第四届高能物理理论与实验融合发展研讨会

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# CP violation in the charged meson decays induced by sterile neutrinos


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# Motivation

The leptonic content in the theory includes three generations of left-handed SM SU(2)_L doublets and n right-handed SM singlets:

$$L_{aL} = \begin{pmatrix} \nu_a \\ l_a \end{pmatrix}_L , \quad N_{bR},$$

The leptonically universal gauge interaction involving neutrinos has the form

$$-\mathcal{L} = \left( \frac{g}{\sqrt{2}} W^+_\mu \sum_{a=1}^{3} \overline{\nu_{aL}}\, \gamma^\mu l_{aL} + \text{h.c.} \right) + \cdots$$

The Yukawa term is

$$-\mathcal{L}_Y = \left( \sum_{a,b=1}^{3} f^l_{ab}\, \overline{L_{aL}}\, H l_{bR} + \sum_{a=1}^{3} \sum_{b=1}^{n} f^\nu_{ab}\, \overline{L_{aL}}\, \hat{H} N_{bR} \right) + \text{h.c.}$$

Atre et al, JHEP05, 030 (2009)

# Motivation

After spontaneous symmetry breaking, the Yukawa term leads to the Dirac masses for the leptons

$$-\mathcal{L}_m^D = \left( \sum_{a,b=1}^{3} \overline{l_{aL}}\, m_{ab}^l\, l_{bR} + \sum_{a=1}^{3} \sum_{b=1}^{n} \overline{\nu_{aL}}\, m_{ab}^\nu\, N_{bR} \right) + \text{h.c.}$$

Consider the heavy Majorana mass term

$$-\mathcal{L}_m^M = \frac{1}{2} \sum_{b,b'=1}^{n} \overline{N_{bL}^c}\, B_{bb'}\, N_{b'R} + \text{h.c.}$$

The full neutrino mass terms have the form

$$-\mathcal{L}_m^\nu = \frac{1}{2} \left( \overline{\nu_L} \;\; \overline{N_L^c} \right) \begin{pmatrix} 0_{3\times3} & m_{3\times n}^\nu \\ m_{n\times3}^{\nu T} & B_{n\times n} \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} + \text{h.c.}$$

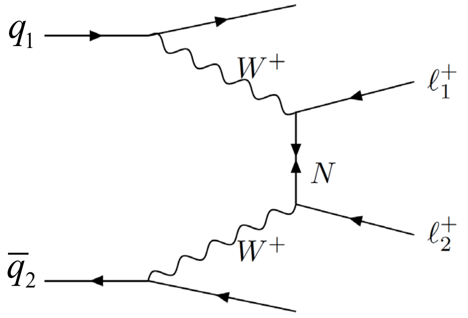
Atre et al, JHEP05, 030 (2009)

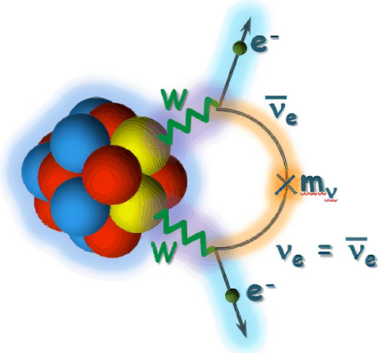
# Motivation

After the unitary transition between gauge interaction eigenstates and mass eigenstates $\begin{pmatrix} \nu_L \\ N_L^c \end{pmatrix} = \mathbb{L} \begin{pmatrix} \nu_L \\ N_L^c \end{pmatrix}_m$ The mass matrix can be diagonalized

Inserting this into the charged current interaction term and considering the unitary transformation of the charged leptons, we get the charged current interactions expressed by the mass eigenstates

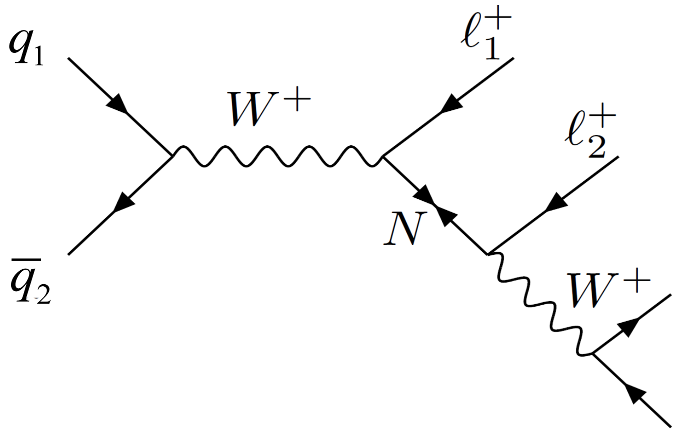
$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}} W_\mu^+ \left( \sum_{\ell=e}^{\tau} \sum_{m=1}^{3} V_{\ell m}^{l\nu *} \overline{\nu_m} \gamma^\mu P_L \ell + \sum_{\ell=e}^{\tau} \sum_{m'=4}^{3+n} U_{\ell m'}^{lN*} \overline{N_{m'}^c} \gamma^\mu P_L \ell \right) + \text{h.c.}$$




Atre et al, JHEP05, 030 (2009)

# Motivation




Atre et al, JHEP05, 030 (2009)
Li et al, Chin.Phys.C43, 023101(2019)
Yuan et al, JHEP08, 066 (2013)
Wang et al, Phys. Lett. B736, 428(2014)
Zhang et al, EPJC71,1715(2011)
Zhang et al, Phys.Rev.D103, 033004 (2021)
Lu et al, Phys.Rev.D104, 115003 (2021)
Liu et al, Phys.Rev.D101, 071701 (2020)
……

Meson decay: K, D, B, Bc 3-body or 4-body

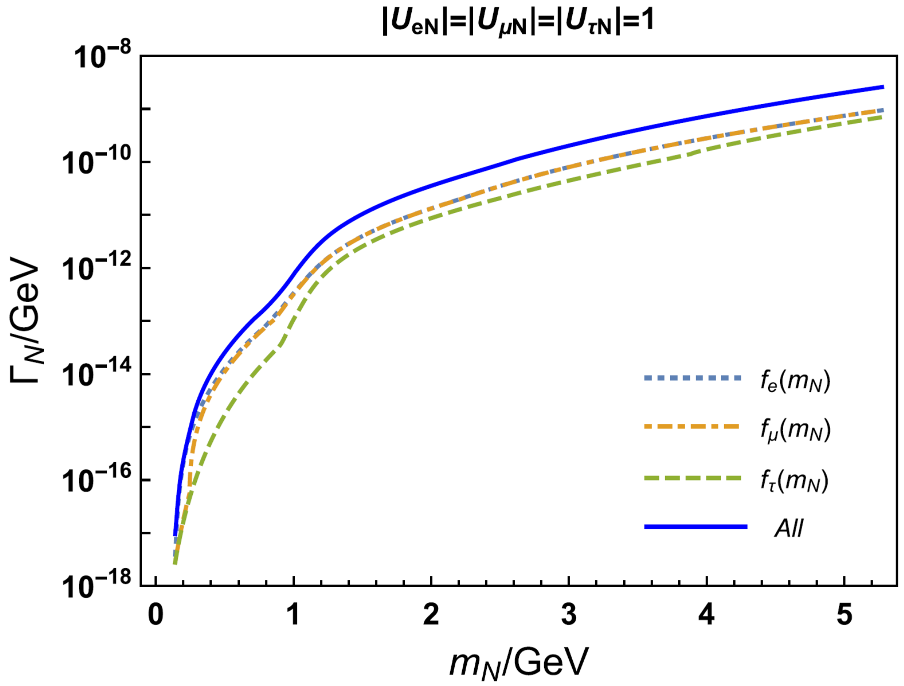
Baryon decay: Lambda_b/c 4-body or 5-body

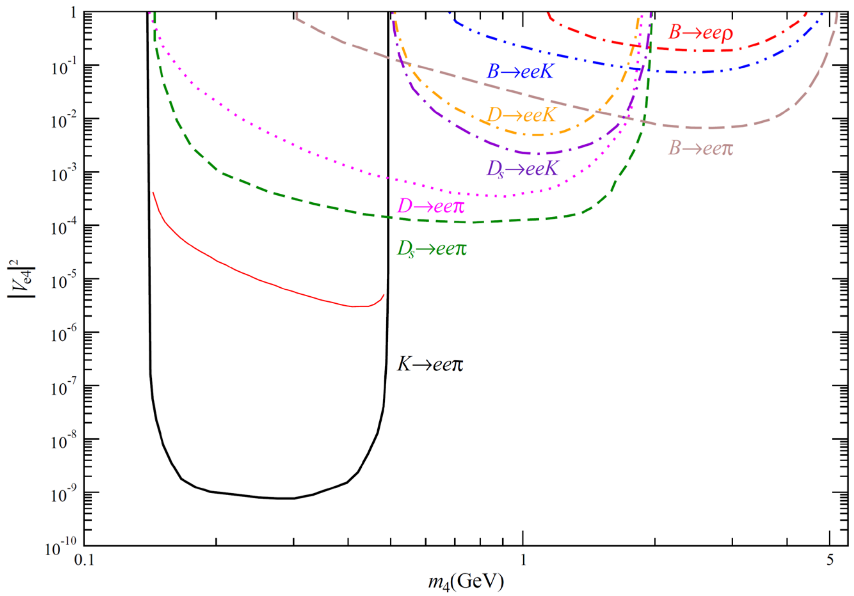
Tauon decay: 3-body

Top decay ……

# Motivation

$$\Gamma_N = |U_{eN}|^2 f_e(m_N) + |U_{\mu N}|^2 f_\mu(m_N) + |U_{\tau N}|^2 f_\tau(m_N)$$




Yuan et al, J. Phys. G45, 065002 (2017)

$\Gamma_N$ is calculated by considering all possible decays of the heavy neutrino, such as $N_4 \to \ell^- P^+$ $N_4 \to \ell_1^- \ell_2^+ \nu_{\ell_2}$

Atre et al, JHEP05, 030 (2009)

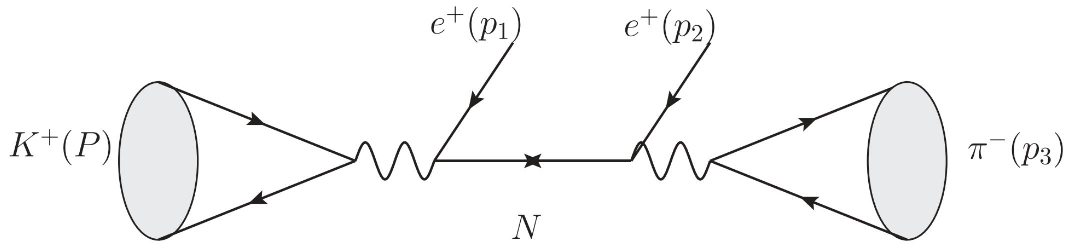
| Decay channel | Braching ratio | Decay channel | Braching ratio |
|---|---|---|---|
| $K^+ \to e^+e^+\pi^-$ | $<6.4 \times 10^{-10}$ | $D^+ \to e^+e^+\pi^-$ | $<1.1 \times 10^{-6}$ |
| $K^+ \to \mu^+\mu^+\pi^-$ | $<8.6 \times 10^{-11}$ | $D^+ \to \mu^+\mu^+\pi^-$ | $<2.2 \times 10^{-8}$ |
| $K^+ \to e^+\mu^+\pi^-$ | $<5.0 \times 10^{-10}$ | $D^+ \to e^+\mu^+\pi^-$ | $<2.0 \times 10^{-6}$ |

Particle Data Group

# Motivation

If the heavy neutrino is a Majorana type, both the lepton number violation processes and lepton flavor violation processes can happen, such as

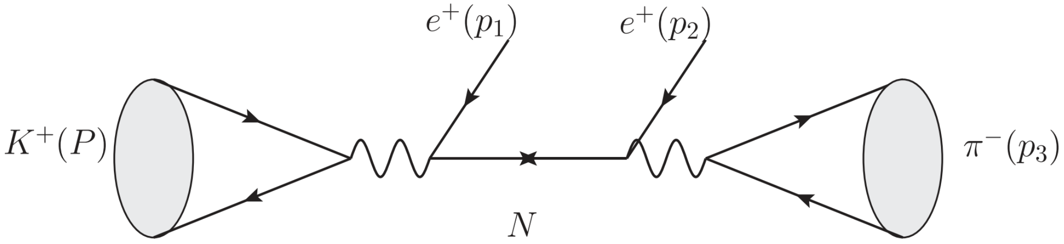
$$K^+ \to \pi^+ e^- \mu^+$$
$$K^+ \to \pi^- e^+ \mu^+$$



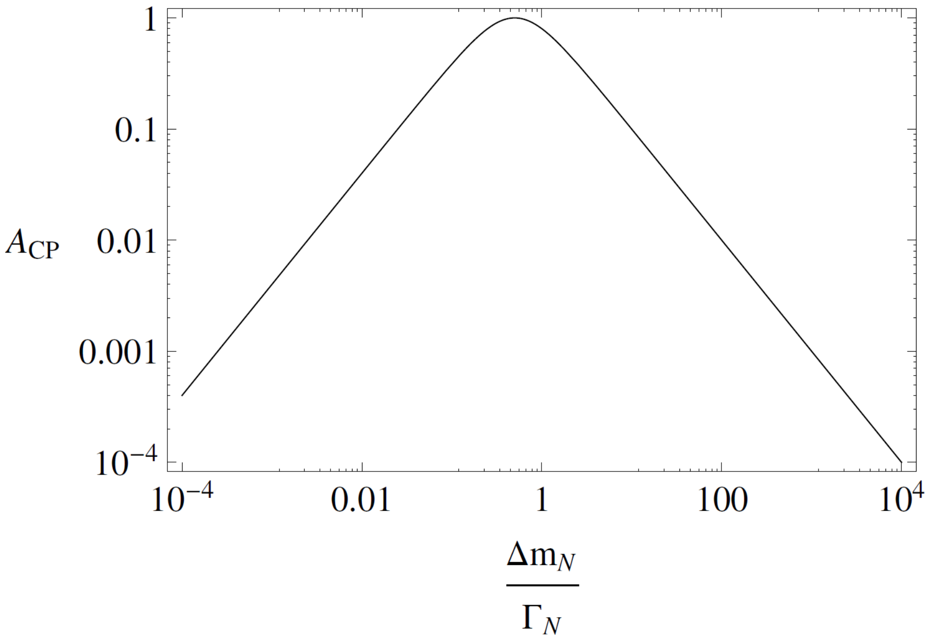

If the heavy neutrino is a Dirac type, only the lepton flavor violation processes can happen

A question is: if the LFV process is detected, how possible it is induced by a Majorana neutrino? [Abada et al, JHEP09, 017 (2019)] If there is only one generation, the LFV and LNV is expected to be of the same order. If there are more than one generation, the CP phase may cause an enhancement of the LNV mode

# Motivation




$$i\mathcal{M} = 2G_F^2 f_K f_\pi V_{ud} V_{us} \left[ \frac{U_{e4}U_{e4}m_4}{s_{23} - m_4^2 + i\Gamma_4 m_4} + \frac{U_{e5}U_{e5}m_5}{s_{23} - m_5^2 + i\Gamma_5 m_5} \right] \bar{u}(p_1) \not{P} \not{p}_3 P_R v(p_2)$$




Narrow width approximation

$$\frac{m_N \Gamma_N}{(p_N^2 - m_N^2)^2 + m_N^2 \Gamma_N^2} \sim \pi\delta(p_N^2 - m_N^2) \qquad \hat{A}_{CP} \sim 2m_N\Gamma_N \frac{\Delta m_N^2}{(\Delta m_N^2)^2 + m_N^2\Gamma_N^2} \leq 1$$

Dib et al, JHEP02, 108 (2015)

# Motivation

- ☐ The partial decay width of the LNV process of the charged meson needs a more careful calculation when there are two quasi-degenerate sterile neutrinos
- ☐ The CP asymmetry may depend on several parameters; a more general result is needed

# Calculation of partial width

Define $k_l = \frac{|U_{l5}|^2}{|U_{l4}|^2}$ $k_{\mu e} = \frac{|U_{\mu 4}|^2}{|U_{e4}|^2}$ $k_{\tau e} = \frac{|U_{\tau 4}|^2}{|U_{\tau 4}|^2}$

$$U_{l4} = |U_{l4}| e^{-i\phi_{l4}}$$

$$U_{l5} = |U_{l5}| e^{-i\phi_{l5}}$$

$$k = \frac{|U_{e5}|^2 f_e(m_5) + |U_{\mu 5}|^2 f_\mu(m_5) + |U_{\tau 5}|^2 f_\tau(m_5)}{|U_{e4}|^2 f_e(m_4) + |U_{\mu 4}|^2 f_\mu(m_4) + |U_{\tau 4}|^2 f_\tau(m_4)}$$

$$\approx \frac{k_e f_e(m_4) + k_{\mu e} k_\mu f_\mu(m_4) + k_{\tau e} k_\tau f_\tau(m_4)}{f_e(m_4) + k_{\mu e} f_\mu(m_4) + k_{\tau e} f_\tau(m_4)}$$

assume

$$\Delta m \equiv m_5 - m_4 \ll m_4$$

# Calculation of partial width

Define $x = (s_{23}^2 - m_4^2)/(\Gamma_4 m_4)$

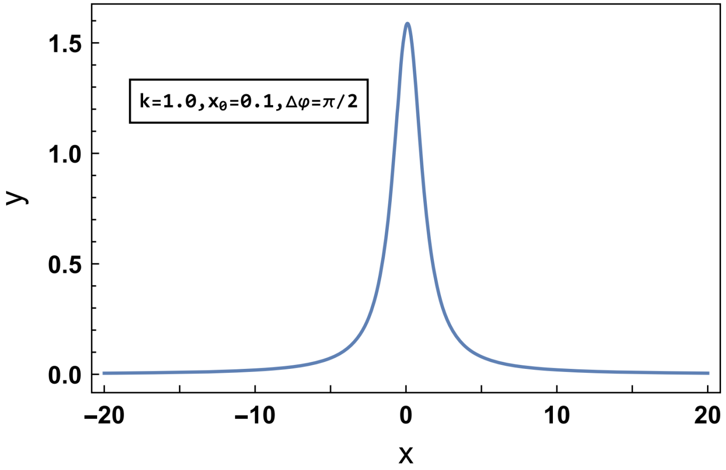
$x_0 = \Delta m/\Gamma_4$

$\Delta\varphi = 2(\phi_{e5} - \phi_{e4})$

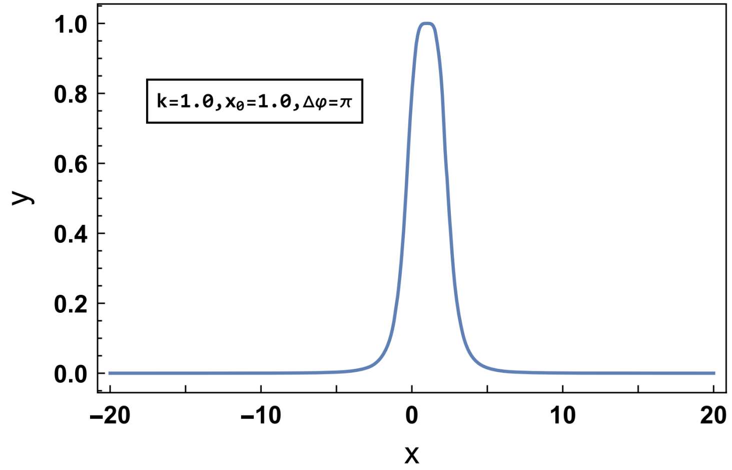
$$y(k_e, k, x_0, \Delta\varphi, x) = \frac{1}{1+x^2}\left\{1 + \frac{k_e^2(x^2+1)}{k^2+(x-2x_0)^2} + \frac{2k_e}{k^2+(x-2x_0)^2} \times [(k+x^2-2xx_0)\cos\Delta\varphi - (kx - x + 2x_0)\sin\Delta\varphi)]\right\}$$

$$\left|\frac{U_{e4}U_{e4}m_4}{s_{23}-m_4^2+i\Gamma_4 m_4} + \frac{U_{e5}U_{e5}m_5}{s_{23}-m_5^2+i\Gamma_5 m_5}\right|^2 = \frac{|U_{e4}|^4}{\Gamma_4^2} y(k_e, k, x_0, \Delta\varphi, x)$$

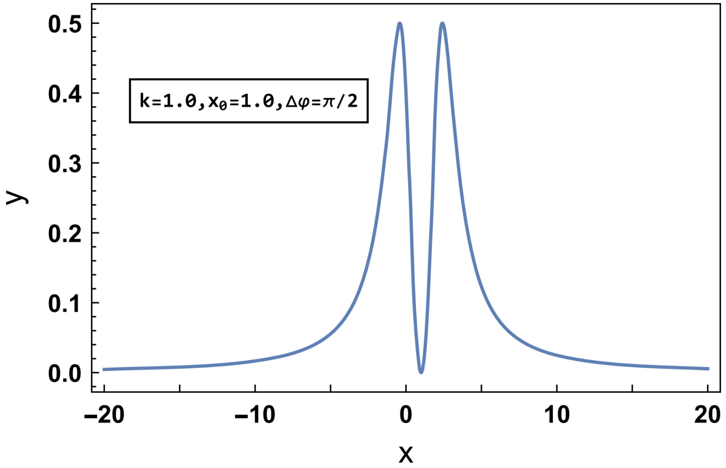
# Calculation of partial width



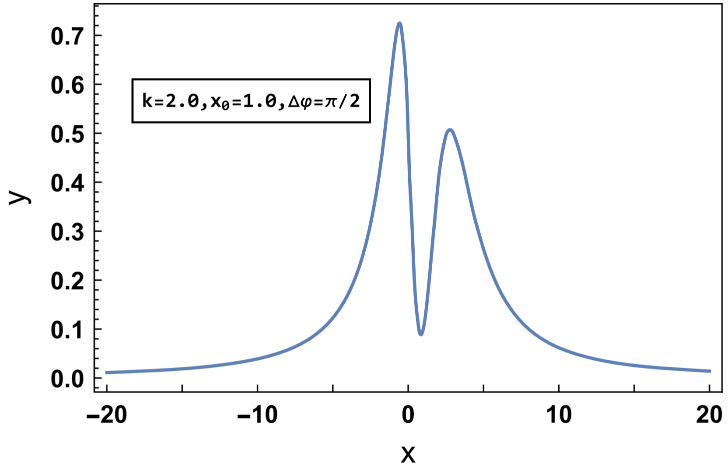

(a)




(b)




(c)




(d)

A simple case:

$|U_{l4}|$ and $|U_{l5}|$ are flavor universal

$$k = k_l$$

$$k_{\mu e} = k_{\tau e} = 1$$

# Calculation of partial width

$$x = (s_{23}^2 - m_4^2)/(\Gamma_4 m_4)$$

$$\Gamma = C_f \frac{|U_{e4}|^4 m_4}{\Gamma_4} \int y(k_e, k, x_0, \Delta\varphi, x) \mathrm{ILT}(s_{23}) dx$$

$$\mathrm{ILT}(s_{23} = m_4^2)$$

$$\Gamma \approx C_f \frac{|U_{e4}|^4 m_4}{\Gamma_4} \int_{-\infty}^{\infty} y(k_e, k, x_0, \Delta\varphi, x) \mathrm{ILT}(m_4^2) dx$$

$$= C_f \frac{|U_{e4}|^4 m_4}{\Gamma_4} \mathrm{Iy}(k_e, k, x_0, \Delta\varphi) \mathrm{ILT}(m_4^2),$$

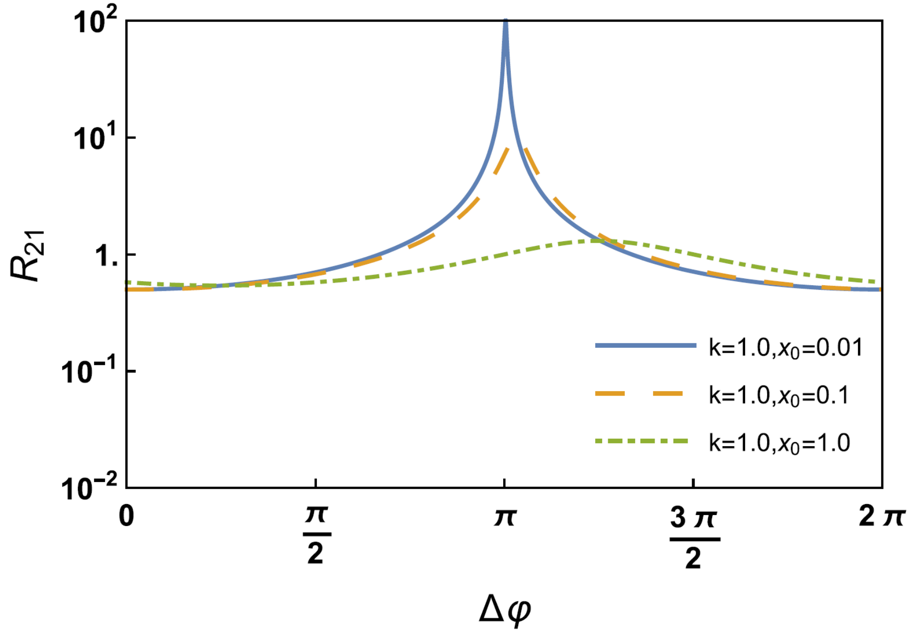
$$\mathrm{Iy}(k_e, k, x_0, \Delta\varphi) = \pi\left(1 + \frac{k_e^2}{k}\right) + \frac{4\pi k_e}{(k+1)^2 + 4x_0^2} \times [(k+1)\cos\Delta\varphi - 2x_0 \sin\Delta\varphi]$$

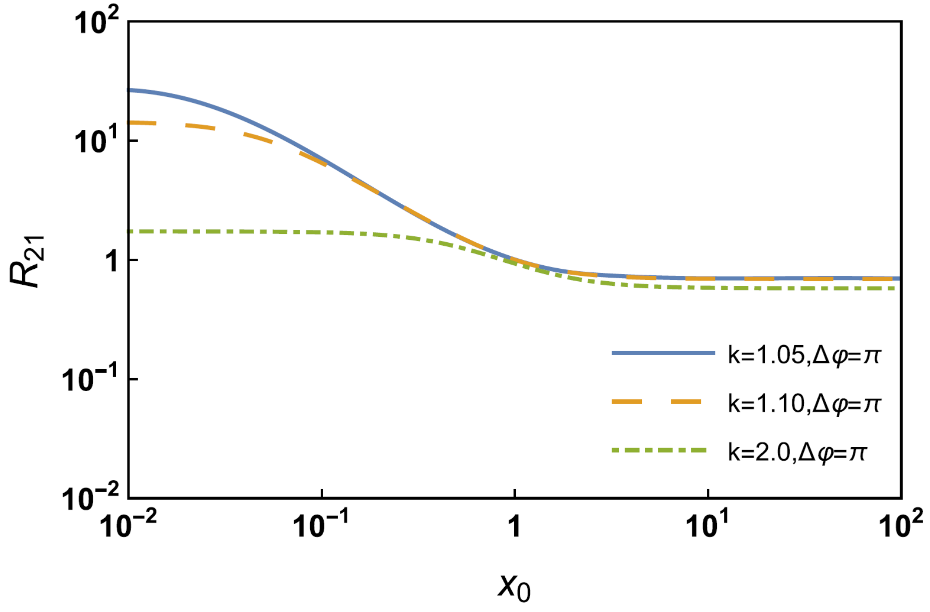
# Calculation of partial width

Case 1: $k = k_l \quad k_{\mu e} = k_{\tau e} = 1$

$$|U_{e4}|^2 = \frac{\mathrm{Br}(K^+ \to e^+ e^+ \pi^-)}{\tau(K^+)} \times \frac{\Gamma_4/|U_{e4}|^2}{C_f m_4 \mathrm{Iy}(k_e, k, x_0, \Delta\varphi)\mathrm{ILT}(m_4^2)}$$

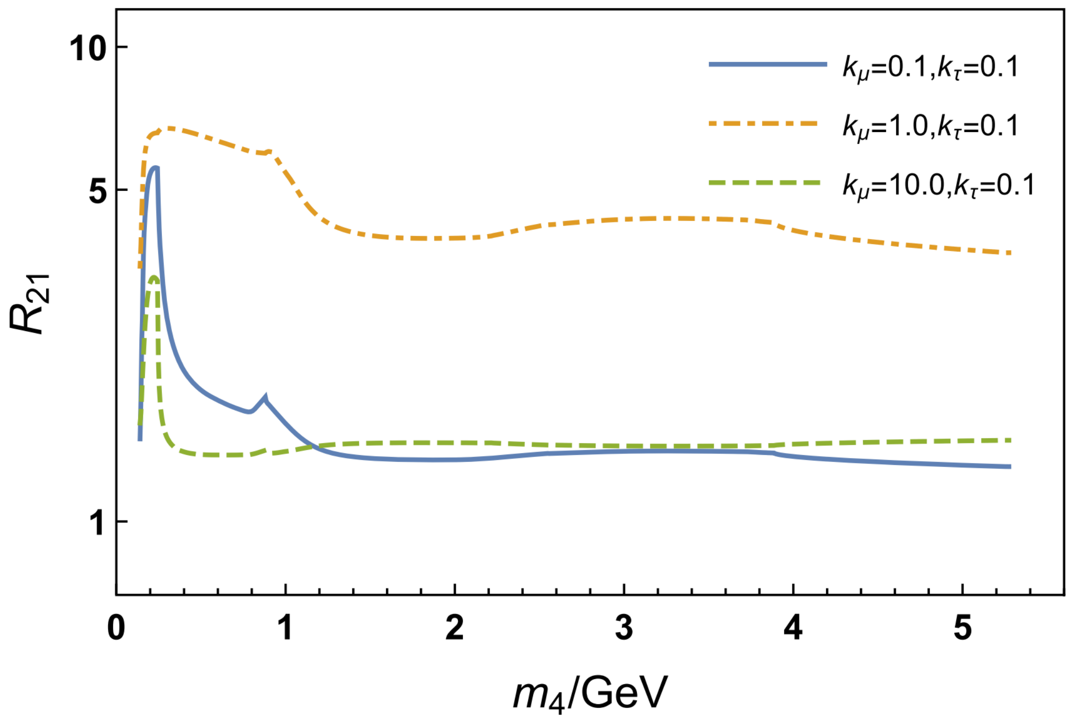
$$R_{21}(k_e, k, x_0, \Delta\varphi) \equiv \frac{|U_{e4}|_{2-\mathrm{gen}}}{|U_{e4}|_{1-\mathrm{gen}}} = \sqrt{\pi/\mathrm{Iy}(k_e, k, x_0, \Delta\varphi)}$$

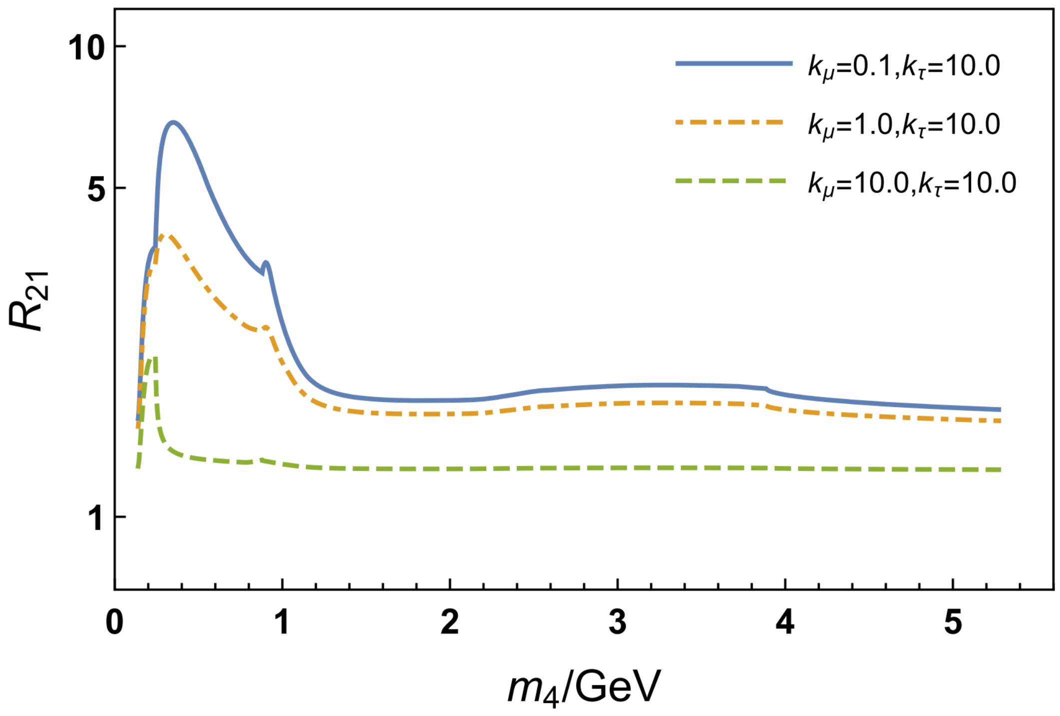


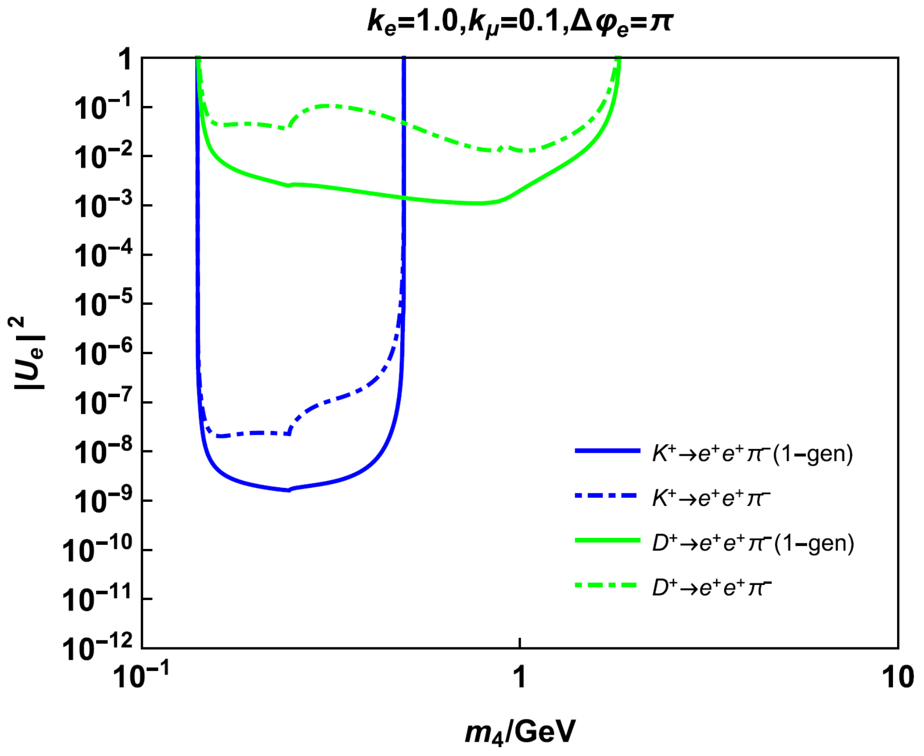
# Calculation of partial width

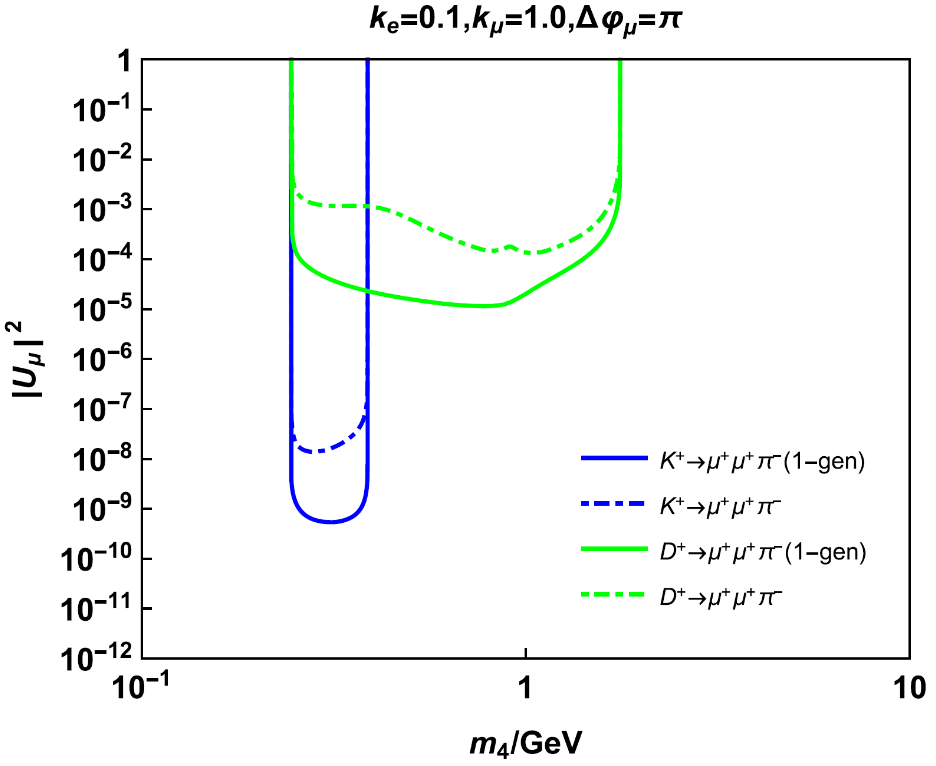
Case 2: $k_{\mu e} = k_{\tau e} = 1$ ; $k_e$ , $k_\mu$ and $k_\tau$ are free parameters



# Calculation of partial width



# CP Violation

$$\Gamma = C_f \frac{|U_{e4}|^4 m_4}{\Gamma_4} \mathrm{Iy}(k_e, k, x_0, \Delta\varphi) \mathrm{ILT}(m_4^2)$$

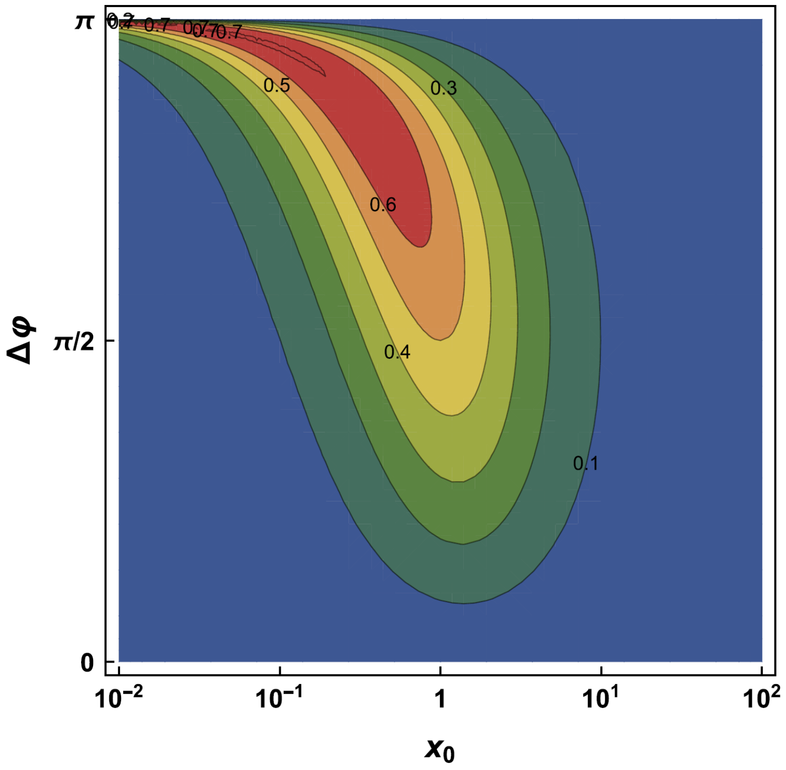
$$\mathcal{A}_{CP} = \frac{\Gamma(K^- \to e^- e^- \pi^+) - \Gamma(K^+ \to e^+ e^+ \pi^-)}{\Gamma(K^- \to e^- e^- \pi^+) + \Gamma(K^+ \to e^+ e^+ \pi^-)}$$

$$= \frac{\mathrm{Iy}(k_e, k, x_0, -\Delta\varphi) - \mathrm{Iy}(k_e, k, x_0, \Delta\varphi)}{\mathrm{Iy}(k_e, k, x_0, -\Delta\varphi) + \mathrm{Iy}(k_e, k, x_0, \Delta\varphi)}$$

$$= \frac{8 k x_0 \sin\Delta\varphi}{(k_e + k/k_e)[(k+1)^2 + 4x_0^2] + 4k(k+1)\cos\Delta\varphi}$$

# CP Violation

Case 1: $k = k_l \quad k_{\mu e} = k_{\tau e} = 1$




$x_0 = \Delta m/\Gamma_4$

$$\mathcal{A}_{CP} = \frac{8kx_0 \sin\Delta\varphi}{(k+1)[(k+1)^2 + 4x_0^2 + 4k\cos\Delta\varphi]}$$

$$\mathcal{A}_{CP}(\pi+\Delta\varphi) = -\mathcal{A}_{CP}(\pi-\Delta\varphi)$$

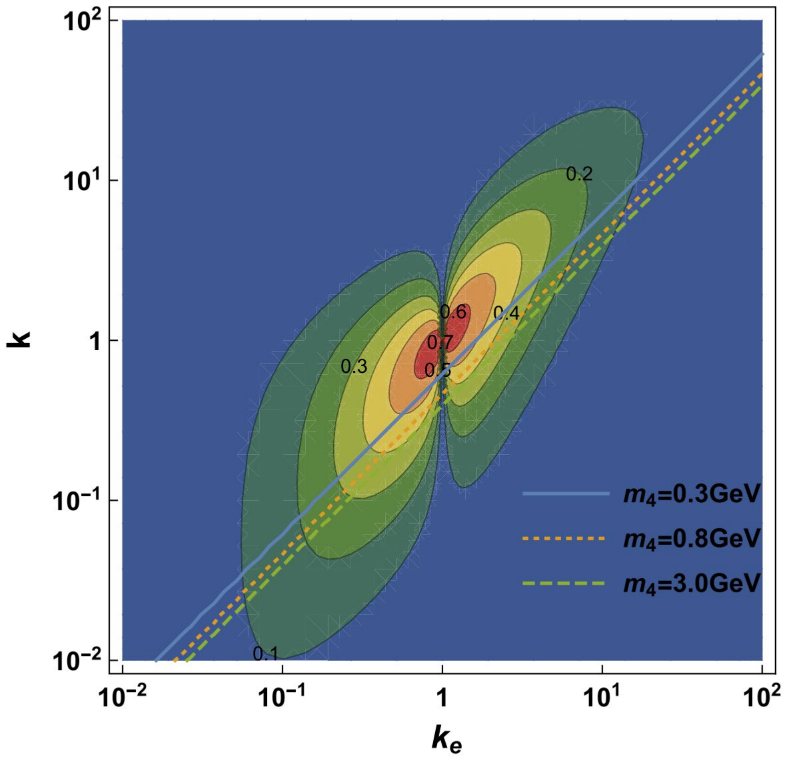
$$\mathcal{A}_{CP}(k=1, x_0, \Delta\varphi = \pi-\alpha) = \frac{x_0 \sin\alpha}{x_0^2 + 1 - \cos\alpha}$$

$$\approx \frac{x_0\alpha}{x_0^2 + \alpha^2/2} = \frac{\beta}{1/2+\beta^2}$$

$$\mathcal{A}_{CP\max} = \frac{\sqrt{2}}{2} \quad \text{when} \quad \beta = \frac{\sqrt{2}}{2}$$

# CP Violation

**Case 2:** $k_{\mu e} = k_{\tau e} = 1$ ; $k_e$, $k_\mu$ and $k_\tau$ are free parameters

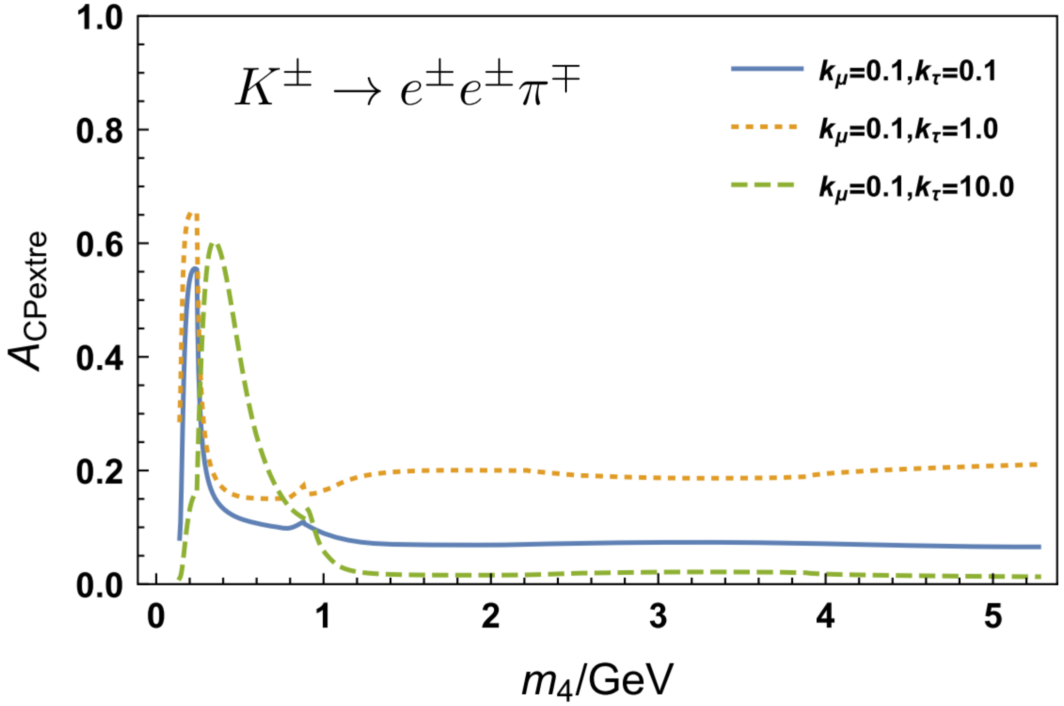


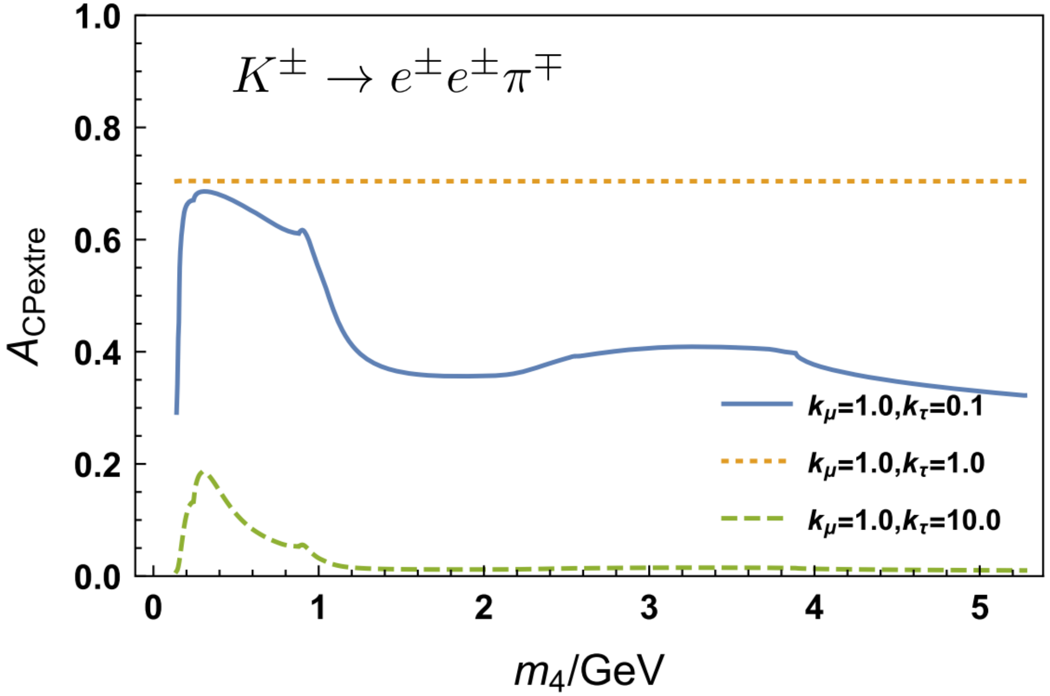

$$\frac{\partial}{\partial x_0}\mathcal{A}_{CP} = 0, \qquad \frac{\partial}{\partial \Delta\varphi}\mathcal{A}_{CP} = 0$$

$$\mathcal{A}_{CP_{\text{extre}}}(k_e, k) = \frac{8kx_0^2|_{\text{extre}}\sqrt{-2k_e\cos\Delta\varphi|_{\text{extre}}}}{4x_0^2|_{\text{extre}}(k^2+2k+k_e^2)+(k+1)(k-k_e)^2}$$

$$k = \frac{k_e f_e(m_4) + k_\mu f_\mu(m_4) + k_\tau f_\tau(m_4)}{f_e(m_4) + f_\mu(m_4) + f_\tau(m_4)} \geq \frac{k_e f_e}{\sum_l f_l}$$

# CP Violation



# Summary

The LNV processes of charged mesons can be induced by two quasi-degenerate heavy sterile Majorana neutrinos

- ☐ The partial decay width are expressed as a function of the mass difference, the phase difference et al
- ☐ The phase difference can greatly affect the results of partial width and the upper bound of the mixing parameters
- ☐ The analytical expression of CP violation is given, and the maximum of the CPV is calculated