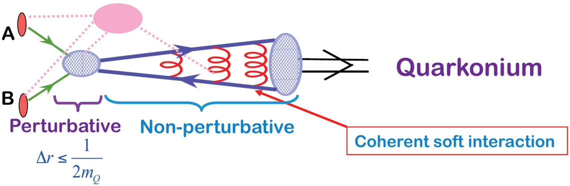
Next-to-leading-order QCD corrections to double prompt J/ψ hadroproduction

Xiao-Bo Jin

Beijing University of Chemical Technology
Based on work in Collaboration with Zhi-Guo He and Bernd A.Kniehl
Phys.Rev.D 111,094040

第四届高能物理理论与实验融合发展研讨会
09.21.2025, Dalian

Calculation framework



NRQCD factorization *Bodwin, Braaten, and Lepage 1995*

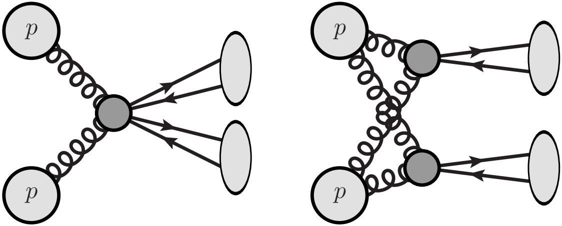
$$d\sigma(A+B \to J/\psi + X) = \sum_n d\hat{\sigma}(A+B \to Q\bar{Q}[n] + X)\langle \mathcal{O}^{J/\psi}[n]\rangle$$

The challenges to NRQCD:

1. The long-standing J/ψ polarization puzzle.
2. The universality of the NRQCD LDMEs for J/ψ production up to QCD NLO.
3. The success of CSM to account for J/ψ production in e^+e^- annihilation, and η_c meson hadroproduction at the LHC.

New features in quarkonium pair hadroproduction

- To test NRQCD factorization and to provide crucial constraint on the long-distance matrix elements. *Barger, Fleming, and Phillips 1996*
- To improve the NRQCD factorization formalism for double P-wave production. *He, Kniehl, and Wang 2018*
- To extract σ_{eff} for DPS process. *Kom, A. Kulesza, and Stirling 2011*
- To understand the new fully heavy tetraquark states ($Q\bar{Q}Q\bar{Q}$). *Aad et al. 2023; Aaij et al. 2020; Hayrapetyan et al. 2024* (See Talk RuiLin Zhu)



Schematical representation of SPS (left) and DPS (right) for a proton-proton collision.
Figure from *Borschensky and Anna Kulesza 2017*

Experimental Status of hadroproduction

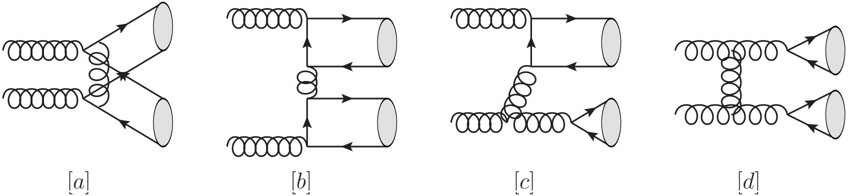
The summary experimental measurements

Experiment	$\sqrt{s}$	states	kinematic condition
D0 (2014)	1.96 TeV	$J/\psi + J/\psi$	$p_T^{J/\psi} > 4.0\text{GeV}, \lvert\eta^{J/\psi}\rvert < 2$
LHCb	7 TeV	$J/\psi + J/\psi$	$P_T^{J/\psi} < 10\text{GeV}$ $2.0 < y^{J/\psi} < 4.5$
(2012)	13 TeV	$J/\psi + J/\psi$	$P_T^{J/\psi} < 10(14)\text{GeV}$
(2017, 2024)		$J/\psi + \psi(2S)$	$2.0 < y^{J/\psi} < 4.5$
CMS (2014)	7 TeV	$J/\psi + J/\psi$	$p_T^{J/\psi} > 4.5\text{GeV}, \lvert y^{J/\psi}\rvert < 2.2$
ATLAS (2017)	8 TeV	$J/\psi + J/\psi$	$p_T^{J/\psi} > 8.5\text{GeV}, \lvert y^{J/\psi}\rvert < 2.1$
ALICE (2023)	13 TeV	$J/\psi + J/\psi$	$p_T^{J/\psi} > 0\text{GeV}, 2.5 < y^{J/\psi} < 4.0$

Rich observables

- σ, $\frac{d\sigma}{dp_T^{\psi}}$, $\frac{d\sigma}{dy^{\psi}}$, $\frac{d\sigma}{dp_T^{\psi\psi}}$, $\frac{d\sigma}{dy^{\psi\psi}}$, $\frac{d\sigma}{dm^{\psi\psi}}$, $\frac{d\sigma}{d\lvert\Delta y^{\psi\psi}\rvert}$, $\frac{d\sigma}{d\lvert\Delta\phi^{\psi\psi}\rvert}$ $\frac{d\sigma}{d\mathscr{A}_T^{\psi\psi}}$.

Theoretical study of J/ψ pair hadroproduction



- The $2(^3S_1^{[8]})$ channel. *Barger, Fleming, and Phillips 1996*
- The $2(^3S_1^{[1]})$ channel. *Qiao 2002*
- Complete NRQCD calculation. *He and Kniehl 2015*
- Relativistic corrections to $2(^3S_1^{[1]})$ and $2(^3S_1^{[8]})$ channels. *He, Jin, and Kniehl 2024; Li, Xu, Liu, and Zhang 2013*
- Parton reggeization approach (PRA) with high-energy resummation. *He, Kniehl, Nefedov, and Saleev 2019*
- QCD corrections to $2(^3S_1^{[1]})$ channel. *Sun, Han, and Chao 2016*

Why are we interested in the NLO QCD corrections to $2({}^3S_1^{[1]})$ channel?

- Theoretically, the NLO QCD corrections become much more complicated when bound states are involved.
- In single J/ψ production, almost all channels were evaluated by more than one group.
- We presented an up-to-date comparison with the latest LHC measurements.

Techniques in NLO corrections I

- The Feynman diagrams are generated by QGRAPH. *Nogueira 1993*
- The Dirac algebra and color matrices are handled by Form. *Vermaseren 2000*
- The phase space integrals are calculated numerically by $c++$ code with the help of cuba package. *Hahn 2005, 2015*

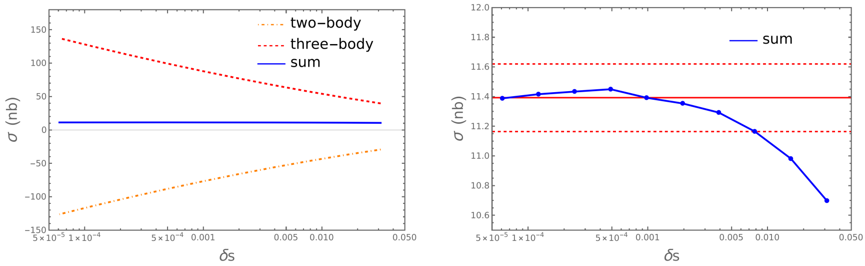
Virtual corrections

- A self-written Mathematica code is developed to perform the partial fraction decomposition.
- The Reduce 2 and FIRE6 is used to perform the IBP reduction translating scalar integrals to master integrals. *Manteuffel and Studerus 2012; Smirnov and Chuharev 2020*
- In total, we have about 130 master integrals, which can all be found in the package package-X 2.0 and QCD-loops. *Ellis and Zanderighi 2008; Patel 2017*

Techniques in NLO corrections II

Real corrections

- The two cutoff phase space slicing method is used to isolate the infrared divergences. *Harris and Owens 2002*
- We found only when $\delta_s = 1 \times 10^{-3}$ and $\delta_c = \frac{\delta_s}{300}$, or they are even smaller, the results will be relative stable.
- Below is dependence of LHCb 13 TeV total cross section on the δ_s with fixed $\delta_c = \frac{\delta_s}{300}$



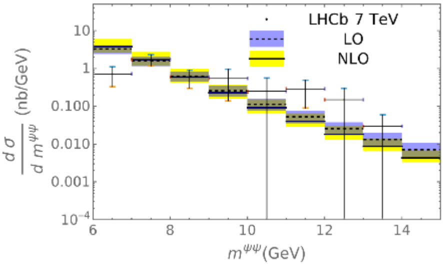
(a) (b)

CS NLO(α_s) @ LHCb 7 TeV

- Total cross section:

$$\sigma^{\rm Tot}_{\rm LHCb} = (5.1 \pm 1.0 \pm 1.1)\ {\rm nb},$$
$$\sigma^{\rm LO}_{\rm CS} = 6.02^{+1.56}_{-1.59}\ {\rm nb}, \sigma^{\rm NLO}_{\rm CS} = 6.46^{+3.99}_{-2.01}\ {\rm nb}.$$

- The K-factor of total cross section is about 1.07.
- The invariant mass spectrum of the J/ψ pair:



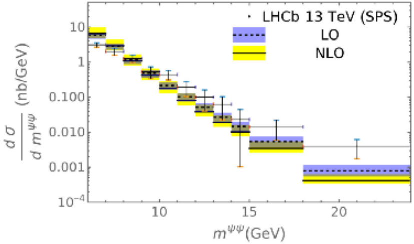
- The NLO QCD corrections near threshold region are positive, and become negative as $m^{\psi\psi}$ increases.

CS NLO(α_s) @ LHCb 13 TeV I

- Total cross section:

$$\sigma^{\rm Tot}_{\rm LHCb} = (16.36 \pm 0.28 \pm 0.88)\,{\rm nb},\ \sigma^{\rm SPS}_{\rm LHCb} = (7.9 \pm 1.2 \pm 1.1)\,{\rm nb},$$
$$\sigma^{\rm LO}_{\rm CS} = 10.9^{+1.4}_{-2.5}\,{\rm nb}, \sigma^{\rm NLO}_{\rm CS} = 11.4^{+5.8}_{-3.2}\,{\rm nb}.$$

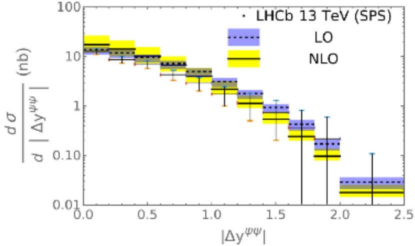
- The K-factor of total cross section is only about 1.05.
- The invariant mass spectrum of the J/ψ pair:



- When $m^{\psi\psi} > 15$GeV, the CS contribution tends to underestimate the LHCb measurements leaving room for CO contribution.

CS NLO(α_s) @ LHCb 13 TeV II

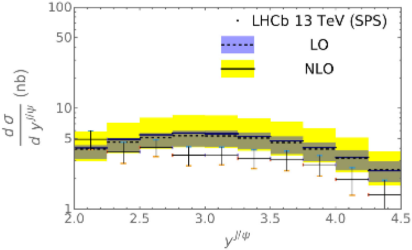
- The $|\Delta y^{\psi\psi}|$ distribution of the J/ψ pair:

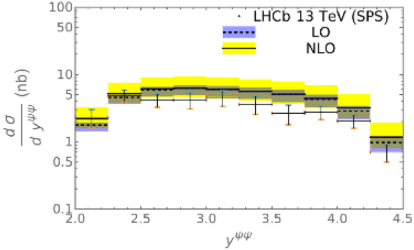


- The NLO QCD corrections barely shift the LO predictions for $|\Delta y^{\psi\psi}| < 1$ region.
- When $|\Delta y^{\psi\psi}| > 1$, the NLO QCD corrections result in a reduction from 30% to 40%.
- Unlike the invariant mass spectrum, in the last bin there is no room for CO contribution.

CS NLO(α_s) @ LHCb 13 TeV III

- The rapidity distributions of single J/ψ (left) and J/ψ pair (right):

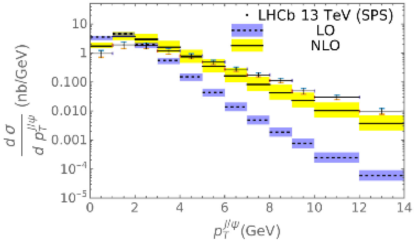




- The tiny effect of NLO QCD corrections are almost uniform in the whole region of $2.0 < y^{\psi}(y^{\psi\psi}) < 4.5$.
- In most bins, the central values of CS predictions lie slightly above the experimental data.
- CS predictions can reasonably well describe the data within errors.

CS NLO(α_s) @ LHCb 13 TeV IV

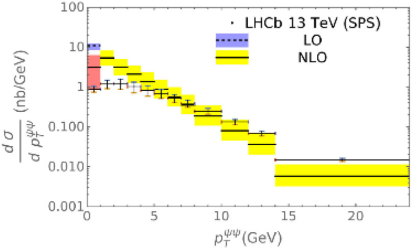
- The p_T distribution of single J/ψ:



- The LO prediction is about 3.7 times larger than LHCb measurements in the first bin, while falls off by more than 2 orders of magnitude in the last bin.
- The NLO QCD corrections bring about −51% decrease and 61 times increase in the first and last bins.
- The NLO QCD corrections greatly moderate the conflict!

CS NLO(α_s) @ LHCb 13 TeV V

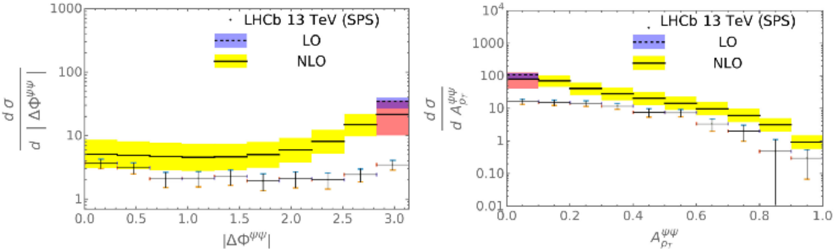
- The p_T distribution of J/ψ pair:



- In collinear parton model (CPM), only the $2 \to 3$ real corrections contribute when $p_T^{\psi\psi} \neq 0$.
- In small $p_T^{\psi\psi}$ region, perturbation theory is spoiled due to soft, hard-collinear gluons, and quarks radiation.
- Yet, as the value of $p_T^{\psi\psi}$ becomes larger, the NLO prediction falls off faster than experimental data.

CS NLO(α_s) @ LHCb 13 TeV VI

- The $|\Delta\Phi^{\psi\psi}|$ (left) and $A_{p_T}^{\psi\psi}$ (right) distributions of J/ψ pair:



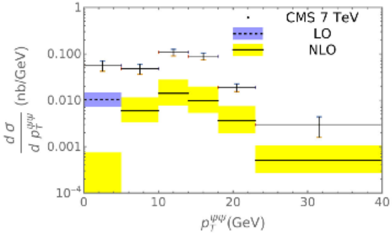
- At LO, $|\Delta\Phi^{\psi\psi}| = \pi$, $A_{p_T}^{\psi\psi} = 0$, so near these end point regions, perturbative results are not reliable too.
- The NLO predictions tends to agree with experimental data away from the end point regions.
- Small p_T resummation approach can improve the results obtained in fixed-order computation.

CS NLO(α_s) @ CMS 7 TeV I

- Total cross section:

$$\sigma_{\rm CMS} = (1.49 \pm 0.07 \pm 0.13)\ \text{nb},$$
$$\sigma_{\rm CS}^{\rm LO} = 0.052^{+0.020}_{-0.016}\ \text{nb}, \sigma_{\rm CS}^{\rm NLO} = 0.147^{+0.136}_{-0.061}\ \text{nb}.$$

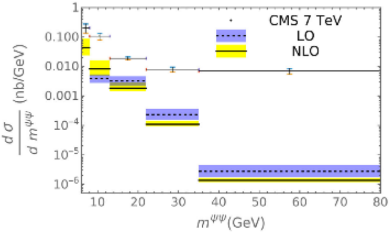
- The CS predictions at QCD NLO are about 10 times smaller than CMS data, even though the K-factor is about 2.8.
- The p_T spectrum of the J/ψ pair:

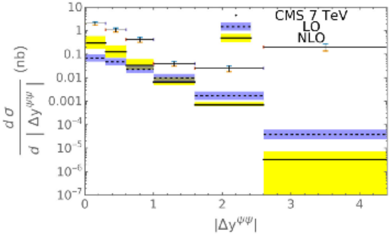


- The CSM predictions lie below the CMS data in the whole $p_T^{\psi\psi}$ range.

CS NLO(α_s) @ CMS 7 TeV II

- The $m^{\psi\psi}$ (left) and $|\Delta y^{\psi\psi}|$ (right) distributions of J/ψ pair:





- The CS contribution at QCD NLO can describe none of the data in experimental bins too.
- The NLO QCD corrections improve theoretical predictions in $m^{\psi\psi} < 13$ GeV and $|\Delta y^{\psi\psi}| < 1$ regions.
- In the residual bins, NLO QCD corrections turn to be negative and lead to 50% and 90% reduction in the last bin, respectively.

CS NLO(α_s) @ ATLAS 8 TeV I

- Total cross section:

$$\sigma(pp \to J/\psi J/\psi + X) = \begin{cases} 82.2 \pm 8.3\,(\text{stat}) \pm 6.3\,(\text{syst}) \pm 0.9\,(\text{BF}) \pm 1.6\,(\text{lumi})\ \text{pb}, & \text{for } |y| < 1.05, \\ 78.3 \pm 9.2\,(\text{stat}) \pm 6.6\,(\text{syst}) \pm 0.9\,(\text{BF}) \pm 1.5\,(\text{lumi})\ \text{pb}, & \text{for } 1.05 \le |y| < 2.1. \end{cases}$$

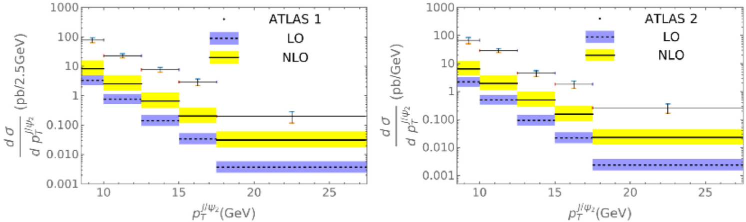
$$\sigma_{\text{CS}}^{\text{LO}} = \begin{cases} 2.97^{+1.42}_{-0.96}\ \text{pb}, & \text{for} |y(J/\psi_2)| < 1.05 \\ 1.98^{+0.97}_{-0.65}\ \text{pb}, & \text{for} 1.05 < |y(J/\psi_2)| < 2.1 \end{cases}$$

$$\sigma_{\text{CS}}^{\text{NLO}} = \begin{cases} 8.61^{+7.87}_{-3.56}\ \text{pb}, & \text{for} |y(J/\psi_2)| < 1.05 \\ 6.64^{+6.36}_{-2.82}\ \text{pb}, & \text{for} 1.05 < |y(J/\psi_2)| < 2.1 \end{cases}$$

- The K-factor are about 2.9 and 3.4 in central and non-central regions.
- However, the same as CMS case, CS contributions are still about 1 order of magnitude smaller than ATLAS measurements.

CS NLO(α_s) @ ATLAS 8 TeV II

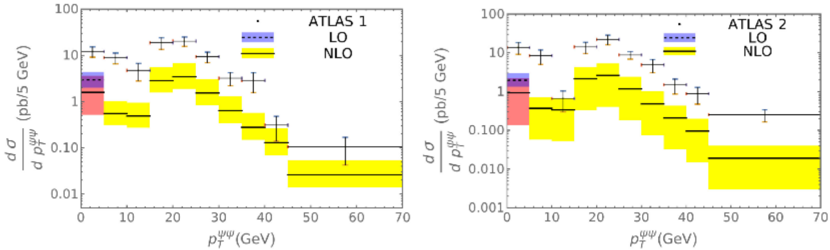
- The p_T distribution of the soft J/ψ in central (left) and forward (right) y^{ψ_2} regions:



- The LO predictions systematically underestimate the ATLAS measurements by upto 2 orders of magnitude.
- The K-factors range from 2.5 and 2.9 in the first bins to 8.5 and 9.8 in the last bins.

CS NLO(α_s) @ ATLAS 8 TeV III

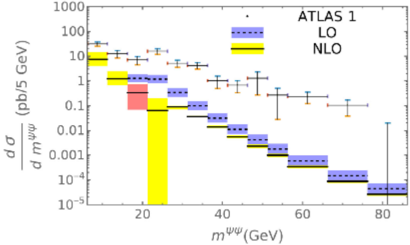
- The p_T distribution of J/ψ pair in central (left) and forward (right) y^{ψ_2} regions:

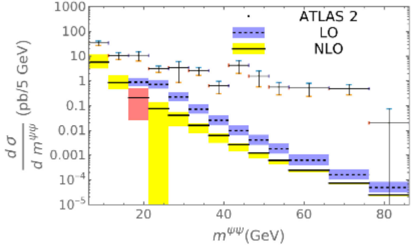


- The CS predictions at QCD NLO nicely reproduce the shapes of $p_T^{\psi\psi}$ distribution similar to CMS case.
- Incidentally, there is agreement within errors in the last two and third bins of the central and forward y^{ψ_2} regions, respectively.

CS NLO(α_s) @ ATLAS 8 TeV IV

- The J/ψ pair invariant mass distributions in central (left) and forward (right) y^{ψ_2} regions:





- The LO predictions show up from the third bin due to kinematics cut, and that is why the NLO results are negative in the third bin.
- From the fourth bins onward, the NLO QCD corrections are negative, ranging from $-95\%(-90\%)$ in the fourth bin to $-40\%(-50\%)$ in the last bins of central (forward) y^{ψ_2} region.

Conclusion and summary

1. The NLO QCD corrections are important to understand the mechanism of J/ψ pair hadroproduction.
2. In LHCb case, the CS contribution can account for most of the experimental measurements although some resummation techniques are needed to improve theoretical predictions.
3. In CMS and ATLAS cases, the CS contribution are far below the experimental measurements, which indicates the CO contribution may play an important role to fill the large discrepancies.
4. Much more theoretical effort are still needed to explain the experimental measurements.

Thank you!