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Exploration of Axions and Standard Model Global Properties

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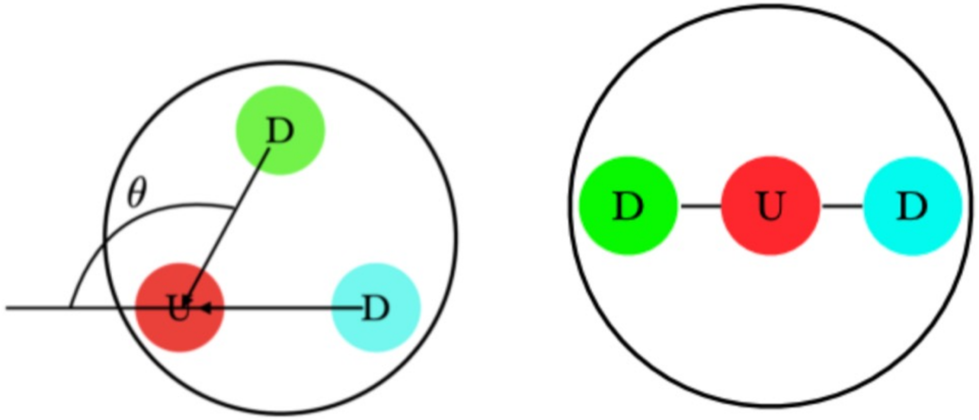
Based on arXiv: 2411.04749, 2506.07546

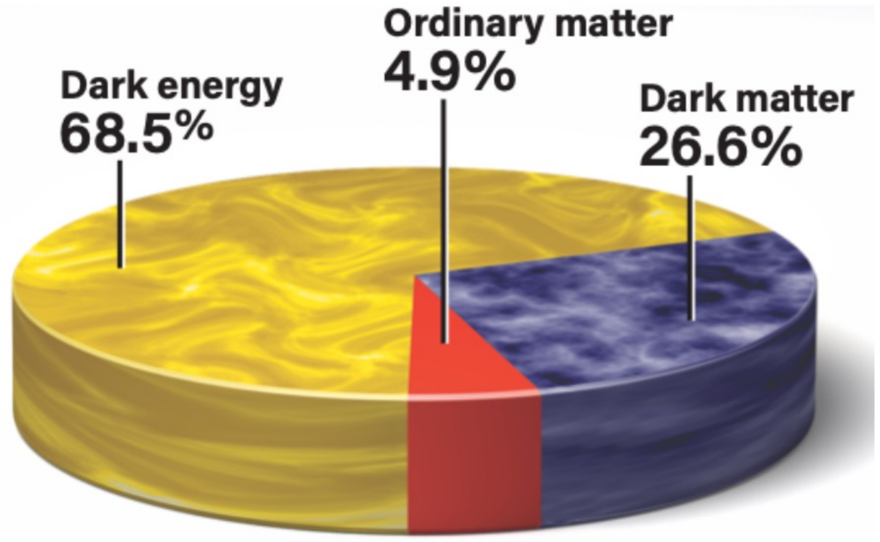
In collaboration with Qing-Hong Cao, Shunshun Cao, Shuailiang Ge, Jinchen Jiang, Jun-Chen Wang and Lijing Shao

The Motivation for the axion—Two Birds with One Stone

Two fundamental puzzles: Strong CP problem and Dark Matter

$\mathcal{L} \supset \theta \tilde{G} G$ $\theta < 10^{-10}$



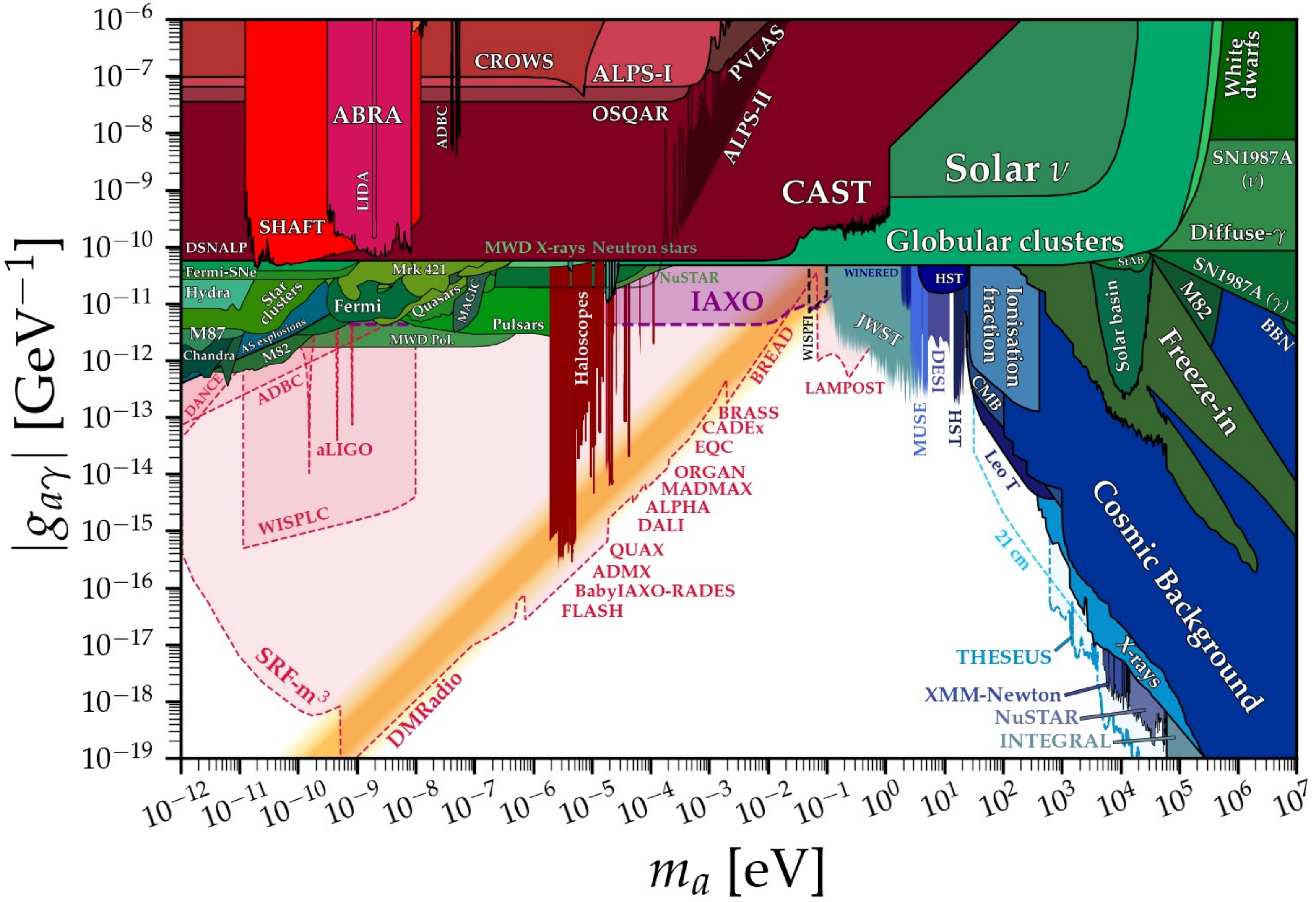


Axion kills two birds with one stone: solves the Strong CP problem and explains Dark Matter

Theoretical and Experimental Advances on the Axion

See the talks by
Prof. Xiaoping Wang,
Zuowei Liu, Yu Gao, and
Houston Nick

Phys.Rev.Lett. 133 (2024) 2, 021005, SHANHE Collaboration;
Sci.Bull. 70 (2025) 661-666, SHANHE Collaboration;
Phys.Rev.Lett. 134 (2025) 7, 071004, PandaX Collaboration;
Phys.Rev.D 95 (2017) 5, 052006,CDEX Collaboration



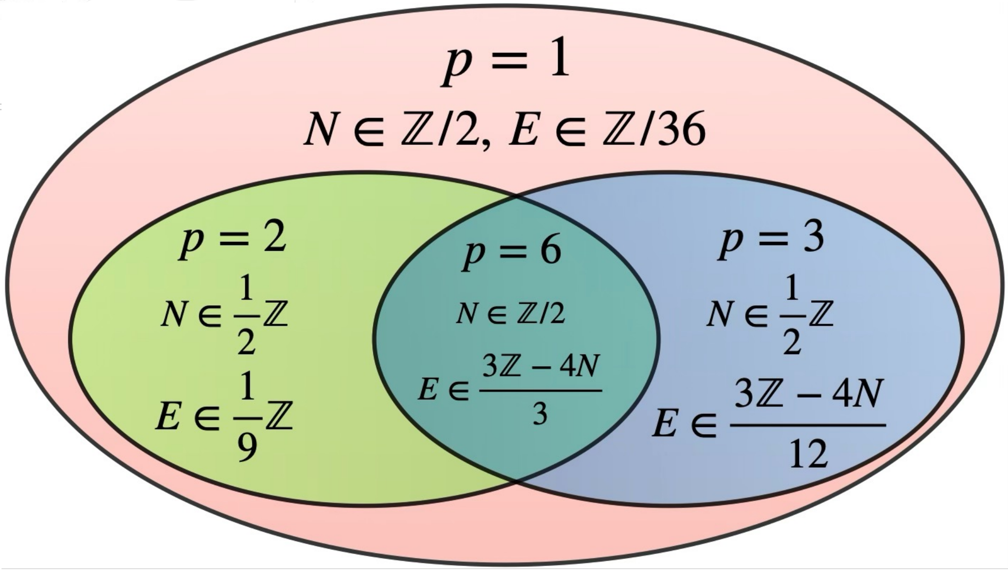
The SM's Hidden Global Structure

$$\mathcal{G} = \mathcal{SU}(3)\times\mathcal{SU}(2)\times\mathcal{U}(1)$$

$$\mathcal{L}_{a\gamma\gamma} = \frac{1}{4}\frac{g_\gamma}{f_a} a \tilde{F}_{\mu\nu} F^{\mu\nu}$$

Quantized coupling

$$g_\gamma = \frac{\alpha_{EM}}{2\pi}\left(\frac{E}{N} - 1.92\right)$$



$$G = SU(3)\times SU(2)\times U(1)/Z_p$$

Hucks Phys. Rev. D 43, 2709(1991)

Choi et al. Phys. Rev. Lett. **132**, 121601
Reece JHEP 10, 116 (2023)
Agrawal JHEP 01, 169 (2024)
Cordova JHEP 05, 325 (2024)

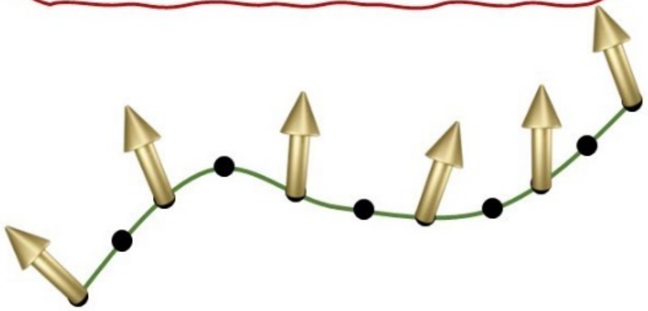
$p = 1$	$p = 2$	$p = 3$
Higher-Group $\frac{48N+36E}{K} \not\equiv 0 \pmod{K}$	Higher-Group $\frac{48N+36E}{(K/2)} \not\equiv 0 \pmod{\frac{K}{2}}$	Higher-Group $\frac{36E}{(K/3)} \not\equiv 0 \pmod{\frac{K}{3}}$
Non-Invertible $36E \not\equiv 0 \pmod{6}$	Non-Invertible $18E \not\equiv 0 \pmod{3}$	Non-Invertible $12E \not\equiv 0 \pmod{2}$

Berry Phase Induced by the Axion Interaction

$$\mathcal{H} \supset \vec{B} \cdot \vec{\sigma}$$

Berry connection

$$\vec{A}_n(\vec{k}) = i\langle n_{\vec{k}} | \nabla_{\vec{k}} | n_{\vec{k}} \rangle$$



Berry phase

$$\gamma_n = \oint_C d\vec{k} \cdot \vec{A}_n(\vec{k})$$

Berry curvature

$$\vec{\Omega}_n(\vec{k}) = \nabla \times \vec{A}_n(\vec{k})$$

Axion Characteristics:

1. T odd
2. Topology: $S^1 \; a \sim a + 2\pi f_a$

Effective Hamiltonian of the Axion Interaction

$$\mathcal{L} \supset \mathcal{L}_{aff} + \mathcal{L}_{a\gamma\gamma}$$

$$\mathcal{L}_{aff} = -\frac{1}{2}\frac{g_f}{f_a}\, a\partial_\mu a \bar{f}\gamma^\mu\gamma^5 f$$

$$\mathcal{L}_{a\gamma\gamma} = \frac{1}{4}\frac{g_\gamma}{f_a} a\, \tilde{F}_{\mu\nu}F^{\mu\nu}$$

$$H_{aff} = \frac{g_f}{2f_a}\left(\nabla a + \partial_t a \frac{\vec{p}}{m_f}\right)\cdot\vec{\sigma}$$

$$H_{a\gamma\gamma} = \frac{g_\gamma}{2f_a}\dot{a}(t)\frac{1}{|\vec{k}|}\vec{k}\cdot\vec{S}$$

$\vec{k}$ phton momentum

$\vec{S}$ spin operator for spin-1

$$H(t) = \vec{V}(t)\cdot\vec{j}$$

$$\mathcal{H} \supset \vec{B}\cdot\vec{\sigma}$$

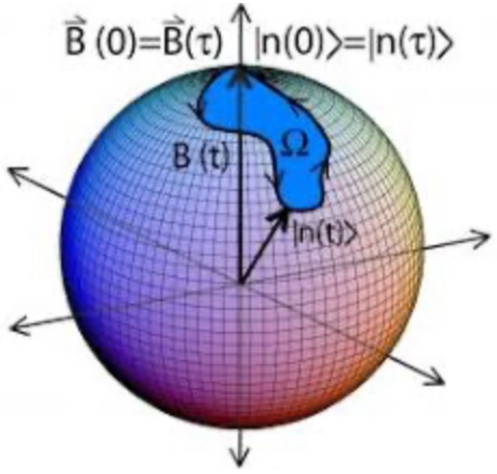
Berry Phase of the Photon

$$H_{a\gamma\gamma} = \frac{g_\gamma}{2f_a}\dot{a}(t)\frac{1}{|\vec{k}|}\vec{k}\cdot\vec{S} \equiv \vec{V}(t)\cdot\vec{j}$$

- Vector's direction changes with time

  Graham et al. Phys. Rev. D **103**, 055010

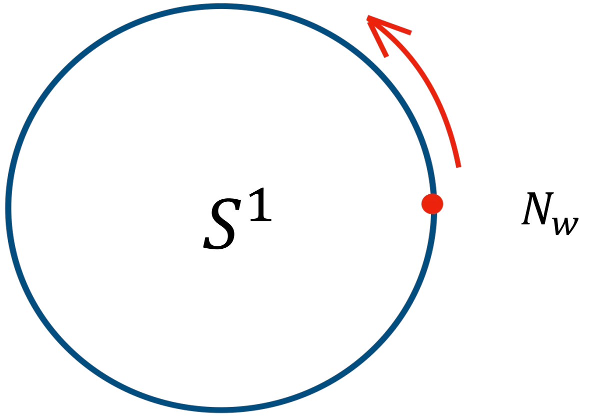
$$\xi_{berry} \propto \frac{g_\gamma}{f_a}$$



- Vector's magnetude changes with time

  M. Pospelov et al. Phys. Rev. Lett. 110, 021803

$$\xi_{berry} = \frac{g_\gamma}{2f_a}\left(\tilde{a}(T) - \tilde{a}(0)\right) = \pi N_w g_\gamma$$



"Berry Phase" of the Photon

$$H_{a\gamma\gamma} = \frac{g_\gamma}{2f_a}\dot{a}(t)\frac{1}{|\vec{k}|}\vec{k}\cdot\vec{S} \equiv \vec{V}(t)\cdot\vec{j}$$

$$\Delta\alpha = \frac{g_\gamma(a_f - a_i)}{f_a} \equiv \frac{g_\gamma}{f_a}\Delta a$$

Harari et al. Phys. Lett. B 289, 67 (1992)
Carrol et al. Phys. Rev. D41, 1231 (1990)

Near the earth: $a_f \simeq 0 \Rightarrow \Delta\alpha = -\frac{g_\gamma}{f_a}a_i$

"Berry Phase" of the Photon

$$H_{a\gamma\gamma} = \frac{g_\gamma}{2f_a}\dot{a}(t)\frac{1}{|\vec{k}|}\vec{k}\cdot\vec{S} \equiv \vec{V}(t)\cdot\vec{j}$$

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Near the earth: $a_f \simeq 0 \Rightarrow \Delta\alpha = -\frac{g_\gamma}{f_a}a_i$

Where $a_i \propto f_a$?

Axion Profile

$a_i \propto f_a$

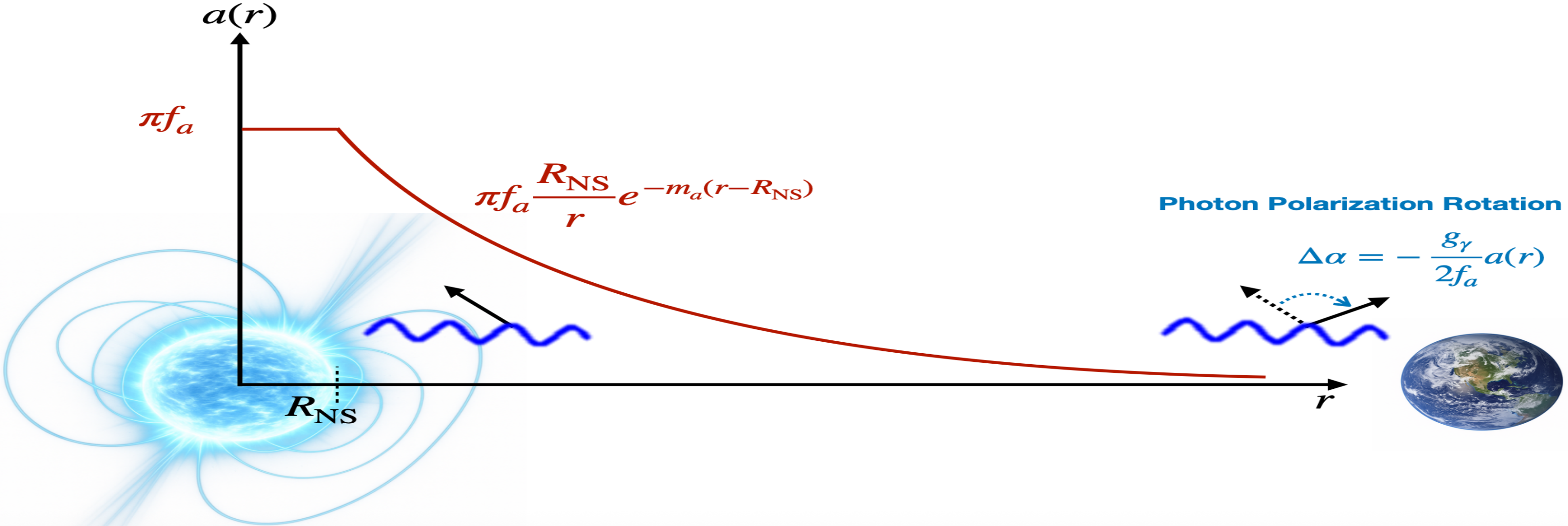
- Domain wall (not-trivial topology)

  M. Pospelov et al. Phys. Rev. Lett. 110, 021803

- Axion cloud (gravitational effects)

  Yifan Chen et al. Phys.Rev.Lett. 124 (2020) 6, 061102
  Yifan Chen et al. *Nature Astron.* 6 (2022) 5, 592-598

- Neutron star (phase transtion)



Axion Potential

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi}\frac{a}{f_a} G^b_{\mu\nu}\tilde{G}^{b,\mu\nu} + \frac{1}{4}\frac{g_\gamma}{f_a} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$V = V_{QCD} + \Delta V$$

$$V_{QCD} = -\mathrm{m}_\pi^2 \mathrm{f}_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\frac{a}{2f_a}} \simeq 2m_q\langle\bar{q}q\rangle |\cos\frac{a}{2f_a}|$$

Grilli et al. JHEP 01, 034 (2016)

$$\Delta V = \sqrt{2}\Lambda^3 f_a |\lambda_1| \cos(\frac{a}{f_a} + \beta_1)$$

Alvey et al. JHEP 01, 032 (2021); Dine 2207.01068

Axion Profile in Neutron Stars

$$V_{QCD} = -\mathrm{m}_\pi^2 \mathrm{f}_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2 \frac{a}{2f_a}} \simeq 2m_q \langle \bar{q} q \rangle | \cos \frac{a}{2f_a} |$$

$$\Delta V = \sqrt{2}\Lambda^3 f_a |\lambda_1| \cos(\frac{a}{f_a} + \beta_1)$$

On Earth

$$\left.\begin{array}{l} \beta_1 = 0 \\ m_q |\langle \bar{q} q \rangle| > 2\sqrt{2}\Lambda^3 f_a |\lambda_1| \end{array}\right\} \quad a = 0$$

Crewther et al. Phys. Lett. B 88, 123 (1979)
Pendlebury et al. Phys. Rev. D 92, 092003 (2015)

On Neutron Stars

$\rho_{NS} \gg \rho_{earth}$ ⟹ Chiral symmetry restoration

Pietroni, Phys. Rev. D 72, 043535 (2005)
Olive et al. Phys. Rev. D 77, 043524(2008)
Hinterbichler et al. Phys. Rev. Lett. 104,231301(2010)

$$m_q |\langle \bar{q} q \rangle| > 2\sqrt{2}\Lambda^3 f_a |\lambda_1|$$ ✗

Axion Profile near Neutron Stars

With $m_q|\langle\bar{q}q\rangle_N| < 2\sqrt{2\Lambda^3 f_a|\lambda_1|} < m_q|\langle\bar{q}q\rangle_{earth}|$

$$a = 0 \quad \Rightarrow \quad a_{\rho_N} = 2f_a \arccos\frac{m_q|\langle\bar{q}q\rangle_N|}{2\sqrt{2\Lambda^3 f_a|\lambda_1|}} \quad \Rightarrow \quad a_{NS} = \pi f_a$$

On Earth

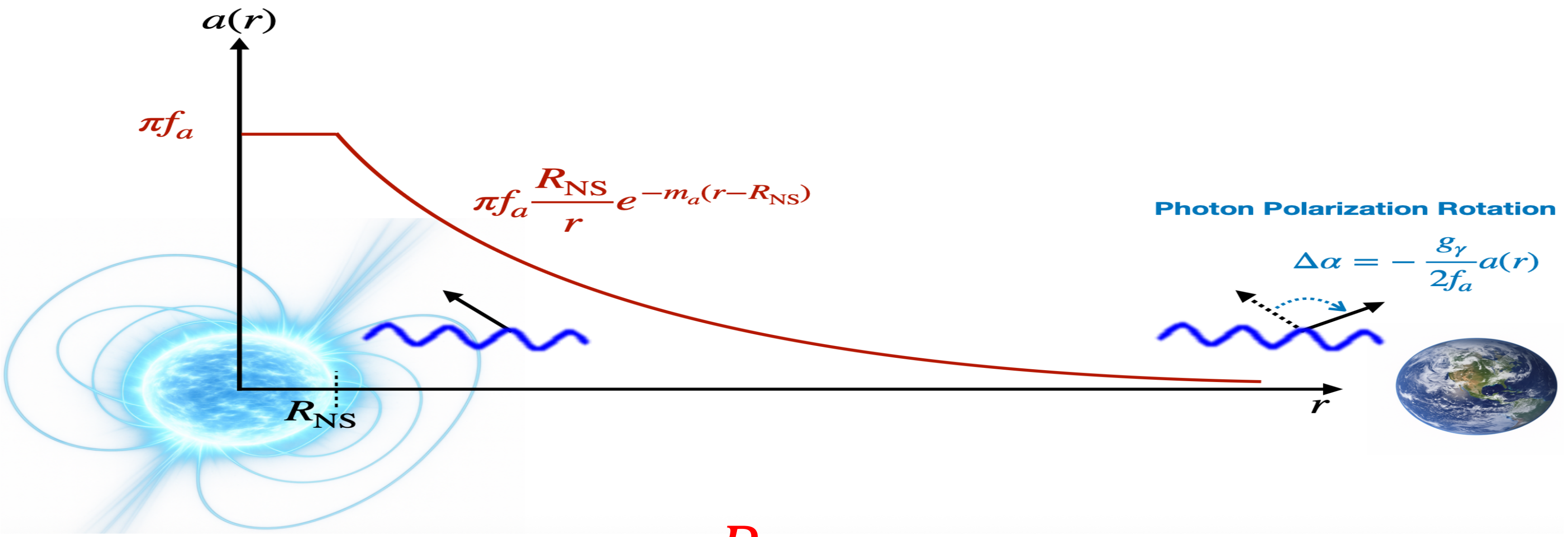
At nuclear density

Inside neutron star

$$a(r) = \begin{cases} \pi f_a & r < R_{NS} \\ \pi f_a \dfrac{R_{NS}}{r} e^{-m_a(r-R_{NS})} & r > R_{NS} \end{cases}$$

Axion-Induced Photon Polarization Rotation

$$a(r) = \begin{cases} \pi f_a & r < R_{NS} \\ \pi f_a \dfrac{R_{NS}}{r} e^{-m_a(r-R_{NS})} & r > R_{NS} \end{cases}$$



$$\Delta\alpha = -g_\gamma \frac{\pi R_{NS}}{2\,r} e^{-m_a(r-R_{NS})}$$

Axion-Induced Photon Polarization Rotation

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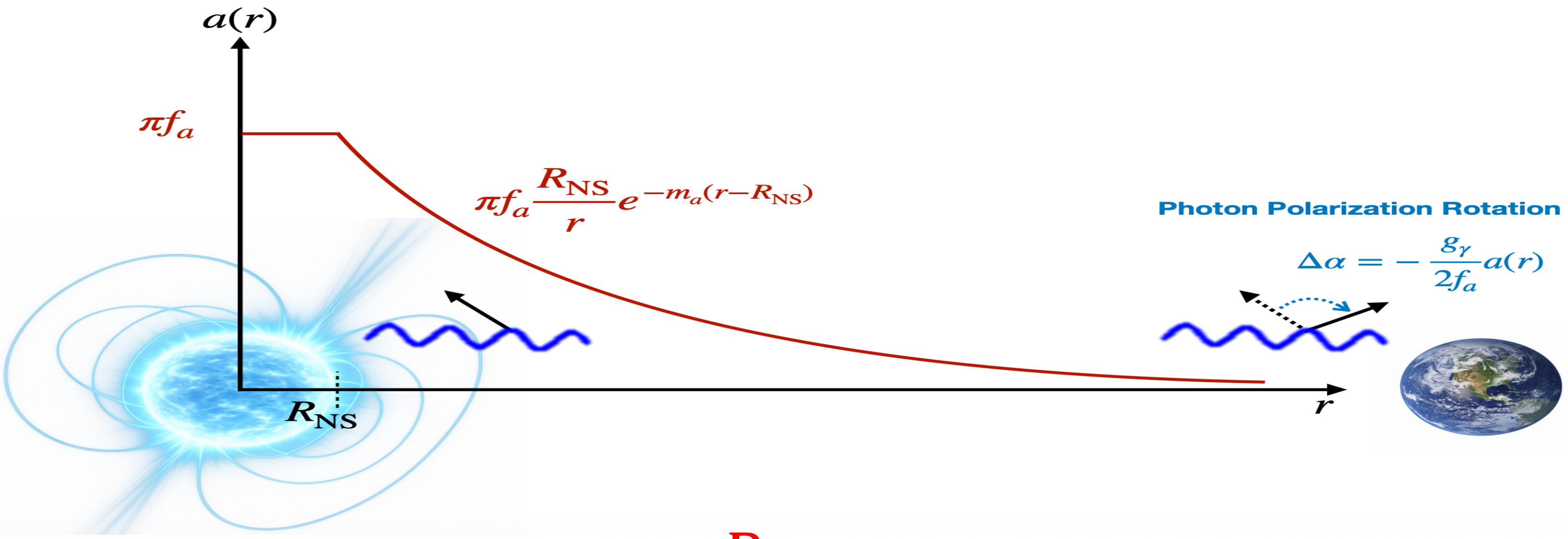

$$\Delta\alpha = -g_\gamma \frac{\pi R_{NS}}{2\,r} e^{-m_a(r-R_{NS})}$$ r difficult to measure

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Radius–Frequency Mapping for Axion Detection

Radius-Frequency Mapping:

photons emitted at different distances away from the neutron star center have varying frequencies

Ruderman et al. Astrophys. J.196, 51 (1975); Lesch et al. Space science reviews68, 349 (1994); Qiu et al. The Astrophysical Journal 958, 78 (2023)

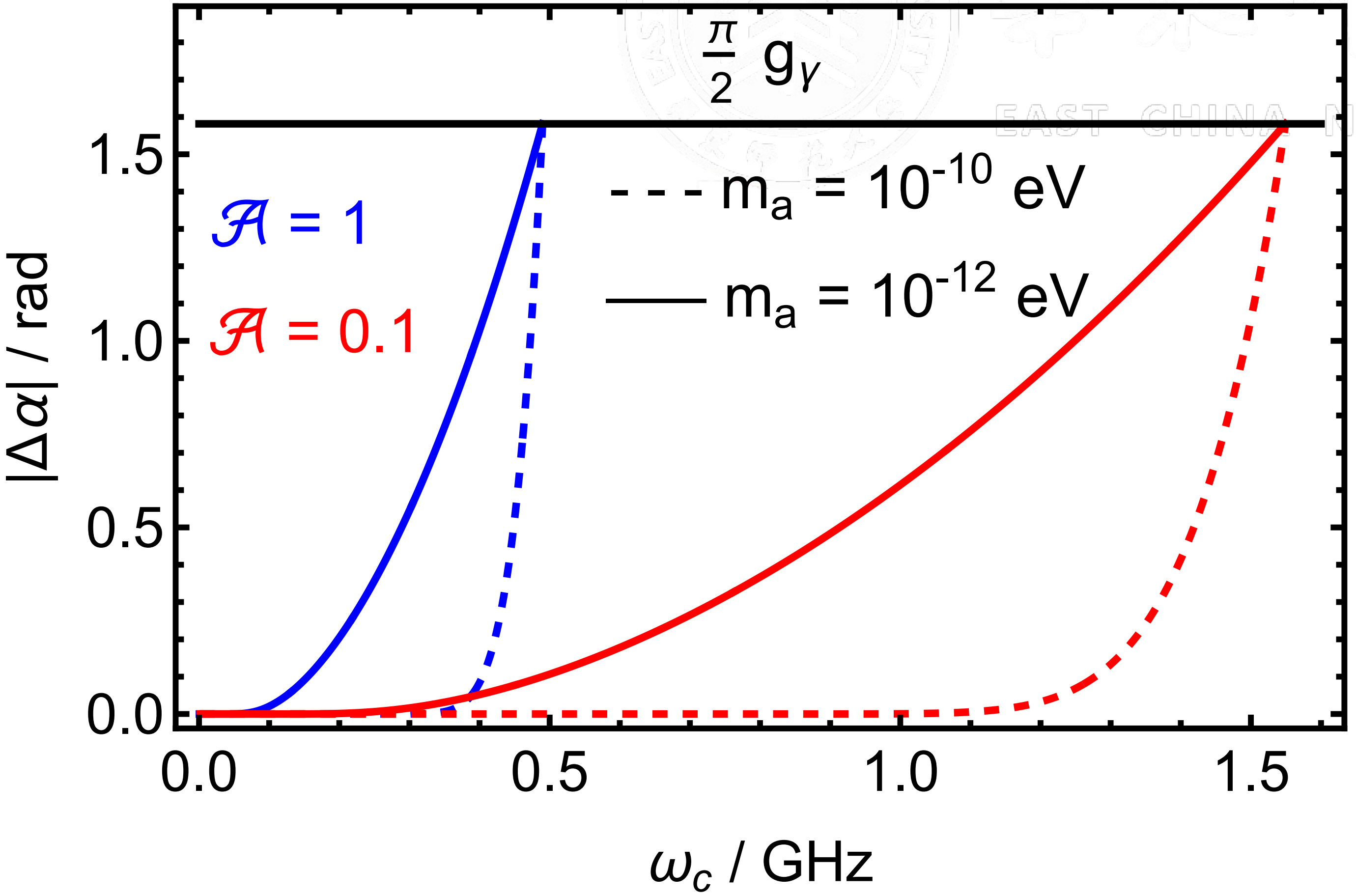
$$\omega_c = \frac{9}{8}(2\pi)^{1/2}\,\gamma^3 P_{NS}^{-\frac{1}{2}} r^{-1/2} \simeq 2.82\frac{\gamma^3}{\sqrt{P_{NS} r}}$$

$$\Delta\alpha(\omega_c) = -g_\gamma \frac{\pi}{2}\frac{R_{NS}P_{NS}\omega_c^2}{7.95\gamma^6} e^{-m_a\left(\frac{7.95\gamma^6}{P_{NS}\omega_c^2} - R_{NS}\right)}$$

P_{NS} rotation period
γ Lorentz factor of the magnetospheric plasma

Ruderman et al. Astrophys. J.196, 51 (1975)

Frequency Dependence of Axion-Induced Polarization

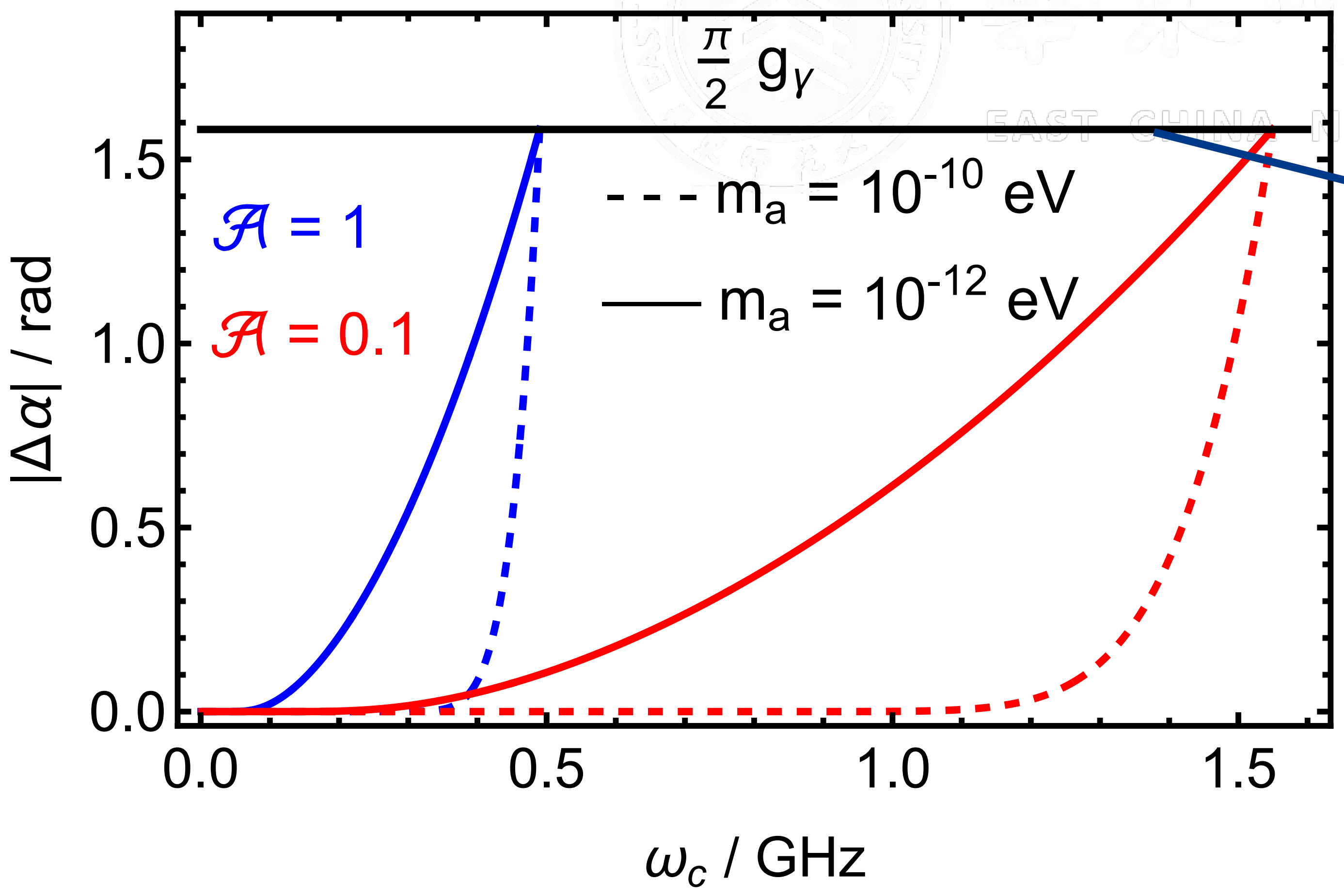


$$\Delta\alpha = -6.59\,\mathcal{A}\left(\frac{\omega_c}{GHz}\right)^2$$

$$\times e^{-m_a R_{NS}\left(\frac{0.24}{\mathcal{A}}\left(\frac{GHz}{\omega_c}\right)^2 - 1\right)}$$

$$\mathcal{A} = \left(\frac{R_{NS}}{10\ km}\right)\left(\frac{P_{NS}}{1\ s}\right)\left(\frac{100}{\gamma}\right)^6$$

Frequency Dependence of Axion-Induced Polarization

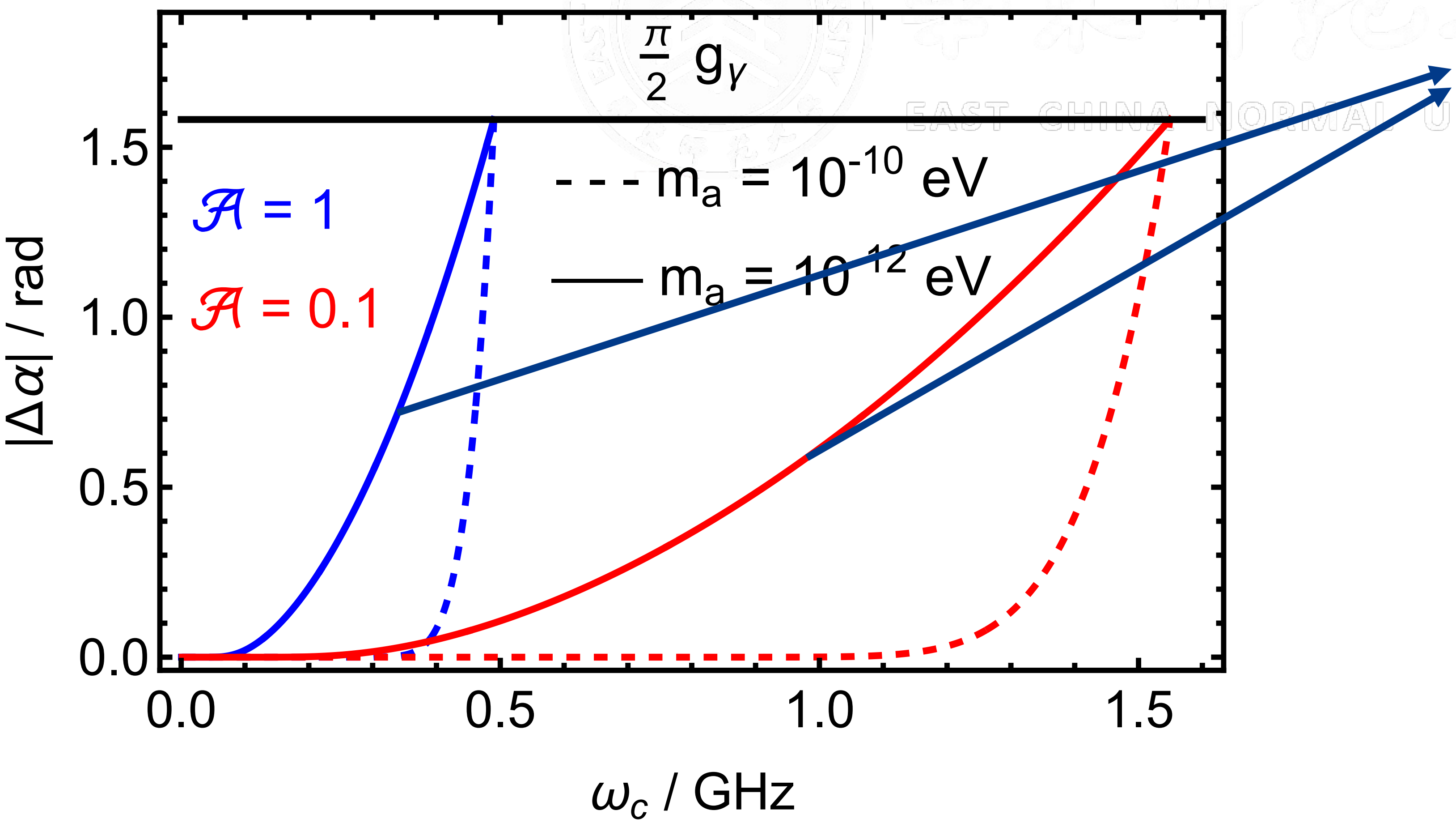


$$|\Delta\alpha|_{max} = g_\gamma \frac{\pi}{2}$$

$$|\Delta\alpha|_{max} = \frac{g_\gamma}{2f_a} a(R_{NS})$$

$$a(R_{NS}) = \pi f_a$$

Frequency Dependence of Axion-Induced Polarization



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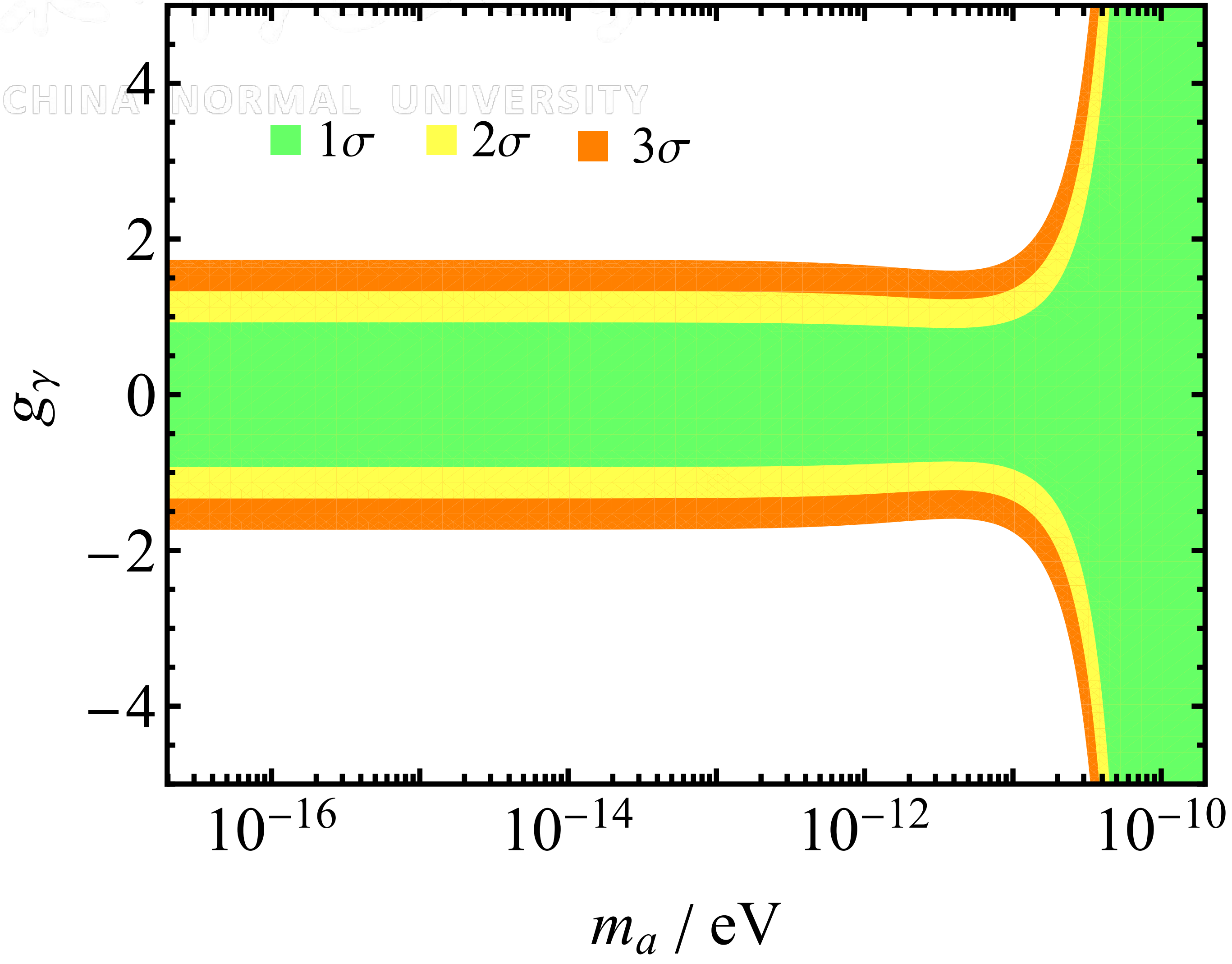
Constraints on Axion–Photon Coupling from FAST Pulsar Data

- α_0 constant initial angle
- Neutron star PSR B1919+21 data from FAST
- 500 MHz bandwidth (1-1.5 GHz)
- Free parameters:

  $g_\gamma, m_a; \alpha_0; \mathcal{A}(R_{NS}, P_{NS}, \gamma)$



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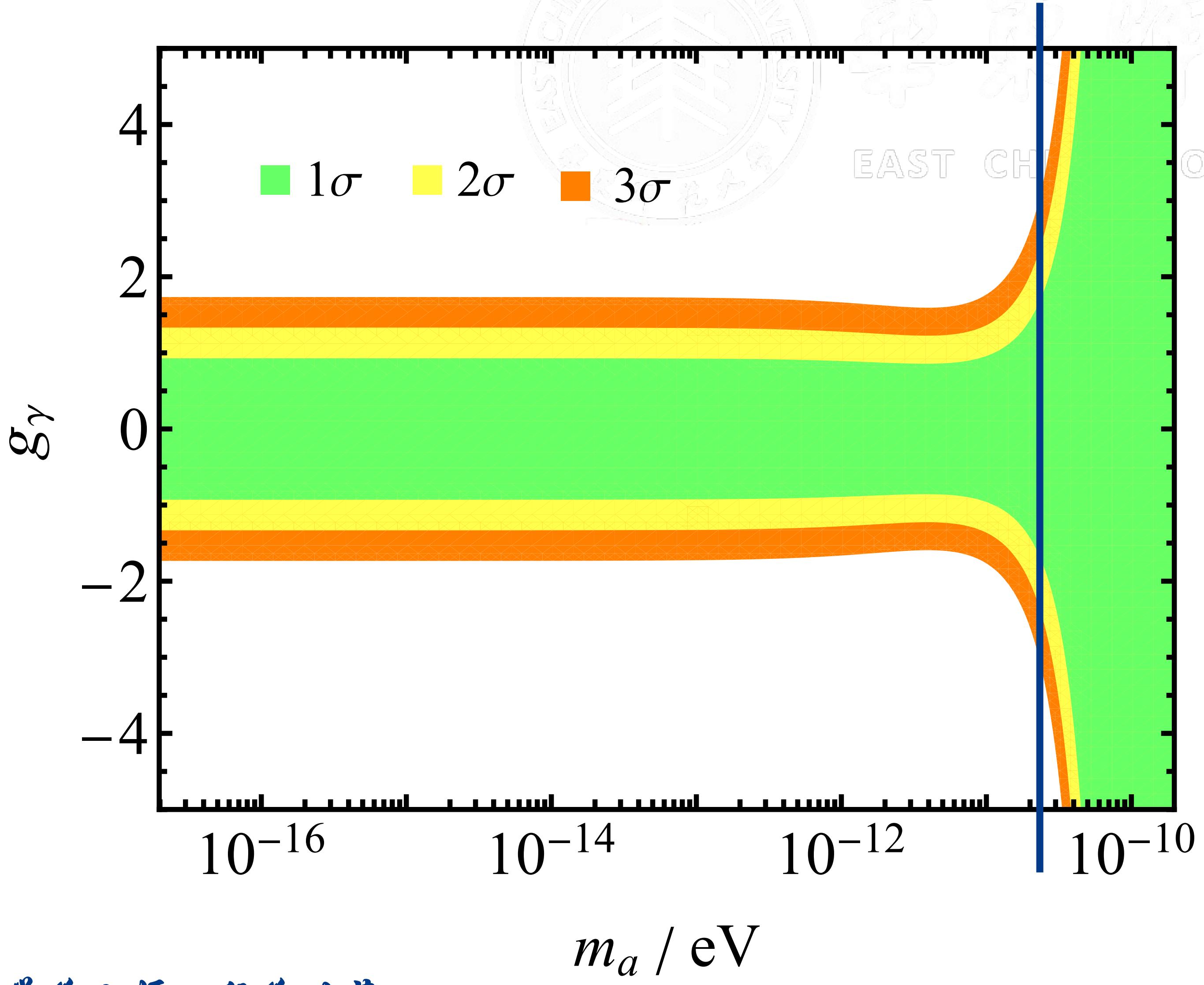

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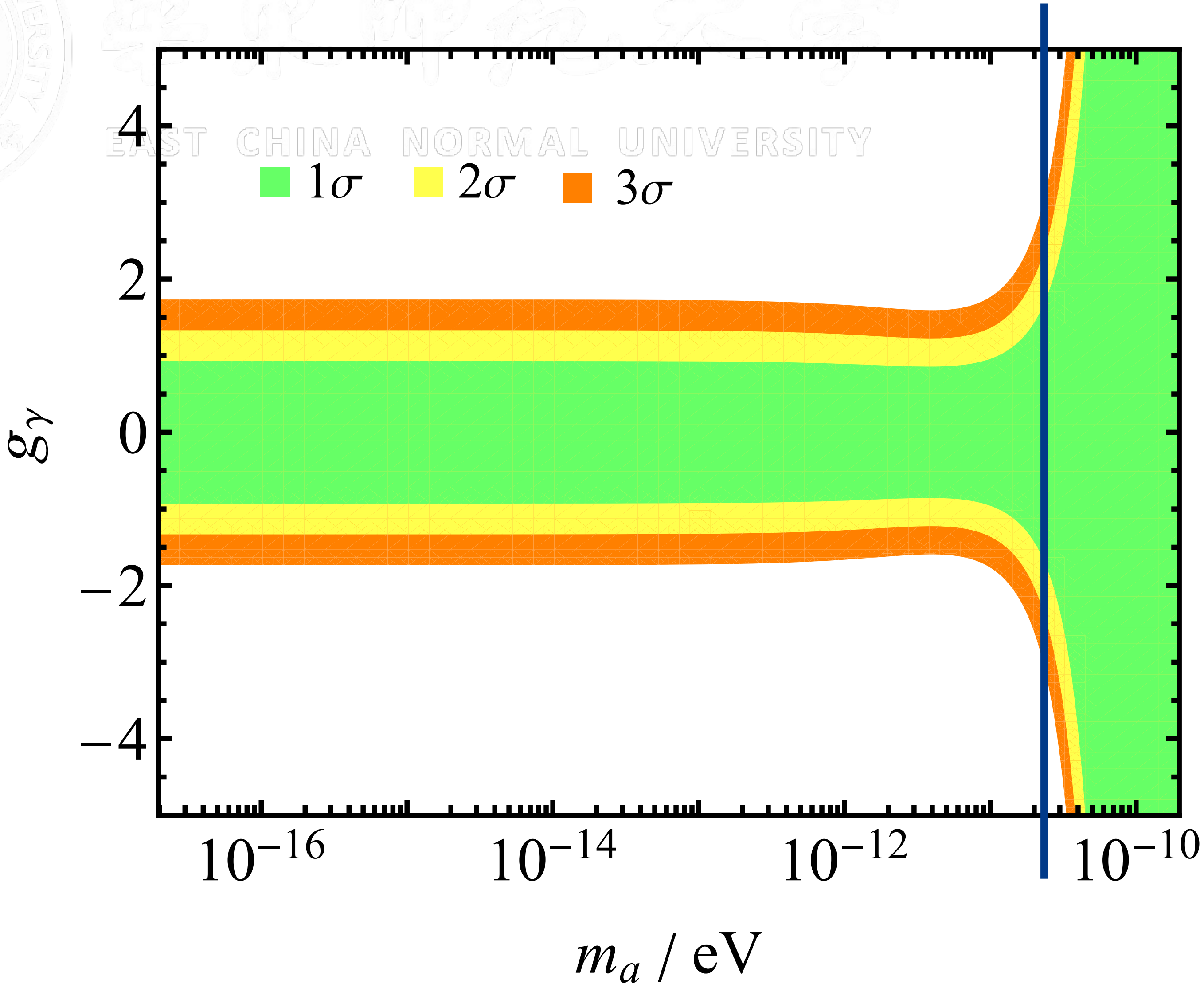
Constraints on Axion–Photon Coupling from FAST Pulsar Data

$$\Delta\alpha = -6.59 g_\gamma \mathcal{A} \left(\frac{\omega_c}{GHz}\right)^2$$

$|g_\gamma| < 0.93$ at 1σ

with $m_a < 10^{-11}$ eV



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$|g_\gamma| < 0.93$ at 1σ

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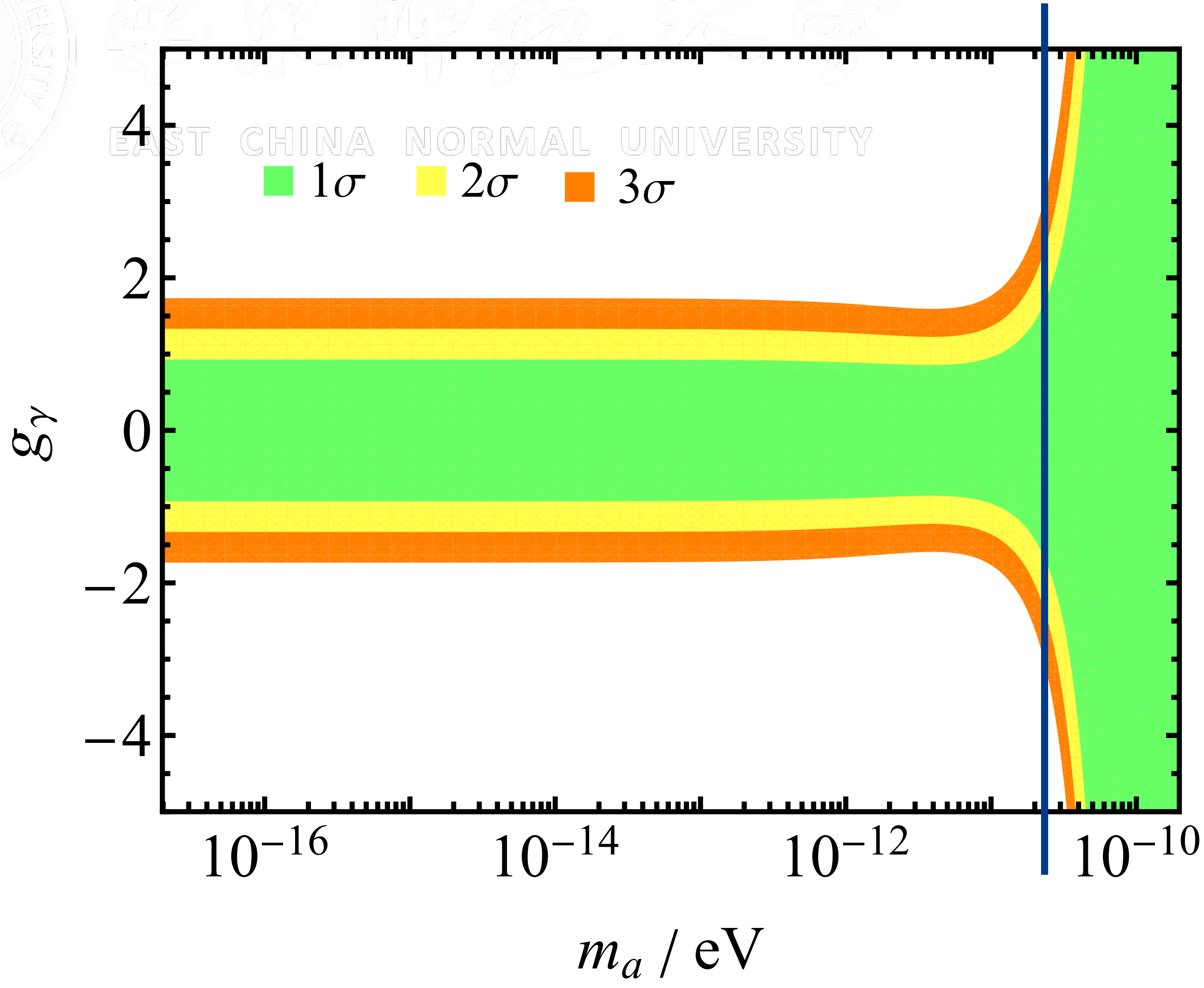
$$g_\gamma = \frac{\alpha_{EM}}{2\pi}\left(\frac{E}{N} - 1.92\right)$$



Summary

- The axion-induced **Berry phase** provides a novel theoretical tool to probe the coupling g_γ without suppression by f_a.
- **Neutron stars** serve as powerful natural laboratories, generating macroscopic axion fields that induce a predictable, frequency-dependent polarization rotation in pulsar signals.
- Using FAST data, we have placed the direct constraint of this kind on $|g_\gamma|$.
- Ultimately, our goal is to perform a direct test of the Standard Model's fundamental global structure, though more precise data and analysis will be required.

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谢谢！

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