$b \to s\nu\bar{\nu}$ 衰变1-Loop过程的新物理研究

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研究背景

近年来，寻找出超出标准模型的新物理(New Physics, NP)是粒子物理领域的主要任务之一。到目前为止，实验上仍然没有直接探测到新物理信号。而B介子稀有衰变被认为是寻找新物理的理想场所。最近，Belle II合作组报告了$B^+ \to K^+\nu\bar{\nu}$分支比的首次测量值[1]，

$$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{\rm exp} = (2.3 \pm 0.7) \times 10^{-5}\,,$$
$$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{\rm SM} = (4.43 \pm 0.31) \times 10^{-6}$$

与标准模型预言值[2]相差2.8σ。在夸克水平上，$B^+ \to K^+\nu\bar{\nu}$衰变与$B^0 \to K^{*0}\nu\bar{\nu}$衰变都是由$b \to s\nu\bar{\nu}$诱导，Belle测量得到[3]

$$\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\rm exp} < 1.8 \times 10^{-5},$$
$$\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\rm SM} = (9.47 \pm 1.40) \times 10^{-6}[2]\text{。}$$

[1] Belle-II Collaboration, I. Adachi et al., Evidence for $B^+ \to K^+\nu\bar{\nu}$ Decays, arXiv:2311.14647.
[2] Belle Collaboration, J. Grygier et al., Search for $B \to h\nu\bar{\nu}$ decays with semileptonic tagging at Belle, Phys. Rev. D 96 (2017), no. 9 091101, [arXiv:1702.03224]. [Addendum: Phys. Rev. D 97, 099902 (2018)].
[3] D. Bečirević, G. Piazza, and O. Sumensari, Revisiting $B^0 \to K^{(*)}\nu\bar{\nu}$ decays in the Standard Model and beyond, Eur. Phys. J. C 83 (2023), no. 3 252, [arXiv:2301.06990].

研究背景

定义

$$R \equiv \frac{\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})}{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})},$$

可以得到目前R值的实验值上限及标准模型理论预言值如下：

$$R_{\rm exp} = 0.50 \pm 0.26,$$

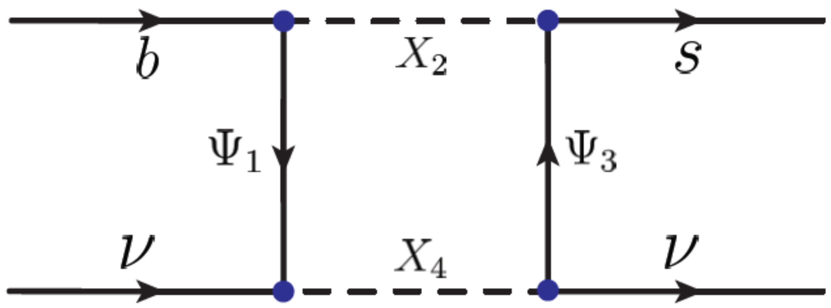
$$R_{\rm SM} = 2.14 \pm 0.35。$$

可以看出实验值明显小于标准模型预测，我们认为其中存在新物理贡献。

研究思路

本课题将对$b \to s\nu\bar{\nu}$衰变过程的单圈(1-loop)图做出系统的研究。在$b \to s\nu\bar{\nu}$衰变的单圈过程中引入新物理粒子，系统地给所有新物理粒子的贡献。

- 有效理论
- 筛选最小模型
- 其他新物理过程限制
- 结论分析



(a)

有效理论

假设中微子是左手的，则 $b \to s\nu\bar{\nu}$ 衰变过程的低能有效拉式量一般写为

$$\mathcal{L}_{\rm eff}^{b\to s\nu\bar{\nu}} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^{*} \frac{\alpha}{4\pi} \sum_{\ell} \left(C_L^{\nu_\ell}\mathcal{O}_L^{\nu_\ell} + C_R^{\nu_\ell}\mathcal{O}_R^{\nu_\ell} \right) + \text{h.c.}\,,$$

其中

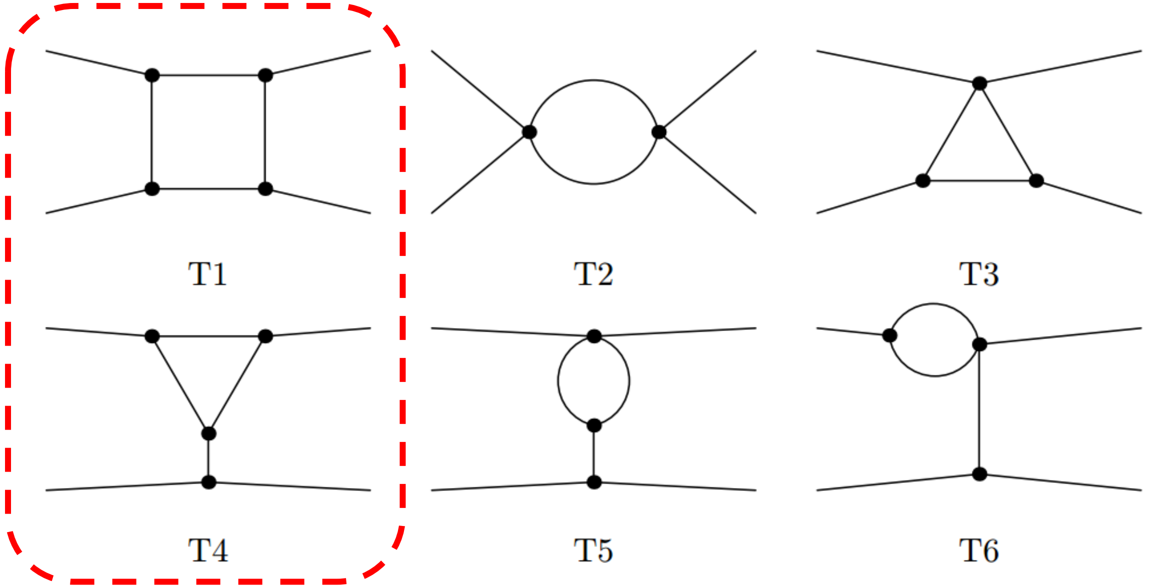
$$\mathcal{O}_L^{\nu_\ell} = (\bar{s}\gamma_\mu P_L b)(\bar{\nu}_\ell\gamma^\mu(1-\gamma_5)\nu_\ell)\,, \qquad \mathcal{O}_R^{\nu_\ell} = (\bar{s}\gamma_\mu P_R b)(\bar{\nu}_\ell\gamma^\mu(1-\gamma_5)\nu_\ell)\,.$$

这里G_F是费米耦合常数， $\alpha = e^2/(4\pi)$是精细结构常数， $C_{L,R}^{\nu_l}$为新物理理算符贡献对应的Wilson系数，

V_{tb}和V_{ts}是Cabbibo-Kobyashi-Maskawa(CKM)矩阵元。

拓扑结构

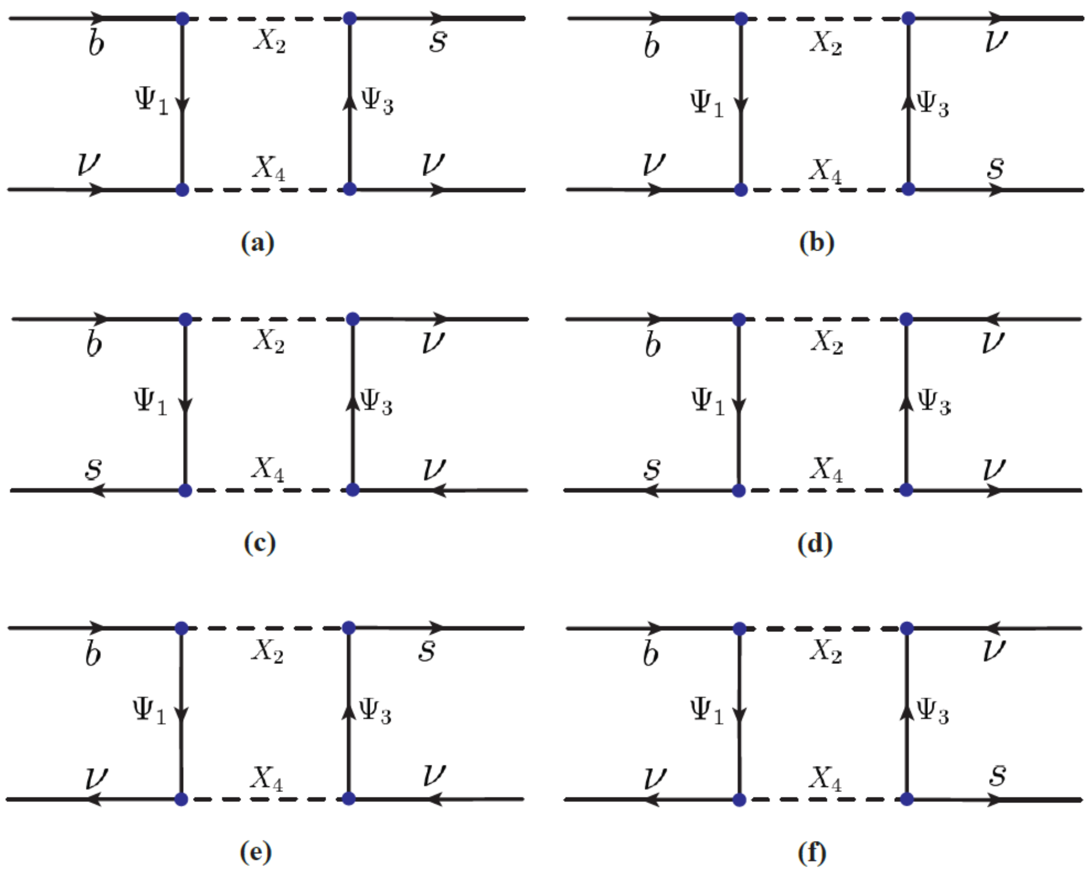
首先，我们利用FeynCalc软件包系统地给出了$b \to s\nu\bar{\nu}$衰变过程的拓扑结构，排除蝌蚪图和自能图，会得到如下图所示的6个拓扑图。进一步地，考虑到我们需要通过loop引入新的未知粒子，根据可重整化要求，只有T1和T4图两种拓扑结构才能满足条件。



外线粒子

在$b \rightarrow s\nu\bar{\nu}$衰变过程中，外线粒子的SU(2)量子数有如下两种可能。其中，第I种方案贡献的流算符是Q_{ld}，第II种方案贡献的流算符是$Q_{lq}^{(1)}$和$Q_{lq}^{(3)}$。假设box图中的新物理粒子为新的重费米子和标量（或矢量）粒子。

SU(2)	b	s	ν	$\bar{\nu}$
I	1	1	2	2
II	2	2	2	2



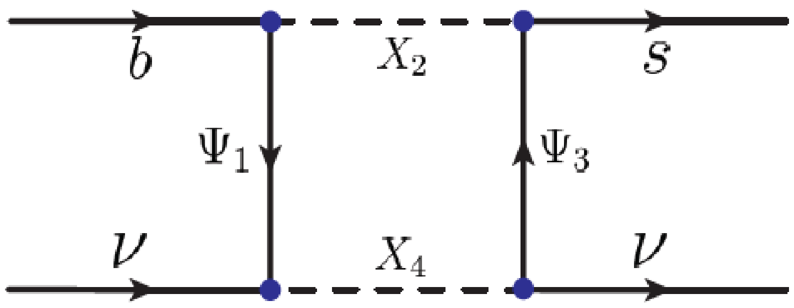
内线粒子

$SU(3)_C$: $3\otimes 3=\bar{3}\oplus 6$， $3\otimes\bar{3}=1\oplus 8$， $3\otimes 6\supset 8$，

$3\otimes\bar{6}\supset\bar{3}$， $3\otimes 8\supset 3\oplus\bar{6}$， $6\otimes 6\supset\bar{6}$，

$6\otimes\bar{6}\supset 1\oplus 8$， $6\otimes 8\supset\bar{3}\oplus 6$， $8\otimes 8\supset 1\oplus 8\oplus 8$。

$SU(2)_L$: $2\otimes 2=1\oplus 3$， $2\otimes 3\supset 2$， $3\otimes 3=1\oplus 3$。



(a)

$SU(3)_c$	Ψ_1	X_2	Ψ_3	X_4
A	1	$\bar{3}$	1	1
B	3	1	3	3
C	3	8	3	3
D	$\bar{3}$	3	$\bar{3}$	$\bar{3}$
E	8	$\bar{3}$	8	8
$SU(2)_L$	Ψ_1	X_2	Ψ_3	X_4
I	1	1	1	2
II	2	2	2	1
III	2	2	2	3
IV	3	3	3	2
Y	Ψ_1	X_2	Ψ_3	X_4
	α	$\alpha+1/3$	α	$\alpha-1/2$

内线粒子

对于引入的新物理方案，需要筛选包含较少新自由度，且能够同时产生Q_{ld}和Q_{lq}两种算符的新物理模型，即最小模型。在计算中发现Q_{lq}可以分为两类，$Q_{lq}^{(1)}$和$Q_{lq}^{(3)}$。

最终，我们找到4种最小模型，分别来自(a)和(e)，(c)$_R$和(c)$_L$，(d)$_R$和(d)$_L$，以及(b)和(c)$_L$。以(a)和(e)为例，

$$\begin{aligned}
[Q_{lq}^{(1)}]_{nmji} &= (\bar\nu^n\gamma_\mu\nu^m)(\bar u^j\gamma^\mu u^i) + (\bar\nu^n\gamma_\mu\nu^m)(\bar d^j\gamma^\mu d^i) \\
&+ (\bar e^n\gamma_\mu e^m)(\bar u^j\gamma^\mu u^i) + (\bar e^n\gamma_\mu e^m)(\bar d^j\gamma^\mu d^i)\,, \\
[Q_{lq}^{(3)}]_{nmji} &= (\bar\nu^n\gamma_\mu\nu^m)(\bar u^j\gamma^\mu u^i) - (\bar\nu^n\gamma_\mu\nu^m)(\bar d^j\gamma^\mu d^i) \\
&- (\bar e^n\gamma_\mu e^m)(\bar u^j\gamma^\mu u^i) + (\bar e^n\gamma_\mu e^m)(\bar d^j\gamma^\mu d^i) \\
&+ 2(\bar\nu^n\gamma_\mu e^m)(\bar d^j\gamma^\mu u^i) + 2(\bar e^n\gamma_\mu\nu^m)(\bar u^j\gamma^\mu d^i)\,.
\end{aligned}$$

$SU(3)_c$	Ψ_1^a	X_2^a	Ψ_3^a	X_4^a	Ψ_1^e	X_2^e	Ψ_3^e	X_4^e
A	1	$\bar 3$	1	1	1	$\bar 3$	1	1
B	3	1	3	3	3	1	3	3
$SU(2)_L$	Ψ_1^a	X_2^a	Ψ_3^a	X_4^a	Ψ_1^e	X_2^e	Ψ_3^e	X_4^e
I	1	1	1	2	1	2	1	2
II	2	2	2	1	2	1	2	1
Y	Ψ_1^a	X_2^a	Ψ_3^a	X_4^a	Ψ_1^e	X_2^e	Ψ_3^e	X_4^e
	α	$\alpha+1/3$	α	$\alpha-1/2$	α'	$\alpha'-1/6$	α'	$\alpha'+1/2$

内线粒子

对比(a)图与(e)图量子数，为最小化模型参数，我们可以令$\alpha=\alpha'$，得到$\psi_1^a=\psi_1^e=\psi_3^a=\psi_3^e$。即可以用以下5个粒子产生(a)图与(e)图的贡献。

$\mathrm{SU}(3)_c$	Ψ_1^a	X_2^a	X_4^a	X_2^e	X_4^e
A	1	$\bar{3}$	1	$\bar{3}$	1
B	3	1	3	1	3
$\mathrm{SU}(2)_L$	Ψ_1^a	X_2^a	X_4^a	X_2^e	X_4^e
I	1	1	2	2	2
II	2	2	1	1	1
Y	Ψ_1^a	X_2^a	X_4^a	X_2^e	X_4^e
i	α	$\alpha+1/3$	$\alpha-1/2$	$\alpha-1/6$	$\alpha+1/2$

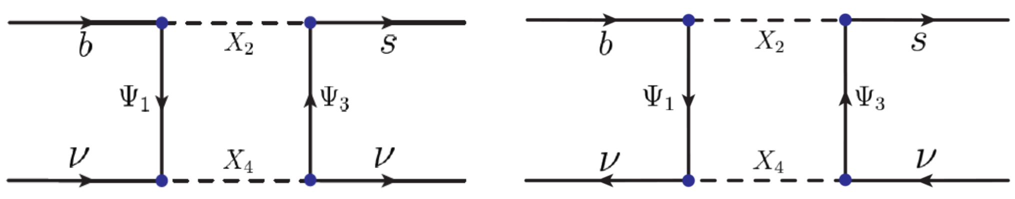
模型计算

- Scalar-rich model:

	Ψ	S_1	S_2	S_3	S_4
$\mathrm{SU}(3)_c$	1	$\bar{3}$	1	$\bar{3}$	1
$\mathrm{SU}(2)_L$	1	1	2	2	2
Y	α	$\alpha+\frac{1}{3}$	$\alpha-\frac{1}{2}$	$\alpha-\frac{1}{6}$	$\alpha+\frac{1}{2}$

(ψ, S_1, S_2)在(a)图产生Q_{ld}算符贡献，(ψ, S_3, S_4)在(e)图产生$Q_{lq}^{(1)}$算符贡献。相互作用顶点的拉氏量为

$$\mathcal{L} = g_1^i \bar{\Psi} P_R d_i S_1 + g_2^i \bar{\Psi} P_R L_i^c S_2 + g_{1'}^i \bar{\Psi} P_L Q_i S_3 + g_{2'}^i \bar{\Psi} P_L L_i S_4 + \text{h.c.}\,,$$



(a) (e)

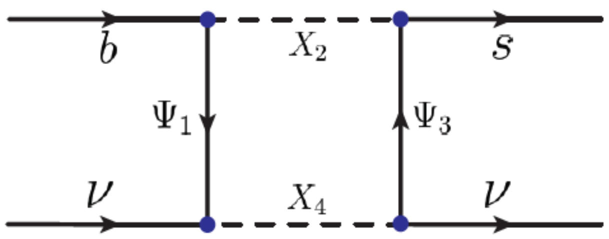
$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \frac{J_4}{4}(m_{S_1}, m_{S_2}, m_\Psi, m_\Psi)(g_1^i g_1^{j*} g_2^m g_2^{n*})\left(\bar{L}_n^{c\alpha}\gamma^\mu d_R^i\right)\left(\bar{d}_R^j \gamma_\mu L_{m\alpha}^c\right)\\ &= \frac{J_4}{4}(m_{S_1}, m_{S_2}, m_\Psi, m_\Psi)(g_1^i g_1^{j*} g_2^m g_2^{n*})\left(\bar{L}_n^{c\alpha}\gamma^\mu L_{m\alpha}^c\right)\left(\bar{d}_R^j \gamma_\mu d_R^i\right)\\ &= -\frac{J_4}{4}(m_{S_1}, m_{S_2}, m_\Psi, m_\Psi)(g_1^i g_1^{j*} g_2^m g_2^{n*})\left(\bar{L}^m\gamma^\mu L^n\right)\left(\bar{d}_R^j \gamma_\mu d_R^i\right)\\ &= -\frac{J_4}{4}(m_{S_1}, m_{S_2}, m_\Psi, m_\Psi)(g_1^i g_1^{j*} g_2^m g_2^{n*})(Q_{ld})_{mnji}\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \frac{J_4}{4}(m_{S_3}, m_{S_4}, m_\Psi, m_\Psi)(g_{1'}^i g_{1'}^{j*} g_{2'}^m g_{2'}^{n*})\left(\bar{L}_\rho^n \gamma_\mu Q^{i\alpha}\right)\left(\overline{Q}_\alpha^j \gamma^\mu L^{m\rho}\right)\\ &= \frac{J_4}{4}(m_{S_3}, m_{S_4}, m_\Psi, m_\Psi)(g_{1'}^i g_{1'}^{j*} g_{2'}^m g_{2'}^{n*})\left(\bar{L}^n \gamma_\mu L^m\right)\left(\overline{Q}^j \gamma^\mu Q^i\right)\\ &= \frac{J_4}{4}(m_{S_3}, m_{S_4}, m_\Psi, m_\Psi)(g_{1'}^i g_{1'}^{j*} g_{2'}^m g_{2'}^{n*})(Q_{\ell q}^{(1)})_{nmji}.\end{aligned}$$

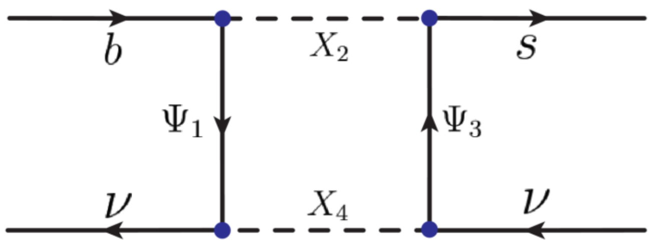
$$\begin{aligned}J_4(M_A, M_B, M_C, M_D) &\equiv \int \frac{\mathrm{d}^d k}{(2\pi)^d i}\frac{k^2}{(k^2-M_A^2)(k^2-M_B^2)(k^2-M_C^2)(k^2-M_D^2)}\\ &= \frac{1}{(4\pi)^2}\left[\frac{M_A^4 \ln\frac{M_D^2}{M_A^2}}{(M_A^2-M_B^2)(M_A^2-M_C^2)(M_A^2-M_D^2)} + \frac{M_B^4 \ln\frac{M_D^2}{M_B^2}}{(M_B^2-M_A^2)(M_B^2-M_C^2)(M_B^2-M_D^2)}\right.\\ &\quad \left. + \frac{M_C^4 \ln\frac{M_D^2}{M_C^2}}{(M_C^2-M_A^2)(M_C^2-M_B^2)(M_C^2-M_D^2)}\right],\end{aligned}$$

模型计算

- Scalar-rich model:



(a)



(e)

若考虑在费米线上插入质量，则会出现：(ψ, S_1, S_4)在(a)图产生Q_{ld}算符贡献，(ψ, S_3, S_2)在(e)图产生$Q_{lq}^{(1)}$算符贡献。拉氏量为

$$\mathcal{L} = g_1^i \bar{\Psi} P_R d_i S_1 + g_{2'}^i \bar{\Psi} P_L L_i S_4 + \mathrm{h.c.}\,,$$

得到

$$\mathcal{L}_{\mathrm{eff}} = -\frac{m_\Psi^2}{2} I_4(m_{S_1}, m_{S_4}, m_\Psi, m_\Psi)(g_1^i g_1^{j*} g_{2'}^m g_{2'}^{n*})(Q_{ld})_{nmji}$$

$$\mathcal{L}_{\mathrm{eff}} = \frac{m_\Psi^2}{2} I_4(m_{S_3}, m_{S_2}, m_\Psi, m_\Psi)(g_{1'}^i g_{1'}^{j*} g_2^m g_2^{n*})(Q_{\ell q}^{(1)})_{mnji}.$$

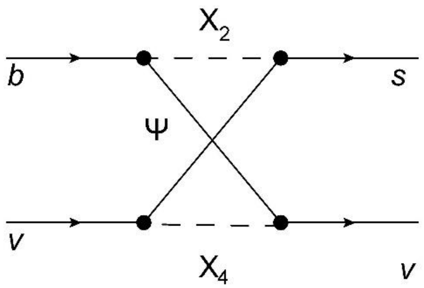
$$\begin{aligned} I_4(M_A, M_B, M_C, M_D) &\equiv \int \frac{\mathrm{d}^d k}{(2\pi)^d i} \frac{1}{(k^2 - M_A^2)(k^2 - M_B^2)(k^2 - M_C^2)(k^2 - M_D^2)} \\ &= \frac{1}{(4\pi)^2}\left[\frac{M_A^2 \ln \frac{M_D^2}{M_A^2}}{(M_A^2 - M_B^2)(M_A^2 - M_C^2)(M_A^2 - M_D^2)} + \frac{M_B^2 \ln \frac{M_D^2}{M_B^2}}{(M_B^2 - M_A^2)(M_B^2 - M_C^2)(M_B^2 - M_D^2)}\right. \\ &\quad \left. + \frac{M_C^2 \ln \frac{M_D^2}{M_C^2}}{(M_C^2 - M_A^2)(M_C^2 - M_B^2)(M_C^2 - M_D^2)}\right], \end{aligned}$$

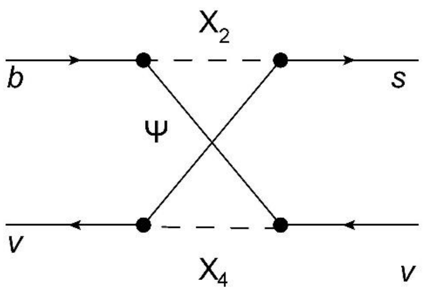
模型计算

	Ψ	S_1	S_2	S_3	S_4
$\mathrm{SU}(3)_c$	1	$\bar{3}$	1	$\bar{3}$	1
$\mathrm{SU}(2)_L$	1	1	2	2	2
Y	α	$\alpha+\frac{1}{3}$	$\alpha-\frac{1}{2}$	$\alpha-\frac{1}{6}$	$\alpha+\frac{1}{2}$

- Scalar-rich model:

若模型中的费米子为Majorana费米子($\alpha = 0$)，需要考虑(a)图和(e)图的费米子交叉图





得到

$$\begin{aligned}\mathcal{L}_{\rm eff}^{M} &= m_\Psi^2 I_4(m_{S_1}, m_{S_2}, m_\Psi, m_\Psi)(g_1^i g_1^{j*} g_2^n g_2^{m*})\left(\bar{L}_n^\alpha d_i\right)\left(\bar{d}_j L_{m\alpha}\right)\\ &= -\frac{1}{2} m_\Psi^2 I_4(m_{S_1}, m_{S_2}, m_\Psi, m_\Psi)(g_1^i g_1^{j*} g_2^n g_2^{m*})\left(\bar{L}_n^\alpha \gamma_\mu L_{m\alpha}\right)\left(\bar{d}_j \gamma^\mu d_i\right)\\ &= -\frac{1}{2} m_\Psi^2 I_4(m_{S_1}, m_{S_2}, m_\Psi, m_\Psi)(g_1^i g_1^{j*} g_2^n g_2^{m*})(Q_{ld})_{nmji}\end{aligned}$$

$$\mathcal{L}_{\rm eff} = -\frac{1}{4} J_4(m_{S_1}, m_{S_4}, m_\Psi, m_\Psi)(g_1^i g_1^{j*} g_{2'}^n g_{2'}^{m*})(Q_{\ell d})_{mnji}.$$

$$\mathcal{L}_{\rm eff} = \frac{1}{4} J_4(m_{S_2}, m_{S_3}, m_\Psi, m_\Psi)(g_{1'}^i g_{1'}^{j*} g_2^n g_2^{m*})(Q_{\ell q}^{(1)})_{nmji}.$$

$$\begin{aligned}\mathcal{L}_{\rm eff}^{M} &= m_\Psi^2 I_4(m_{S_3}, m_{S_4}, m_\Psi, m_\Psi)(g_{1'}^i g_{1'}^{j*} g_{2'}^n g_{2'}^{m*})\left(\bar{L}_\rho^{nc} Q^{i\alpha}\right)\left(\overline{Q}_\alpha^j L^{mc\rho}\right)\\ &= -\frac{1}{2} m_\Psi^2 I_4(m_{S_3}, m_{S_4}, m_\Psi, m_\Psi)(g_{1'}^i g_{1'}^{j*} g_{2'}^n g_{2'}^{m*})\left(\bar{L}_\rho^{nc} \gamma^\mu L^{mc\rho}\right)\left(\overline{Q}_\alpha^j \gamma_\mu Q^{i\alpha}\right)\\ &= \frac{1}{2} m_\Psi^2 I_4(m_{S_3}, m_{S_4}, m_\Psi, m_\Psi)(g_{1'}^i g_{1'}^{j*} g_{2'}^n g_{2'}^{m*})\left(\bar{L}^m \gamma^\mu L^n\right)\left(\overline{Q}^j \gamma_\mu Q^i\right)\\ &= \frac{1}{2} m_\Psi^2 I_4(m_{S_3}, m_{S_4}, m_\Psi, m_\Psi)(g_{1'}^i g_{1'}^{j*} g_{2'}^n g_{2'}^{m*})(Q_{\ell q}^{(1)})_{mnji}.\end{aligned}$$

模型计算

- Scalar-rich model:

综合以上所有情况，可以得到此新物理模型对Q_{ld}和$Q_{lq}^{(1)}$算符的贡献

$$\mathcal{L}_{\rm eff} = [C_{\ell q}^{(1)}]_{mnji} Q_{\ell q}^{(1)} + [C_{\ell d}]_{mnji} Q_{\ell d} \,,$$

$$\begin{aligned}
[C_{\ell d}]_{mnji} = & -\frac{(g_1^i g_1^{j*} g_2^m g_2^{n*})}{4} \left[J_4(m_{S_1}, m_{S_2}, m_\Psi, m_\Psi) + 2\eta^M m_\Psi^2 I_4(m_{S_1}, m_{S_2}, m_\Psi, m_\Psi) \right] \\
& - \frac{(g_1^i g_1^{j*} g_{2'}^n g_{2'}^{m*})}{4} \left[2m_\Psi^2 I_4(m_{S_1}, m_{S_4}, m_\Psi, m_\Psi) + \eta^M J_4(m_{S_1}, m_{S_4}, m_\Psi, m_\Psi) \right] \,,
\end{aligned}$$

$$\begin{aligned}
[C_{\ell q}^{(1)}]_{mnji} = & \frac{(g_{1'}^i g_{1'}^{j*} g_{2'}^n g_{2'}^{m*})}{4} \left[J_4(m_{S_3}, m_{S_4}, m_\Psi, m_\Psi) + 2\eta^M m_\Psi^2 I_4(m_{S_3}, m_{S_4}, m_\Psi, m_\Psi) \right] \\
& + \frac{g_{1'}^i g_{1'}^{j*} g_2^m g_2^{n*}}{4} \left[2m_\Psi^2 I_4(m_{S_2}, m_{S_3}, m_\Psi, m_\Psi) + \eta^M J_4(m_{S_2}, m_{S_3}, m_\Psi, m_\Psi) \right] \,.
\end{aligned}$$

其中，当ψ为Majorana费米子时，$\eta^M = 1$， ψ非Majorana费米子时，$\eta^M = 0$。

模型计算

- Fermion-rich model:

	Ψ_1	S	Ψ_2	Ψ_3
$\mathrm{SU}(3)_c$ (A)	1	$\bar{3}$	$\bar{3}$	$\bar{3}$
$\mathrm{SU}(3)_c$ (B)	3	1	1	3
$\mathrm{SU}(2)_L$	1	1	2	2
Y (cc)	α	$\alpha+1/3$	$\alpha-1/6$	$\alpha+1/2$

与Scalar-rich model相似，此模型(B)中要考虑标量S为实标量($\alpha=-\frac{1}{3}$)的交叉图贡献。

$$\mathcal{L}_{\text{eff}} = [C_{\ell q}^{(1)}]_{mnji} Q_{\ell q}^{(1)} + [C_{\ell d}]_{mnji} Q_{\ell d}\,,$$

$$[C_{\ell d}]_{mnji} = \frac{(g_1^i g_1^{j*} g_2^m g_2^{n*})}{4} J_4(m_{\Psi_1}, m_{\Psi_2}, m_S, m_S)\left(1-\eta^M\right)\,,$$

$$[C_{\ell q}^{(1)}]_{mnji} = \frac{(g_{1'}^i g_{1'}^{j*} g_2^m g_2^{n*})}{4} J_4(m_{\Psi_2}, m_{\Psi_3}, m_S, m_S)\left(1-\eta^M\right)\,.$$

模型限制

- $b \to s\nu_l\bar{\nu}_l\ (l = e, \mu, \tau)$：

低能有效拉氏量

$$\mathcal{L} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \frac{\alpha}{4\pi} \sum_\ell \left(C_L^{\nu_\ell}\mathcal{O}_L^{\nu_\ell} + C_R^{\nu_\ell}\mathcal{O}_R^{\nu_\ell}\right) + h.c.,$$

$$\mathcal{O}_L^{\nu_\ell} = (\bar{s}\gamma_\mu P_L b)(\bar{\nu}_\ell\gamma_\mu(1-\gamma_5)\nu_\ell)\,, \quad \mathcal{O}_R^{\nu_\ell} = (\bar{s}\gamma_\mu P_R b)(\bar{\nu}_\ell\gamma_\mu(1-\gamma_5)\nu_\ell)\,.$$

分支比[2]

$$\mathcal{B}(B^+ \to K^+\nu\bar{\nu}) = 3.46 \times 10^{-8} \sum_\ell |C_L^{\nu_\ell} + C_R^{\nu_\ell}|^2\,,$$

$$\mathcal{B}(B^0 \to K^*\nu\bar{\nu}) = 6.84 \times 10^{-8} \sum_\ell |C_L^{\nu_\ell} - C_R^{\nu_\ell}|^2 + 1.36 \times 10^{-8} \sum_\ell |C_L^{\nu_\ell} + C_R^{\nu_\ell}|^2\,.$$

Belle II合作组给出了$B^+ \to K^+\nu\bar{\nu}$分支比的测量值[1]，

$$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{\text{exp}} = (2.3 \pm 0.7) \times 10^{-5}\,,$$

对$B^0 \to K^{*0}\nu\bar{\nu}$衰变，Belle测量得到[3]

$$\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\text{exp}} < 1.8 \times 10^{-5}。$$

[1] Belle-II Collaboration, I. Adachi et al., Evidence for $B^+ \to K^+\nu\bar{\nu}$ Decays, arXiv:2311.14647.
[2] F.-Z. Chen, Q. Wen, and F. Xu, Correlating$B \to K^{(*)}\nu\bar{\nu}$and flavor anomalies in SMEFT, arXiv: 2401.11552 [hep-ph].
[3] D. Bečirević, G. Piazza, and O. Sumensari, Revisiting $B^0 \to K^{(*)}\nu\bar{\nu}$ decays in the Standard Model and beyond, Eur. Phys. J. C 83 (2023), no. 3 252, [arXiv:2301.06990].

模型限制

- $b \to s\nu_l\bar{\nu}_l\ (l = e, \mu, \tau)$:

拉氏量中的$O_L^{\nu_l}$贡献包含SM和NP两部分，即$C_L^{\nu_l} = C_{L,SM}^{\nu_l} + C_{L,NP}^{\nu_l}$。其中$C_{L,SM}^{\nu_l} = -6.32(7)$ [4-6]。对比新物理模型的拉氏量表达式，得到模型中Wilson系数 $C_{lq}^{(1)}$和C_{ld}与$C_L^{\nu_l}$和$C_R^{\nu_l}$的关系，

$$C_{lq}^{(1)} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \frac{\alpha}{4\pi} \cdot 2C_L^{\nu_l},$$

$$C_{ld} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \frac{\alpha}{4\pi} \cdot 2C_R^{\nu_l},$$

通过分支比的实验值可以限制$C_L^{\nu_l}$和$C_R^{\nu_l}$，进而得到$C_{lq}^{(1)}$和C_{ld}中的耦合系数g 以及NP粒子的质量的限制。为初步得到耦合系数g 的限制，我们参考ATLAS合作组给出的质量限制[7]，$m_\psi = 600\text{GeV}$, $m_{S_i}(i = 1,2,3,4) = 700\text{GeV}$。得到的耦合系数$g$ 最小在$[3, 4]$。

[4] A. J. Buras, J. Girrbach-Noe, C. Niehoff, and D. M. Straub, $B \to K^{(*)}\nu\bar{\nu}$ decays in the Standard Model and beyond, JHEP 02 (2015) 184.
[5] J. Brod, M. Gorbahn, and E. Stamou, Updated Standard Model Prediction for $K\to \pi\nu\bar{\nu}$ and ϵ_K , PoS BEAUTY2020 (2021) 056.
[6] R. Bause, H. Gisbert, and G. Hiller, Implications of an enhanced $B \to K\nu\bar{\nu}$ branching ratio, Phys. Rev. D 109 no. 1, (2024) 015006.
[7] M. Aaboud et al. [ATLAS], Search for dark matter and other new phenomena in events with an energetic jet and large missing transverse momentum using the ATLAS detector, JHEP01(2018), 126.

模型限制

- 所有数值分析计算中涉及的输入参数：

$G_F = 1.1663788 \times 10^{-5}\ \mathrm{GeV}^{-2}$	$f_{B_s} = 227.7 \pm 4.5\ \mathrm{MeV}$
$m_W = 80.377 \pm 0.012\ \mathrm{GeV}$	$f_{B_c^+} = 427 \pm 6\ \mathrm{MeV}$
$m_\tau = 1776.86 \pm 0.12\ \mathrm{MeV}$	$f_{B^+} = 190.0 \pm 1.3\ \mathrm{MeV}$
$m_{K^0} = 497.611 \pm 0.013\ \mathrm{MeV}$	$f_{D_s^+} = 249.9 \pm 0.5\ \mathrm{MeV}$
$m_{K^+} = 493.677 \pm 0.016\ \mathrm{MeV}$	$\tau_{B_s} = 1.527 \pm 0.011\ \mathrm{ps}$
$m_{K^{*0}} = 895.55 \pm 0.20\ \mathrm{MeV}$	$\tau_{B^0} = 1.519 \pm 0.004\ \mathrm{ps}$
$m_{B_s} = 5366.77 \pm 0.24\ \mathrm{MeV}$	$\tau_{B^+} = 1.638 \pm 0.004\ \mathrm{ps}$
$m_{B^+} = 5279.34 \pm 0.12\ \mathrm{MeV}$	$\tau_{B_c^+} = 0.510 \pm 0.009\ \mathrm{ps}$
$m_{B_c^+} = 6274.47 \pm 0.32\ \mathrm{MeV}$	$\tau_{D_s^+} = 0.504 \pm 0.004\ \mathrm{ps}$
$m_{B^0} = 5279.58 \pm 0.17\ \mathrm{MeV}$	$\tau_{D^+} = 1.033 \pm 0.005\ \mathrm{ps}$
$m_{D^0} = 1864.84 \pm 0.05\ \mathrm{MeV}$	$\tau_{D^0} = 0.4103 \pm 0.0010\ \mathrm{ps}$
$m_{D^+} = 1869.66 \pm 0.05\ \mathrm{MeV}$	$\lvert V_{ub}\rvert = (3.70 \pm 0.11) \times 10^{-3}$
$m_{D_s^+} = 1968.35 \pm 0.07\ \mathrm{MeV}$	$\lvert V_{cb}\rvert = (42.22 \pm 0.51) \times 10^{-3}$
$m_{\pi^0} = 134.9768 \pm 0.0005$	$\lvert V_{tb}\rvert = 1.014 \pm 0.029$
$m_{\pi^+} = 139.57039 \pm 0.00018$	$\lvert V_{ts}\rvert = (41.5 \pm 0.9) \times 10^{-3}$
$m_b(m_b) = 4.18 \pm 0.03\ \mathrm{GeV}$	$\lvert V_{cs}\rvert = 0.975 \pm 0.006$
$m_s(2\ \mathrm{GeV}) = 93.4^{+8.6}_{-3.4}\ \mathrm{MeV}$	$A_+^{D^0\pi^0} = 0.9 \times 10^{-8}$
$\alpha(4.18\ \mathrm{GeV}) = 1/132.1$	$A_+^{D^+\pi^+} = 3.6 \times 10^{-8}$
$\alpha(2\ \mathrm{GeV}) = 1/133.2$	$A_+^{D_s^+K^+} = 0.7 \times 10^{-8}$

Particle Data Group Collaboration, R. L. Workman et al., Review of Particle Physics, PTEP 2022 (2022) 083C01.

模型限制

- $B_s \to \tau^+\tau^-$：

低能有效拉氏量 $$\mathcal{L}_{\text{eff}}^{b\to s\ell^+\ell^-} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \frac{\alpha}{4\pi} \sum_{i=9}^{10} (C_i^\ell \mathcal{O}_i^\ell + C_i^{\ell\prime} \mathcal{O}_i^{\ell\prime}) + \text{h.c.}$$

$$\mathcal{O}_9^{\ell(\prime)} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell)\,, \qquad \mathcal{O}_{10}^{\ell(\prime)} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)\,,$$

分支比

$$\mathcal{B}(B_s \to \tau^+\tau^-) = \tau_{B_s} \frac{G_F^2 \alpha^2}{16\pi^3} |V_{ts}V_{tb}^*|^2 f_{B_s}^2 m_{B_s} \sqrt{1 - \frac{4m_\tau^2}{m_{B_s}^2}}\, m_\tau^2 |C_{10}^\tau - C_{10}^{\tau\prime}|^2\,,$$

在标准模型中，O_9^l和O_{10}^l的贡献是非零的，有[2]

$$C_{9,\text{SM}}^\ell\big|_{\text{NNLL}} = 4.2607\,, \quad C_{10,\text{SM}}^\ell\big|_{\text{NNLL}} = -4.2453\,.$$

LHCb合作组给出了$B_S \to \tau^+\tau^-$分支比的上限[1]，

$$\mathcal{B}(B_s \to \tau^+\tau^-)_{\text{exp}} < 6.8 \times 10^{-3}$$

此实验值给出的参数约束极弱，对模型起不到很好的限制作用。 $\bar{B} \to K l^+ l^-$衰变过程与其类似，不再单独讨论。

[1] LHCb Collaboration, R. Aaij et al., Search for the decays $B_s^0 \to \tau^+\tau^-$ and $B^0 \to \tau^+\tau^-$, Phys. Rev. Lett. 118 no. 25, (2017) 251802, arXiv:1703.02508 [hep-ex].

[2] W.-S. Hou, M. Kohda, and F. Xu, Rates and asymmetries of $B \to \pi l^+ l^-$ decays, Phys. Rev. D 90 no. 1, (2014) 013002, arXiv:1403.7410 [hep-ph].

模型限制

- $\Delta \boldsymbol{F} = \mathbf{2}$:

$\Delta F = 2$过程的哈密顿量为[1] $$\mathcal{H}^{ij}_{|\Delta F|=2} = \sum_a C^{ij}_a(\mu) Q^{ij}_a + h.c.\,,$$

式中$i, j = s, b$ 时为 $B_s - \bar{B}_s$ Mixing，四费米子算符定义[1,2]

$$\begin{aligned}
Q_1^{\mathrm{VLL}} &= (\overline{d_i}\gamma_\mu P_L d_j)(\overline{d_i}\gamma^\mu P_L d_j)\,,\\
Q_1^{\mathrm{LR}} &= (\overline{d_i}\gamma_\mu P_L d_j)(\overline{d_i}\gamma^\mu P_R d_j)\,,\\
Q_2^{\mathrm{LR}} &= (\overline{d_i} P_L d_j)(\overline{d_i} P_R d_j)\,,\\
Q_1^{\mathrm{SLL}} &= (\overline{d_i} P_L d_j)(\overline{d_i} P_L d_j)\,,\\
Q_2^{\mathrm{SLL}} &= -(\overline{d_i}\sigma_{\mu\nu} P_L d_j)(\overline{d_i}\sigma^{\mu\nu} P_L d_j)\,,
\end{aligned}$$

$$\begin{aligned}
Q_1^{ij} &= Q^{ij}_{\mathrm{VLL}},\\
Q_2^{ij} &= Q^{ij}_{\mathrm{SLL},1},\\
Q_4^{ij} &= Q^{ij}_{\mathrm{LR},2},\\
Q_3^{ij} &= [\bar{d}_i^\alpha P_L d_j^\beta][\bar{d}_i^\beta P_L d_j^\alpha] = -\frac{1}{2}Q^{ij}_{\mathrm{SLL},1} + \frac{1}{8}Q^{ij}_{\mathrm{SLL},2},\\
Q_5^{ij} &= [\bar{d}_i^\alpha P_L d_j^\beta][\bar{d}_i^\beta P_R d_j^\alpha] = -\frac{1}{2}Q^{ij}_{\mathrm{LR},1},
\end{aligned}$$

得到$\Delta F = 2$过程的Wilson系数C与各模型耦合系数g 以及NP粒子的质量的关系。

B_L^0与B_H^0间的质量差$\Delta M_0 = 2|M_{12}^0|$，而M_{12}^0与$\Delta F = 2$过程的哈密顿量相关：

$$M_{12}^{ij} \equiv [M_{12}^{ij}]_{\mathrm{SM}} + [M_{12}^{ij}]_{\mathrm{BSM}} = \frac{\langle B_q|\mathcal{H}^{ij}_{|\Delta F|=2}|\bar{B}_q\rangle}{2M_{B_q}}\,,$$

[1] J. Aebischer, C. Bobeth, A. J. Buras, and J. Kumar, SMEFT ATLAS of $\Delta F = 2$ transitions, JHEP 12 (2020) 187, arXiv:2009.07276 [hep-ph].
[2] A. J. Buras, S. Jager, and J. Urban, Master formulae for Delta F=2 NLO QCD factors in the standard model and beyond, Nucl. Phys. B 605 (2001) 600–624, arXiv:hep-ph/0102316.

模型限制

- $\Delta F = 2$:

HFLAV给出的B_L^0与B_H^0间的质量差$\Delta M_0 = (3.334 \pm 0.013) \times 10^{-10}\mathrm{MeV}$。在SM中[3]，

$$[\mathcal{H}^{ij}_{|\Delta F|=2}]_{\rm SM} = [C^{ij}_{\rm VLL}(\mu)]_{\rm SM} Q^{ij}_{\rm VLL} + h.c.,$$

$$[C^{ij}_{\rm VLL}(\mu_{ew})]_{\rm SM} = \frac{G_F^2}{4\pi^2} m_W^2 (V^*_{ti} V_{tj}) \hat{\eta}_B S_0(x_t),$$

$$S_0(x) = \frac{x(4 - 11x + x^2)}{4(x-1)^2} + \frac{3x^3 \ln x}{2(x-1)^3} = 2.35,$$

式中，$x_t = m_t^2/m_W^2$， $\hat{\eta}_B = 0.8393 \pm 0.0034$

对于BSM部分，

$$[M^{ij}_{12}]_{\rm BSM} = \frac{1}{2M_{B_q}} \sum_a C^{ij}_a(\mu) \langle Q^{ij}_a \rangle(\mu) \, .$$

$$\langle Q^{ij}_a \rangle(\mu) \equiv \langle B_q | Q^{ij}_a | \bar{B}_q \rangle(\mu).$$

ij	$\mu_{\rm had}$ [GeV]	N_f	$\langle Q_1^{ij}\rangle$ [GeV^4]	$\langle Q_2^{ij}\rangle$ [GeV^4]	$\langle Q_3^{ij}\rangle$ [GeV^4]	$\langle Q_4^{ij}\rangle$ [GeV^4]	$\langle Q_5^{ij}\rangle$ [GeV^4]
sd	3.0	3	0.002156(34)	−0.0420(16)	0.0128(6)	0.0930(30)	0.0241(14)
cu	3.0	4	0.0806(56)	−0.1442(72)	0.0452(31)	0.2745(140)	0.1035(74)
db	4.16	5	0.56(2)	−0.53(3)	0.106(8)	0.96(5)	0.51(2)
sb	4.16	5	0.86(3)	−0.85(5)	0.174(11)	1.40(6)	0.74(3)

Table 2: The values of the matrix elements in the SUSY basis in the $\overline{\rm MS}$-NDR scheme at the low-energy scale $\mu_{\rm had}$ for number of flavours N_f.

其中强子矩阵元具体数值参考右表，因此我们可以得到其Wilson系数与模型的Wilson系数关系，进而对耦合系数g做出限制。初步计算分析后，发现$B_s - \bar{B}_s$ Mixing过程约束较弱。

[3] A. Lenz, U. Nierste, J. Charles, S. Descotes-Genon, A. Jantsch, C. Kaufhold, H. Lacker, S. Monteil, V. Niess, and S. T'Jampens, Anatomy of New Physics in $B - \bar{B}$ mixing, Phys. Rev. D 83 (2011) 036004, arXiv:1008.1593 [hep-ph].

模型限制

- $b \to s\gamma, b \to sg$:

$b \to s\gamma, sg$过程分支比[1]

$$\overline{\mathcal{B}}(b \to s\gamma)_{E_\gamma > E_0 = 1.6\,\mathrm{GeV}} = \overline{\mathcal{B}}(b \to s\gamma)^{\mathrm{SM}} + \delta\overline{\mathcal{B}}(b \to s\gamma)$$

$$\begin{aligned}\delta\overline{\mathcal{B}}(b \to s\gamma) = 10^{-4} \times \left(\frac{r_V}{0.9626}\right) \mathrm{Re}\Big[& -8.100\,\mathcal{C}_7^{\mathrm{LO}} - 2.509\,\mathcal{C}_8^{\mathrm{LO}} + 2.767\,\mathcal{C}_7^{\mathrm{LO}}\mathcal{C}_8^{\mathrm{LO}*} \\ & + 5.348\,\left|\mathcal{C}_7^{\mathrm{LO}}\right|^2 + 0.890\,\left|\mathcal{C}_8^{\mathrm{LO}}\right|^2 - 0.085\,\mathcal{C}_7^{\mathrm{NLO}} - 0.025\,\mathcal{C}_8^{\mathrm{NLO}} \\ & + 0.095\,\mathcal{C}_7^{\mathrm{LO}}\mathcal{C}_7^{\mathrm{NLO}*} + 0.008\,\mathcal{C}_8^{\mathrm{LO}}\mathcal{C}_8^{\mathrm{NLO}*} + 0.028\,\left(\mathcal{C}_7^{\mathrm{LO}}\mathcal{C}_8^{\mathrm{NLO}*} + \mathcal{C}_7^{\mathrm{NLO}}\mathcal{C}_8^{\mathrm{LO}*}\right)\Big]\end{aligned}$$

其中， $r_V = \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2$

实验值[2] $\mathcal{B}(B \to X_s\gamma)^{\mathrm{exp}}_{E_\gamma > 1.6\,\mathrm{GeV}} = \left(3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03\right) \times 10^{-4}$,

标准模型理论值[3,4]

$$\mathcal{B}(B \to X_s\gamma) = (2.98 \pm 0.26) \times 10^{-4}$$

[1] T. Enomoto and R. Watanabe, Flavor constraints on the Two Higgs Doublet Models of Z symmetric and aligned types, JHEP 05 (2016) 002.
[2] E. Lunghi and J. Matias, Huge right-handed current effects in $B \to \pi K^*$(K pi)$l^+ l^-$ in supersymmetry, JHEP 04 (2007) 058.
[3] M. Misiak et al., Estimate of $B(\bar{B} \to X_s\gamma)$ at $\mathcal{O}(\alpha_s^2)$, arXiv:hep-ph/0609232.
[4] T. Becher and M. Neubert, Analysis of $Br(\bar{B} \to X_s\gamma)$ at NNLO with a Cut on Photon Energy, arXiv:hep-ph/0610067.

模型限制

- $b \to s\gamma, b \to sg$:

在scalar-rich model中，计算得到的C_7和C_8如下

$$C_7 = -\frac{1}{2m_S^2}\left\{\left(g^i_{1'}g^{j*}_{1'} + g^i_1 g^{j*}_1\frac{m_s}{m_b}\right)\left(Q_S\widetilde{F}_7(x) + Q_\Psi F_7(x)\right) - 4g^{j*}_{1'}g^i_1\frac{m_\Psi}{m_b}\left(Q_S\widetilde{G}_7(x) + Q_\Psi G_7(x)\right)\right\},$$

$$C'_7 = -\frac{1}{2m_S^2}\left\{\left(g^i_{1'}g^{j*}_{1'}\frac{m_s}{m_b} + g^i_1 g^{j*}_1\right)\left(Q_S\widetilde{F}_7(x) + Q_\Psi F_7(x)\right) - 4g^i_{1'}g^{j*}_1\frac{m_\Psi}{m_b}\left(Q_S\widetilde{G}_7(x) + Q_\Psi G_7(x)\right)\right\},$$

$$C_8 = -\frac{1}{2m_S^2}\left\{\left(g^i_{1'}g^{j*}_{1'} + g^i_1 g^{j*}_1\frac{m_s}{m_b}\right)\left(\chi_S\widetilde{F}_7(x) + \chi_\Psi F_7(x)\right) - 4g^{j*}_{1'}g^i_1\frac{m_\Psi}{m_b}\left(\chi_S\widetilde{G}_7(x) + \chi_\Psi G_7(x)\right)\right\},$$

$$C'_8 = -\frac{1}{2m_S^2}\left\{\left(g^i_{1'}g^{j*}_{1'}\frac{m_s}{m_b} + g^i_1 g^{j*}_1\right)\left(\chi_S\widetilde{F}_7(x) + \chi_\Psi F_7(x)\right) - 4g^i_{1'}g^{j*}_1\frac{m_\Psi}{m_b}\left(\chi_S\widetilde{G}_7(x) + \chi_\Psi G_7(x)\right)\right\},$$

其中， $x = m_\Psi^2/m_{S_i}^2\,(i = 1,3)$，$\chi_\Psi = 0$，$\chi_S = 1$（ Ψ 是色单态，S是色三重态）。

$$F_7(x) = \frac{x^3 - 6x^2 + 6x\log x + 3x + 2}{12(x-1)^4},$$

$$\widetilde{F}_7(x) = x^{-1}F_7(x^{-1}),$$

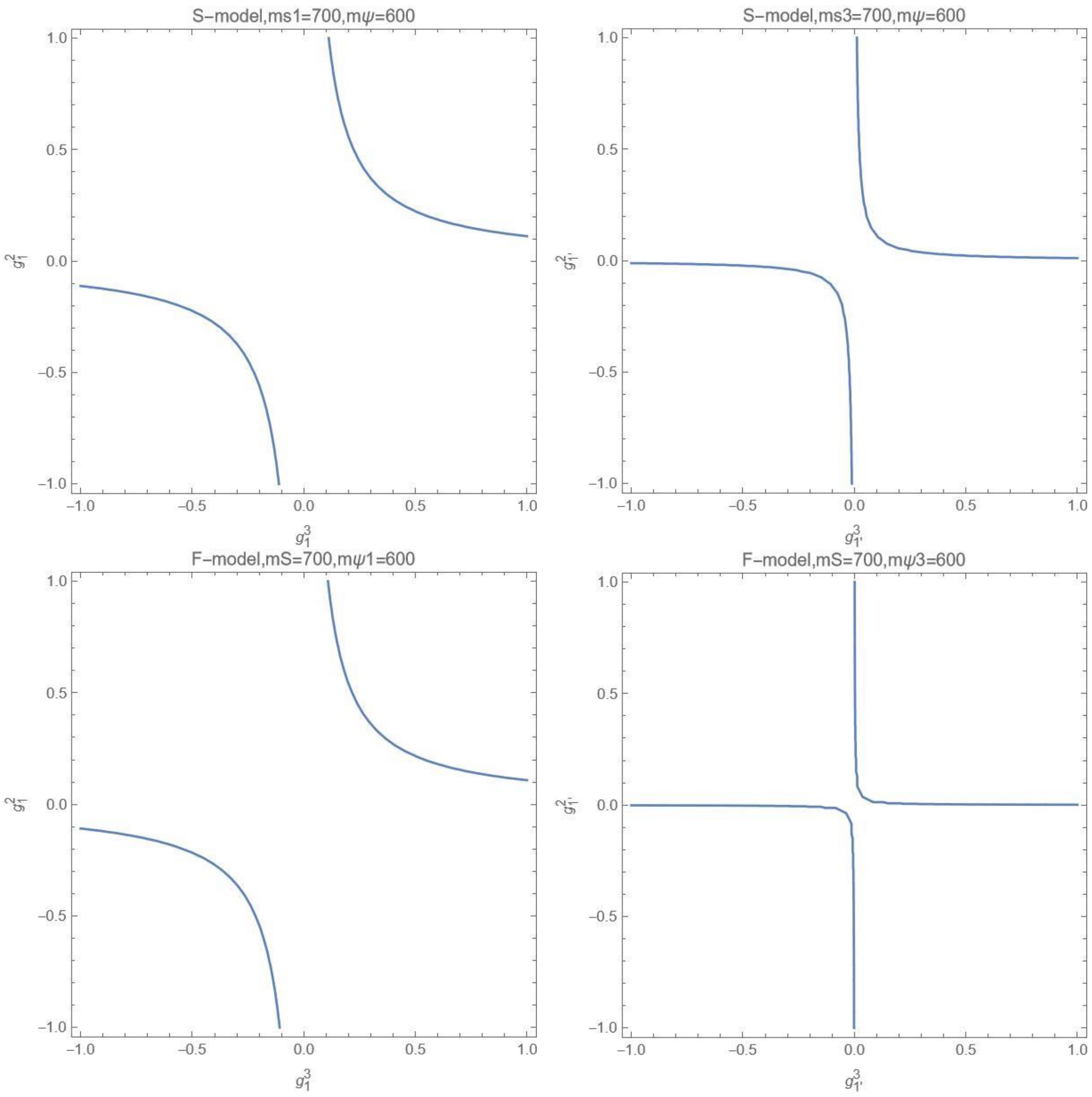
$$G_7(x) = \frac{x^2 - 4x + 3 + 2\log x}{8(x-1)^3},$$

$$\widetilde{G}_7(x) = \frac{x^2 - 2x\log x - 1}{8(x-1)^3}.$$

模型限制

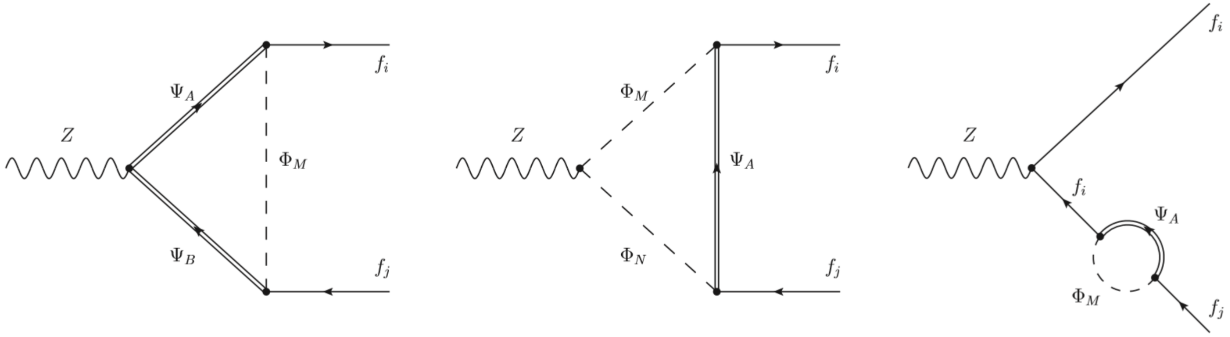
- $b \rightarrow s\gamma, b \rightarrow sg$:

各模型耦合系数g间的限制图：



模型限制

- Z couplings



定义$Z\bar{f}f$顶点的相互作用拉氏量[1]

$$\mathcal{L}_Z = \frac{g}{c_W}\bar{f}_i\gamma^\mu\left[g_L^{ij}(q^2)P_L + g_R^{ij}(q^2)P_R\right]f_j Z_\mu + h.c.,$$

$$g_{L(R)}^{ij}(q^2) = g_{(LR)}^{\text{SM}}\delta_{ij} + \Delta g_{L(R)}^{ij}(q^2)$$

其中，g是SU(2)规范耦合系数， $c_W = \cos\theta_W$ ， θ_W 即Weinberg角，q 是Z玻色子动量。我们可以将Z与NP标量粒子和费米子的相互作用顶点拉氏量写作

$$\mathcal{L}_Z = \frac{g}{c_W}Z_\mu\left(\bar{\Psi}_A\gamma^\mu\left[g_{A,B}^{\Psi,L}P_L + g_{A,B}^{\Psi,R}P_R\right]\Psi_B + g_{MN}^{\Phi}\Phi_M^\dagger i\overleftrightarrow{\partial^\mu}\Phi_N\right) + h.c.$$

对本文讨论的$b\to s\nu\bar{\nu}$ 衰变过程，Z玻色子耦合会有两种： $Z\to bs,\ Z\to ll$。

[1] P. Arnan, A. Crivellin, M. Fedele, and F. Mescia, Generic Loop Effects of New Scalars and Fermions in $\boldsymbol{b\to sl^+l^-}$, (g−2)μ and a Vector-like 4th Generation, JHEP 06 (2019) 118, arXiv:1904.05890 [hep-ph].

模型限制

- Z couplings($Z \to bs$)

在scalar-rich model中， S_1和S_3可以对此过程产生贡献。电弱破缺后，

$$
\begin{aligned}
& g_1^i \bar{\Psi} P_R d_i S_1 + g_{1'}^i \bar{\Psi} P_L Q_i S_3 \\
= & g_1^i \bar{\Psi} P_R d_i S_1 + g_{1'}^i \bar{\Psi} P_L V^\dagger u_i S_3^- - g_{1'}^i \bar{\Psi} P_L d_i S_3^+ .
\end{aligned}
$$

将S_1和S_3^+写成

$$
S = \begin{pmatrix} S_1 \\ S_3^+ \end{pmatrix}
$$

ZS^+S相互作用拉氏量

$$
L_z = \frac{g}{c_W} \hat{T}_S Z_\mu S^\dagger i \overleftrightarrow{\partial^\mu} S + \text{h.c.}
$$

$$
\hat{T}_S = \begin{pmatrix} -Q_\Phi s_W^2 & 0 \\ 0 & T_3 - Q_\Phi s_W^2 \end{pmatrix}
$$

Yukawa相互作用顶点拉氏量

$$
\mathcal{L}_{\text{int}} = \left[\bar{\Psi}_A \left(L_{AM}^b P_L b + L_{AM}^s P_L s \right) S_M + \bar{\Psi}_A \left(R_{AM}^b P_R b + R_{AM}^s P_R s \right) S_M \right].
$$

$$
\begin{aligned}
L_{AM}^b &= -g_{1'}^3 \delta_{M,2}, \quad & L_{AM}^s &= -g_{1'}^2 \delta_{M,2}, \\
R_{AM}^b &= g_1^3 \delta_{M,1}, \quad & R_{AM}^s &= g_1^2 \delta_{M,1}.
\end{aligned}
$$

模型限制

- Z couplings $(Z\to bs)$

得到

$$\Delta g_L^{sb}(q^2) = \frac{L_{AM}^{s*}L_{AM}^{b}}{16\pi^2}\Bigg[q^2\left(\frac{Q_{S_M}}{m_{S_M}^2}\widetilde{H}_Z(x_{AM}) + \frac{Q_{\Psi_R}}{m_{S_M}^2}\widetilde{F}_Z(x_{AM}) - \frac{Q_{\Psi_L}}{m_{S_M}^2}\widetilde{G}_Z(x_{AM})\right) + \frac{1}{2}Q_{S_M}H_Z(x_{AM}) + \frac{1}{2}Q_{\Psi_R}F_Z(x_{AM}) + Q_{\Psi_L}G_Z(x_{AM}) + Q_L I_Z(x_{AM})\Bigg]$$

$$\Delta g_R^{sb}(q^2) = \Delta g_L^{sb}(L\leftrightarrow R)$$

其中

$$\widetilde{H}_Z(x) = \frac{2-9x+18x^2-11x^3+6x^3\log x}{36(-1+x)^2},$$

$$\widetilde{F}_Z(x) = \frac{11-18x+9x^2-2x^3+6\log x}{18(-1+x)^4},$$

$$\widetilde{G}_Z(x) = \frac{2+3x-6x^2+x^3+6x\log x}{12(-1+x)^4},$$

$$H_Z(x,m) = \frac{(-1+x)(-1+3x)-2x^2\log x}{2(-1+x)^2} + \Delta(m),$$

$$F_Z(x,m) = \frac{-1+x^2+2(-2+x)x\log x}{2(-1+x)^2} - \Delta(m),$$

$$G_Z(x) = \frac{x(1-x+\log x)}{(-1+x)^2},$$

$$I_Z(x) = \frac{1-4x+3x^2-2x^2\log x}{4(-1+x)^2} + \frac{\Delta(m)}{2},$$

$$\boxed{\Delta(m) \equiv \frac{1}{\epsilon} + \log\left[\frac{\mu^2}{m^2}\right].}$$

$$Q_{S_1} = -Q_\Phi s_W^2,\quad Q_{S_2} = \frac{1}{2} - Q_\Phi s_W^2,$$

$$Q_{\Psi_L} = Q_{\Psi_R} = -Q_\Psi s_W^2,\quad Q_L = -\frac{1}{2} - Q_d s_W^2,\quad Q_R = -Q_d s_W^2\,.$$

模型限制

- Z couplings$(Z \to bs)$

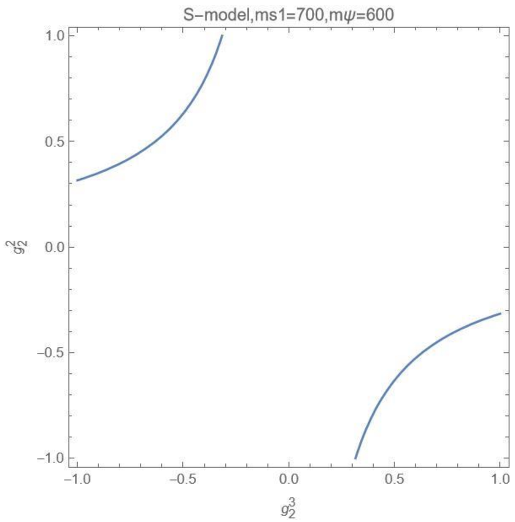
实验给出Z couplings的有关NP贡献[1-2]：

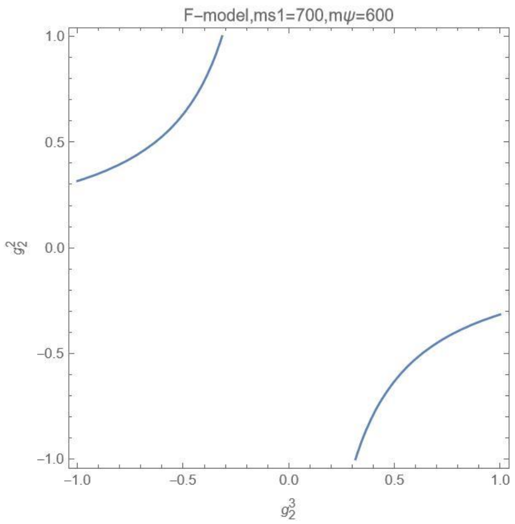
$$\boxed{\Delta g_{\mu_L}(m_Z^2) = -(0.1 \pm 1.1) \times 10^{-3}\,,} \qquad \Delta g_{\mu_R}(m_Z^2) = (0.0 \pm 1.3) \times 10^{-3}\,,$$

$$\Delta g_{b_L}(m_Z^2) = -(0.33 \pm 0.16) \times 10^{-2}\,, \qquad \Delta g_{b_R}(m_Z^2) = -(2.30 \pm 0.82) \times 10^{-3}\,,$$

$$\Delta g_{\nu_L}(m_Z^2) = (0.40 \pm 0.21) \times 10^{-2}\,,$$

$Z \to ll$过程对耦合系数g间的限制图





[1] ALEPH, DELPHI, L3, OPAL, SLD, LEP Electroweak Working Group, SLD Electroweak Group, SLD Heavy Flavour Group collaboration, Precision electroweak measurements on the Z resonance, Phys. Rept. 427 (2006) 257 [hep-ex/0509008].
[2] A. Efrati, A. Falkowski and Y. Soreq, Electroweak constraints on flavorful effective theories,JHEP 07 (2015) 018 [1503.07872].

Thanks

请各位老师同学

批评指正