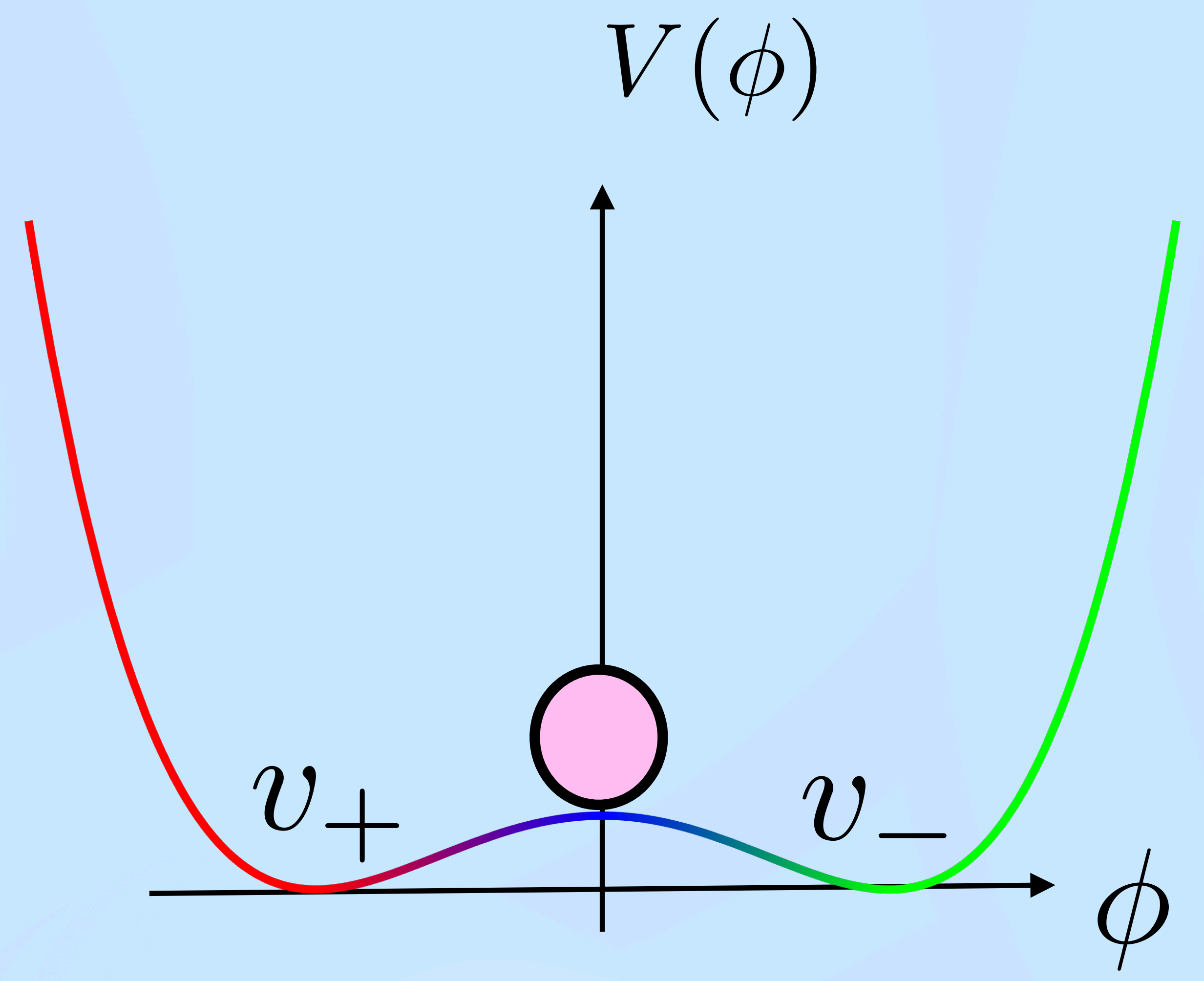
Non-Abelian Domain walls

付博文 20 Sep 2025
高能物理理论与实验融合发展研讨会

Based on **BF**, S. F. King, L. Marsili, S. Pascoli, J. Turner, Y-L Zhou, 2409.16359

Domain walls

Consider a real scalar field with $\mathbb{Z}_2$-symmetric potential



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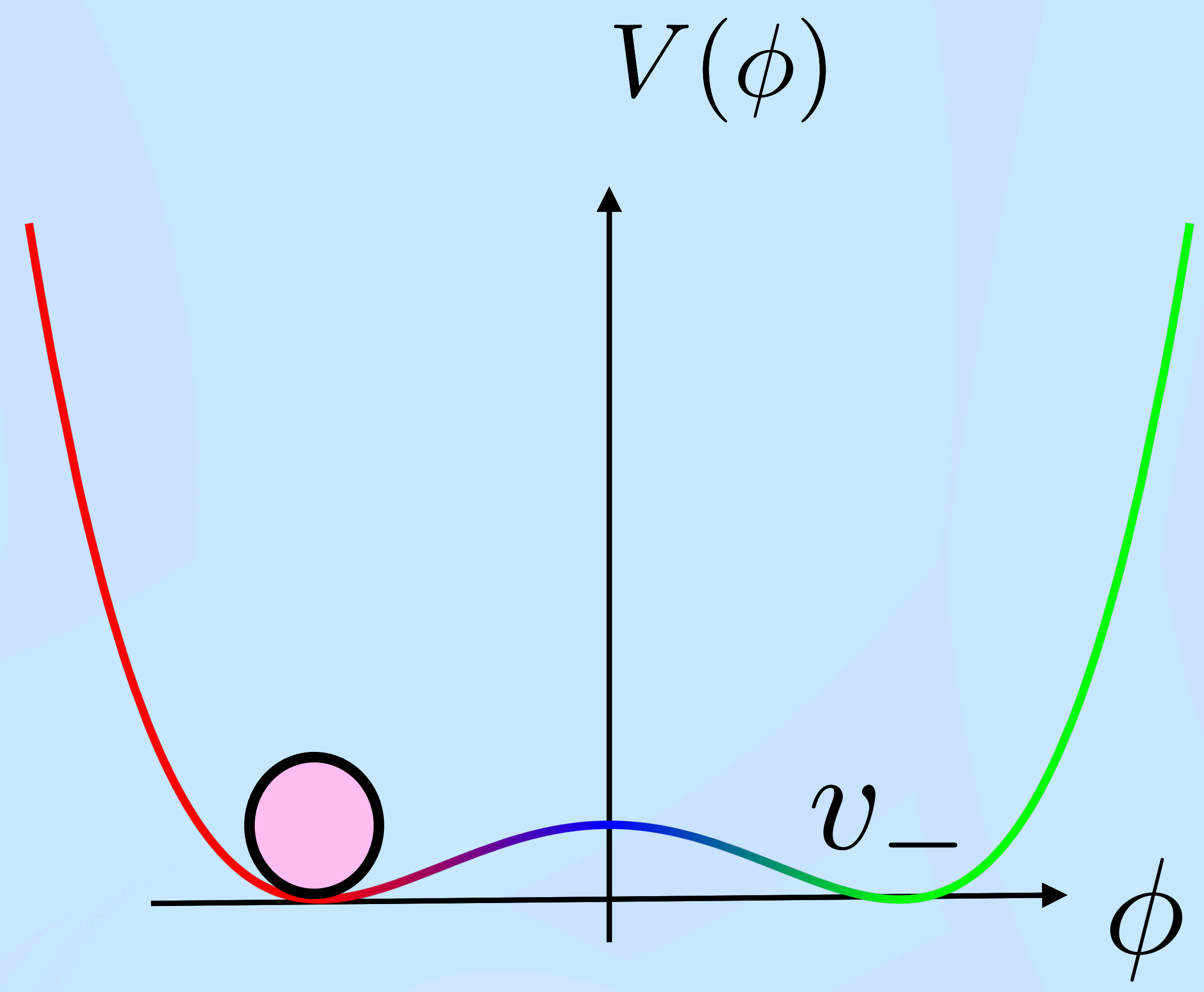

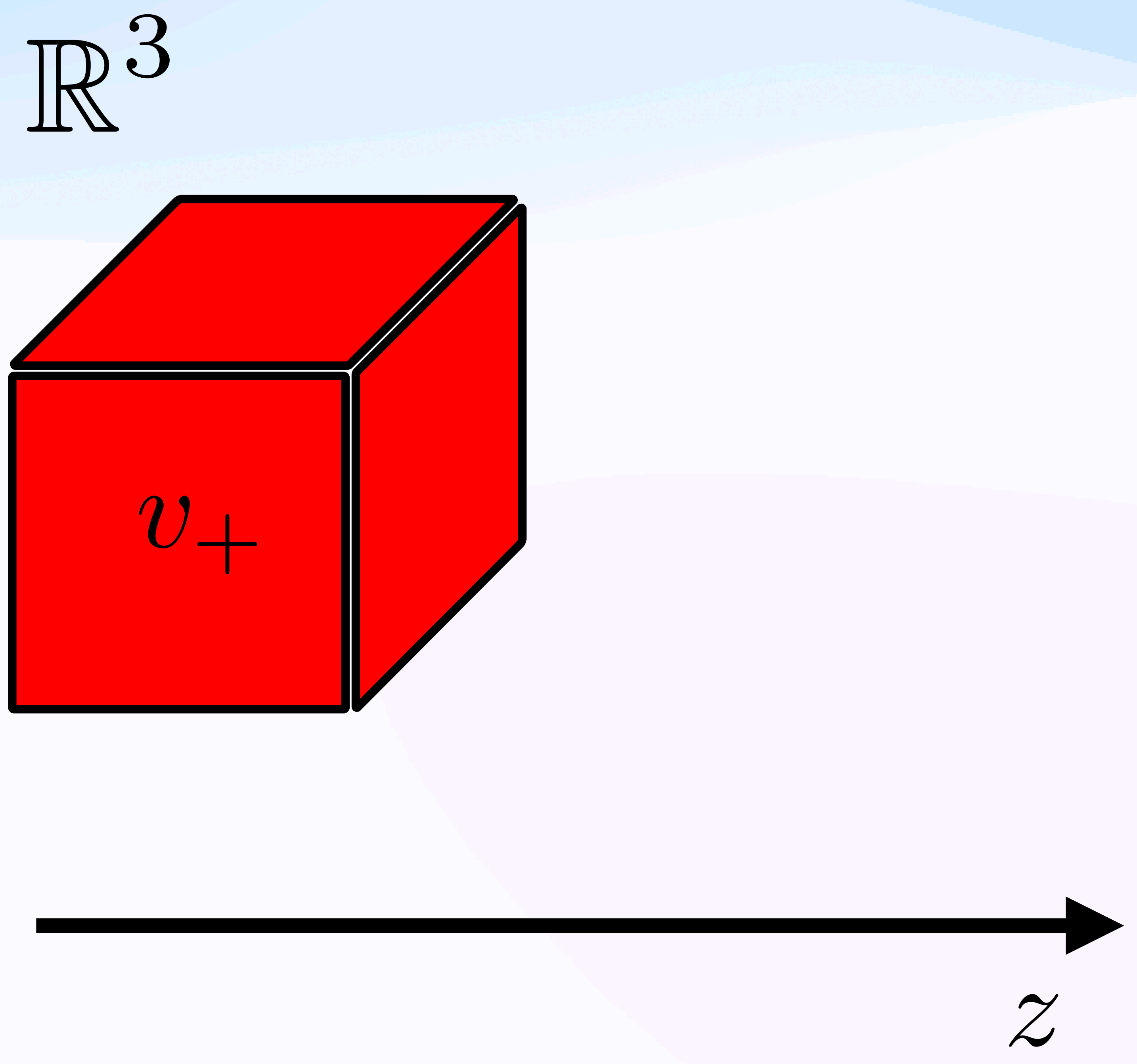

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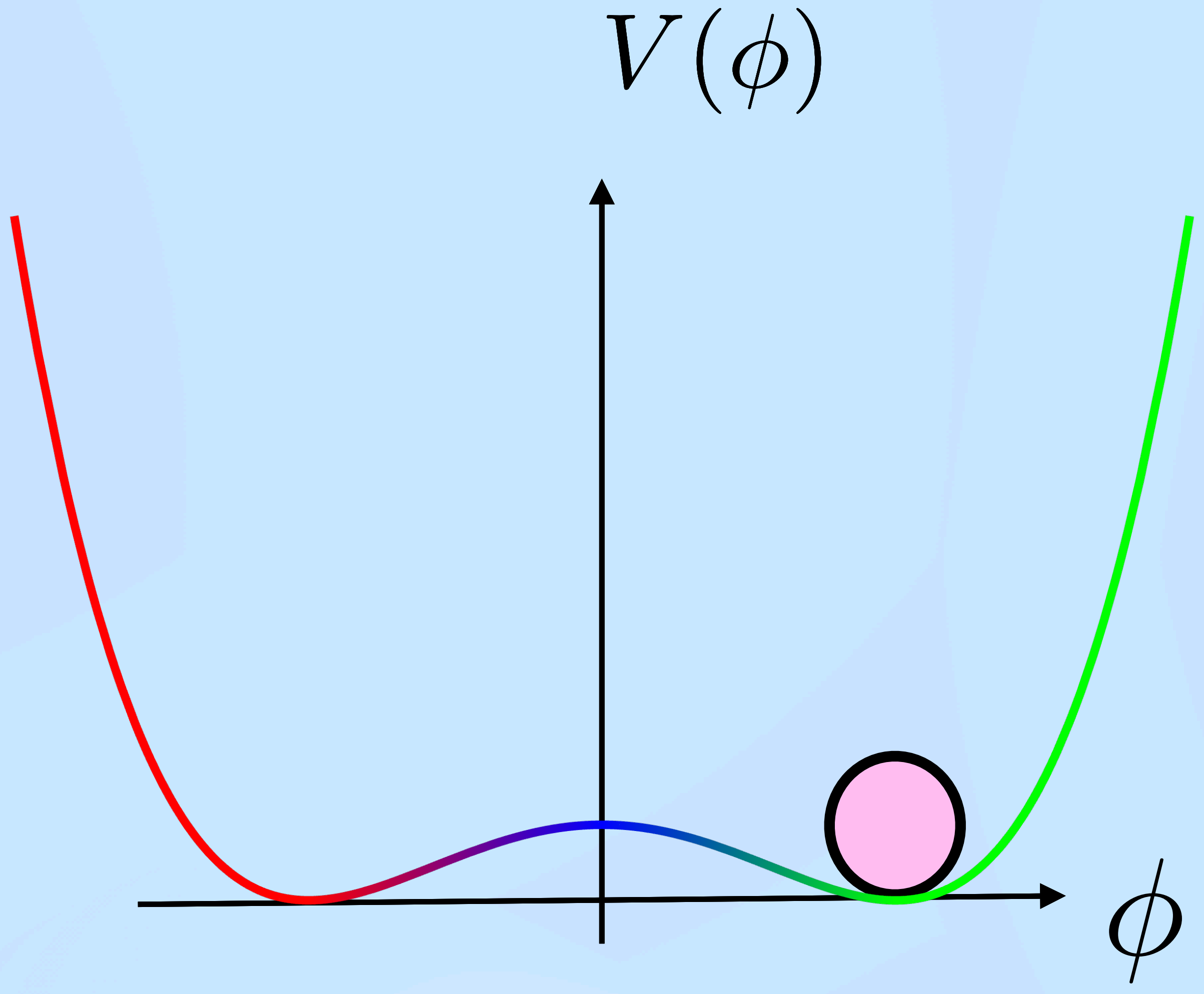

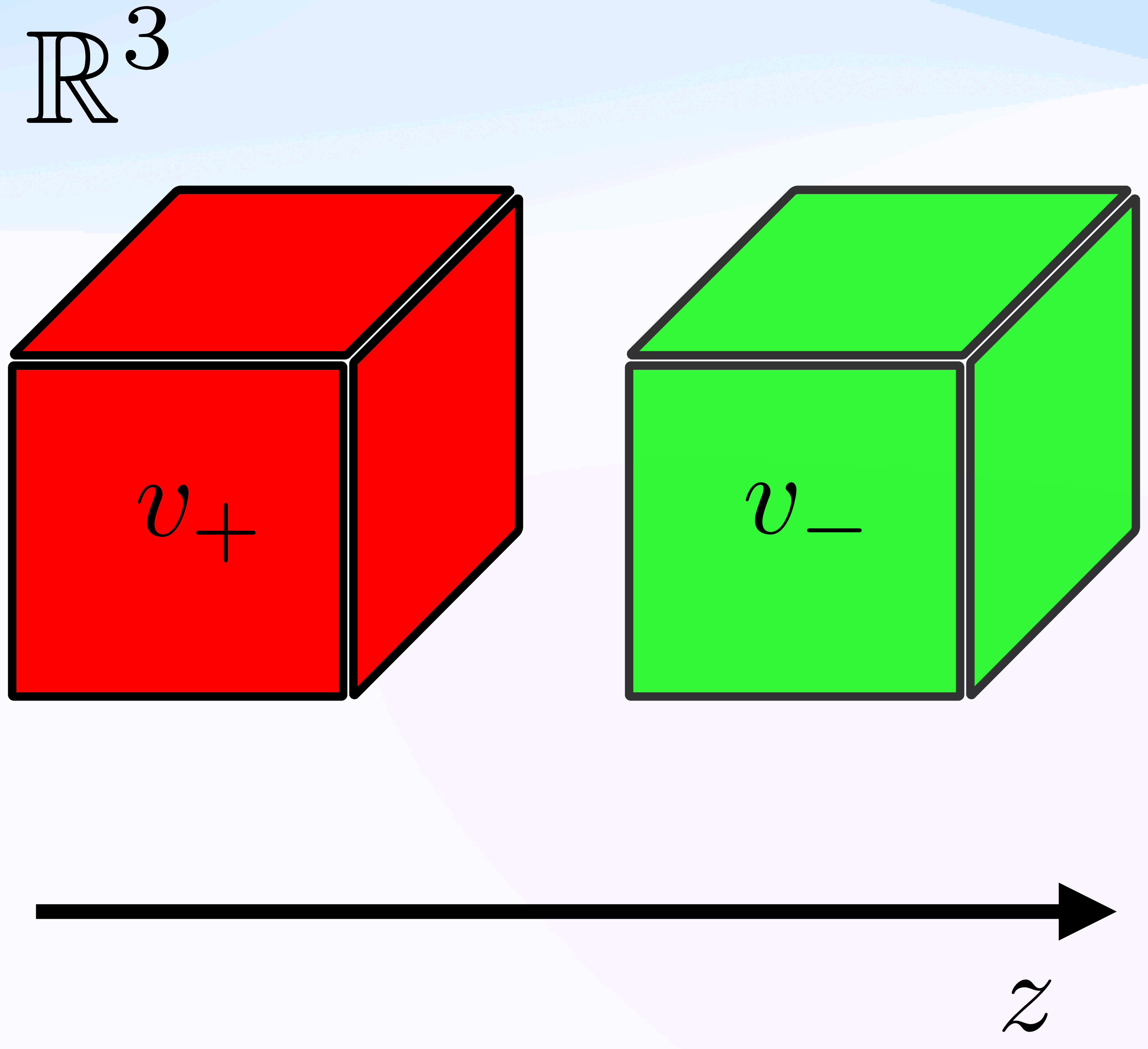

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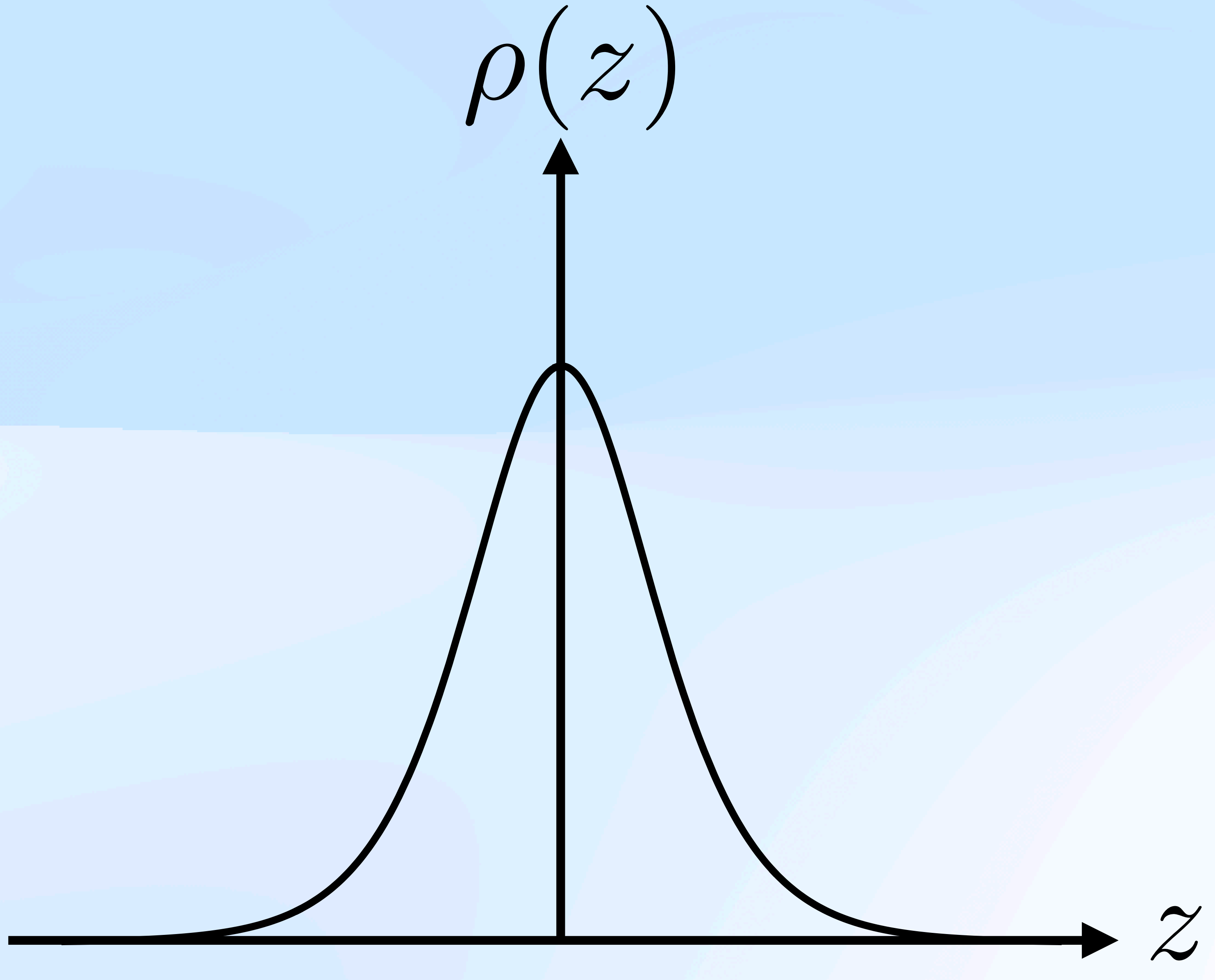
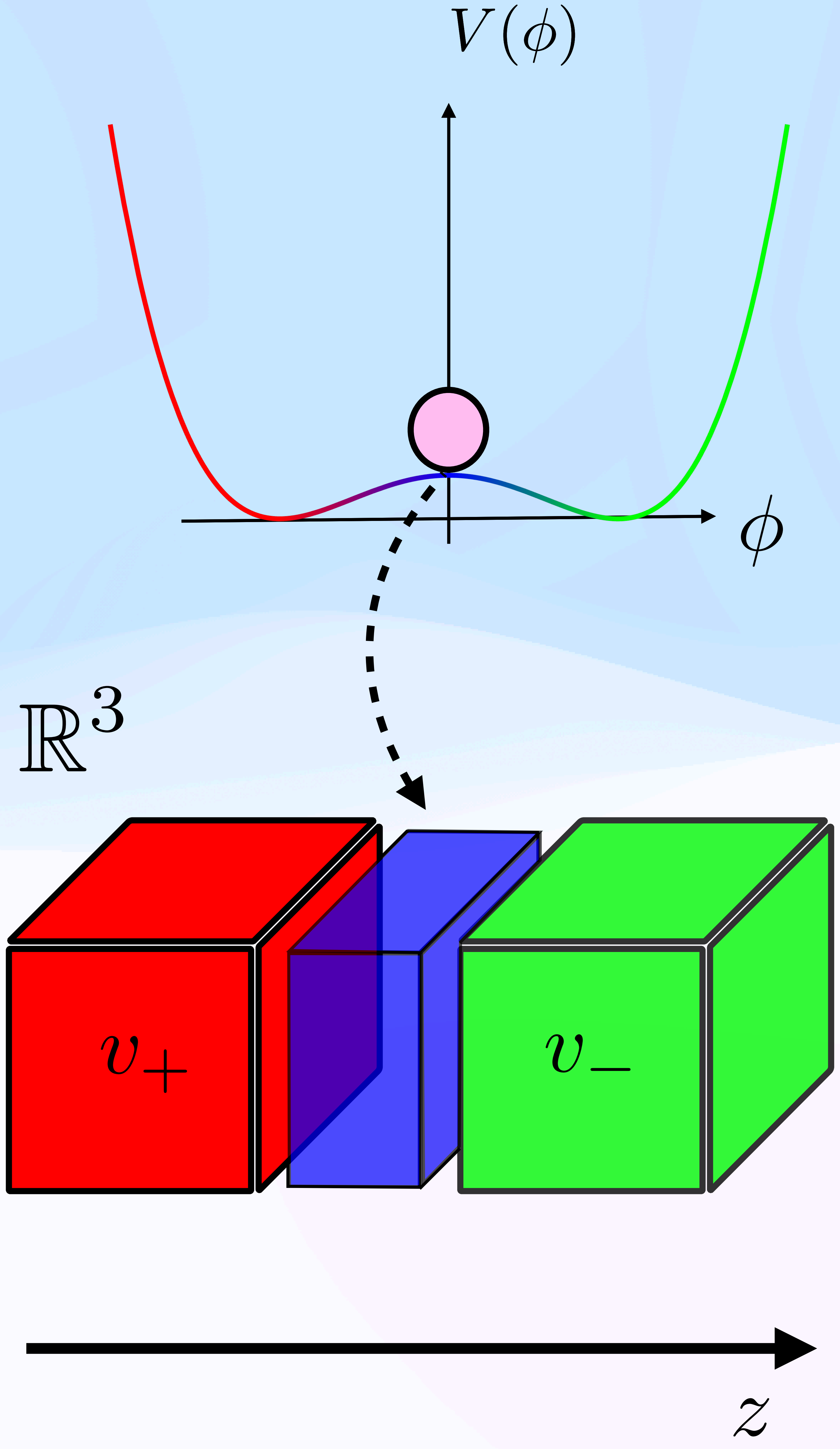
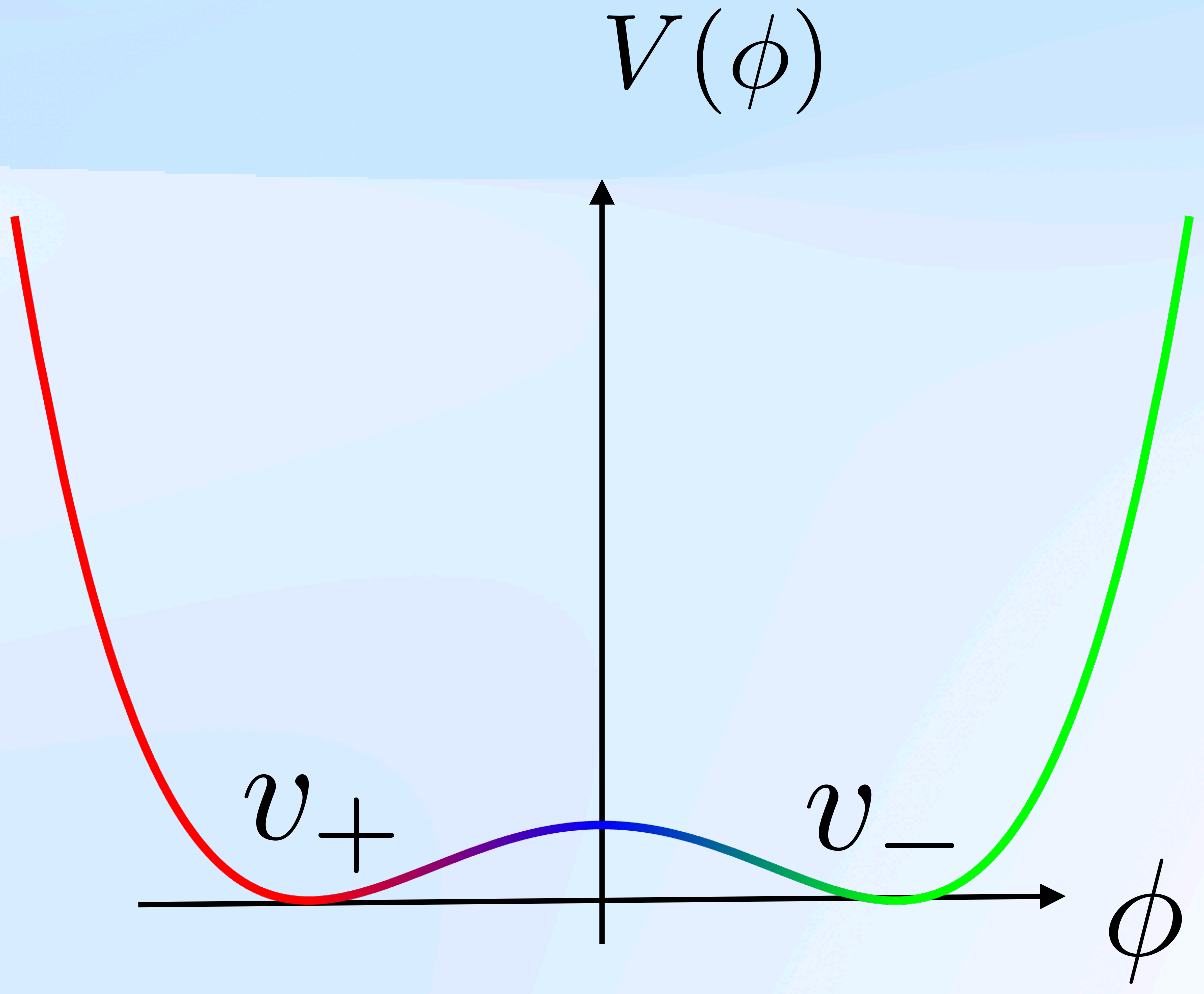
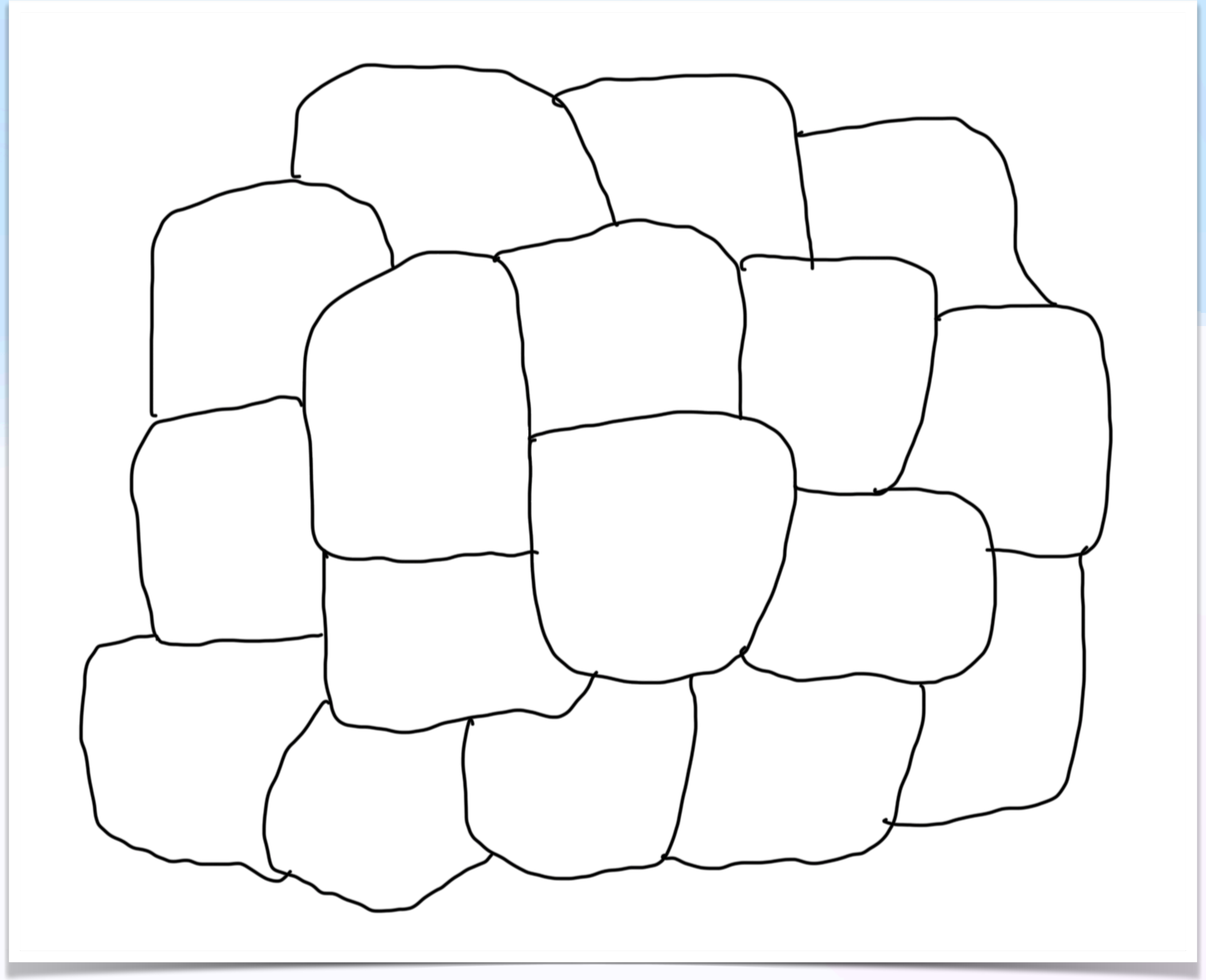




Domain wall formation

Kibble mechanism





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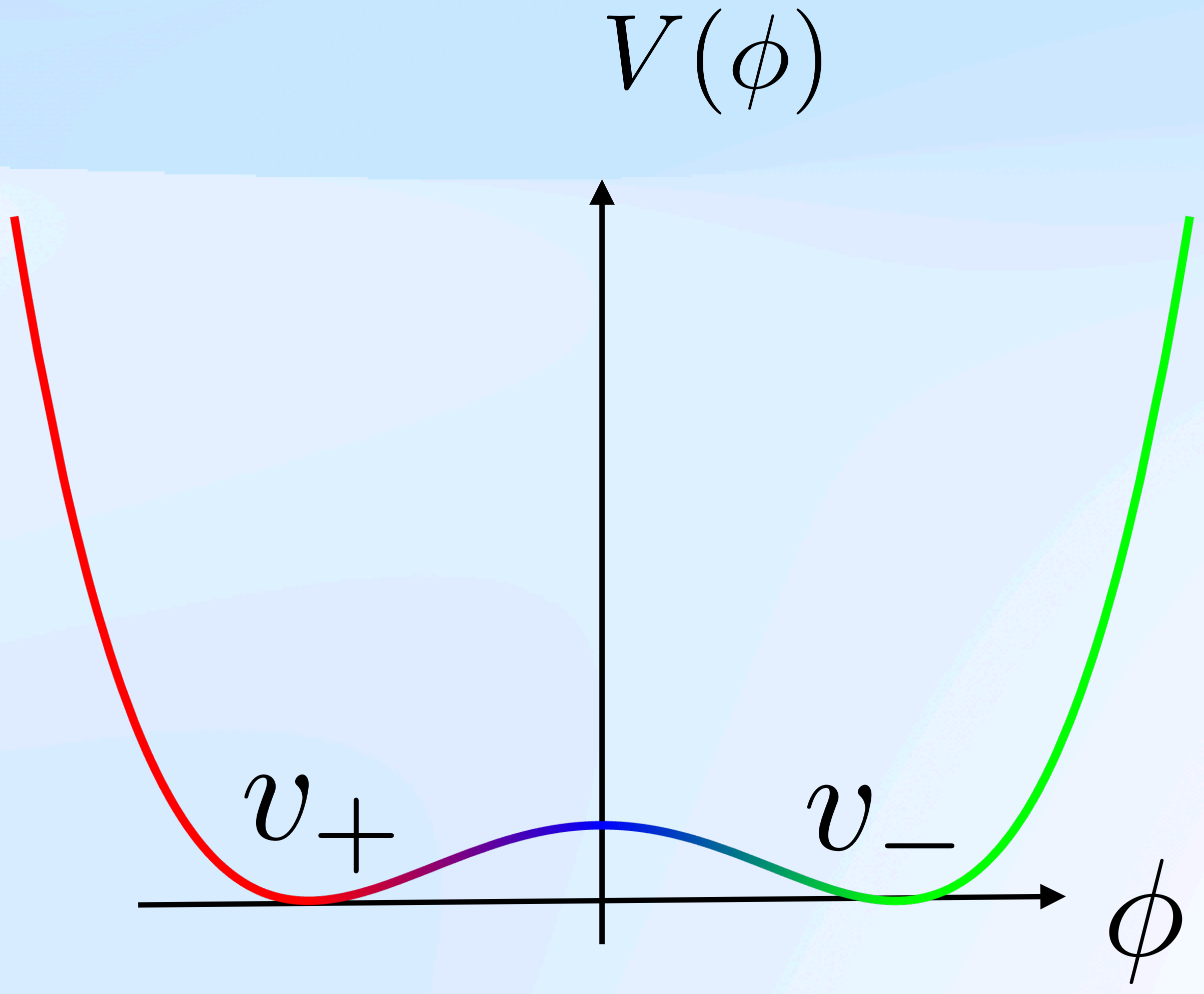

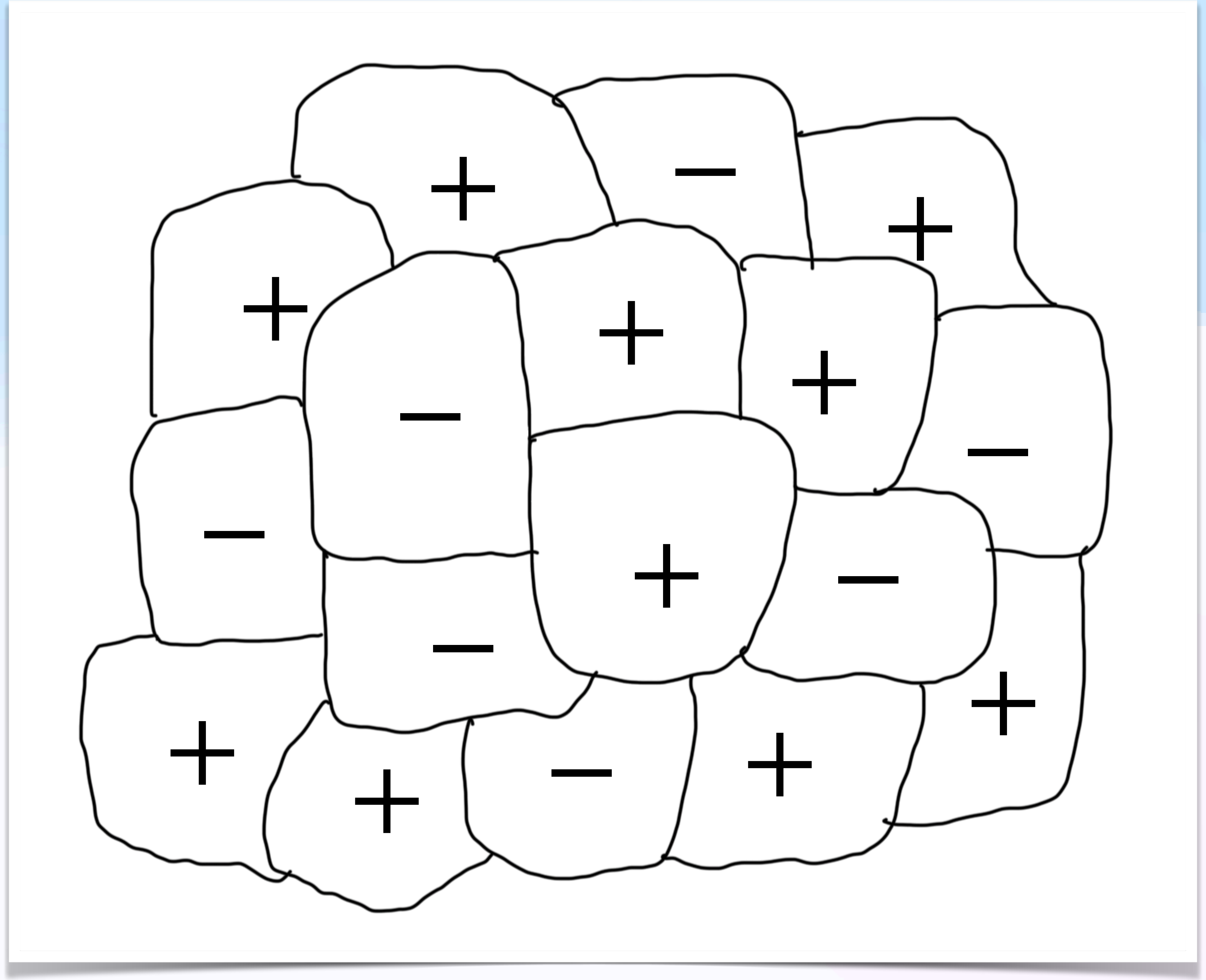

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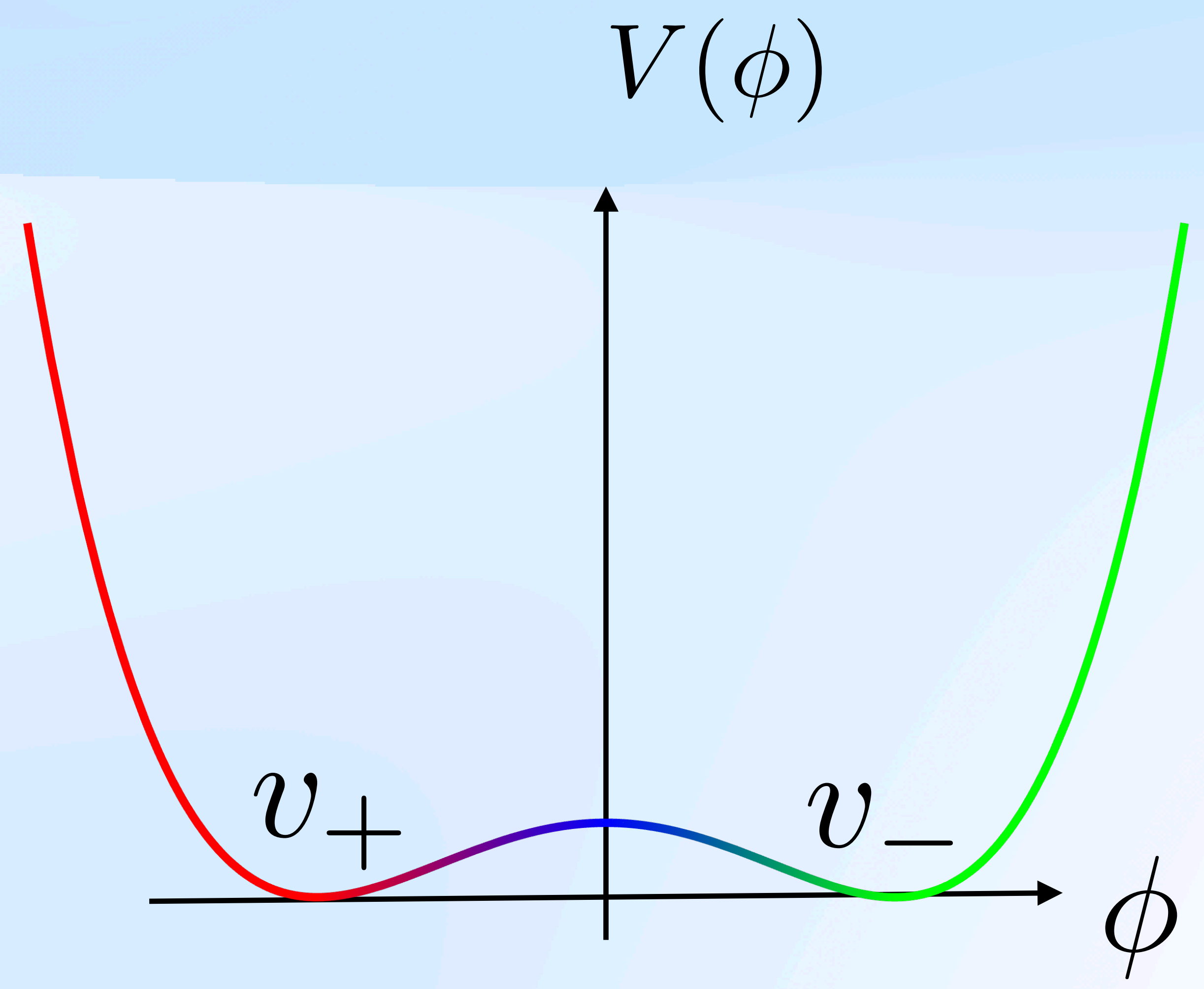
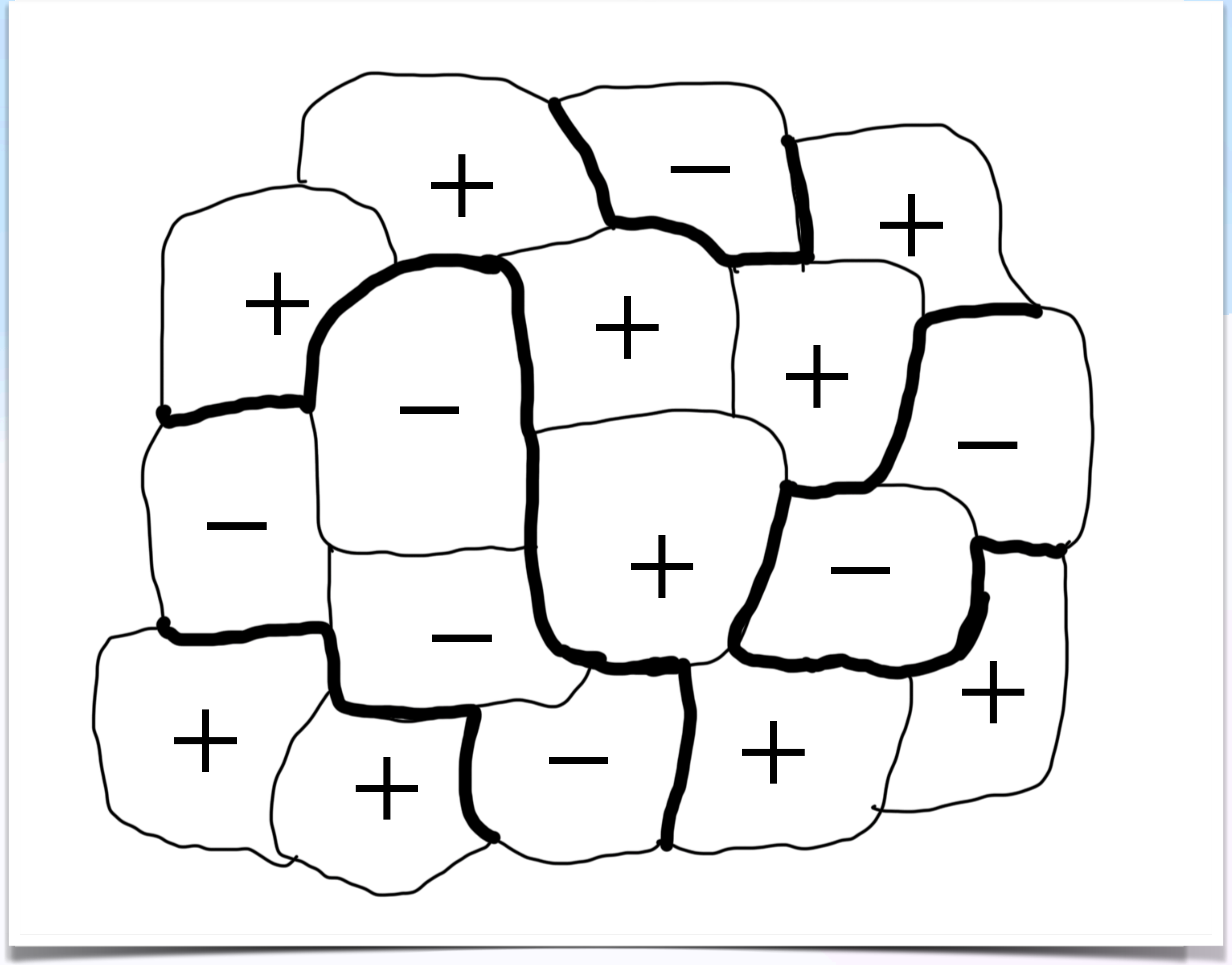
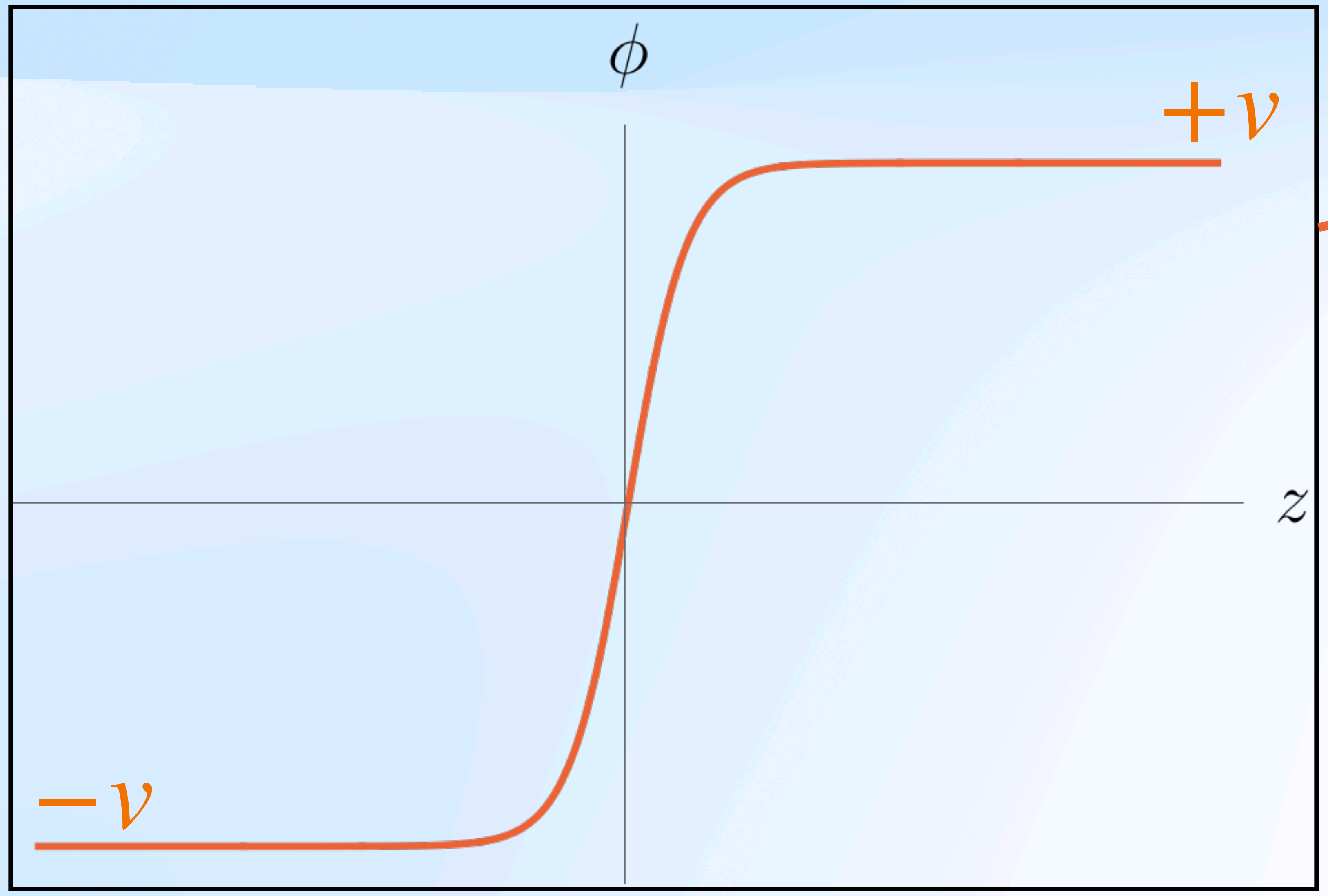


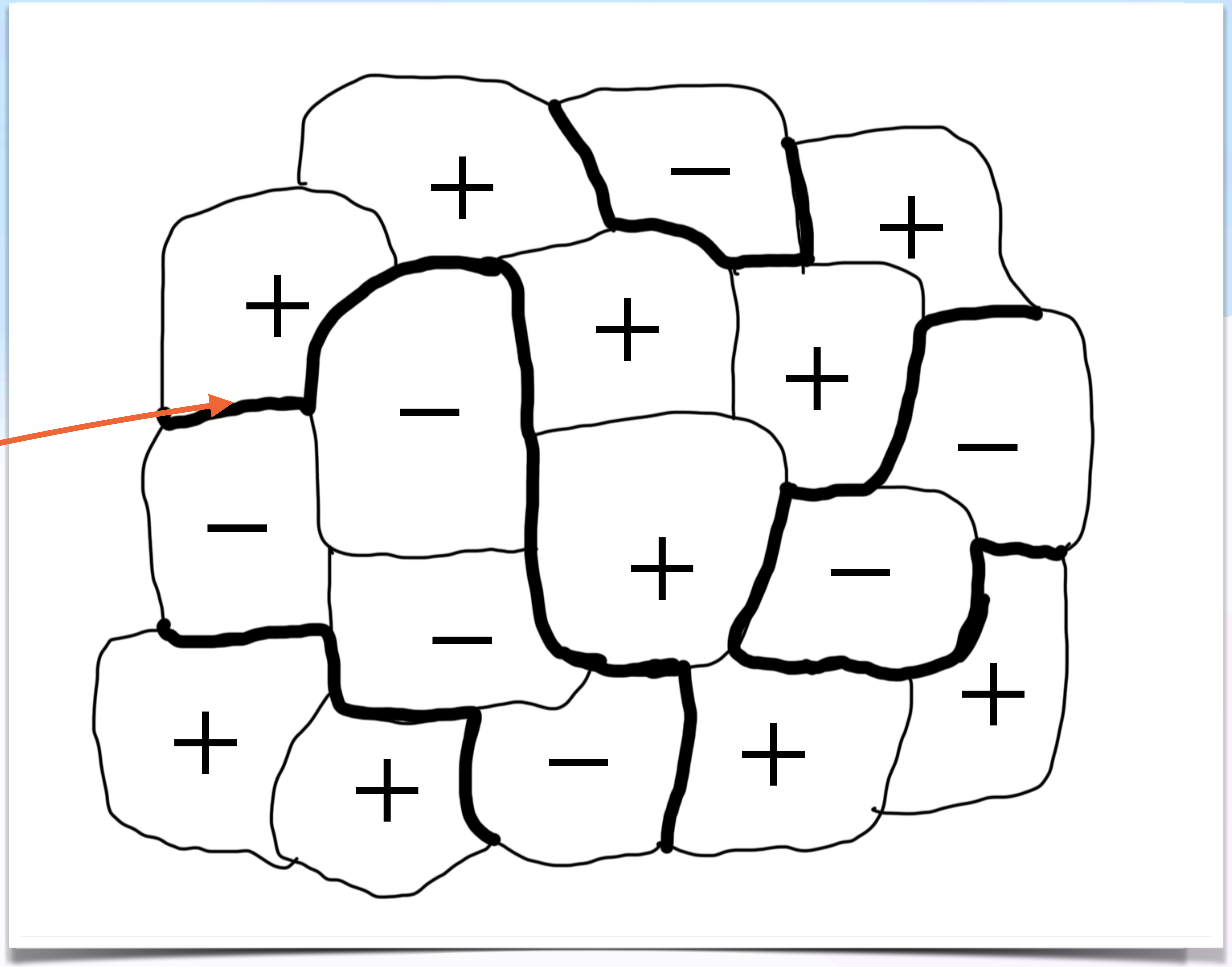


Domain wall formation

$$\frac{\partial^2 \phi}{\partial z^2} = \frac{\partial V}{\partial \phi}, \quad \phi(\pm\infty) = \pm v$$

$$\phi(z) = v \tanh \frac{z}{\Delta} \qquad \Delta = \sqrt{\frac{2}{\lambda v^2}}$$





Discrete Symmetries

- Abelian: Z_n
- Non-Abelian： $A_n, S_n, \Delta(27)......$

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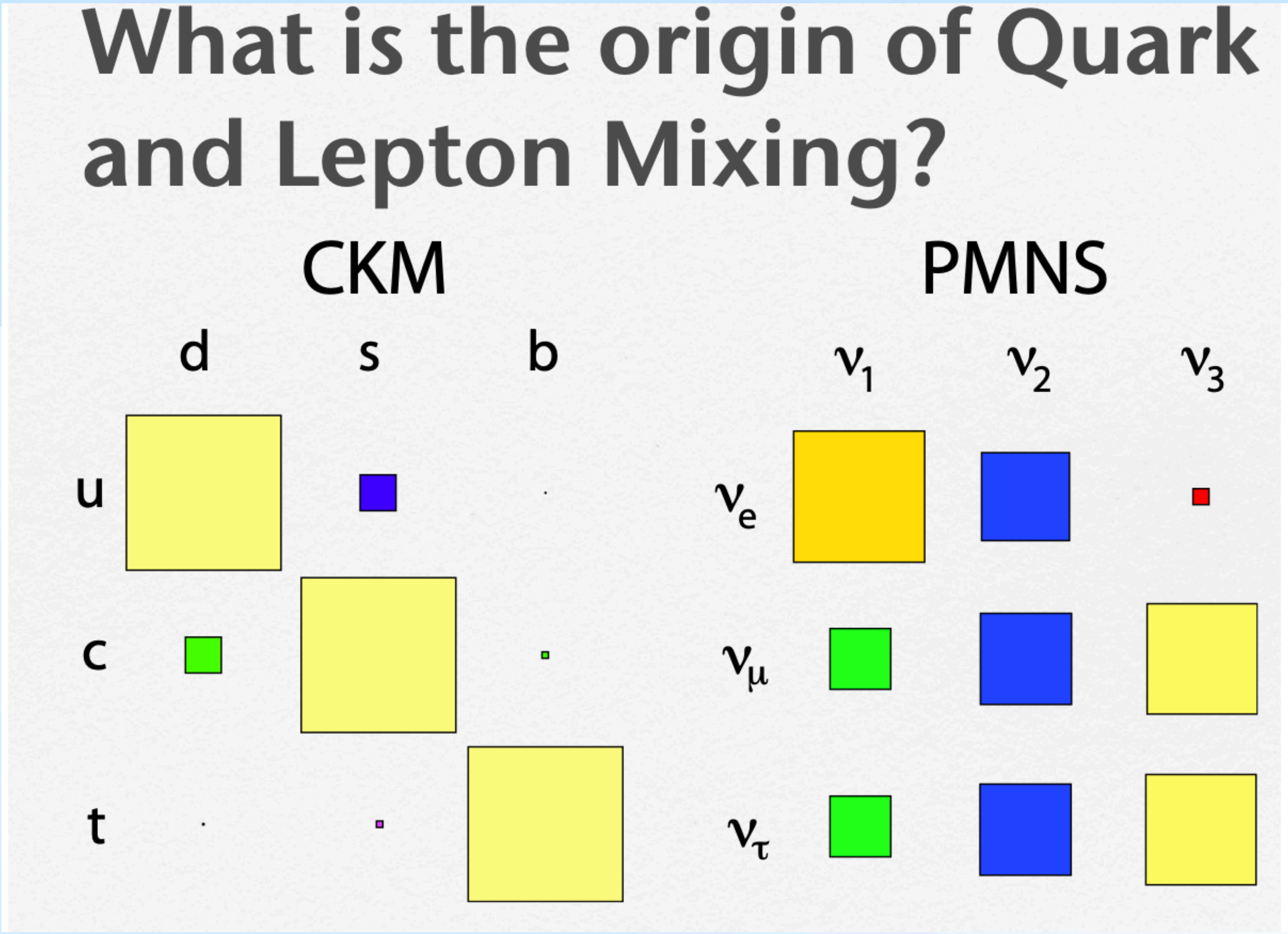

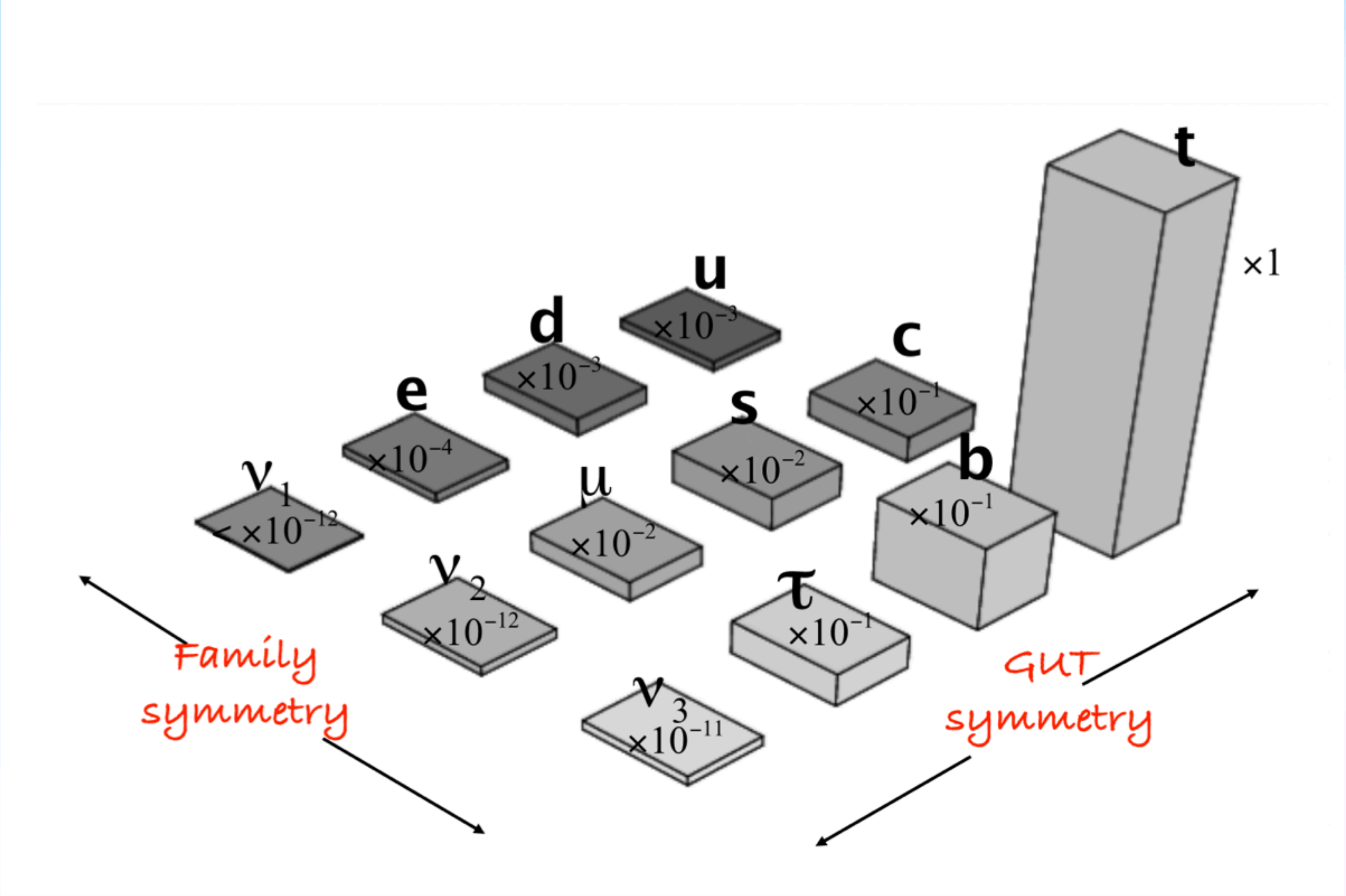

King 2015

Discrete Symmetries

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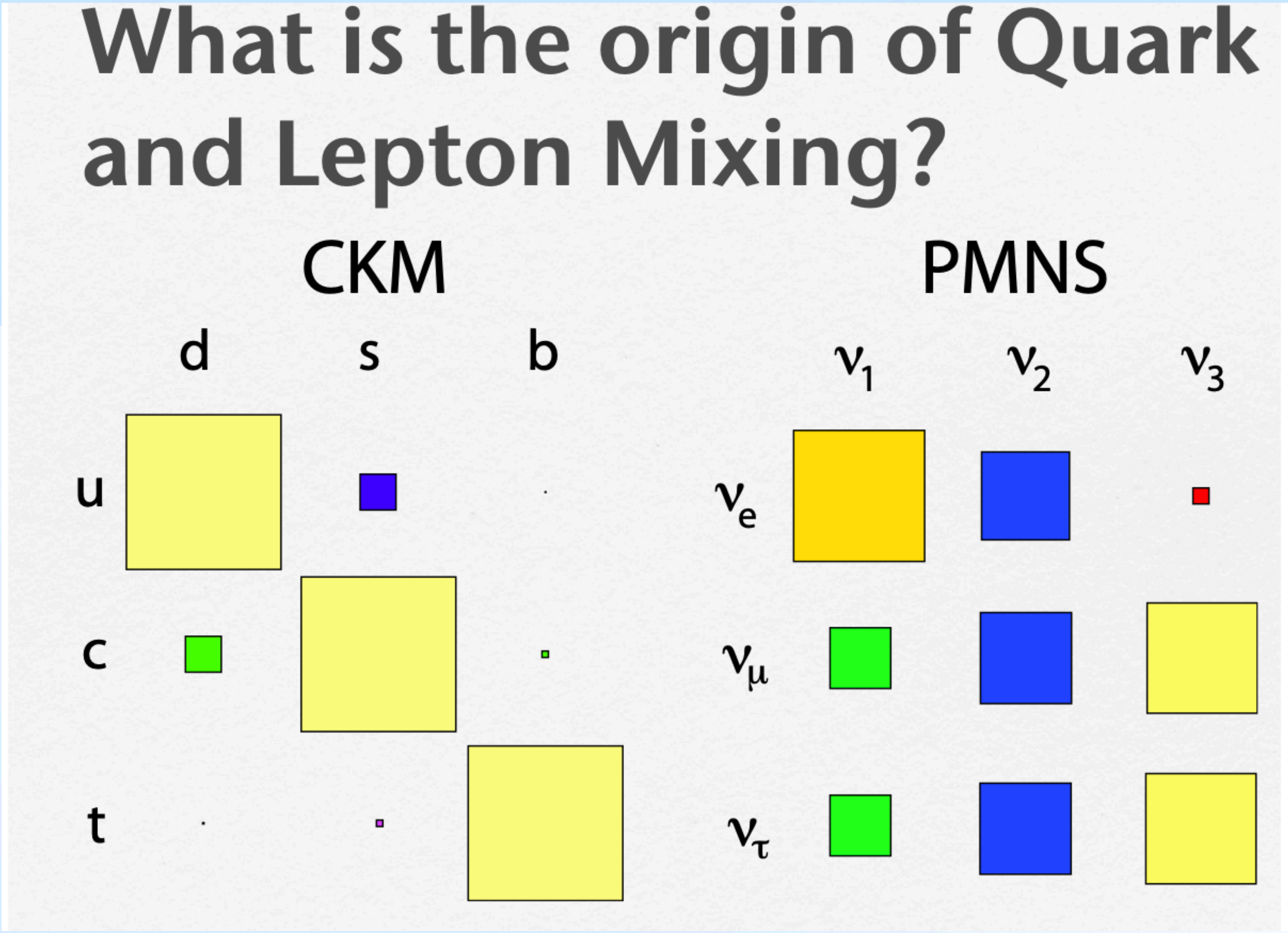

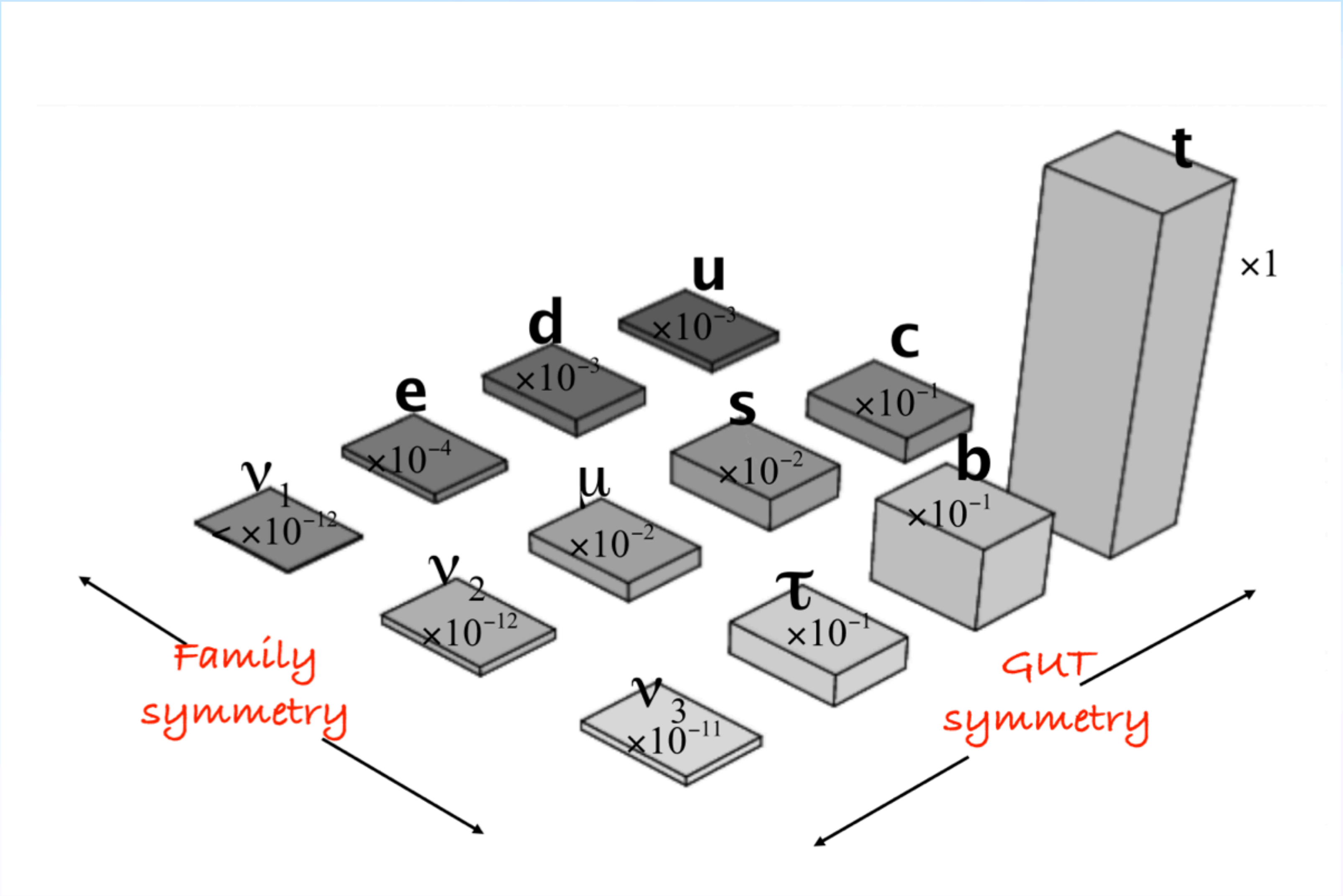

King 2015

flavour symmetries

Altarelli&Feruglio 1002.0211 Petcov 1711.10806

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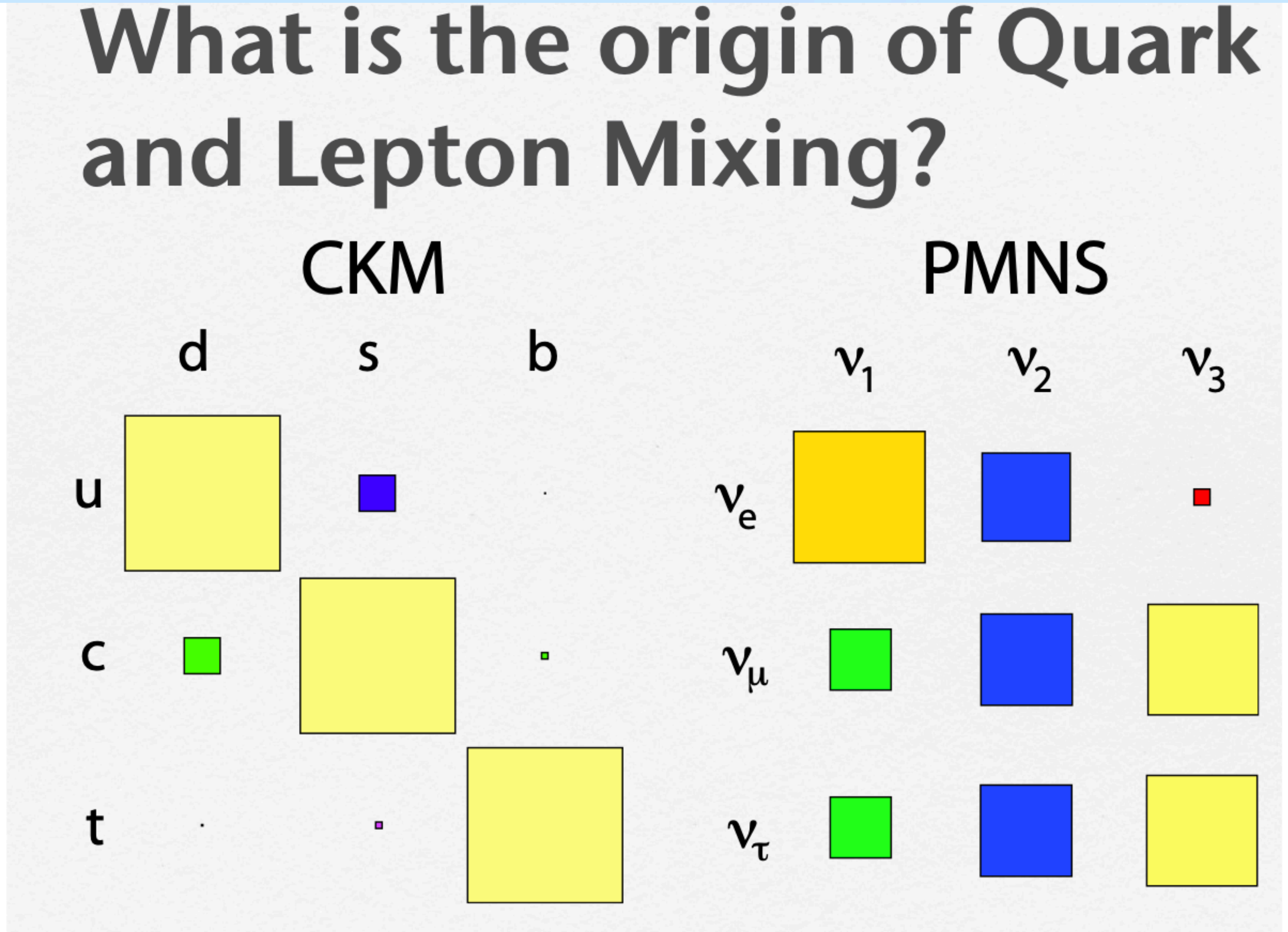

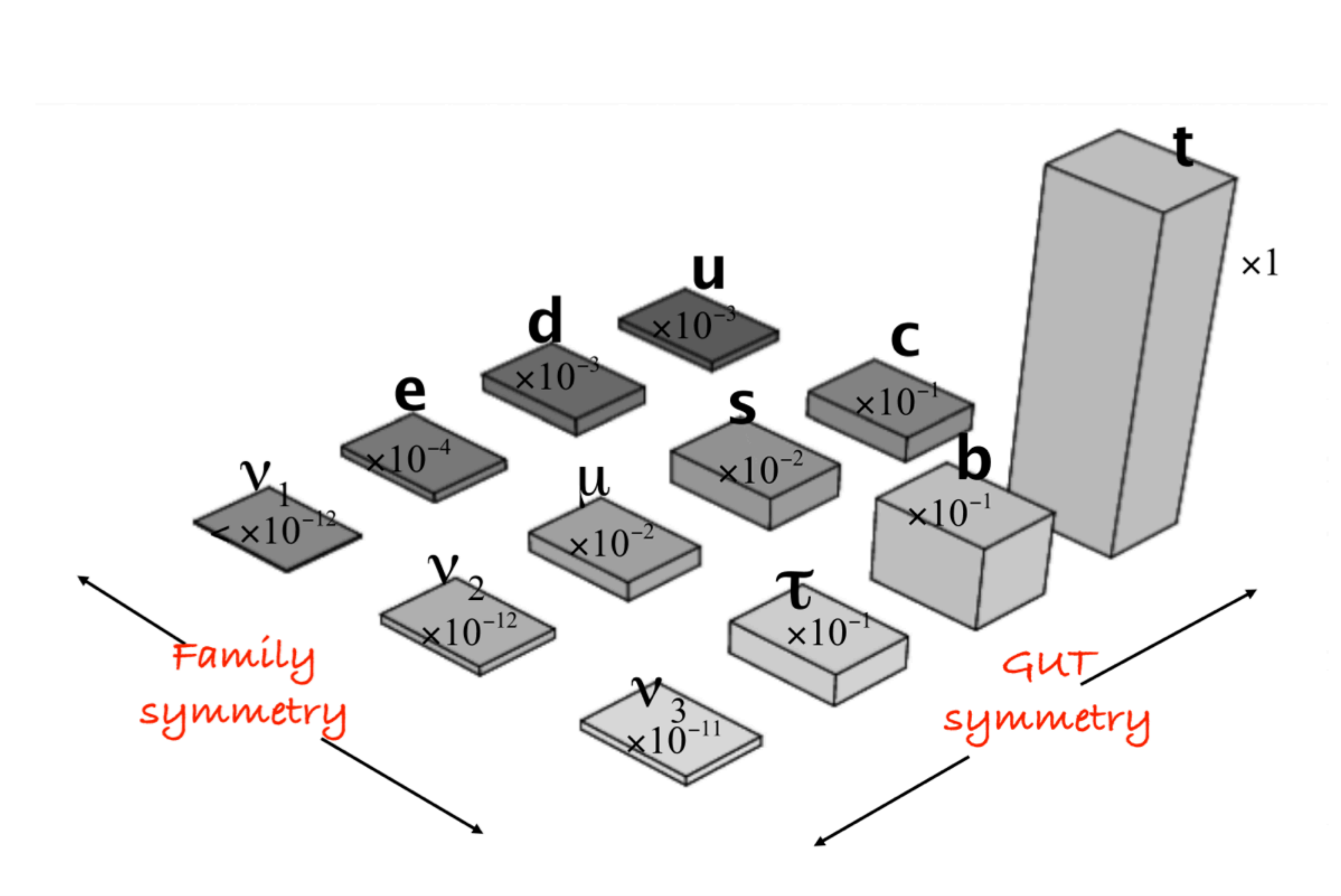

King 2015

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Altarelli&Feruglio 1002.0211 Petcov 1711.10806

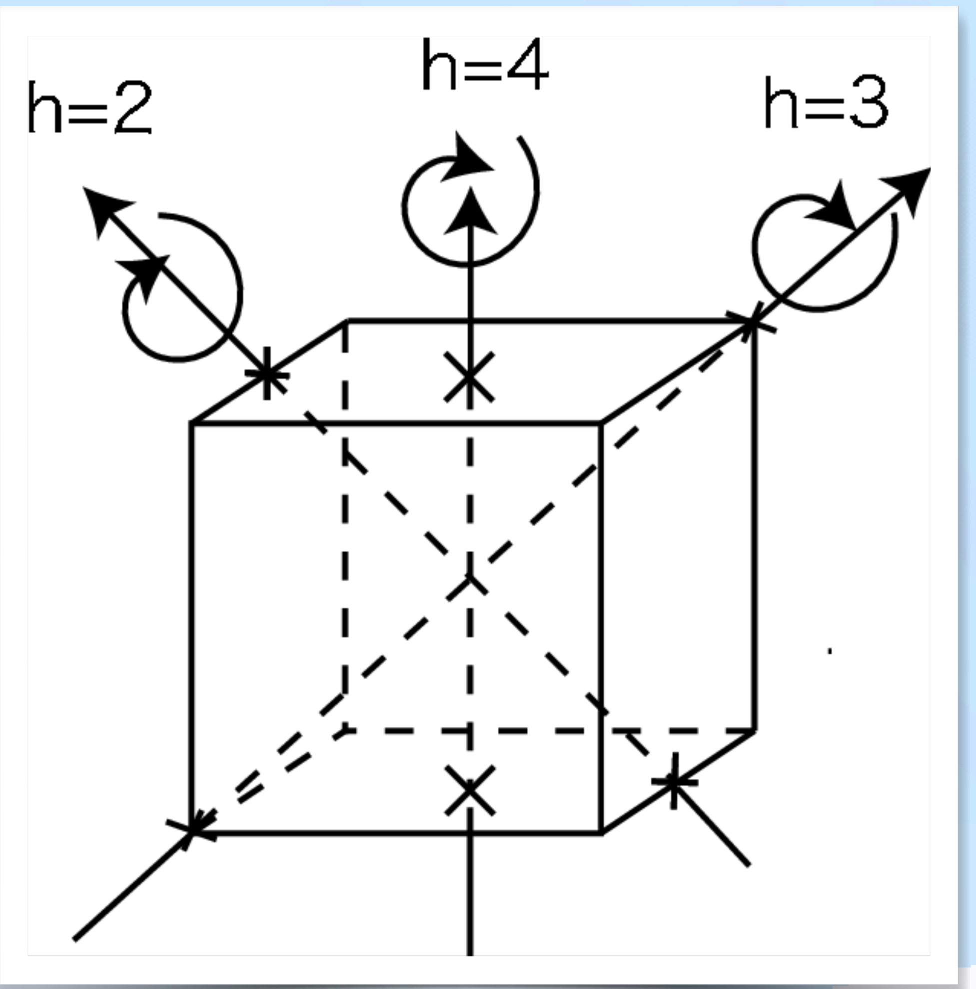
dark matter

Hirsch etc 1007.0871

S_4 scalar theory

- The octahedral/cube group S_4:

$$T=\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix},\quad S=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-1\end{pmatrix},\quad U=\pm\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$



Ishimori, etc 1003.3552

- The most general renormalisable flavon potential:

$$V(\phi)=-\frac{\mu^2}{2}I_1+\frac{g_1}{4}I_1^2+\frac{g_2}{2}I_2$$

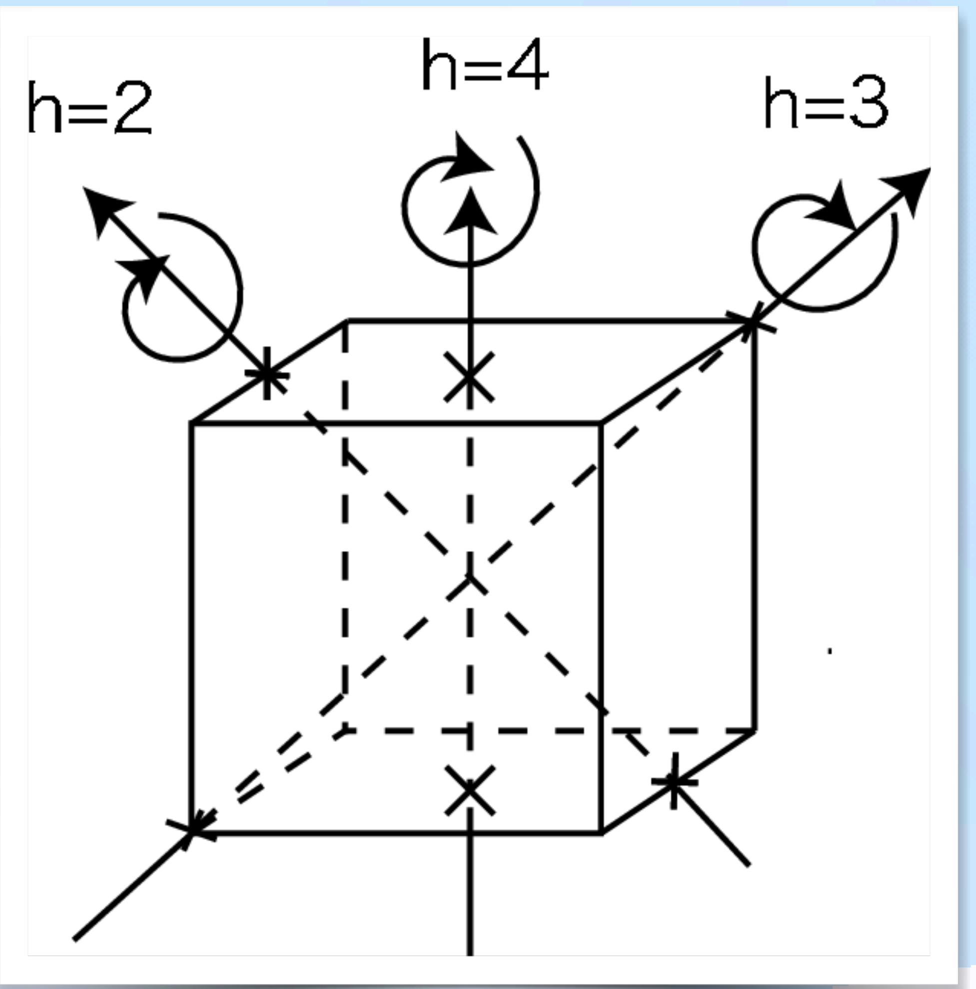
$$I_1=\phi_1^2+\phi_2^2+\phi_3^2\,,\quad I_2=\phi_1^2\phi_2^2+\phi_2^2\phi_3^2+\phi_3^2\phi_1^2$$

Also for $A_4\times Z_2$

S_4 scalar theory

- The octahedral/cube group S_4:

$$T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad U = \pm \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$



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- The most general renormalisable flavon potential:

$$V(\phi) = -\frac{\mu^2}{2} I_1 + \frac{g_1}{4} I_1^2 + \frac{g_2}{2} I_2$$

$$\beta = \frac{g_2}{g_1}$$

$$I_1 = \phi_1^2 + \phi_2^2 + \phi_3^2, \quad I_2 = \phi_1^2\phi_2^2 + \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2$$

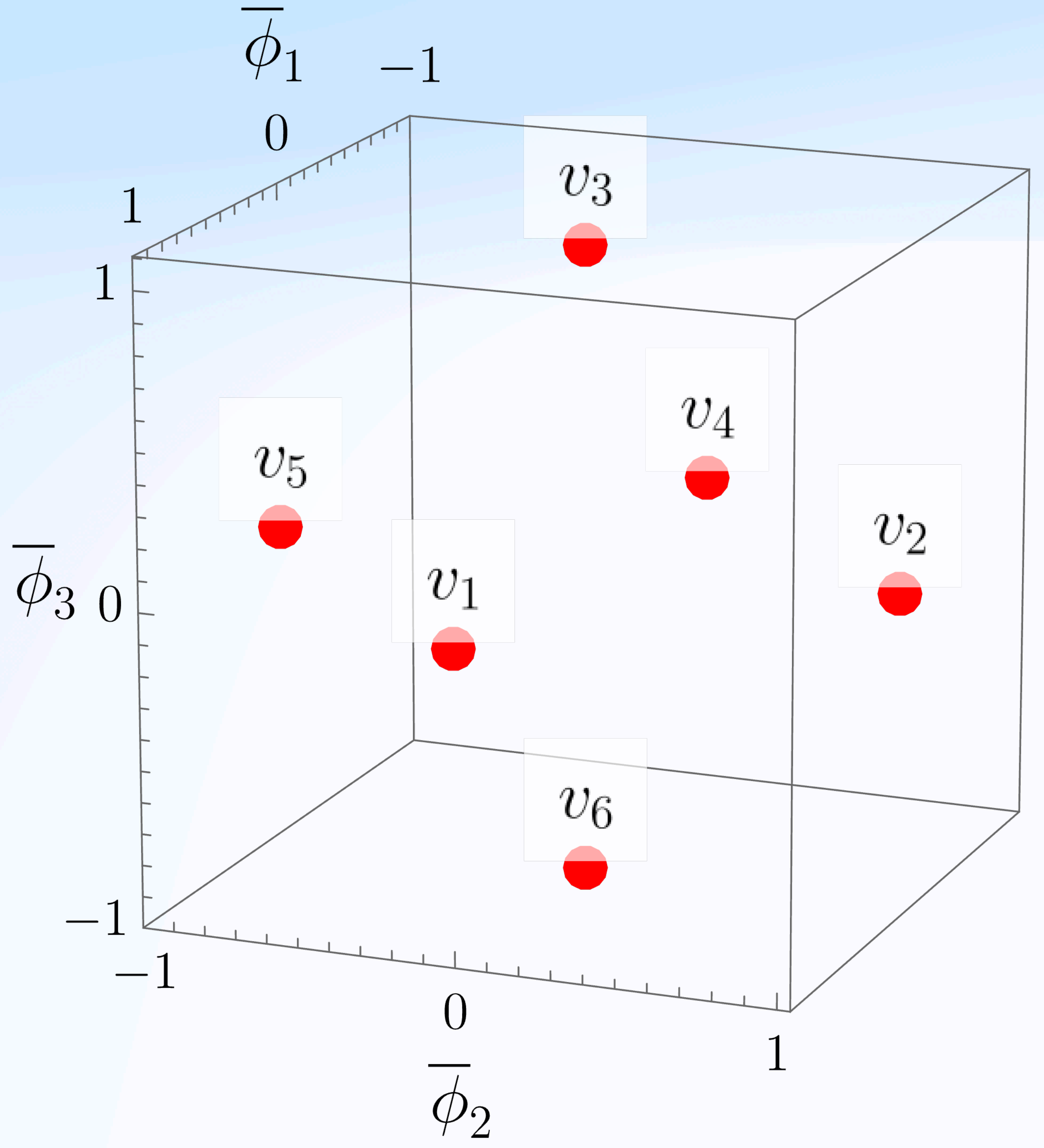
Also for $A_4 \times Z_2$

S_4 vacuum structure

2409.16359 Fu, King, Marsili, Pascoli, JT, Zhou

$$\left\{\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix},\begin{pmatrix}-1\\0\\0\end{pmatrix},\begin{pmatrix}0\\-1\\0\end{pmatrix},\begin{pmatrix}0\\0\\-1\end{pmatrix}\right\}v$$

$$v = \frac{\mu}{\sqrt{g_1}}$$

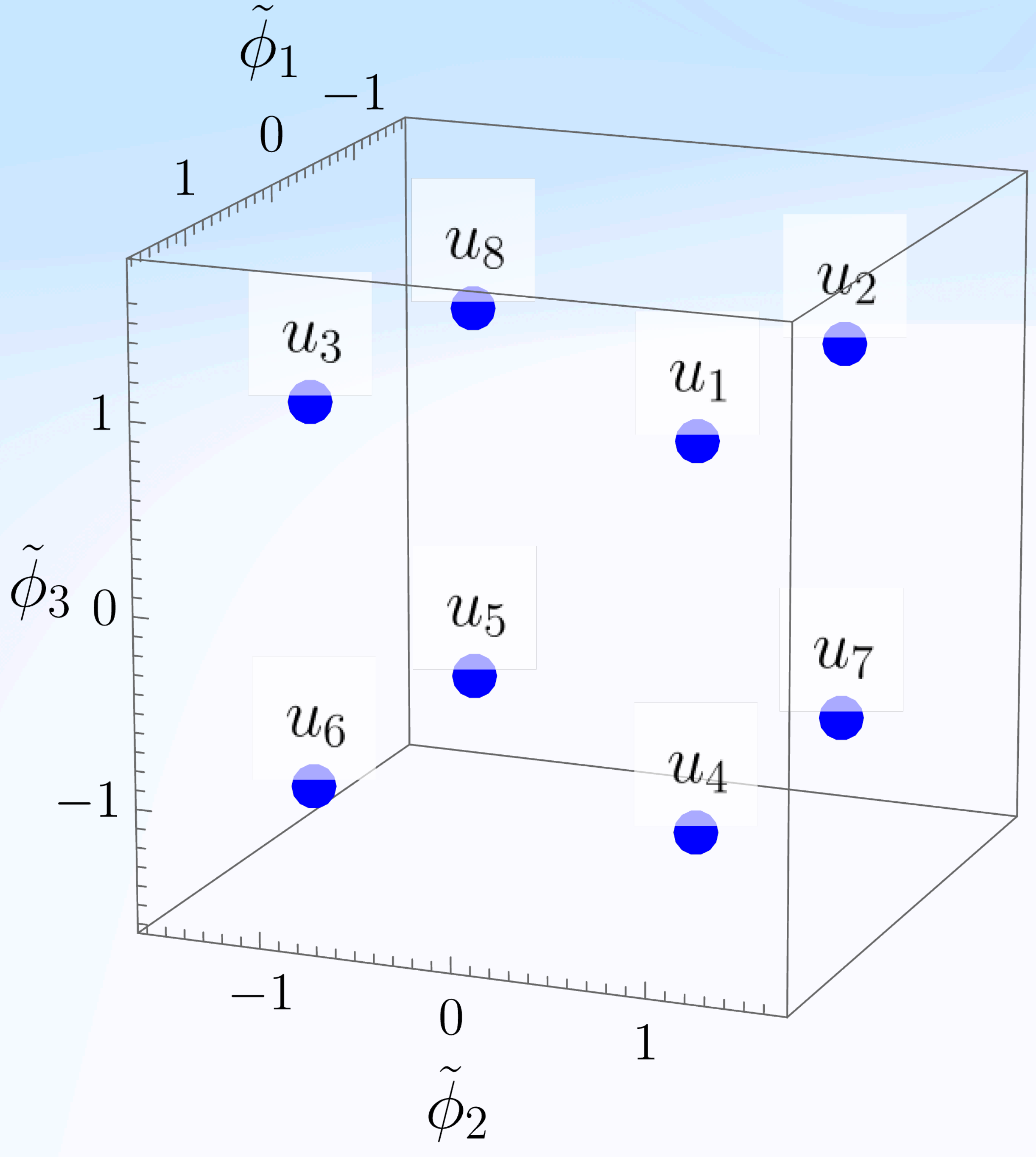


S_4 vacuum structure

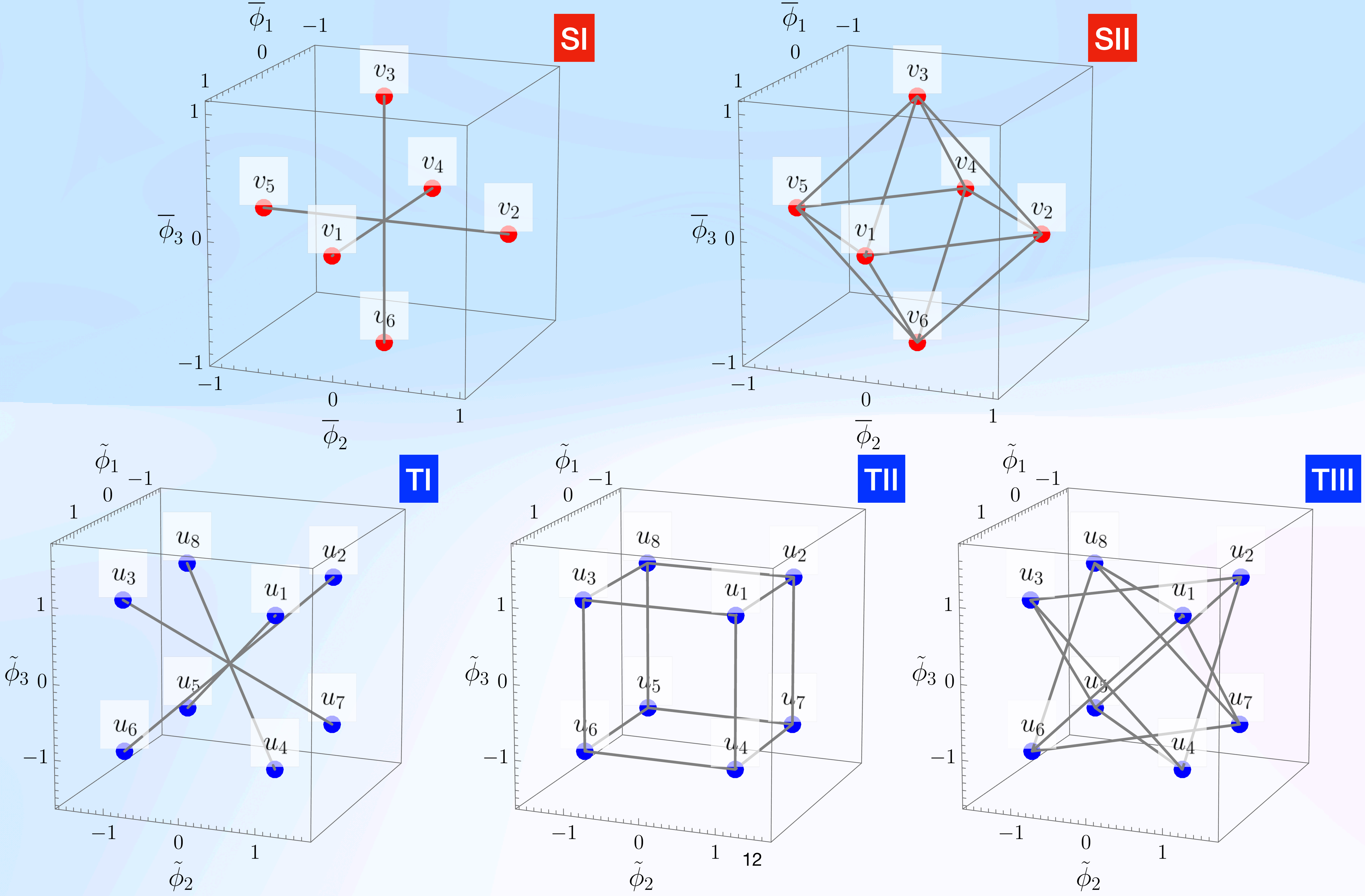
[2409.16359](2409.16359) Fu, King, Marsili, Pascoli, JT, Zhou

$$\left\{\begin{pmatrix}1\\1\\1\end{pmatrix},\begin{pmatrix}-1\\1\\1\end{pmatrix},\begin{pmatrix}1\\-1\\1\end{pmatrix},\begin{pmatrix}1\\1\\-1\end{pmatrix},\begin{pmatrix}-1\\-1\\-1\end{pmatrix},\begin{pmatrix}1\\-1\\-1\end{pmatrix},\begin{pmatrix}-1\\1\\-1\end{pmatrix},\begin{pmatrix}-1\\-1\\1\end{pmatrix}\right\}u$$

$$u = \frac{\mu}{\sqrt{3g_1 + 2g_2}}$$



S_4 domain walls



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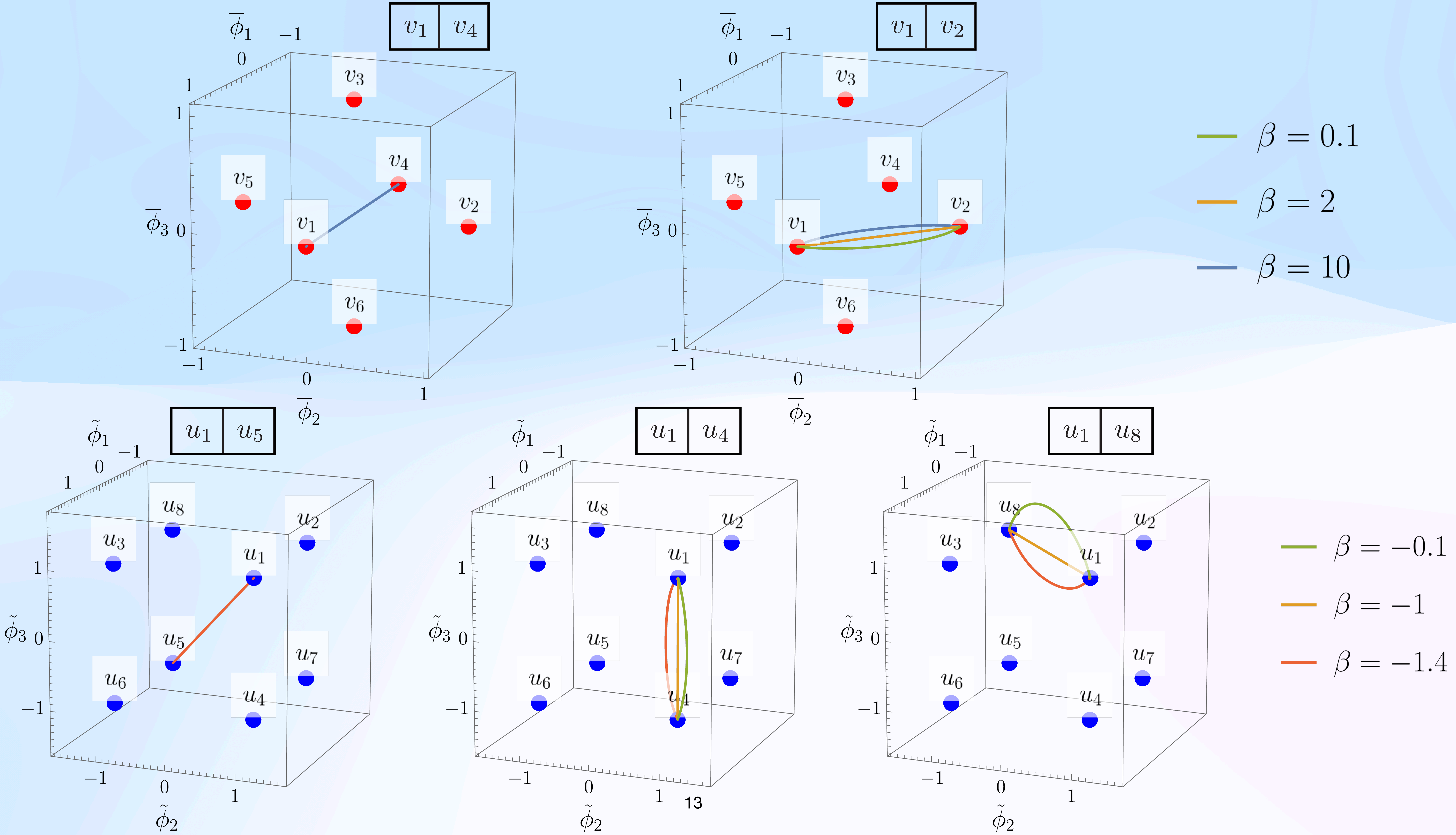
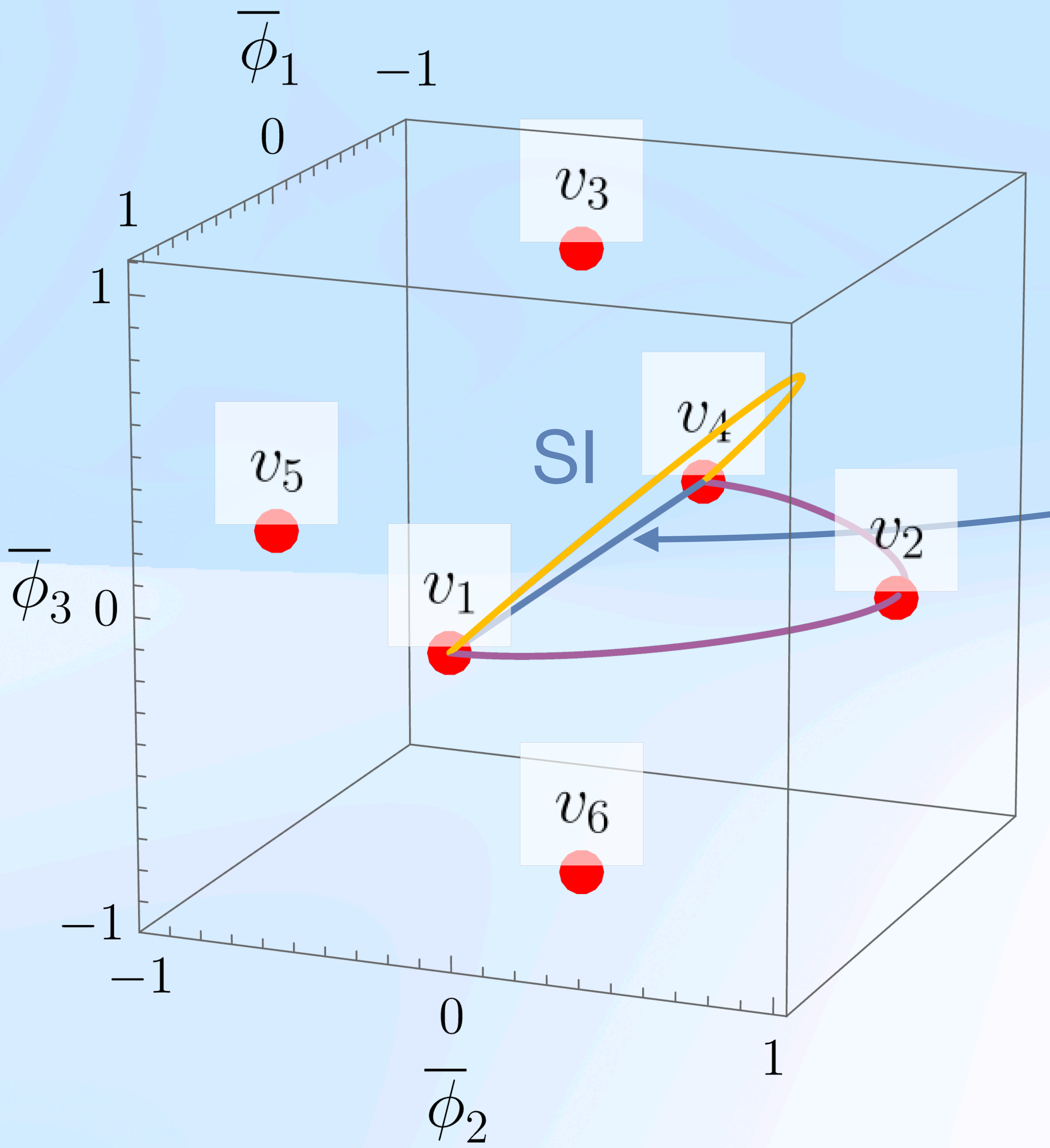


Stability of DWs



Straight line SI solution

Independent of $\beta = g_2/g_1$

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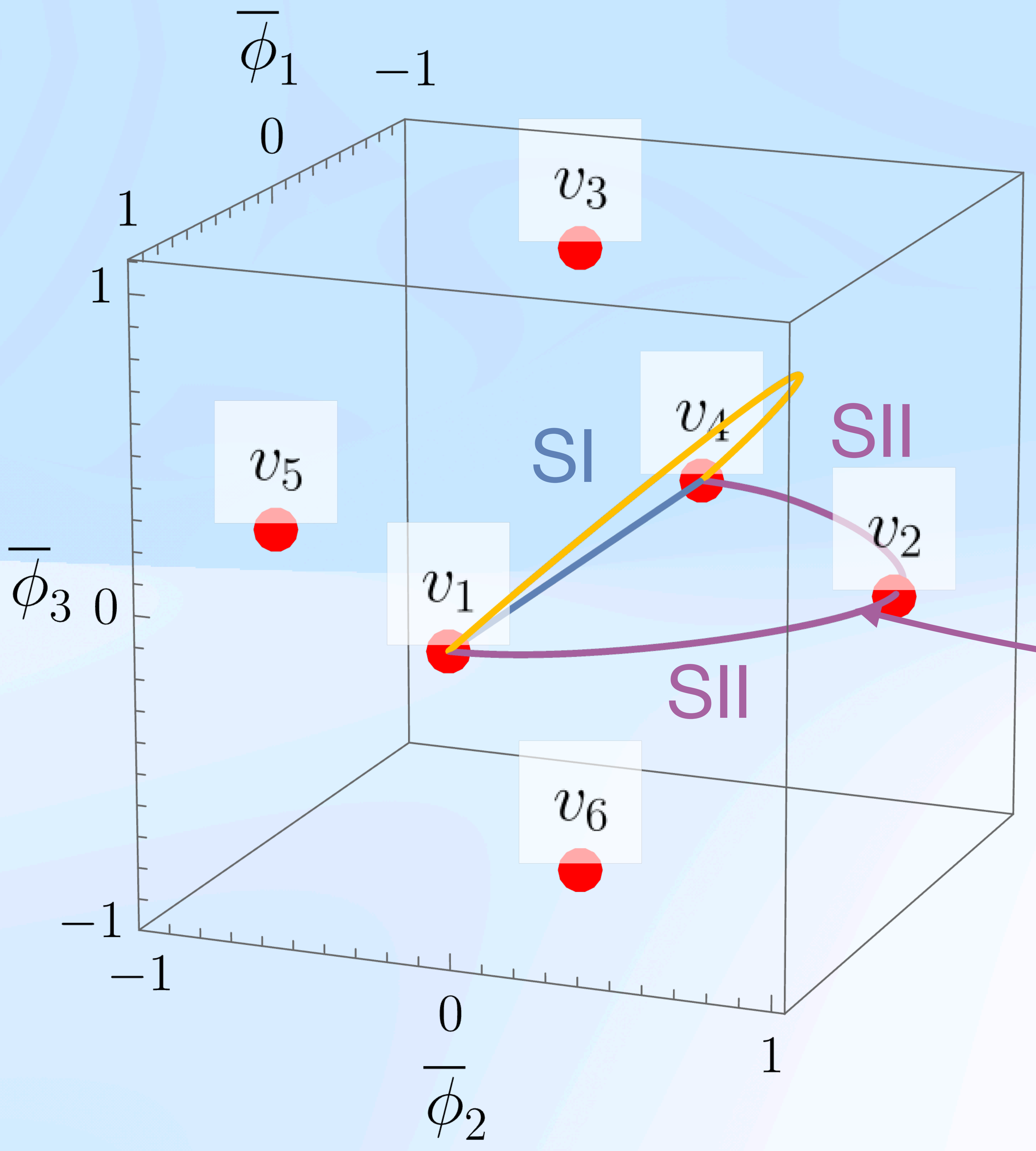

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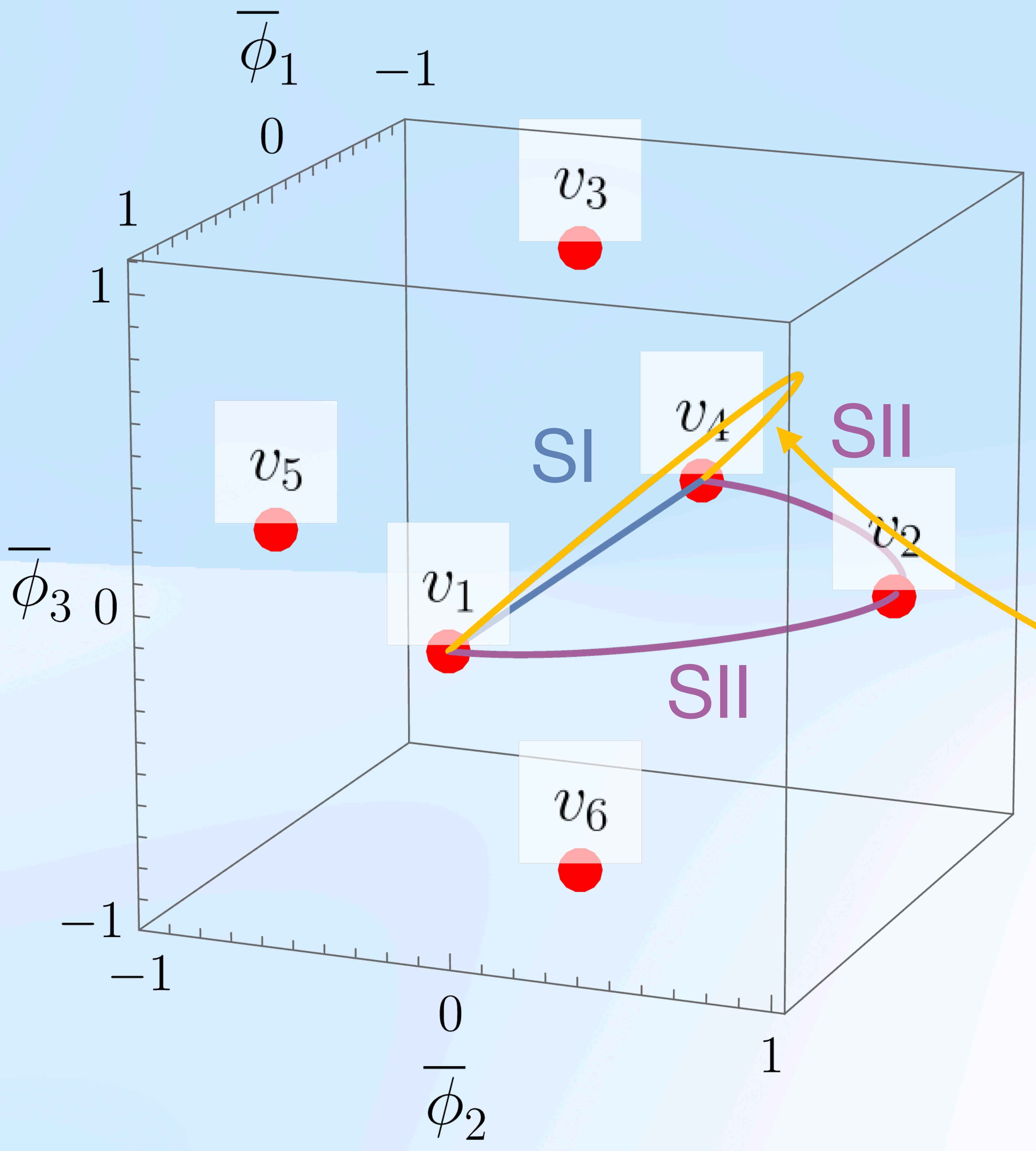
Two SIII solutions with pitstop at v_2

Stability of DWs



Straight line SI solution
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Two SIII solutions with
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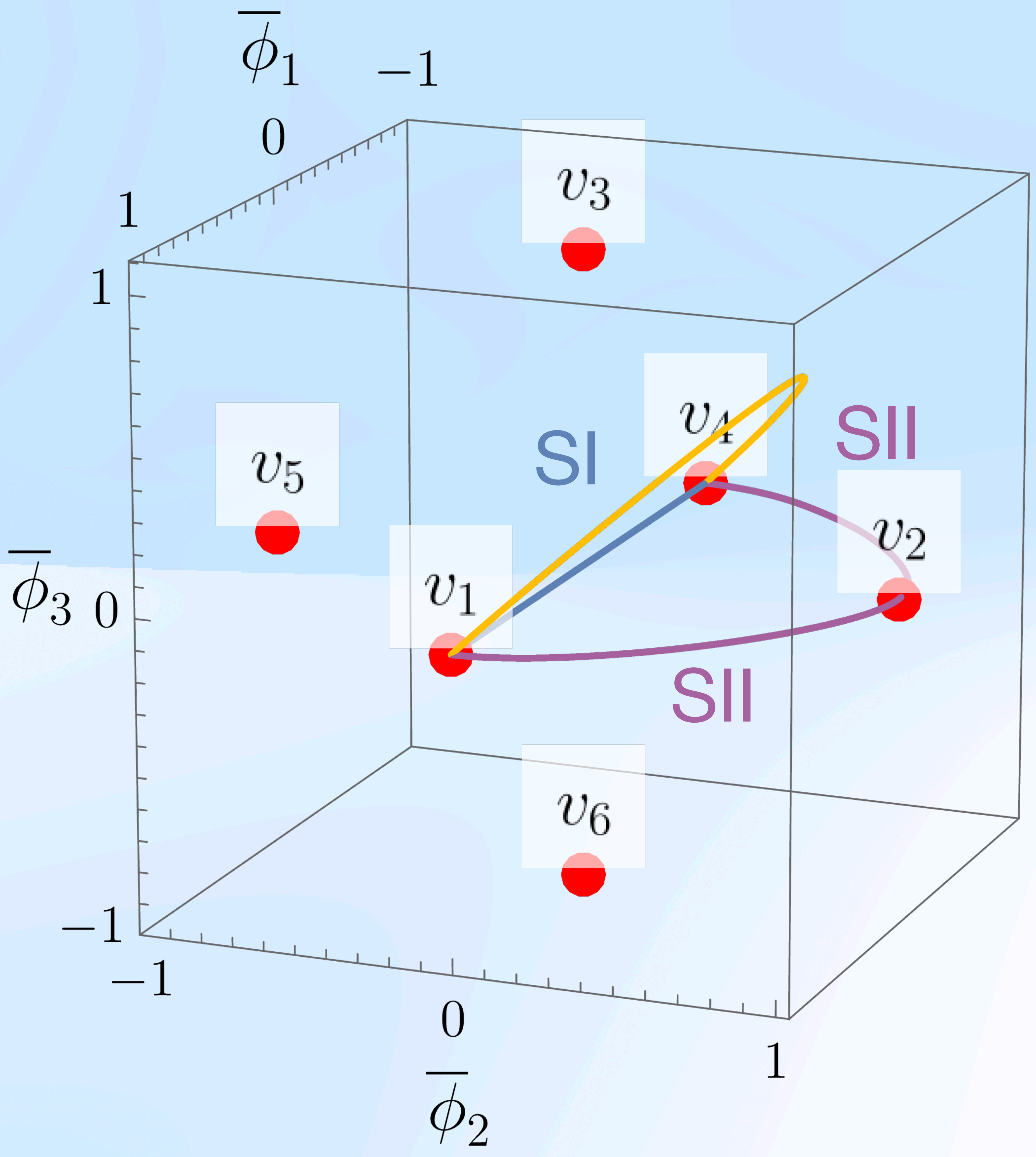
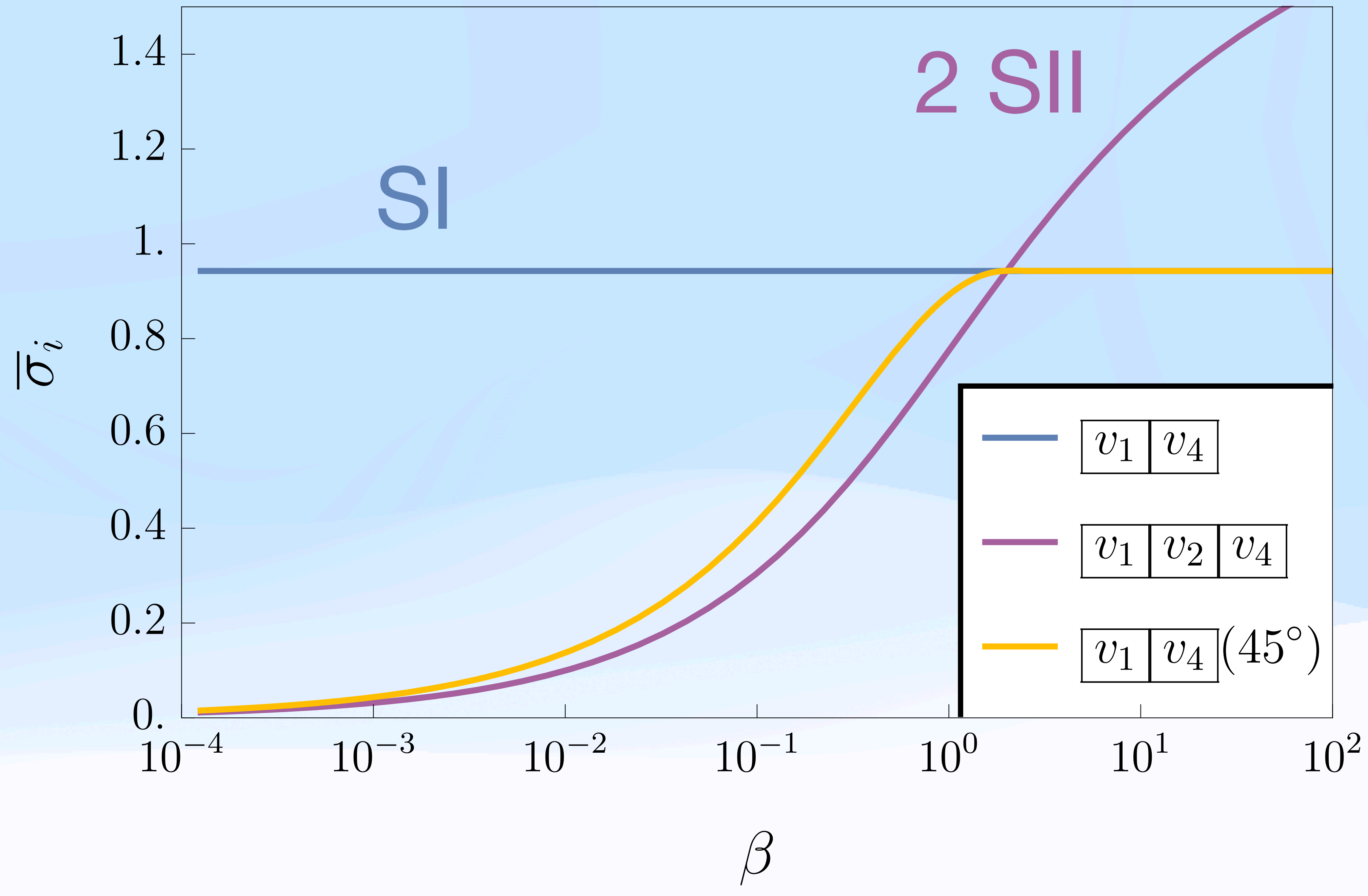
Intermediate solution (still
satisfies EoM)

Stability of DWs

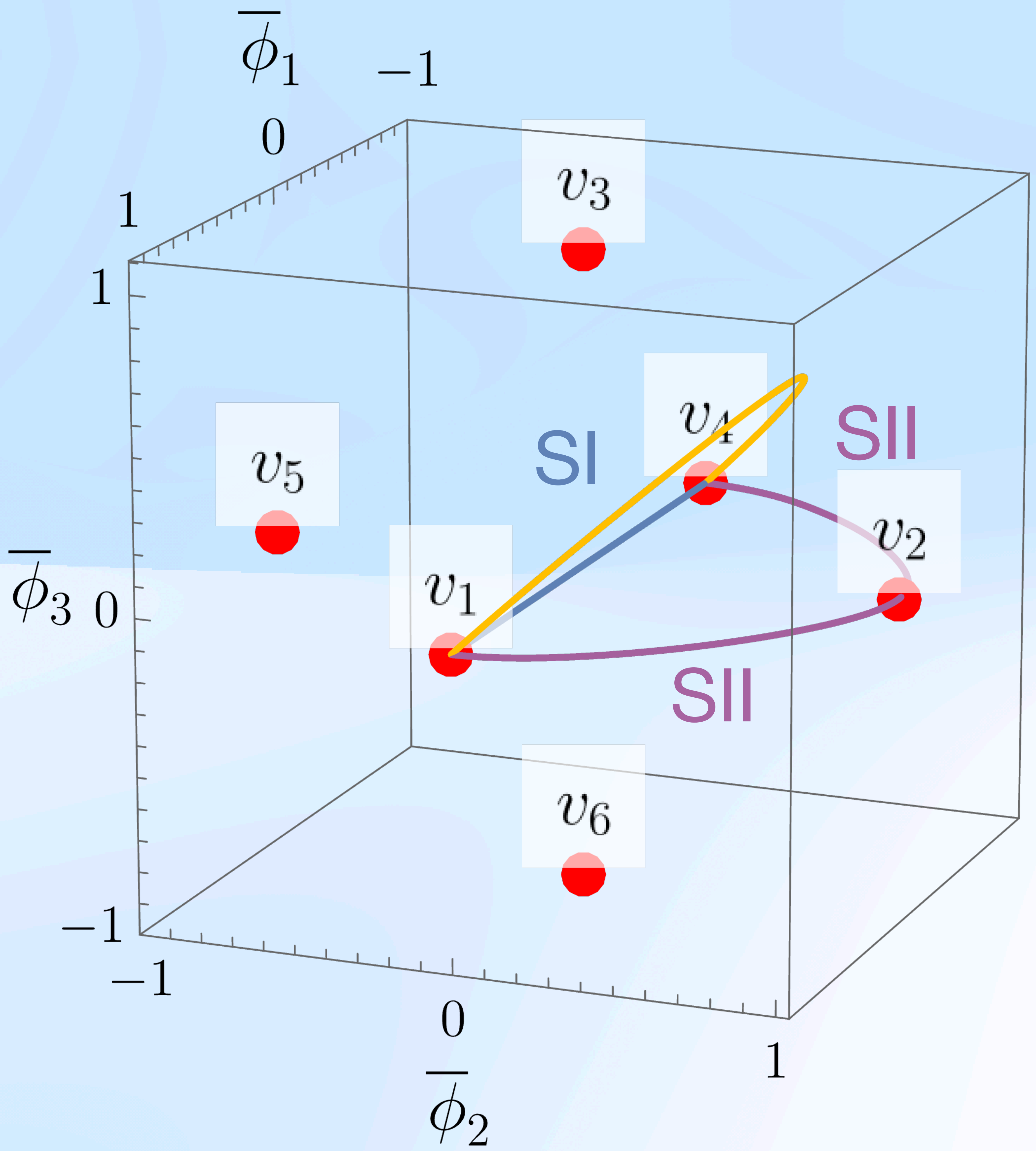


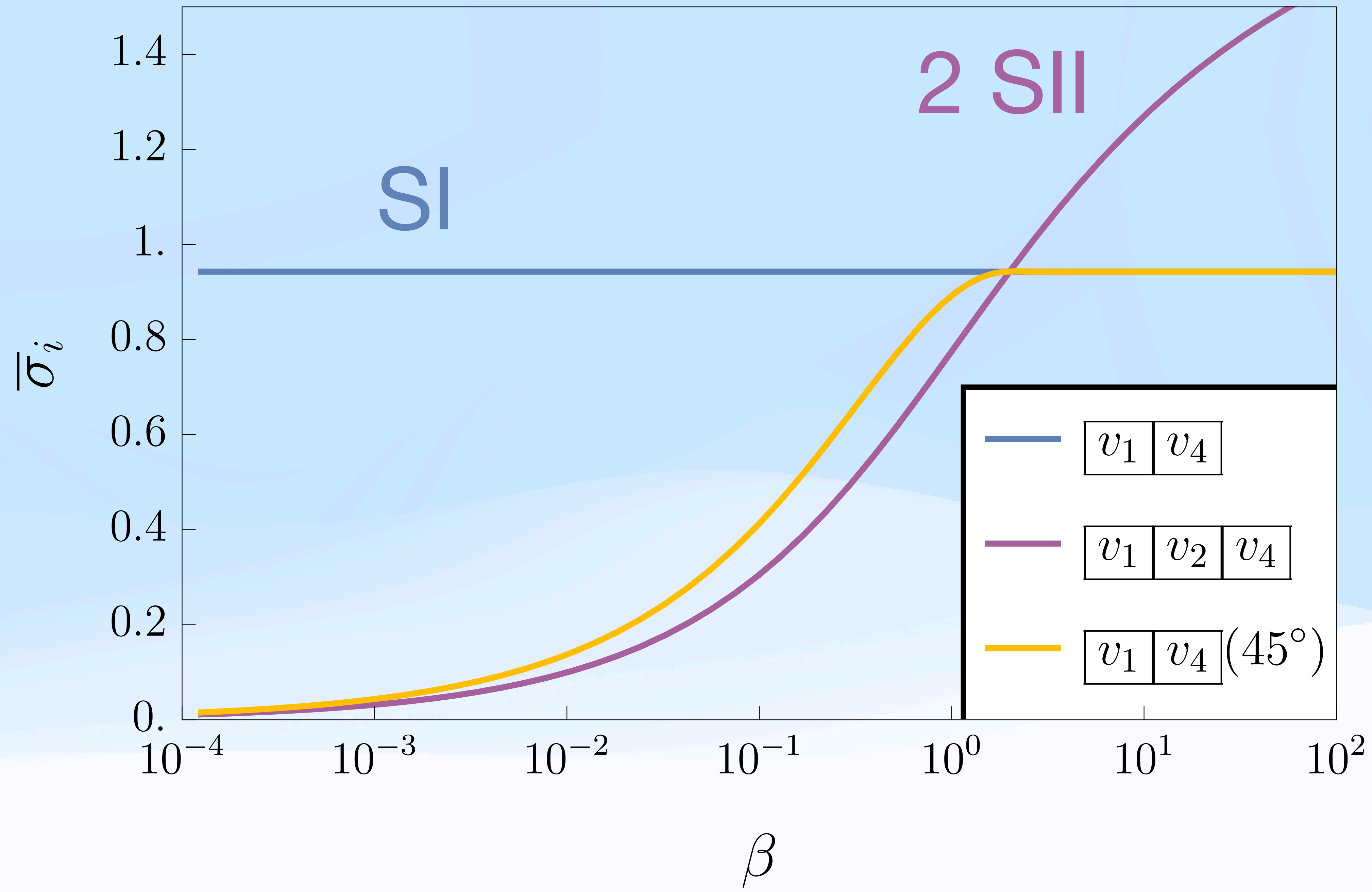


$$\beta \ll 2 : \sigma\,(\mathrm{SI}) > 2\sigma\,(\mathrm{SII})$$

SI DW **unstable** & would decay to SII

Stability of DWs

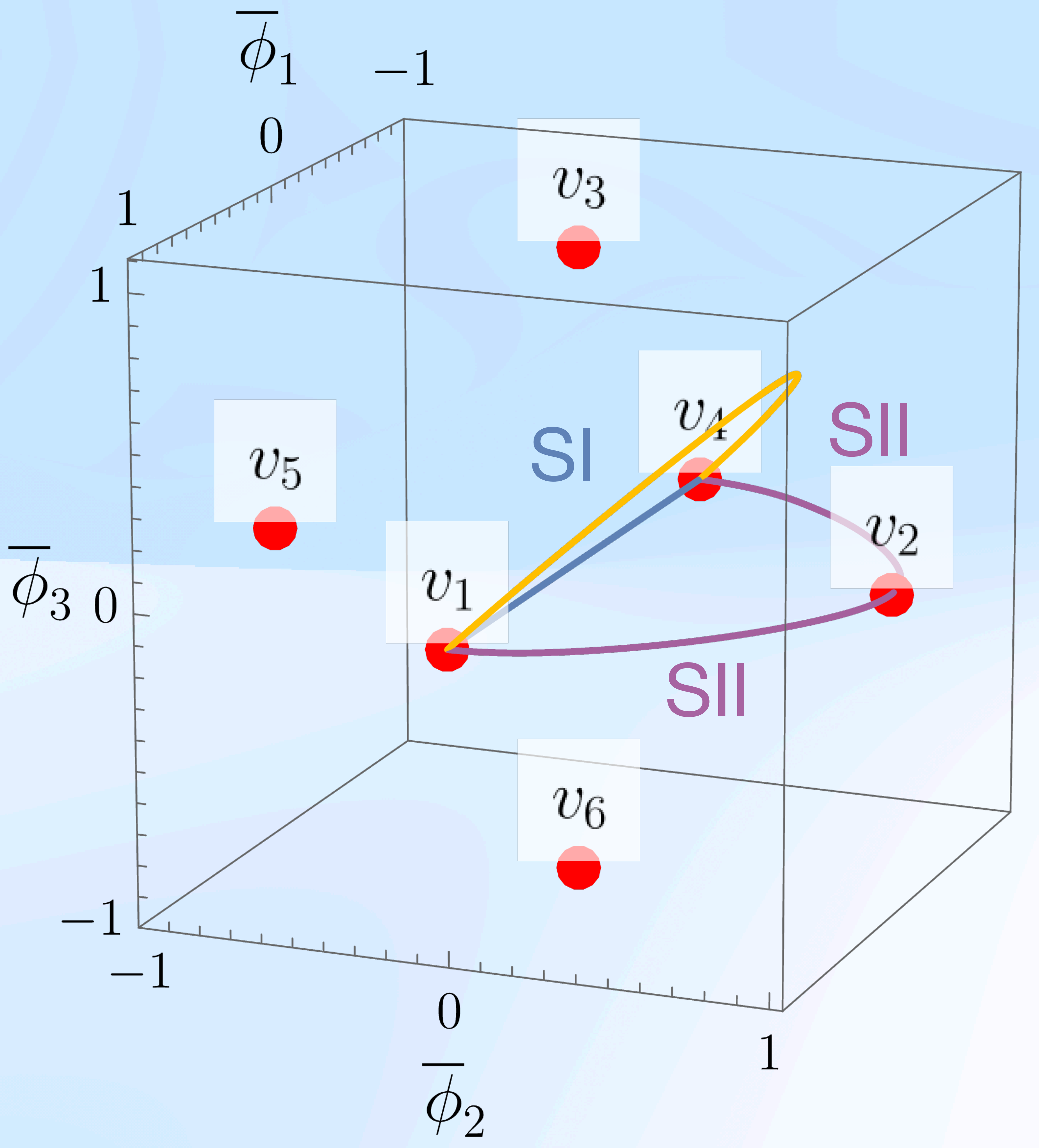


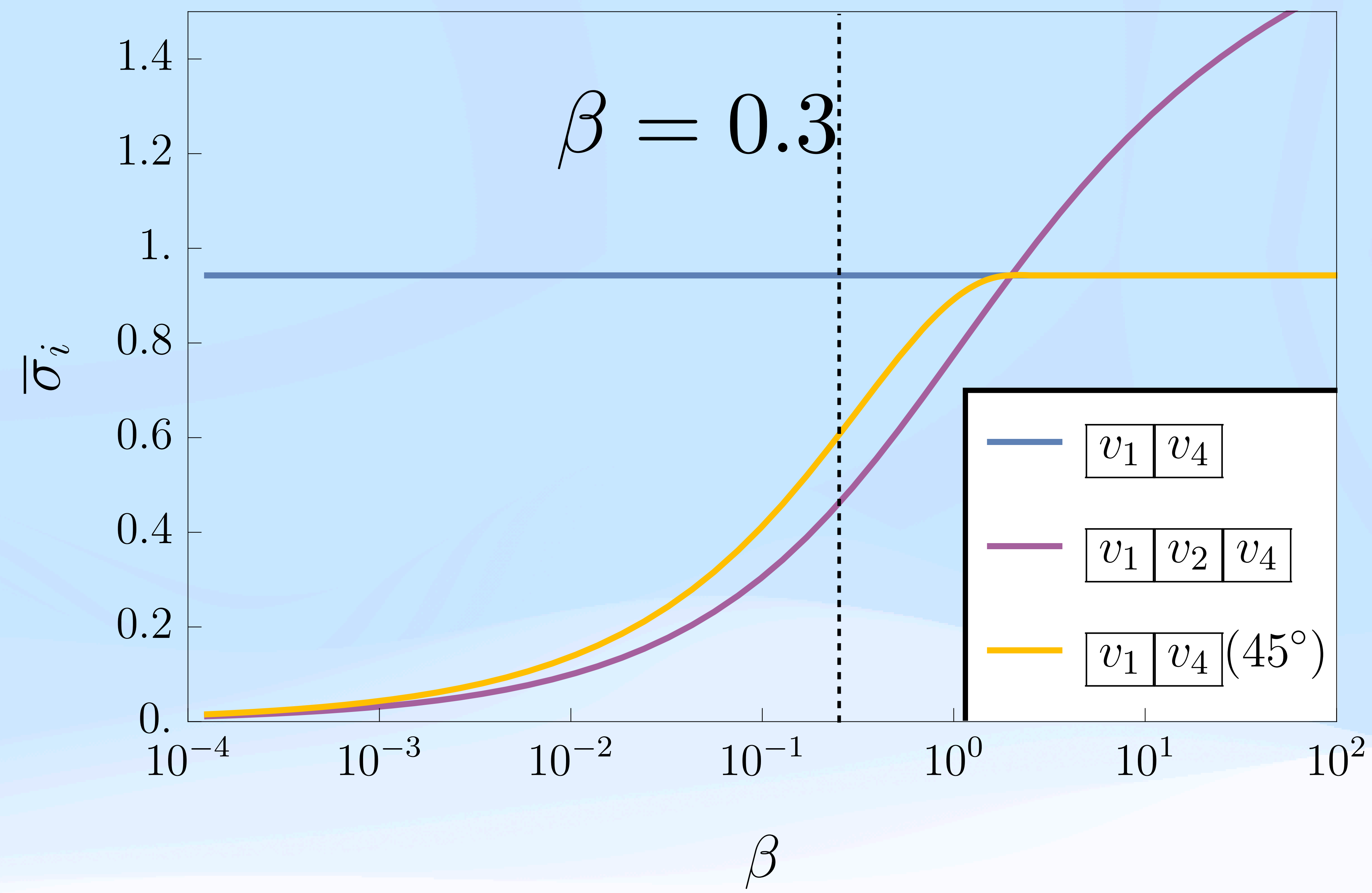


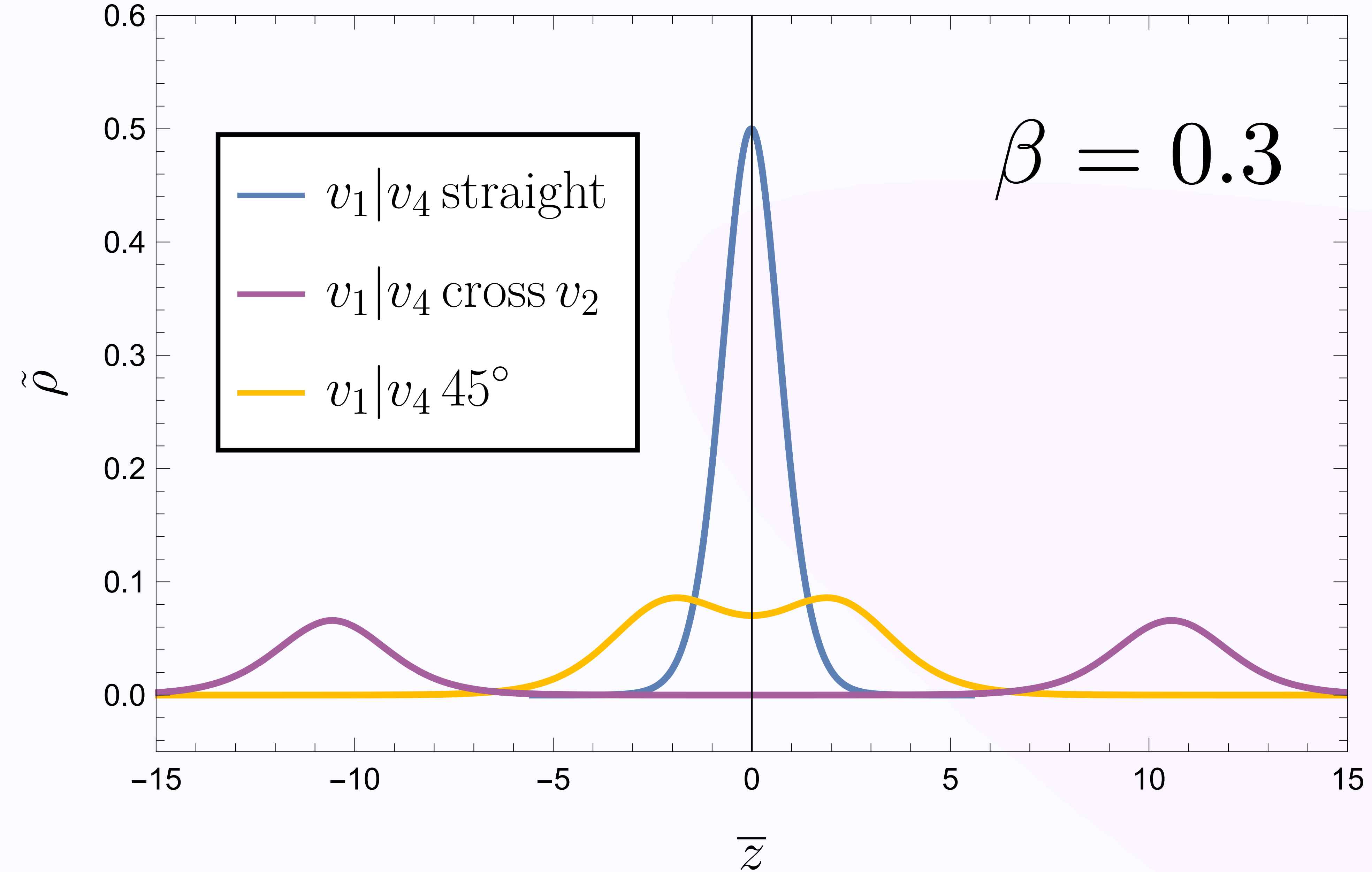
$$\beta \gg 2 : \sigma(\text{SI}) < 2\sigma(\text{SII})$$

SI DW **stable**

Stability of DWs







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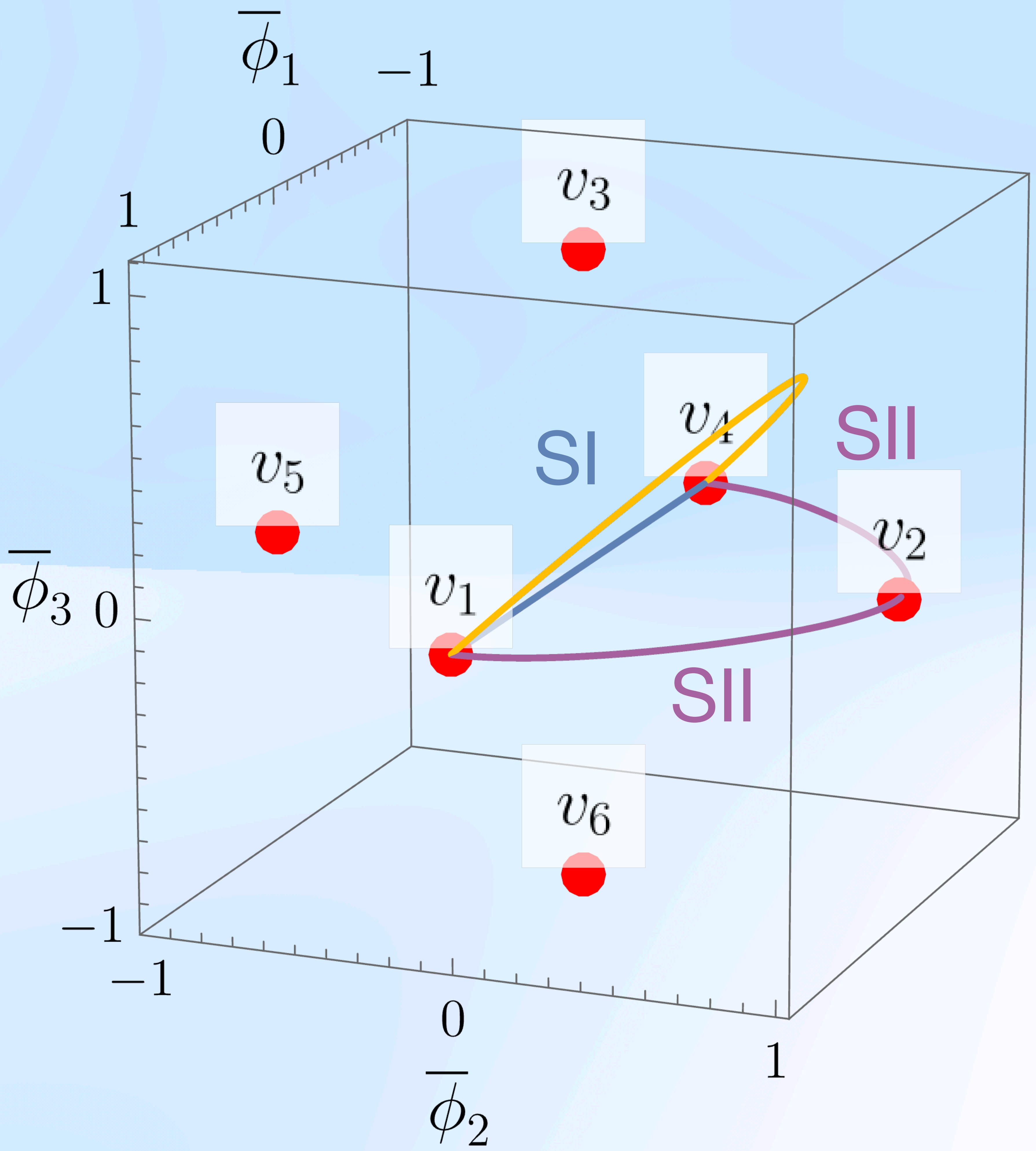
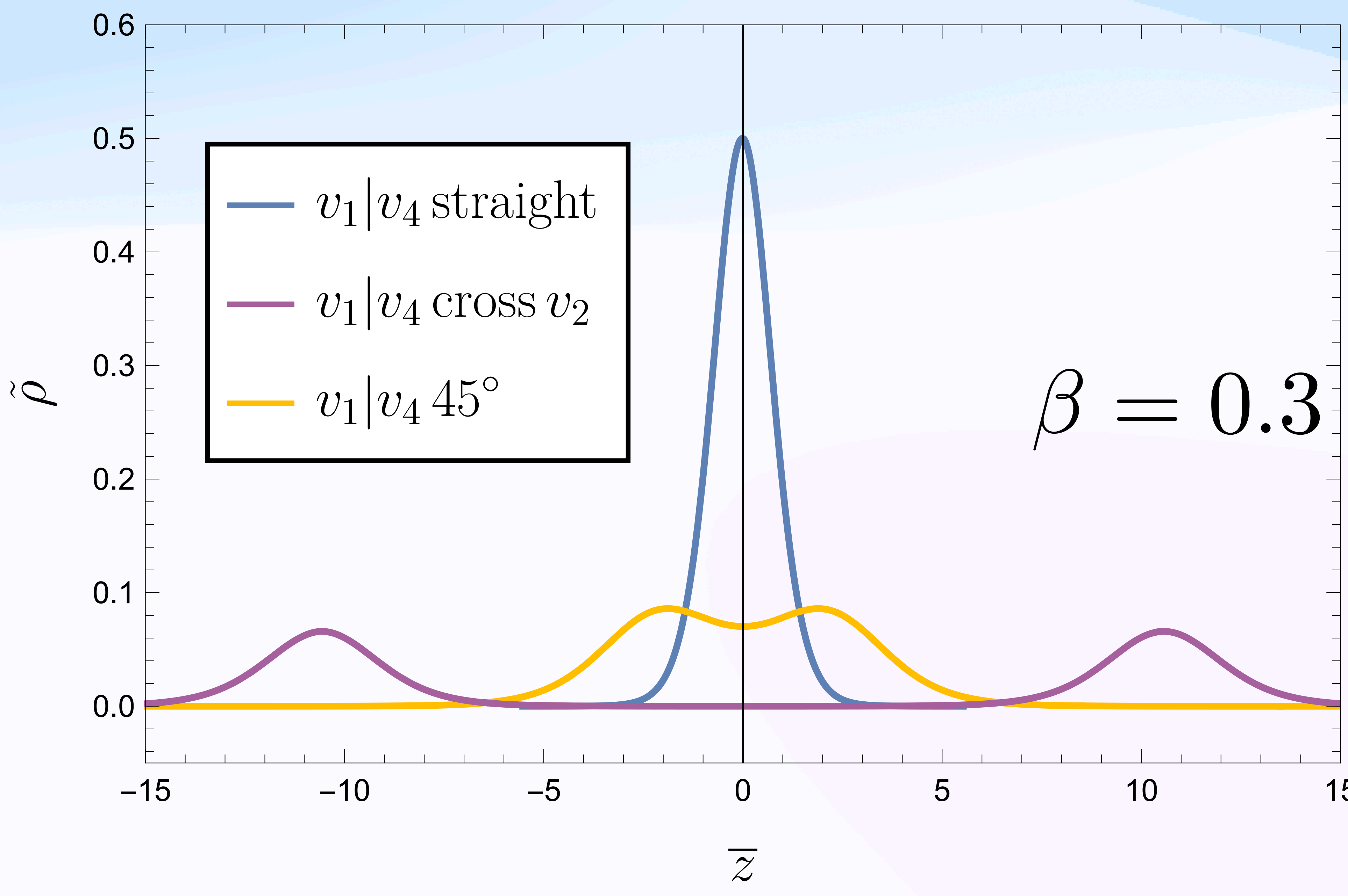
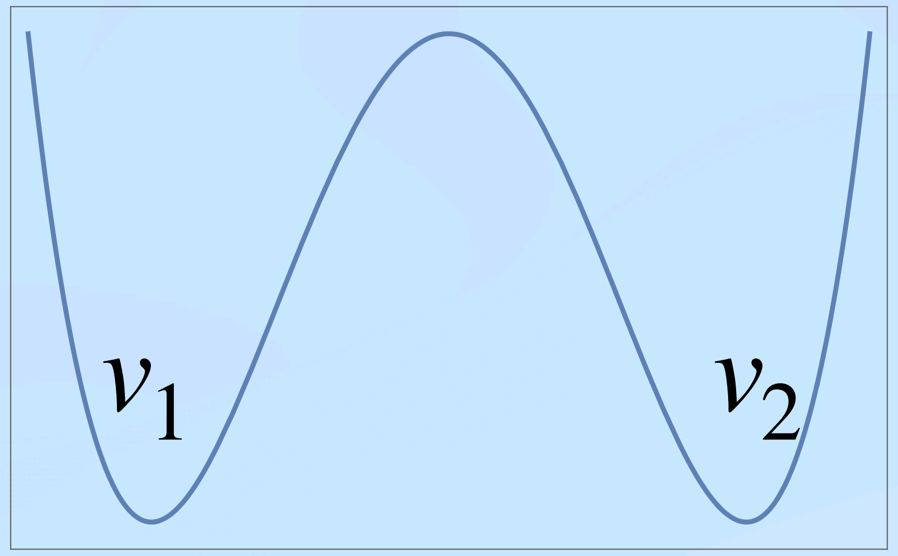


For $\beta = 0.3$, the SI-type DW will decay to two SII type DWs

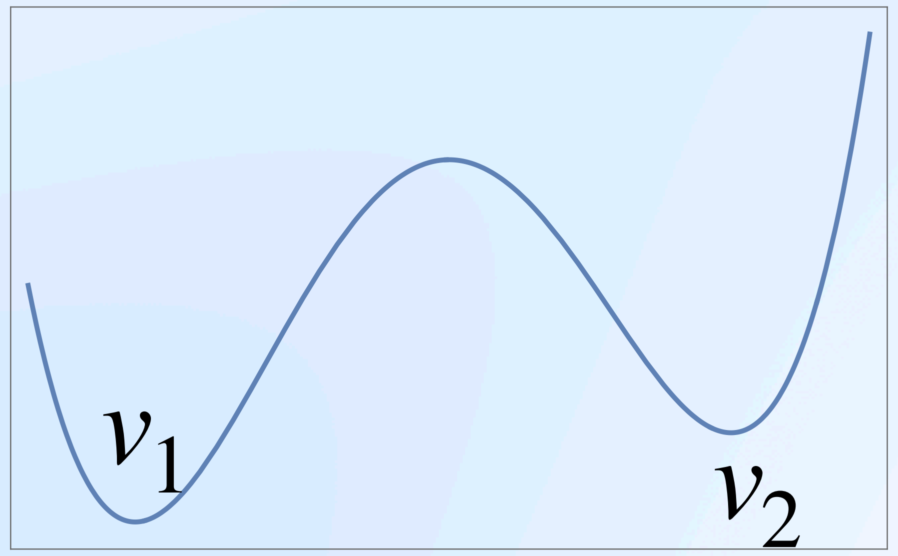


Gravitational wave from DWs

- Exact discrete symmetry $\Longrightarrow$ stable DWs



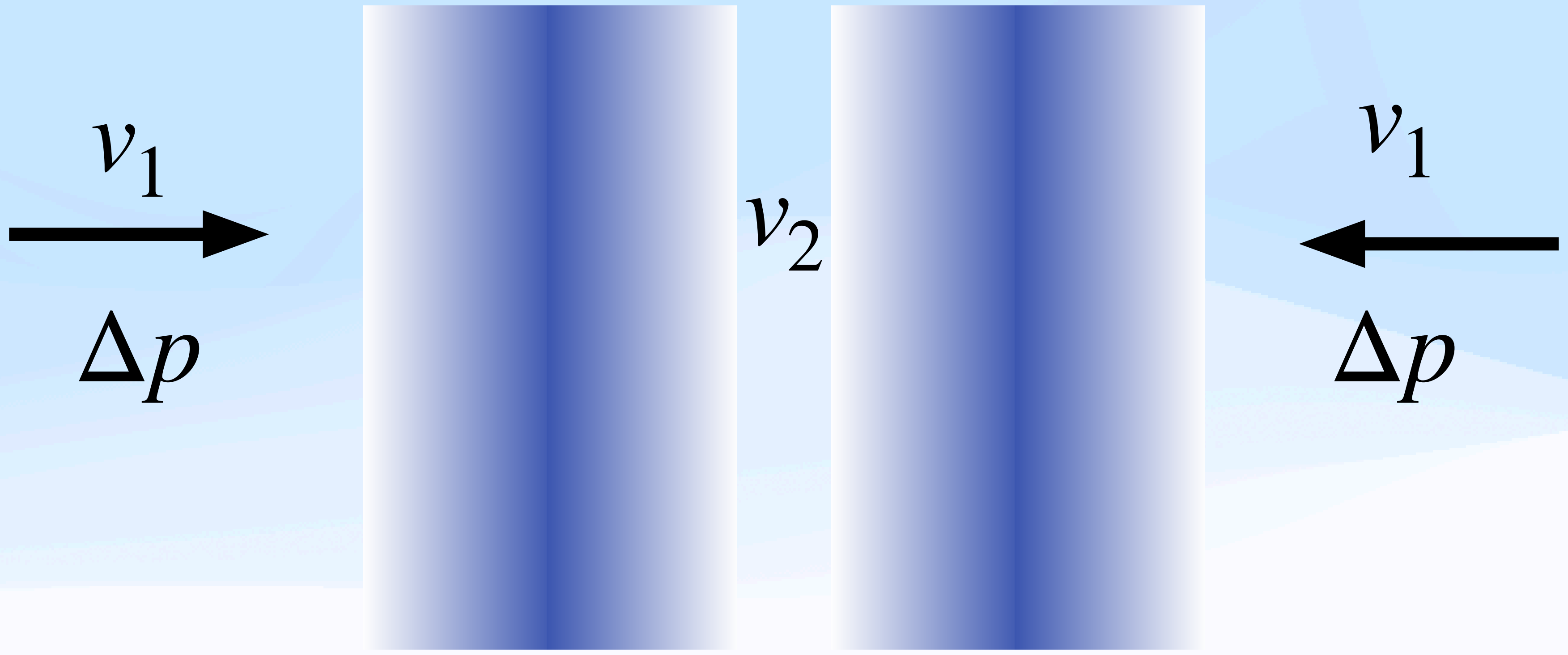
- Bias $\Longrightarrow$ unstable DWs



$$V_{\rm bias} \equiv V(v_2) - V(v_1)$$

Saikawa 1703.02576

Pressure difference $\Delta p \propto V_{\rm bias}$



$$\Omega_{\rm GW}^{\rm peak} \propto \frac{\sigma^4}{V_{\rm bias}^2} \qquad f_{\rm peak} \propto \sqrt{\frac{V_{\rm bias}}{\sigma}}$$

$$\begin{cases} f > f_{\rm peak}, \Omega_{\rm GW} \propto f^{-1} \\ f < f_{\rm peak}, \Omega_{\rm GW} \propto f^3 \end{cases}$$

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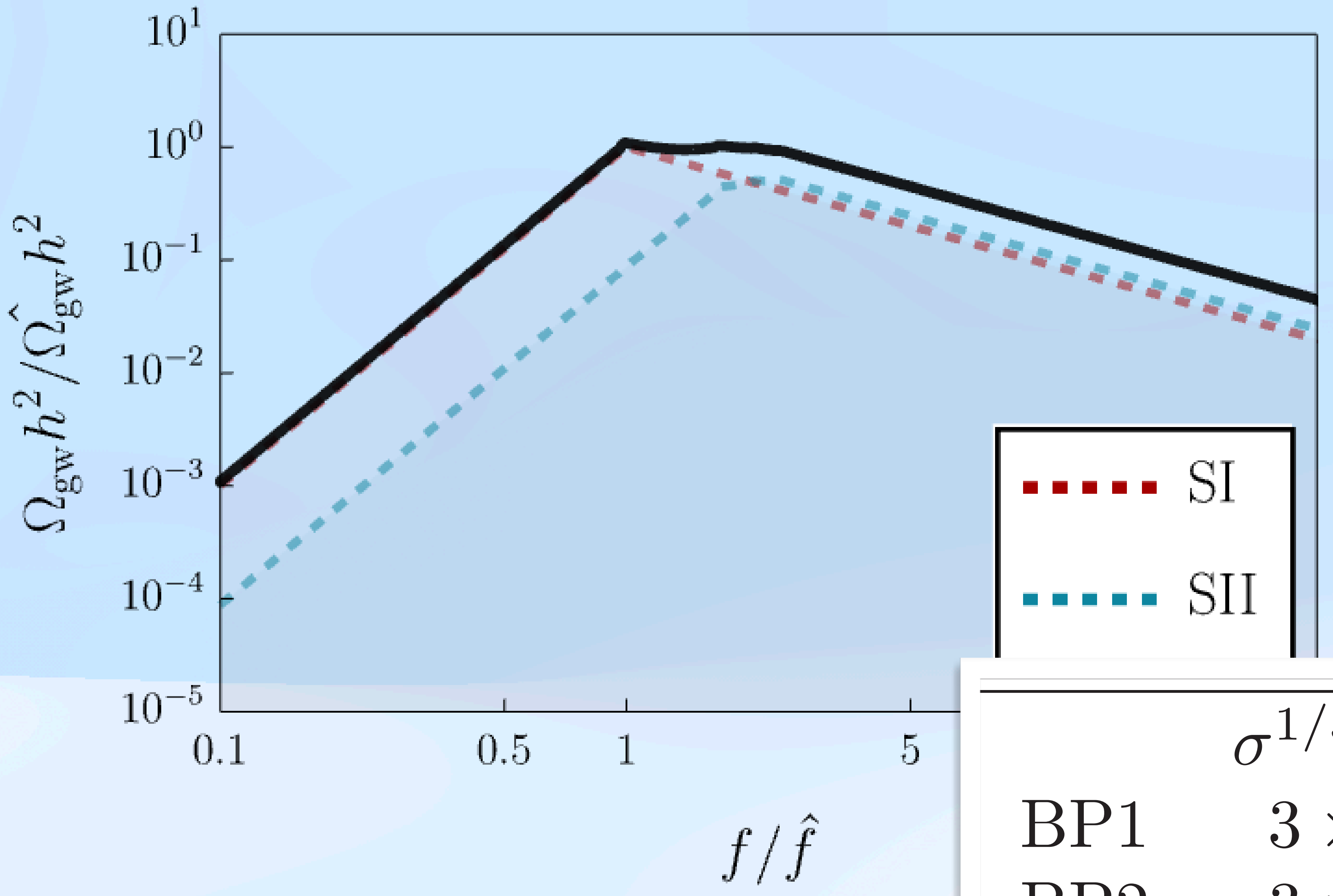
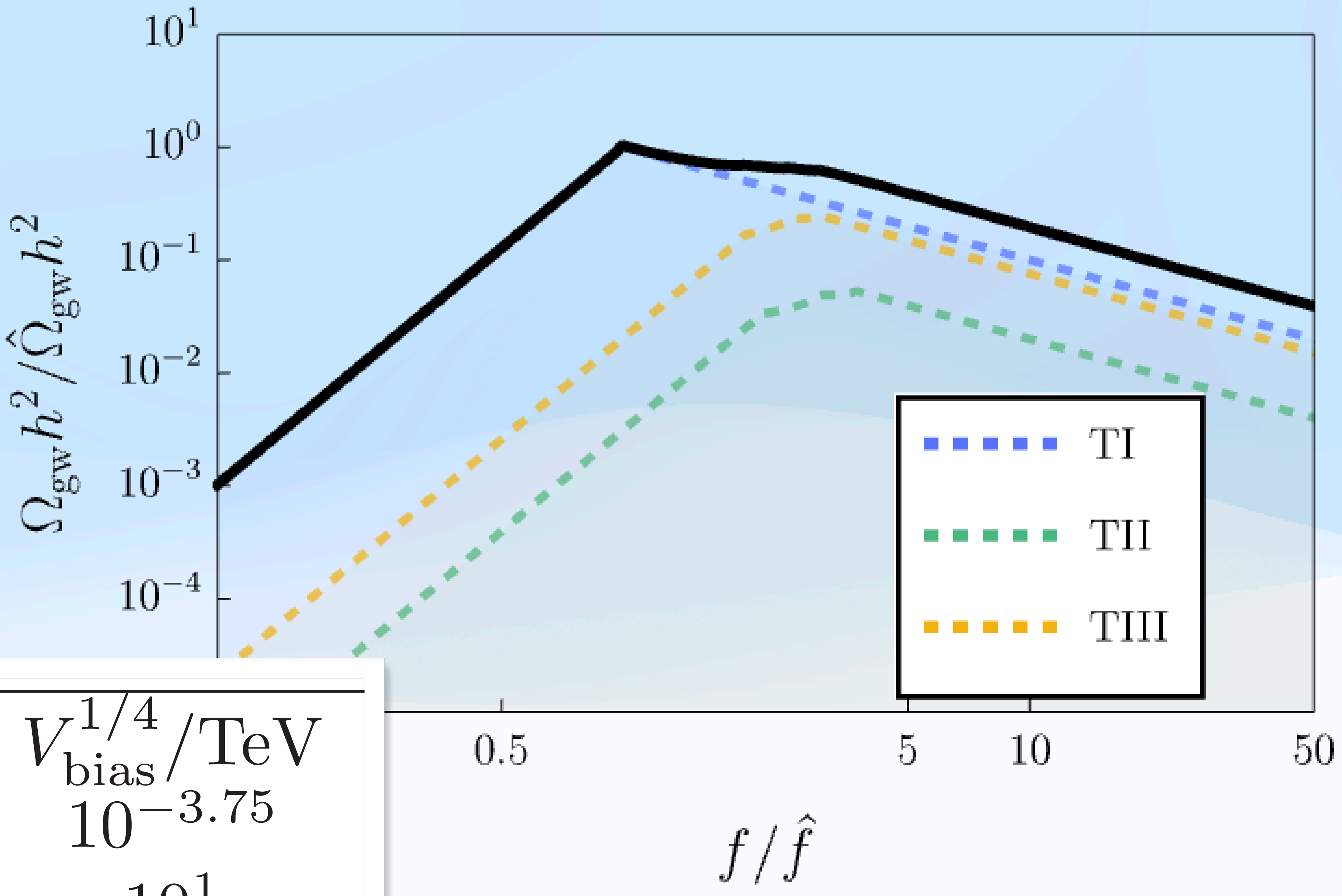
Gravitational wave

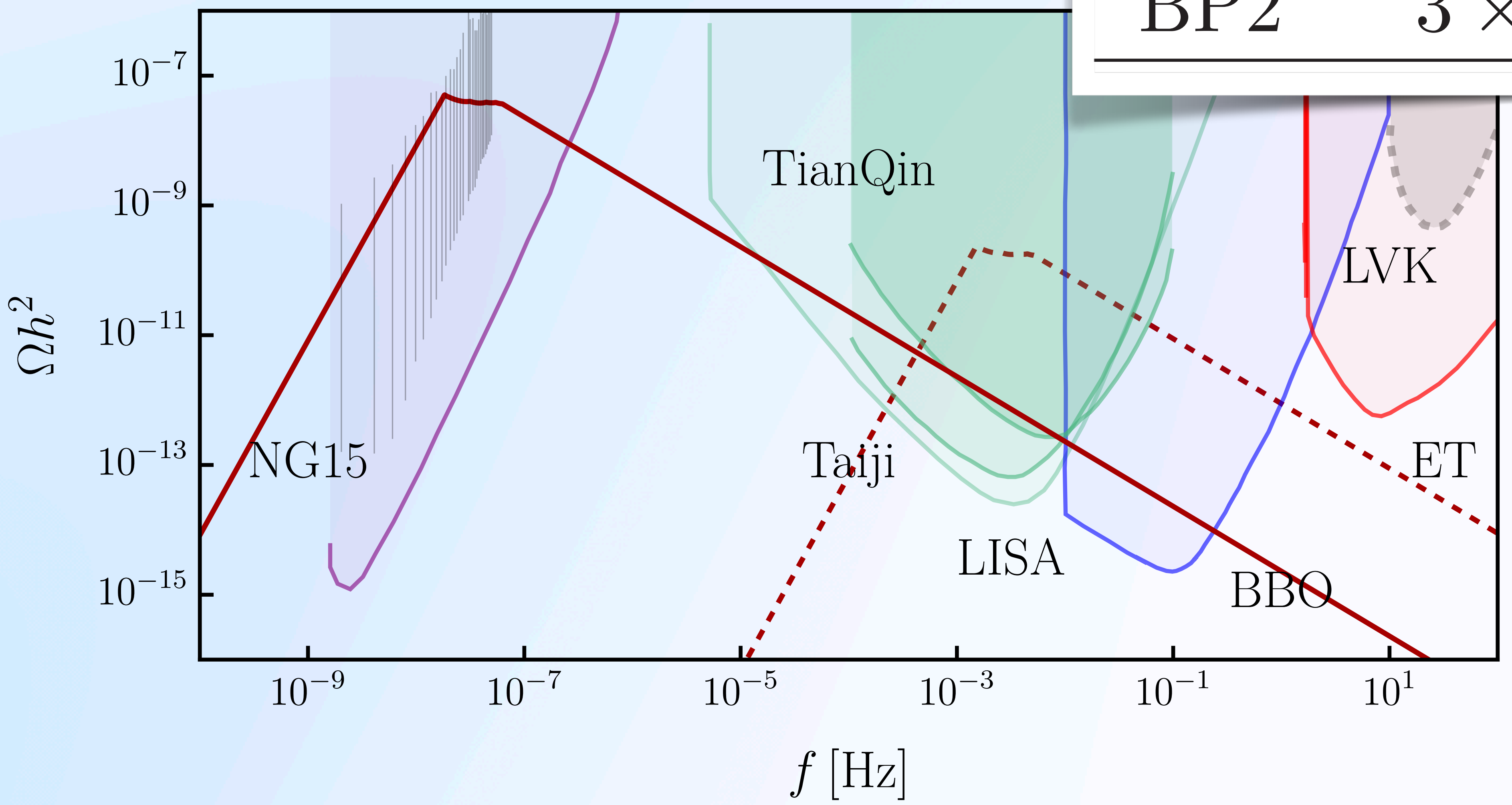
ϵ^v_{12}	ϵ^v_{13}	ϵ^v_{14}	ϵ^v_{15}	ϵ^v_{16}
$2\hat{\epsilon}$	$3\hat{\epsilon}$	$\hat{\epsilon}$	$4\hat{\epsilon}$	$5\hat{\epsilon}$

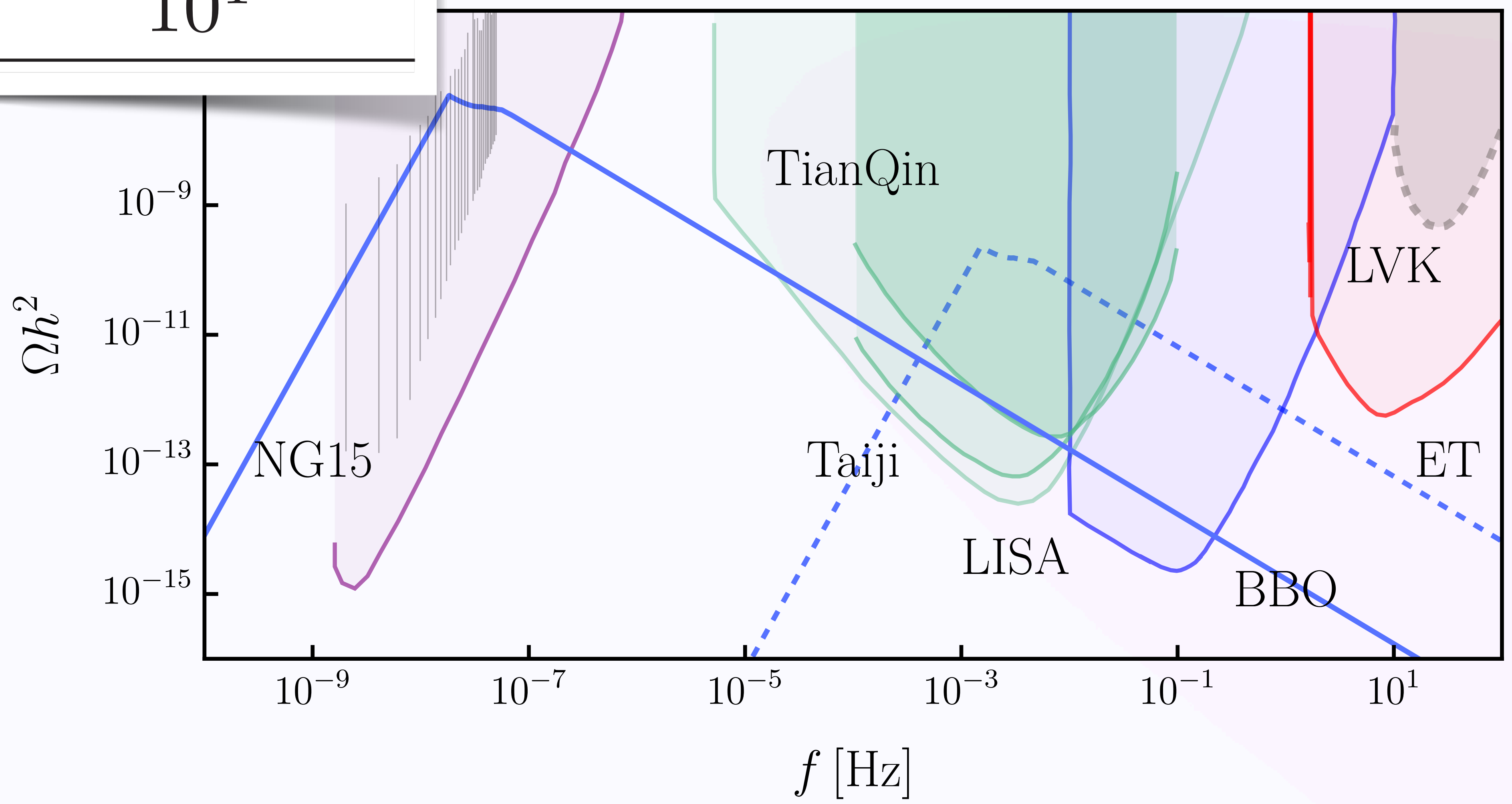
ϵ^u_{12}	ϵ^u_{13}	ϵ^u_{14}	ϵ^u_{15}	ϵ^u_{16}	ϵ^u_{17}	ϵ^u_{18}
$2\hat{\epsilon}$	$4\hat{\epsilon}$	$6\hat{\epsilon}$	$\hat{\epsilon}$	$3\hat{\epsilon}$	$5\hat{\epsilon}$	$7\hat{\epsilon}$





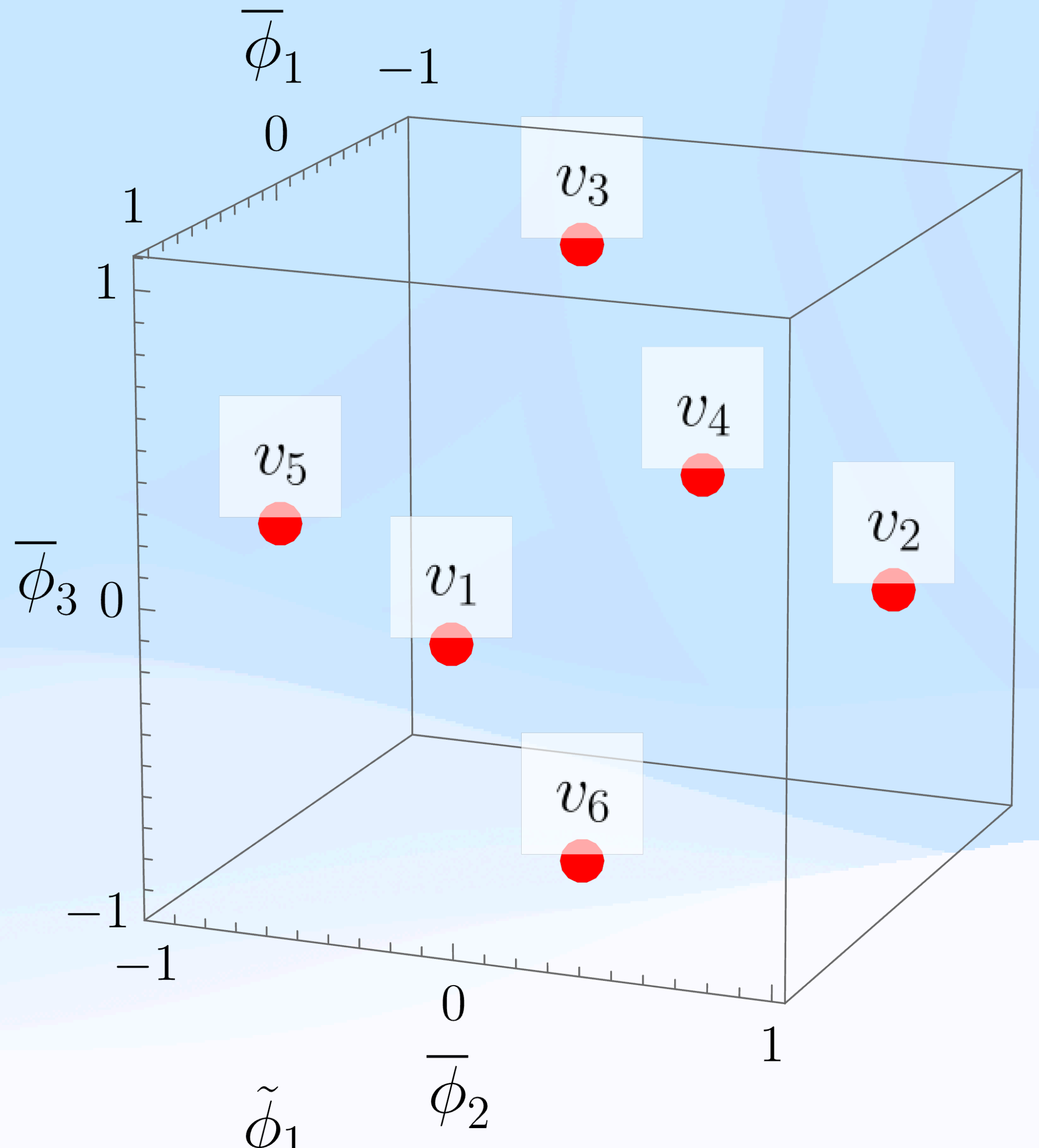
	$\sigma^{1/3}$/TeV	$V_{\rm bias}^{1/4}$/TeV
BP1	3×10^2	$10^{-3.75}$
BP2	3×10^5	10^1





Summary and Outlook

- Non-abelian DWs have more interesting and non-trivial structure and phenomena
- In certain range of parameter space, unstable DWs can show up
- If the DWs are stable, they can give rise to a unique multi-peak GW signal



- The signature of GW raised by unstable domain walls is still unexplored
- In realistic flavour models, the flavon fields are complex scalars and there may be more than one multiplets