Precision Calculation of the Heavy Meson and Light Vector Meson Couplings from LCSRs

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Introduction

Coupling constants $g_{H^{(*)}HV}$, where $H=(B_{(s)}, D_{(s)}), V=(\rho, K^*, \omega, \phi)$

- offer valuable insights into the non-perturbative aspects of the long-distance dynamics in strong interactions
- play a crucial role in extracting the low-energy constants λ and β in **HMχPT**, which are essential for describing heavy-light meson interactions at low energies
- enter the residue of the $H \to V$ transition form factor V, A_0 and T_1 , which are key to extracting the **CKM** and the **LFU** parameters

$$g_{H_1 H_2 V} = \frac{2m_V}{c_V f_{H_1}} \lim_{q^2 \to m_{H_1}^2} \left[\left(1 - \frac{q^2}{m_{H_1}^2}\right) A_0(q^2) \right]$$

- help understand the final-state interactions (**FSIs**), potentially contributing strong phases and paving the way for direct CP violation in non-leptonic B meson decays

By including NLO corrections and higher-twist contributions, we accurately determined these **coupling constants** via **LCSRs**

LCSRs for the $H^*_{(s)} H_{(s)} V$ couplings

- Def. with the eff. Lagrangian[Casalbuoni,et al.,95'][Yan,et al.,92'][Cheng,et al.,05']

$$\langle V(p,\eta^*)H^*(p+q,\varepsilon)|\mathcal{L}_{eff}|H(q)\rangle \equiv -g_{H^*HV}\epsilon_{\alpha\beta\rho\sigma}\eta^{*\alpha}\varepsilon^{*\beta}p^\rho q^\sigma ,$$

- Vacuum$-V$ correlation function

$$F_\mu(p,q)=i\int d^4x e^{-i(p+q)\cdot x}\langle V(p,\eta^*)|T\{j_\mu(x),j_5(0)\}|0\rangle=\epsilon_{\mu\nu\rho\sigma}\eta^{*\nu}p^\rho q^\sigma F((p+q)^2,q^2),$$

where, $j_\mu = \bar{q_1}\gamma_\mu Q$ and $j_5 = (m_Q+m_{q_2})\bar{Q}\, i\,\gamma_5\, q_2$,

- Double dispersion relation

$$F((p+q)^2,q^2)=-\frac{g_{H^*HV}f_{H^*}f_H m_H^2 m_{H^*}}{\left[m_{H^*}^2-(p+q)^2\right]\left[m_H^2-q^2\right]}+\iint_\Sigma \frac{\rho_{cont}(s_1,s_2)\,ds_1 ds_2}{[s_1-(p+q)^2]\,(s_2-q^2)}+\cdots .$$

The relevant matrix elements used to define the heavy meson decay constants

$$\langle 0\,|j_5|\,H(p)\rangle = f_H m_H^2, \qquad \langle H^*(p+q,\varepsilon)\,|j_\mu|\,0\rangle = f_{H^*}m_{H^*}\varepsilon_\mu .$$

$\rho_{cont}(s_1,s_2)$ captures the combined spectral density of excited and continuum states
Σ denotes the duality region occupied by these states in the (s_1,s_2)-plane

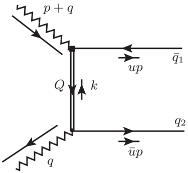
LCSRs for the $H^*_{(s)} H_{(s)} V$ couplings

- light-cone OPE expressed as a double dispersion relation

$$F^{(\mathrm{OPE})}((p+q)^2, q^2) = \iint ds_1\, ds_2 \frac{\rho^{(\mathrm{OPE})}(s_1, s_2)}{(s_1 - (p+q)^2)(s_2 - q^2)} ,$$

where the involved dual spectral density $\rho^{(\mathrm{OPE})}(s_1, s_2)$ refers to

$$\rho^{(\mathrm{OPE})}(s_1, s_2) \equiv \frac{1}{\pi^2} \mathrm{Im}_{s_1} \mathrm{Im}_{s_2} F^{(\mathrm{OPE})}(s_1, s_2) ,$$



- performing the double Borel transformation with respect to the variables $(p+q)^2 \to M_1^2$ and $q^2 \to M_2^2$

$$f_H f_{H^*}\, g_{H^* HV} = -\frac{1}{m_H^2 m_{H^*}} \iint^{\widetilde{\Sigma}} ds_1\, ds_s \exp\left(\frac{m_{H^*}^2 - s_1}{M_1^2} + \frac{m_H^2 - s_2}{M_2^2} \right) \rho^{(\mathrm{OPE})}(s_1, s_2) .$$

Boundary $\widetilde{\Sigma}$ dual to ground state, arises from subtracting continuum contrib. using the parton-hadron duality ansatz

Double spectral density at LO

- Sum contributions from individual twists up to twist-5

$$F^{(\mathrm{LO})} = F^{(\mathrm{LO})}_{2p,tw2} + F^{(\mathrm{LO})}_{2p,tw3} + F^{(\mathrm{LO})}_{2p,tw4} + F^{(\mathrm{LO})}_{2p,tw5} + F^{(\mathrm{LO})}_{3p,tw4}$$

- Compact form through the power expansion

$$F^{(\mathrm{LO})} = \frac{1+\hat{m}_{q_2}}{m_Q} \sum_{i=0}^{4} \sum_{j=1}^{4} \Big\{ \int_0^1 du\, \delta_V{}^i\, \mathcal{C}(r_1, r_2, u)^j\, \mathcal{A}^{\mathrm{LO}}_{2\mathrm{p},\,ij}(u) + \int_0^1 dv \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_3\, \delta_V{}^i\, \mathcal{C}(r_1, r_2, \alpha)^j\, \mathcal{A}^{\mathrm{LO}}_{3\mathrm{p},\,ij}(v, \underline{\alpha}) \Big\}$$

where $\delta_V = m_V/m_Q$, $\underline{\alpha} = (\alpha_1, \alpha_3)$, $\hat{m}_{q_2} = m_{q_2}^2/m_Q^2$

$$\mathcal{C}(r_1, r_2, u) = \frac{1}{\bar{u}r_1 + ur_2 - 1},$$

and $r_1 = (p+q)^2/m_Q^2$, $r_2 = q^2/m_Q^2$

Double spectral density at LO

- The part involving soft functions is denoted by the following notations

$$\mathcal{A}^{(\mathrm{LO})}_{2\mathrm{p},\,01}(u) = f_V^{\perp}\,\phi^{\perp}_{2;V}(u)\,,\quad \mathcal{A}^{(\mathrm{LO})}_{2\mathrm{p},\,12}(u) = -\frac{1}{2} f_V^{\parallel}\,\psi^{\perp}_{3;V}(u)\,,$$

$$\mathcal{A}^{(\mathrm{LO})}_{2\mathrm{p},\,22}(u) = \frac{f_V^{\perp}}{4}\left[4u\bar{u}\phi^{\perp}_{2;V}(u) + \phi^{\perp}_{4;V}(u)\right],\quad \mathcal{A}^{(\mathrm{LO})}_{2\mathrm{p},\,23}(u) = -\frac{1}{2} f_V^{\perp}\,\phi^{\perp}_{4;V}(u)\,,$$

$$\mathcal{A}^{(\mathrm{LO})}_{2\mathrm{p},\,33}(u) = -f_V^{\parallel}\,u\bar{u}\psi^{\perp}_{3;V}(u)\,,\quad \mathcal{A}^{(\mathrm{LO})}_{2\mathrm{p},\,34}(u) = \frac{3}{4} f_V^{\parallel}\,\psi^{\perp}_{5;V}(u),$$

$$\mathcal{A}^{(\mathrm{LO})}_{2\mathrm{p},\,43}(u) = \frac{f_V^{\perp}}{2}\,u\bar{u}\left[2u\bar{u}\phi^{\perp}_{2;V}(u) + \phi^{\perp}_{4;V}(u)\right],\quad \mathcal{A}^{(\mathrm{LO})}_{2\mathrm{p},\,44}(u) = -\frac{3}{2} f_V^{\perp}\,u\bar{u}\phi^{\perp}_{4;V}(u)\,,$$

$$\mathcal{A}^{(\mathrm{LO})}_{3\mathrm{p},\,22}(v,\underline{\alpha}) = f_V^{\perp}\Big\{2\bar{v}\left[\Phi^{\perp(2)}_{4;V}(\underline{\alpha}) - \Phi^{\perp(1)}_{4;V}(\underline{\alpha})\right] - \Psi^{\perp}_{4;V}(\underline{\alpha}) - (v-\bar{v})\widetilde{\Psi}^{\perp}_{4;V}(\underline{\alpha})$$
$$-\frac{2(v-\bar{v})}{\bar{\alpha}}\left[\widehat{\Phi}^{\perp(2)}_{4;V}(\underline{\alpha}) - \widehat{\Phi}^{\perp(1)}_{4;V}(\underline{\alpha}) + \widehat{\Phi}^{\perp(3)}_{4;V}(\underline{\alpha}) - \widehat{\Phi}^{\perp(4)}_{4;V}(\underline{\alpha})\right]\Big\},$$

$$\mathcal{A}^{(\mathrm{LO})}_{3\mathrm{p},\,23}(v,\underline{\alpha}) = f_V^{\perp}\,\frac{2(v-\bar{v})(r_2-1)}{\bar{\alpha}}\left[\widehat{\Phi}^{\perp(2)}_{4;V}(\underline{\alpha}) - \widehat{\Phi}^{\perp(1)}_{4;V}(\underline{\alpha}) + \widehat{\Phi}^{\perp(3)}_{4;V}(\underline{\alpha}) - \widehat{\Phi}^{\perp(4)}_{4;V}(\underline{\alpha})\right].$$

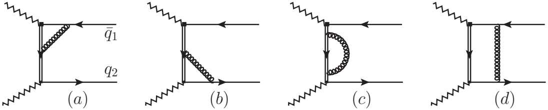
- the LO double spectral density

$$\rho^{\mathrm{LO}}(s_1, s_2) = \frac{1}{\pi^2}\,\mathrm{Im}_{s_1}\,\mathrm{Im}_{s_2}\,F^{(\mathrm{LO})}((p+q)^2, q^2).$$

Double spectral density at NLO

- To achieve NLO precision, we express the invariant amplitude

$$F^{(\mathrm{OPE})}((p+q)^2,q^2) = F^{(\mathrm{LO})}((p+q)^2,q^2) + \frac{\alpha_s C_F}{4\pi} F^{(\mathrm{NLO})}((p+q)^2,q^2),$$



- factorization formula for twist-2 contributions at NLO

$$F^{(\mathrm{NLO})}((p+q)^2,q^2) = (1+\hat{m}_{q_2}) f_V^{\perp}(\mu) \int_0^1 du\, \mathcal{C}(r_1,r_2,u) \cdot \mathcal{T}^{(1)}(r_1,r_2,\mu) \cdot \phi_{2;V}^{\perp}(u,\mu),$$

- the NLO double spectral density:

$$\rho^{\mathrm{NLO}}(s_1,s_2) = \frac{1}{\pi^2} \mathrm{Im}_{s_1} \mathrm{Im}_{s_2} F^{(\mathrm{NLO})}((p+q)^2,q^2).$$

Double spectral density at NLO

- Combining all one-loop contrib., the results agree with a previous study [H.D. Li,et al.,20']

$$
\begin{aligned}
&\mathcal{T}^{(1)}(r_1, r_2, \mu) \\
&= \frac{\alpha_s C_F}{4\pi}\left\{(-2)\left[\frac{1-r_1}{r_3-r_1}\ln\frac{1-r_3}{1-r_1} + \frac{1-r_2}{r_3-r_2}\ln\frac{1-r_3}{1-r_2} + \frac{3}{1-r_3}\right]\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m_Q^2}\right)\right. \\
&+ 2\left[\left(\frac{1-r_1}{r_3-r_1} + \frac{1-r_2}{r_3-r_2}\right)\mathrm{Li}_2(r_3) - \frac{1-r_1}{r_3-r_1}\mathrm{Li}_2(r_1) - \frac{1-r_2}{r_3-r_2}\mathrm{Li}_2(r_2)\right] \\
&+ 2\left[\left(\frac{1-r_1}{r_3-r_1} + \frac{1-r_2}{r_3-r_2}\right)\ln^2(1-r_3) - \frac{1-r_1}{r_3-r_1}\ln^2(1-r_1) - \frac{1-r_2}{r_3-r_2}\ln^2(1-r_2)\right] \\
&+ \left[\frac{1-r_3}{r_3}\left(\frac{1-r_3-2r_1}{r_3-r_1} - \frac{2(1-r_3+r_2)}{r_3-r_2}\right) + \frac{1-6r_3}{r_3^2} + 1\right]\ln(1-r_3) + \frac{1-9r_3}{r_3(1-r_3)} \\
&\left. - \frac{(1-r_1)(1-3r_1)}{r_1(r_3-r_1)}\ln(1-r_1) + \frac{2(1-r_2)}{r_2(r_3-r_2)}\ln(1-r_2) - 3\right\},
\end{aligned}
$$

- double spectral density

$$
\rho^{(\mathrm{OPE})}(s_1, s_2) = \rho^{(\mathrm{LO})}(s_1, s_2) + \frac{\alpha_s C_F}{4\pi}\rho^{(\mathrm{NLO})}(s_1, s_2)
$$

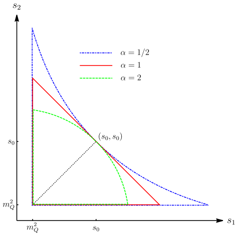
Quark-hadron duality and sum rules

- parameterization of the boundaries [Khodjamirian,Melić,Wang, Wei, 20']

$$\left(\frac{s_1}{s_*}\right)^\alpha + \left(\frac{s_2}{s_*}\right)^\alpha \leq 1\,, \qquad s_1\,,\, s_2 \geq m_Q^2\,.$$

probe the three regions, generated at

$$\alpha = 1\,, \qquad s_* = 2s_0\,, \qquad \text{(triangle)};$$
$$\alpha = \frac{1}{2}\,, \qquad s_* = 4s_0\,, \qquad \text{(concave)};$$
$$\alpha = 2\,, \qquad s_* = \sqrt{2}s_0\,, \qquad \text{(convex)};$$



- Assuming equal Borel parameters $M_1^2 = M_2^2 = 2M^2$ allows us to rewrite the sum rule

$$f_H f_{H^*}\, g_{H^* HV} = -\frac{1}{m_H^2 m_{H^*}} \exp\left(\frac{m_H^2 + m_{H^*}^2}{2M^2}\right)\left[\mathcal{F}^{(\mathrm{LO})}(M^2, s_0) + \frac{\alpha_s\, C_F}{4\pi}\,\mathcal{F}^{(\mathrm{NLO})}(M^2, s_0)\right].$$

- define the integral over the triangular region

$$\mathcal{F}(M^2, s_0) \equiv \int_{-\infty}^{\infty} ds_1 \int_{-\infty}^{\infty} ds_2\; \theta(2s_0 - s_1 - s_2)\, \exp\left(-\frac{s_1 + s_2}{2M^2}\right) \rho(s_1\,, s_2)\,.$$

LCSRs for the $H_{(s)} H_{(s)} V$ couplings

- Def. with the eff. Lagrangian[Casalbuoni,et al.,95'][Yan,et al.,92'][Cheng,et al.,05']

$$\langle V(p,\varepsilon^*) H_2(p+q) | \mathcal{L}_{eff} | H_1(q) \rangle \equiv g_{H_1 H_2 V} (q\cdot\varepsilon^*),$$

- Vacuum$-V$ correlation function

$$\Pi((p+q)^2, q^2) = i \int d^4x \, e^{iq\cdot x} \langle V(p,\varepsilon^*) | T\{ j_5^\dagger(x), j_5(0) \} | 0 \rangle,$$

where, $j_5 = (m_Q + m_{q_2}) \bar{Q} \, i\gamma_5 \, q$

- Double dispersion relation

$$\Pi((p+q)^2, q^2) = \frac{(q\cdot\varepsilon^*) g_{H_1 H_2 V} f_{H_1} f_{H_2} m_{H_1}^2 m_{H_2}^2}{[m_{H_1}^2 - q^2][m_{H_2}^2 - (p+q)^2]} + \iint_\Sigma \frac{\rho_{cont}(s_1, s_2) \, ds_1 \, ds_2}{[s_1 - q^2][s_2 - (p+q)^2]} + \cdots$$
$$\equiv (q\cdot\varepsilon^*) \, F((p+q)^2, q^2)$$

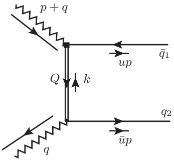
LCSRs for the $H_{(s)} H_{(s)} V$ couplings

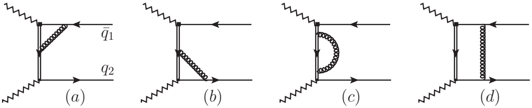
- light-cone OPE expressed as a double dispersion relation

$$\Pi^{\rm OPE}((p+q)^2, q^2) = (q \cdot \varepsilon^*) \iint ds_1\, ds_2 \frac{\rho^{\rm OPE}(s_1, s_2)}{(s_1 - (p+q)^2)(s_2 - q^2)}$$

where the involved dual spectral density $\rho^{\rm (OPE)}(s_1, s_2)$ refers to

$$\rho^{\rm (OPE)}(s_1, s_2) \equiv \frac{1}{\pi^2} {\rm Im}_{s_1} {\rm Im}_{s_2} \Pi^{\rm (OPE)}(s_1, s_2)\,,$$





- performing the double Borel transformation with respect to the variables $q^2 \to M_1^2$ and $(p+q)^2 \to M_2^2$

$$f_{H_1} f_{H_2} g_{H_1 H_2 V} = \frac{1}{m_{H_1}^2 m_{H_2}^2} \int_{-\infty}^{+\infty} ds_1 \int_{-\infty}^{2s_0 - s_2} ds_2 \exp\left[\frac{m_{H_1}^2 - s_1}{M_1^2} + \frac{m_{H_2}^2 - s_2}{M_2^2}\right] \rho^{\rm OPE}(s_1, s_2)\,.$$

confined to the triangular duality region with $\alpha = 1$ and $s_* = 2s_0$, wherein $s_2 \le 2s_0 - s_1$

Hard-collinear factorization for the correlation function at LP

- factorization formula:

$$\Pi^{\rm OPE}_{\rm LP}((p+q)^2, q^2) = q^\mu \left(\mathcal{T}_V^{(0)} + \mathcal{T}_V^{(1)}\right) \otimes \langle O^V_\mu \rangle$$

- matrix element of the vector light-cone operator

$$\langle O^V_\mu \rangle = \langle V(p,\varepsilon^*)|\bar{q}'(x)\gamma_\mu q(0)|0\rangle \,, O^V_\mu = \bar{q}'(x)\gamma_\mu q(0) \overset{\rm LP}{=} n_\mu \bar{\chi}(x)\frac{\not{\bar{n}}}{2}\chi(0),$$

$$O^V_\mu = n_\mu \int d\omega_1 d\omega_2 \, e^{\frac{i}{2}\omega_1 (n\cdot x)} \mathcal{J}_V(\vec{\omega}) \equiv n_\mu \mathcal{O}_V \,, \mathcal{J}_V(\vec{\omega}) \equiv \bar{\chi}_{n,\,\omega_1} \frac{\not{\bar{n}}}{2} \chi_{n,\,\omega_2}$$

- Therefore, rewrite the factorization formula

$$\Pi^{\rm OPE}_{\rm LP}((p+q)^2, q^2) = (n\cdot q)\int_0^1 du \left(\mathcal{T}_V^{(0)} + \mathcal{T}_V^{(1)}\right) \langle \mathcal{J}_V(\vec{\omega})\rangle \,.$$

where

$$\langle \mathcal{O}_V \rangle = \langle V(p,\varepsilon^*)| \int d\omega_1 d\omega_2 \, e^{\frac{i}{2}\omega_1 (n\cdot x)} \mathcal{J}_V(\vec{\omega})|0\rangle \,, \langle \mathcal{J}_V(\vec{\omega})\rangle = \frac{\bar{n}\cdot\varepsilon^*}{2} m_V f_V^{\parallel} \phi^{\parallel}_{2;V}(u) \,,$$

Hard-collinear factorization for the correlation function at LP

- NLO hard function

$$\mathcal{T}_V^{(1)}(r_1, r_2, u) = \frac{\alpha_s C_F}{4\pi}\left[\mathcal{T}_V^{(1),\,a} + \mathcal{T}_V^{(1),\,b} + \mathcal{T}_V^{(1),\,c} + \mathcal{T}_V^{(1),\,d}\right],$$

- the NLO renormalized hard function New

$$\begin{aligned}
\mathcal{T}_V^{(1)}(r_1, r_2, u) &= \frac{\alpha_s C_F}{4\pi}\frac{1}{r_3-1}\Bigg\{\Bigg[2\left(2-\frac{1-r_1}{r_3-r_1}\ln\frac{1-r_3}{1-r_1}-\frac{1-r_2}{r_3-r_2}\ln\frac{1-r_3}{1-r_2}\right) \\
&-2(r_3-1)\left(\frac{(r_3-1)\ln(1-r_3)}{(r_3-r_1)(r_3-r_2)}+\frac{(1-r_1)\ln(1-r_1)}{(r_3-r_1)(r_1-r_2)}+\frac{(1-r_2)\ln(1-r_2)}{(r_3-r_2)(r_2-r_1)}\right)-\frac{r_3-7}{r_3-1}\Bigg]\ln\frac{\mu^2}{m_Q^2} \\
&+\frac{2\left[r_3^2-r_3(r_1+r_2)+2r_1r_2-r_1-r_2+1\right]}{(r_3-r_1)(r_3-r_2)}\left[\mathrm{Li}_2(r_3)+\ln^2(1-r_3)\right] \\
&-\frac{2(r_1-1)(r_3-r_1+r_2-1)}{(r_3-r_1)(r_1-r_2)}\left[\mathrm{Li}_2(r_1)+\ln^2(1-r_1)\right] \\
&-\frac{2(r_2-1)(r_3+r_1-r_2-1)}{(r_3-r_2)(r_2-r_1)}\left[\mathrm{Li}_2(r_2)+\ln^2(1-r_2)\right] \\
&-\left[\frac{r_1^2+r_3(2-3r_1-3r_2)+3r_1r_2+1}{(r_3-r_1)(r_3-r_2)}+\frac{r_3(2r_1r_2-r_1-r_2)-r_1r_2}{r_3^2(r_3-r_1)(r_3-r_2)}\right]\ln(1-r_3) \\
&-\frac{2(r_1-1)(r_3r_1-r_2)}{r_1(r_3-r_1)(r_1-r_2)}\ln(1-r_1)-\frac{2(r_2-1)(r_3r_2-r_1)}{r_2(r_3-r_2)(r_2-r_1)}\ln(1-r_2)+\frac{3r_1^2+6r_3-1}{r_3(r_3-1)}\Bigg\},
\end{aligned}$$

subtracting the UV and IR divergence in the $\overline{\mathrm{MS}}$ scheme

The factorization scale independence for the correlation function

- The derivative of the twist-two NLO correlation function on the scale μ

$$\frac{d}{d\ln\mu}\Pi^{\rm OPE}_{\rm LP}(r_1,r_2,\mu) = -m_V f_V^{\parallel}(q\cdot\varepsilon^*)\int_0^1 du\left\{\left(\frac{d}{d\ln\mu}\phi^{\parallel}_{2;V}(u,\mu)\right)\left[\mathcal{T}_V^{(0)}(r_1,r_2,u,\mu)+\mathcal{T}_V^{(1)}(r_1,r_2,u,\mu)\right]\right.$$
$$\left. + \phi^{\parallel}_{2;V}(u,\mu)\frac{d}{d\ln\mu}\left[\mathcal{T}_V^{(0)}(r_1,r_2,u,\mu)+\mathcal{T}_V^{(1)}(r_1,r_2,u,\mu)\right]\right\},$$

- the evolution equations of the twist-2 LCDA and the heavy quark mass

$$\frac{d}{d\ln\mu}\phi^{\parallel}_{2;V}(u,\mu) = 2\int_0^1 du'\, V(u,u')\phi^{\parallel}_{2;V}(u',\mu)\,,$$
$$\frac{d}{d\ln\mu}m_Q(\mu) = -6\frac{\alpha_s C_F}{4\pi}m_Q(\mu)\,,$$

where the evolution kernel is given by[Efremov and Radyushkin, 80']

$$V(u,u') = \frac{\alpha_s C_F}{4\pi}\left[\frac{2\bar u}{\bar u'}\left(1+\frac{1}{u-u'}\right)\theta(u-u')\Big|_+ + \frac{2u}{u'}\left(1+\frac{1}{u'-u}\right)\theta(u'-u)\Big|_+\right],$$

the plus distribution function is defined as

$$V(u,u')\Big|_+ = V(u,u') - \delta(u-u')\int_0^1 dt V(t,u')\,.$$

The factorization scale independence for the correlation function

- Inserting these evolution equations, then we obtain the first term

$$\int_0^1 du \left(\frac{d}{d\ln\mu}\phi_{2;V}^{\parallel}(u,\mu)\right)\left[\mathcal{T}_V^{(0)}(r_1,r_2,u,\mu)+\mathcal{T}_V^{(1)}(r_1,r_2,u,\mu)\right]$$
$$= 4\cdot\frac{\alpha_s C_F}{4\pi}\int_0^1 du\,\frac{\phi_{2;V}^{\parallel}(u,\mu)}{r_3-1}\left\{\left[\frac{1-r_1}{r_3-r_1}\ln\frac{1-r_3}{1-r_1}+\frac{1-r_2}{r_3-r_2}\ln\frac{1-r_3}{1-r_2}+2\right]\right.$$
$$\left.+\left[-\frac{1}{2}+\frac{(r_3-1)^2\ln(1-r_3)}{(r_3-r_1)(r_3-r_2)}-\frac{(r_3-1)(r_1-1)\ln(1-r_1)}{(r_3-r_1)(r_1-r_2)}+\frac{(r_3-1)(r_2-1)\ln(1-r_2)}{(r_3-r_2)(r_1-r_2)}\right]\right\},$$

and the second term reads

$$\int_0^1 du\,\phi_{2;V}^{\parallel}(u,\mu)\frac{d}{d\ln\mu}\left[\mathcal{T}_V^{(0)}(r_1,r_2,u,\mu)+\mathcal{T}_V^{(1)}(r_1,r_2,u,\mu)\right]$$
$$= 4\cdot\frac{\alpha_s C_F}{4\pi}\int_0^1 du\,\frac{\phi_{2;V}^{\parallel}(u,\mu)}{r_3-1}\left\{\left[-\frac{1-r_1}{r_3-r_1}\ln\frac{1-r_3}{1-r_1}-\frac{1-r_2}{r_3-r_2}\ln\frac{1-r_3}{1-r_2}-2\right]\right.$$
$$\left.+\left[-\frac{(r_3-1)^2\ln(1-r_3)}{(r_3-r_1)(r_3-r_2)}+\frac{(r_3-1)(r_1-1)\ln(1-r_1)}{(r_3-r_1)(r_1-r_2)}-\frac{(r_3-1)(r_2-1)\ln(1-r_2)}{(r_3-r_2)(r_1-r_2)}\right]+\frac{1}{2}\right\}.$$

- Put them together, we obtain

$$\frac{d}{d\ln\mu}\Pi_{\rm LP}^{\rm OPE}(r_1,r_2,\mu)=O(\alpha_s^2),$$

Input parameters

- Decay constants [Gelhausen,et al.13']

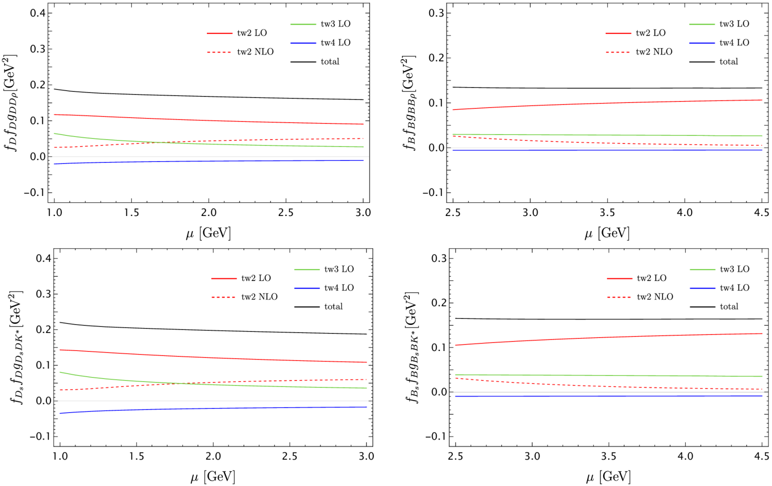
Methods	charmed meson		bottom meson	
	f_D [MeV]	f_{D_s} [MeV]	f_B [MeV]	f_{B_s} [MeV]
two-point QCDSRs	201^{+12}_{-13}	238^{+13}_{-23}	207^{+17}_{-09}	242^{+17}_{-12}
LQCD	212.0 ± 0.7	249.9 ± 0.5	190.0 ± 1.3	230.3 ± 1.3
Experiment	207.3 ± 6.2	249.5 ± 3.16	201.6 ± 21.2	$--$

- Scale, Borel and threshold parameters

Parameter	default value (interval)	[Ref.]	Parameter	default value (interval)	[Ref.]
charmed meson sum rules			bottom meson sum rules		
μ (GeV)	1.5 (1.0 - 3.0)	[48]	μ (GeV)	3.0 (2.5 - 4.5)	[49]
M^2 (GeV^2)	4.5 (3.5 - 5.5)		M^2 (GeV^2)	16.0 (12.0 - 20.0)	
s_0 (GeV^2)	7.0 (6.5 - 7.5)		s_0 (GeV^2)	37.5 (35.0 - 40.0)	

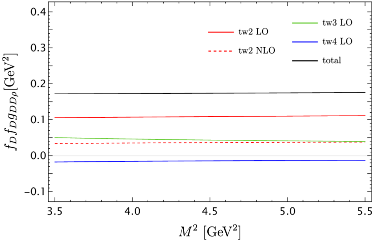
Scale dependence

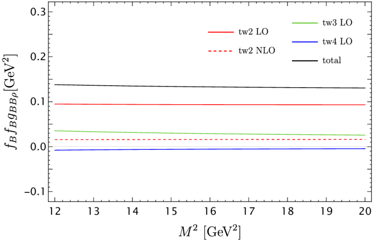
- The μ dependence



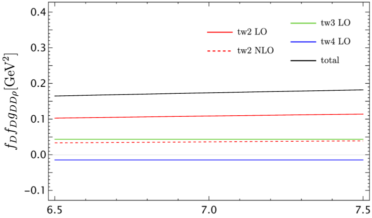
Borel and threshold dependence

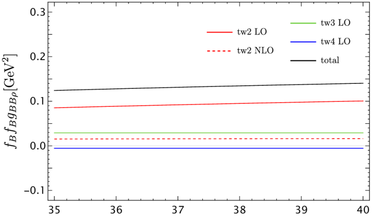
- The Borel parameter dependence





- The threshold parameter dependence





Numerical analysis for the $H^{(*)}_{(s)} H_{(s)} V$ couplings

- Power corrections

Power	δ_ϕ^0	δ_ϕ^1	δ_ϕ^2	δ_ϕ^3	δ_ϕ^4	total
$g_{D_s^* D_s \phi}$	2.41	0.63	−0.48	−0.045	0.020	2.54
$g_{B_s^* B_s \phi}$	2.47	0.53	−0.48	−0.016	0.020	2.86

Table 5: Power corrections for the strong couplings $g_{H^* H \phi}$ using the decay constats from LQCD, in units of $[\mathrm{GeV}^{-1}]$. The expansion extends to the power of δ_ϕ ranging from 0 to 4, where $\delta_\phi = \delta_\phi^{(Q)} = m_\phi / m_Q$, for the coupling with $D_s^{(*)}$ meson $\delta_\phi^{(c)} = 0.85$ and for the coupling $B_s^{(*)}$ meson $\delta_\phi^{(b)} = 0.23$.

Power	δ_V^1	$\delta_V^1 \alpha_s$	δ_V^2	δ_V^3	δ_V^4	total
$g_{DD\rho}$	2.76	0.80	0.65	−0.34	−0.022	3.86
$g_{BB\rho}$	2.69	0.44	0.65	−0.10	−0.006	3.68
$g_{D_s D_s \phi}$	2.52	0.77	0.65	−0.59	−0.037	3.31
$g_{B_s B_s \phi}$	2.39	0.41	0.66	−0.16	−0.011	3.29

Table 4. Power corrections for the strong couplings $g_{HH\rho}$ and $g_{HH\phi}$ using the decay constants from LQCD. The expansion extends to the power of δ_V ranging from 1 to 4, where $\delta_V = \delta^{(Q)} = m_V/m_Q$

Numerical analysis for the $D^*_{(s)} D_{(s)} V$ couplings

Method	$g_{D^* D\rho}$	$g_{D^* D\omega}$	$g_{D_s^* DK^*}$	$g_{D^* D_s K^*}$	$g_{D_s^* D_s \phi}$
LCSR[8]	$3.80^{+0.59}_{-0.45}$	–	–	–	–
LCSR[19]	3.56 ± 0.60	–	–	4.04 ± 0.8	3.28 ± 0.64
LCSR[17]	4.17 ± 1.04	–	–	–	–
QCDSR[21]	–	–	3.74 ± 1.38	–	–
LCSR (this work)	$3.53^{+0.61}_{-0.57}$	$2.25^{+0.42}_{-0.41}$	$3.55^{+0.65}_{-0.61}$	$3.87^{+0.73}_{-0.68}$	$2.69^{+0.55}_{-0.52}$
	$3.40^{+0.49}_{-0.43}$	$2.17^{+0.34}_{-0.32}$	$3.30^{+0.52}_{-0.48}$	$3.34^{+0.54}_{-0.53}$	$2.54^{+0.40}_{-0.39}$

Table 11: Numerical values of coupling $g_{D^*_{(s)} D_{(s)} V}(\mathrm{GeV}^{-1})$ from several methods.

Numerical analysis for the $D_{(s)}D_{(s)}V$ couplings

Method	$g_{DD\rho}$	$g_{DD\omega}$	$g_{D_sDK^*}$	$g_{D_sD_s\phi}$	$g_{BB\rho}$
LCSR[10]	2.62 ± 0.58	–	3.22 ± 0.64	2.9 ± 0.68	
LCSR[11]	4.61		–	–	5.34 ± 1.15
QCDSR[12]	−−	2.9	–	–	
QCDSR[13]	−−	–	3.26 ± 0.43	–	
QCDSR[14]	−−	–	–	4.00 ± 1.09	
LQCD[8]	4.84 ± 0.34	–	−−	–	
LCSR (this work)	$4.30^{+0.82}_{-0.72}$	$2.80^{+0.54}_{-0.48}$	$4.30^{+0.80}_{-0.67}$	$3.65^{+0.93}_{-0.59}$	$3.10^{+0.40}_{-0.54}$
	3.86 ± 0.49	2.52 ± 0.33	$3.88^{+0.52}_{-0.53}$	$3.31^{+0.39}_{-0.41}$	$3.68^{+0.34}_{-0.35}$
	4.21 ± 0.58	2.74 ± 0.39	3.98 ± 0.55	$3.32^{+0.40}_{-0.42}$	$3.27^{+0.87}_{-0.67}$

Table 10. Numerical values of coupling $g_{D_{(s)}D_{(s)}V}$ from several methods.

The $H_{(s)}H_{(s)}V$ couplings from $B \to V$ transition form factors

Coupling	decay constants	This work	$A_0(q^2)$ [24]	$A_0(q^2)$ [28]
$g_{BB\rho}$	two-point QCDSRs	$3.10^{+0.40}_{-0.54}$	$6.11^{+1.17}_{-1.23}$	$6.15^{+2.47}_{-2.05}$
	LQCD	$3.68^{+0.34}_{-0.35}$	$6.65^{+1.25}_{-1.25}$	$6.70^{+2.79}_{-2.18}$
	Experiment	$3.27^{+0.87}_{-0.67}$	$6.27^{+1.39}_{-1.32}$	$6.32^{+2.73}_{-2.14}$
$g_{BB\omega}$	two-point QCDSRs	$2.02^{+0.27}_{-0.36}$	$4.24^{+1.04}_{-1.07}$	$4.74^{+2.01}_{-1.53}$
	LQCD	$2.39^{+0.24}_{-0.24}$	$4.62^{+1.12}_{-1.12}$	$5.17^{+2.18}_{-1.63}$
	Experiment	$2.13^{+0.57}_{-0.44}$	$4.36^{+1.17}_{-1.13}$	$4.87^{+2.13}_{-1.60}$
$g_{B_sBK^*}$	two-point QCDSRs	$3.26^{+0.39}_{-0.46}$	$6.30^{+1.47}_{-1.49}$	$7.46^{+3.10}_{-2.22}$
	LQCD	$3.73^{+0.37}_{-0.37}$	$6.62^{+1.50}_{-1.50}$	$7.84^{+3.23}_{-2.27}$
$g_{B_sB_s\phi}$	two-point QCDSRs	$2.98^{+0.45}_{-0.49}$	$7.13^{+1.43}_{-1.46}$	−−
	LQCD	$3.29^{+0.36}_{-0.34}$	$7.49^{+1.45}_{-1.45}$	−−

Table 11. Coupling constant $g_{B_{(s)}B_{(s)}V}$ extracted from the residue of the transition form factors A_0 in the framework of LCSR fit, compared with our obtained values.

1. Introduction
2. LCSRs for the $H^*_{(s)}H_{(s)}V$ couplings
3. LCSRs for the $H_{(s)}H_{(s)}V$ couplings
4. Numerical analysis
5. Conclusions and Outlook

Conclusions and Outlook

- In this talk, we present our recent progresses for the calculation of the strong H^*HV and HHV couplings from LCSRs via involving the NLO QCD corrections at leading twist and higher power contributions at leading order.

- The further improvements are necessary from two aspects:
  - More perturbation corrections and higher power contributions need to be included.
  - The dispersion relation relating the strong couplings and the residue of the $H \to V$ form factors need to be clarified.

Thanks!