



The NPC analysis on fragmentation functions of light hadrons

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based on works of the NPC collaboration:

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arXiv: 2305.14620, 2401.02781, 2407.04424, 2407.10059, 2502.17837, 2503.21311

Flavor Physics Lecture 2025-7

Online 2025/4/17



Outline

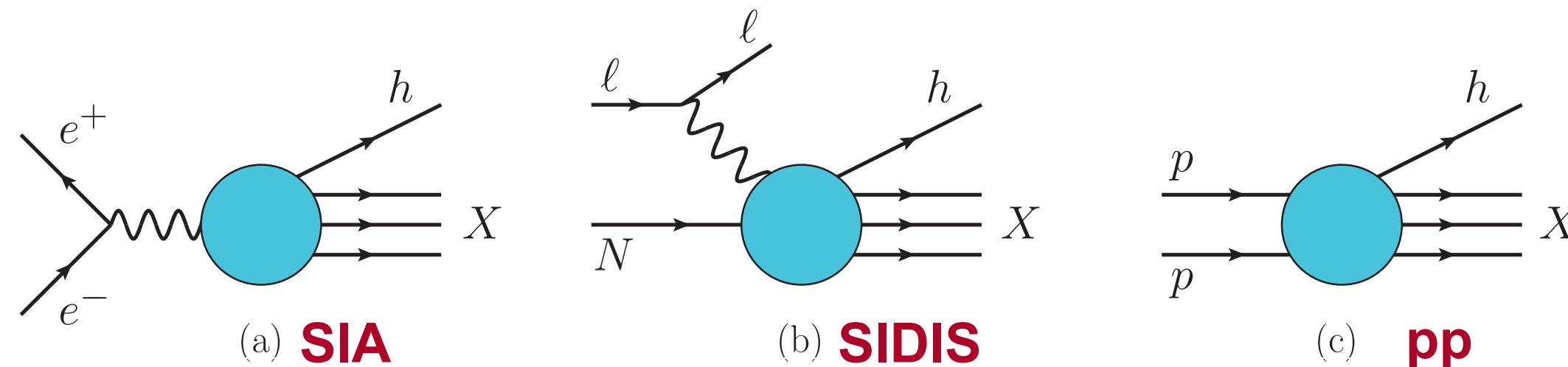
- ◆ 1. Introduction to fragmentation functions
- ◆ 2. Automation of fragmentation calculations at NLO
- ◆ 3. NPC23 FFs to light charged hadrons at NLO (pp/SIDIS/SIA)
- ◆ 4. NPC23 FFs to light neutral hadrons at NLO (pp/SIDIS/SIA)
- ◆ 5. NPC23 FFs to light charged hadrons at NNLO (SIDIS/SIA)
- ◆ 6. Outlook and summary

Single inclusive hadron production

- ✦ In its simplest form, fragmentation functions (FFs) describe number density of the identified hadron wrt the fraction of momentum of the initial parton it carries, as measured in single inclusive hadron production, e.g., from single-inclusive annihilation (SIA), semi-inclusive DIS (SIDIS), pp collisions

single inclusive hadron production/observable

[1607.02521]



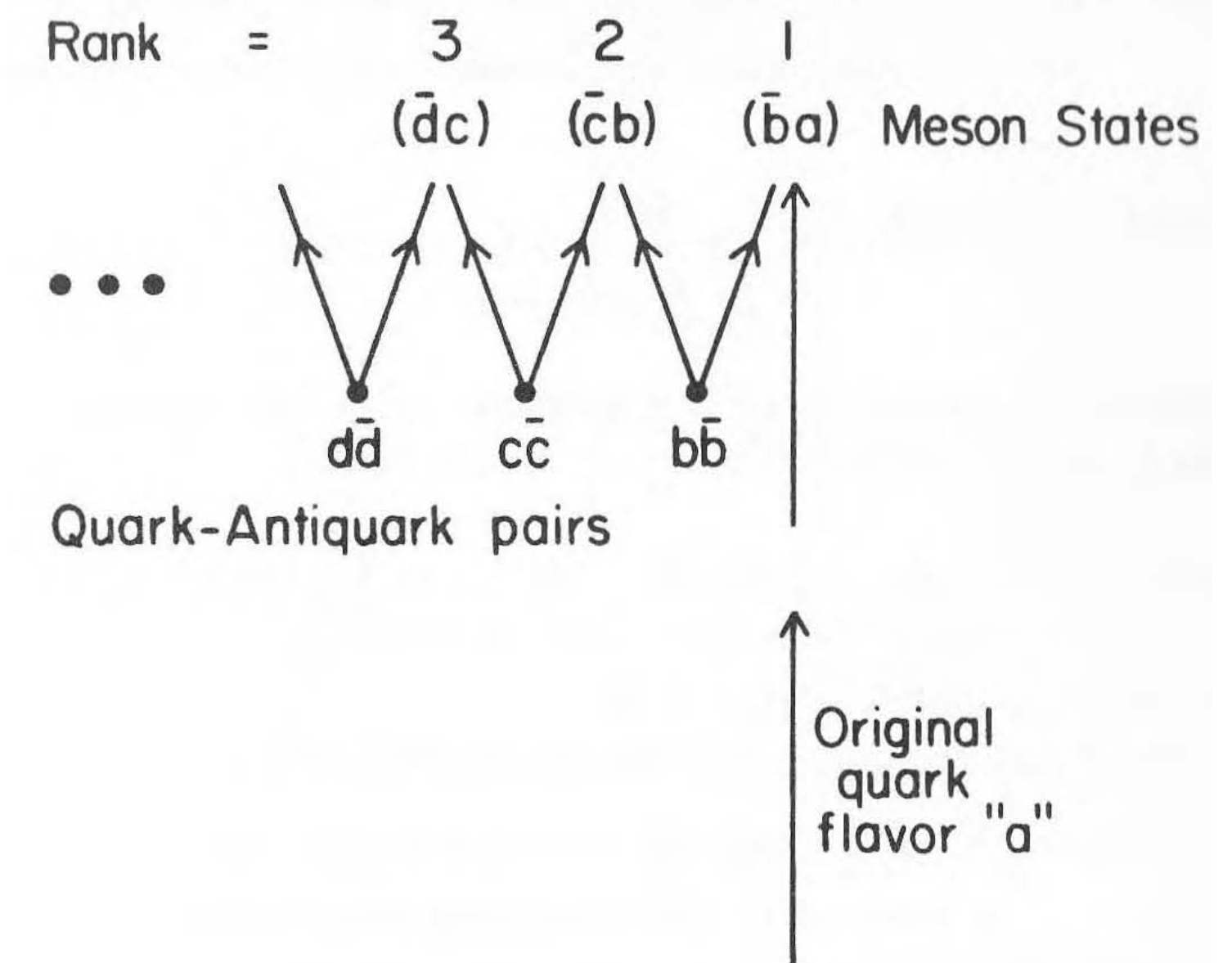
$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma^{e^+e^- \rightarrow hX}}{dz} = F^h(z, Q^2), \quad z = \frac{2E_h}{\sqrt{s}}$$

exp. definition of unpolarized collinear FFs

other forms: polarized FFs, TMD FFs, di-hadron FFs

quark parton model [Field&Feynman]

Hierarchy of Final Mesons

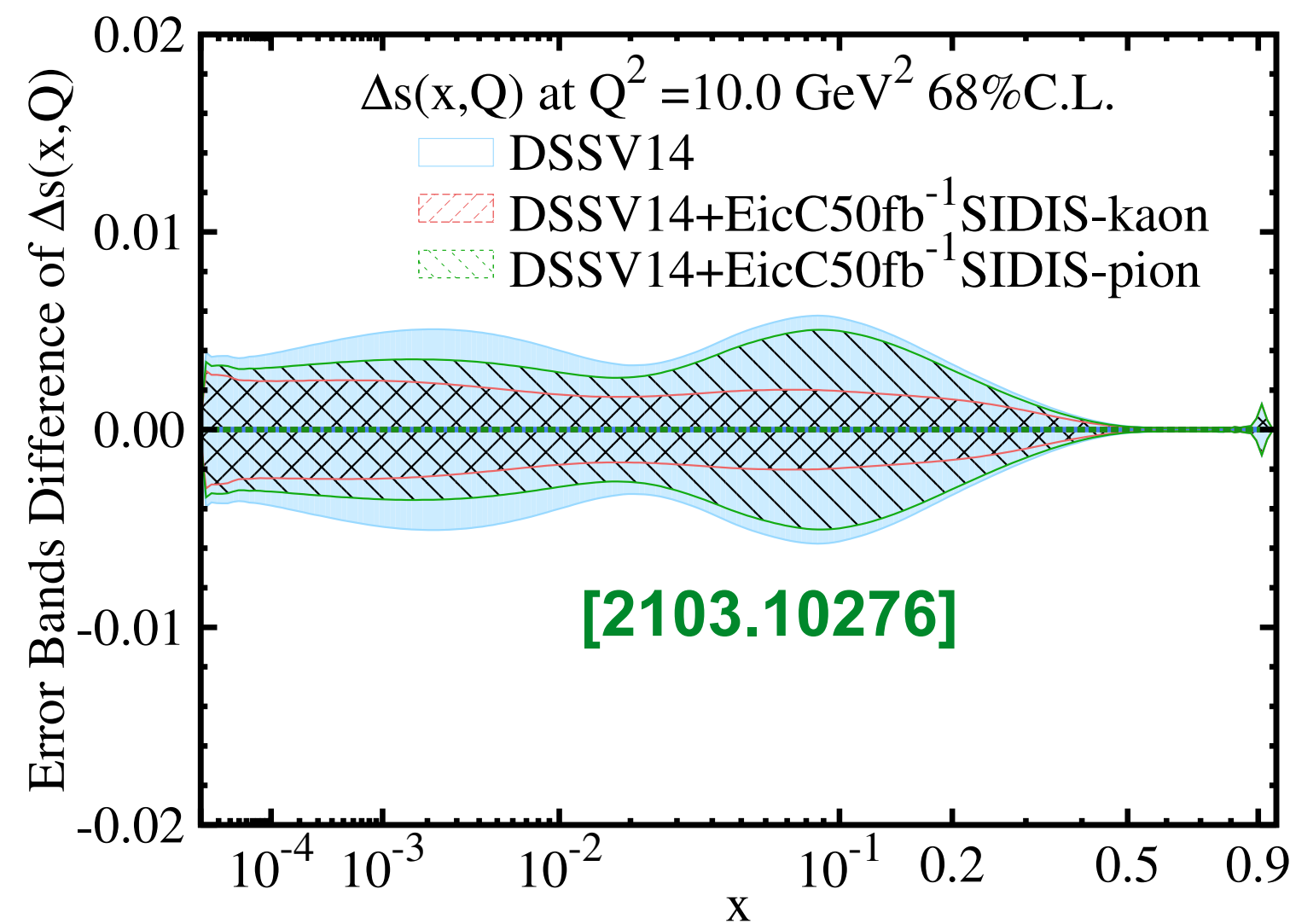


quark decaying functions to mesons via creation of quark-antiquark pairs in cascade

Phenomenology implications

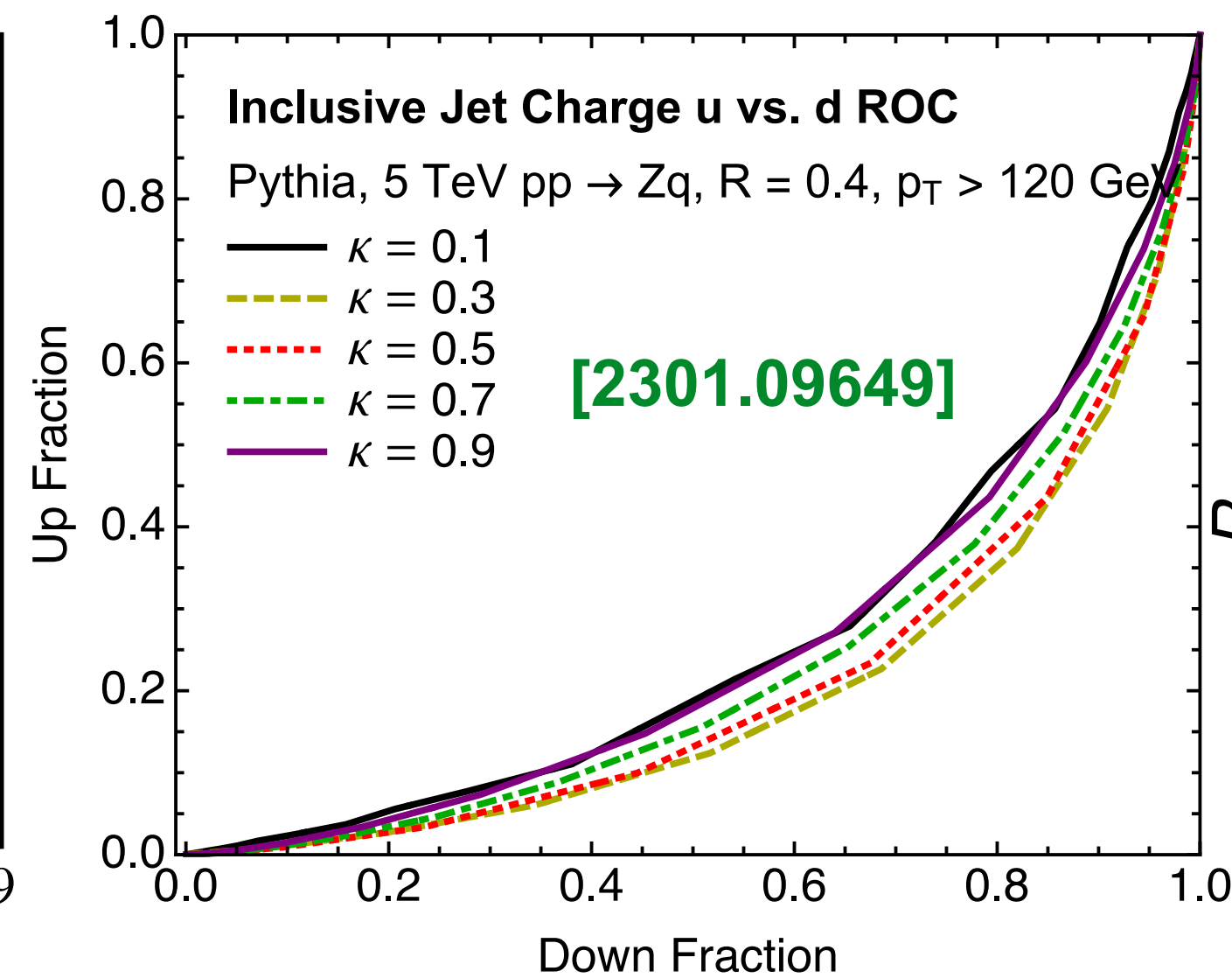
- ✦ Apart from being central topic of QCD hadronization and confinement, FFs also have wide applications in various programs of nuclear and particle physics, e.g., in studies of nucleon structure including polarized PDFs, jet flavor tagging , jet transportation properties in QGP matters, etc.

polarized PDFs of strange quark



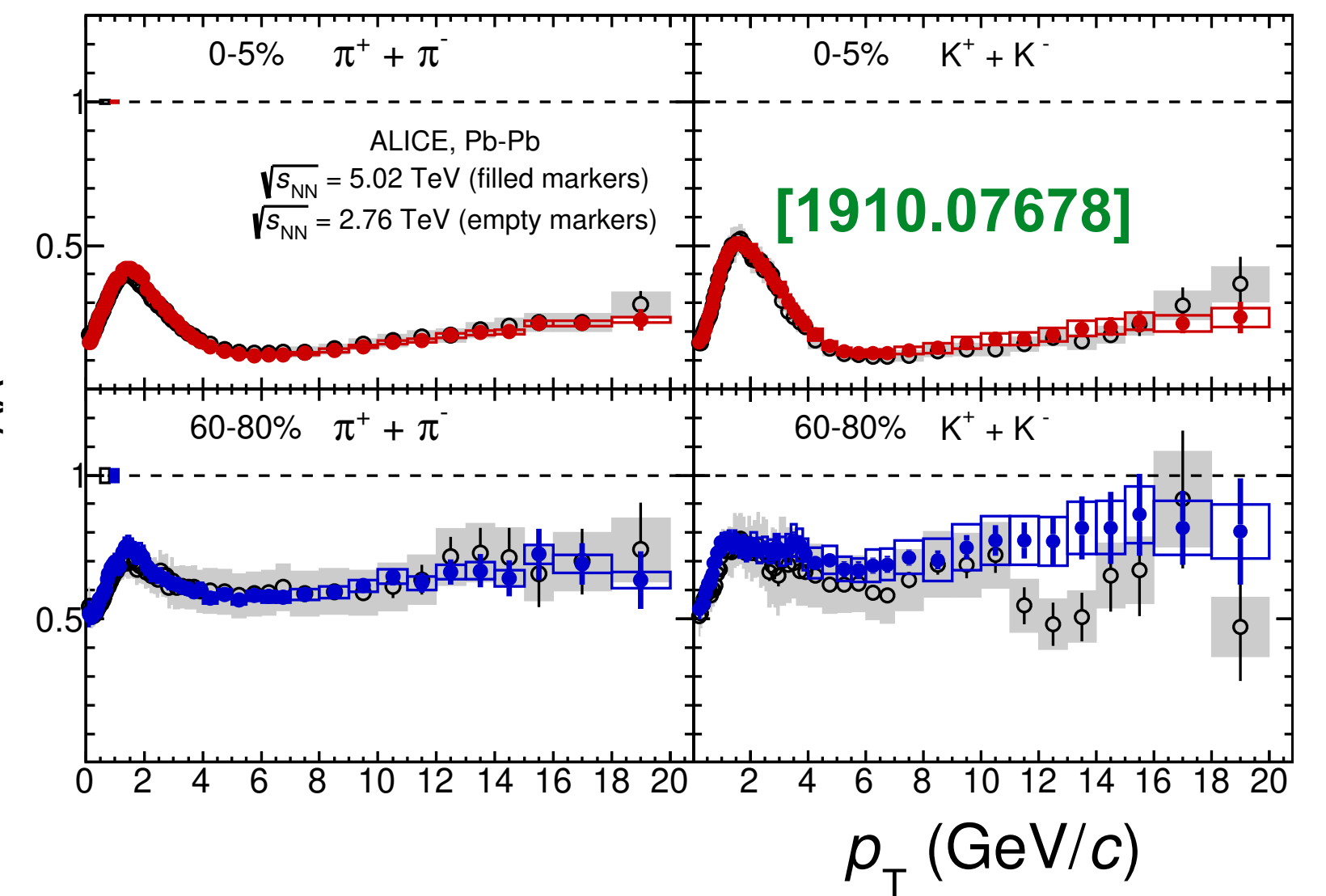
impact of SIDIS pseudo-data

u/d quark jet separation



jet charge from constituent tracks

hadron production: Pb-Pb vs pp collisions



hadron suppression factor

$$F_1^h(x, Q^2, z) = \frac{1}{2} \sum_q e_q^2 [q(x, Q^2) D^{q \rightarrow h}(Q^2, z) + \bar{q}(x, Q^2) D^{\bar{q} \rightarrow h}(Q^2, z)],$$

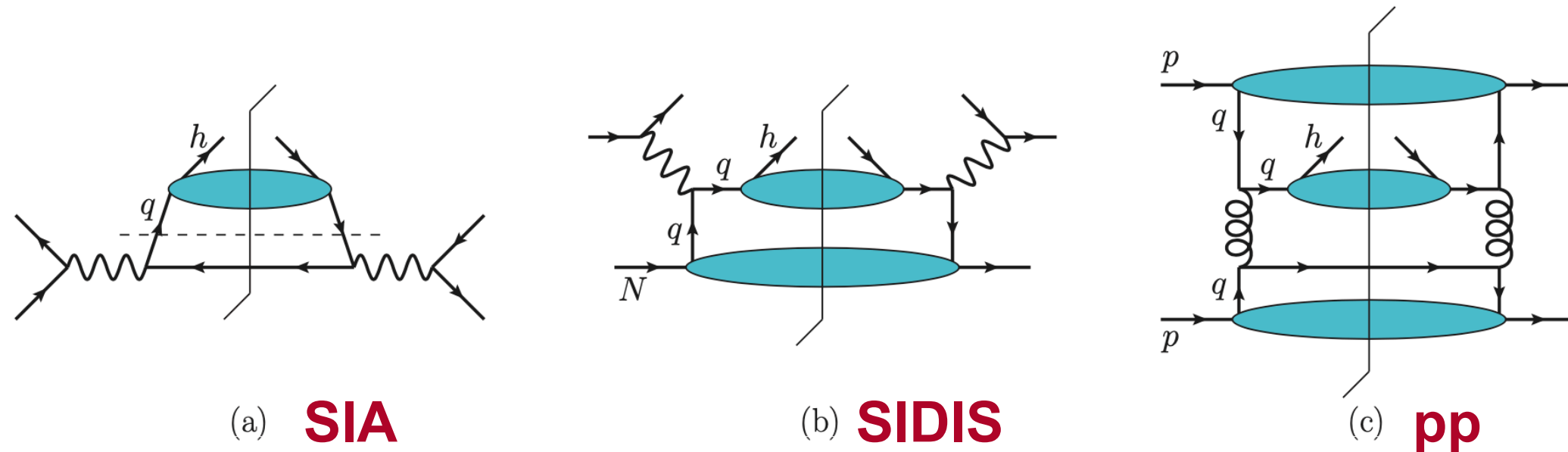
$$g_1^h(x, Q^2, z) = \frac{1}{2} \sum_q e_q^2 [\Delta q(x, Q^2) D^{q \rightarrow h}(Q^2, z) + \Delta \bar{q}(x, Q^2) D^{\bar{q} \rightarrow h}(Q^2, z)].$$

$$Q_J = \frac{1}{(p_{T,J})^\kappa} \sum_{i \in \text{Tracks}} q_i \times (p_{T,i})^\kappa$$

$$R_{AA}(p_T) = \frac{dN^{AA}/dp_T}{\langle N_{\text{coll}} \rangle dN^{PP}/dp_T}$$

QCD collinear factorization

- ✦ QCD collinear factorization ensures universal separation of long-distance and short-distance contributions in high energy scatterings involving initial/final state hadrons, and enables predictions on cross sections



- ✦ coefficient functions, hard scattering; infrared (IR) safe, calculable in pQCD, independent of the hadron
- ✦ FFs/PDFs, reveal inner structure of hadrons or parton-hadron transition; NP origin, universal, e.g. DIS vs. pp collisions; fitted from data
- ✦ runnings of FFs/PDFs with scales μ_D/μ_f are governed by the DGLAP equation

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma^{e^+e^- \rightarrow hX}}{dz} = \sum_q e_q^2 (2F_1^h(z, Q^2) + F_L^h(z, Q^2))$$

$$2F_1^h(z, Q^2) = \sum_q e_q^2 \left(D_1^{h/q}(z, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \left(C_1^q \otimes D_1^{h/q} + C_1^g \otimes D_1^{h/g} \right) (z, Q^2) \right)$$

$$\frac{d^3\sigma^{\ell p \rightarrow \ell hX}}{dx dy dz} = \frac{2\pi\alpha_{\text{em}}^2}{Q^2} \left(\frac{1 + (1-y)^2}{y} 2F_1^h(x, z, Q^2) + \frac{2(1-y)}{y} F_L^h(x, z, Q^2) \right)$$

$$2F_1^h(x, z, Q^2) = \sum_q e_q^2 \left(f_1^{q/p} D_1^{h/q} + \frac{\alpha_s(Q^2)}{2\pi} \left(f_1^{q/p} \otimes C_1^{qq} \otimes D_1^{h/q} + f_1^{q/p} \otimes C_1^{gq} \otimes D_1^{h/g} + f_1^{g/p} \otimes C_1^{qg} \otimes D_1^{h/q} \right) \right),$$

unpolarized collinear FFs, operator definition

$$D_1^{h/q}(z) = \frac{z}{4} \sum_X \int \frac{d\xi^+}{2\pi} e^{ik^-\xi^+} \text{Tr} \left[\langle 0 | \mathcal{W}(\infty^+, \xi^+) \psi_q(\xi^+, 0^-, \vec{0}_T) | P_h, S_h; X \rangle \times \langle P_h, S_h; X | \bar{\psi}_q(0^+, 0^-, \vec{0}_T) \mathcal{W}(0^+, \infty^+) | 0 \rangle \gamma^- \right].$$

$$\frac{d}{d \ln \mu^2} D_1^{h/i}(z, \mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \sum_j \int_z^1 \frac{du}{u} P_{ji}(u, \alpha_s(\mu^2)) D_1^{h/j} \left(\frac{z}{u}, \mu^2 \right)$$

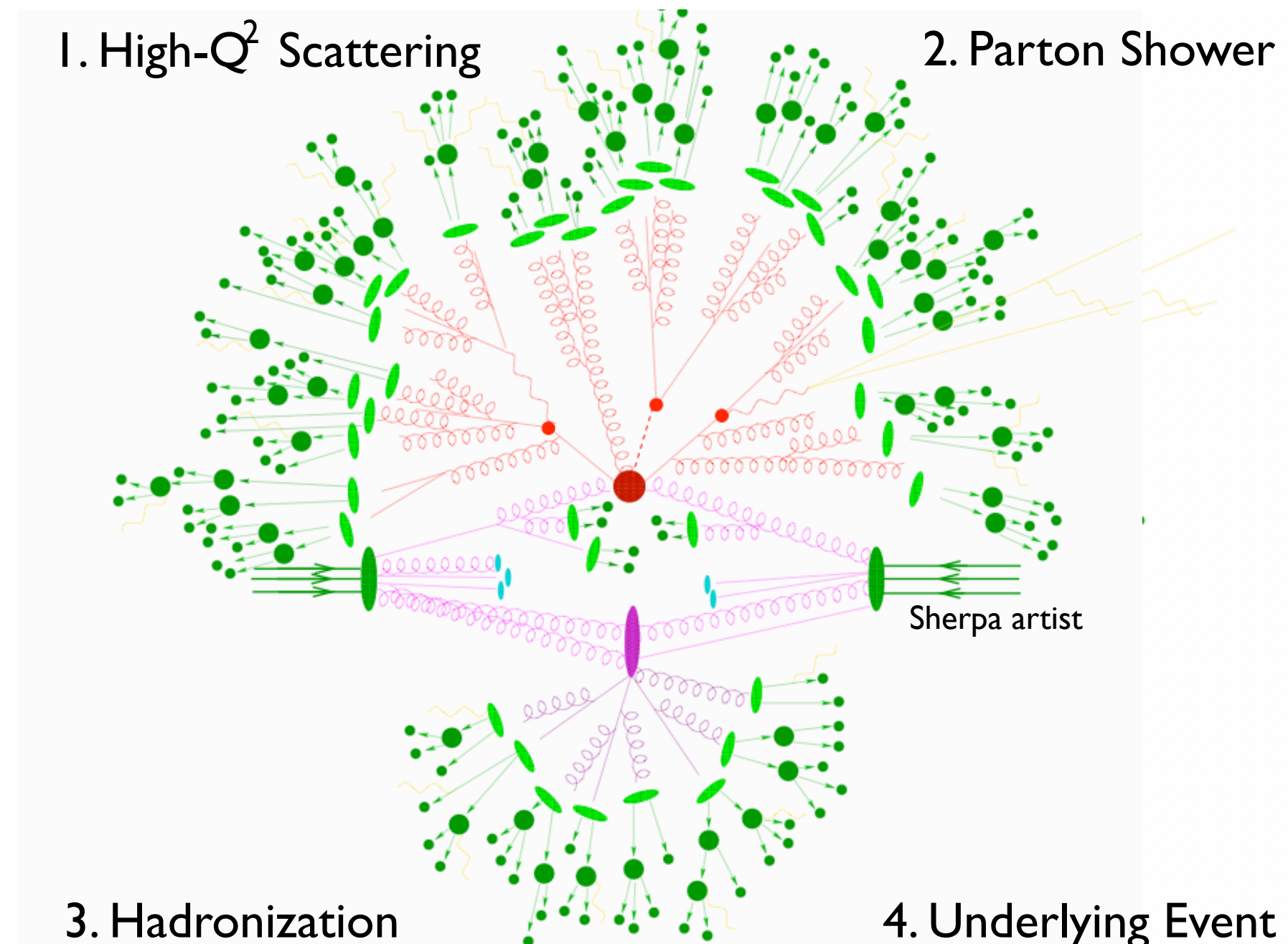
[Collins, Soper, Sterman]

FFs vs. MC generators

- FFs provides a better and controlled precision for specific observables related to hadron productions based on first principle of QCD while MC generators with string/cluster models aim at a more dynamic and full description of all final state hadrons

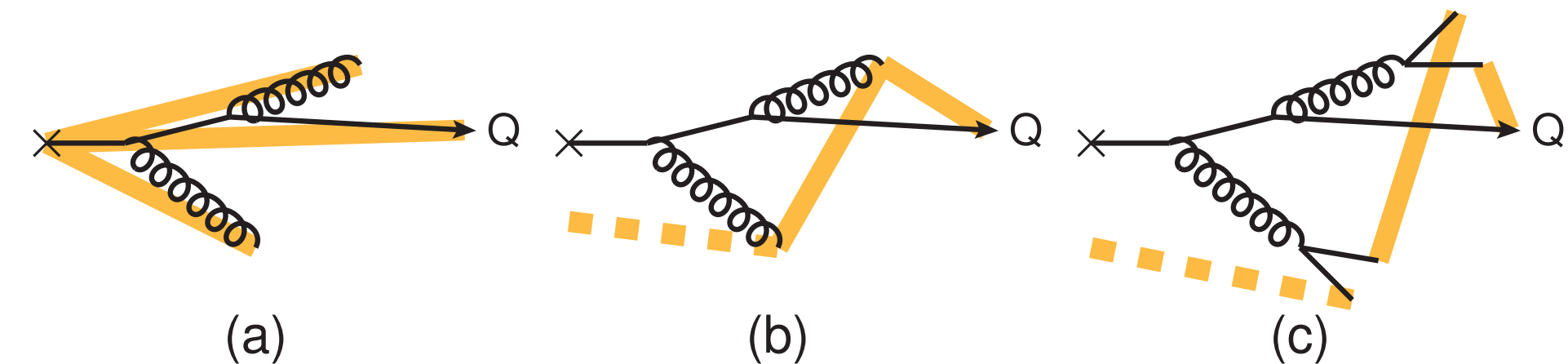
full description of final state hadrons

[Pythia, Herwig, Sherpa]



MCs rely on models and PS at the lowest order

FFs vs. string/cluster fragmentation models



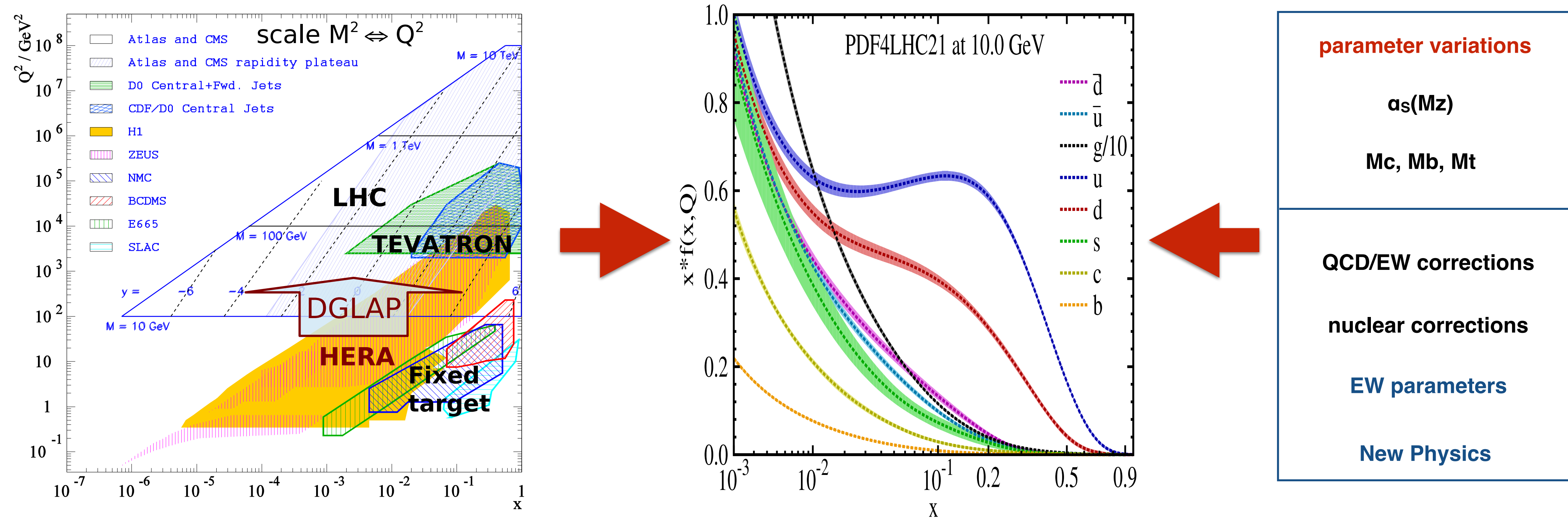
[T. Sjostrand, 2024]

- Fragmentation functions are static. They may work for some simple tasks, but do not offer a full picture.
- Event generators with strings/clusters are not perfect, but they offer a more realistic and dynamic approach.
- Generator uncertainty from many issues: N^{th} LO, PDFs, $m_{c,b}$, α_s , shower, match&merge, colour reconnection, ...

FFs provides better precision on specific tasks

Global analysis of PDFs

- Proton PDFs are usually extracted from global analysis on variety of data, e.g., DIS, Drell-Yan, jets and top quark productions at fixed-target and collider experiments, with increasing weight from LHC, together with SM QCD parameters [see 1709.04922, 1905.06957 for recent reviews]

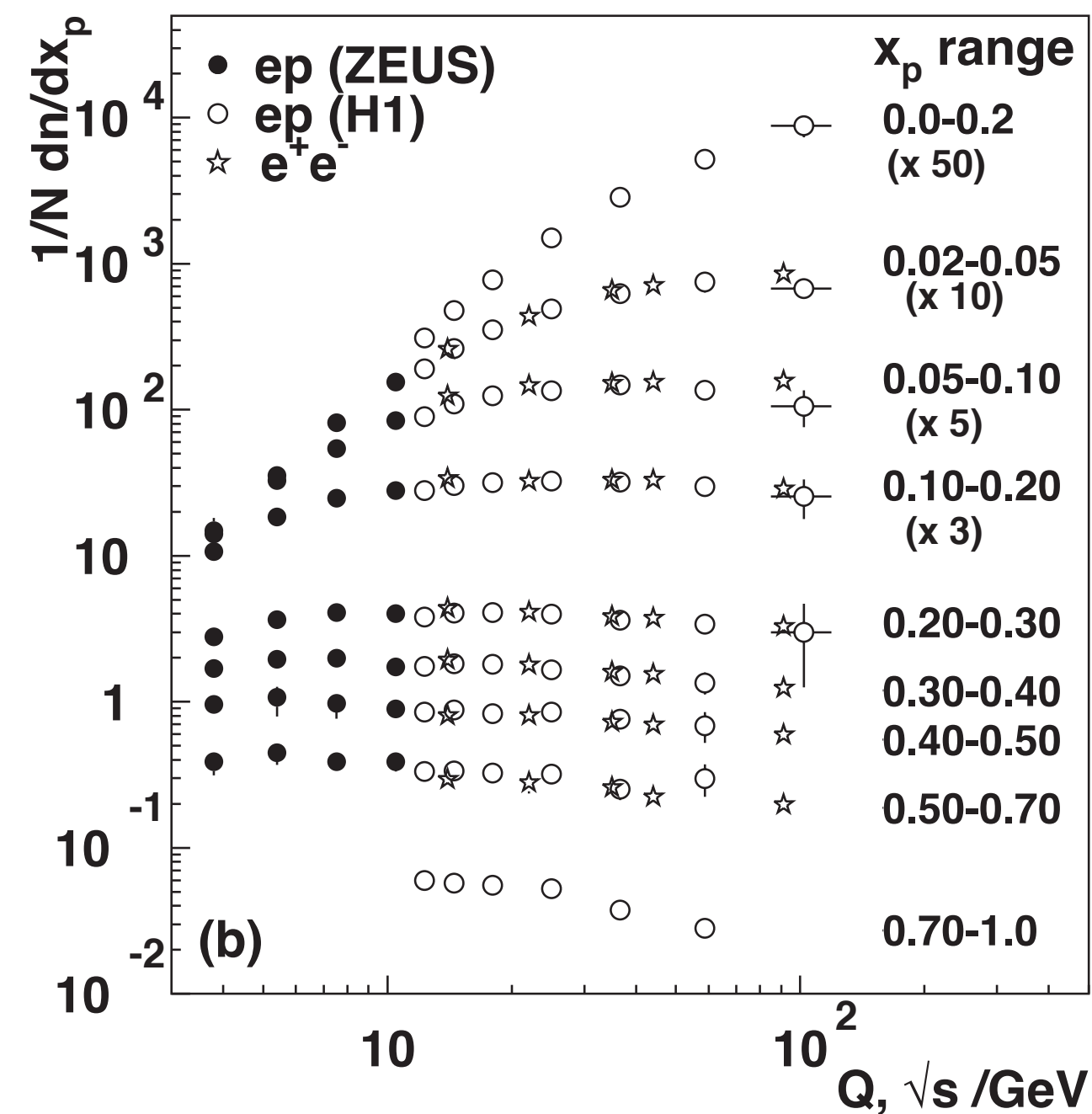


- Modern PDF analyses are carried out at NNLO/aN3LO in QCD as demanded by the LHC precision programs; diversity of the PDF analyses/groups are important to avoid theoretical/experimental bias
- analysis groups [CT, MSHT, NNPDF, ABM, HERAPDF, ATLASpdf, CJ, JAM...] use different heavy-quark schemes, data, and methodologies; extensive benchmarks exist (PDF4LHC); alternative approach from lattice simulations [2004.03543]

Global analysis of FFs

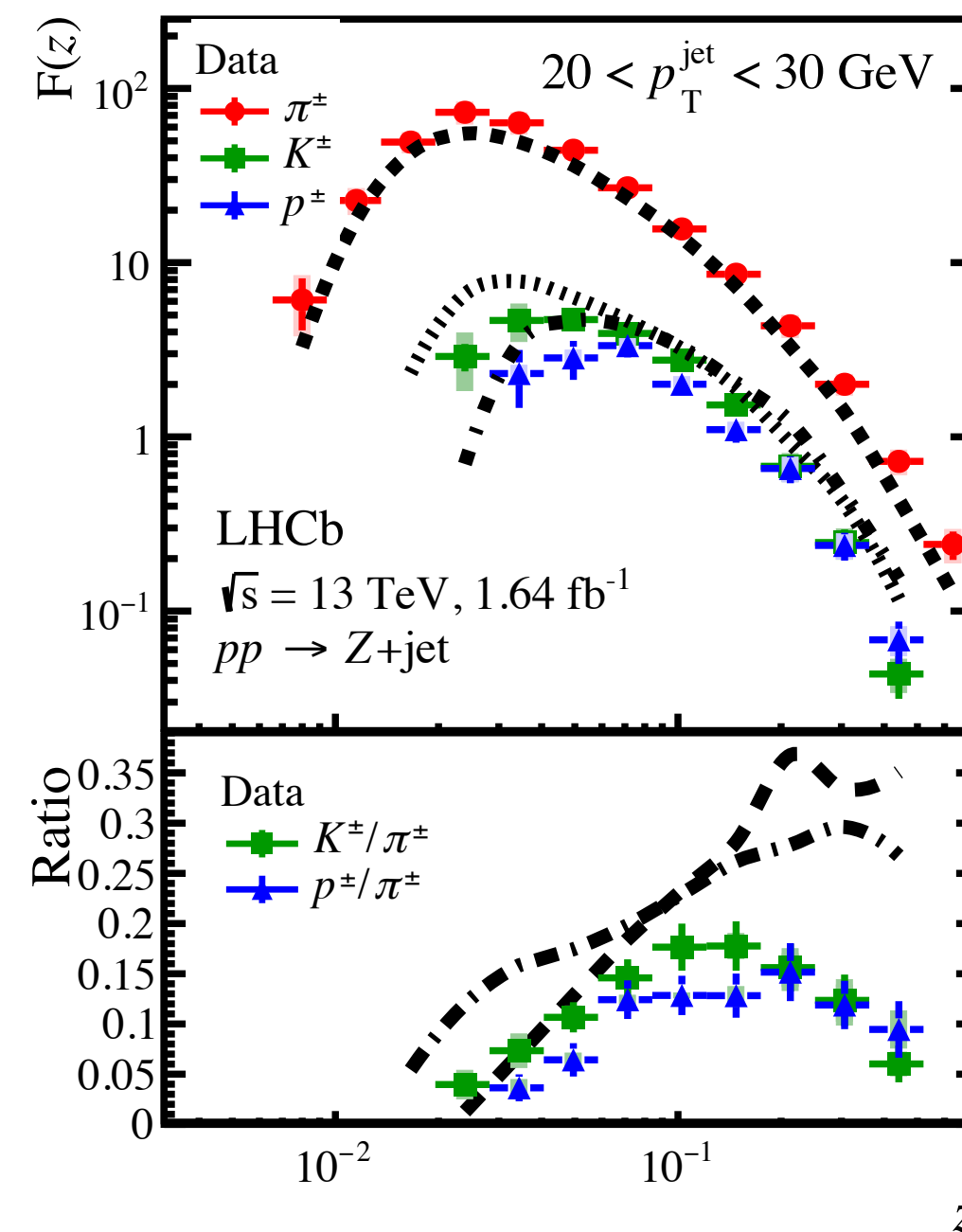
- Measurements are available from colliders SLAC, LEP, HERA, RHIC, LHC and fixed-target HERMES, COMPASS experiments for various light charged hadrons as well as neutral hadrons; many groups provide phenomenological FFs from global analysis at NLO/NNLO in QCD

single incl. production of unidentified charged hadrons (SIA & SIDIS)



[Particle data group]

jet fragmentation to light charged hadrons (LHCb)



[2208.11691]

$$z = \frac{\mathbf{p}_{\text{had}} \cdot \mathbf{p}_{\text{jet}}}{|\mathbf{p}_{\text{jet}}|^2}, \quad F(z) = \frac{1}{N_{Z+\text{jet}}} \frac{dN_{\text{had}}(z)}{dz}$$

global analysis

[1607.02521 for a review]

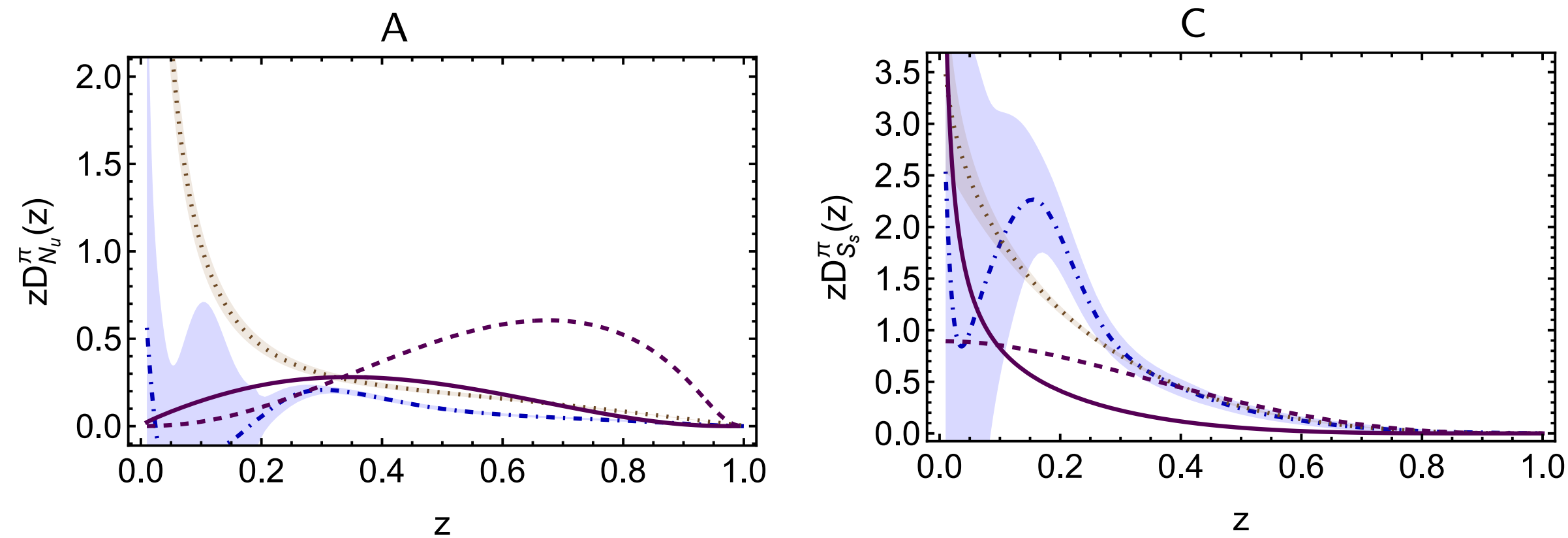
- major groups/families include BKK, AKK, HKNS, DSS, NNFF, MAPFF, JAM, SAK etc.
- mostly done at NLO in QCD since exact NNLO coefficient functions only known recently for SIDIS
- different determination can be quite different due to selection of data sets as well as theory treatments, not converge as well as the case of PDF fits

pinning down uncertainties of FFs is crucial for EIC(c) physics !

Computations of FFs

- Recent developments exist on constructing FFs from hadron scale distribution functions with analytical continuation (Drell-Levy-Yan relation) or direct computation of FFs in simplified models (e.g., 't Hooft, NJL) analytically or using quantum simulations

CSM pion FFs compared with phenomenological FFs



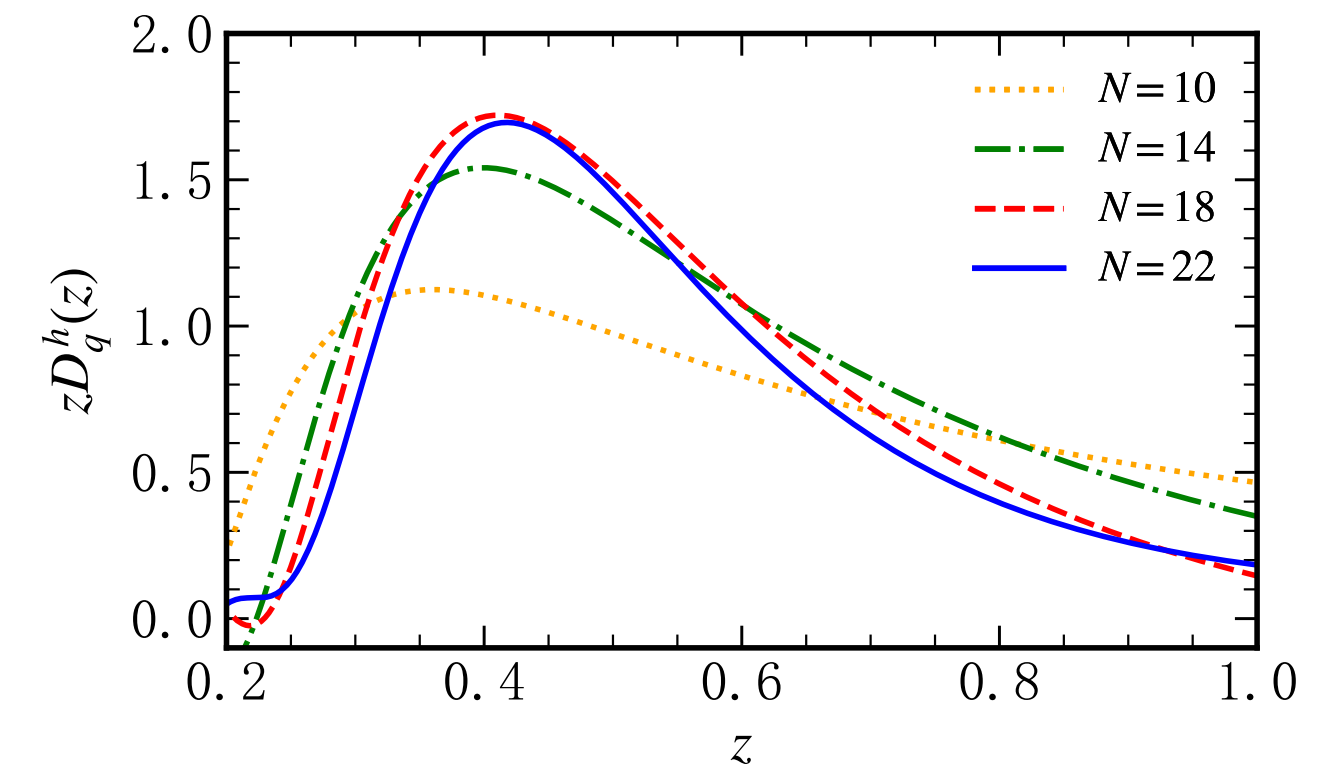
[Xing, Bian, Cui, Roberts, 2504.08142]

$$u_\pi(x; \zeta_{\mathcal{H}}) = n_\pi \ln \left[1 + \frac{1}{\rho_\pi^2} x^2 (1-x)^2 \right. \\ \left. \times \left(1 + \frac{1}{2} \gamma_\pi^2 [(1-x)^{2\beta_\pi} + x^{2\beta_\pi}] \right) \right]$$

$$d_u^{\pi^+}(z) = z u_\pi(1/z; \zeta_{\mathcal{H}}), \\ d_u^{K^+}(z) = z u_{K^+}(1/z; \zeta_{\mathcal{H}}), \\ d_s^{K^-}(z) = z s_{K^-}(1/z; \zeta_{\mathcal{H}}).$$

- Effective FFs are obtained from CSM PDFs with DLY relations, then evolved with AO+DGLAP equation to get CSM FFs

FFs vs. #qubits in NJL model



[Li, Xing, Zhang, 2406.05683]

quark to meson FFs in 't Hooft model

$$D_{q \rightarrow H}(z) = \frac{4\lambda^2}{N_c} \frac{(1-z)^2}{z \left[(z^2(m^2 - 2\lambda) + (1-z)M_H^2)^2 + \epsilon^2 \right]} \left[\int_0^1 dy \frac{\phi_n(y)}{(y - 1/z)^2} \right]^2$$

[Jia, Mo, Xiong, 2310.17640]

✦ 2. Automation of fragmentation calculations at NLO

Hadron production cross sections at NLO

- Generic algorithms on NLO calculations of jet production cross sections have been developed for long times, based on local subtraction or phase-space slicing method; especially automation of NLO jet cross sections exists, e.g., in MG5 and Sherpa

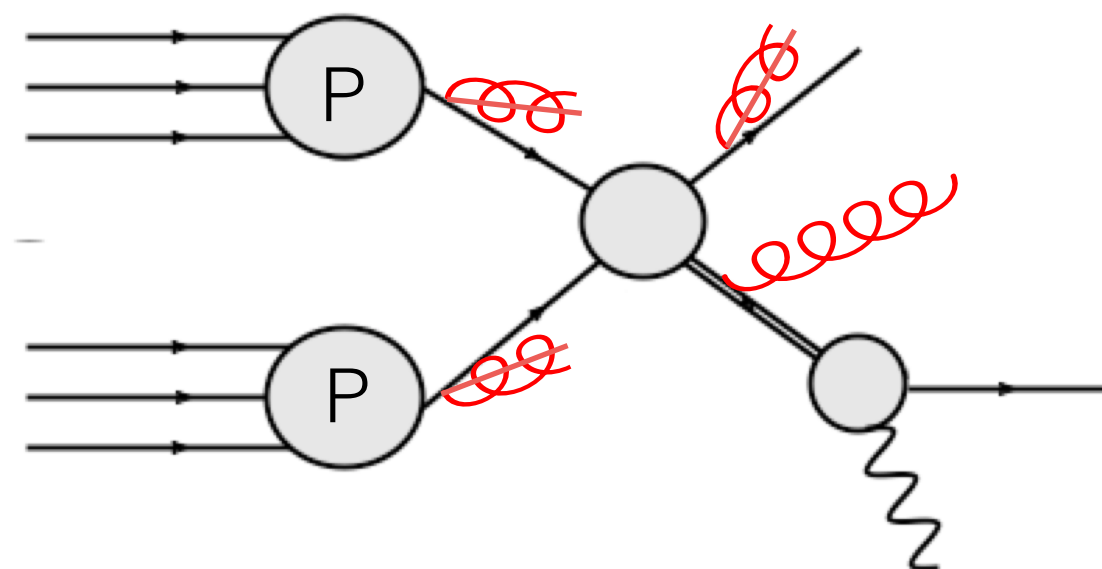
local subtraction

$$\frac{d\sigma}{dF} = \int dPS_m \left[|M|_{B,m}^2 + |M|_{V,m}^2 + |\tilde{\mathcal{I}}_m|^2 \right] \delta(\hat{F}(p_m; f_m) - F) \\ + \int dPS_{m+1} \left[|M|_{R,m+1}^2 \delta(\hat{F}(p_{m+1}; f_{m+1}) - F) - |\mathcal{I}_{m+1}|^2 \delta(\hat{F}(\tilde{p}_m; \tilde{f}_m) - F) \right]$$

phase-space slicing

$$\frac{d\sigma}{dF} = \int dPS_m \left[|M|_{B,m}^2 + |M|_{V,m}^2 \right] \delta(\hat{F}(p_m; f_m) - F) \\ + \int dPS_{m+1} (\Theta(C - \lambda) + \Theta(\lambda - C)) \left[|M|_{R,m+1}^2 \delta(\hat{F}(p_{m+1}; f_{m+1}) - F) \right]$$

QCD radiations



- ❖ FKS subtraction (jet), [\[Frixione, Kunszt, Signer\]](#), as implemented in Madgraph5
- ❖ Dipole subtraction (jet & hadron), [\[Catani, Seymour\]](#), as implemented in MCFM
- ❖ Two-cutoff slicing (jet & hadron), [\[Harris, Owens\]](#)
- ❖ Antenna subtraction (jet & hadron), [\[2406.09925\]](#); alternative subtraction [\[2403.14574\]](#); semi-analytical calculations, [\[1903.01529, BigkT\]](#)
- ❖ STRIPPER (jet & hadron at NNLO), [\[2503.11489\]](#)

public tools and automation on the hadron cross sections calculations are very much limited!

Hadron production cross sections at NLO

- ✦ We developed a hybrid scheme for NLO calculations of hadron production cross sections, using slicing of collinear radiation region of phase space, combined with any local subtraction scheme for calculations of jet cross sections

a hybrid scheme for hadron production

$$\begin{aligned} \frac{d\sigma}{dp_{T,h}} = & \int dx \int dPS_m \left[|M|_{B,m}^2 + |M|_{V,m}^2 + |\tilde{\mathcal{I}}|_m^2 \right] \sum_{i=1}^m \delta(p_{T,h} - xp_{T,i}) D_{h/i}^0(x) \\ & + \int dx \int dPS_{m+1} (\Theta(\lambda - C) + \Theta(C - \lambda)) \left[|M|_{R,m+1}^2 \sum_{i=1}^{m+1} \delta(p_{T,h} - xp_{T,i}) D_{h/i}^0(x) \right. \\ & \left. - |\mathcal{I}|_{m+1}^2 \sum_{\tilde{i}=1}^m \delta(p_{T,h} - x\tilde{p}_{T,\tilde{i}}) D_{h/\tilde{i}}^0(x) \right] \end{aligned}$$

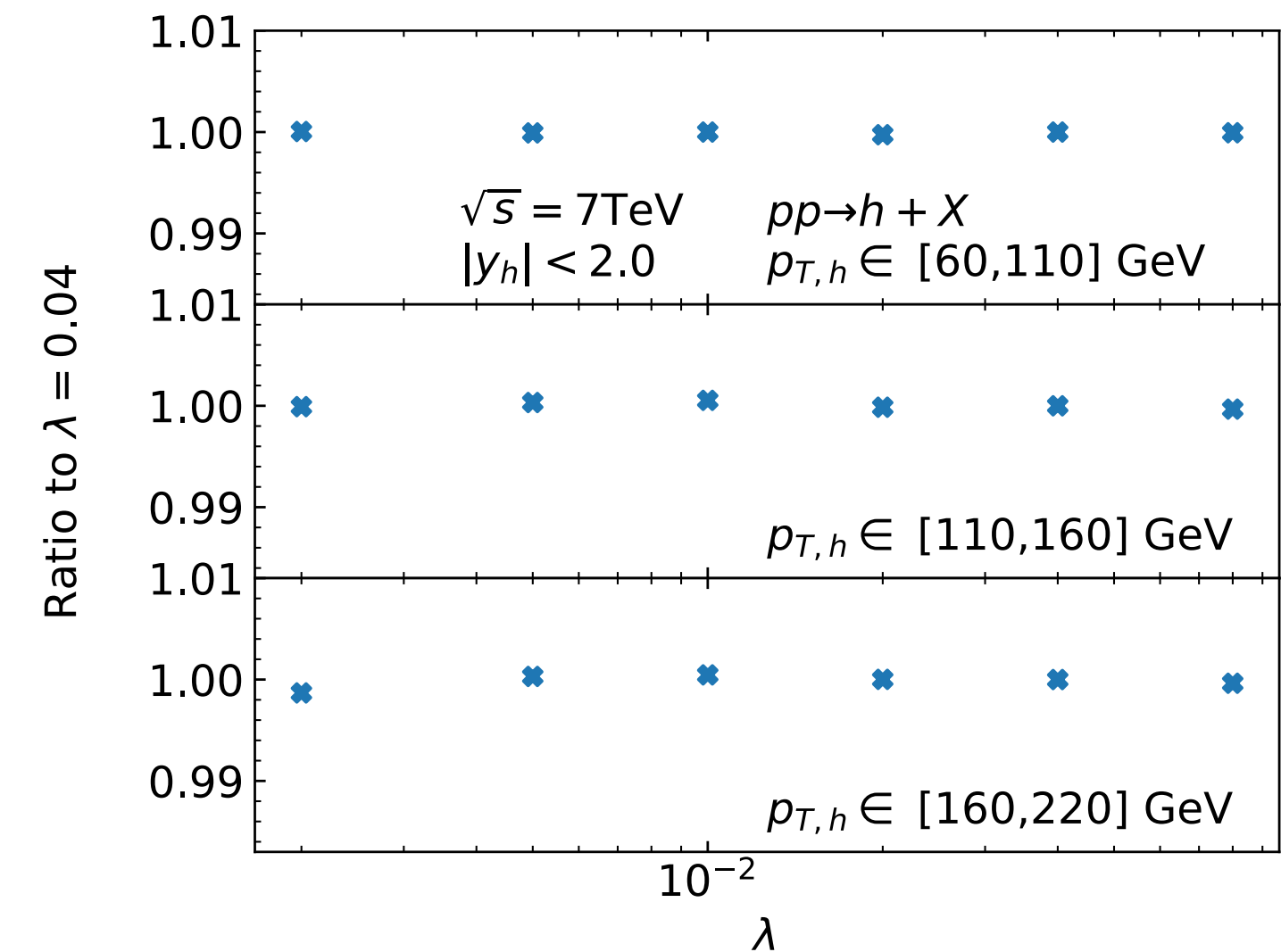
$$\begin{aligned} = & \int dx \int dPS_m \left[|M|_{B,m}^2 + |M|_{V,m}^2 + |\tilde{\mathcal{I}}|_m^2 \right] \sum_{i=1}^m \delta(p_{T,h} - xp_{T,i}) D_{h/i}^0(x) \\ & + \int dx \int dPS_{m+1} \Theta(C - \lambda) \left[|M|_{R,m+1}^2 \sum_{i=1}^{m+1} \delta(p_{T,h} - xp_{T,i}) D_{h/i}^0(x) \right. \\ & \left. - |\mathcal{I}|_{m+1}^2 \sum_{\tilde{i}=1}^m \delta(p_{T,h} - x\tilde{p}_{T,\tilde{i}}) D_{h/\tilde{i}}^0(x) \right] + |\tilde{\mathcal{J}}|_m^2, \end{aligned}$$

$$\begin{aligned} |\tilde{\mathcal{J}}|_m^2 = & \int dx \int dPS_m |M|_{B,m}^2 \left(\frac{\alpha_S(\mu_R)}{2\pi} \right) \sum_{i=1}^m \delta(p_{T,h} - xp_{T,i}) \\ & \times \left[\left(\frac{4\pi\mu_R^2 e^{-\gamma_E}}{\mu_D^2} \right)^\epsilon \left(-\frac{1}{\epsilon} + \ln \frac{\lambda^2 p_{T,i}^2}{\mu_D^2} \right) \sum_j P_{ji}^{+(0)} \otimes D_{h/j}(x, \mu_D) + \tilde{D}_{h/i}(x, \mu_D) \right] \end{aligned}$$

[JG, Liu, Shen, Zhou, 2305.14620 (JHEP)]

- ✦ key observation: hadron vs. jet, only the final state collinear singularity need further regularization
- ✦ can be bridged with any public program used for calculations of jet cross sections

slicing variable, e.g., : $\Delta\theta = \min\{\Delta\theta_{ij}\}$



pp inc. hadron cross sections vs. cutoff

FMNLO (fragmentation at NLO in QCD)

- FMNLO is a program for automated and fast calculations of fragmentation cross sections of arbitrary hard processes. It is based on a hybrid scheme of phase-space slicing method and local subtraction method, accurate to NLO in QCD

- automation of fragmentation calculations for arbitrary hard processes up to NLO, within SM and BSMs via MG5_aMC@NLO
- fast convolution of partonic cross sections with FFs without repeating the time consuming MC integrations using interpolation grids
- future goal/generalizations: transverse observables, NNLO corrections

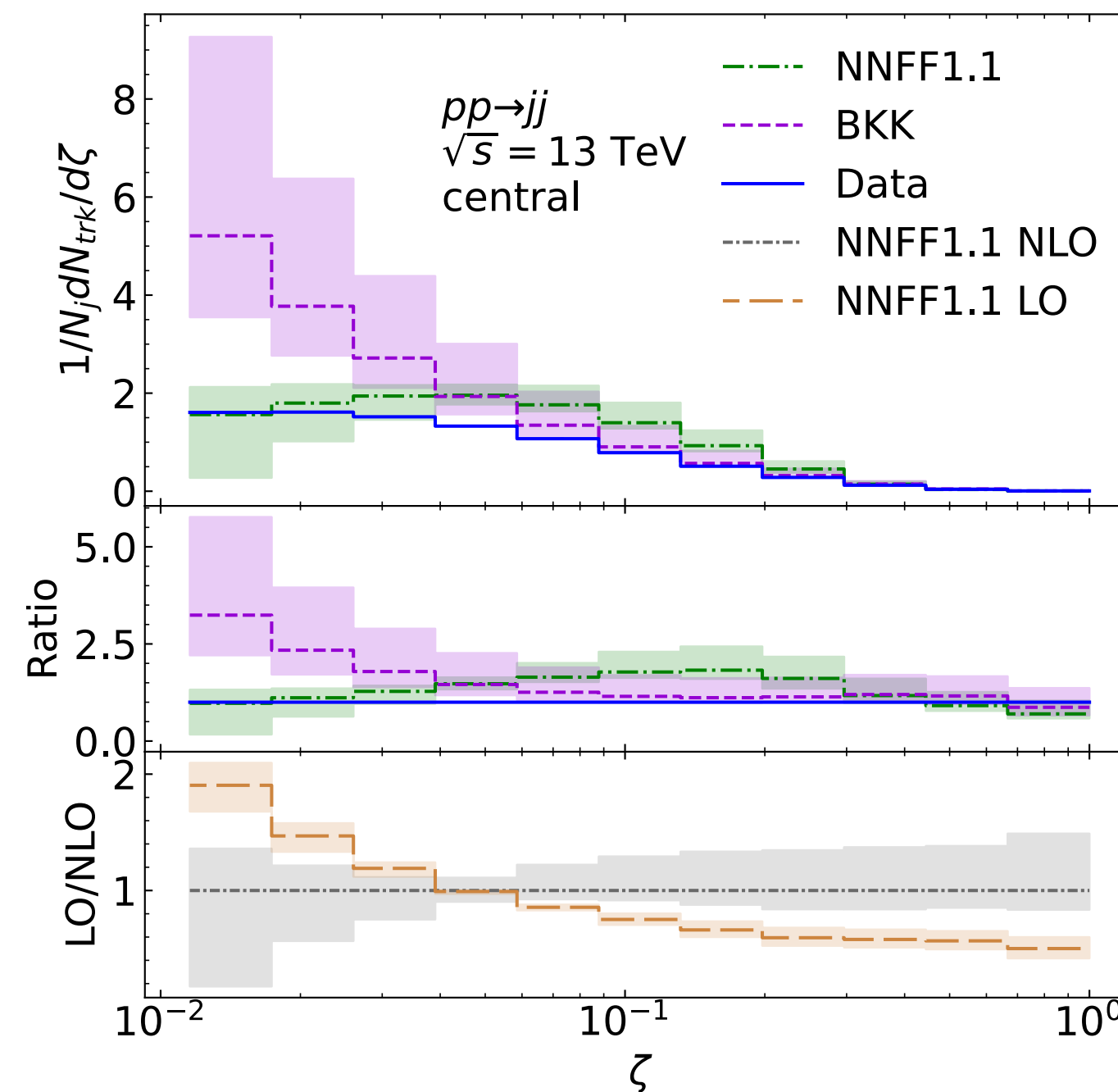
🔥 News

2024.07: 🎉🎉 [FMNLOv2.1](#) NNLO calculations are available for limited cases, SIA, decay of the Higgs boson to gluons, and SIDIS.

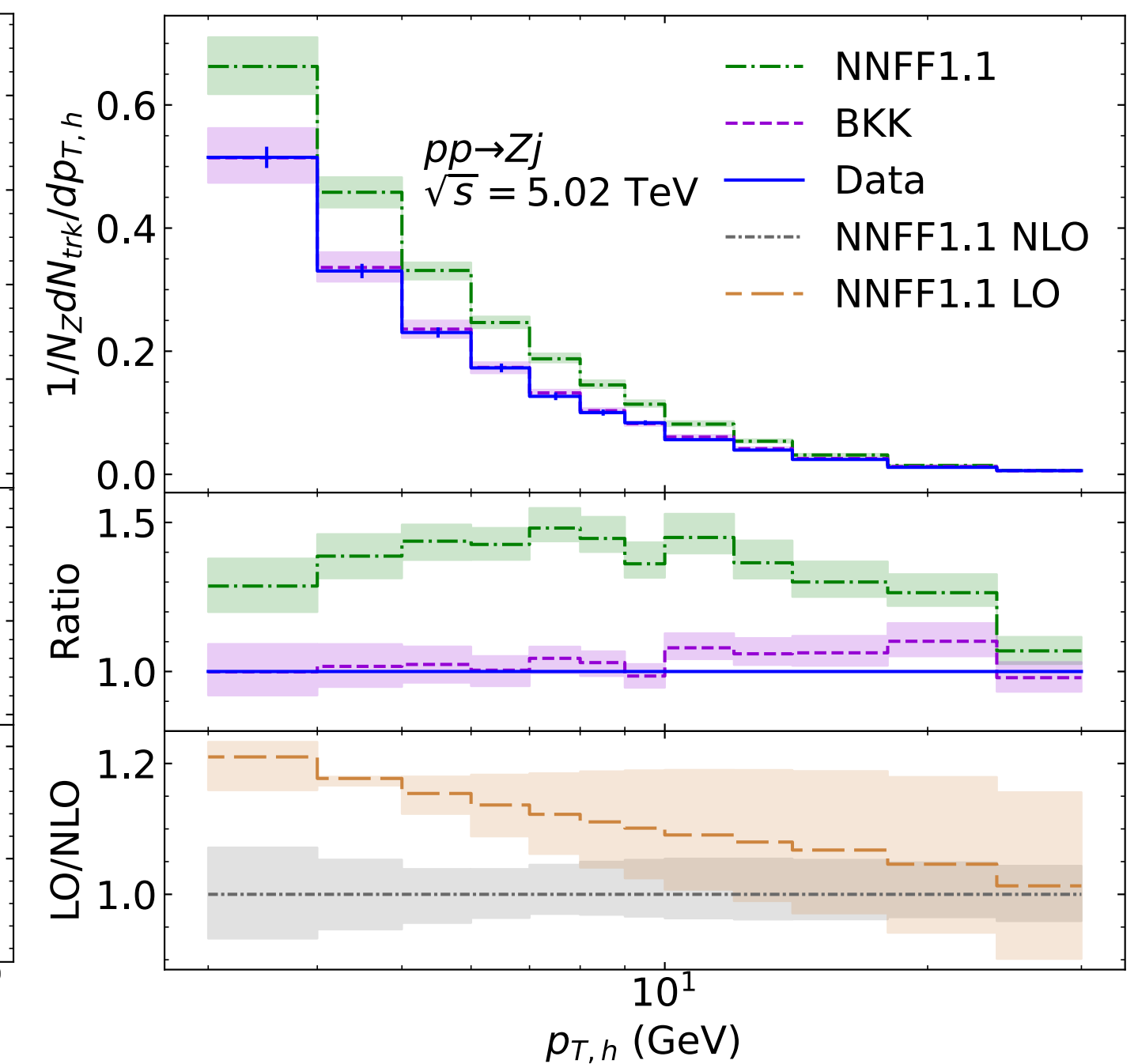
2024.07: 🎉🎉 [FMNLOv2.0](#) include a **SIDIS** module for calculations of SIDIS at NLO.

2023.05: 🎉🎉 [FMNLOv1.0](#) first release of **FMNLO** interfaced with **MG5_aMC@NLO**.

QCD inclusive dijets at LHC



Z-boson tagged jet



<https://fmnlo.sjtu.edu.cn/~fmnlo/>

[JG, Liu, Shen, Zhou, 2305.14620 (JHEP)]

FMNLO (fragmentation at NLO in QCD)

- ✦ A two-step tutorial for usage of FMNLO: 1, generation of an interpolation table for hard coefficient functions; 2, fast convolution with arbitrary FFs and arbitrary binning of the experimental distribution

create a MG5 module for a hard scattering process and set parameters in proc.run

```
# main input for generation of NLO fragmentation grid file by MG5
process A180104895
# subgrids with name tags
grid pp
obs 4
cut 0.02
pta1 60.0
pta2 10000.0
ptj1 30.0
ptj2 10000.0
# in MG5 format
set lpp1 1
set lpp2 1
set ebeam1 2510.0
set ebeam2 2510.0
set lhaid 13100
set iseed 11
set muR_over_ref 1.0
set muF_over_ref 1.0
end
```

execute ./mgen.sh

setup parameters for cross section calculations

```
1 # loop for D fun (1/2 -> LO/NLO) | evo for D fun (0/1 -> internal/hoppet)
2 # followed by >=1/0 -> internal/LHAPDF | FFID | FFmember
3 2 0
4 0 NNFF11_HadronSum_nlo 0
5 # normalization | grid file | binnig file
6 # 0/1/2 -> absolute dis./normalized to corresponding order/leading order
7 # can include multiple entries in several lines
8 1 "../grid/A180104895_pp.fmg" "../grid/1801-04895.Bin"
```

example of binning

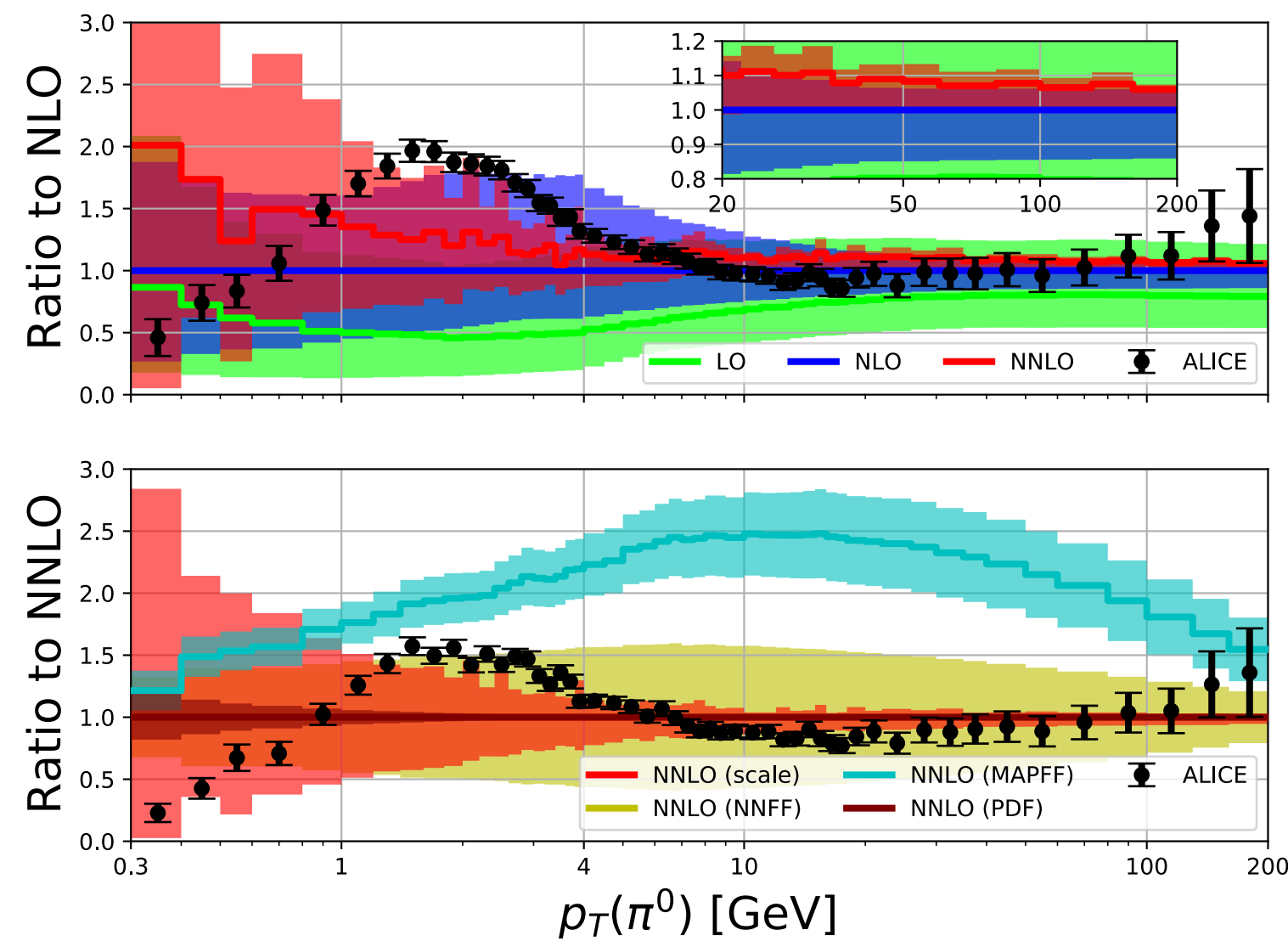
```
8
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5
```

cross section output **execute ./fmnlo**

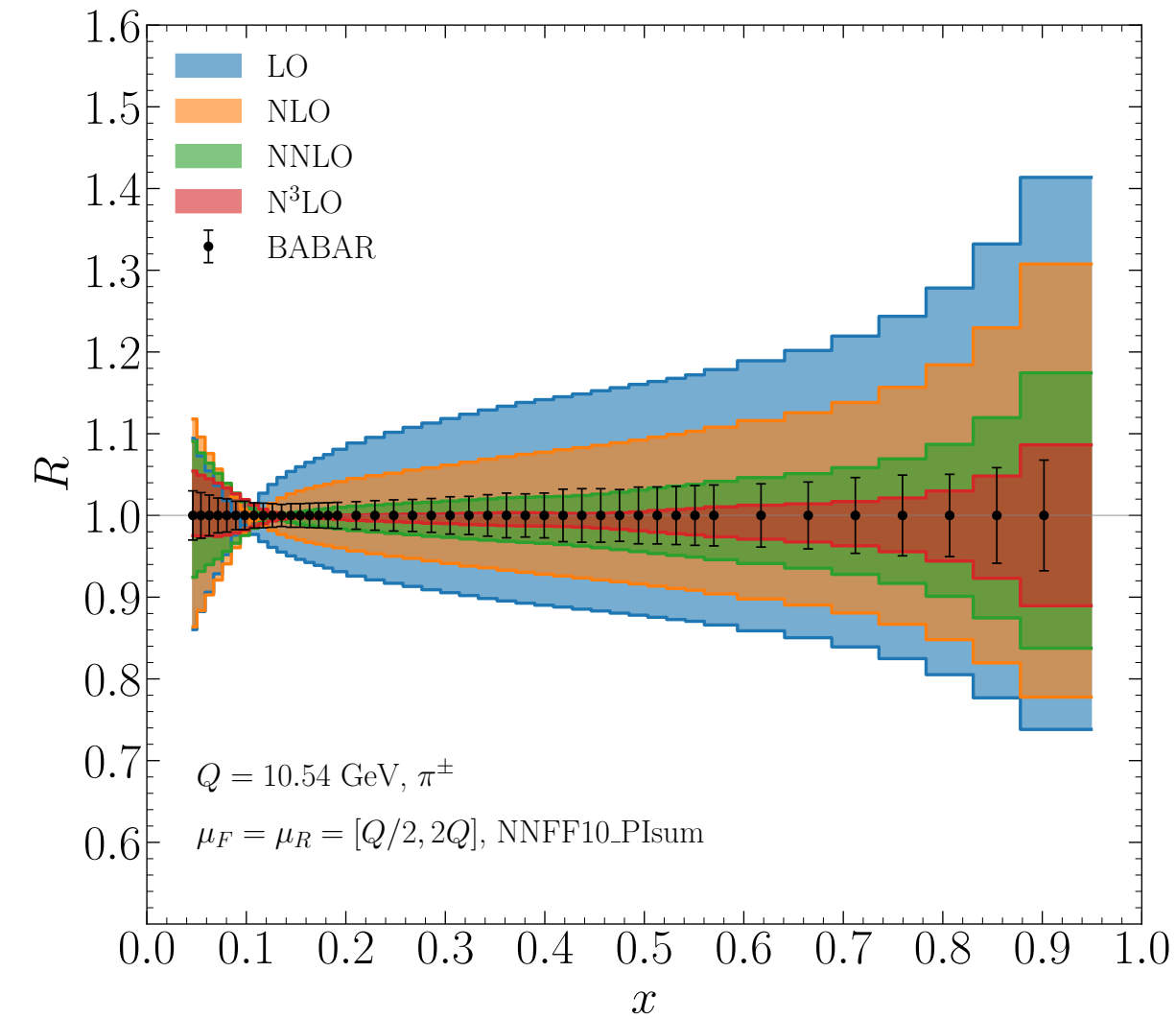
ID(1/x dx/dkv)	zd	zu	LO*{1,0.5,2}	NLO*{1,0.5,2}	NLO/LO
1 5.00000E-01	1.00000E+00	2.45041E-01 ..	2.61256E-01 ..	1.066 ..	
2 1.00000E+00	1.50000E+00	6.69355E-01 ..	7.61367E-01 ..	1.137 ..	
3 1.50000E+00	2.00000E+00	1.33705E+00 ..	1.61524E+00 ..	1.208 ..	
4 2.00000E+00	2.50000E+00	2.12904E+00 ..	2.51633E+00 ..	1.182 ..	
5 2.50000E+00	3.00000E+00	2.93954E+00 ..	3.23861E+00 ..	1.102 ..	
6 3.00000E+00	3.50000E+00	3.65409E+00 ..	3.60505E+00 ..	0.987 ..	
7 3.50000E+00	4.00000E+00	4.21952E+00 ..	3.12574E+00 ..	0.741 ..	
8 4.00000E+00	4.50000E+00	2.98053E+00 ..	1.44477E+00 ..	0.485 ..	

Progress on HO perturbative calculations

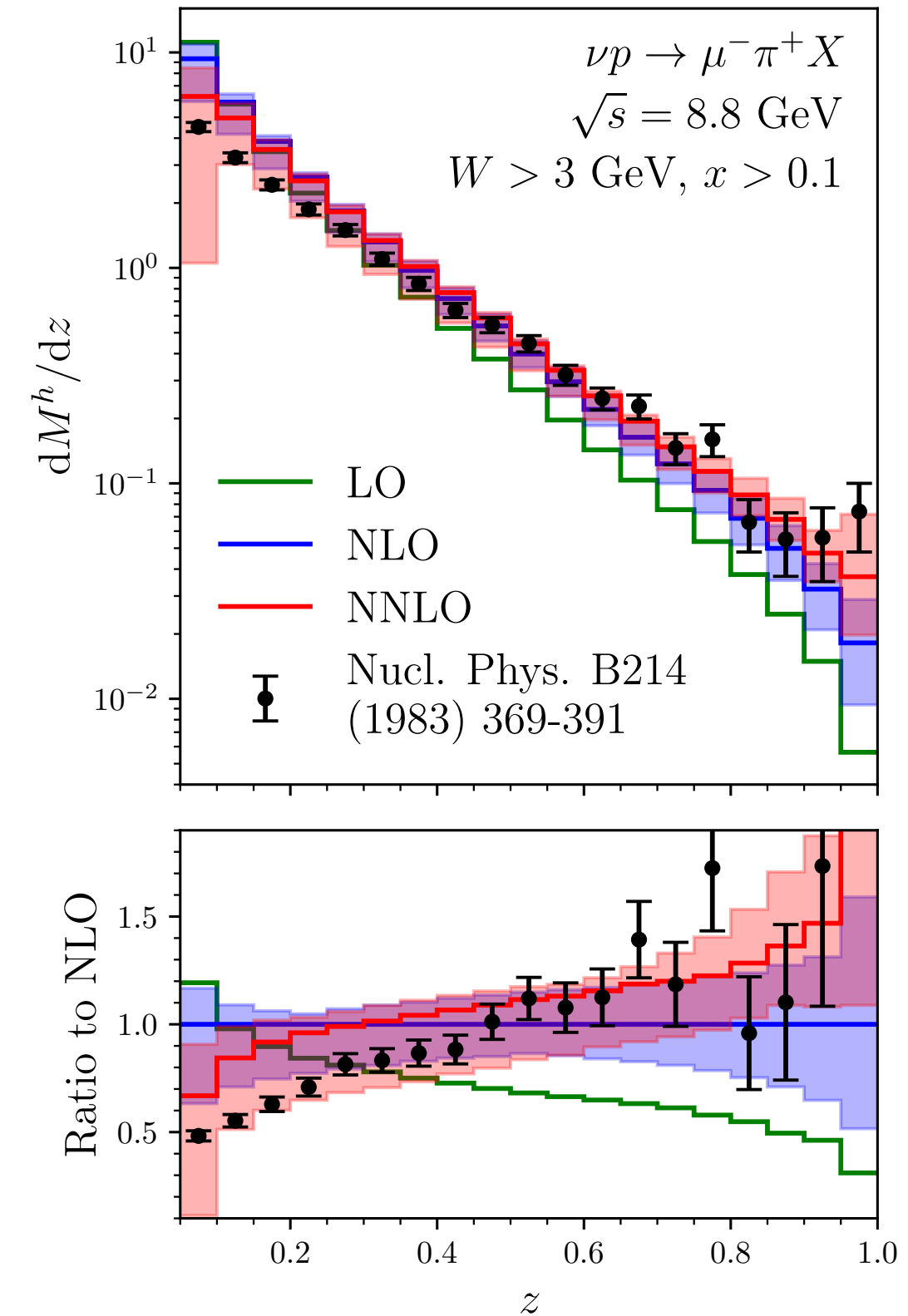
- ✦ In this year a lot of progresses have been made on higher-order perturbative calculations of coefficient functions on single hadron production at pp collision, CC-SIDIS and SIA, at NNLO or even N3LO, which we plan to implement in future



[Czakon, Mitov, Generet, Poncelet, 2503.11489]



[He, Xing, Yang, Zhu, 2503.20441]



[Bonino, Gehrmann, Stagnitto, Lochner, Schonwald, 2504.05376]

✦ 3. NPC23 FFs to light charged hadrons at NLO (pp/SIDIS/SIA)

Overview of the NPC23 analysis of FFs

- ✦ Establishing a new framework on global analysis of fragmentation functions to identified charged hadrons, including charged pion, kaon and proton, using most recent data from SIA, SIDIS, and pp collisions

parametrization of FFs to charged pion/kaon/
proton at an initial scale ($Q=5$ GeV):

$$zD_i^h(z, Q_0) = z^{\alpha_i^h} (1-z)^{\beta_i^h} \exp \left(\sum_{n=0}^m a_{i,n}^h (\sqrt{z})^n \right)$$

parton-to- π^+	avored	α	β	a_0	a_1	a_2	d.o.f.
u	Y						5
$d \simeq u$	Y	-	-		-	-	1
$\bar{u} = d$	N					x	4
$s = \bar{s} \simeq \bar{u}$	N	-				x	3
$c = \bar{c}$	N					x	4
$b = b$	N					x	4
g	N		F				4

parton-to- K^+	avored	α	β	a_0	a_1	a_2	d.o.f.
u	Y					x	4
$\bar{s} \simeq u$	Y	-	-		-	x	1
$\bar{u} = d = \bar{d} = s$	N					x	4
$c = \bar{c}$	N					x	4
$b = b$	N					x	4
g	N		F			x	3

parton-to- p	avored	α	β	a_0	a_1	a_2	d.o.f.
$u = 2d$	Y					x	4
$\bar{u} = d = s = \bar{s}$	N				x	x	3
$c = \bar{c}$	N					x	4
$b = b$	N					x	4
g	N		F			x	3

- ✦ a **joint determination** of FFs to charged pion, kaon and proton (via ratios or sum) at NLO in QCD (63 parameters) including estimation of uncertainties with Hessian sets
- ✦ apply a **strong selection criteria** on kinematics of the fragmentations to ensure validity of LT factorization and perturbative calculations ($E_h/p_{T,h} > 4$ GeV, and $z > 0.01$)
- ✦ including **theory uncertainties** (residual scale variations) into the covariance matrix
- ✦ use fast interpolation techniques as in FMNLO which largely increase efficiency of the fit

[JG, Liu, Shen, Xing, Zhao, 2401.02781 (PRL), 2407.04424 (PRD Editor's suggestion)]

most complete FFs of light hadrons with
inclusion of extensive data sets at high energies!

Selection of data

- ✦ For the first time the jet fragmentation data from LHC have been incorporated into the global analysis of FFs to light charged hadrons, including from processes of incl. jet, dijet, Z or photon tagged jet productions, due to the development of FMNLO

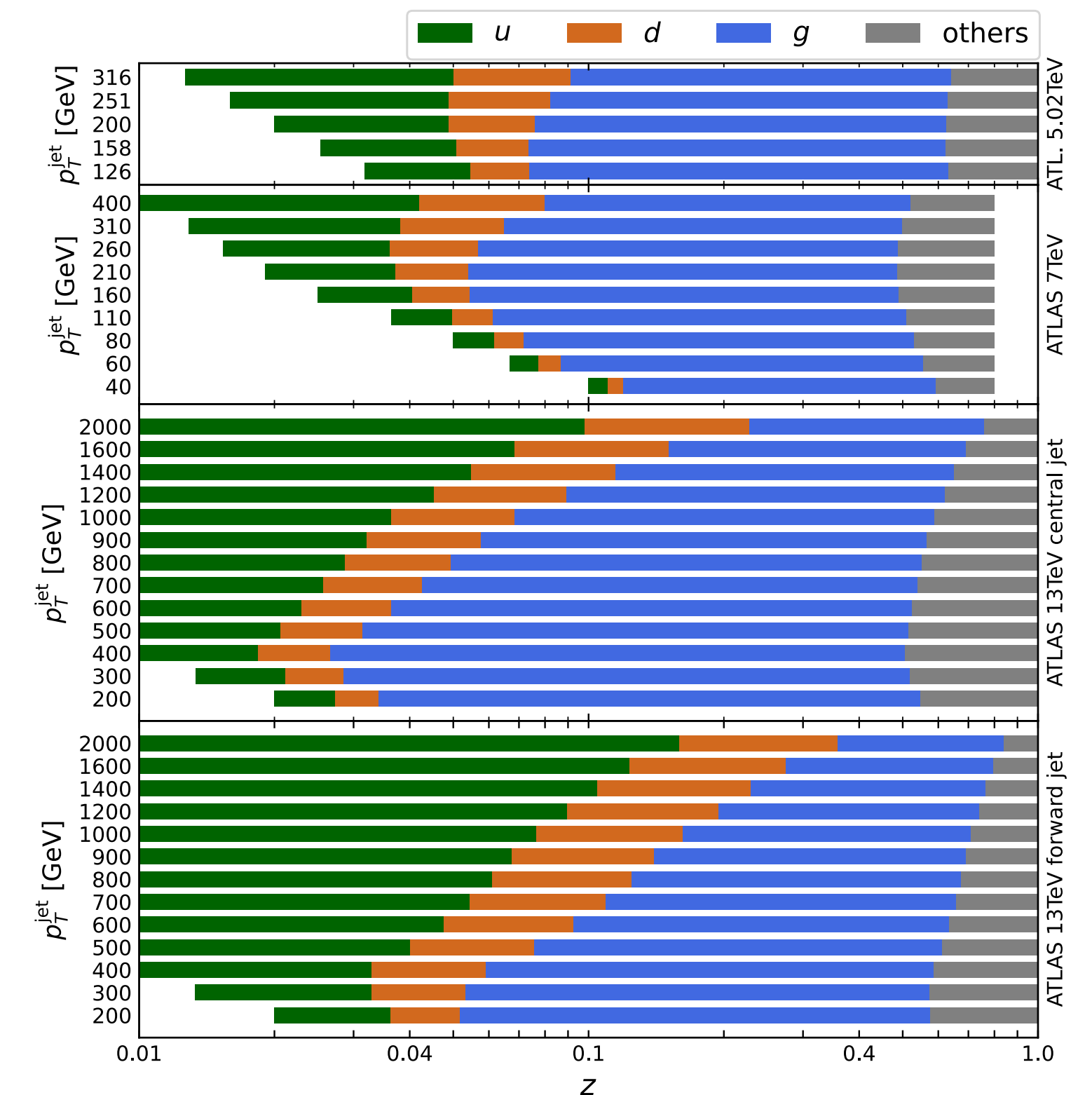
LHC measurements for hadron inside jet measurements (jet fragmentation)

exp.	$\sqrt{s}$ (TeV)	luminosity	hadrons	final states	R_j	cuts for jets/hadron	observable	N_{pt}
ATLAS[60]	5.02	25 pb ⁻¹	$h^\pm$	$\gamma + j$	0.4	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}$	$\frac{1}{N_{\text{jet}}} \frac{dN_{\text{ch}}}{dp_{T,h}}$	6
CMS[61]	5.02	27.4 pb ⁻¹	$h^\pm$	$\gamma + j$	0.3	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}, \Delta R_{h,j} < R_j$	$\frac{1}{N_{\text{jet}}} \frac{dN_{\text{ch}}}{d\xi}$	4
ATLAS[62]	5.02	260 pb ⁻¹	$h^\pm$	$Z + h$	no jet	$\Delta\phi_{h,Z} > \frac{3}{4}\pi$	$\frac{1}{n_Z} \frac{dN_{\text{ch}}}{dp_{T,h}}$	9
CMS[63]	5.02	320 pb ⁻¹	$h^\pm$	$Z + h$	no jet	$\Delta\phi_{h,Z} > \frac{7}{8}\pi$	$\frac{1}{n_Z} \frac{dN_{\text{ch}}}{dp_{T,h}}$	11
LHCb[64]	13	1.64 fb ⁻¹	$\pi^\pm, K^\pm, p/\bar{p}$	$Z + j$	0.5	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}, \Delta R_{h,j} < R_j$	$\frac{1}{n_Z} \frac{dN_{\text{ch}}}{d\xi}$	20
ATLAS[65]	5.02	25 pb ⁻¹	$h^\pm$	inc. jet	0.4	-	$\frac{1}{N_{\text{jet}}} \frac{dN_{\text{ch}}}{d\xi}$	63
ATLAS[66]	7	36 pb ⁻¹	$h^\pm$	inc. jet	0.6	$\Delta R_{h,j} < R_j$	$\frac{1}{N_{\text{jet}}} \frac{dN_{\text{ch}}}{d\xi}$	103
ATLAS[67]	13	33 fb ⁻¹	$h^\pm$	dijet	0.4	$p_T^{\text{lead}}/p_T^{\text{sublead}} < 1.5$	$\frac{1}{N_{\text{jet}}} \frac{dN_{\text{ch}}}{d\xi}$	280

❖ LHC measurements on hadron inside jet provide essential inputs for u/d/g flavor separation with wide kinematic coverages, both in energy scale Q and in momentum fraction z

❖ In dijets or inclusive jets production, low p_T and central (high p_T and forward) jets are mostly initiated by g(u-quark); Z or photon tagged jets are more likely from u/d quarks

kinematic/flavor coverage (LO) for ATLAS jet fragmentation



direct probe of gluon FFs!

Selection of data

- Other data include ratios of inclusive hadron production measured in pp collisions, single incl. hadron production from SIA (w/wo heavy-flavor tagging) mostly at Z-pole, and incl. hadron production in SIDIS from HERA and COMPASS, for identified or unidentified charged hadrons

incl. hadron production at RHIC and LHC (pp)

exp.	$\sqrt{s_{NN}}$ (TeV)	# events (million)	$p_{T,h}$	hadrons	observable	N_{pt}
ALICE[58]	13	40-60(pp)	[2, 20] GeV	π, K, p, K_S^0	$K/\pi, p/\pi, K_S^0/\pi$	49
ALICE[58]	7	150(pp)	[3, 20] GeV	π, K, p	13TeV/7TeV for π, K, p	37
ALICE[57]	5.02	120(pp)	[2, 20] GeV	π, K, p	$K/\pi, p/\pi$	34
ALICE[56]	2.76	40(pp)	[2, 20] GeV	π, K, p	$K/\pi, p/\pi$	27
STAR[68]	0.2	14(pp)	[3, 15] GeV	π, K, p, K_S^0	$K/\pi, p/\pi^+, \bar{p}/\pi^-, K_S^0/\pi, \pi^-/\pi^+, K^-/K^+$	60

incl. hadron production mostly at Z-pole (SIA)

exp.	$\sqrt{s}$	lum. (n_Z)	final states	hadrons	N_{pt}
OPAL[51]	m_Z	780 000	$Z \rightarrow q\bar{q}$	$\pi^\pm, K^\pm$	20
ALEPH[52]	m_Z	520 000	$Z \rightarrow q\bar{q}$	$\pi^\pm, K^\pm, p(\bar{p})$	42
DELPHI[53]	m_Z	1 400 000	$Z \rightarrow q\bar{q}$	$\pi^\pm, K^\pm, p(\bar{p})$	39
			$Z \rightarrow b\bar{b}$	$\pi^\pm, K^\pm, p(\bar{p})$	39
SLD[77]	m_Z	400 000	$Z \rightarrow q\bar{q}$	$\pi^\pm, K^\pm, p(\bar{p})$	66
			$Z \rightarrow b\bar{b}$	$\pi^\pm, K^\pm, p(\bar{p})$	66
			$Z \rightarrow c\bar{c}$	$\pi^\pm, K^\pm, p(\bar{p})$	66
TASSO[75]	34GeV	77 pb ⁻¹	inc. had.	$\pi^\pm, K^\pm, p(\bar{p})$	3
TASSO[75]	44GeV	34 pb ⁻¹	inc. had.	$\pi^\pm, \pi^0$	5
TPC[76]	29GeV	70 pb ⁻¹	inc. had.	$\pi^\pm, K^\pm$	12
OPAL[54]	201.7GeV	433 pb ⁻¹	inc. had.	$h^\pm$	17
DELPHI[55]	189GeV	157.7 pb ⁻¹	inc. had.	$\pi^\pm, K^\pm, p(\bar{p})$	9

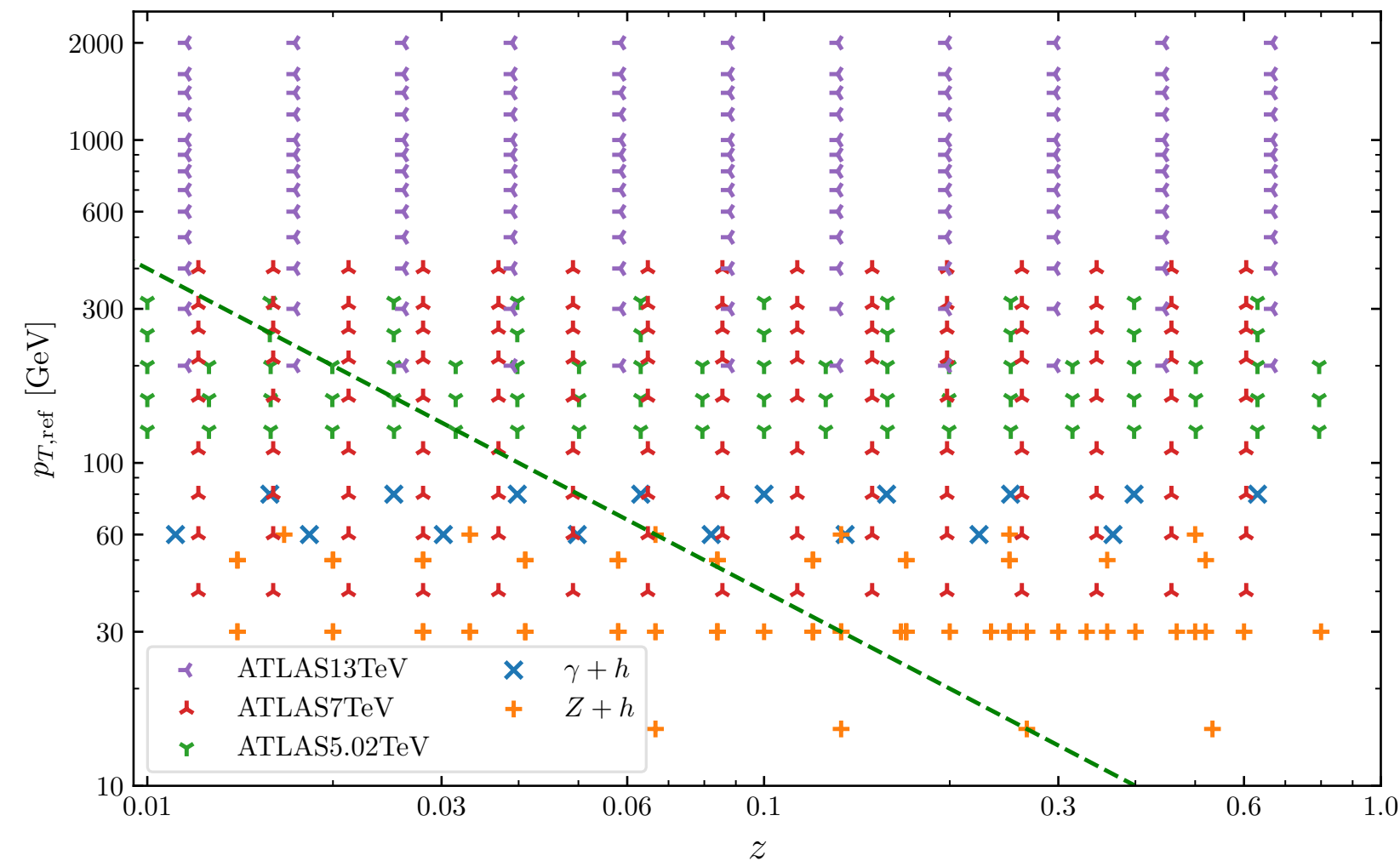
incl. hadron production at HERA and COMPASS (SIDIS)

exp.	$\sqrt{s}$ (GeV)	luminosity	kinematic cuts	hadrons	obs	N_{pt}
H1[69]	318	44 pb ⁻¹	$Q^2 \in [175, 20000]$ GeV ²	$h^\pm$	$D \equiv \frac{1}{N} \frac{dn_{h^\pm}}{dx_p}$	16
H1[70]	318	44 pb ⁻¹	$Q^2 \in [175, 8000]$ GeV ²	$h^\pm$	$A \equiv \frac{D^+ - D^-}{D^+ + D^-}$	14
ZEUS[71]	300,318	440 pb ⁻¹	$Q^2 \in [160, 40960]$ GeV ²	$h^\pm$	D	32
COMPASS06[72, 73]	17.3	540 pb ⁻¹	$x \in [0.14, 0.4], y \in [0.3, 0.5]$	π, K, h	$\frac{dM^h}{dz}$	124
COMPASS16[74]	17.3	-	$x \in [0.14, 0.4], y \in [0.3, 0.5]$	π, K, p	$\frac{dM^h}{dz}$	97

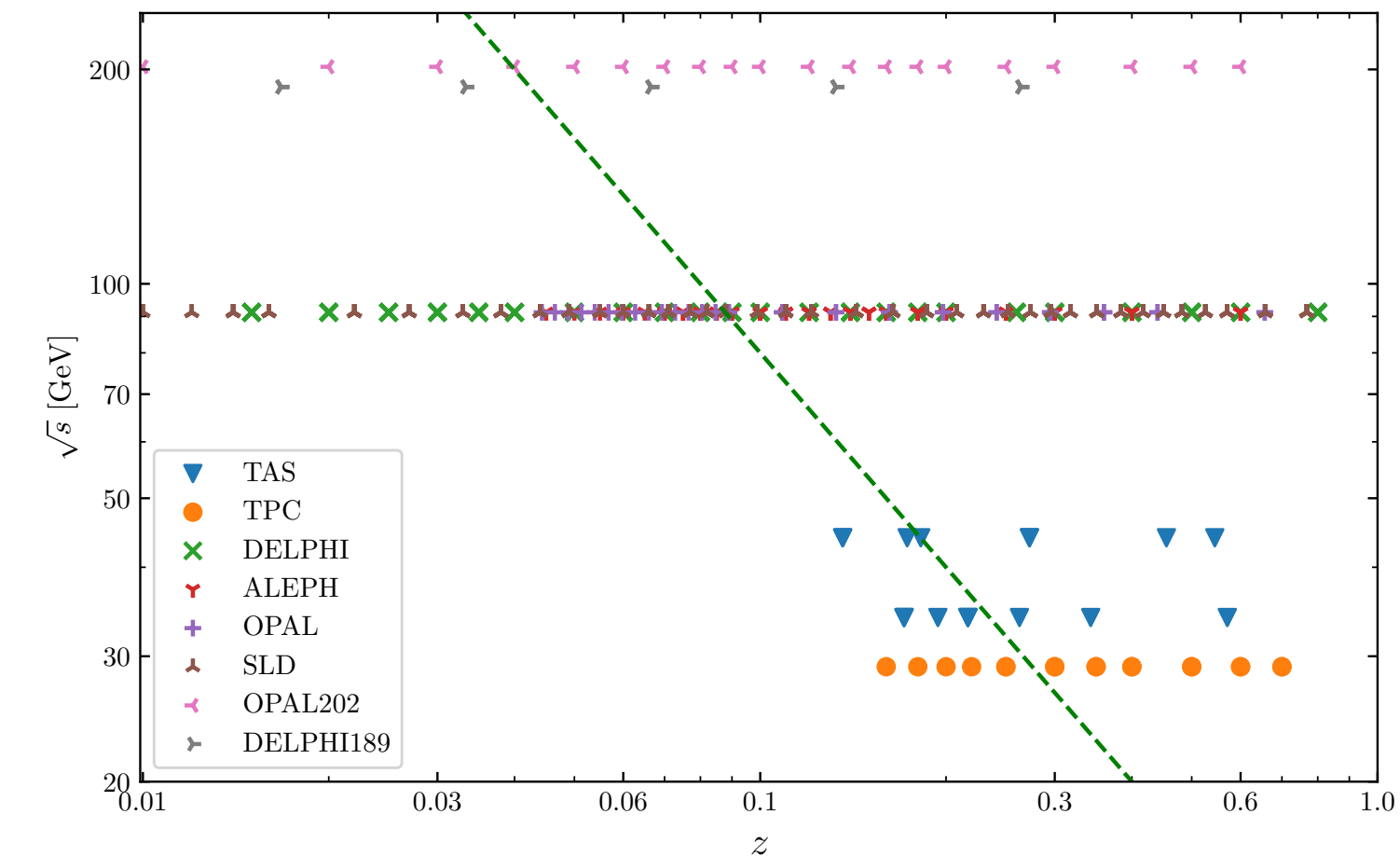
Selection of data

- Other data include ratios of inclusive hadron production measured in pp collisions, single incl. hadron production from SIA (w/wo heavy-flavor tagging) mostly at Z-pole, and incl. hadron production in SIDIS from HERA and COMPASS, for identified or unidentified charged hadrons

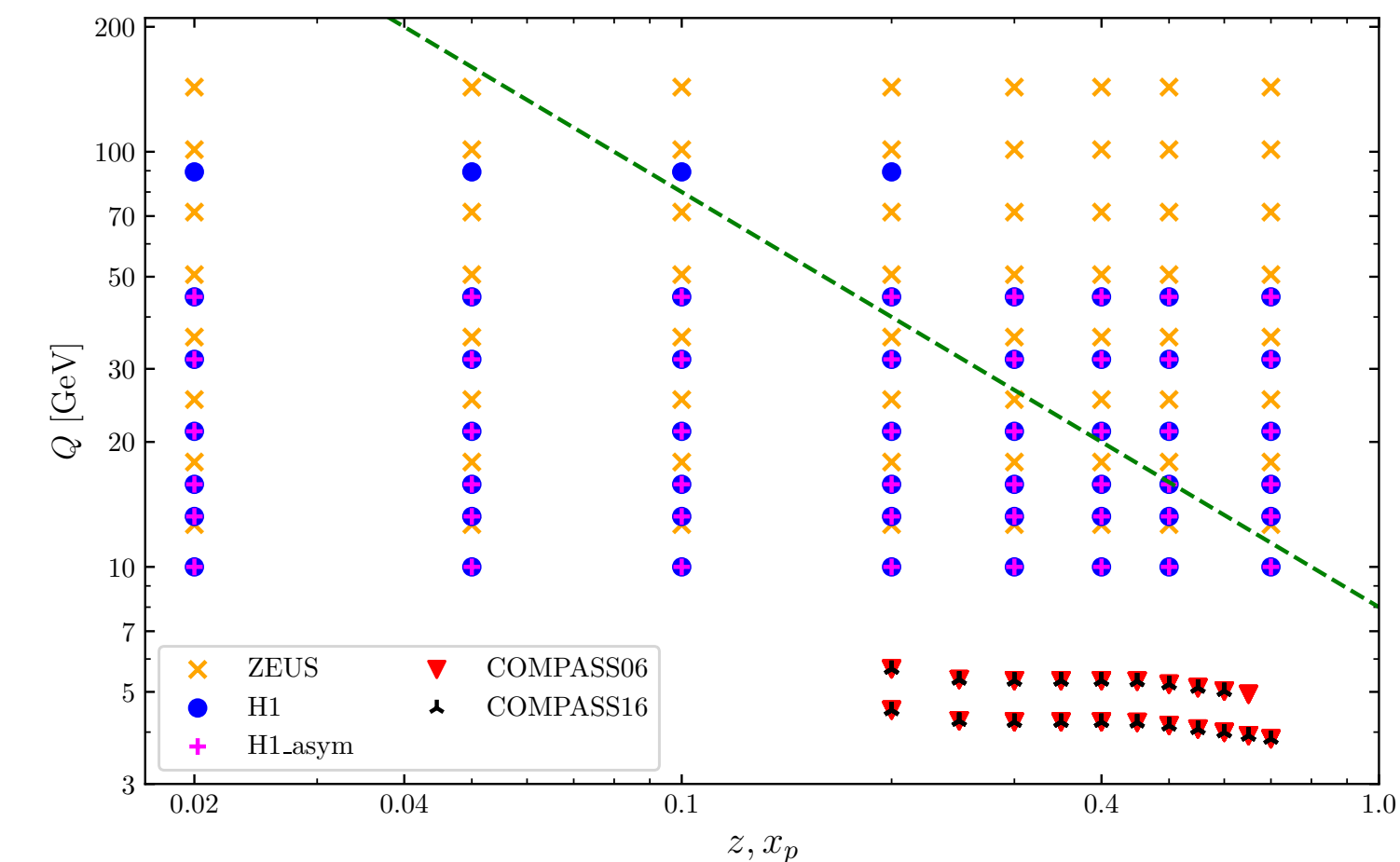
kinematic cut on LHC jet fragmentation



kinematic cut on SIA



kinematic cut on SIDIS



- after kinematic cuts (E_h or $p_{Th} > 4$ GeV) data probe z values down to 0.01 for LHC jet fragmentation, ~ 0.09 for SIA, and ~ 0.2 for HERA
- an exception is made for COMPASS data where we keep those bins with highest Q values though they failed the energy cut

Quality of the fit

- ✦ A best-fit with good agreements to the global data sets (1370 points in total) are found, χ^2/N well below 1; individual agreements to the 138 sub-datasets are also tested, motivating usage of a tolerance $\Delta\chi^2 \sim 2$ in determination of Hessian uncertainties

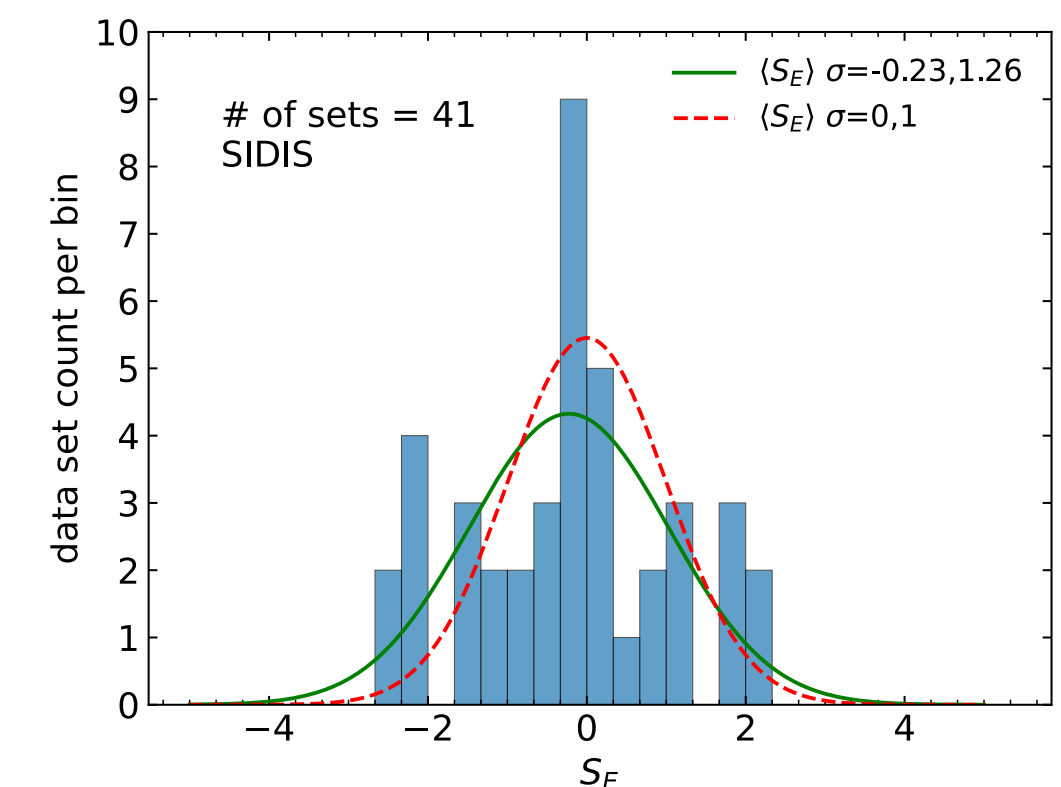
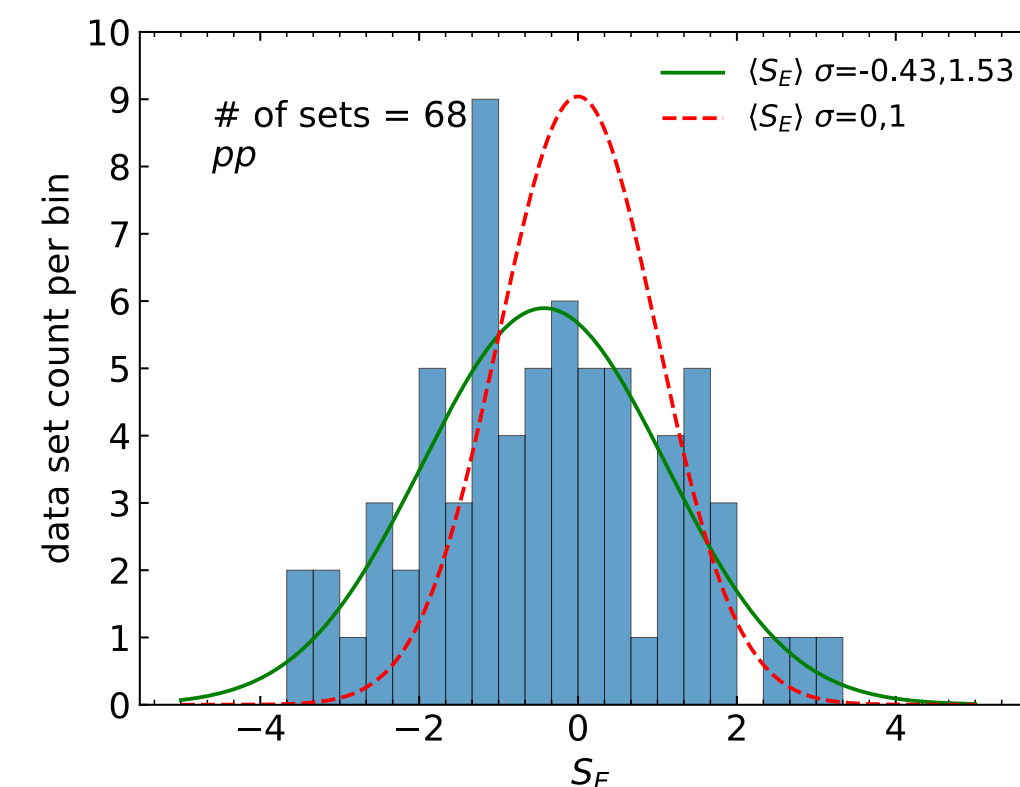
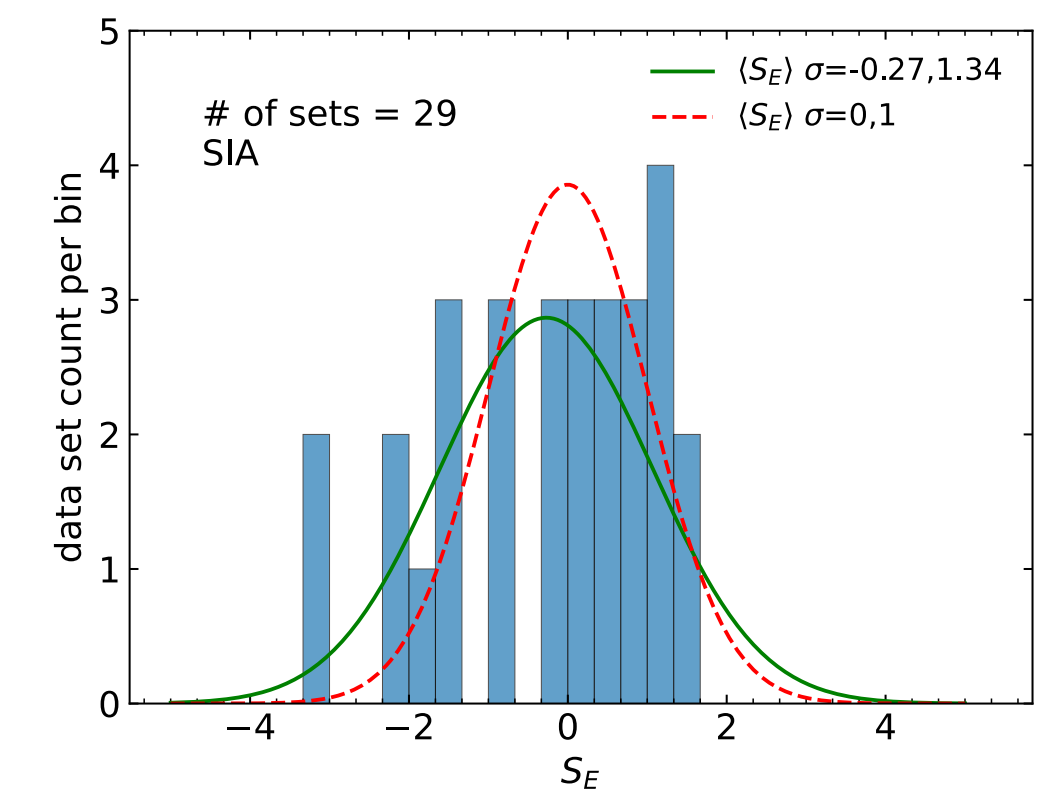
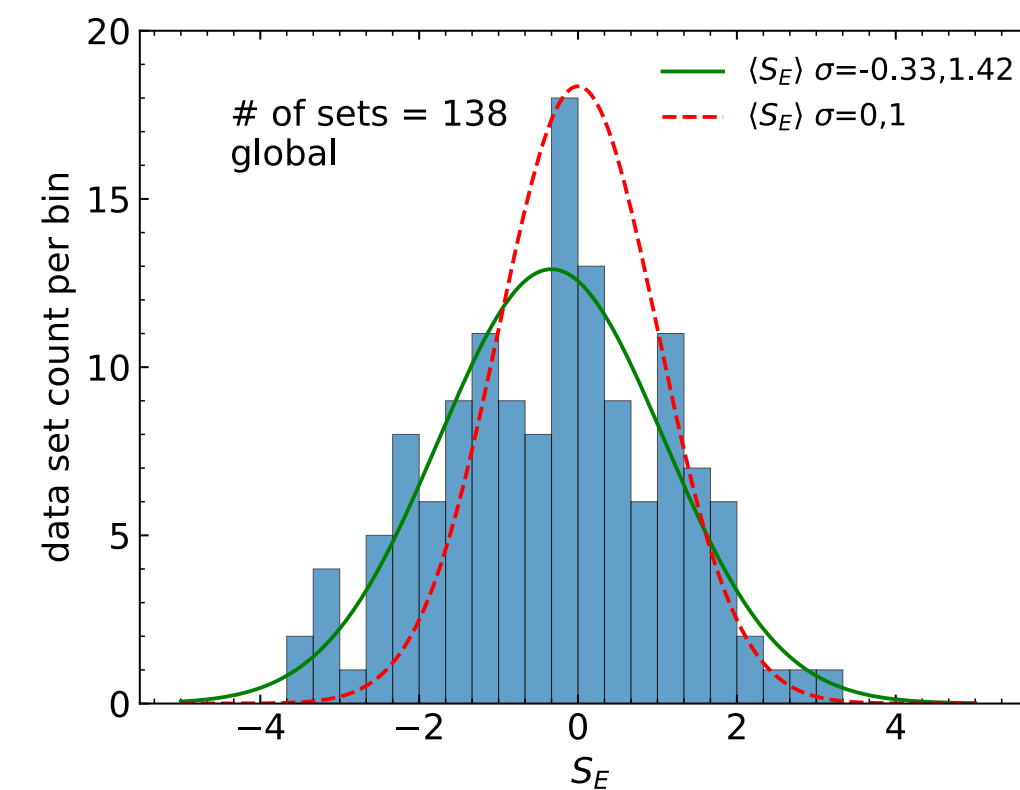
[CTEQ-TEA]

overall agreement: χ^2 breakdown to sub-groups for the best-fit

Experiments	N_{pt}	χ^2	χ^2/N_{pt}
ATLAS jets [†]	446	350.8	0.79
ATLAS Z/ γ +jet [†]	15	31.8	2.12
CMS Z/ γ +jet [†]	15	17.3	1.15
LHCb Z+jet	20	30.6	1.53
ALICE inc. hadron	147	150.6	1.02
STAR inc. hadron	60	42.2	0.70
pp sum	703	623.3	0.89
TASSO	8	7.0	0.88
TPC	12	11.6	0.97
OPAL	20	16.3	0.81
OPAL (202 GeV) [†]	17	24.2	1.42
ALEPH	42	31.4	0.75
DELPHI	78	36.4	0.47
DELPHI (189 GeV)	9	15.3	1.70
SLD	198	211.6	1.07
SIA sum	384	353.8	0.92
H1 [†]	16	12.5	0.78
H1 (asy.) [†]	14	12.2	0.87
ZEUS [†]	32	65.5	2.05
COMPASS (06I)	124	107.3	0.87
COMPASS (16p)	97	56.8	0.59
SIDIS sum	283	254.4	0.90
Global total	1370	1231.5	0.90

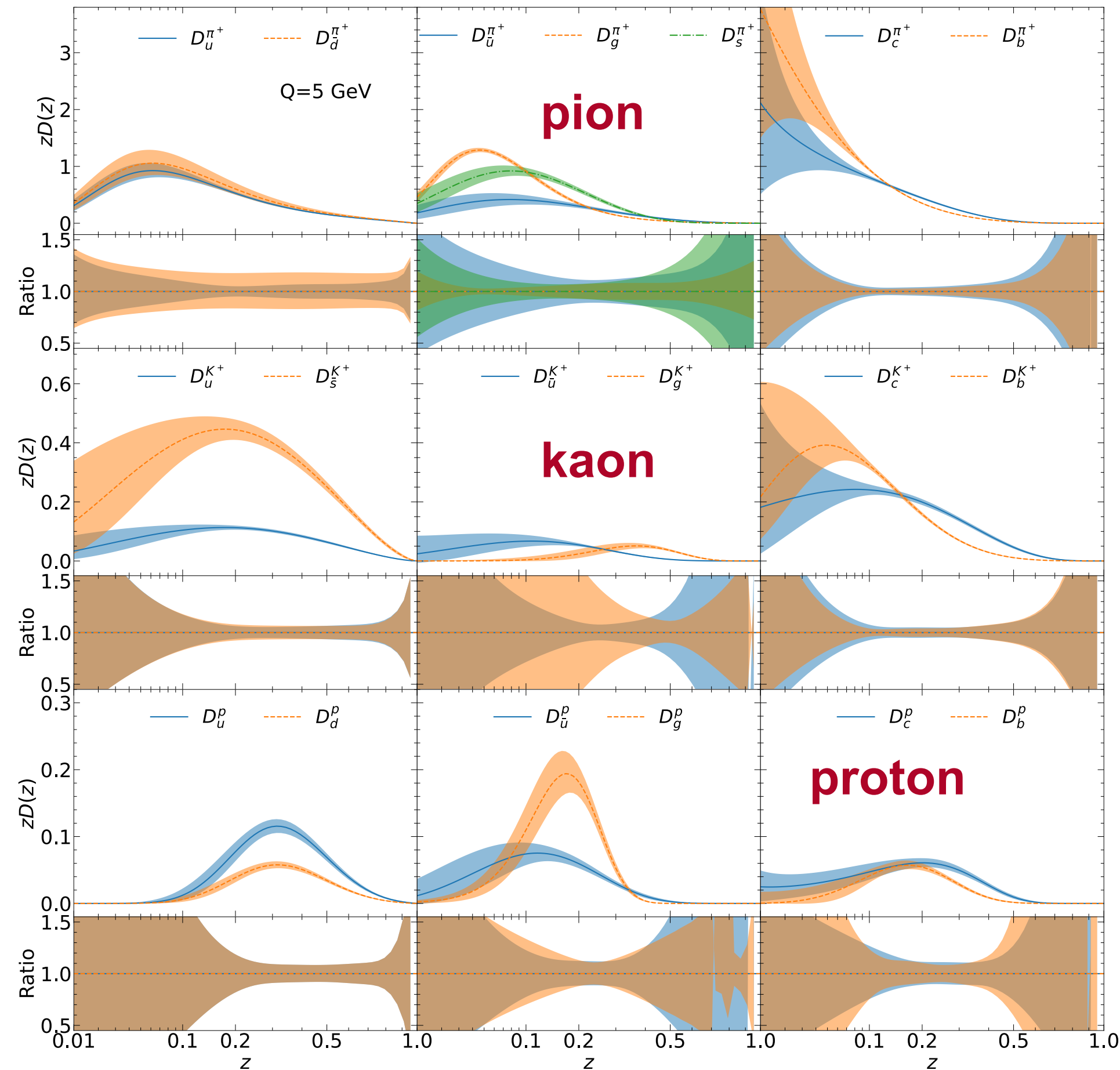
individual agreement: distributions of the effective Gaussian variable

$$S_E = \frac{(18N_{pt})^{3/2}}{18N_{pt} + 1} \left\{ \frac{6}{6 - \ln(\chi^2/N_{pt})} - \frac{9N_{pt} - 1}{9N_{pt}} \right\}$$



NPC23 FFs to charged hadrons

- ✦ We arrive at a best-fit of the charged pion, kaon and proton FFs together with 126 Hessian error FFs, two for each of the eigenvector direction; FFs are generally well constrained in the region with $z \sim 0.1-0.7$

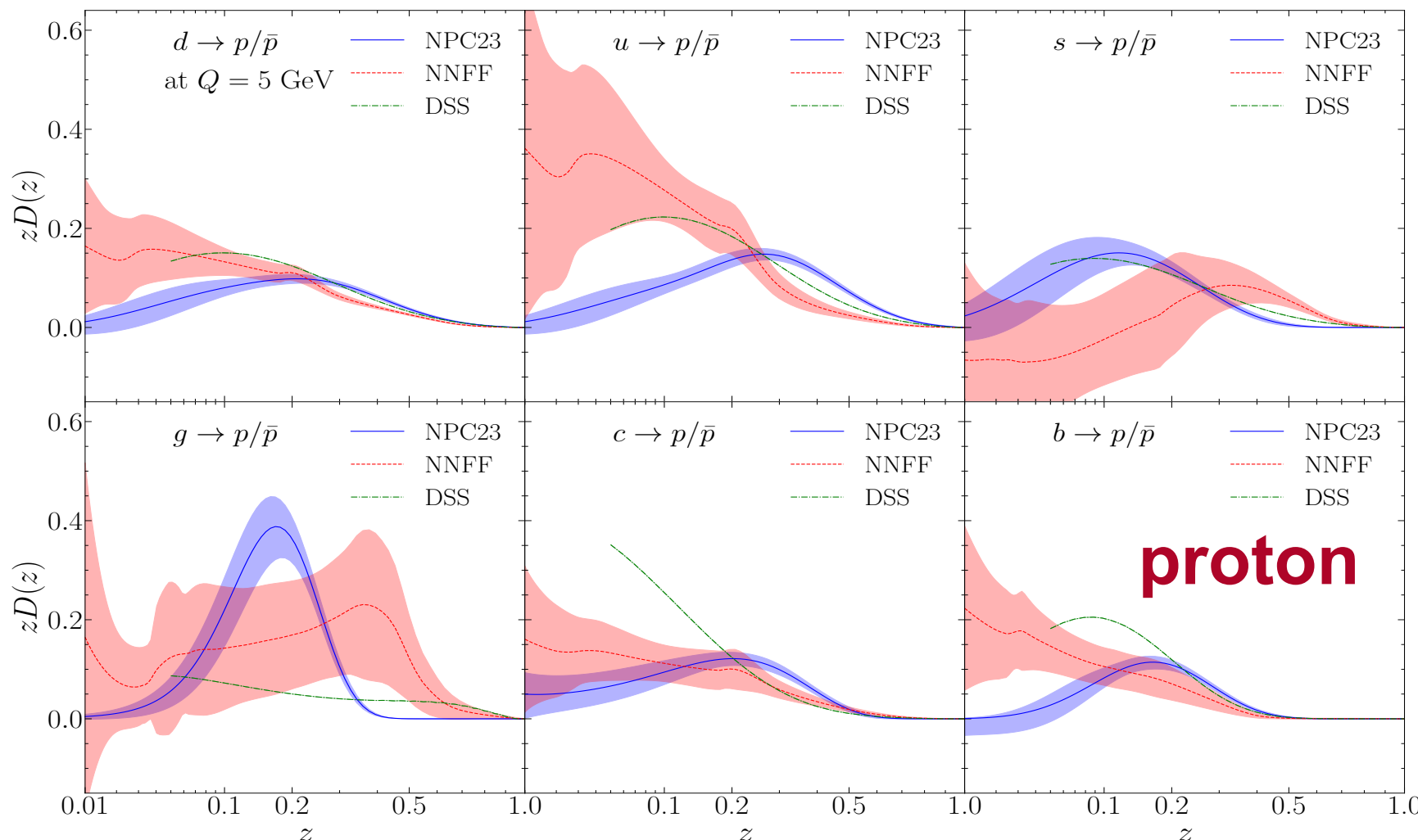
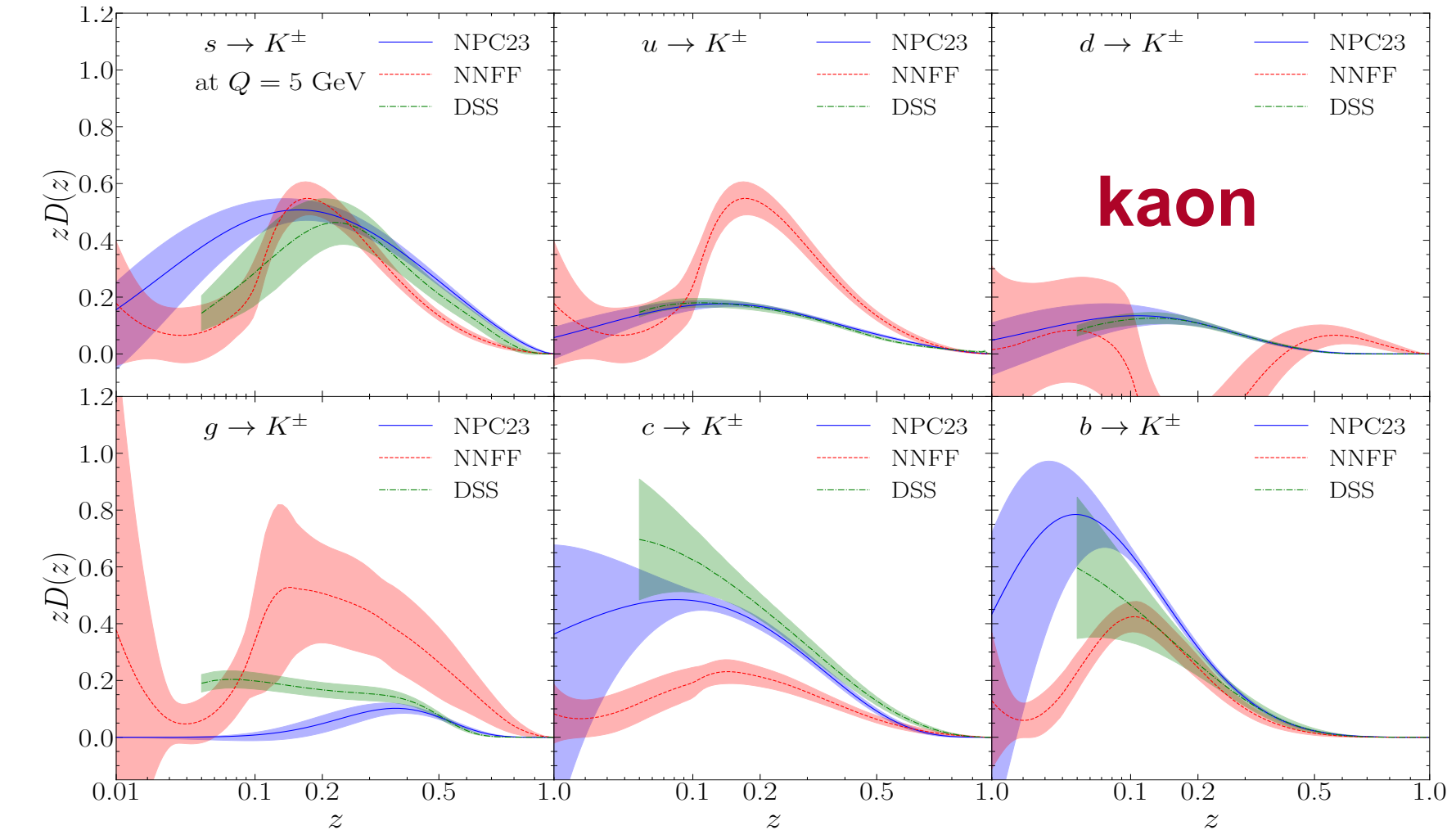
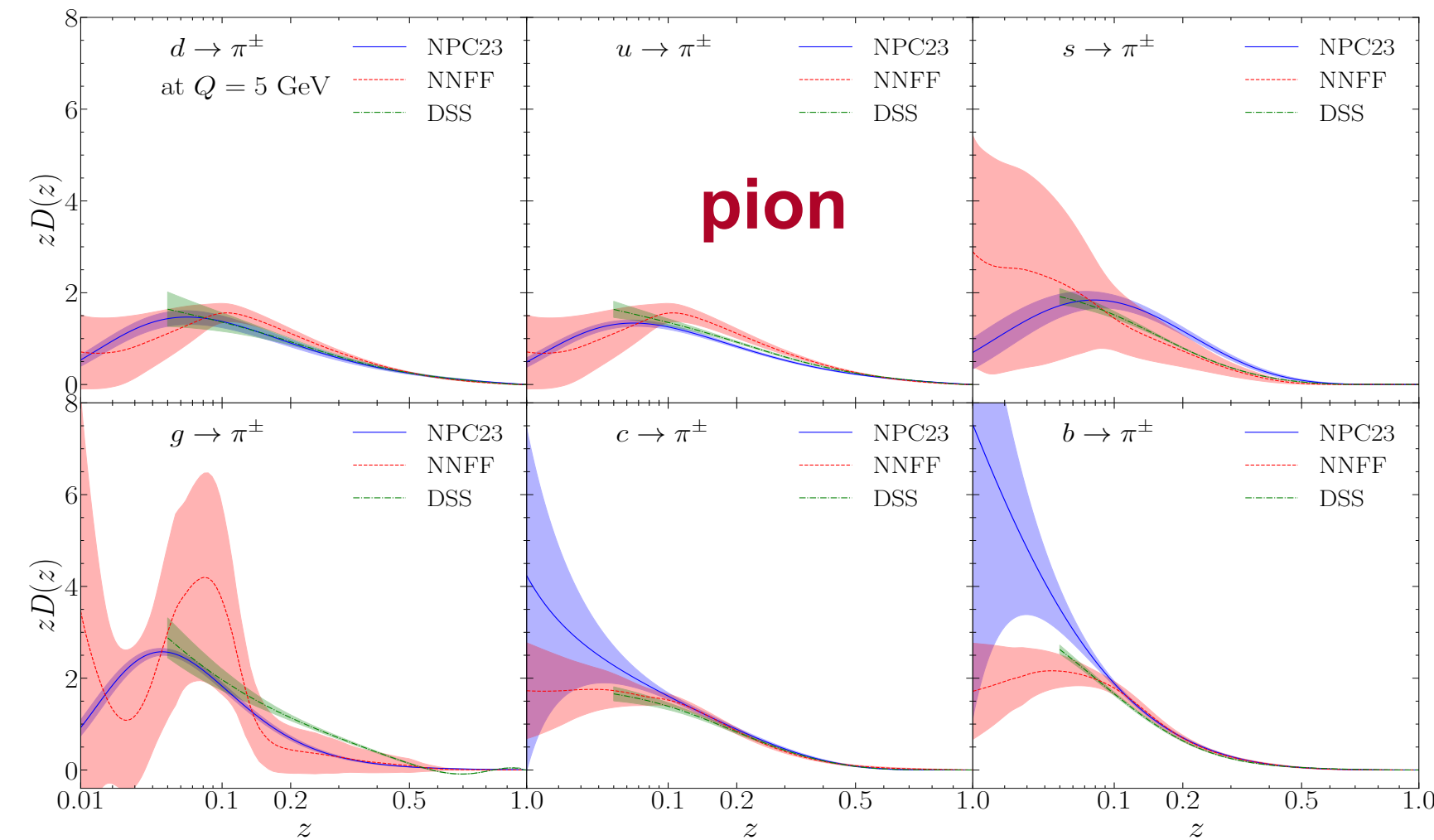


FFs (positively charged) vs. momentum fraction

- ✦ our results show an uncertainty of 3%, 4% and 8% for FFs of gluon to pion at $z=0.05, 0.1$ and 0.3 , respectively
- ✦ similarly an uncertainty of 4%, 4% and 7% for FFs of u-quark to pion, kaon and proton at $z=0.3$, respectively
- ✦ FFs of heavy-quarks are well constrained for z between $0.1 \sim 0.5$ due to the tagged SIA events of Z-pole measurements
- ✦ a preference for larger FFs of s quark to pion due to pulls from SIA data
- ✦ high precision of gluon FFs is mostly due to the data of jet fragmentation from the LHC

Comparison to other determinations

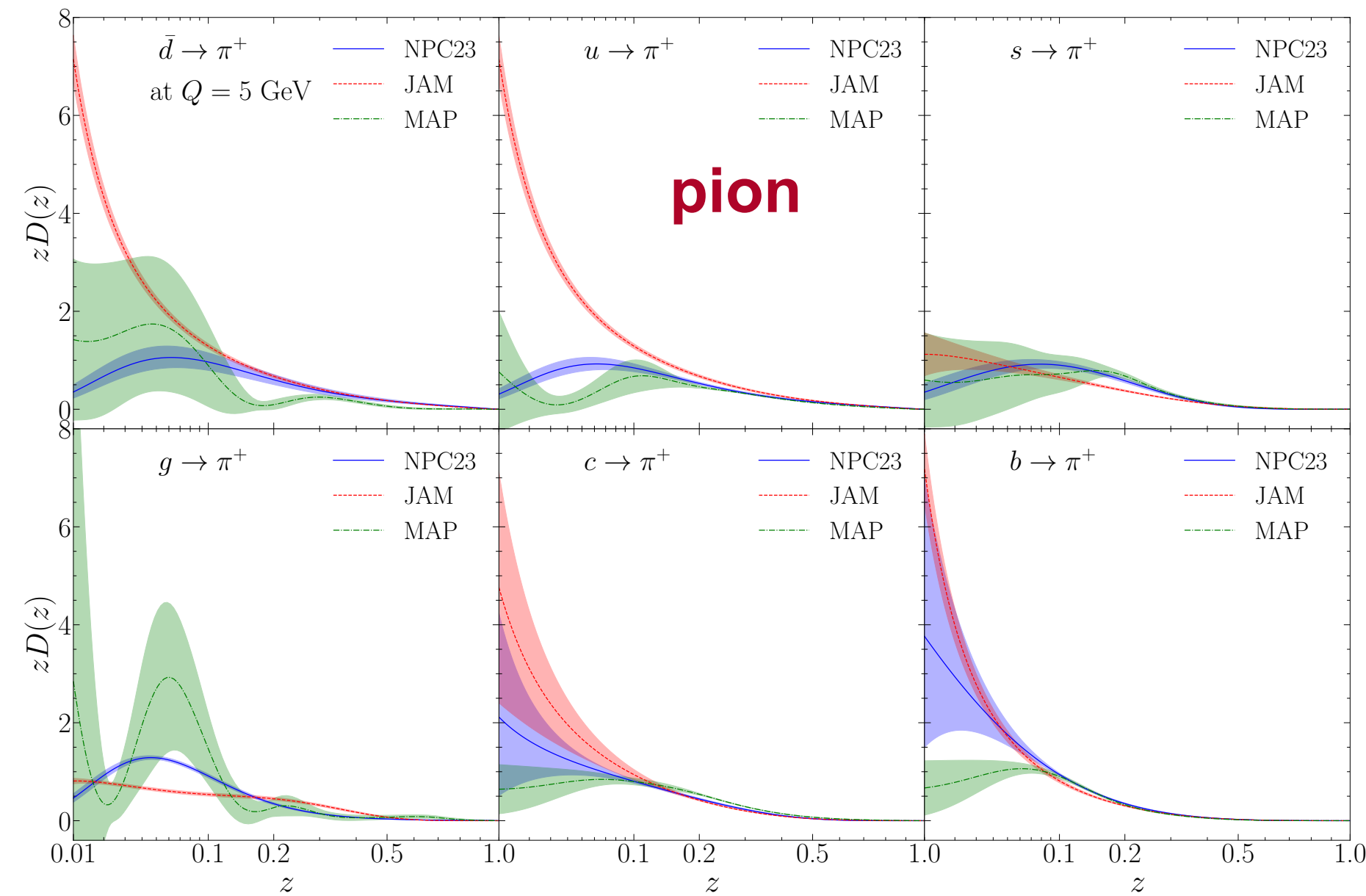
- Our new extractions on FFs are compared to previous determinations from other groups (e.g., DSS and NNFF) for the charge-summed pion, kaon and proton; discrepancies are found and further investigations will be needed
- FFs (charge-summed) vs. momentum fraction **[BDSSV21, DSS17, DSS07, NNFF1.0]**



- ❖ We find general agreement between ours with DSS for FFs of u and d quarks to pion, and of u, d and s quark to kaon
- ❖ however, discrepancies are found for FFs to protons and for FFs of gluon to all three charged hadrons
- ❖ NNFF1.0 show larger uncertainties in general since only SIA data were used
- ❖ future benchmark works involving different groups will be needed for investigation on discrepancies

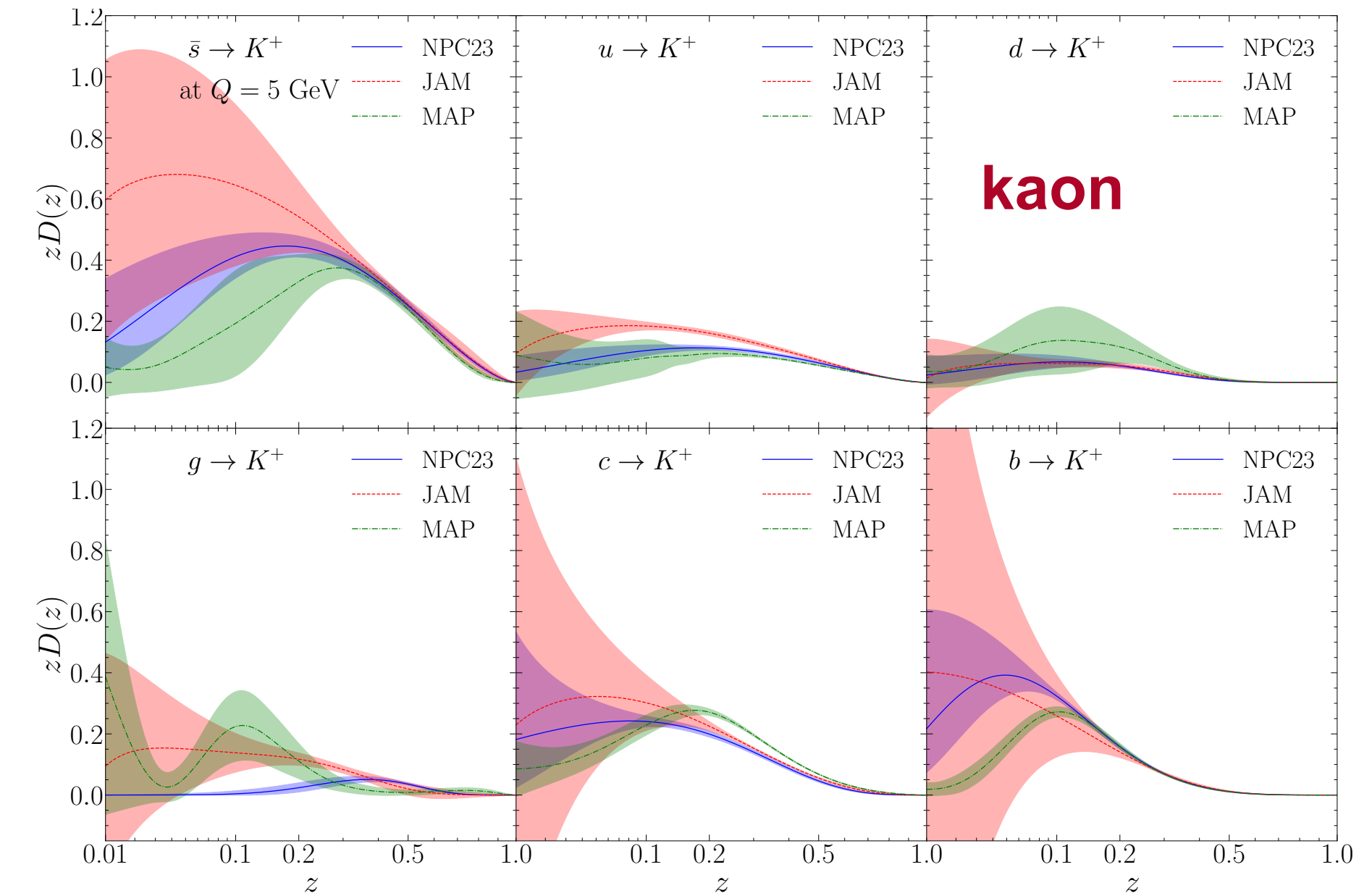
Comparison to other determinations

- Our new extractions on FFs are compared to previous determinations from other groups (e.g., JAM20 and MAPFF) for the positively charged pion and kaon; discrepancies are found and further investigations will be needed



FFs (positively charged) vs. momentum fraction

[JAM20, MAPFF]

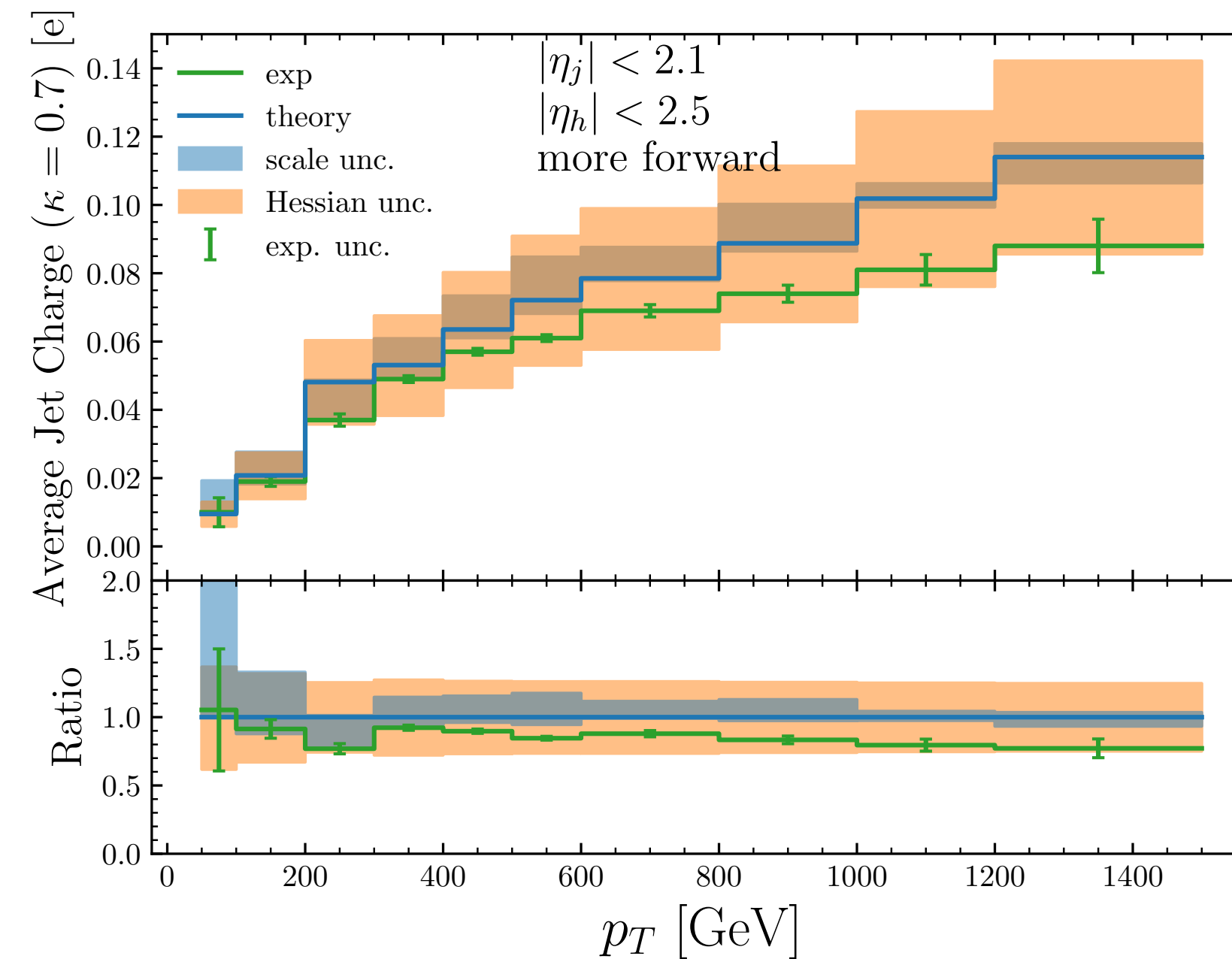
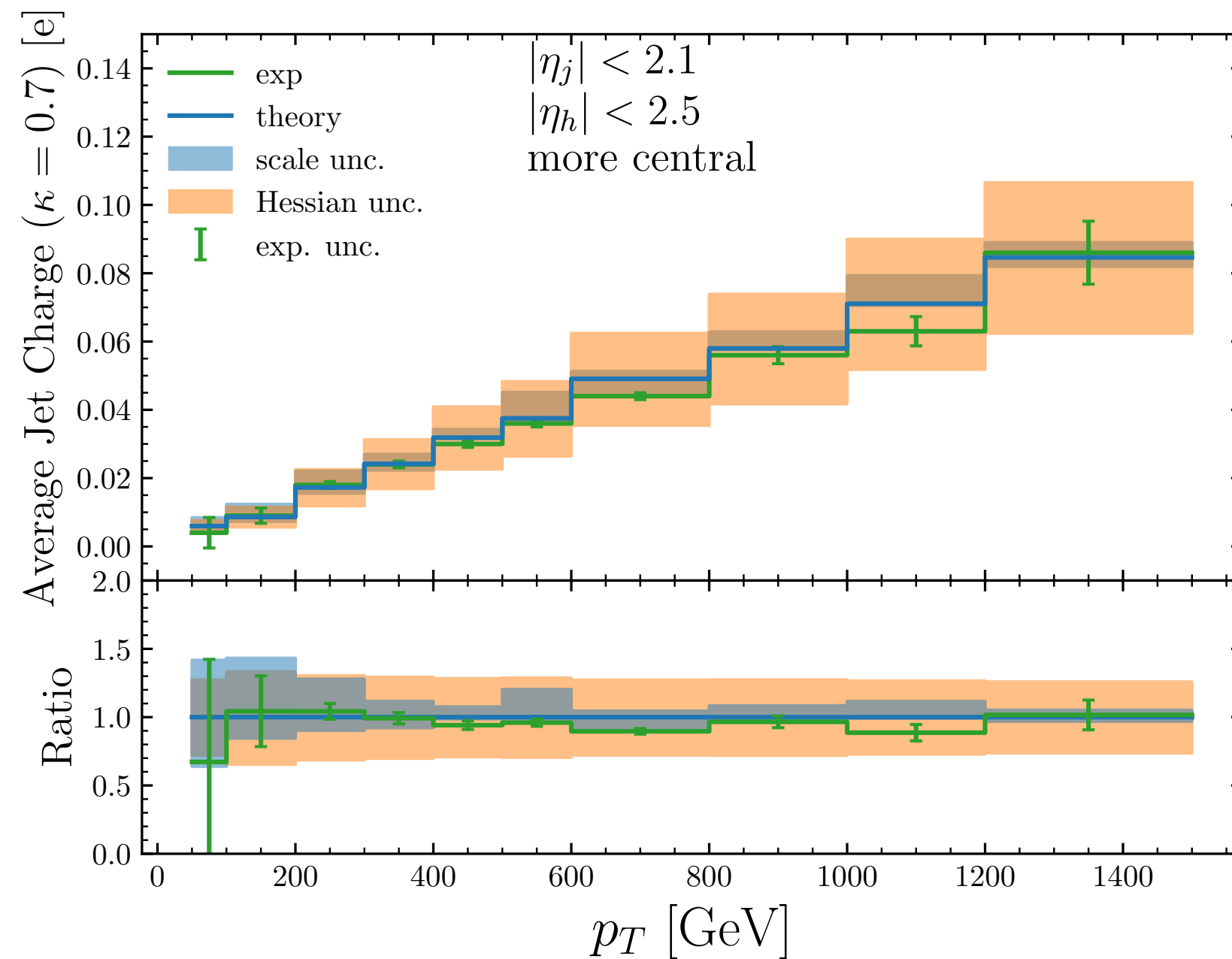


- similar agreements between ours with MAPFF for FFs of u and s quarks to pion, and with JAMFF for s and d quark to kaon
- discrepancies are found with JAMFF for FFs of u quark to pion at low z, and FFs of u quark to kaon
- FFs of charm quark to kaon are quite different for MAPFF as well as NNFF; spread of gluon FFs remains large

Average Jet charge at the LHC

- ✦ Our NLO predictions on average jet charge agree well with the ATLAS measurements; larger uncertainties from FFs comparing to experimental precision indicate potential strong constraints from data on jet charge

average jet charge for forward/central jet at LHC 8 TeV



[ATLAS 8 TeV]

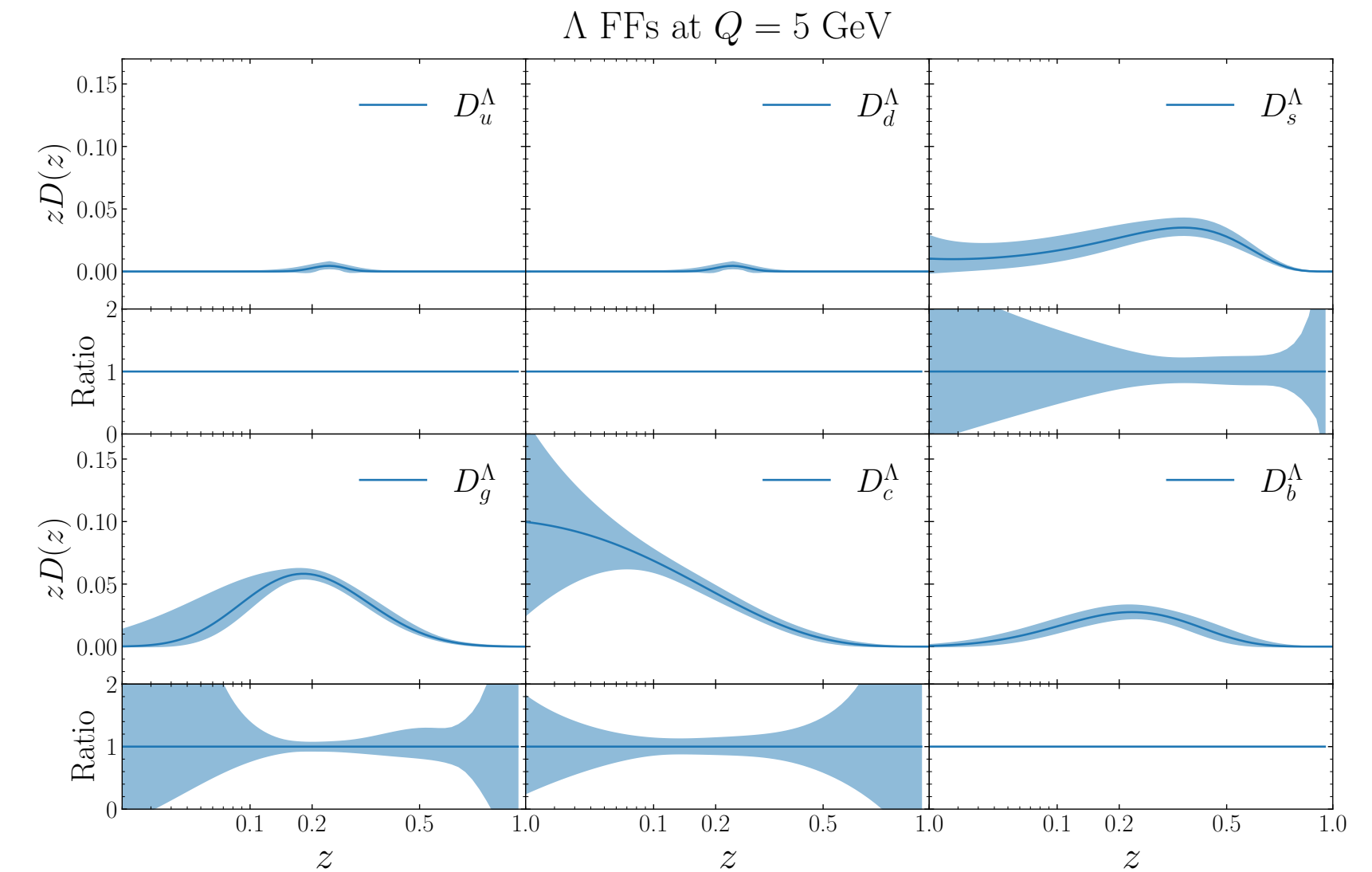
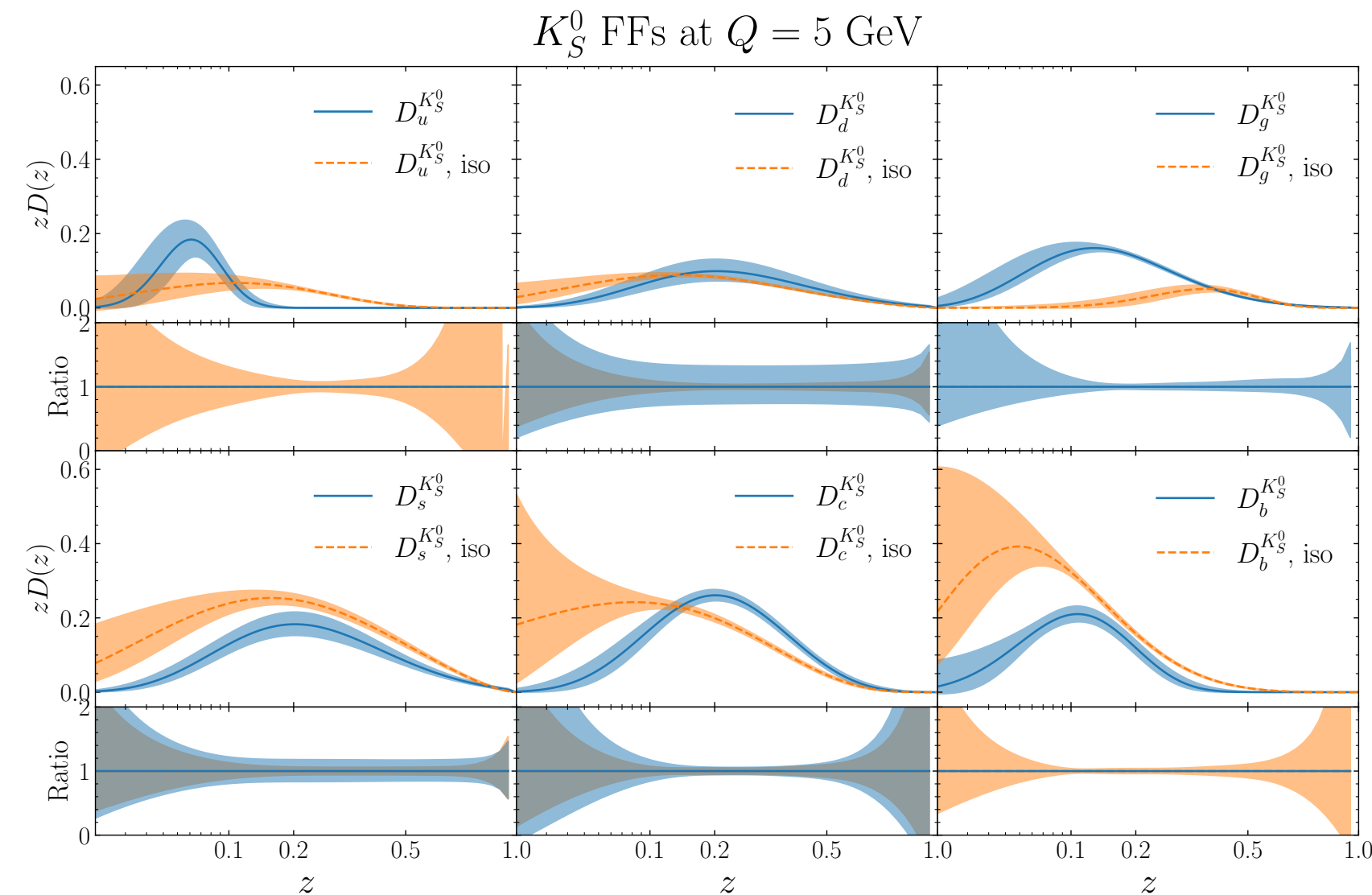
$$Q_J = \sum_{i \in J} \left(\frac{p_{T,i}}{p_{T,J}} \right)^\kappa Q_i$$

$$\langle Q_J \rangle = \int_{z_0}^1 dz_h z_h^\kappa \frac{1}{\sigma_J} \left(\frac{d\sigma_{h^+}}{dz_h} - \frac{d\sigma_{h^-}}{dz_h} \right)$$

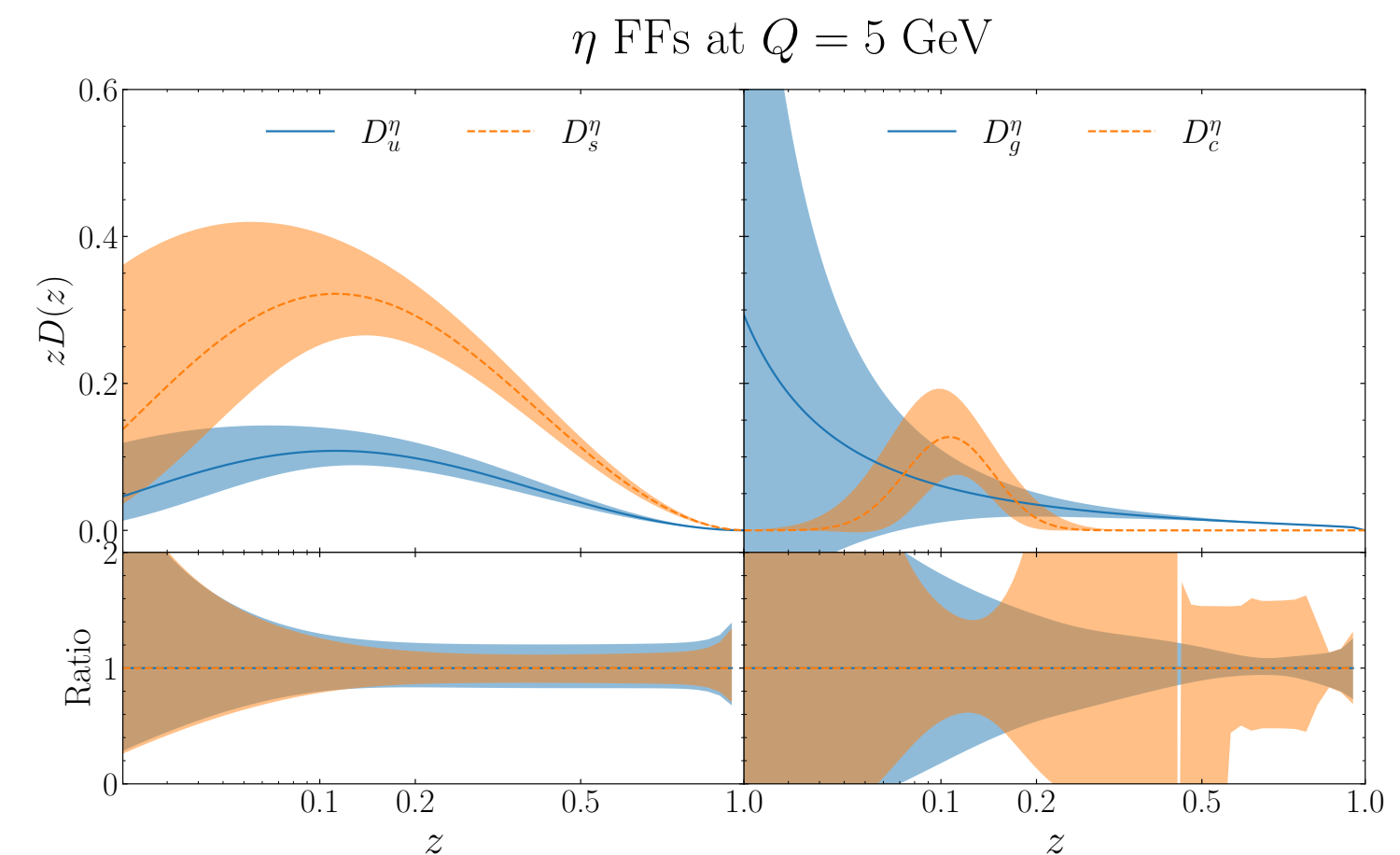
✦ 4. NPC23 FFs to light neutral hadrons at NLO (pp/SIDIS/SIA)

NPC23 analysis on neutral hadrons at NLO

- ✦ Global analysis of FFs to light neutral hadrons at NLO with high-energy data from SIA, SIDIS and pp collisions, including kaon, eta, lambda; FFs of neutral pion are constructed via isospin symmetry and find very good agreement with high-energy data
[JG, Li, Shen, Xing, Zhao, Zhou, Zhou, 2503.21311]



- ❖ similar setup as the charged hadron fit with high-energy data, except relaxing the SIA cut to $z > 0.05$
- ❖ FFs of K_S from direct fit are compared to those constructed via isospin symmetry [Chen, Liang, Pan, Song, Wei]; further measurements are needed for testing isospin symmetry in kaons FFs



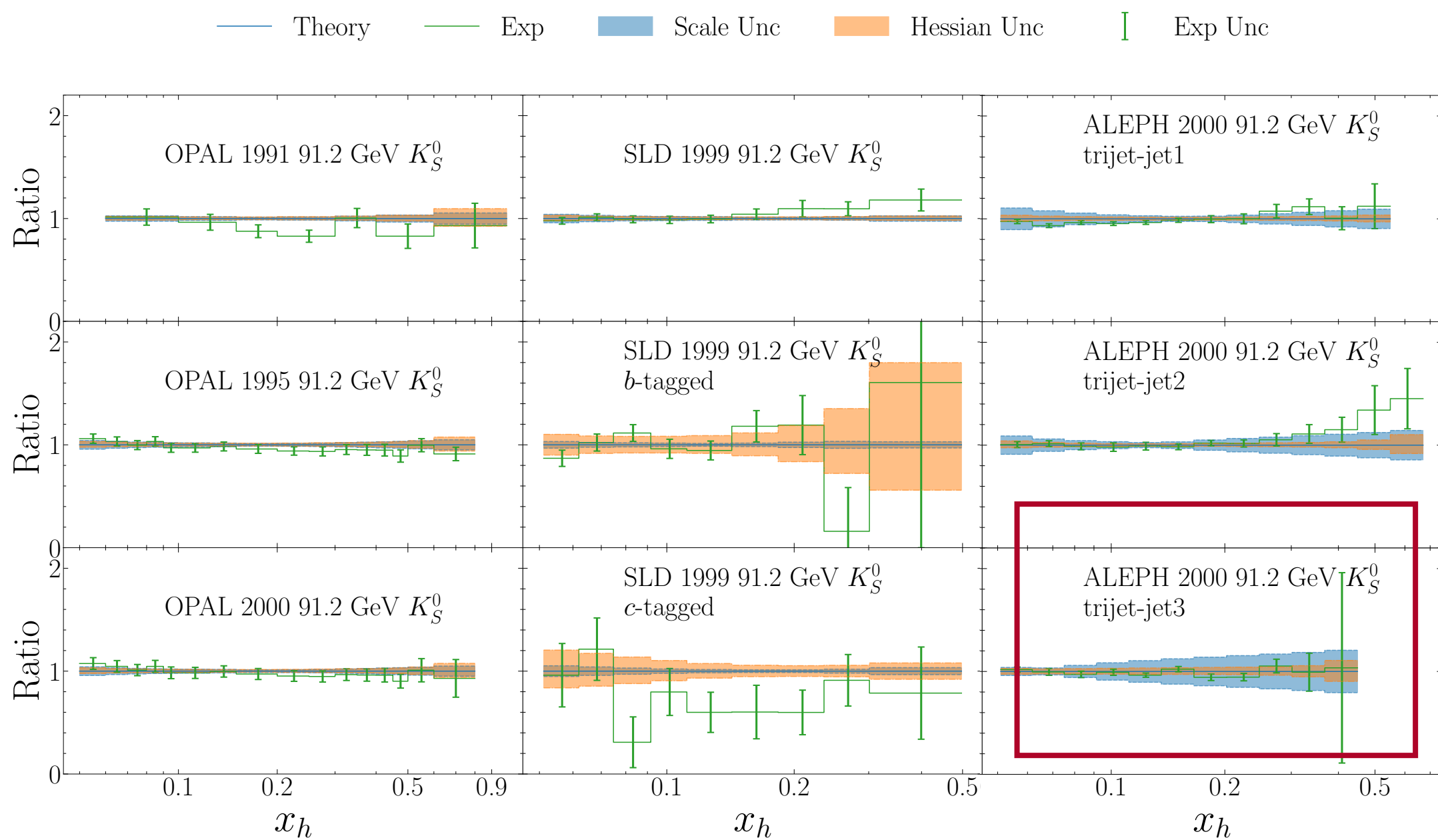
NPC23 analysis on neutral hadrons at NLO

- ✦ Data on neutral hadron production from SIDIS and pp are very much limited; while SIA data have poor flavor separations, e.g., d vs. s-quark in case of neutral kaons; direct constraints on gluon FFs from jet fragmentation in SIA

data sets in fit of neutral kaon

collaboration	year	$\sqrt{s}$ [GeV]	final state	observable	N_{pt}
TASSO [84]	1985	14, 22 & 34	K_S^0	$\frac{s}{\beta} \frac{d\sigma}{dx_h}$	9, 6 & 13
TASSO [85]	1990	14.8, 21.5, 34.5, 35 & 42.6	K_S^0	$\frac{s}{\beta} \frac{d\sigma}{dx_h}$	9, 6, 13, 13 & 13
TPC [82]	1984	29	K_S^0	$\frac{1}{\beta\sigma_h} \frac{d\sigma}{dx_h}$	8
MARK II [83]	1985	29	K_S^0	$\frac{1}{\beta\sigma_h} \frac{d\sigma}{dx_h}$	17
HRS [86]	1987	29	K_S^0	$\frac{s}{\beta} \frac{d\sigma}{dx_h}$	12
CELLO [87]	1990	35	K_S^0	$\frac{1}{\beta\sigma_h} \frac{d\sigma}{dx_h}$	9
TOPAZ [88]	1995	58	K_S^0	$\frac{1}{\sigma_h} \frac{d\sigma}{d\xi}$	4
OPAL [77]	1991	91.2	K_S^0	$\frac{1}{\beta\sigma_h} \frac{d\sigma}{dx_h}$	7
OPAL [78]	1995	91.2	K_S^0	$\frac{1}{\sigma_h} \frac{d\sigma}{dx_h}$	16
OPAL [79]	2000	91.2	K_S^0	$\frac{1}{\sigma_h} \frac{d\sigma}{dx_h}$	16
ALEPH [80]	1998	91.2	K_S^0	$\frac{1}{\sigma_h} \frac{d\sigma}{dx_p}$	16
ALEPH [89]	2000	91.2	K_S^0	$\frac{1}{\sigma_h} \frac{d\sigma}{d\xi}$	14
ALEPH [89]	2000	91.2	$K_S^0(\text{jet})$	$\frac{1}{\sigma_{\text{jet}}} \frac{d\sigma}{d\xi}$	12, 13 & 11
DELPHI [81]	1995	91.2	K_S^0	$\frac{1}{\sigma_h} \frac{d\sigma}{dx_p}$	13
SLD [73]	1999	91.2	K_S^0	$\frac{1}{\sigma_h} \frac{d\sigma}{dx_p}$	9
SLD (tagged) [73]	1999	91.2	$K_S^0(\text{tagged})$	$\frac{1}{\sigma_h} \frac{d\sigma}{dx_p}$	9 & 9

collaboration	year	$\sqrt{s}$ [GeV]	final state	observable	N_{pt}
ZEUS [71]	2012	318	K_S^0	$\frac{1}{N_{\text{DIS}}} \frac{dN_{K_S^0}}{dz_p}$	5, 5 & 2
ALICE [92]	2021	13000 & 7000	K_S^0	$\frac{dN_{K_S^0}^{13 \text{ TeV}}}{dp_T} \bigg/ \frac{dN_{K_S^0}^{7 \text{ TeV}}}{dp_T}$	10
ALICE [92]	2021	13000	K_S^0 & $\pi^\pm$	$\frac{dN_{K_S^0}}{dp_T} \bigg/ \frac{dN_{\pi^\pm}}{dp_T}$	15



- ✦ assumption on hierarchy of FFs of s and d-quarks to neutral kaon are made in order to obtain a stable fit
- ✦ fragmentation of the third jet in trijet production measured at ALEPH leads to strong constraints on the gluon FFs

Test on momentum sum rule

- FFs have the interpretation of number densities of hadrons and satisfy various fundamental sum rules as derived from first principle, including momentum sum rule, charge sum rule, etc. **[Collins, Rogers, 2023]**; momentum sum rules are tested with the extracted FFs

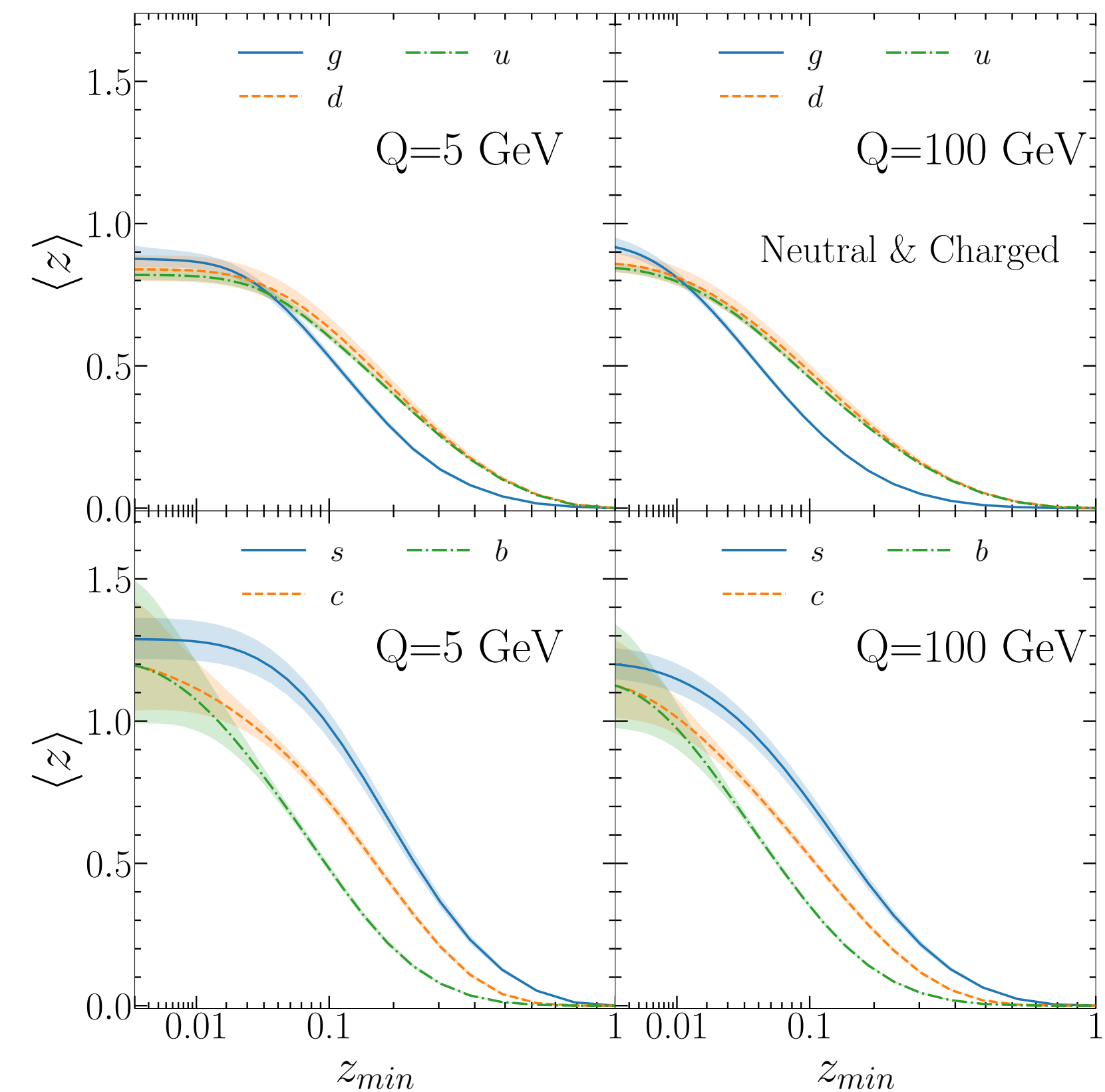
momentum sum rule:
$$\sum_h \int_0^1 dz z D_i^h(z, Q) = 1$$

with finite cutoff:
$$\langle z \rangle_i^h = \int_{z_{\min}}^1 dz z D_i^h(z, Q)$$

hadron	$g (z_{\min} = 0.05)$	$d (z_{\min} = 0.05)$	$u (z_{\min} = 0.05)$	$s (z_{\min} = 0.05)$
$K_S^0 + K_L^0$	$0.0932^{+0.0058}_{-0.0037}$	$0.0857^{+0.0285}_{-0.0229}$	$0.0187^{+0.0057}_{-0.0053}$	$0.1586^{+0.0270}_{-0.0241}$
$\Lambda + \bar{\Lambda}$	$0.0357^{+0.0050}_{-0.0029}$	$0.0008^{+0.0007}_{-0.0005}$	$0.0008^{+0.0007}_{-0.0005}$	$0.0331^{+0.0071}_{-0.0056}$
η	$0.0192^{+0.0083}_{-0.0077}$	$0.0420^{+0.0092}_{-0.0068}$	$0.0420^{+0.0092}_{-0.0068}$	$0.1247^{+0.0189}_{-0.0174}$
π^0	$0.1577^{+0.0072}_{-0.0072}$	$0.1829^{+0.0076}_{-0.0065}$	$0.1829^{+0.0076}_{-0.0065}$	$0.1954^{+0.0160}_{-0.0156}$
sum	$0.3058^{+0.0134}_{-0.0115}$	$0.3113^{+0.0309}_{-0.0248}$	$0.2443^{+0.0133}_{-0.0108}$	$0.5119^{+0.0373}_{-0.0340}$

momentum sharing to neutral hadrons

- total momentum sharing of strange quark to light hadrons turns to be above one as caused by the unexpected large FFs of s-quark to pions; further investigations are needed



total momentum vs. cutoff, for light charged and neutral hadrons

✦ 5. NPC23 FFs to light charged hadrons at NNLO (SIDIS/SIA)

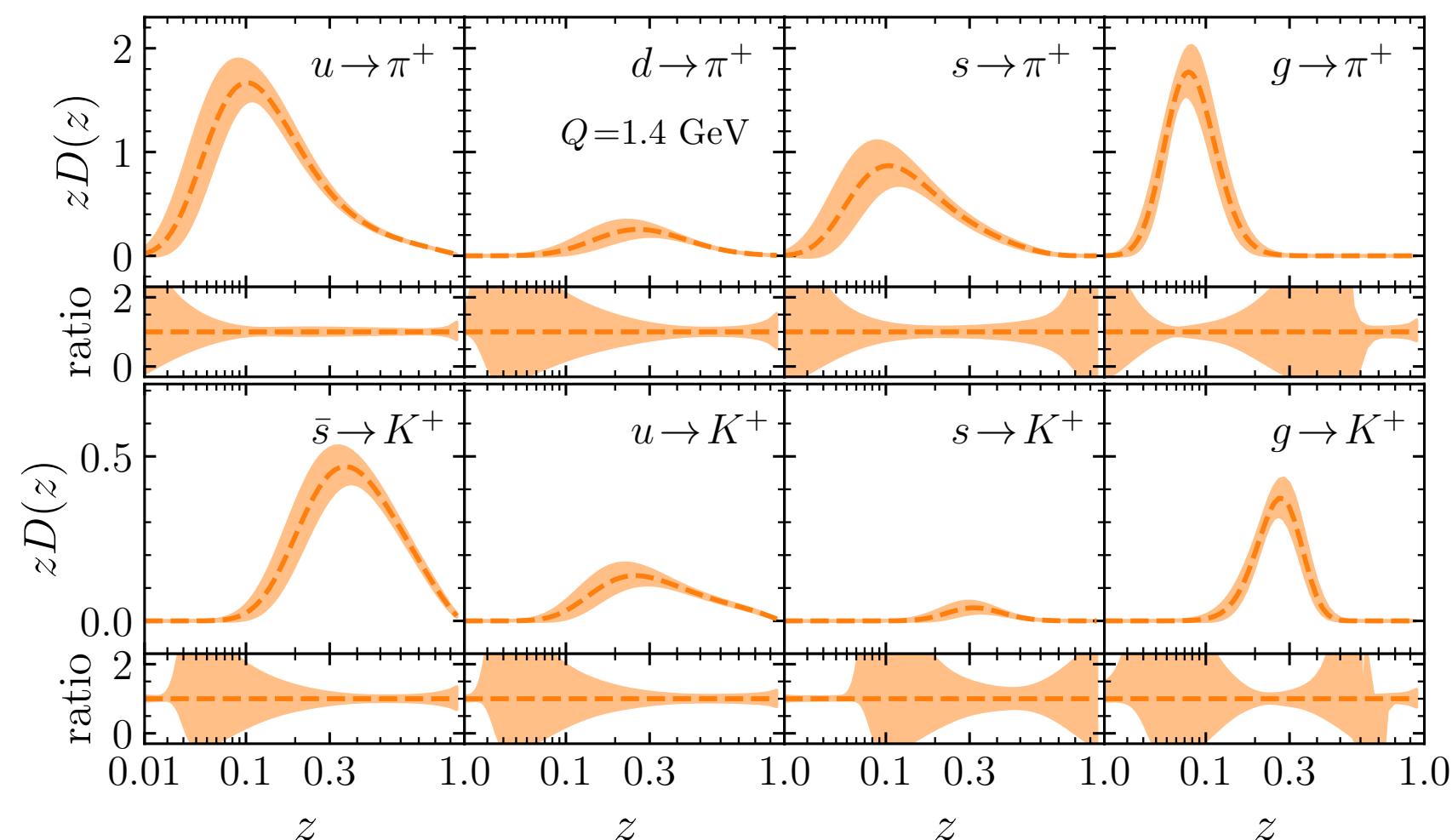
NPC23 analysis at NNLO

- ✦ Global analysis of FFs to light charged hadrons at full NNLO with SIA and SIDIS data, especially with the recent BESIII measurements, focusing on low momentum transfer region and test of collinear factorization

quality of NNLO fit with varying hadron energy cut

NNLO	BESIII		COMPASS		B-factories		HE-SIA		global		
$E_{h,\min}[\text{GeV}]$	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2	χ^2/N_{pt}
0.5	242	1.26	358	1.65	233	1.06	426	1.19	1259	1650.2	1.31
0.6	212	1.21	290	1.59	228	0.92	423	0.97	1153	1338.8	1.16
0.7	182	1.11	214	1.47	223	0.61	413	0.84	1032	997.2	0.97
0.8	152	0.98	142	1.30	218	0.53	407	0.82	919	781.8	0.85
0.9	122	1.05	94	1.29	213	0.52	407	0.80	836	687.1	0.82
1.0	98	1.14	54	0.97	209	0.49	403	0.80	764	587.2	0.77

FFs to charged pion/kaon at an initial scale ($Q=1.4$ GeV)



[JG, Shen, Xing, Zhao, Zhou, 2502.17837]

- ✦ a joint determination of FFs to charged pion, kaon at NNLO in QCD (54 parameters) including estimation of uncertainties with Hessian sets
- ✦ parametrization at $Q_0=1.4$ GeV for light flavors and at mass threshold for heavy quarks; 3-loop DGLAP evolution from [Mitov, Moch, Vogt, Almasy; Chen, Yang, Zhu, Zhu] implemented in HOPPET
- ✦ SIDIS coefficient functions from [Bonino, Gehrmann, Stagnitto; Goyal, Moch, Pathak, Rana, Ravindran]; SIA with hadron mass corrections; include residual scale variations into the covariance matrix
- ✦ select SIA and SIDIS data with $z>0.01$, $Q>2$ GeV, and $E_h>E_{h,\min}$ (0.8 GeV default) with later varied

first FFs of light hadrons at full NNLO ensuring test of factorization at low Q^2 !

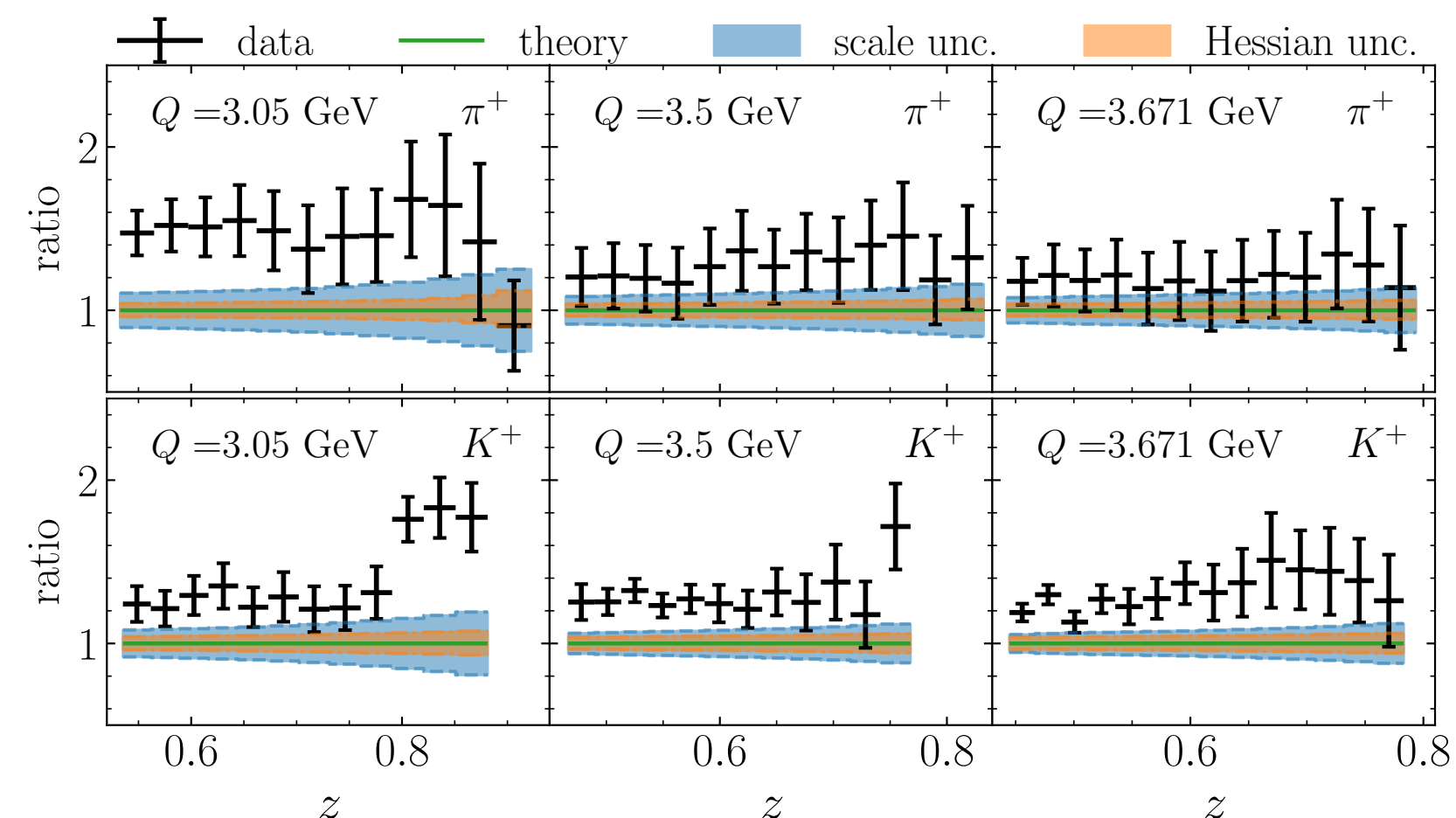
NPC23 analysis at NNLO

- ✦ Satisfactory agreements between NNLO theory and data are found for both the SIA and SIDIS data with $Q \sim 2\text{-}3$ GeV, though the two seem to pull the FFs in different directions; note exp. uncertainties are mostly correlated systematic uncertainties

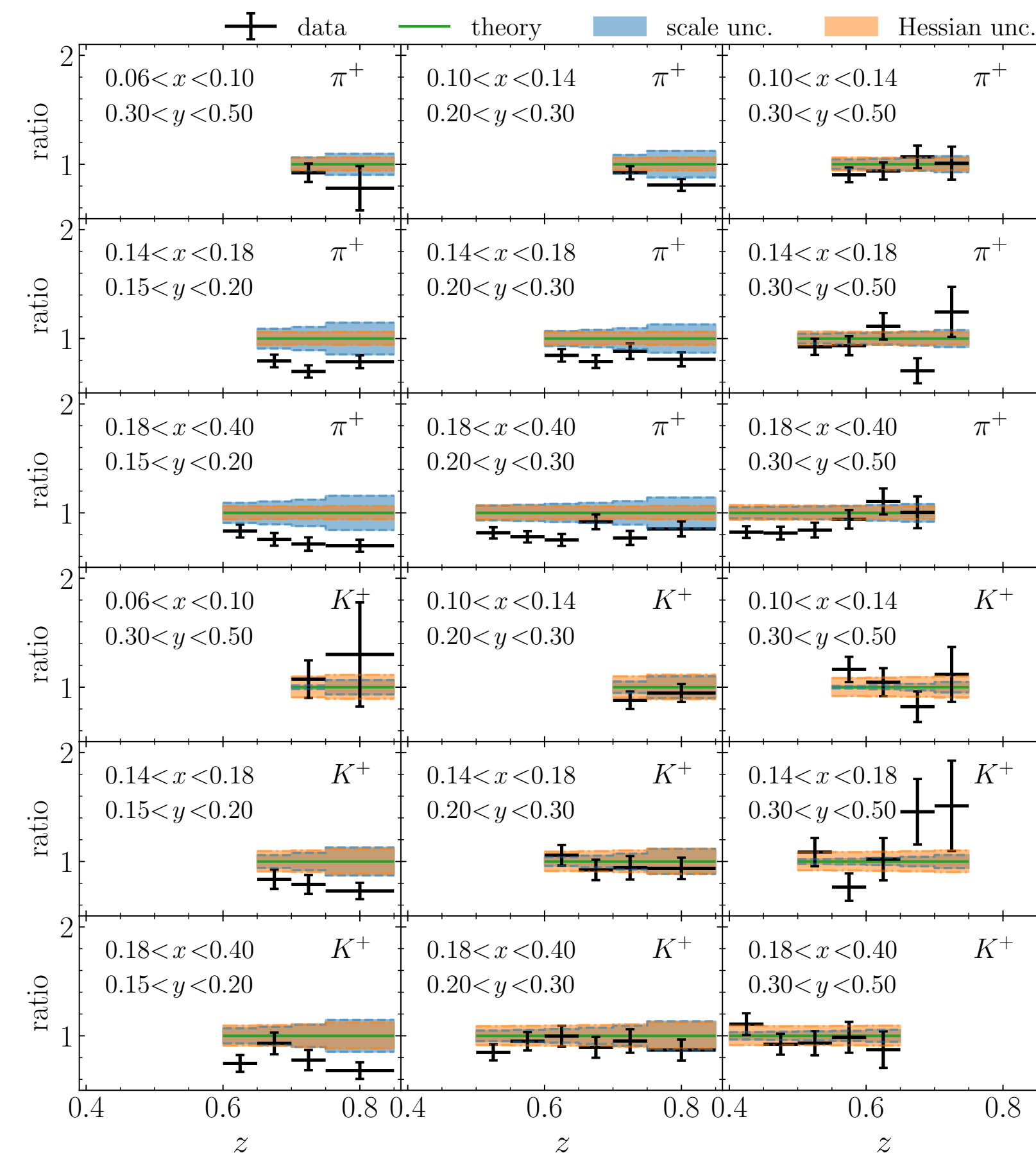
BESIII charged pion/kaon [2502.16084]

$\sqrt{s}$ (GeV)	$\mathcal{L}$ (pb $^{-1}$)	$N_{\text{had}}^{\text{tot}}$	N_{bkg}
2.0000	10.074	350298 ± 592	8722 ± 94
2.2000	13.699	445019 ± 668	10737 ± 104
2.3960	66.869	1869906 ± 1368	47550 ± 219
2.6444	33.722	817528 ± 905	21042 ± 146
2.9000	105.253	2197328 ± 1483	56841 ± 239
3.0500	14.893	283822 ± 533	7719 ± 88
3.5000	3.633	62670 ± 251	1691 ± 42
3.6710	4.628	75253 ± 275	6461 ± 81

theory vs data for the NNLO fit (BESIII)



theory vs data for the NNLO fit (COMPASS06)



NPC23 analysis at NNLO

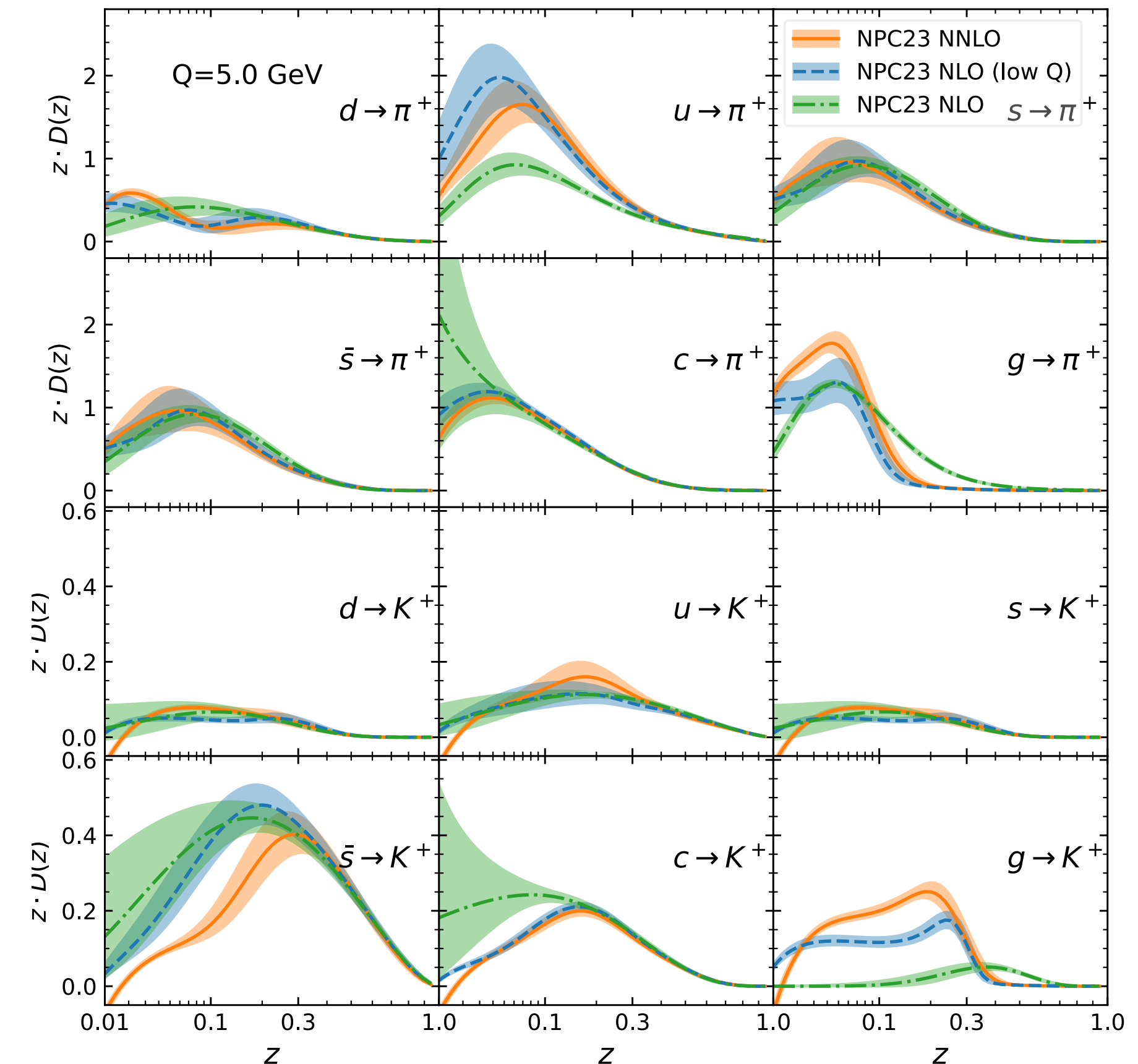
- ✦ The NNLO FFs are compared to its NLO version (low Q), as well as previous NLO FFs based on high-energy data; the observed differences largely reflect the interplay of different data sets, for instance, from BESIII/B-factories vs. from LEP/SLD/LHC

quality of NLO fit with varying hadron energy cut

NLO	BESIII		COMPASS		B-factories		HE-SIA		global		
$E_{h,min}[\text{GeV}]$	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2	χ^2/N_{pt}
0.5	242	1.38	358	1.50	233	1.01	426	1.23	1259	1631.2	1.30
0.6	212	1.26	290	1.44	228	0.87	423	1.06	1153	1333.2	1.16
0.7	182	1.12	214	1.43	223	0.67	413	0.97	1032	1057.9	1.03
0.8	152	1.03	142	1.26	218	0.54	407	0.85	919	801.6	0.87
0.9	122	1.08	94	1.22	213	0.52	407	0.84	836	697.5	0.83
1.0	98	1.18	54	0.93	209	0.49	403	0.83	764	603.7	0.79

- ✦ The NLO fits show similar trends as the NNLO with slightly worse chi2 in general; the resulted FFs differs with NNLO ones mostly for gluon and for low-z region
- ✦ The gluon FFs are different comparing to the NLO ones from high-energy data which are constrained mostly by LHC jet fragmentations; the u-quark FFs are also different due to pulls from BESIII&B-factories

NNLO vs. NLO (low Q) vs. NLO FFs (Q=5 GeV)



NPC23 analysis at NNLO

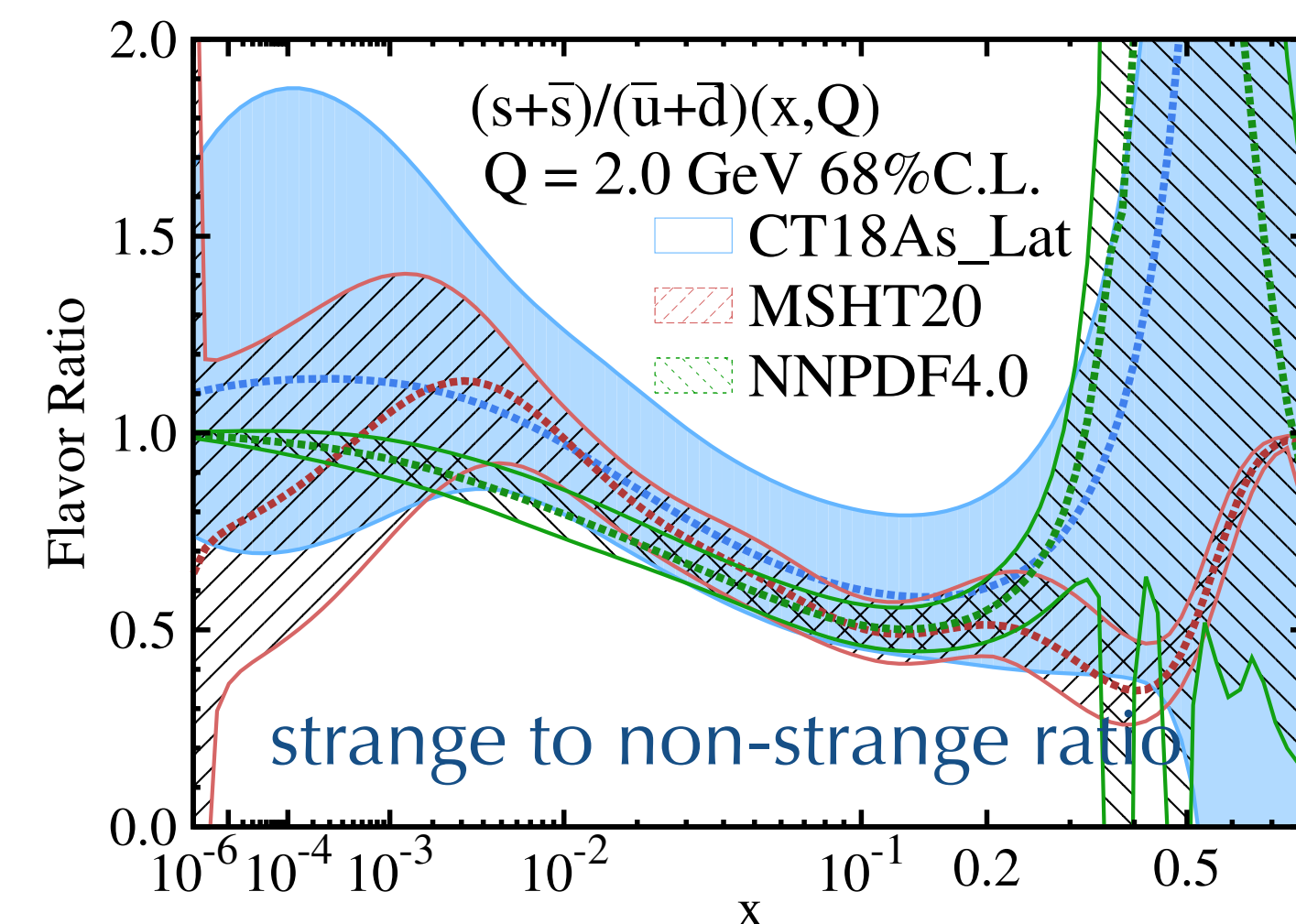
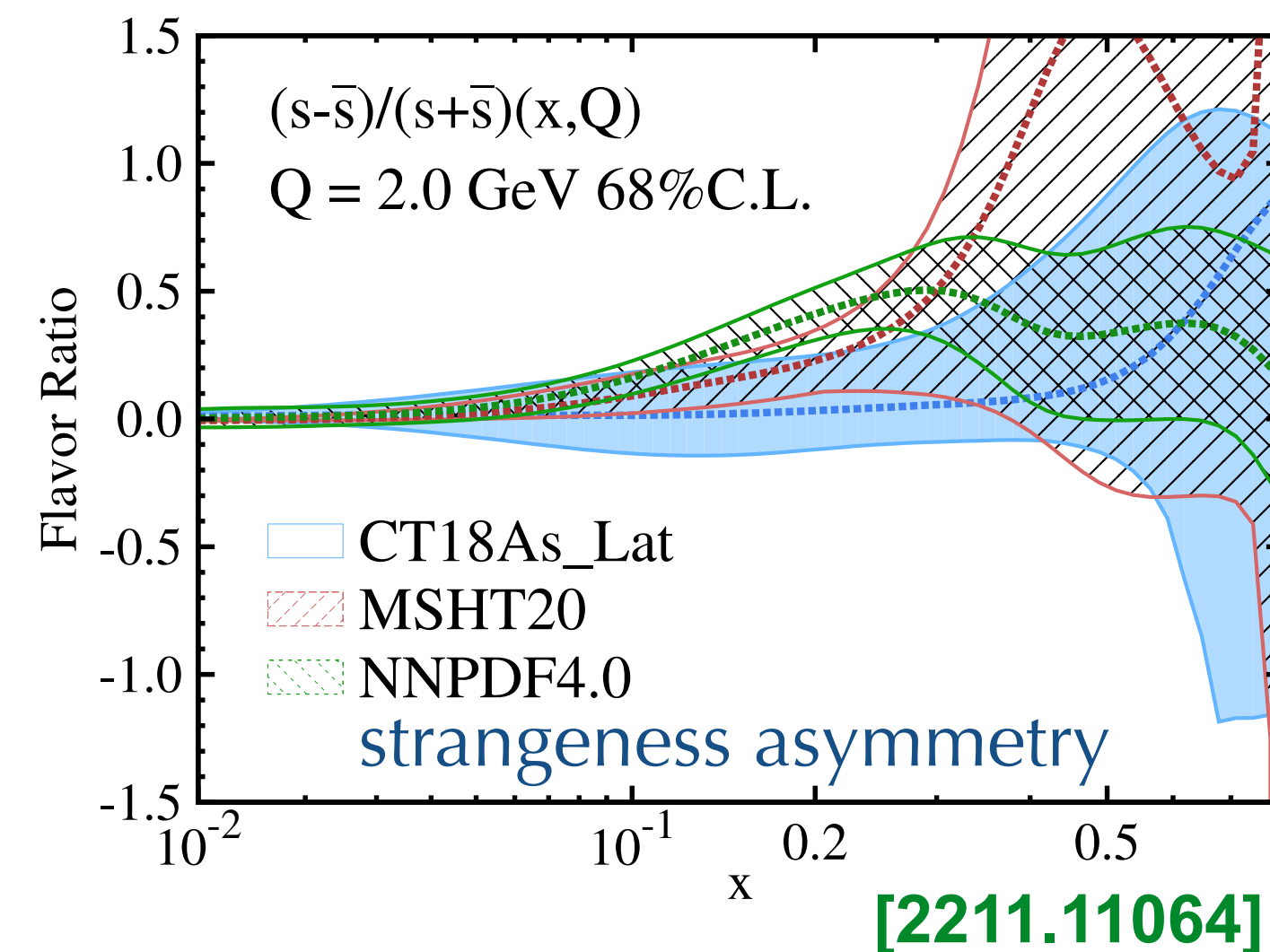
- ✦ We can explore complementary constraints from SIDIS data on the state-of-the-art proton PDFs at NNLO in QCD, especially for the sea quark separations and the strangeness asymmetry in PDFs

cross section difference of charged kaons with
iso-scalar target

$$\frac{d^3\sigma^{K^+-K^-}}{dx dy dz} \propto 2(u_v(x) + d_v(x))(D_u^{K^+}(z) - D_{\bar{u}}^{K^+}(z)) \\ + s_v(x)(D_s^{K^+}(z) - D_{\bar{s}}^{K^+}(z))$$

$$d_v \equiv d - \bar{d}, \quad r_s \equiv \frac{s + \bar{s}}{\bar{u} + \bar{d}}, \quad r_a \equiv \frac{s - \bar{s}}{s + \bar{s}}$$

- ✦ CT18, ABMP16 and ATLAS21 assume zero strangeness asymmetry at initial scale; MSHT20 and NNPDF4.0 find a preference of positive asymmetry (at large-x) from inclusive data
- ✦ The chi2 of best-fit to SIDIS and SIA data varies by ~40 units with the choice of PDFs and show a preference of reduced strangeness asymmetry, as well as further strange-quark suppression at x=0.2



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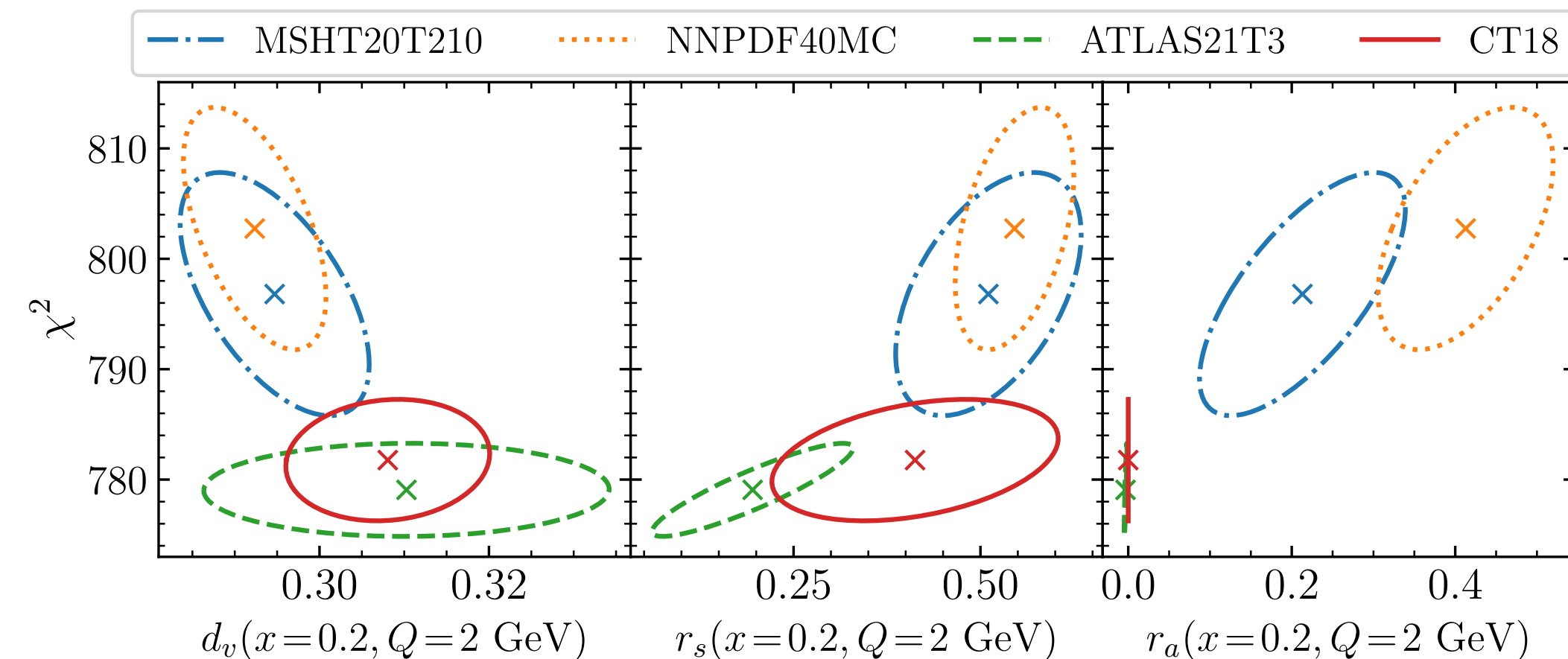
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correlation of FF best-fit chi2 and PDF variables



PDF variables before/after reweighting/profiling

	$d_v(x = 0.2, Q = 2\text{GeV})$	$r_s(x = 0.2, Q = 2\text{GeV})$	$r_a(x = 0.2, Q = 2\text{GeV})$
NNPDF4.0	0.2924 ± 0.0084	0.547 ± 0.079	0.408 ± 0.107
NNPDF4.0(reweighting)	0.3021 ± 0.0069	0.438 ± 0.066	0.281 ± 0.086
MSHT20	0.295 ± 0.011	0.511 ± 0.124	0.213 ± 0.126
MSHT20(profiling)	0.298 ± 0.011	0.481 ± 0.121	0.167 ± 0.136

◆ 6. Outlook and summary

Opportunities at future lepton colliders

- High luminosity and high energies of future lepton colliders open new opportunities for precision determination of FFs, especially with production of the W boson pairs and the Higgs boson with hadronic decays
[Zhou, JG, 2407.10059 (JHEP)]

hadron multiplicity measurements from
annihilation to quarks

e^+e^- annihilation							
$\sqrt{s}$ (GeV)	luminosity (ab ⁻¹)			final state	kinematic cuts	hadrons	N_{pt}
	CEPC	FCC- ee	ILC				
91.2	60	150	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	132
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	65
160	4.2	-	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	168
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	83
161	-	10	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	168
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	83
240	13	5	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	186
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	92
250	-	-	2	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	186
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	92
350	-	0.2	0.2	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	198
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	98
360	0.65	-	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	198
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	98
365	-	1.5	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	198
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	98
500	-	-	4	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	198
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	98

hadron multiplicity measurements from
decays of W or Higgs bosons

W boson decay channels							
$\sqrt{s}$ (GeV)	# events (million)			final state	kinematic cuts	hadrons	N_{pt}
	CEPC	FCC- ee	ILC				
80.419	116	68	62	$W^-W^{+*} \rightarrow W^-q\bar{q}$	-	$h^{+,-}$	120
	58	34	31	$W^-W^{+*} \rightarrow W^-c\bar{s}$			
Higgs boson decay channels							
$\sqrt{s}$ (GeV)	# events (million)			final state	kinematic cuts	hadrons	N_{pt}
	CEPC	FCC- ee	ILC				
125	0.23	0.09	0.07	gg	-	$h^\pm$	77
	0.08	0.03	0.02	$c\bar{c}$			
	1.53	0.59	0.47	$b\bar{b}$			

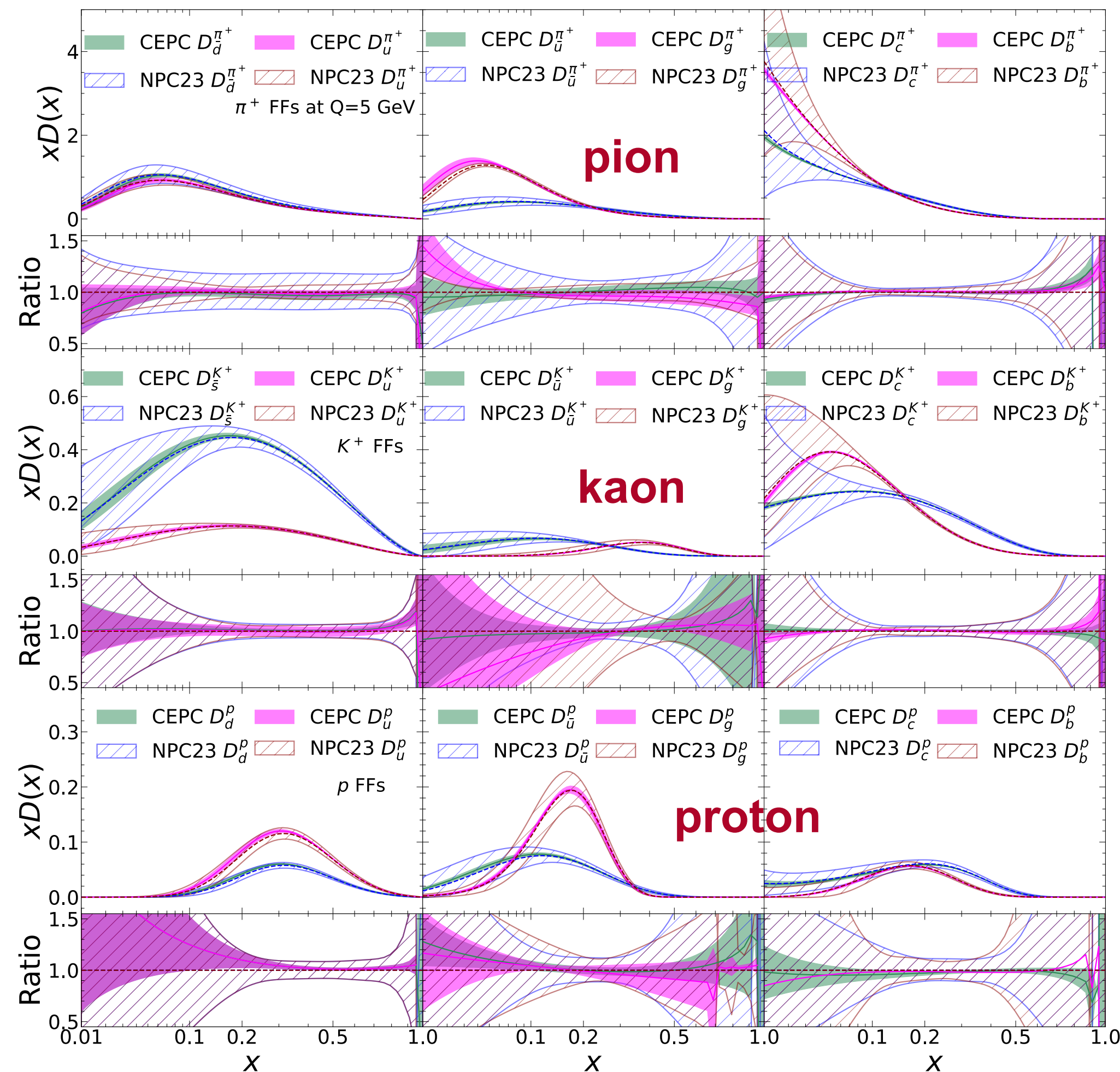
- ❖ (anti-)quark flavor separation from different energies, angular distributions, and heavy-flavor tagging
- ❖ d/s quark separation from W boson decays; probe of gluon fragmentation from Higgs boson decays

Higgs factory alone are ideal machine for
fragmentation functions!

Projection for constraints on FFs

- ◆ Pseudo-data on the proposed measurements are constructed using NPC23 FFs as the truth theory; fits to FFs at NLO in QCD are carried out with data solely from future electron-positron colliders

[Zhou, JG, 2407.10059 (JHEP)]



FFs (positively charged) vs. momentum fraction

- ◆ assuming same (un-)correlated systematic uncertainties as in SLD; statistical errors calculated based on prescribed luminosities
- ◆ fits using the same fitting framework as NPC23 including theoretical uncertainties
- ◆ best-fit agrees well with the truth FF; uncertainties are greatly reduced taking the CEPC as an example
- ◆ W boson data are essential for quark flavor separation; similarly Higgs boson data for constraining gluon FFs
- ◆ removal of theoretical uncertainties leads to reduction of FF uncertainties by more than a factor of two in many cases
- ◆ ILC, FCC-ee and CEPC give quite similar results except in regions statistics are limited;

Summary

- ✦ Fragmentation functions (FFs) are essential non-perturbative inputs for precision calculations of hadron production cross sections in high energy scattering from first principle of QCD
- ✦ FMNLO is a program for automated and fast calculations of fragmentation cross sections at NLO in QCD, which is desirable for global analysis of FFs providing much improved efficiency and capability for arbitrary hard processes, and is publicly available
- ✦ We perform a joint global analysis of FFs to light charged/neutral hadrons at NLO in QCD, using high-energy data from SIA, SIDIS, and pp collisions; constraints on gluon FFs are much improved; NPC23 FFs are publicly available in LHAPDF6 format
- ✦ A global analysis of FFs to charged pion and kaon at NNLO in QCD are performed based on SIA and SIDIS data, including recent BESIII measurements, allowing a test of collinear factorization at low-Q; impact of fragmentation data on the NNLO PDFs of proton are also studied

Thank you for your attention!