_{y1}$$

$$2\pi\frac{\mathrm{d}\alpha_{y1'}}{\mathrm{d}t}\bigg|_{\rm KK} = \left[2\alpha_{y1} + \frac{3}{2}\alpha_{y6} + 12\alpha_{y1'} + \frac{15}{2}\alpha_{y10} + \frac{9}{4}\alpha_{y6'} - \frac{129}{8}\alpha_4 - \frac{45}{8}(\alpha_{2L} + \alpha_{2R})\right]\alpha_{y1'}$$

$$2\pi\frac{\mathrm{d}\alpha_{y15}}{\mathrm{d}t}\bigg|_{\rm KK} = \left[2\alpha_{y1} + \frac{3}{2}\alpha_{y6} + 9\alpha_{y15} + \frac{3}{2}\alpha_{y10} + \frac{9}{4}\alpha_{y6'} - \frac{129}{8}\alpha_4 - \frac{45}{8}(\alpha_{2L} + \alpha_{2R})\right]\alpha_{y15}$$

Explicitly running Yukawa couplings from EW scale to UV

$$\begin{array}{lcccc} \mu: & M_Z \longrightarrow & M_{\text{PS}} \longrightarrow & M_{\text{KK}} \longrightarrow & \text{UV} \\ \text{Yukawas}: & y_t, y_b, y_\tau & y_1, y_1', y_{15} & y_1, y_1', y_{15} & y_{\mathbf{10}}, y_{\mathbf{120}} \end{array}$$

Matching

$$\boxed{\begin{aligned} y_t &= y_1 c^u_{\mathbf{10}} + y_1' c^{d'}_{\mathbf{120}} + \frac{1}{2\sqrt{3}} y_{15} c^d_{\mathbf{120}}\,, \\ y_b &= y_1 c^d_{\mathbf{10}} + y_1' c^{d'}_{\mathbf{120}} + \frac{1}{2\sqrt{3}} y_{15} c^d_{\mathbf{120}}\,, \\ y_\tau &= y_1 c^d_{\mathbf{10}} + y_1' c^{d'}_{\mathbf{120}} - \frac{\sqrt{3}}{2} y_{15} c^d_{\mathbf{120}}\,, \end{aligned}}$$

Explicitly running Yukawa couplings from EW scale to UV

$$\begin{array}{lcccc} \mu: & M_Z \longrightarrow & M_{\rm PS} \longrightarrow & M_{\rm KK} \longrightarrow & \mathrm{UV} \\ \text{Yukawas}: & y_t, y_b, y_\tau & y_1, y_1', y_{15} & y_1, y_1', y_{15} & y_{\mathbf{10}}, y_{\mathbf{120}} \end{array}$$

Matching

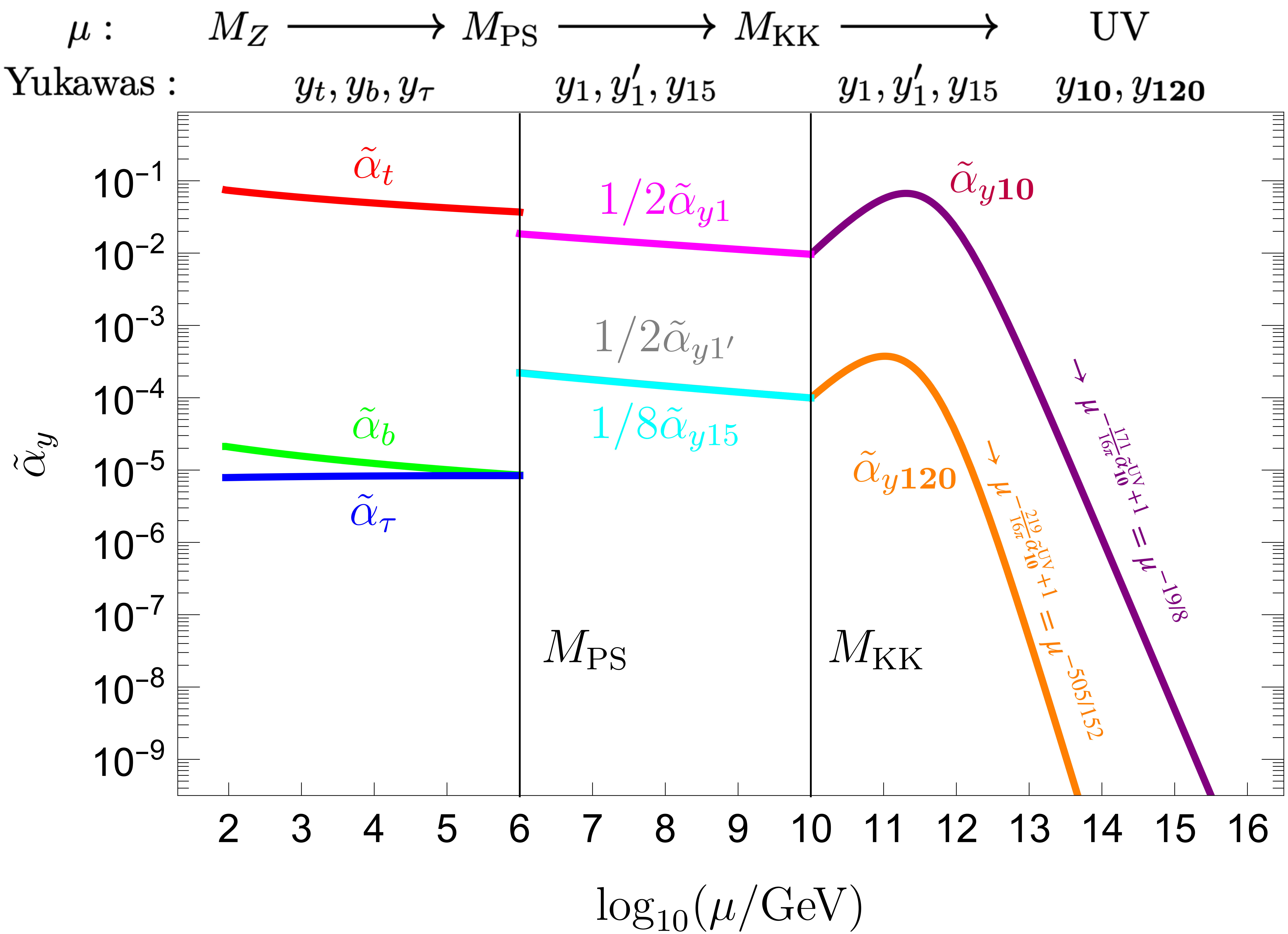
$$\boxed{\begin{array}{rcl} \frac{1}{2}\alpha_{y1},\ \frac{1}{4}\alpha_{y6} & \to & \alpha_{y\mathbf{10}}\,, \\ \frac{1}{2}\alpha_{y1'},\ \frac{1}{8}\alpha_{y15},\ \frac{1}{8}\alpha_{y10},\ \frac{1}{16}\alpha_{y6'} & \to & \alpha_{y\mathbf{120}}\,, \end{array} \quad \text{in the UV limit,}}$$

Asymptotic unification?

$$\boxed{\begin{array}{rcl} \frac{1}{2}\alpha_{y1},\ \frac{1}{4}\alpha_{y6} & = & \alpha_{y\mathbf{10}}\,, \\ \frac{1}{2}\alpha_{y1'},\ \frac{1}{8}\alpha_{y15},\ \frac{1}{8}\alpha_{y10},\ \frac{1}{16}\alpha_{y6'} & = & \alpha_{y\mathbf{120}}\,, \end{array} \quad \text{at } \mu = M_{\rm KK}\,.}$$

Exact unification?

Explicitly running Yukawa couplings from EW scale to UV

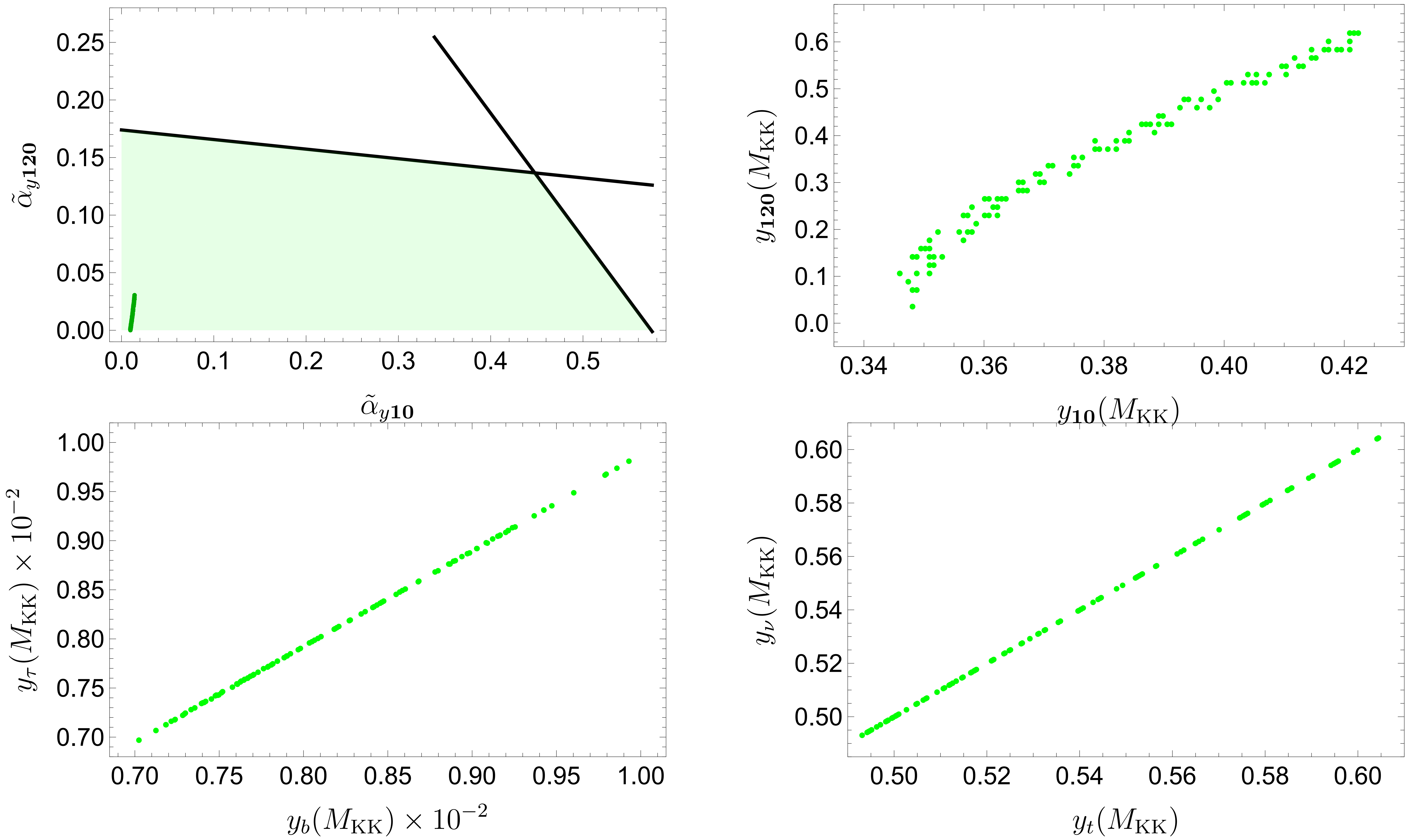


$$\tilde{\alpha}_{yr}(t) = \alpha_{yr}(t)S(t)$$

$$\alpha_{yr} = \frac{y_r^2}{4\pi}$$

$$\tilde{\beta}_y \sim (c_1\tilde{\alpha}_y - c_2\tilde{\alpha}_{\bf 10})\,\tilde{\alpha}_y$$

Parameter space for asymptotically free Yukawa couplings



Conclusion

- GUT is a hot topic for its rich phenos, in particular facing to the future neutrino and GW measurements. These studies focus at / below the GUT scale. Its UV behaviour has been paid less attention.
- GUT also provides a plateau to discuss fundamental properties of QFT, Asymptotic GUT (aGUT) is one of them.
- We demonstrate that aGUT is realised in a 5D SO(10) with minimal and realistic field content.

$$\text{5D SO(10)} \xrightarrow[\text{BC}]{M_{\rm KK} \sim 10^{10}\ \rm GeV} \text{4D Pati-Salam} \xrightarrow[\mathbf{16}]{M_{\rm PS} \sim 10^{6}\ \rm GeV} \text{SM}$$

  - Gauge couplings are unified asymptotically in the deep UV regime.
  - Yukawa couplings should be unified exactly at the KK scale. Their UV behaviour are very model-dependent, with a realistic content: complex $\mathbf{10}$, real $\mathbf{120}$, and $\mathbf{16}$, asymptotic freedom for Yukawa couplings are achieved in some parameter space.
  - The rep $\overline{\mathbf{126}}$ is not welcome in 5D SO(10) aGUT.

Thanks and questions ⋯