Quantum Simulation of Particle Scattering

Yahui Chai

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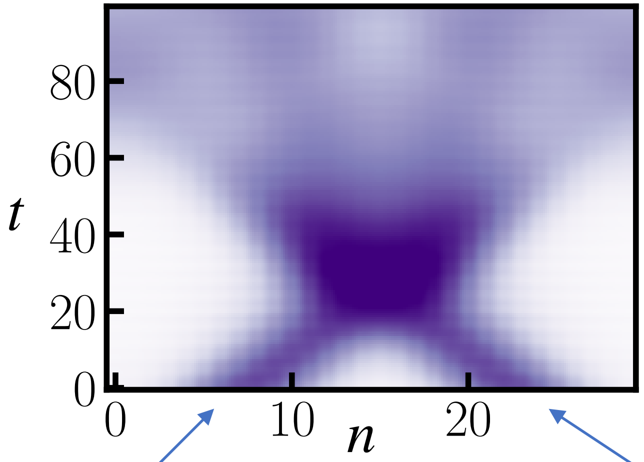
IBM Quantum

Outline

1. Quantum algorithm for meson scattering

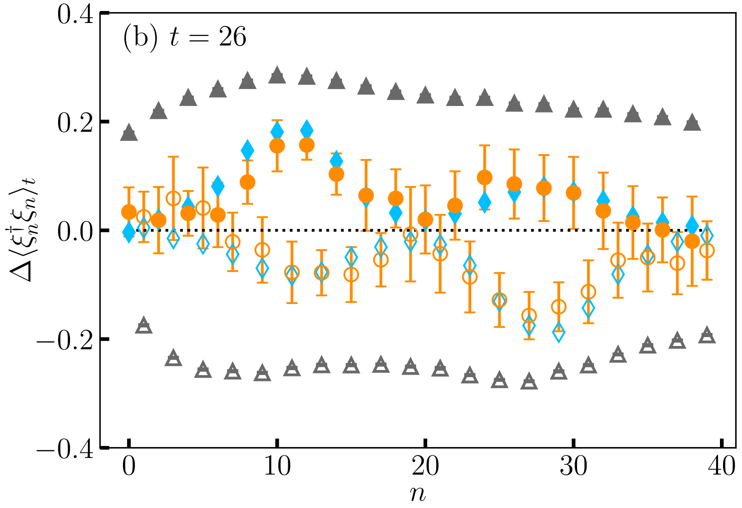


- Simulation of dynamics
- Quantum subspace expansion (QSE) for stable particle construction
- Quantum circuit decomposition

https://arxiv.org/abs/2505.21240

2. Hardware run for fermion scattering

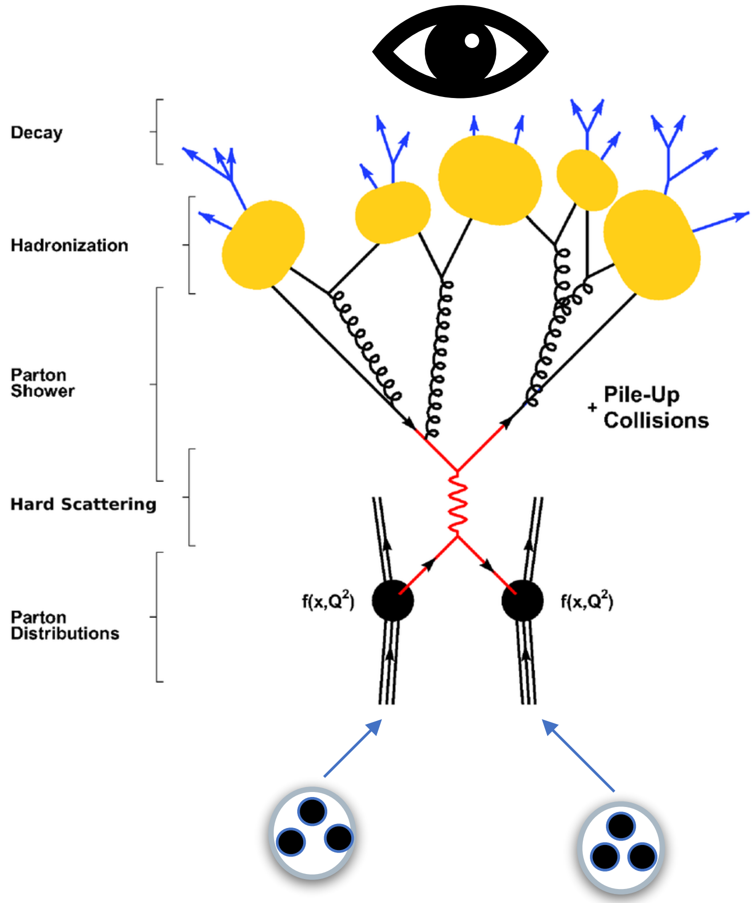
- Tensor network circuit compilation
- Hardware run for 40 and 80 qubits



https://arxiv.org/abs/2507.17832

Motivation

- Experiments on colliders: verify the theory, probe the particle structure.



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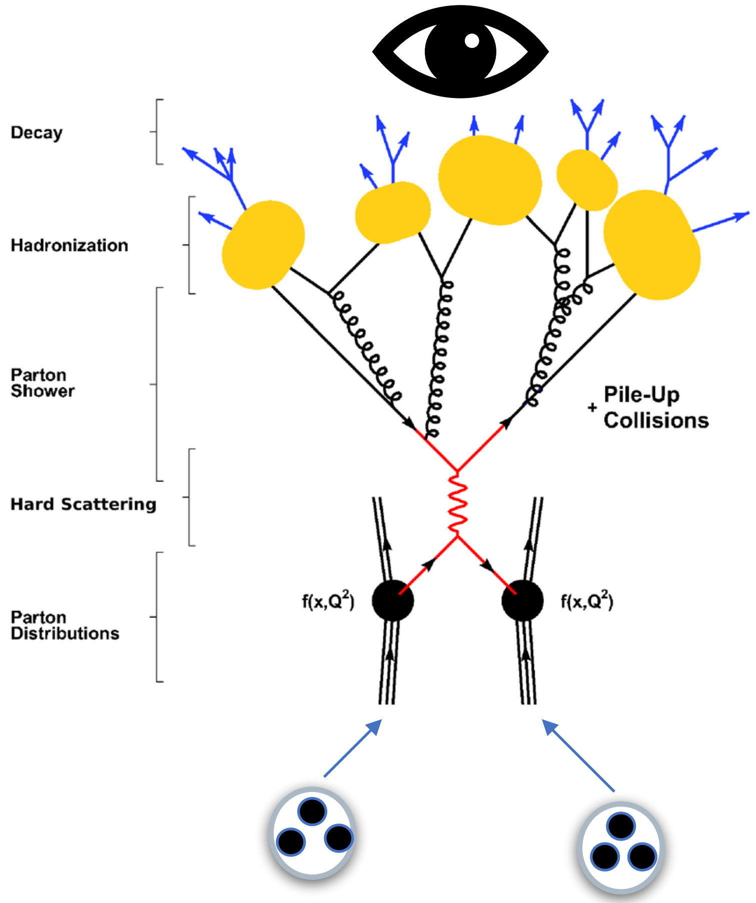
- Real time dynamics in classical simulation
    - Conventional Monte Carlo: sign problem
    - Tensor Network : increasing entanglement with time-- increasing computation resource



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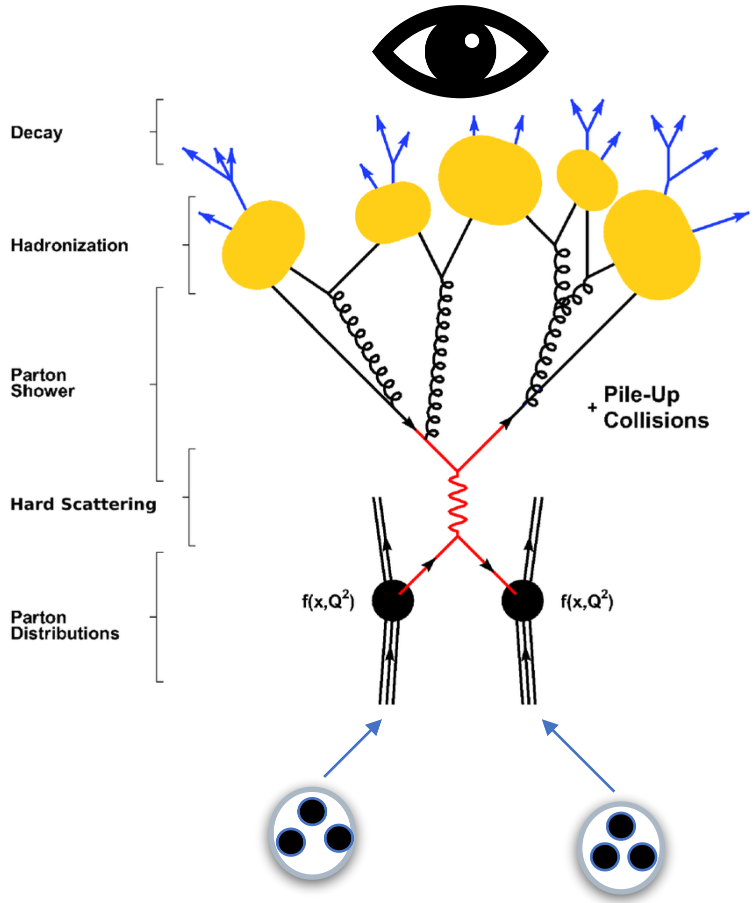
- Quantum computers promise to efficiently simulate real-time dynamics



Scattering by QC and TN

Fermionic wave packet scattering: a quantum computing approach

Yahui Chai[1], Arianna Crippa[1,2], Karl Jansen[1,3], Stefan Kühn[1], Vincent R. Pascuzzi[4], Francesco Tacchino[5], and Ivano Tavernelli[5]

Real-time scattering in the lattice Schwinger model

Irene Papaefstathiou,[1,2] Johannes Knolle,[3,2,4] and Mari Carmen Bañuls[1,2]

Real-Time Simulation of Asymmetry Generation in Fermion-Bubble Collisions

Marcela Carena,[1, 2, 3, 4, 5, *] Ying-Ying Li,[6, 7, †] Tong Ou,[3, 5, ‡] and Hersh Singh[2, §]

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Quantum computation of hadron scattering in a lattice gauge theory

Zohreh Davoudi,[1,2,3,4,*] Chung-Chun Hsieh,[1,2,3,†] and Saurabh V. Kadam[5,‡]

Digital quantum simulations of scattering in quantum field theories using W stat

Roland C. Farrell,[1,2,*] Nikita A. Zemlevskiy,[3,†] Marc Illa,[3,‡] and John Preskill[1,2,4,§]

Real-time scattering and freeze-out dynamics in Rydberg-atom lattice gauge theory

De-Sheng Xiang,[1,*] Peng Zhou,[1,*] Chang Liu,[2,*] Hao-Xiang Liu,[1] Yao-Wen Zhang,[1] Dong Yuan,[3,4] Kuan Zhang,[1] Biao Xu,[1] Marcello Dalmonte,[5,†] Dong-Ling Deng,[3,2,6,‡] and Lin Li[1,7,§]

Scattering process on QC

1. Vacuum state $|\Omega\rangle$: ground state of Hamiltonian

QC: VQE 😎

2. Initial state $|\psi(t=0)\rangle = B_1^\dagger B_2^\dagger |\Omega\rangle$: wave packets of particles

QC: only unitary operator ☹️

3. Time evolution: $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$

QC: trotterization. 😎

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Lattice $\mathbb{Z}_2$ gauge theory in 1+1 D

- Staggered fermion coupled with $\mathbb{Z}_2$ gauge field, periodic condition

$$H = \frac{1}{2a}\sum_{n=1}^{L}\left(\xi_n^{\dagger} Z_{g,n}\xi_{n+1} + \text{h.c.}\right) + m\sum_{n=1}^{L}(-1)^n \xi_n^{\dagger}\xi_n + \varepsilon\sum_{n=1}^{L} X_{g,n}$$

Kinetic term

Mass term

Electric field term

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Kinetic term — Mass term — Electric field term

- Quantum number: charge conjugation and momentum

$$C\xi_n C^{-1} = (-1)^n \xi_{n+1}^\dagger \qquad CZ_{g,n}C^{-1} = Z_{g,n+1}$$

Lattice $\mathbb{Z}_2$ gauge theory in 1+1 D

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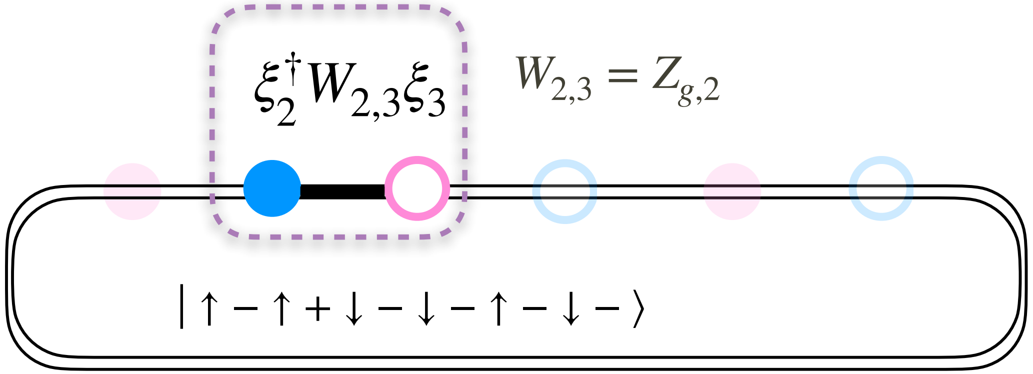
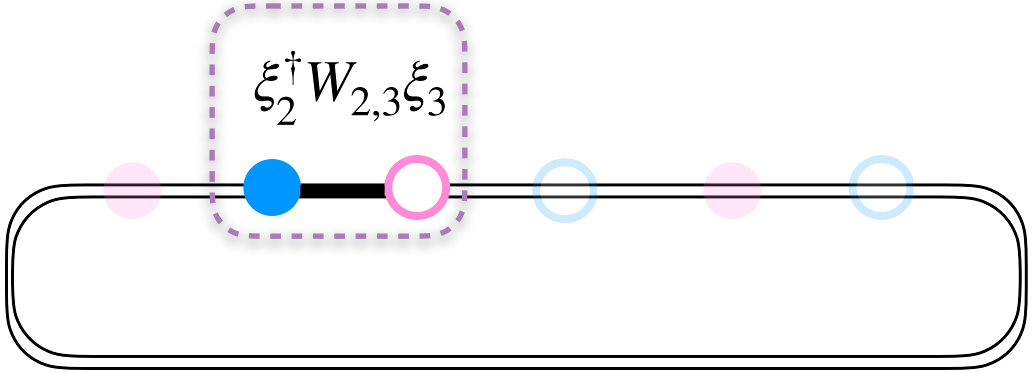
- Strong coupling limit $\varepsilon \to \infty$, periodic boundary condition

Lightest meson:



How to create a meson state?

- Strong coupling limit $\varepsilon \to \infty$



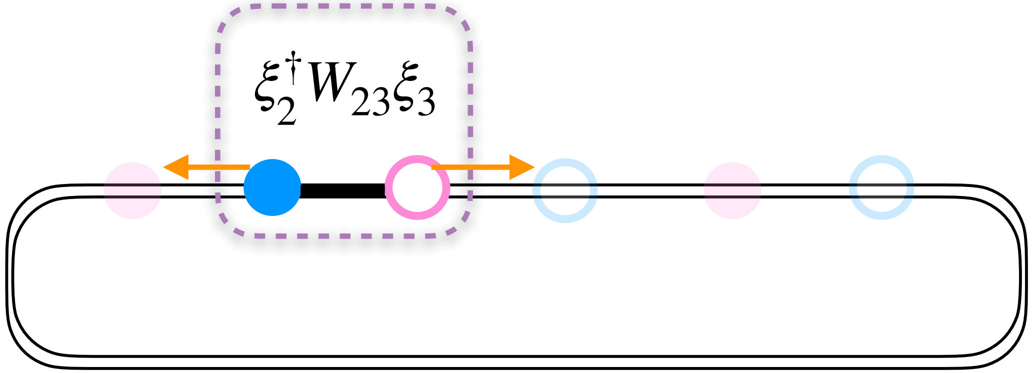
- Meson eigenstate

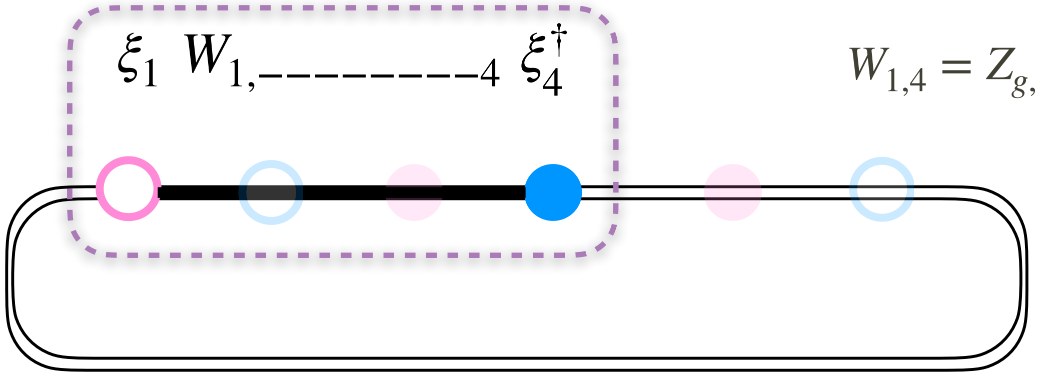
$$|k = 0,\ c = -1\rangle_b = \sum_n \left(\xi_n^\dagger W_{n,n+1}\xi_{n+1} - \text{h.c.}\right)|\Omega\rangle_\infty$$

Momentum   Charge conjugation

How to create a meson state?

- General coupling ε





$W_{1,4} = Z_{g,1} \ Z_{g,2} \ Z_{g,3}$

Meson Creation Operator

$$b^\dagger_{k,c} = \sum_I a_I^{(k,c)} M_I$$

Operators, $M_I \equiv M_{(n,l)} \in \{\xi_n^\dagger W_{n,l} \xi_l \,|\, n, l = 1,2,\cdots L\}$

Meson Creation Operator

$$b^{\dagger}_{k,c} = \sum_{I} a_I^{(k,c)} \; M_I$$

$$|\, k, c\rangle_b = b^{\dagger}_{k,c} \,|\, \Omega\rangle = \sum_{I} a_I^{(k,c)} \; M_I \,|\, \Omega\rangle$$

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$$H|k, c\rangle_b = E|k, c\rangle_b$$

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$$H|\,k, c\rangle_b = E|\,k, c\rangle_b$$

$$C|\,k, c\rangle_b = ce^{ik}|\,k, c\rangle_b$$

$c = -1$, vector meson;
$c = 1$, scalar meson

Quantum subspace expansion

$$b_{k,c}^{\dagger} = \sum_I a_I^{(k,c)} M_I$$

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$$\mathscr{H}_{I,J} = \langle \Omega | M_I^{\dagger} \, H \, M_J | \Omega \rangle$$

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$$\mathscr{S}_{I,J} = \langle \Omega | M_I^{\dagger} \; M_J | \Omega \rangle$$

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- QSE equation

$$\left(\mathscr{H} + \mathscr{C} \right) \vec{a}^{(k,c)} = \lambda \; \mathscr{S} \vec{a}^{(k,c)},$$

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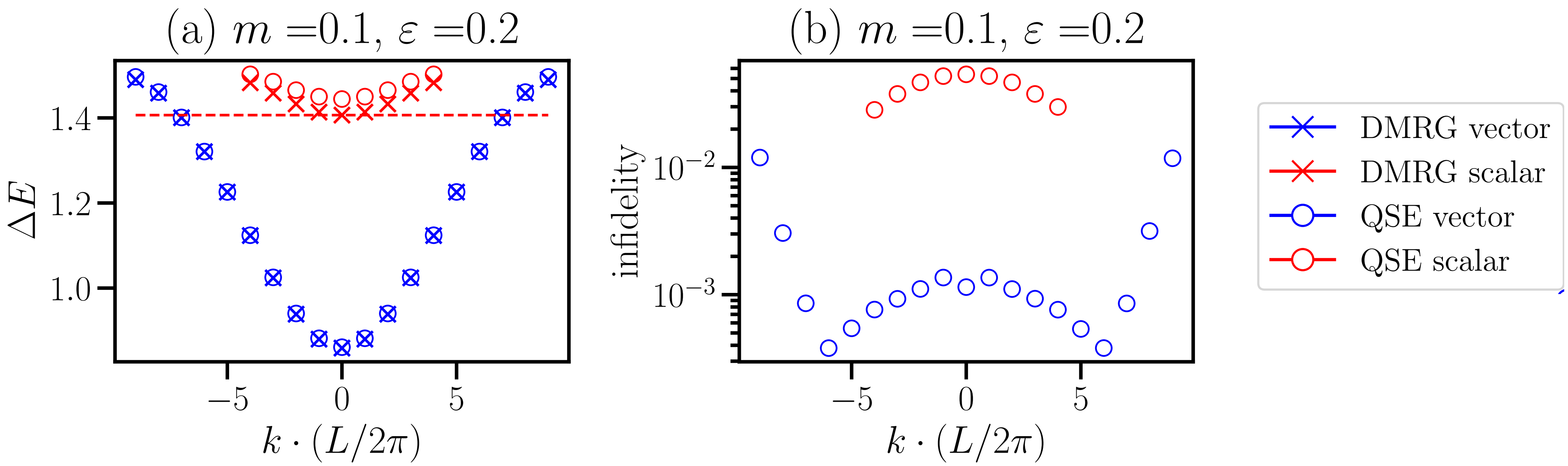
$$\vec{a}^{k,c} = (a_{1,1}^{k,c}, a_{1,2}^{k,c}, \cdots)$$

Results from QSE for $L = 30$

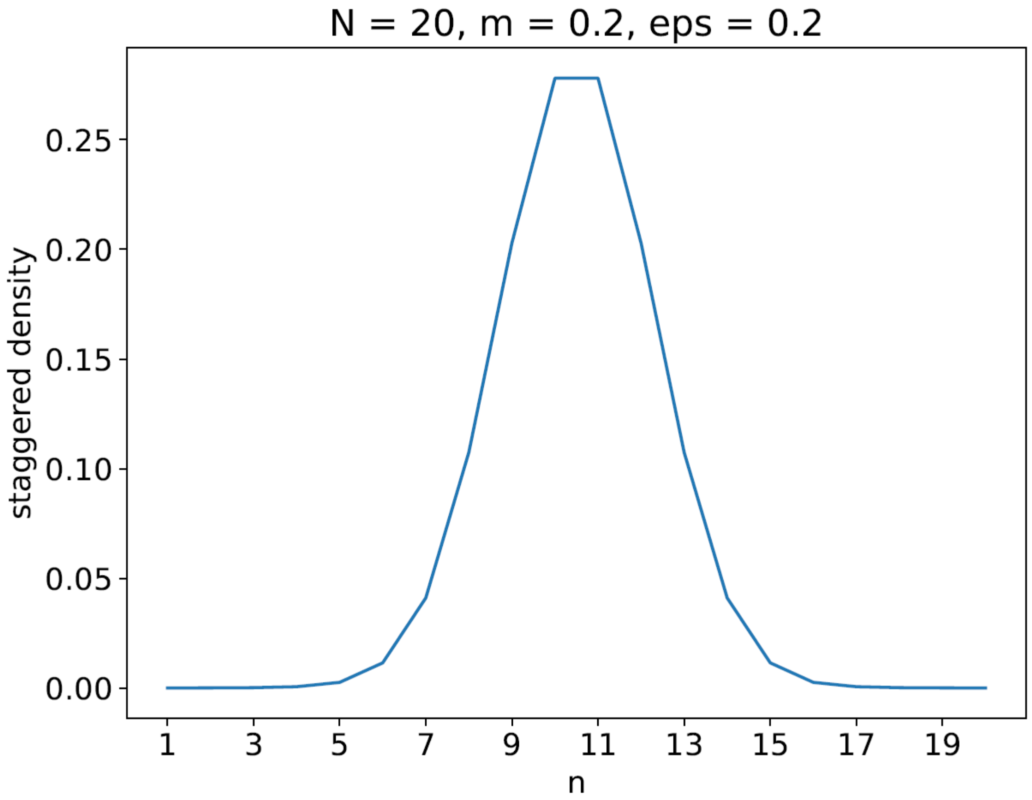
- With coefficients from QSE: $|k, c\rangle_b = \sum_I a_I^{k,c} M_I \, |\Omega\rangle$



Wave packet of meson

$$B^\dagger_{\bar{k},\bar{x}} = \sum_{k\in\Lambda^*} \phi(k)_{\bar{k},\bar{x}}\ b^\dagger_{k,-1} \qquad \phi(k)_{\bar{k},\bar{x}} = \frac{1}{\sqrt{\mathcal{N}_\phi}} \exp(-ik\bar{x})\ \exp\left(-(k-\bar{k})^2/(4\sigma_k^2)\right)$$

$B^\dagger_{\bar{k},\bar{x}}|\Omega\rangle$:



Scattering

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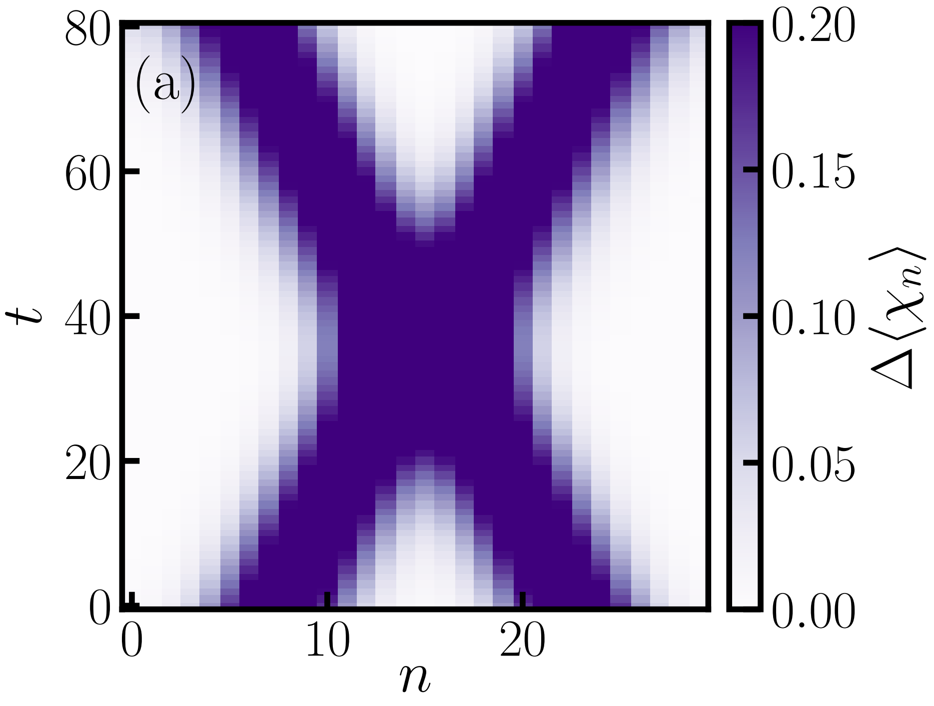
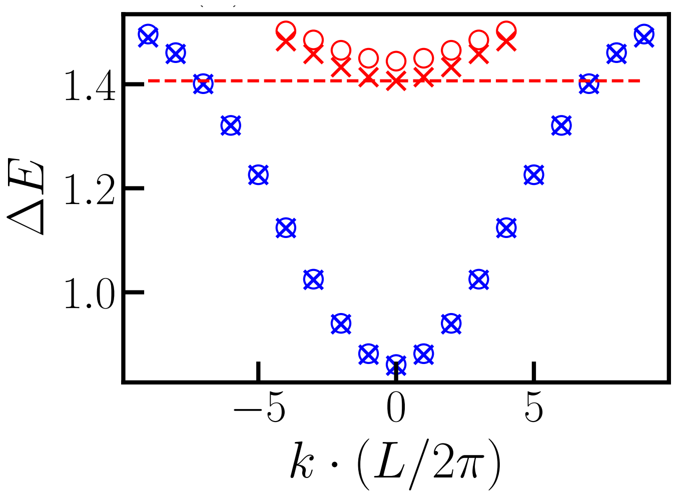
- Scattering:
  1. Initial state: $|\psi(t=0)\rangle = B^\dagger_{\bar{k},\bar{x}_1} B^\dagger_{-\bar{k},\bar{x}_2} |\Omega\rangle$
  2. Time evolution: $|\psi(t)\rangle = e^{-iHt} |\psi(t=0)\rangle$

Trotterization

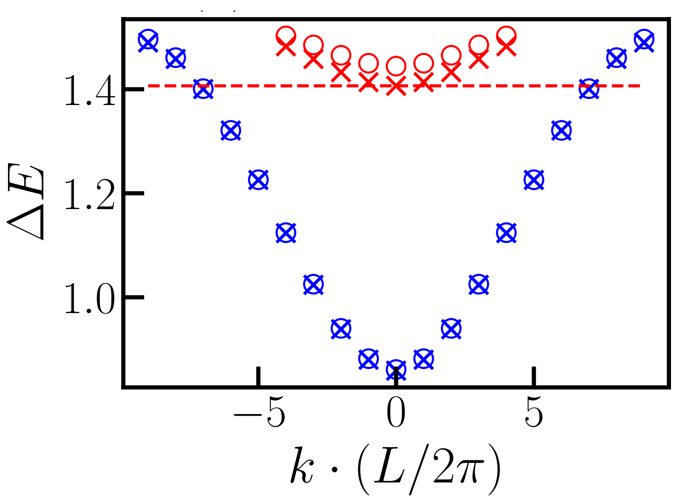


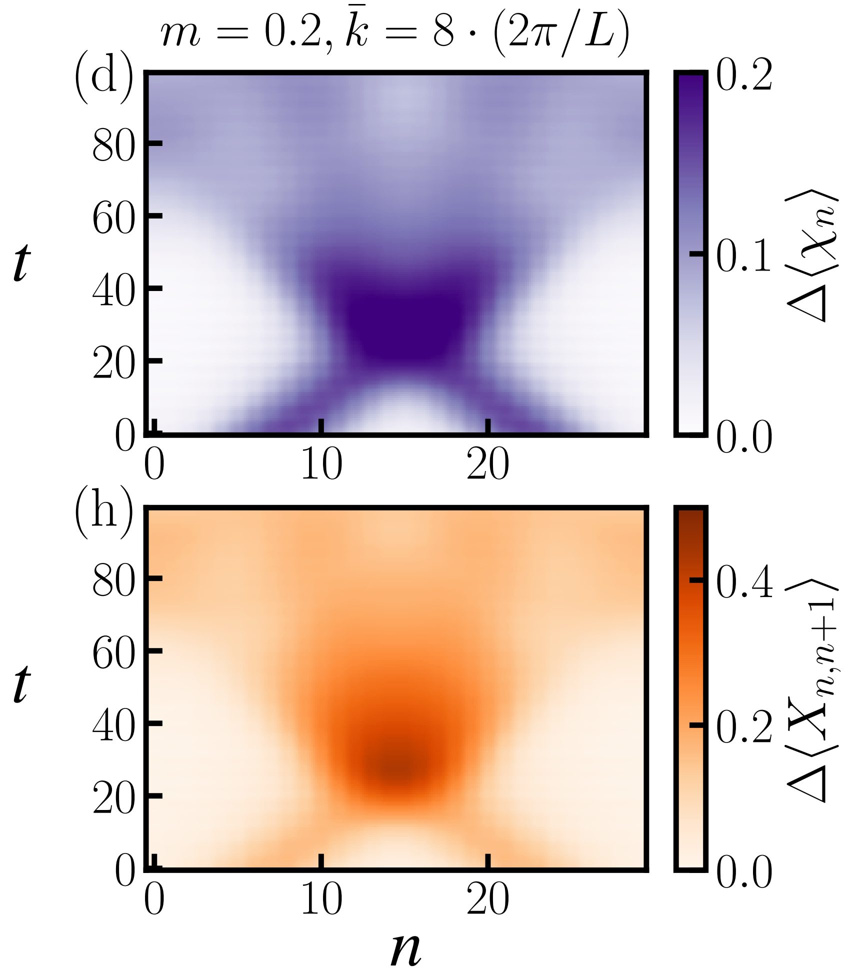
Elastic Scattering

Inelastic Scattering



Inelastic Scattering





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Quantum circuit for wave packet

- Non-unitary $$B^\dagger_{\bar{k},\bar{x}} = \sum_{k\in\Lambda^*} \phi(k)_{\bar{k},\bar{x}}\, b^\dagger_{k,-1},$$ $$= \sum_{k\in\Lambda^*} \sum_{nl} \phi(k)_{\bar{k},\bar{x}} a^{k,-1}_{nl}\ \xi^\dagger_n W_{n,l} \xi_l,$$

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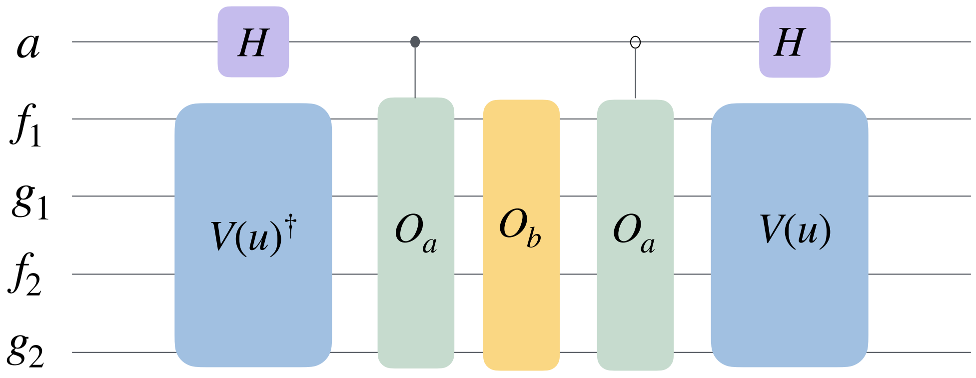
- Decomposition via Givens rotation



	CNOT number	CNOT depth
$V(u,\tilde{\xi})$ or $V(u,\tilde{\xi})^\dagger$	$2L(L-1)$	$4(2L-3)$
O_a	$4(L-1)$	$4(L-1)$
O_b	$12(L-1)$	$12(L-1)$
Total	$4L^2+16L-20$	$36L-44$

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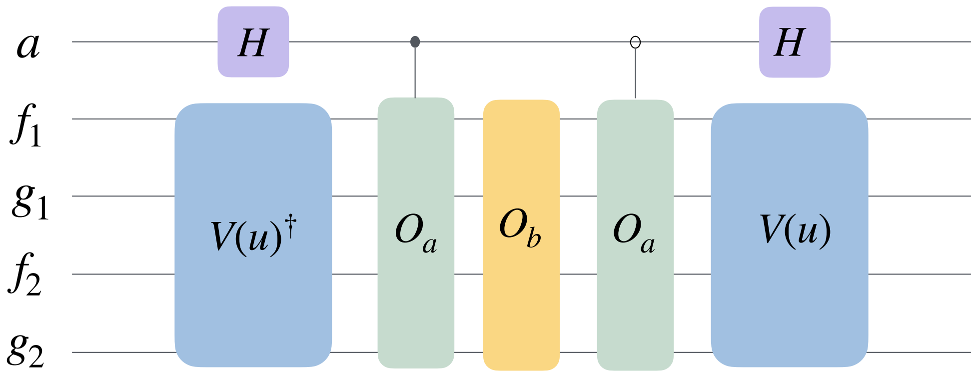

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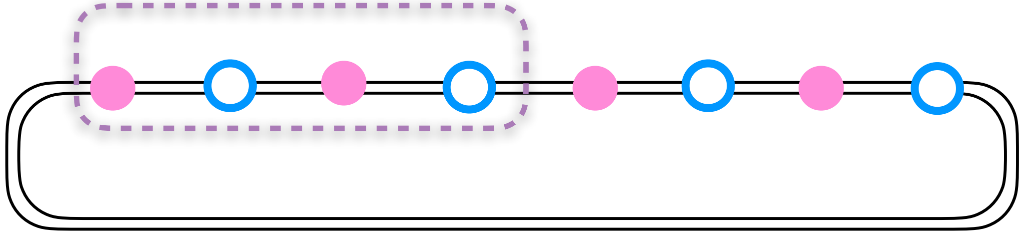
500~1000

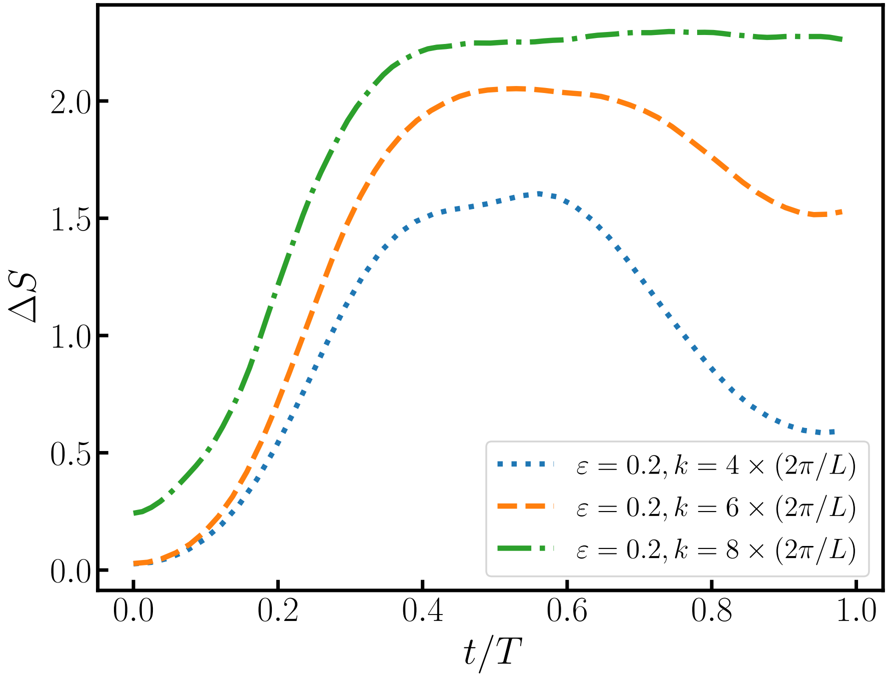
Hardware run?

Entropy

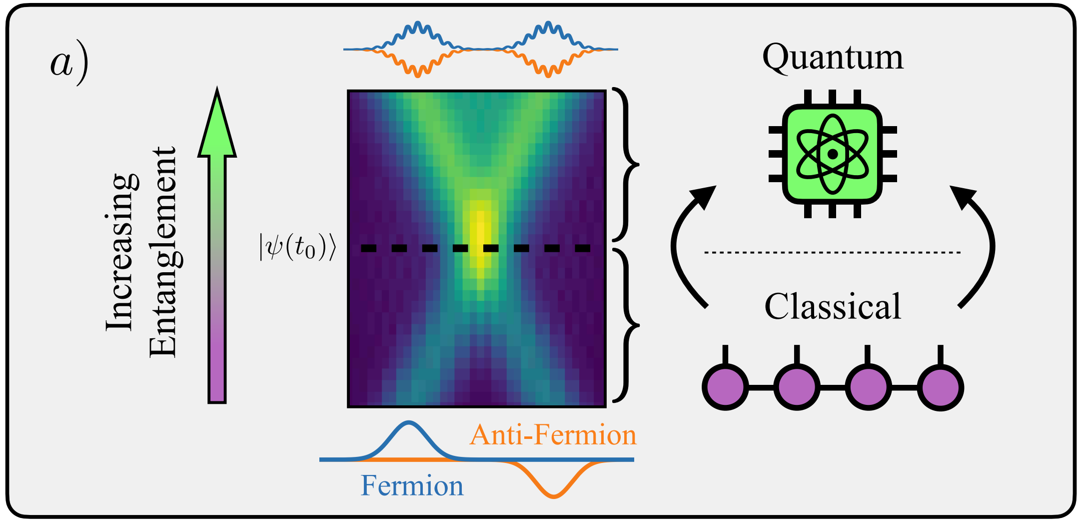
$$S(t) = -Tr[\rho(t)\log_2 \rho(t)]$$

Larger S, harder TN





TN + QC for fermion scattering



b)

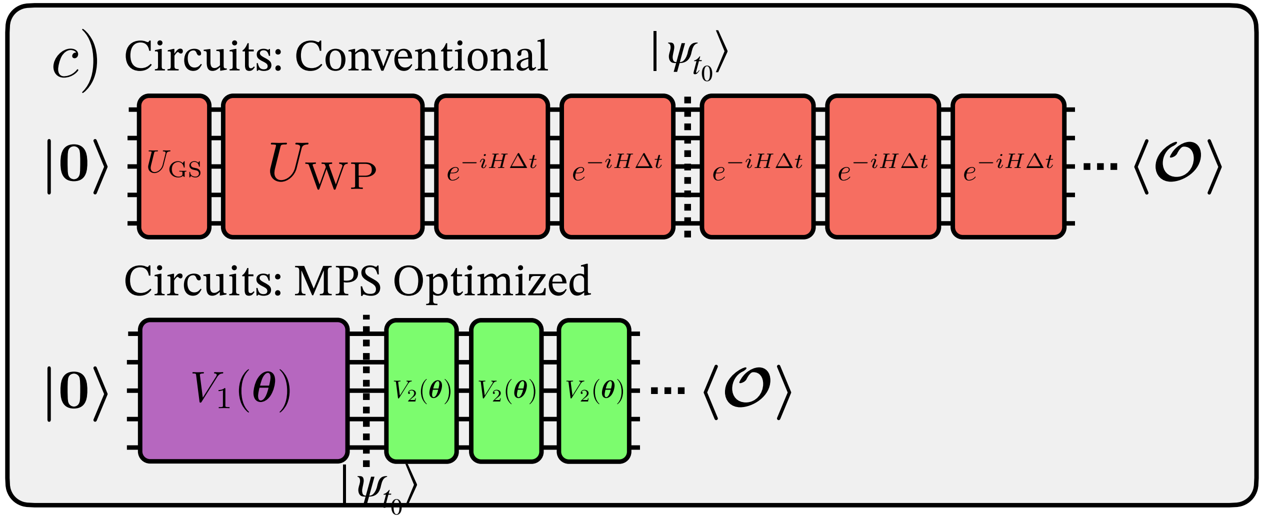
MPS-based optimization generates efficient circuits

Application	Optimize
State Preparation	$V_1(\boldsymbol{\theta})\|\mathbf{0}\rangle \approx \|\psi(t_0)\rangle$
Unitary Evolution	$V_2(\boldsymbol{\theta}) \approx e^{-iHt}$

Variationally minimization by TN

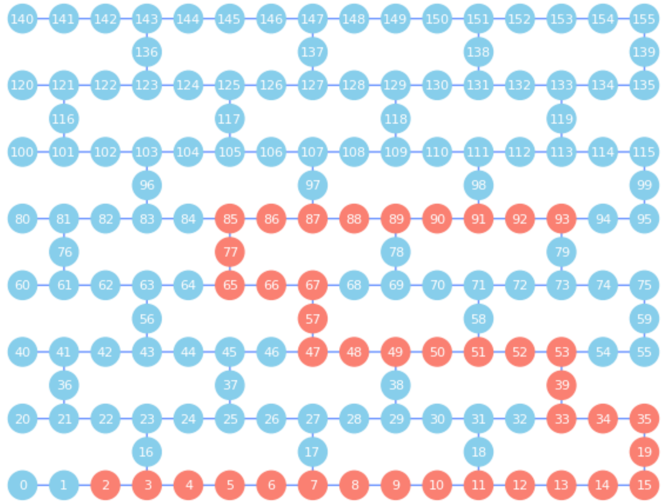
$$C_{\mathrm{State}}(\boldsymbol{\theta}) = 1 - \left|\langle\psi_{t_0}|V_1(\boldsymbol{\theta})|0\rangle\right|^2$$

$$C_{\mathrm{Uni}}(\boldsymbol{\theta}) = 1 - \frac{1}{2^{2N}}\left|\mathrm{Tr}(V_2(\boldsymbol{\theta})^\dagger e^{-iHt})\right|$$



Hardware run of fermion scattering for 40 qubits

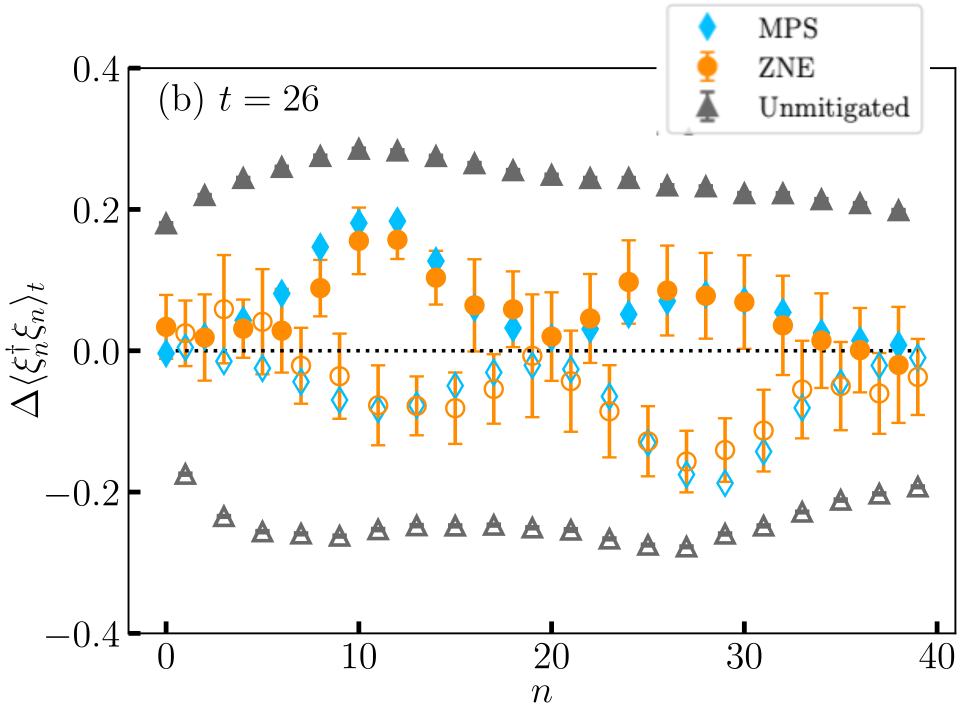
- IBM device ibm_fez
- Error mitigation: zero-noise extrapolation (ZNE)



$N = 40,\ T = 26,\ \Delta t = 2/3$		
	CNOT layers	CNOT gates
$\lvert\psi(t_0)\rangle$	36 (241)	702 (3371)
e^{-i2H}	12 (18)	234 (351)
In total	96 (331)	1872 (5126)

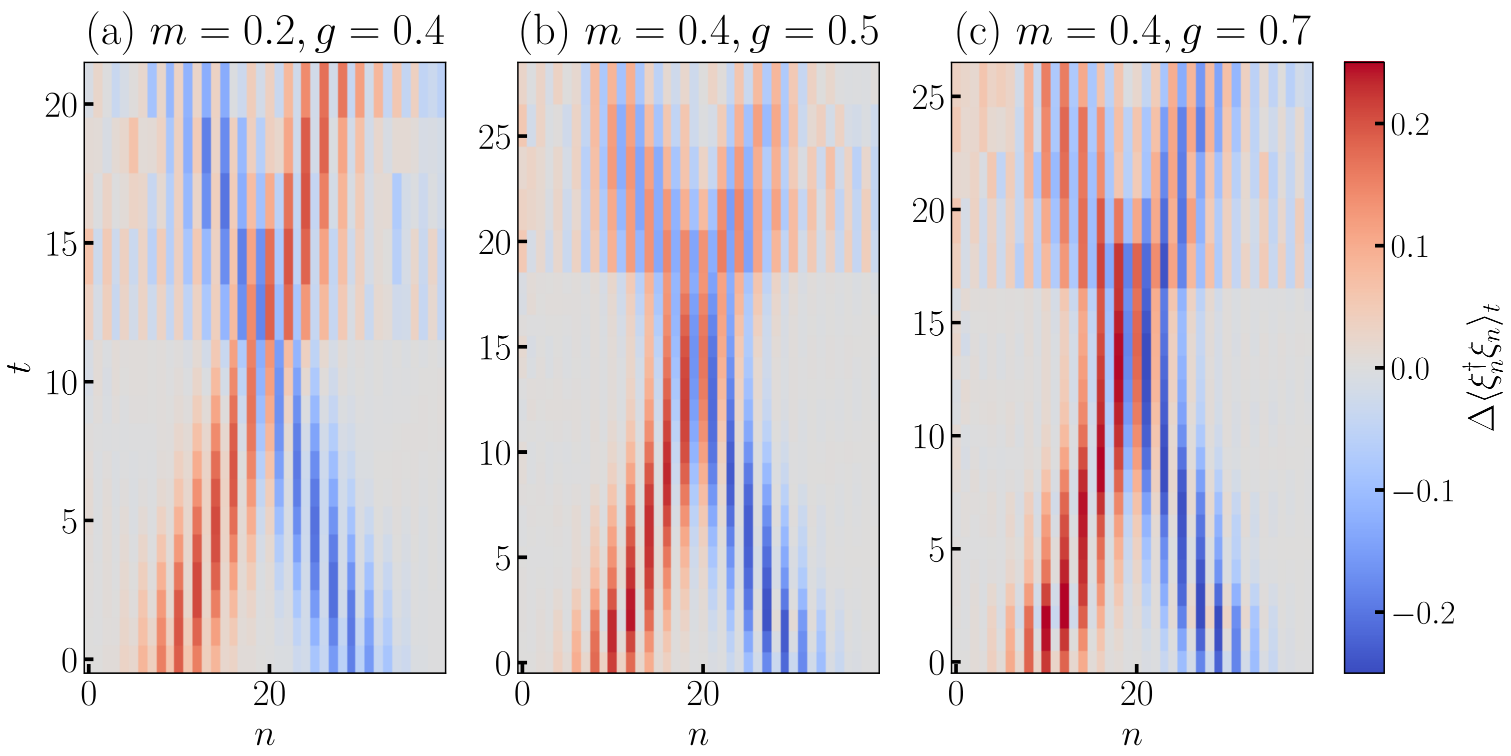
Our optimized circuit

Conventional approach



Hardware run for 40 qubits: full dynamics

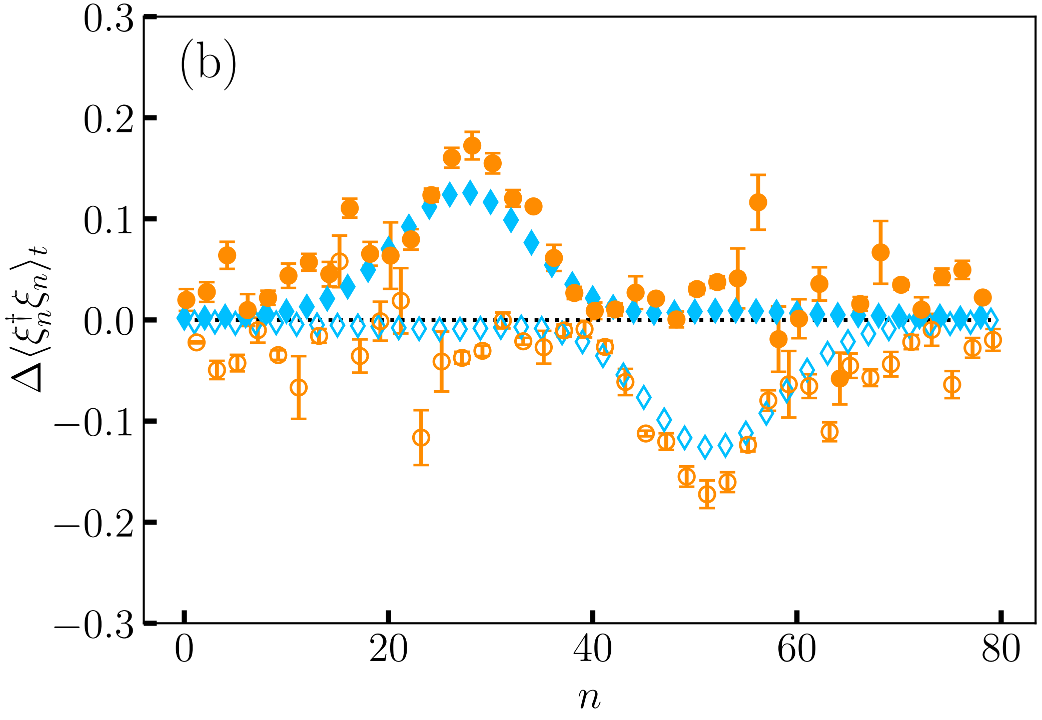
- IBM device ibm_fez
- Error mitigation: zero-noise extrapolation (ZNE)



Hardware run for 80 qubits: state preparation

- IBM device ibm_fez
- Error mitigation: zero-noise extrapolation (ZNE)

$N = 80,\ t_0 = 10,\ \Delta t = 2/3$		
	CNOT layers	CNOT gates
$\lvert\psi(t_0)\rangle$	25 (249)	948 (3987)



Summary

- Algorithm development for meson scattering:
    - QSE for eigenstates with specific momentum and charge conjugation
    - Circuit decomposition
    - Inelastic scattering: new particle production, string breaking, hadronization…
- Hardware run for fermion scattering, optimized circuit by TN
    - Full scattering for 40 qubits
    - State preparation for 80 qubits

Outlook

- Other Gauge theory, e.g., QED
- Higher dimension (2+1)D
- Improvement QSE

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- Higher dimension (2+1)D
- Improvement QSE

柴雅卉
山西 运城

扫一扫上面的二维码图案，加我为朋友。

yahui.chai@desy.de

Thank you!

Arianna Crippa
(CQTA)

Karl Jansen
(CQTA)

Stefan Kühn
(CQTA)

Vincent R.
Pascuzzi (IBM, NY)

Francesco
Tacchino (IBM
Zürich)

Ivano Tavernelli
(IBM Zürich)

Yibin Guo
(CQTA)

Joe Gibbs
University of Surrey

Zoe Holmes
EPFL

First fermion scattering paper

Y. Chai, A. Crippa, K. Jansen, S. Kühn, V. R. Pascuzzi, F. Tacchino, and I. Tavernelli, Quantum **9**, 1638 (2025).

Meson scattering paper

arXiv:2505.21240

Fermion scattering hardware run paper

arXiv:2507.17832

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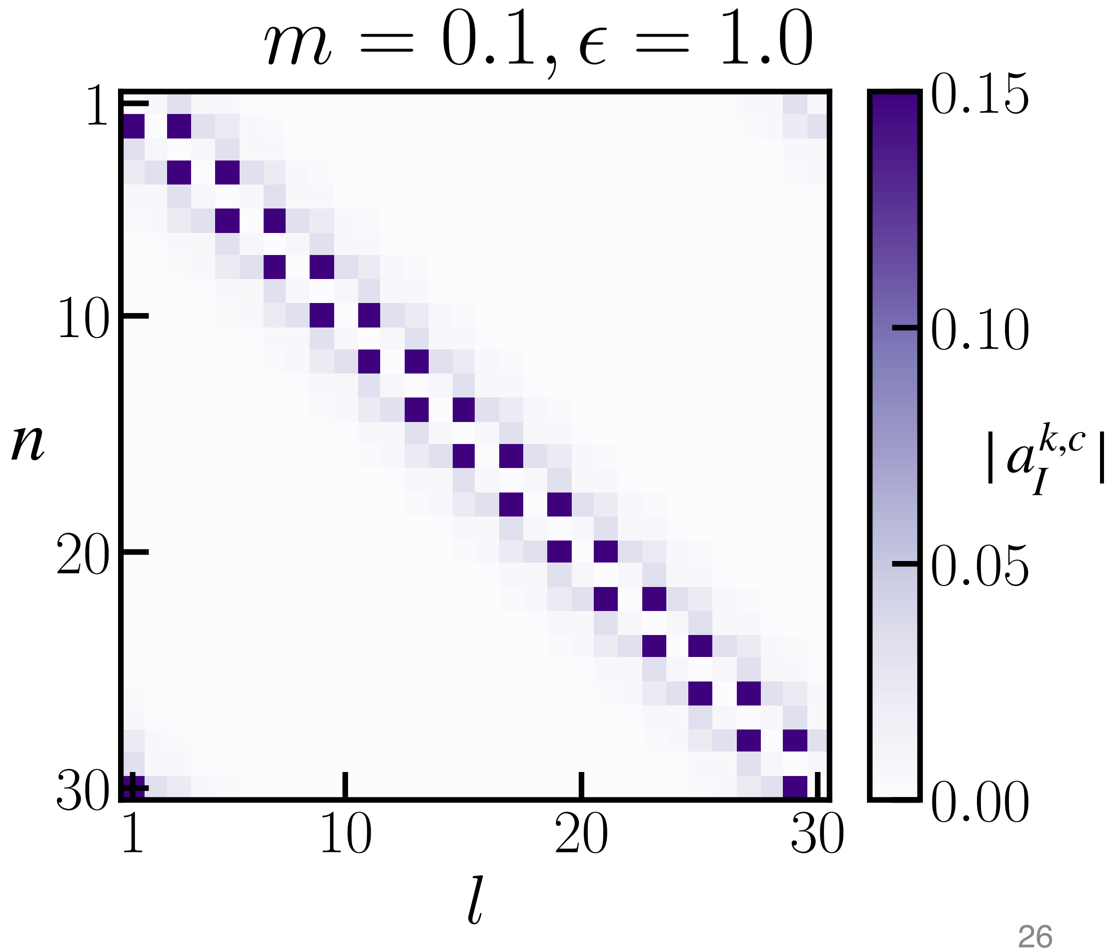
Results from QSE

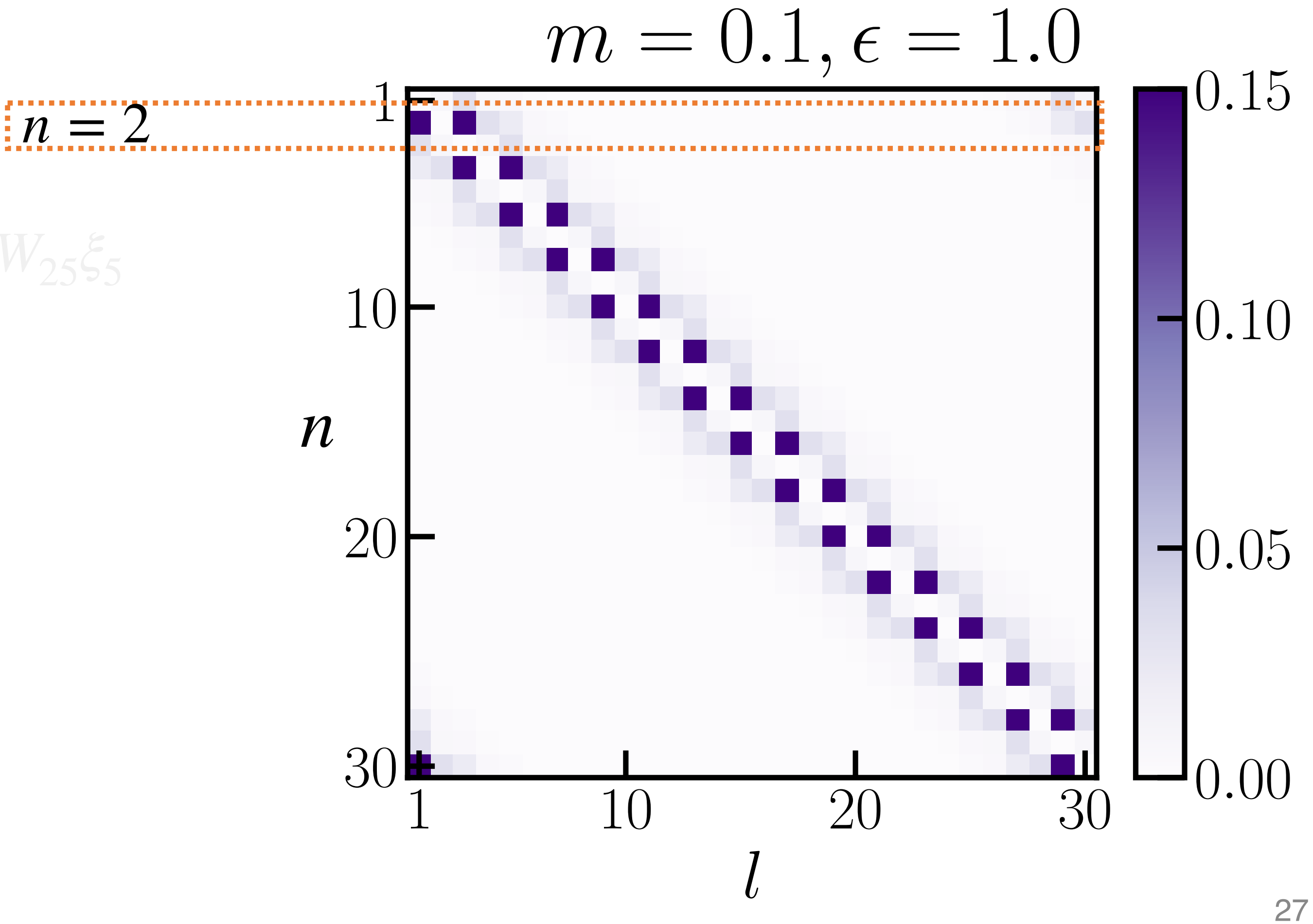
- With coefficients from QSE: $|k, c\rangle_b = \sum_I a_I^{k,c} M_I \, |\Omega\rangle$



Results from QSE

- With coefficients from QSE: $|k, c\rangle_b = \sum_I a_I^{k,c} M_I \, |\Omega\rangle$

$\xi_2^\dagger W_{21} \xi_1 \quad \xi_2^\dagger \xi_2 \quad \xi_2^\dagger W_{23} \xi_3 \quad \xi_2^\dagger W_{24} \xi_4 \quad \xi_2^\dagger W_{25} \xi_5$

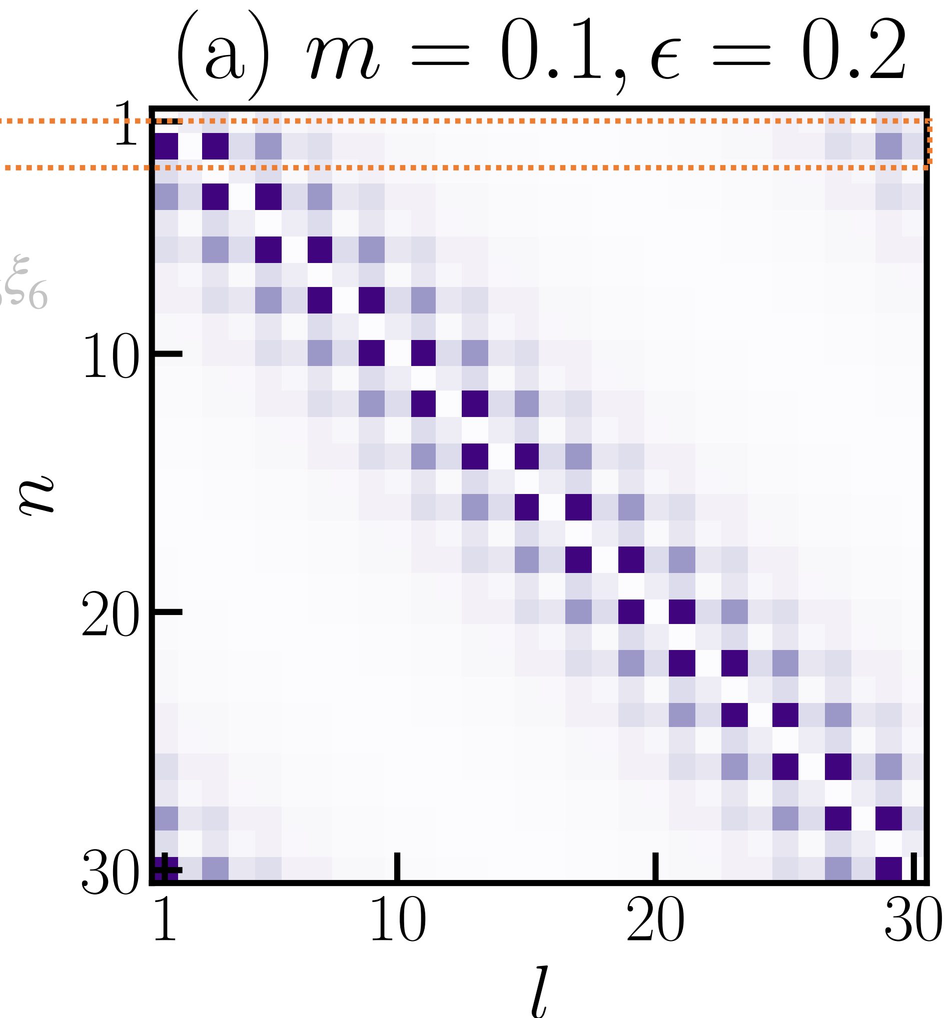


Results from QSE

- With coefficients from QSE: $$|k,c\rangle_b = \sum_I a_I^{k,c} M_I |\Omega\rangle$$

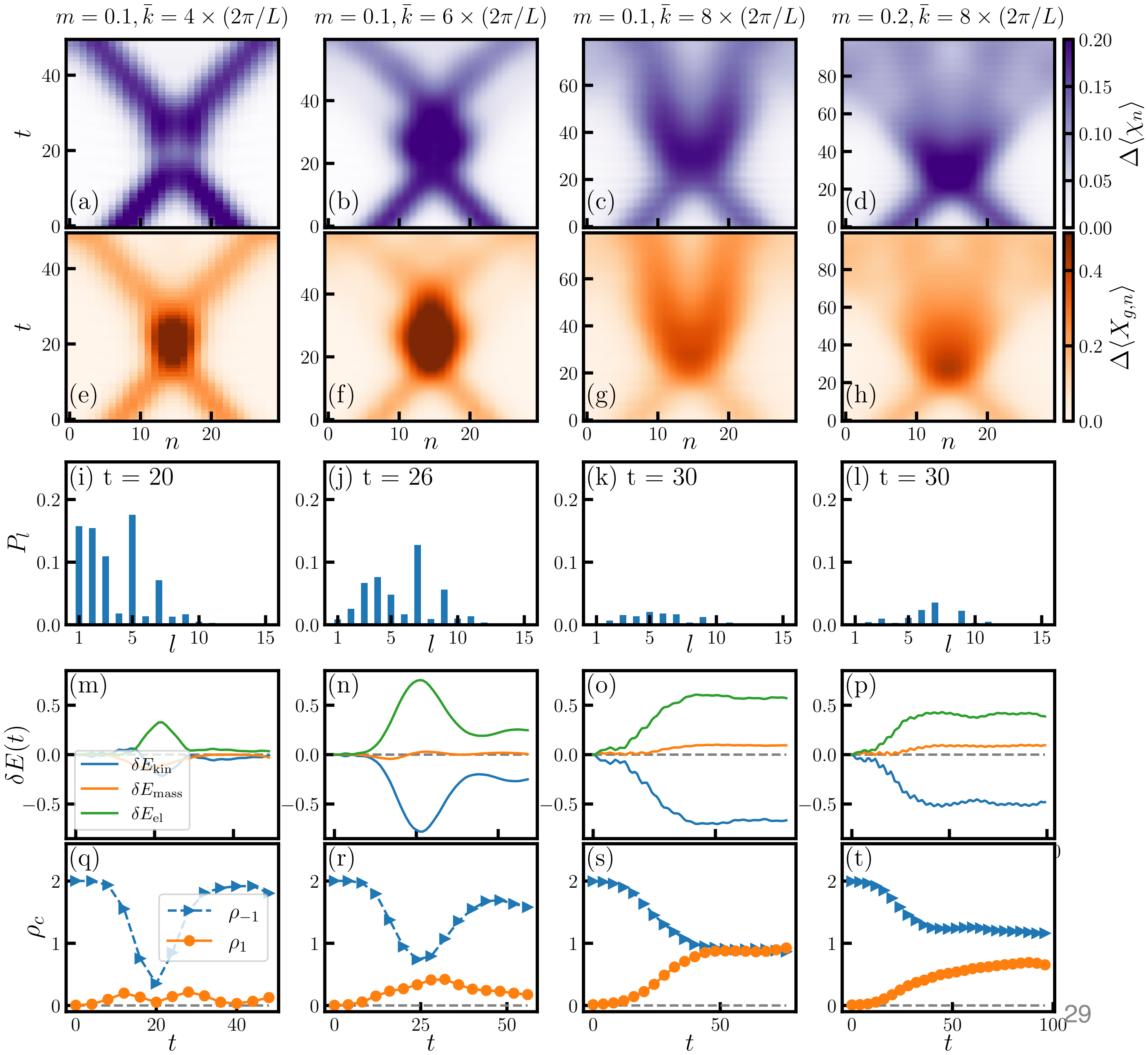
$n = 2$

$\xi_2^\dagger W_{21}\xi_1$ $\xi_2^\dagger \xi_2$ $\xi_2^\dagger W_{23}\xi_3$ $\xi_2^\dagger W_{24}\xi_4$ $\xi_2^\dagger W_{25}\xi_5$ $\xi_2^\dagger W_{26}\xi_6$

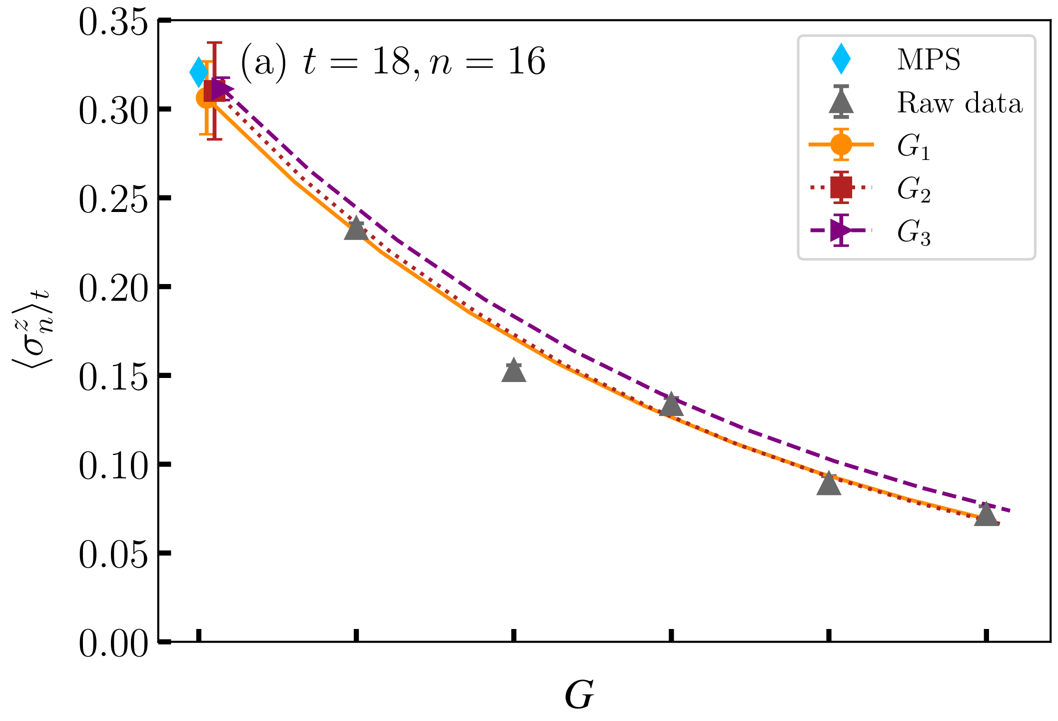


Scattering

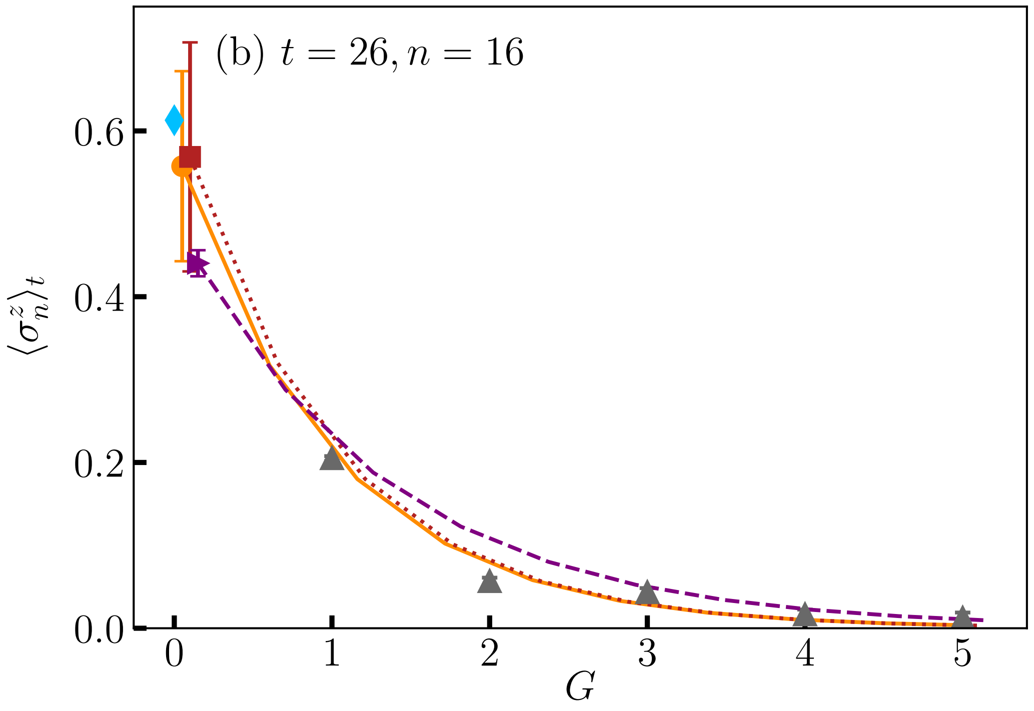
$$P_l = \sum_{n=1}^{L-l} |\langle \psi(t) | \xi_n^\dagger W_{n,n+l} \xi_{n+l} | \Omega \rangle|^2 + \sum_{n=l+1}^{L} |\langle \psi(t) | \xi_n^\dagger W_{n,n-l} \xi_{n-l} | \Omega \rangle|^2$$



ZNE



(a) $t = 18, n = 16$
MPS
Raw data
G_1
G_2
G_3
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
$\langle \sigma_n^z \rangle_t$
G



(b) $t = 26, n = 16$
0.6
0.4
0.2
0.0
0
1
2
3
4
5
$\langle \sigma_n^z \rangle_t$
G